HOW CAN BUYERS ENGAGE SUPPLIERS TO BE MORE SOCIALLY AND ENVIRONMENTALLY RESPONSIBLE?

by

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DISSERTATION ABSTRACT

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Title: How Can Buyers Engage Suppliers to Be More Socially and Environmentally Responsible?

A major concern for buyers in a given tier of the supply chain continues to be the challenge of balancing the economic benefits of outsourcing with the loss in their ability to control and influence sustainability performance of their suppliers. The overarching question in this dissertation is how buyers can engage suppliers to improve social and environmental performance of the supply chain. A combination of analytical and empirical models are developed and analyzed to offer a buyer guidance at strategic (i.e., managing trust in buyer-supplier relationships) and tactical (i.e., designing suitable contracting mechanisms) levels on how to make suppliers more socially and environmentally responsible. In the first essay, we consider a buyer who enjoys the pricing power and also has the ability to commit to contract terms. We investigate how such a buyer's commitment to contract terms affects the sustainability and financial performance of the supply chain. The second essay focuses on understanding the impact of supplier competition on the buyer's ability to influence suppliers' compliance when suppliers have more parity in contracting power. Unlike the first essay, wherein the buyer stipulates both price and quantity, this essay

considers situations wherein the supplier offers a wholesale price and the buyer is limited to only offering the quantity in a wholesale contract. In the third essay, we propose a framework to investigate the role of specific nature of trust (i.e., calculative and relational trust) between buyers and suppliers in influencing the impact of their supplier relationship management strategies on suppliers' sustainability performance.

This dissertation includes previously unpublished co-authored material.

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CHAPTER I

INTRODUCTION

According to United Nations Global Compact, many well-known global manufacturers and brands are well into sustainability initiatives, and yet face some significant roadblocks in various tiers of their supply chain. While many brands outsource their entire production, several major manufacturers outsource a large fraction of components in their product to lower tiers of the supply chain. Their ability to influence sustainability in their supply chain is curtailed since they only have direct control over a fraction of their tiers. Consequently, an overwhelming majority of sustainability challenges stem from their supplier base, wherein suppliers can fall seriously behind the curve (UNGC, 2015). Manufacturers are interested in finding effective ways to push their sustainability efforts upstream using incentives, business leverage, training programs, and auditing to improve their suppliers' sustainability performance. A major concern for buyers in a given tier of the supply chain continues to be the challenge of balancing the economic benefits of outsourcing with the loss in their ability to control and influence sustainability performance of their suppliers.

The overarching question in this dissertation is how buyers can engage suppliers to improve social and environmental performance of the supply chain. A combination of analytical and empirical models are developed and analyzed to offer a buyer guidance at strategic (i.e., managing trust in buyer-supplier relationships) and tactical (i.e., designing suitable contracting mechanisms) levels on how to make suppliers more socially and environmentally responsible. The dissertation includes three essays that consider several key questions that together contribute to answering the overarching question of the dissertation. This dissertation has been partially supported by the Center for Sustainable Business Practices at the Lundquist College of Business.

The first major set of research questions that are amenable to economic modeling are addressed using an analytical framework. In the first essay, we investigate how buyer's policy of commitment to contract terms affects the sustainability and financial performance of the supply chain. We consider a framework wherein the buyer audits the supplier for the supplier's compliance to a code of conduct. The degree of the buyer's commitment to contract terms is defined by his commitment to price and/or order quantity in a wholesale contract prior to knowing the outcome of auditing *vis-à-vis* making it conditional to the outcome of auditing. To the best of our knowledge, this lever has not been considered hitherto in the sustainability literature. The specific research questions include 1) How does the buyer's commitment affect the supplier's compliance to the code of conduct and the overall sustainability performance of the supply chain? 2) How does the buyer's commitment affect the financial performance of the buyer and the supplier? 3) What is the interaction between different ways in which the buyer can progressively commit to price and/or order quantity? 4) What is the effect of increasing the standard of the code of conduct on the buyer's auditing, the supplier's compliance, and the overall sustainability performance of the supply chain?

Four multistage game-theoretic models that consider a Nash equilibrium are developed and analyzed to fully characterize the effect of the buyer's commitment to contract terms and the auditing effort on the sustainability and financial performance of the supply chain. The study provides managers with yet another effective tool to consider improving compliance in their supplier base. Our results show that commitment provides the buyer with significant opportunity to advance both social good and profit in a win-win manner. We find that increasing the level of commitment improves the supplier's likelihood of compliance to the sustainability standard. Interestingly, we also find that both contracting opportunity and profit for the buyer increase monotonically with the degree of commitment. Additionally, committing to only the price or only the quantity (*vis-à-vis* no-commitment) has an asymmetric effect. We show that committing to the price and committing to the quantity are complementary strategies for the buyer and substitutes for the supplier. This essay is an unpublished co-authored work with Dr. Nagesh Murthy and Dr. Zhibin (Ben) Yang.

The second essay focuses on understanding the impact of supplier competition on the buyer's ability to influence suppliers' compliance when suppliers have more parity in contracting power. Unlike the first essay, wherein the buyer stipulates both price and quantity, this essay considers situations wherein the supplier offers a wholesale price and the buyer is limited to only offering the quantity in a wholesale contract. Depending on whether auditing precedes contracting or follows contracting, it determines whether suppliers first compete on their compliance efforts or wholesale prices. We develop a framework to investigate if and how the sequence in which the supplier competition manifests, influences buyers auditing effort, suppliers compliance efforts, and financial performance of parties.

We consider two suppliers who are asymmetric in their production and compliance costs. One supplier has a lower production cost and higher compliance cost, while the other has a higher production cost and lower compliance cost. We analyze multi-stage game theoretic models to find the buyer's optimal auditing effort, suppliers' compliance efforts, and profits of parties. Our results indicate that when the auditing precedes the contracting, the buyer does not have to exert significant auditing effort since suppliers find it beneficial to be compliant with the code of conduct in the presence of competition. In the process the supplier is also able to squeeze out the buyer's profit to zero. When the demand in the market is high, the supplier with the lower cost of production (but higher compliance cost) wins the competition. In contrast, when the market price is high while demand is low, the supplier with the advantage in compliance cost wins the competition. We also show that the supplier competition leads to increasing the contracting opportunity.

The third essay focuses on a broader research question pertaining to buyer-supplier relationship management strategies to influence supplier's sustainability performance. The primary research question considers how the specific nature of trust (i.e., calculative vs. relational trust) between the buyer and the supplier influences the impact of supplier relationship management strategies (transactional and collaborative) on supplier's sustainability performance. This research question is not easily amenable to analytical modeling, and hence addressed by developing a conceptual and empirical framework. In this essay, we propose a conceptual framework to lay a foundation for developing a theoretical lens to understand this phenomena for promoting socially and environmentally responsible behavior in supply chains. After a careful examination and unification of the literature we develop a set of hypotheses to enable a formalized study that can address the above research question.

The remainder of this dissertation is organized as follows. In next chapter we present our analytical model to examine the role of commitment in enhancing social and environmental compliance of the supplier. In chapter 3, we investigate the impact of supplier competition on buyer's auditing, suppliers' compliance, and financial performance of parties. In chapter 4, we develop our empirical framework to examine the role of buyer's transactional and collaborative approaches on suppliers' social and environmental performance. Our conceptual framework focuses on the moderating role of trust in this relationship. Finally, we provide concluding remarks in chapter 5.

CHAPTER II

COMMITTING TO CONTRACT FOR A SUPPLIER'S SOCIAL AND ENVIRONMENTAL COMPLIANCE

This work was submitted to the journal of Manufacturing & Service Operations Management with co-authorship of Dr. Nagesh Murthy and Dr. Zhibin (Ben) Yang.

Introduction

Major brands in many industries outsource their production and hence lose direct control in managing sustainability in their supplier base. It is not surprising that an overwhelming majority (i.e., as much as 70%) of social and environmental violations emanate from suppliers (UNGC, 2015). A key finding from an international survey on different industries in America, Europe and Asia shows that customers demand more sustainable supply chains (DNVGL, 2014). Citizens at large also engage in shaping public policy to raise the stringency of national and international standards for sustainability (Bartley, 2003; Toffel et al., 2015; Thorlakson et al., 2018). Consumers hold brands and major manufacturers accountable when their suppliers' sustainability violations become known in the marketplace. Although brands may not be legally responsible for their suppliers' social and environmental misconduct, they can incur significant costs for their suppliers' violations (Caro et al., 2018). The damage to reputation and brand from revelations of social and environmental misconduct in the supplier base may lead to loss of revenue and market (Guo et al., 2016; Plambeck and Taylor, 2016).

Given the strategic implications of sustainability in supply chains, the number of S&P 500 companies publishing sustainability reports has increased from 20% in

2011 to 82% in 2016 (Coppola, 2017). Brands and major manufacturers adopt a code of conduct that sets expectations for their suppliers' social and environmental practices. These expectations often increase over time (Nike, 2018; Apple, 2018). Hence, brands are constantly engaged in enhancing compliance of their suppliers to an existing code of conduct or to a new, upcoming, and more stringent sustainability standard. Brands audit their suppliers for compliance to the code of conduct and proactively or reactively strive to reduce the likelihood of social and environmental violations in their supplier base (GIIRS, 2014). Yet, the challenge persists as the compliance in the supply base varies significantly across industries and firms, with significant consequences for the buyer (Thorlakson et al., 2018).

While auditing is an important tool for buyers to ascertain their suppliers' compliance with the buyers' sustainability standard, a key challenge remains: how to motivate their suppliers to invest, in the first place, in sustainability capabilities that can enhance the likelihood of their compliance to the requisite (i.e., prevalent or upcoming) sustainability standard (e.g., the supplier's social and environmental code of conduct). Assuring compliance may require an inordinate (i.e., costly) effort for suppliers. At the same time, catching a non-compliant supplier with certainty may require an inordinate auditing effort for the buyer. Thus, it becomes important to design a suitable contracting mechanism that, when used in conjunction with auditing, can further align the incentives for both parties. Such a mechanism needs to consider consequences of sustainability violations, while accounting for the costs associated with varying levels of compliance and costs for varying levels of auditing effort, including the cost of any corrective action to be undertaken by the supplier when one fails an audit. One option for a buyer to incentivize the supplier is to consider committing to contract terms before both parties invest in their respective efforts, and before the auditing outcome (i.e., whether the supplier passed or failed the audit) is known with regards to the supplier's compliance status. The buyer's commitment to contract terms (i.e., degree of commitment) can manifest in terms of an *a priori* commitment to only price, only quantity, or both.

The potential benefit of the buyer's commitment to similar contract terms to induce supplier's investment prior to resolution of a key underlying uncertainty has been studied under different settings in the operations management literature. Taylor and Plambeck (2007) study a scenario wherein the supplier has to invest in capacity for producing a product still under development, hence faces significant demand uncertainty. Their result indicates that the buyer should commit to both price and quantity if the production cost is low and either the capacity cost is low or the cost of capital is high. Otherwise, the buyer should commit to price only. Hu et al. (2013) study a scenario wherein a buyer is interested in motivating a supplier to invest for capacity restoration capability when faced with a supply chain disruption. They find that *ex ante* (i.e., prior to disruption) commitment to price and quantity dominates the *ex post* (i.e., after disruption) commitment, with both parties being better off. Kim and Netessine (2013) consider a scenario involving development of an innovative product wherein there exists uncertainty about component production cost. They examine the effect of commitment to only price, only quantity, and both on collaborative efforts of parties to reduce the uncertainty about component production cost and, in turn, lower the expected cost. They find that under all three types of commitment, neither party exerts collaborative effort.

The potential efficacy of a buyer's commitment to contract terms has not been examined in the sustainability literature. Given the promising and yet mixed results of the benefit of commitment for a buyer, we examine the efficacy of a buyer's commitment to price and/or quantity when used in conjunction with auditing. The efficacy for sustainability is measured in terms of the ability to enhance both the supplier's likelihood of compliance to the code of conduct prior to auditing outcome, and *overall sustainability compliance* in the marketplace (which also accounts for the corrective action when the supplier fails an audit). We consider the progression from "no-commitment" to "commitment to only the wholesale price" to "commitment to both wholesale price and order quantity" to represent the increasing level of the buyer's commitment in a wholesale contract. We develop, analyze, and compare multi-stage game-theoretic models to represent varying levels of a buyer's commitment. We solve for the subgame-perfect equilibrium and characterize the contract terms, the buyer's auditing effort, and the supplier's compliance effort at the equilibrium. Our results show that the commitment provides the buyer with significant opportunity to advance both social good and profit in a *win-win* manner. Interestingly, the transition from "no-commitment" to "commitment to only the wholesale price" policy *vis-à-vis* a transition to "commitment to only the quantity" has an asymmetric effect on sustainability and profit metrics.

We find that increasing the level of commitment improves the supplier's likelihood of compliance to the sustainability standard (i.e, the code of conduct). We show that overall sustainability compliance in the market also improves, in a great majority of cases, with an increase in the commitment level. Interestingly, we find that both contracting opportunity and profit for the buyer increase monotonically with the degree of commitment. Our results indicate that committing to quantity is financially valuable for the buyer only if the buyer has already committed to the wholesale price. Thus, committing to the price and committing to the quantity are complementary strategies for the buyer. In contrast, we find that the supplier's profit is not changing monotonically by advancing the level of commitment. Our results show that the buyer's strategies of commitment to wholesale price and commitment to quantity have the effect of being substitutes for the supplier. We also find that when the buyer transitions from "no-commitment" to "commitment to only the wholesale price" policy, it can result in a *win-win-win* scenario for both the buyer and the supplier. In this case not only does the buyer's profit improve, along with an improvement in overall sustainability compliance (i.e., *win-win* for the buyer); but also, the supplier makes a positive profit (i.e., *win* for the supplier). Lastly, we examine the sensitivity of the supplier's compliance and overall sustainability compliance to an increase in the standard for the code of conduct. We identify conditions (i.e., scenarios) in which, notably, the supplier's compliance and overall sustainability compliance increase with an increase in the standard for the code of conduct.

The remainder of chapter is organized as follows. First, we review the relevant literature. Then, we introduce the model set-up. After that, we analyze the "no-commitment" policy as a benchmark. Next, we investigate the effect of *partial commitment* policies. After that, we analyze the policy of "commitment to both wholesale price and quantity" and investigate the interplay between the buyer's policies that consider increasing level of commitment. Next, we use our analysis in previous sections and study the effect of raising the standard of the code of conduct on sustainability and financial metrics. Finally, we provide concluding remarks. The proofs of all Lemmas and Propositions are provided in Appendix.

Literature Review

Our paper is related to research focused on examining the efficacy of sourcing strategies, supply chain structures, incentives and penalties in contract design, and auditing schemes as levers for enhancing suppliers' social and environmental responsibility.

With regard to the related literature on sourcing strategies and supply chain structures, Guo et al. (2016) study responsible sourcing in supply chains when faced with the choice of a responsible supplier who is costly *vis-à-vis* a less expensive supplier who may face responsibility violations. Agrawal and Lee (2017) study the effect of sourcing policies in influencing a supplier's decision to adopt sustainable practices. de Zegher et al. (2017a) investigate the interplay between sourcing channel and contract design to incorporate responsible sourcing strategies while also creating economic value. Letizia and Hendrikse (2016) study the impact of supply chain structures using horizontal or vertical alliances among supply chain members to incentivize suppliers to adopt corporate social responsibility activities. Orsdemir et al. (2018) investigate vertical integration as a strategy for ensuring corporate social and environmental responsibility. Huang et al. (2017) study the issue of how a buyer should manage social responsibility in lower tiers of its supply chain by examining a scenario wherein a tier 0 buyer either works directly with a tier 2 supplier for enabling its compliance or delegates it to the tier 1 supplier. Zhang et al. (2017) examine equilibrium sourcing decisions in a three-tier network comprised of manufacturers, smelters, and mines to understand implications of imposing penalties on manufacturers for failing to curb usage of conflict minerals.

The next research stream has focused on the effect of incentives and penalties in contract design for sustainability. Babich and Tang (2012) consider a buyer's moral hazard problem with hidden action to investigate the effectiveness of using only deferred payment, only inspection, or both as mechanisms to tackle the problem of adulteration by its supplier. They find that deferred payment mechanism is preferable to inspection if the threat of adulteration is low. Chen and Lee (2016) consider an adverse selection problem with hidden information to study effectiveness of supplier certification, process audits, and contingent payments as tools for enhancing a supplier's compliance to the social and environmental standard. Their results indicate that while process audit and contingency payment alone can directly increase the supplier's compliance, they are not as effective as a supplier certification instrument for screening suppliers with different levels of ethics. Karaer et al. (2017) study the effect of offering wholesale price premium and sharing the cost of a supplier's sustainability effort in enhancing the supplier's ability to produce a more environmentally responsible product. Under a single-supplier setting, they find that if it is optimal for the buyer to offer a price premium, then the buyer also fully subsidizes the supplier's cost for investing in environmental quality. Cho et al. (2018) investigate the effect of offering price premium, and the effect of whether inspection information is disclosed, as tools for curbing the use of child labor in supply chains. They find that the firm's pricing and inspection strategies behave as substitutes in curbing child labor. de Zegher et al. (2017b) consider a 3-tier supply chain to study the efficacy of offering contingent advance payments by palm oil buyers, when contracting with the mill and farmers, to enhance compliance for sustainable sourcing commitment.

Another stream of research has focused on examining the efficacy of a variety of auditing schemes in influencing a supplier's social or environmental compliance. Plambeck and Taylor (2016) consider a buyer-supplier setting with an exogenous wholesale contract in order to investigate the degree to which the buyer's actions related to increasing auditing effort, to publicizing negative auditing reports, and to providing a loan to a supplier can backfire, in terms of increased evasive (i.e., hiding) action by the supplier. Caro et al. (2018), Fang and Cho (2016), and Chen et al. (2017) build on the framework in Plambeck and Taylor (2016) to examine the efficacy of various auditing approaches (e.g., independent, shared, or joint audit) to influence a supplier's sustainability effort and ensuing compliance. Caro et al. (2018) investigate the impact of two non-competing buyers either jointly auditing a common supplier or sharing results of one's independent audit with the other buyer. Fang and Cho (2016) consider blocks of competing buyers sourcing from a common supplier wherein buyers in each block either jointly audit the supplier or audit the supplier independently and share results with buyers in the block. However, the contract with the supplier in this setting is exogenous, and no corrective action is required by the supplier upon failing an audit. Chen et al. (2017) consider a scenario with two identical buyers and three suppliers (with an exogenous wholesale price offered to each supplier). Each buyer sources two components, with one of them being from the common supplier and the other from a non-common supplier. Motivated by a budget-constrained setting, they investigate whether buyers should prioritize the auditing effort based on the degree of supplier's centrality, i.e., whether to emphasize the audit for the common supplier *vis-à-vis* their respective non-common suppliers. In their setting, the supplier incurs a correction cost upon failing an audit; however that cost is not linked to the sustainability effort of the supplier.

In contrast to the extant literature, we focus on understanding the efficacy of commitment to a wholesale contract on enhancing sustainability compliance in the market. To the best of our knowledge, this lever has not been considered hitherto in the sustainability literature. Thus, with a different focus, we investigate the effect of the interplay between degree of commitment to an endogenized wholesale contract and the buyer's auditing effort on the supplier's compliance to the code of conduct and ensuing overall sustainability compliance in the market. We modify the basic setting in Plambeck and Taylor (2016), wherein the supplier, upon failing the audit, is expected to take a corrective action that is commensurate with his compliance effort. Further, we now also solve for the subgame-perfect equilibrium and characterize the contract terms, the buyers auditing effort, and the supplier's compliance effort at the equilibrium. Lastly, we study the effect of raising the standard of the code of conduct. We present details of the modeling framework in the next section.

Model Setting

We consider a supply chain in which a buyer sources a product from a supplier and then resells the product to the consumer market. It costs c per unit for the supplier to manufacture the product. The buyer offers to the supplier a take-it-or-leave-it wholesale-price contract (w, q), where w is the wholesale price and q is the order quantity. The market for the product has the demand size of D and the selling price of p. To focus on the analysis of efficacy of degree of commitment on sustainability in the supply chain, we simplify p to be exogenous and D to be deterministic. We assume that $c \leq w \leq p$ and $0 \leq q \leq D$.

The buyer puts in place a supplier code of conduct to guide and audit the supplier for socially and environmentally responsible business practice. We assume that the buyer, the supplier, and the market have a common understanding of what constitutes a violation of this sustainability code of conduct. The market expects the buyer to source from a compliant supplier. When a supplier is prone to social or environmental misconduct (i.e., violations) in its business practice, it adversely affects the buyer's demand in the market.

Without loss of generality, we consider that the supplier is initially non-compliant with the code of conduct and can make an effort to be compliant. The supplier's compliance effort produces a random outcome: the supplier becomes compliant with the code of conduct with probability e and remains to be non-compliant with probability $1 - e$. Intuitively, a higher compliance effort leads to a higher probability of being compliant. We follow the literature, such as Plambeck and Taylor (2016) and Chen et al. (2017), to use the probability e also as a proxy for the supplier's compliance effort. The supplier's compliance effort entails the cost of $K_s(e)$ for $e \in [0, 1]$, which is continuous and strictly increasing in e with $K_s(0) = 0$ and is differentiable.

The supplier's compliance status is invisible to the buyer. The buyer has to audit the supplier to improve its knowledge about the supplier's compliance status, but only imperfectly. If the supplier is compliant with the code of conduct, then the auditing process reliably concludes with a "pass". With a non-compliant supplier, the buyer's auditing process yields a random outcome. With probability a, the buyer correctly identifies a non-compliant supplier, and the supplier fails the audit. With probability $1 - a$, a non-compliant supplier passes the audit. Intuitively, the buyer's probability of correctly identifying a non-compliant supplier increases with the level of auditing effort, which can be increased, for example, by assembling a more experienced auditing team. We use the probability of identifying a non-compliant supplier, a, as a proxy for the buyer's auditing effort. The buyer's auditing effort has the cost of $K_b(a)$, which is continuous and strictly increasing in $a \in [0, 1]$ with $K_b(0) = 0$ and is differentiable.

After the completion of the supplier's compliance effort and the buyer's auditing, the buyer observes "pass" with probability $e+(1-e)(1-a)$ or "fail" with probability $(1-e)a$. We assume that the auditing outcome—"pass" or "fail"—is verifiable by the buyer and by the supplier, so the buyer can make use of the auditing outcome in dealing with the supplier. Whenever possible and necessary, the buyer designs the contract to be contingent on the auditing outcome. We use (w_p, q_p) and (w_f, q_f) to denote the contracts to be used when the auditing outcome is "pass" and "fail", respectively. Furthermore, if the buyer's audit successfully identifies a non-compliant supplier, the buyer requests the supplier to take corrective action at the supplier's own expense. We assume that the buyer's audit uncovers all violations, and hence, a corrective action makes a non-compliant supplier perfectly compliant. The supplier's cost of correction depends on the level of initial compliance effort and is denoted as $K_c(e)$, which is continuous and strictly decreasing in $e \in [0, 1]$ with $K_c(1) = 0$ and is differentiable.

When the buyer fails to identify a non-compliant supplier, we assume that the market can perfectly identify non-compliance of the supplier in a timely manner, e.g., via a NGO investigation or via media coverage. The market reacts to non-compliance with market disruption. Since our focus is on examining the effect of degree of commitment to contract terms, we make a simplifying assumption that the buyer loses all demand D with probability 1 due to market disruption.

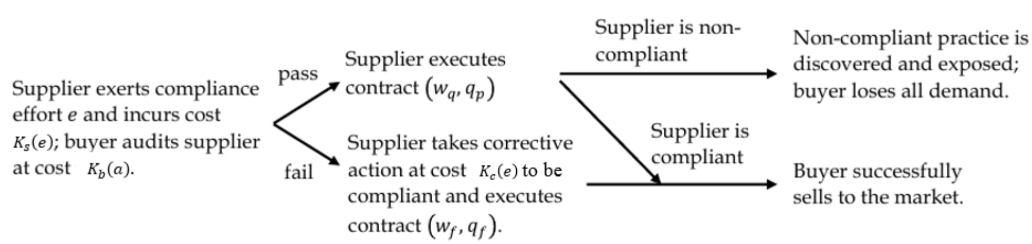


FIGURE 1. Sequence of Actions.

We model a one-period problem, in which the buyer and the supplier contract for one-time supply and take a sequence of actions. The timing of the actions is illustrated in Figure 1. First, the supplier exerts the compliance effort e, and the buyer exerts an auditing effort a. We assume that at the time of auditing the buyer and supplier cannot observe each other's effort level. However, after the auditing the supplier's compliance effort becomes verifiable to the buyer. Following the literature, such as Kim and Netessine (2013), we treat the buyer's and the supplier's decisions about their respective levels of effort as a simultaneous-move game and solve for Nash equilibrium. Next, the supplier takes necessary actions to fulfill the contract requirement, contingent on the auditing outcome. If the supplier passes the audit, then the buyer requires the supplier to execute the contract of (w_p, q_p). If the supplier fails the audit, then the audit indicates that the supplier is non-compliant. The buyer invokes the contract of (w_f, q_f) and requests the supplier to take corrective action before executing the contract. The supplier may turn down the buyer's request for correction and renege on the contract of (w_f, q_f). We assume that the supplier's reservation profit of non-participation is zero. Finally, the buyer attempts selling the product to the market. The outcome of selling depends on the supplier's true compliance status. Working with a compliant supplier fully eliminates the risk of an adverse market response. As such, the buyer sells successfully to the market and collects the revenue of $(p - w_i)q_i$ (with $q_i \leq D$). The subscript $i = p$ if the supplier passes the audit; and $i = f$ if the supplier fails the audit and is corrected later on to become compliant. If the supplier remains non-compliant at the time of selling, market disruption occurs, and the buyer makes zero profit from the market.

As the sequence of actions evolves, the buyer and the supplier incur costs, collect revenues, and receive information (i.e., auditing outcome and any market disruption).

Looking forward at different time epochs, the buyer and the supplier see different profit functions for the rest of the game. We now elaborate on the buyer's and the supplier's profit-to-go at two time epochs: after the conclusion of auditing and at the inception of auditing.

After auditing concludes, the buyer observes the outcome to be either "pass" or "fail", but not the supplier's true compliance status. If the supplier passes the audit, there is a probability that the supplier is, actually, non-compliant and the buyer experiences market disruption and collects zero revenue. The buyer invokes the contract of (w_p, q_p) and anticipates the following expected profit from the rest of the game:

$$\pi_B^p(w_p, q_p, e, a) = \frac{e}{e + (1-e)(1-a)} p\, q_p - w_p q_p, \tag{2.1}$$

where $\frac{e}{e+(1-e)(1-a)}$ is the conditional probability that the supplier is compliant and the market is not disrupted, given that the supplier passes the audit. Note that the buyer's expected profit depends on e and a, because they affect the conditional probability. The supplier collects the following profit from the supply contract:

$$\pi_S^p(w_p, q_p) = (w_p - c)q_p. \tag{2.2}$$

In the case that the supplier fails the audit, the buyer invokes the contract of (w_f, q_f) and requests the supplier to take the corrective action to be compliant. Note that "fail" is an unambiguous indication of the supplier' non-compliance, so it resolves the uncertainty in the auditing outcome. The supplier's corrective action resolves the uncertainty in the market's response. The buyer collects the profit of selling to the market:

$$\pi_B^f(w_f, q_f) = (p - w_f)q_f, \tag{2.3}$$

under the condition that the supplier agrees to take corrective action to be fully compliant. The supplier's profit of staying in the contract is its income from the supply contract less the cost of correction:

$$\pi_S^f(w_f, q_f, e) = (w_f - c)q_f - K_c(e). \tag{2.4}$$

Before auditing begins, the buyer faces the uncertainties of auditing outcome and the market's response later on. The buyer's expected profit is the expectation of profit to be collected after auditing less the upfront cost of auditing:

$$\Pi_B(w_p, q_p, w_f, q_f, e, a) = [e+(1-e)(1-a)]\pi_B^p(w_p, q_p, e, a)+[(1-e)a]\pi_B^f(w_f, q_f)-K_b(a). \tag{2.5}$$

Using the expressions for $\pi_B^p(w_p, q_p, e, a)$ and $\pi_B^f(w_f, q_f)$ in (2.1) and (2.3), one can transform (2.5) into the more explicit form:

$$\Pi_B(w_p, q_p, w_f, q_f, e, a) = e(p-w_p)q_p-[(1-e)(1-a)]w_pq_p+(1-e)a(p-w_f)q_f-K_b(a). \tag{2.6}$$

This form of $\Pi_B(w_p, q_p, w_f, q_f, e, a)$ helps us understand the buyer's trade-off in the face of an uncertain outcome of auditing. The buyer anticipates three possible situations looking forward. If the supplier is compliant (with probability e), then the buyer uses the contract of (w_p, q_p), sells to the market, and collects the profit of $(p - w_p)q_p$. If the supplier is non-compliant but passes the audit (with probability $(1-e)(1-a)$), then the buyer pays the amount of w_pq_p for the supply but experiences a market disruption. If the supplier is non-compliant and fails the audit (with probability $(1 - e)a$), then the buyer uses the contract of (w_f, q_f) and collects the profit of $(p - w_f)q_f$.

Similarly, the supplier's expected profit is:

$$\Pi_S(w_p, q_p, w_f, q_f, e, a) = [e+(1-e)(1-a)]\pi_S^p(w_p, q_p) + [(1-e)a]\pi_S^f(w_f, q_f, e) - K_s(e), \tag{2.7}$$

which can be written more explicitly as:

$$\Pi_S(w_p, q_p, w_f, q_f, e, a) = [e+(1-e)(1-a)]q_p(w_p-c)+(1-e)a\big[q_f(w_f-c)-K_c(e)\big]-K_s(e). \tag{2.8}$$

For tractability, we assume that the buyer's auditing cost, $K_b(a)$ is linear in $a \in [0, 1]$, and that the supplier's compliance effort cost and correction cost, $K_s(e)$ and $K_c(e)$ respectively, are linear in $e \in [0, 1]$. Specifically, we assume $K_b(a) = \gamma \times a$, $K_s(e) = \alpha \times e$ and $K_c(e) = \beta \times (1 - e)$, where γ, α and β are cost coefficients and positive constants.

The buyer can choose to commit to the wholesale price or to the order quantity before the inception of auditing, and the buyer has commitment power. In this paper, the commitment to a contract term means that the buyer decides the contract term and offers it to the supplier before auditing the supplier. The buyer offers to honor this *a priori* contracting decision regardless of the outcome of the audit, so long as the supplier is willing to take corrective upon failing the audit. There are four scenarios of contract commitment, depending on whether the buyer commits to the wholesale price or to the order quantity. In the first scenario, the buyer makes no-commitment at all. All contracting decisions are made after auditing outcome is obtained. In the second scenario, the buyer commits to only the order quantity with $q = q_p = q_f$, leaving the wholesale prices w_p and w_f to be decided after auditing. In the third scenario, the buyer commits to only the wholesale price with $w = w_p = w_f$, leaving the order quantities q_p and q_f to be decided after auditing. Finally, in the fourth scenario, the buyer commits to both the wholesale price and order quantity with $w = w_p = w_f$ and $q = q_p = q_f$.

The Benchmark: "No-Commitment Policy"

We first analyze the scenario wherein the buyer does not commit to either the wholesale price or the order quantity. That is, the buyer decides the wholesale price and the order quantity only after the auditing outcome is revealed, but before the execution of the contract. We will use this scenario as a benchmark for further analysis. We analyze this benchmark model backward and solve for the subgame perfect Nash equilibrium, starting with the buyer's contracting decisions after auditing.

Optimal Contracting Decisions

We first analyze the buyer's contracting decision, which is made after auditing is concluded. Given the buyer's and the supplier's efforts, respectively a and e, the buyer decides a menu of two contracts (w_p, q_p) and (w_f, q_f) to be used in the respective cases where auditing yields the outcome of "pass" and "fail".

In the case that the supplier passes the audit, the buyer offers the contract of (w_p, q_p) to maximize its expected profit:

$$\max_{0 \leq q_p \leq D,\, c \leq w_p \leq p} \pi_B^p(w_p, q_p, e, a), \tag{2.9}$$

where $\pi_B^p(w_p, q_p, e, a)$ is defined in (2.1). We present the buyer's optimal contract in Lemma 1.

Lemma 1. *When the buyer makes no contract commitment, and given that the supplier passes the audit, the buyer's optimal wholesale price and order quantity* (w_p^*, q_p^*) *are:*

– *If* $\frac{\left(1-\frac{p}{c}e\right)^+}{1-e} \leq a \leq 1$, *or equivalently* $\frac{1-a}{\frac{p}{c}-a} \leq e \leq 1$, *then the buyer's optimal contract is* $(w_p^*, q_p^*) = (c, D)$.

– *If* $0 \leq a < \frac{\left(1-\frac{p}{c}e\right)^+}{1-e}$, *or equivalently* $0 \leq e < \frac{1-a}{\frac{p}{c}-a}$, *then the buyer does not contract with the supplier.*

At the optimal contract, the supplier's profit-to-go is $\pi_S^p(w_p^*, q_p^*) = 0$.

Proof. All proofs are relegated to the Appendix. □

In the case that the supplier fails the audit, the buyer offers the supplier the contract of (w_f, q_f) to maximize its profit of selling to the market, subject to the supplier taking corrective action:

$$\begin{aligned} \max_{0 \leq q_f \leq D,\, c \leq w_f \leq p} \quad & \pi_B^f(w_f, q_f) \\ \text{subject to} \quad & \pi_S^f(w_f, q_f, e) \geq 0, \end{aligned} \tag{2.10}$$

where $\pi_B^f(w_f, q_f)$ and $\pi_S^f(w_f, q_f, e)$ are defined in (2.3) and (2.4), respectively. Recall that, after auditing the supplier's compliance effort becomes verifiable to the buyer. Lemma 2 presents the optimal contract for the case that the supplier fails the audit.

Lemma 2. *When the buyer makes no contract commitment, and given that the supplier fails the audit, the buyer's optimal wholesale price and order quantity* (w_f^*, q_f^*) *are:*

– *If* $\left[1 - \frac{(p-c)D}{\beta}\right]^+ \leq e \leq 1$, *then the buyer's optimal contract is* $(w_f^*, q_f^*) = \left(c + \frac{\beta(1-e)}{D}, D\right)$.

– *If* $0 \leq e < \left[1 - \frac{(p-c)D}{\beta}\right]^+$, *then the buyer does not contract with the supplier.*

At the optimal contract, the supplier's profit-to-go is $\pi_S^f(w_f^*, q_f^*, e) = 0$.

Lemmas 1 and 2 show that the buyer contracts with the supplier only if the supplier's initial compliance effort e or the buyer's auditing effort a is sufficiently high. With a high initial compliance effort, the supplier's *ex post* correction cost is low, so the supplier can afford remaining in the contract. Conversely, with a high upfront auditing effort, there is a low chance that the supplier is non-compliant at the end of auditing. So, the buyer is confident to contract with the supplier.

Furthermore, Lemmas 1 and 2 show that the supplier collects zero profit after auditing. This is because the buyer has the contracting power and sets the wholesale price to drive the supplier's profit exactly to its reservation profit. When the supplier passes the audit, the buyer sets the wholesale price to be the production cost. When the supplier fails the audit, the buyer sets the wholesale price to exactly offset the production cost and the cost of correction.

Sub-Game Perfect Nash Equilibrium of Auditing and Compliance

Next, we derive the buyer's decision of auditing effort and the supplier's decision of compliance effort, given that the buyer will optimally design the contracts of (w_p^*, q_p^*) and (w_f^*, q_f^*) after auditing. We solve for Nash equilibrium of a and e. To do that, we first solve for the buyer's and the supplier's best response functions to each other's effort level.

From (2.7), the supplier's total expected profit at the inception of auditing is

$$\Pi_S(w_p^*, q_p^*, w_f^*, q_f^*, e, a) = [e + (1-e)(1-a)]\pi_S^p(w_p^*, q_p^*) + [(1-e)a]\pi_S^f(w_f^*, q_f^*, e) - K_s(e), \tag{2.11}$$

in which, as Lemmas 1 and 2 show, $\pi_S^p(w_p^*, q_p^*) = \pi_S^f(w_f^*, q_f^*, e) = 0$. Since the supplier's profit-to-go after auditing is zero in the optimal contract, his expected profit is negative for any positive level of compliance effort. Hence, it is optimal for the supplier to make zero compliance effort upfront, that is, $e = 0$. As a result, $\Pi_S(w_p^*, q_p^*, w_f^*, q_f^*, e, a) \equiv 0$ for all a. The buyer extracts all profit of the supply chain for the entire game.

The buyer chooses the auditing effort a to maximize its expected profit:

$$\max_{0 \le a \le 1} \Pi_B(w_p^*, q_p^*, w_f^*, q_f^*, e, a) \tag{2.12}$$

where $\Pi_B(w_p^*, q_p^*, w_f^*, q_f^*, e, a)$ is shown in (2.6). We substitute $e = 0$, (w_p^*, q_p^*), and (w_f^*, q_f^*) into (2.12) and solve for the optimal a. Proposition 1 presents the Nash equilibrium and its outcome.

Proposition 1. *Consider that the buyer does not commit to either the wholesale price or the order quantity. At the equilibrium, the supplier's compliance effort e^* and the buyer's auditing effort a^*, the buyer's optimal contracts (w_p^*, q_p^*) and (w_f^*, q_f^*), and the buyer's and the supplier's total profit π_B^* and π_S^* are:*

- *If $(p-c)D > \beta+\gamma$, then we have $(e^*, a^*) = (0, 1)$, $(w_p^*, q_p^*) = (c, D)$, $(w_f^*, q_f^*) = (c + \frac{\beta}{D}, D)$, $\pi_B^* > 0$ and $\pi_S^* = 0$.*
- *If $(p-c)D \le \beta + \gamma$, then the buyer does not contract with the supplier.*

$D(p-c)$ is the supply chain's total contribution margin (i.e., excluding cost of efforts) for producing the product and selling it to the market, and $\beta+\gamma$ is the supply chain's total cost of efforts (to be eventually borne by the buyer for contracting to occur) when the supplier does not exert any compliance effort. Proposition 1 shows that the buyer contracts with the supplier if and only if the supply chain's total contribution margin is sufficiently high to offset the cost of auditing and correction when the supplier does not expend any compliance effort upfront.

Furthermore, Proposition 1 shows that, while the supplier has no incentive to exert any compliance effort (i.e., $e^* = 0$), the buyer audits the supplier at the highest possible level with $a^* = 1$. The buyer does so to produce verifiable evidence for holding the supplier accountable and enforcing the corrective action to avoid market disruption. Thus, a basic managerial insight for a buyer who does not commit to contract terms but is able to enforce corrective action when the supplier fails an audit is that the buyer cannot enhance the upfront compliance of a supplier.

Partial Commitment Policies

In this section we investigate the effect of the buyer's partial commitment (i.e., when the buyer commits to only quantity or only the wholesale price). In this section first, we briefly present and analyze the policy in which the buyer commits to only the quantity. We compare this policy to "no-commitment" (benchmark policy) to characterize the effect of commitment to only the order quantity. We find that "commitment to only the order quantity" has identical equilibrium outcomes *vis-à-vis* the policy of "no-commitment". Next, we present and analyze the policy in which the buyer commits to only the wholesale price. Then, we compare this policy to the "no-commitment" policy to characterize in detail the effect of commitment to only the wholesale price.

Committing to Only Order Quantity

In this section, we analyze the policy in which the buyer commits to only the order quantity of the contract. Before the buyer and the supplier exert their respective efforts, the buyer decides the order quantity q and commits to $q_f = q_p = q$ to be invoked upon execution of the contract. The buyer decides the wholesale prices w_p and w_f after auditing concludes.

The analysis of this policy closely resembles that for the policy without commitment in §2.4, and the equilibrium outcomes of the two policies are identical. Hence, we skip all technicalities and refer the reader to Lemmas 1 and 2, and Proposition 1 for the equilibrium outcomes. Since the buyer still sets the wholesale price after the conclusion of auditing, similar to that in the policy without commitment, the buyer extracts the entire profit of the supply chain. So, under both policies the buyer and the supplier behave identically with the implication being that "committing to only the order quantity" makes no difference at all. This offers an added insight for a buyer who is now able to commit to quantity while being able to enforce corrective action upon the supplier failing an audit. The buyer still cannot enhance the upfront compliance of a supplier with this commitment.

Committing to Only the Wholesale Price

In this section, we analyze the policy in which the buyer commits to only the wholesale price of the contract. Before the buyer and the supplier exert their respective efforts, the buyer decides the wholesale price w and commits to $w_f = w_p = w$ to be invoked upon execution of the contract. The buyer decides order quantities q_p and q_f after auditing concludes.

We analyze the model backward, starting with the buyer's order quantity decisions after observing the auditing outcome to be either "pass" or "fail". The optimization programs representing the buyer's decision are identical to programs (2.9) and (2.10) for the model without contract commitment, except that w_p and w_f are dropped out of the buyer's decision at this time point. We present the buyer's optimal q_p and q_f in Lemma 3.

Lemma 3. *Consider the situation wherein the buyer commits to only the wholesale price.*

If the supplier passes the audit, then the buyer's optimal order quantity q_p is such that

- *If $\frac{\left(1-\frac{p}{w}e\right)^+}{1-e} \leq a \leq 1$, or equivalently $\frac{1-a}{\frac{p}{w}-a} \leq e \leq 1$, then the buyer's optimal order quantity is $q_p^* = D$; and the supplier's profit at this stage is $(w-c)D$.*
- *If $0 \leq a < \frac{\left(1-\frac{p}{w}e\right)^+}{1-e}$, then the buyer does not order, i.e., $q_p^* = 0$.*

If the supplier fails the audit, then the buyer's optimal order quantity q_f is such that

- *If $\left[1 - \frac{(w-c)D}{\beta}\right]^+ \leq e \leq 1$, then the buyer's optimal order quantity is $q_f^* = D$; and the supplier's profit at this stage is $(w-c)D - \beta(1-e)$.*
- *If $0 \leq e < \left[1 - \frac{(w-c)D}{\beta}\right]^+$, then the buyer does not order, i.e., $q_f^* = 0$.*

The insight from Lemma 3 is similar to that from Lemmas 1 and 2 when there is no-commitment. The buyer orders from the supplier only if the supplier's initial compliance effort e or the buyer's auditing effort a is sufficiently high. A key difference is that the supplier need not make zero profit, as the buyer may find it beneficial to set the wholesale price w to be high (in the first stage) and commit to it.

Sub-Game Perfect Nash Equilibrium of Auditing and Compliance

We move one step backward in time to immediately after the buyer's decision of w, and solve for the Nash equilibrium of the buyer's auditing effort a and the supplier's compliance effort e. The buyer and the supplier simultaneously choose a and e to maximize their expected profit: $\Pi_B(w_p, q_p^*, w_f, q_f^*, e, a)$ and $\Pi_S(w_p, q_p^*, w_f, q_f^*, e, a)$. In the buyer's and the supplier's profit functions, $w_p = w_f = w$ are considered as a given parameter at this stage, and (q_p^*, q_f^*) as optimally decided according to Lemma 3. There are many combinations of optimal q_f and q_p in Lemma 3, depending on the values of a, e, as well as the model parameters of D, p, w, α, β, and γ. So, to solve for the Nash equilibrium of a and e, there are a plethora of cases to be considered, each depending on the values of D, p, w, α, β, and γ. We relegate details of the solution procedure to the Appendix and present the equilibrium outcome in Lemma 4.

Lemma 4. *Consider the scenario that the buyer commits to only the wholesale price before auditing. At the Nash Equilibrium(NE), the buyer's auditing effort a^* and the supplier's compliance effort e^* are:*

- *When $0 < \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ or $p < \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$*
    - *and under $c + \frac{(4\beta-\alpha)\alpha}{4\beta D} \leqslant w \leqslant p - \frac{\gamma}{D}$ then $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$, which is NE form I;*
    - *otherwise, the buyer's profit is zero and the supplier's profit is non-positive.*
- *When $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$*
    - *and under $c+\frac{(4\beta-\alpha)\alpha}{4\beta D} \leqslant w \leqslant p-\frac{\gamma}{D}$ and $D \geqslant \frac{2\beta\gamma}{p\alpha}$ then $(e^*, a^*) = (1-\frac{\alpha}{2\beta}, 1)$, which is NE form I;*

- *and under* $\hat{w}(D,p,c,\alpha,\beta,\gamma) \leqslant w \leqslant p - \frac{\gamma}{D}$ *and* $D < \frac{2\beta\gamma}{p\alpha}$ *then* $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$*, which is NE form II;*

- *otherwise, the buyer's profit is zero and the supplier's profit is non-positive.*

  where $\hat{w}(D,p,c,\alpha,\beta,\gamma) = \frac{D\gamma(4\beta-\alpha)+2D^3p(p+c)-\sqrt{(D\gamma(4\beta-\alpha)+2D^3p(p+c))^2-16D^3p((D^3p^2C)+(2Dp-\gamma)\beta\gamma)}}{4D^3p}$. *The buyer's profit and the supplier's profit associated with each forms of NE are:*

– *If NE form I holds, which is* $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$*, then*

$$q_f^* = \begin{cases} 0 & c \leqslant w < c + \frac{\alpha}{2D} \\ D & c + \frac{\alpha}{2D} \leqslant w \leqslant p \end{cases}, \quad q_p^* = D,$$

$$\Pi_B = \begin{cases} (1 - \frac{\alpha}{2\beta})D(p-w) - \gamma & c \leqslant w < c + \frac{\alpha}{2D} \\ D(p-w) - \gamma & c + \frac{\alpha}{2D} \leqslant w \leqslant p \end{cases}, \qquad \text{(cor. 1)}$$

$$\text{and } \Pi_S = \begin{cases} (1 - \frac{\alpha}{2\beta})D(w-c) - \alpha(1 - \frac{\alpha}{2\beta}) & c \leqslant w < c + \frac{\alpha}{2D} \\ D(w-c) - \frac{\alpha(4\beta-\alpha)}{4\beta} & c + \frac{\alpha}{2D} \leqslant w \leqslant p \end{cases}. \qquad \text{(cor. 2)}$$

– *If NE form II holds, which is* $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$*, then*

$$q_f^* = \begin{cases} 0 & c \leqslant w < c + \frac{\beta\gamma}{D^2 p} \\ D & c + \frac{\beta\gamma}{D^2 p} \leqslant w \leqslant p \end{cases}, \quad q_p^* = D,$$

$$\Pi_B = \begin{cases} \frac{1}{2\beta} D(p-w)(2\beta - \alpha) - \gamma & c \leqslant w < c + \frac{\beta\gamma}{D^2 p} \\ D(p-w) - \gamma & c + \frac{\beta\gamma}{D^2 p} \leqslant w \leqslant p \end{cases}, \qquad \text{(cor. 3)}$$

$$\textit{and } \Pi_S = \begin{cases} \frac{1}{2\beta} D(w-c)(2\beta - \alpha) - (\alpha - \frac{\alpha\gamma}{Dp}) & c \leqslant w < c + \frac{\beta\gamma}{D^2 p} \\ D(w-c) - (\alpha - \frac{\alpha\gamma}{2Dp}) & c + \frac{\beta\gamma}{D^2 p} \leqslant w \leqslant p \end{cases}. \qquad \text{(cor. 4)}$$

– *If NE form III holds, which is* $(e^*, a^*) = (\frac{w}{p}, 0)$*, then* $(q_f^*, q_p^*) = (0, 0)$ *and* $(\Pi_S, \Pi_B) = (-\gamma\frac{w}{p}, 0)$.

– *If NE form IV holds, which is* $(e^*, a^*) = (0, 0)$*, then* $(q_f^*, q_p^*) = (0, 0)$ *and* $(\Pi_S, \Pi_B) = (0, 0)$.

Figure 2 shows the graphic representation of Lemma 4. The buyer is able to justify a full auditing effort in situations when the demand is high while being able to offer a wholesale price to the supplier that is neither too low nor too high (i.e., as represented in *region of NE form I* in Figure 2-a). In situations when the market price is low, the buyer exerts full auditing effort regardless of γ (i.e., even when it is high) when the demand is high, and doing so offers a positive profit by avoiding any risk of market disruption (i.e., as in *region of NE form I* in Figure 2-a). When the auditing cost coefficient is high, the buyer exerts full effort only when both demand and market price are high (i.e., as in *region of NE form I* in Figure 2-b). However, when the demand is moderate, even with a high market price the buyer can no longer

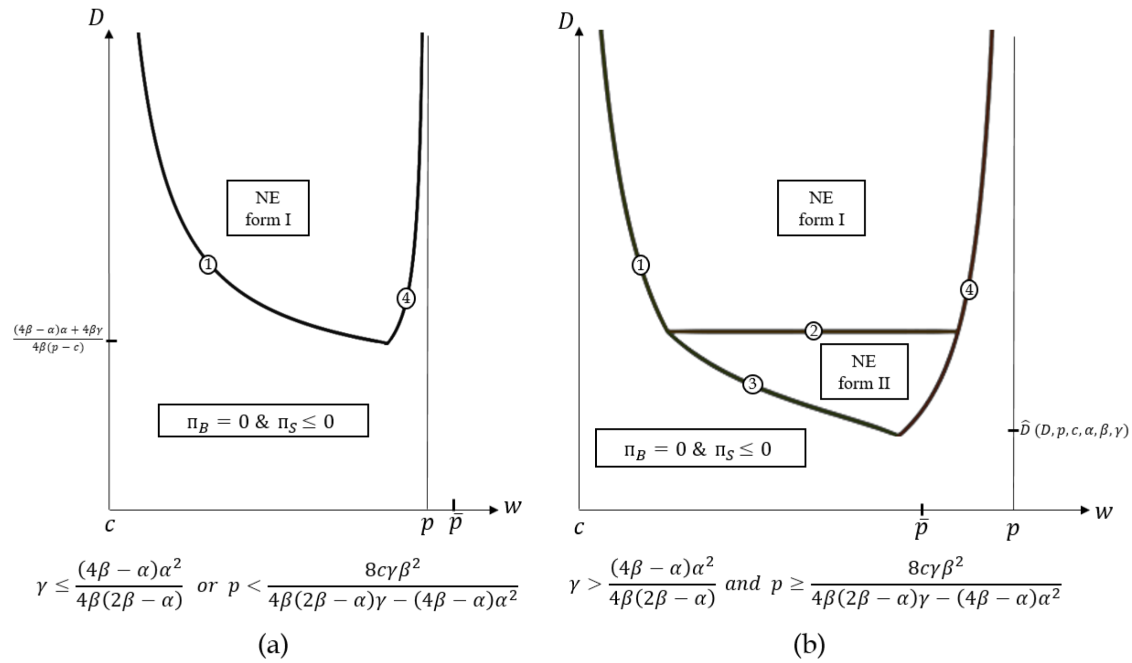


FIGURE 2. Nash Equilibrium Parameter Space

where line 1 represents $D = \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)}$, line 2 represents $D = \frac{2\beta\gamma}{p\alpha}$, line 3 represents $w = \hat{w}(D,p,c,\alpha,\beta,\gamma)$, line 4 represents $D = \frac{\gamma}{p-w}$, and $\bar{p} = \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$.

justify a full auditing effort (i.e., as in *region of NE form II* in Figure 2-b). In both *regions of NE form I and II*, the supplier exerts some level of compliance effort since he receives a contract with a wholesale price w that is neither too low nor too high.

We next consider the region that is outside of the regions represented by NE forms I & II. When the wholesale price is high, the buyer has no willingness to exert any effort, but the supplier exerts some level of compliance effort. The buyer makes zero profit, and the supplier loses money by exerting compliance effort. When either demand or wholesale price is low, neither the buyer nor the supplier can justify their respective efforts. The optimal profit-to-go for both parties is zero. Hence, we consider the region outside of those NE forms I & II as a *no-contracting* region in our further analysis.

Optimal Wholesale Price Decision

We move one step backward in time to solve for the buyer's decision of w. At this step, before the buyer and the supplier exert their respective efforts, the buyer decides the wholesale price w and commits to $w_f = w_p = w$ to be invoked upon execution of the contract.

Optimization program (2.13) shows that the buyer chooses the wholesale price w to maximize its profit-to-go, which is presented in function (2.6), subject to the (*ex ante*) supplier's participation in the contracting. The (*ex ante*) supplier's participation constraint is governed by the supplier's profit-to-go which is presented in function (2.8). Note that for these functions, we assume that $w_f = w_p = w$.

$$\begin{aligned} \max_{c \le w \le p} \quad & \Pi_B(w_p, q_p^*, w_f, q_f^*, e^*, a^*) \\ \text{subject to} \quad & \Pi_S(w_p, q_p^*, w_f, q_f^*, e^*, a^*) \ge 0 \end{aligned} \tag{2.13}$$

Proposition 2 presents the buyer's optimal wholesale price, along with model equilibrium for committing to only the wholesale price.

Proposition 2. *Consider the situation wherein the buyer commits to only the wholesale price. At the equilibrium, the supplier's compliance effort* e^* *and the buyer's auditing effort* a^**, the buyer's optimal contracts* (w^*, q_f^*, q_p^*)*, and the buyer's and the supplier's respective total profit* π_B^* *and* π_S^* *are:*

- *Form (i): no-contracting;*
- *Form (ii):*$(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$*,* $w^* = c + \frac{(4\beta-\alpha)\alpha}{4\beta D}$*,* $(q_p^*, q_f^*) = (D, D)$*,and* $\pi_B^* > 0$ *,* $\pi_S^* = 0$*;*

– *Form (iii):* $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha D p}{2\beta\gamma}),\ w^* = \hat{w}(D, p, c, \alpha, \beta, \gamma)$,

$$(q_p^*, q_f^*) = \begin{cases} (D, 0), & \text{if } D < \frac{(2\beta+\alpha)\gamma}{2p\alpha} \\ (D, D), & \text{otherwise} \end{cases}, \text{ and } \pi_B^* > 0 \ , \ \pi_S^* > 0.$$

The above forms are defined by the following conditions:

– *When* $0 < \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ *or* $p < \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$*:*

  - *If* $0 \leqslant D < \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$*, then form (i) occurs.*

  - *Otherwise, form (ii) occurs.*

– *When* $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ *and* $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$ *:*

  - *If* $0 \leqslant D \leqslant \hat{D}(p, c, \alpha, \beta, \gamma)$*, then form (i) occurs.*

  - *If* $D \geqslant \frac{2\gamma\beta}{p\alpha}$*, then form (ii) occurs.*

  - *If* $\hat{D}(p, c, \alpha, \beta, \gamma) < D < \frac{2\gamma\beta}{p\alpha}$*, then form (iii) occurs,*

Figure 3 shows the graphic representation of regions of equilibrium forms for Proposition 2. When the market conditions are not favorable (i.e., either the demand or the market price is low) we observe a no-contracting region that expands with increase in auditing cost coefficient (*region of equilibrium form i* in Figure 3). On the other hand, when demand and market price are both high, the buyer and the supplier exert their respective efforts, with the buyer also exerting full auditing effort (*region of equilibrium form ii*). Interestingly, when the demand and price are moderate and the auditing cost coefficient is high (i.e., $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$), it leads to a new behavior (*region of equilibrium form iii*) not observed when the auditing cost coefficient is low. The buyer is now only able to justify a partial auditing effort. In both *regions of equilibrium forms ii and iii*, the supplier's compliance effort is not zero, as the supplier receives a contract and has incentive to exert some level of compliance effort.

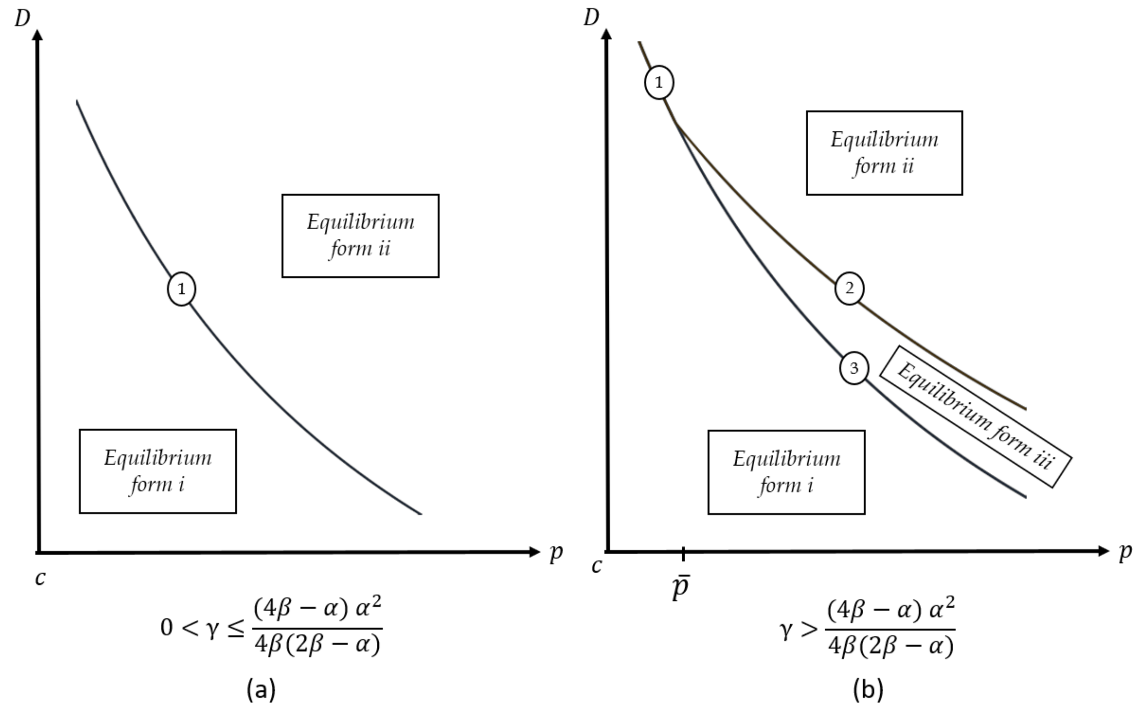


FIGURE 3. Illustration of the Equilibrium Forms in Commitment to Wholesale Price Policy

Where line 1 represents $D = \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$, line 2 represents $D = \frac{2\beta\gamma}{p\alpha}$, line 3 represents $D = \hat{D}(p, c, \alpha, \beta, \gamma)$, and $\bar{p} = \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$.

The buyer makes a positive profit in all regions wherein contracting exists. The supplier's profit is squeezed out to zero in *region of equilibrium form ii*; and surprisingly, the supplier's profit is positive in *region of equilibrium form iii*. We next provide some intuition for it. It should be noted that there are two lower bounds for wholesale price that are relevant to our game-theoretic analysis to ensure the supplier's participation. One lower bound for wholesale price is determined from the *ex ante* consideration, to ensure that the supplier is able to justify exerting compliance effort upfront. The second lower bound for wholesale price is determined from the *ex post* consideration, to ensure that that the supplier can justify corrective action upon failing an audit, even after having exerted compliance effort upfront.

When the lower bound for the wholesale price w determined from the *ex ante* participation constraint is greater than the lower bound for wholesale price (w) determined from *ex post* participation constraint, the buyer can set the wholesale price in the first step in a manner that squeezes out the supplier's expected profit to zero. On the other hand, when the lower bound for the wholesale price for the supplier's *ex ante* participation is less than that for the supplier's *ex post* participation, the buyer is no longer able to squeeze the supplier's profit to zero while setting wholesale price in the first step.

To further understand how the above-mentioned lower bounds are affected by compliance and auditing cost coefficients, we first consider the scenario in which cost coefficients for compliance α and auditing γ are low (i.e., you are in *region of equilibrium form ii*, Figure 3-a). Additionally, α is low enough such that the equilibrium behavior moves to *region of equilibrium form iii* in Figure 3-b when the auditing cost coefficient γ is increased to beyond the threshold (i.e., $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$), *ceteris paribus*.

For α considered in the above scenario, when γ is below the threshold (i.e, $\gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and wherein you are in *region of equilibrium form ii*, Figure 3-a), the supplier's compliance effort is high since α is low (see Proposition 2). A high compliance effort leads to a lower correction cost for *ex post* consideration. The supplier's *ex ante* compliance cost dominates *ex post* correction cost. Hence, in *region of equilibrium form ii*, the lower bound for the wholesale price for the supplier's *ex ante* participation is greater than that for the supplier's *ex post* participation, thus allowing the buyer to squeeze the supplier's profit to zero. But when the auditing cost coefficient is increased to beyond the threshold (i.e., $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$), *ceteris paribus*, the equilibrium behavior is now represented by *region of equilibrium form iii* in

Figure 3-b. The supplier's optimal compliance effort decreases since γ has increased (see Proposition 2). As a result, the compliance cost decreases. A lower compliance effort now leads to a higher correction cost for *ex post* consideration. The supplier's *ex ante* compliance cost is now dominated by *ex post* correction cost. Hence in *region of equilibrium form iii* the lower bound for the wholesale price for the supplier's *ex ante* participation is lower than that for the supplier's *ex post* participation, thus preventing the buyer from squeezing the supplier's profit to zero.

Lastly, when the cost coefficient for compliance α is significantly high (and equilibrium is represented by *region of equilibrium form ii* in Figure 3), the supplier's *ex ante* compliance cost dominates the supplier's *ex post* correction cost despite reduced level of compliance effort. Hence, the lower bound for the wholesale price for the supplier's *ex ante* participation is greater than that for the supplier's *ex post* participation, thus allowing the buyer to squeeze the supplier's profit to zero. A key insight for the buyer from this policy is that, when auditing is not costly, by committing to only the wholesale price the buyer is always able to justify a full auditing effort. This ensures full overall sustainability compliance in the market, resulting from the supplier's upfront compliance and any subsequent corrective action. However, when auditing is costly, the buyer can no longer justify a full auditing effort in all cases; and the buyer loses the ability to ensure full overall sustainability compliance when the auditing effort is partial.

Effect of Committing to Price Only

In this section we consider the metrics of the supplier's compliance effort e, the buyer's auditing effort a, ensuing overall sustainability compliance in the market O_{sc}, the buyer's profit π_B, and the supplier's profit π_S in order to compare the *benchmark* *model* (i.e., with no-commitment to either wholesale price or order quantity) with the "commitment to only the wholesale price". The overall sustainability compliance in the market is represented by $e + (1 - e)a$, which also accounts for the additional compliance guaranteed by corrective action when a supplier fails the audit. So, it measures the likelihood of the buyer's product not experiencing a market disruption (i.e., loss of demand) on account of any sustainability violation in its supply base. The change in each metric is measured based on the value of the corresponding metric when one transitions from a policy of "no-commitment" to a policy of "commitment to only the wholesale price". The change is indicated by using a sign to show whether it increased, decreased, or remained unchanged.

Based on Proposition 1 and Proposition 2, a comparison of "no-commitment" policy with "commitment to only the wholesale price" policy for varying levels of auditing cost coefficient γ generates three cases (i.e., $0 \leqslant \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$, $\frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)} < \gamma < \frac{\alpha\beta}{2\beta-\alpha}$, and $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$). We first present the outcome of comparison under large γ (i.e., $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$) in Proposition 3, as this case provides a super-set of all scenarios observed during the transition. We present details for the other cases of γ after Proposition 3.

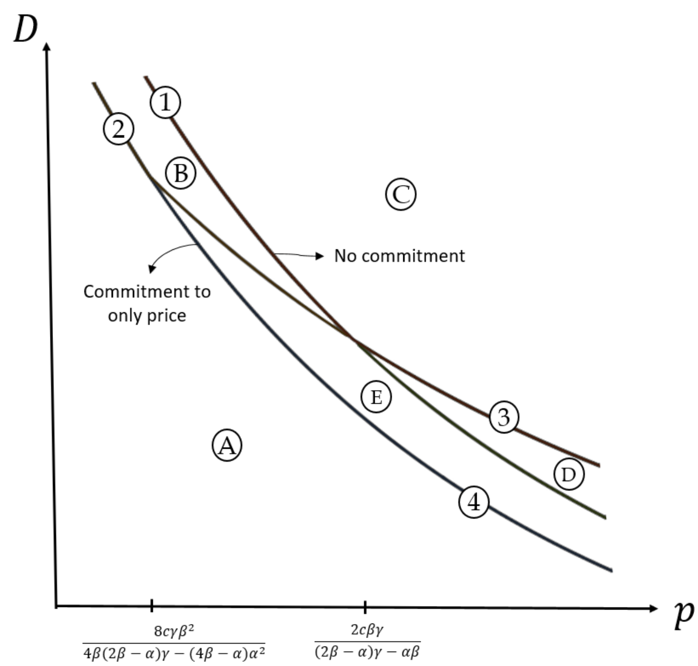
Proposition 3. *Consider the situation wherein the buyer increases the level of commitment and transitions from a "no-commitment" policy to a policy with "commitment to only the wholesale price". When γ is large (i.e., $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$), the following metrics change as below for all p and D:*

- *The supplier's compliance* (e) *increases monotonically with increase in level of commitment as stipulated above.*

– *The buyer's auditing effort* (a) *increases monotonically with increase in level of commitment as stipulated above, except when* $\frac{\beta+\gamma}{p-c} < D < \frac{2\beta\gamma}{p\alpha}$ *and* $p > \frac{2c\beta\gamma}{(2\beta-\alpha)\gamma-\alpha\beta}$.

– *The overall sustainability compliance* O_{sc} *increases monotonically with increase in level of commitment as stipulated above, except when* $\frac{\beta+\gamma}{p-c} < D < \frac{2\beta\gamma}{p\alpha}$ *and* $p > \frac{2c\beta\gamma}{(2\beta-\alpha)\gamma-\alpha\beta}$.

– *The buyer's profit* π_B *and the supplier's profit* π_S *increase monotonically with increase in level of commitment as stipulated above.*



$if \quad \gamma \geq \frac{\alpha\beta}{2\beta-\alpha}$

Region	e	a	O_{SC}	π_B	π_S
A	-	-	-	-	-
B	↑	↑	↑	↑	-
C	↑	-	-	↑	-
D	↑	↓	↓	↑	↑
E	↑	↑	↑	↑	↑

FIGURE 4. From "no-commitment" to a "commitment to only the wholesale price" when $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$

where line 1 belongs to "no-commitment" model and represents $D = \frac{\beta+\gamma}{p-c}$. Other lines belong to "commitment to only the wholesale price", where line 2 represents $D = \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$, line 3 represents $D = \frac{2\beta\gamma}{p\alpha}$, and line 4 represents $D = \hat{D}(p, c, \alpha, \beta, \gamma)$.

Figure 4 provides a graphic representation of Proposition 3. We present the change of metrics in Figure 4 when the auditing cost is high and the buyer advances the level of commitment from "no-commitment" to "commitment to only the wholesale price". In *region A* under both above-mentioned commitment policies, we do not observe contracting because of low level of demand and market price. Therefore, all metrics remain unchanged.

In all regions where contracting occurs after committing to price, both the buyer's profit and the supplier's compliance effort increase with the transition to "commitment to only the wholesale price" policy. This transition also increases the contracting opportunity. *Region B* and *region E* in Figure 4 are regions that now have contracting. This new contracting opportunity also results in improvement of overall sustainability compliance in these two regions. The supplier profit is squeezed to zero in *Region B*. Interestingly, in *region E*, we observe a *win-win-win* situation in which not only does the buyer's profit improve, along with an improvement in overall sustainability compliance; but also, the supplier makes a positive profit. The intuition for this behavior was discussed in wholesale price setting stage , wherein the buyer has to set the wholesale price high enough (*ex ante*) to give the supplier incentive to participate in corrective action (*ex post*). In this case, the buyer is thus unable to squeeze out the supplier's profit to zero. In *region C* the buyer exerts full auditing effort for both policies (with no change). The ensuing perfect auditing results in maximum overall sustainability compliance for both policies. The supplier's compliance increases because committing to price is an incentive for the supplier. With a commitment to only the wholesale price, the buyer experiences increased profit by being able to command a lower wholesale price (*vis-à-vis* no-commitment policy).

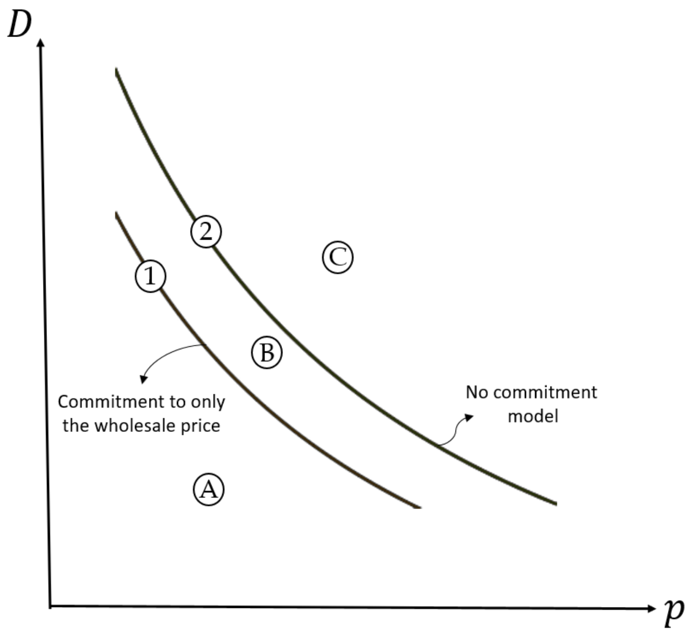
In *region D* we observe a financially *win-win* case for the buyer and the supplier, who see increase in their respective profits. When the buyer commits to only the wholesale price, unlike for the "no-commitment" policy, when in *region D*, the buyer is unable to set the contract terms before auditing in a manner that can squeeze out the supplier's profit to zero. However, the overall sustainability compliance decreases, despite an increase in the supplier's compliance, since the buyer no longer exerts full auditing effort.

Thus, in all cases where there is contracting, we observe that advancing the level of commitment policy from "no-commitment" to "commitment to only the wholesale price" increases the buyer's profit. Additionally, the supplier's compliance effort also increases with the advancement in the level of commitment in all these cases. However, the supplier's profit only increases monotonically. In contrast, note that there is no change in any of the metrics as one transitions from a policy of "no-commitment" to "commitment to only the quantity" . Interestingly, the transition to "commitment to only the wholesale price" policy *vis-à-vis* transition to "commitment to only the quantity" has an asymmetric effect on sustainability and profit metrics.

Other Cases (i.e., for $0 < \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ *and* $\frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)} < \gamma < \frac{\alpha\beta}{2\beta-\alpha}$*) for Comparing Transition from "no-commitment" policy to "Commitment to only the Wholesale Price" policy*

In *region A* under both above-mentioned commitment policies, we do not observe contracting because of low level of demand and market price. Therefore, all metrics remain unchanged.

In all regions where contracting occurs, both the buyer's profit and the supplier's compliance effort increase with the transition to "commitment to only the wholesale price" policy. This transition also increases the contracting opportunity. *Region B* in Figure 5 is the region that now has contracting. This new contracting opportunity



if $0 < \gamma \leq \dfrac{(4\beta - \alpha)\,\alpha^2}{4\beta(2\beta - \alpha)}$

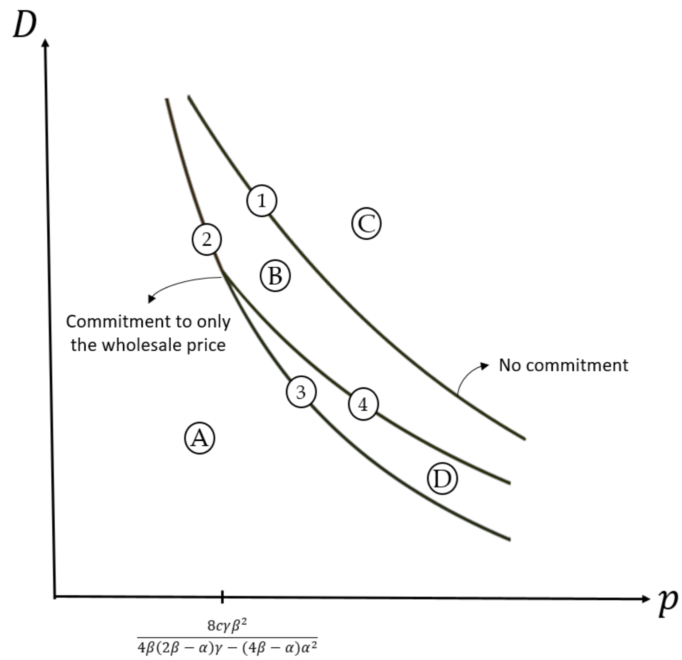
Region	e	a	O_{SC}	π_B	π_S
A	-	-	-	-	-
B	↑	↑	↑	↑	-
C	↑	-	-	↑	-

FIGURE 5. From "no-commitment" to "commitment to only the wholesale price" when $0 < \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$

where line 1 belongs to "commitment to only the wholesale price" model and represents $D = \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$. Line 2 belongs to "no-commitment" and represents $D = \frac{\beta+\gamma}{p-c}$.

also results in improving overall sustainability compliance in these two regions. The supplier profit is squeezed to zero in *Region B*. In *region C* the buyer exerts full auditing effort for both policies (hence, no change). The ensuing perfect auditing results in maximum overall sustainability compliance for both policies. The supplier's compliance increases because committing to price is an incentive for the supplier. With a commitment to only the wholesale price, the buyer experiences increased profit by being able to command a lower wholesale price (*vis-à-vis* no-commitment policy).

When auditing cost coefficient is moderate (i.e., $\frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)} < \gamma < \frac{\alpha\beta}{2\beta-\alpha}$), we observe *regions A, B, and C* with above-mentioned characteristics in Figure 6. We also observe a new region which represents the new contracting opportunity. Interestingly, in this region, *region D*, we observe a *win-win-win* situation in which



$if \quad \frac{(4\beta-\alpha)\,\alpha^2}{4\beta(2\beta-\alpha)} < \gamma < \frac{\alpha\beta}{2\beta-\alpha}$

Region	e	a	O_{SC}	π_B	π_S
A	-	-	-	-	-
B	↑	↑	↑	↑	-
C	↑	-	-	↑	-
D	↑	↑	↑	↑	↑

FIGURE 6. From "no-commitment" to a "commitment to only the wholesale price" when $\frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)} < \gamma < \frac{\alpha\beta}{2\beta-\alpha}$

where line 1 belongs to "no-commitment" model and represents $D = \frac{\beta+\gamma}{p-c}$. Other lines belong to "commitment to only the wholesale price", where line 2 represents $D = \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$, line 3 represents $D = \frac{2\beta\gamma}{p\alpha}$, and line 4 represents $D = \hat{D}(p, c, \alpha, \beta, \gamma)$.

not only does the buyer's profit improve along with an improvement in overall sustainability compliance but also the supplier makes a positive profit. The intuition for this behavior was discussed in §2.5, wherein the buyer has to set the wholesale price high enough (*ex ante*) to give the supplier incentive to participate in corrective action (*ex post*). In this case, the buyer is thus unable to squeeze out the supplier's profit to zero.

Thus, in all cases where there is contracting, we observe that advancing the level of commitment policy from *no-commitment* to *commitment to only the wholesale price* increases the buyer's profit as well as the supplier's compliance effort. However the supplier's profit only increases monotonically with this transition.

Full Commitment Policy

We investigate the effect of the buyer's full commitment (i.e., the buyer commits to both wholesale price and quantity) in this section. First , we present and analyze the "full commitment" policy. Subsequently , we compare this policy to "commitment to only the wholesale price", in order to characterize the effect of also committing to quantity when the buyer has already committed to only the wholesale price.

The Buyer's Commitment to Price and Quantity

In this section, we analyze the policy wherein the buyer commits to both the wholesale price and the order quantity in the contract (i.e., *full commitment*). The buyer now decides contract terms (i.e., the wholesale price w and order quantity q) before the buyer and the supplier exert their respective efforts. The buyer commits to $w_f = w_p = w$ and $q_f = q_p = q$ to be invoked upon execution of the contract. We analyze the model backward, by first solving for the Nash Equilibrium of the buyer's and the supplier's efforts. Then we analyze the buyer's problem to choose the optimal wholesale price and order quantity.

Sub-Game Perfect Nash Equilibrium of Auditing and Compliance

The buyer and the supplier simultaneously choose their respective efforts a and e to maximize their respective expected profits: $\Pi_B(w_p, q_p, w_f, q_f, e, a)$ and $\Pi_S(w_p, q_p, w_f, q_f, e, a)$. In the buyer's and the supplier's profit functions, $w_p = w_f = w$ and $q_f = q_p = q$ are considered as given at this stage. Details of solution procedures for finding the best response functions are relegated to the Appendix. We present the equilibrium outcome in Lemma 5.

Lemma 5. *Consider the situation wherein the buyer commits to both wholesale price and order quantity. At the equilibrium, the supplier's compliance effort* e^* *and the buyer's auditing effort* a^* *are in one of the following cases:*

- *If* $0 \leqslant q < \frac{\gamma}{p}$*, then* $(e^*, a^*) = (0, 0)$*,*
- *If* $\frac{\gamma}{p} \leqslant q \leqslant \frac{2\gamma\beta}{p\alpha}$*, then* $(e^*, a^*) = (1 - \frac{\gamma}{pq}, \frac{\alpha pq}{2\gamma\beta})$*, and*
- *If* $q > \frac{2\gamma\beta}{p\alpha}$*, then* $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$*.*

Lemma 5 shows that for a low level of order quantity, the supplier has no willingness to exert any level of compliance effort. The supplier exerts compliance effort when the order quantity is beyond a threshold (i.e., $q \geqslant \frac{\gamma}{p}$). Rearranging the above cases based on the auditing cost coefficient γ, we can see that the buyer does not exert any auditing effort when γ is beyond a threshold (i.e., $\gamma > pq$). The buyer's auditing effort increases with a decrease in γ.

Optimal Contracting Decisions

We move one step backward in time to solve for the buyer's decision of (w, q). The optimization program (2.14) shows that the buyer chooses the wholesale price w and quantity q to maximize its profit-to-go, which is presented in function (2.6), subject to the supplier's participation in the contracting. The supplier's participation constraint is determined by the supplier's profit-to-go, which is presented in function (2.8). Note that for these functions, we assume that $w_f = w_p = w$ and $q_f = q_p = q$.

$$\begin{aligned} \max_{\substack{0 \leqslant q \leqslant D \\ c \leqslant w \leqslant p}} \quad & \Pi_B(w_p, q_p, w_f, q_f, e^*, a^*) \\ \text{subject to} \quad & \Pi_S(w_p, q_p, w_f, q_f, e^*, a^*) \geq 0 \end{aligned} \tag{2.14}$$

We present the buyer's optimal decisions, along with model equilibrium, in Proposition 4.

Proposition 4. *Consider the situation wherein the buyer commits to both wholesale price and quantity. At the equilibrium, the buyer's optimal wholesale price w^* and order quantity q^*, the supplier's compliance effort e^* and the buyer's auditing effort a^*, and the buyer's and the supplier's respective total profits π_B^* and π_S^* in the following regions are:*

- *Form (i): no-contracting.*
- *Form (ii):* $(w^*, q^*) = (c + \frac{(4\beta-\alpha)\alpha}{4\beta D}, D)$, $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$, *and* $\pi_B^* > 0$, $\pi_S^* = 0$.
- *Form (iii):* $(w^*, q^*) = (c + \frac{\alpha}{D} - \frac{\alpha\gamma}{2pD^2}, D)$, $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$, *and* $\pi_B^* > 0, \pi_S^* = 0$.

Above forms are defined based on following conditions:

- $0 < \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ *or* $p < \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$:
  - *If* $0 \leqslant D < \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$, *then form (i) occurs.*
  - *Otherwise, form (ii) occurs.*
- *When* $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ *and* $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$:
  - *If* $0 \leqslant D \leqslant \tilde{D}(p, c, \alpha, \beta, \gamma)$, *then form (i) occurs.*
  - *If* $D \geqslant \frac{2\gamma\beta}{p\alpha}$, *then form (ii) occurs.*
  - *If* $\tilde{D}(p, c, \alpha, \beta, \gamma) < D < \frac{2\gamma\beta}{p\alpha}$, *then form (iii) occurs,*

where $\tilde{D}(p, c, \alpha, \gamma) = \frac{p(\alpha+\gamma)+\sqrt{p(2c\alpha\gamma+p(\alpha^2+\gamma^2))}}{2p(p-c)}$.

The graphical representation for Proposition 4 (for the *full commitment policy*) is identical to Figure 3, which provides a graphical representation for Proposition 3 (*commitment to only the wholesale price*), *with one difference*. Line 3 (as seen in Figure 3) for Proposition 4 would now represent $D = \tilde{D}(p, c, \alpha, \beta, \gamma)$, instead. Thus, after accounting for the one aforementioned difference, the conditions for the corresponding regions for Proposition 4 are identical to that for Proposition 3. The supplier's compliance effort and the buyer's auditing effort are identical in corresponding regions. The rationale for the buyer's and the supplier's efforts in corresponding regions is similar for both policies. Therefore, for the sake of brevity, we do not discuss the behavior in these regions again.

In all regions wherein contracting exists, the buyer makes a positive profit, and the supplier's profit is now squeezed out to zero. The buyer now has to only consider the *ex ante* participation constraint for the supplier and is thus able to set the wholesale price and quantity in a manner that squeezes out the supplier's profit to zero, even in *region of equilibrium form iii*.

Effect of Committing to Quantity in Addition to Price

In this section we consider the metrics that we introduced in §2.5 to compare the *commitment to only the wholesale price* policy with the *full commitment* policy. Based on Proposition 2 and Proposition 4, a comparison of *commitment to only the wholesale price* policy with *full commitment* policy for varying levels of auditing cost coefficient γ generates two cases. We present the change of metrics for these two cases in Proposition 5.

Proposition 5. *Consider the situation wherein the buyer increases the level of commitment and transitions from a "commitment to only the wholesale price" policy* *to a policy with "full commitment". The following metrics change as below for all p and D:*

When $\gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$,

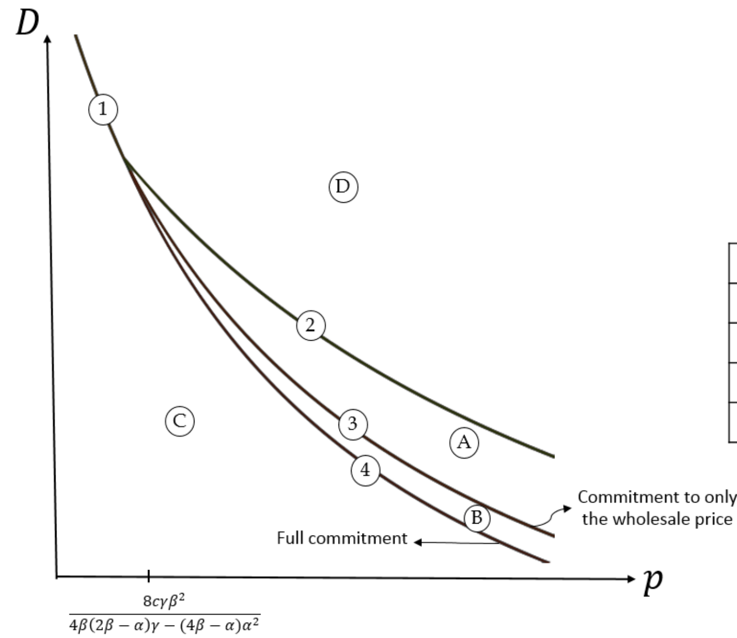
- *The supplier's compliance e, the buyer's auditing effort a, the overall sustainability compliance* O_{sc}*, and the buyer's and the supplier's respective profits* π_B*,* π_S *do not experience any change.*

When $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$,

- *The supplier's compliance e, the buyer's auditing effort a, the overall sustainability* O_{sc}*, and the buyer's profit* π_B *increase monotonically in level of commitment, as stipulated above.*

- *The supplier's profit* π_S *does not change, except it decreases when* $\tilde{D}(p, c, \alpha, \gamma) < D < \hat{D}(p, c, \alpha, \beta, \gamma)$ *and* $p > \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$.

When the level of commitment is advanced from *commitment to only the wholesale price* to *full commitment* policy, if the auditing cost coefficient γ is low, the equilibrium outcomes for the two policies are identical. Hence, this transition does not change any of the metrics considered. In Figure 7, we provide a graphic representation of Proposition 5 when γ is large. Two regions (i.e., A and B) exhibit a behavior that is of significant interest to the buyer. In region A, unlike under *commitment to only the wholesale price* policy, the buyer is now able to squeeze the supplier's profit to zero under *full commitment* policy. The compliance and auditing efforts remain unchanged. Interestingly, in this transition the buyer is able to command a lower wholesale price that leads to higher profit, even without any change in compliance or auditing efforts. This transition also helps to increase the contracting opportunity. *Region B* represents the new contracting opportunity,



$if \ \frac{(4\beta-\alpha)\,\alpha^2}{4\beta(2\beta-\alpha)} < \gamma$

Region	e	a	O_{SC}	π_B	π_S
A	-	-	-	↑	↓
B	↑	↑	↑	↑	-
C	-	-	-	-	-
D	-	-	-	-	-

FIGURE 7. From "commitment to only the wholesale price" to "full commitment" when $\frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)} < \gamma$.

Line 1 and 2 belong to both commitment policies and represent $D = \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$ and $D = \frac{2\gamma\beta}{p\alpha}$ respectively. Line 3 belongs to commitment to only the wholesale price policy and represents $D = \hat{D}(p, c, \alpha, \beta, \gamma)$. Line 4 belongs to full commitment and represents $D = \tilde{D}(p, c, \alpha, \gamma)$.

wherein both parties exert partial effort and the buyer is able to squeeze the supplier's profit to zero. The equilibrium outcomes in regions C and D are identical for both policies. Hence, this transition does not change any of the metrics in these two regions. An important counter-intuitive managerial insight for the buyer is that an ability to fully commit to contract terms (*vis-à-vis* price only) can improve the contracting opportunity and ensuing profit only when the auditing is costly.

In summary, we find that committing to quantity is valuable for the buyer only if the buyer has already committed to the wholesale price. Committing to the price and committing to the quantity are complementary strategies for the buyer. When the buyer commits to the quantity, the supplier is worse off only if the buyer has already committed to the wholesale price. Thus, in contrast, the buyer's strategies of commitment to price and commitment to quantity have the effect of being substitutes for the supplier. Lastly, the overall sustainability compliance from the buyer's commitment to quantity improves only if the buyer has already committed to the wholesale price.

We present the analysis of the effect of raising the standard of the code of conduct on sustainability and financial metrics, and ensuing discussion in the next section.

Effect of Raising the Standard for the Code of Conduct

In addition to increasing suppliers' compliance effort and overall sustainability compliance to an existing standard, brands strive to progressively improve sustainability standards for their suppliers' processes and facilities. They do so to meet the increasing expectations for sustainability in the market by increasing the scope of the code of conduct and/or strengthening the standard for the code of conduct (Nike, 2018; Apple, 2018). Next, we examine the effect of raising the standard for the code of conduct on all metrics of interest in this study.

Model Set-up and Notations

We introduce d to represent a quantifiable proxy for the standard of the code of conduct for the supplier. A higher d implies a higher standard of the code of conduct. We consider d to be a continuous variable, to model the notion of raising the standard, and also to modify the definition of cost functions associated with compliance, auditing, and correction. We modify the supplier's compliance cost to $K_s(e, d) = \alpha(d) \times e$, the supplier's correction cost to $K_c(e, d) = \beta(d) \times (1-e)$, and the buyer's auditing cost to $K_b(a, d) = \gamma(d) \times a$. We allow for more generalized definitions of cost coefficient functions and only assume that $\alpha(d)$, $\beta(d)$, and $\gamma(d)$ are increasing in d. By substituting $\alpha(d)$ for α, $\beta(d)$ for β, and $\gamma(d)$ for γ in model sections (i.e., §2.4 to §2.6) we are able to obtain the results for this new set-up using the previous analysis. Next, we develop additional insights regarding when the buyer raises the standard of the code of conduct (hereafter simply referred to as "standard").

Results and Discussions

One can see that if the standard increases, the contracting opportunity decreases for all commitment policies (i.e., region of no-contracting expands). Therefore, we focus on analyzing how an increase in the standard affects the supplier's and the buyer's effort and the overall sustainability. These metrics remain invariant to an increase in the standard under "no-commitment" and "commitment to only quantity" policies. Hence, we focus on the implications for *regions of equilibrium forms ii and iii* (i.e., where contracting occurs) under the policies of "commitment to only the wholesale price" and "full commitment". In *region of equilibrium form ii*, we analyze the implications for compliance effort to a change in standard, since only the supplier exerts partial effort in this region; and the overall sustainability is maximum, because of full auditing effort in this region. In *region of equilibrium form iii*, both the buyer and the supplier exert partial efforts, with the overall sustainability compliance being less than the maximum. Hence, for *region of equilibrium form iii*, we analyze the effect of raising the standard on the buyer's auditing, the supplier's compliance, and the overall sustainability compliance. We present the results in Proposition 6.

Proposition 6. *Consider the situation wherein the buyer commits to "only the wholesale price" or to "both wholesale price and quantity". Also, consider associated* *regions with these commitment policies (referring to regions of equilibrium forms ii & iii in Propositions 2 and 4).*

I) In region of equilibrium form ii, the supplier's compliance effort is increasing in the standard, if and only if $\frac{\beta'(d)}{\beta(d)} > \frac{\alpha'(d)}{\alpha(d)}$. *The buyer's effort is constant in the standard.*

II) In region of equilibrium form iii, the buyer's auditing effort is increasing in the standard, if and only if $0 < \frac{\gamma'(d)}{\gamma(d)} < \frac{\alpha'(d)}{\alpha(d)} - \frac{\beta'(d)}{\beta(d)}$. *Also, the supplier's compliance effort is always decreasing in the standard.*

III) The overall sustainability compliance in region of equilibrium form iii is increasing in the standard, if and only if $Dp > \frac{2\beta(d)\gamma'(d)}{\alpha(d)(\frac{\alpha'(d)}{\alpha(d)} - \frac{\beta'(d)}{\beta(d)})}$ *and* $\frac{\alpha'(d)}{\alpha(d)} > \frac{\beta'(d)}{\beta(d)}$.

In *region of equilibrium form ii*, the buyer always exerts full auditing effort. Thus, due to an increase in compliance cost and correction cost coefficients with an increase in standard, the supplier's equilibrium compliance effort has to balance any savings from a reduction in *ex ante* compliance effort with an increase in *ex post* correction cost on account of increased chance of failing the audit. Hence, the supplier's compliance effort in equilibrium decreases (increases) with an increase in standard when the compliance cost is more (less) sensitive to an increase in standard relative to the correction cost. This offers a valuable insight for the buyer as to when to expect an increase in the supplier's compliance effort with an increase in standard, without any change in auditing effort.

One can see from Propositions 2 and 4 that in *region of equilibrium form iii*, the supplier's effort decreases with an increase in the standard. The buyer can increase the auditing effort with an increase in standard only in situations wherein the supplier still finds it beneficial to participate in corrective action, despite an increase in the likelihood of failing the audit. The supplier does so when compliance cost is more sensitive to an increase in the standard relative to the correction cost. The buyer is able to do so when increased costs of auditing and the compensation for the supplier's correction are more than offset by the savings from market disruption prevented by the supplier's corrective action. Hence, the buyer's auditing effort increases in the standard, if and only if $0 < \frac{\gamma'(d)}{\gamma(d)} < \frac{\alpha'(d)}{\alpha(d)} - \frac{\beta'(d)}{\beta(d)}$.

Our analysis shows that in *region of equilibrium form iii*, the overall sustainability compliance increases with an increase in the standard, if and only if the market conditions are sufficiently favorable (i.e., the demand or the market price is high) and the sensitivity of correction cost coefficient to increase in the standard is less than that for compliance cost coefficient. The supplier finds it more beneficial to undertake corrective action if compensated for it rather than having to increase compliance effort. Only when the market is favorable enough is the buyer able to compensate the supplier for corrective action. The requisite threshold for favorability of market conditions increases as the relative difference in sensitivity of the supplier's cost coefficients decreases. The market thus needs to be even more favorable when the supplier's sensitivity for correction cost is not sufficiently lower than that to compliance cost.

When one is in the *region of equilibrium form ii*, and sufficiently far from the boundary (i.e., line $D = \frac{2\beta\gamma}{p\alpha}$), the nature of the results are not affected by an increase in the standard. However, the nature of results changes for points close to the boundary, as described in Proposition 7 below.

Proposition 7. *Consider a situation wherein the buyer commits to only the wholesale price and you are in the region of equilibrium form ii of Proposition 2. Select a point very close to and above the line* $D = \frac{2\beta\gamma}{p\alpha}$ *such that, with an infinitesimal* *increment in d, the point would now be in the region of equilibrium form iii, if and only if* $\frac{\alpha'(d)}{\alpha(d)} - \frac{\beta'(d)}{\beta(d)} < \frac{\gamma'(d)}{\gamma(d)}$.

Proposition 7 states that when the auditing cost is sufficiently sensitive to an increase in the standard, for the points considered above, the equilibrium behavior moves from *region of equilibrium form ii* to *region of equilibrium form iii* of Proposition 2 (i.e., for *committing to only the wholesale price*). This has several implications. The overall sustainability compliance decreases. The buyer is no longer able to squeeze the supplier's profit to zero.

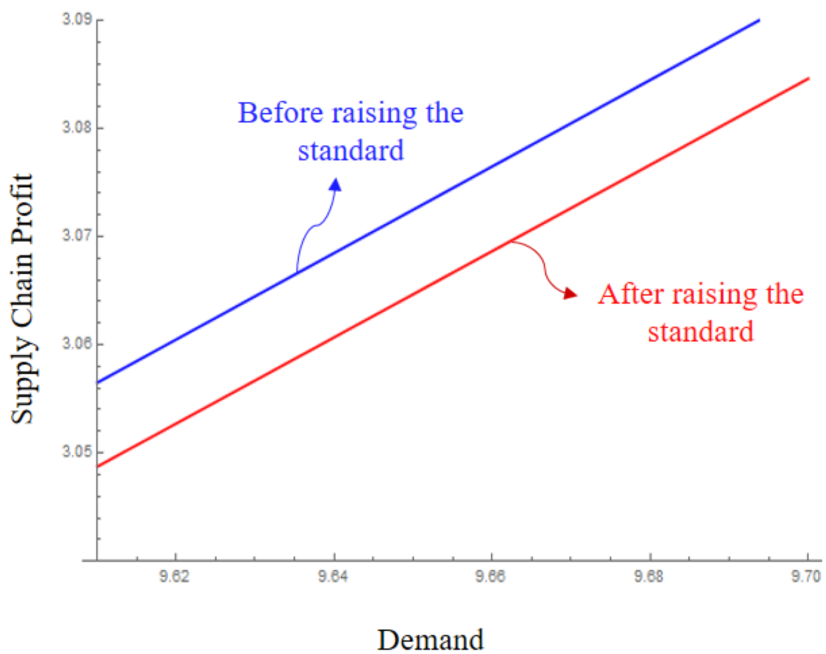


FIGURE 8. Effect of Raising the Standard of Code of Conduct on Supply chain profit

Considering linear cost function and changing the standard from $d = 1$ to $d = 1.01$, with $\alpha = 0.2, \beta = 0.8, \gamma = 0.6, c = 0.1, p = 0.5$

Additionally, even though both the buyer and the supplier now have positive profits, our numerical analysis (Figure 8) shows that the buyer is worse off, also, with a decrease in profit. We know that from Proposition 2, the supplier's profit in *region of equilibrium form ii* is zero. As we go to *region of equilibrium form iii*, it increases to a positive profit. Therefore, the buyer's profit in this transition decreases. In situations wherein correction cost is more sensitive to an increase in the standard *vis-à-vis* compliance cost, the buyer is always worse off in the above-mentioned scenario, regardless of the sensitivity of the auditing cost. However, when the compliance cost is more sensitive, then the buyer is worse off only when the sensitivity of the auditing cost is larger than the sensitivity differential between compliance and correction cost coefficients. This provides a cautionary insight for the buyer who is considering raising the standard of the code of conduct.

The proofs of all Lemmas and Propositions are provided in Appendix.

Summary & Conclusion

We investigate the efficacy of the buyer's commitment to contract terms as yet another tool for the buyer that has not been considered in the literature to enhance a supplier's social and environmental compliance. Thus, with a different focus, we investigate the effect of the interplay between degree of commitment to an endogenized wholesale contract and the buyer's auditing effort on the supplier's compliance to the code of conduct and ensuing overall sustainability compliance in the market. We consider a game-theoretic framework to investigate the impact of policies with varying degrees of commitment on the supplier's compliance, sustainability in the marketplace, and the buyer's and the supplier's profits. We solve for the subgame-perfect equilibrium and characterize the contract terms, the buyer's auditing effort, and the supplier's compliance effort at the equilibrium. We also examine the sensitivity of the supplier's compliance and overall sustainability compliance to an increase in the standard for the code of conduct.

The interplay between the buyer's auditing cost and the supplier's compliance and correction costs under varying levels of commitment provides significant managerial insights. Our results show that the commitment provides the buyer with significant opportunity to advance both social good and profit in a *win-win* manner. We find that increasing the level of commitment improves the supplier's likelihood of compliance to the sustainability standard. We show that overall sustainability compliance in the market also improves, in a great majority of cases, with an increase in the commitment level. Our results indicate that both contracting opportunity and profit for the buyer increase monotonically with the degree of commitment.

When the buyer commits to only the wholesale price and auditing is not costly, we find that the buyer is always able to justify a full auditing effort. This ensures full overall sustainability compliance in the market, resulting from the supplier's upfront compliance and any subsequent corrective action. However, when auditing is costly, the buyer can no longer justify a full auditing effort in all cases and loses the ability to ensure full overall sustainability compliance. In cases wherein the buyer exerts partial auditing effort, we find that in spite of experiencing high auditing cost, the buyer can improve the contracting opportunity and ensuing profit by fully committing to contract terms. This offers an important counter-intuitive insight for the buyer.

Interestingly, the transition from "no-commitment" to "commitment to only the wholesale price" policy *vis-à-vis* a transition to "commitment to only the quantity" has an asymmetric effect on sustainability and profit metrics. The overall sustainability compliance from the buyer's commitment to quantity improves only if the buyer has already committed to the wholesale price. With regard to profits, we find that committing to quantity is valuable for the buyer only if the buyer has already committed to the wholesale price. Committing to the price and committing to the quantity are complementary strategies for the buyer. When the buyer commits to quantity, the supplier is worse off only if the buyer has already committed to the wholesale price. Thus, the buyer's strategies of commitment to price and commitment to quantity have the effect of being substitutes for the supplier. Additionally, when the buyer transitions from "no-commitment" to "commitment to only the wholesale price" policy, we find a *win-win-win* scenario in which not only does the buyer's profit improve along with an improvement in overall sustainability compliance but also the supplier makes a positive profit. Lastly, we identify conditions in which, interestingly, the supplier's compliance and overall sustainability compliance increase with an increase in the standard for the code of conduct. We also provide a cautionary insight for the buyer who is considering raising the standard of the code of conduct. Going beyond our focus in this study, it will be beneficial for future research to examine the effect of competition on the efficacy of commitment.

Bridge to Next Chapter

In this chapter, we consider a buyer who enjoys the pricing power and also has the ability to commit to contract terms. We investigate how such a buyer's commitment to contract terms affects the sustainability and financial performance of the supply chain. In the next chapter, we consider a different scenario wherein the buyer and the supplier(s) have more parity in their contracting power, with the supplier offering the price. Hence, the buyer's power is limited. To consider a more realistic business environment, we also introduce the supplier competition to our study.

CHAPTER III

IMPACT OF SUPPLIER COMPETITION ON SUPPLIER'S SOCIAL AND ENVIRONMENTAL COMPLIANCE

Introduction

In the previous chapter, we investigated the role of buyer's commitment to contract terms in enhancing supplier's social and environmental compliance. This represented buyer-supplier dyads wherein the buyer enjoys significant pricing power in determining the wholesale price and order quantity for the supplier. These same buyers may not enjoy such pricing power across their other business units. For example, while major brands in the footwear industry such as Nike and Adidas enjoy significant pricing power on their captive suppliers in the footwear industry, these brands share their suppliers for their apparel business with several competitors. Hence, there is more pricing parity enjoyed by their apparel suppliers. Smaller and upcoming brands do not even enjoy the pricing power with their major suppliers since they often account for a small fraction of their supplier's business. Thus, it is important to understand how a buyer can influence the supplier to be more sustainable when a supplier enjoys the pricing power. In this situation, the buyer can induce competition between suppliers in conjunction with the use of auditing to influence sustainability of suppliers.

Hence, the second essay focuses on understanding the impact of supplier competition on the buyer's ability to influence suppliers' compliance when suppliers have more parity in the contracting power. Unlike the first essay, wherein the buyer stipulates both price and quantity, this essay considers situations wherein the supplier offers a wholesale price and the buyer is limited to only offering the quantity in a wholesale contract. Depending on whether auditing precedes contracting or follows contracting, it determines whether suppliers first compete on their compliance efforts or wholesale prices. We develop a framework to investigate if and how the sequence in which the supplier competition manifests, influences buyers auditing effort, suppliers compliance efforts, and financial performance of parties. Based on the above discussion, we ask following research questions:

1) How does competition between suppliers affect the supplier's compliance to a code of conduct and the financial performance of the buyer and suppliers?

2) How do timing of contracting and auditing affect the buyer's auditing, the suppliers' compliance, and the financial performance of the parties?

We introduce a framework to investigate the above research questions. This framework has two dimensions. In one dimension, we consider the timing of contracting: Either auditing precedes contracting or auditing follows contracting. The second dimension presents the structure of supplier base: Single supplier (as a benchmark) or two competing suppliers. When we consider both dimensions together, it creates four different scenarios. We show these four scenarios in Figure 9. The single supplier model is a benchmark model (i.e., we have one buyer who sources from a supplier) under each timing. We also consider under each timing the scenario wherein the buyer can source from two competing suppliers. We can compare these four scenarios in two different ways. First in each timing of contracting, we compare the benchmark model with the supplier competition model. This comparison shows the effect of supplier competition on the supplier's compliance, the buyer's auditing, and the financial performance of parties. The second comparison happens within each structure of supplier base. For instance in a single supplier, we compare model 1 with model 3. It shows the effect of timing of contracting on the supplier's compliance, the buyer's auditing, and the financial performance of parties. By analyzing this framework, we provide a guideline to a buyer who does not have the leverage to set the wholesale price, but he can manage the timing of contracting or the structure of supplier base.

		Timing of Contracting	
		Auditing → Contracting	Contracting → Auditing
Supplier Base	Single Supplier	Model (1)	Model (3)
	Two Suppliers	Model (2)	Model (4)

FIGURE 9. Comprehensive Research Framework

In this chapter, we present and analyze models for when auditing precedes contracting. The analysis for the scenario wherein the contracting precedes auditing is in progress. We present multi-stage games to analyze models analytically. We solve for the subgame-perfect equilibrium and characterize the contract terms, the buyer's auditing effort, and the supplier's compliance effort at the equilibrium.

The remainder of this chapter is organized as follows. First, we review the relevant literature. Then, we introduce the model set-up. After that, we analyze the single supplier scenario wherein auditing precedes contracting. Next, we investigate the effect of supplier competition in the above case wherein the auditing precedes contracting. Then, we compare the above mentioned scenarios to understand the effect of competition when auditing precedes contracting. Finally, we provide concluding remarks.

Literature Review

Our paper is related to research focused on examining the efficacy of sourcing strategies, supply chain structures, incentives and penalties in contract design, auditing schemes, and competition between suppliers as levers for enhancing suppliers' social and environmental responsibility. We have already presented the literature review for the efficacy of sourcing strategies, supply chain structures, incentives and penalties in contract design, and auditing schemes in the previous chapter. Here, we review the related literature for competition and point out our differentiation with it.

One stream of research has focused on investigating the effect of suppliers' competition in influencing a supplier's social or environmental compliance. Supplier competition has been addressed in a variety of papers such as Cachon and Zhang (2007) and Jin and Ryan (2012). What differentiates our research from these papers in supply chain management literature is that the wholesale price decision is a decision for suppliers; and more importantly, the compliance effort of the supplier is a continuous decision variable, and the consequence of violation is not modeled like defects in quality literature. In sustainability literature, Karaer et al. (2017) study the effect of offering a wholesale price premium by the buyer and sharing the cost of a supplier's sustainability effort in enhancing the supplier's ability to produce a more environmentally responsible product. Under a single supplier setting, they find that if it is optimal for the buyer to offer a price premium, then the buyer also fully subsidizes the supplier's cost for investing in environmental quality. Under supplier competition, they assume that suppliers are identical except production cost. Also, they assume that there already exists a wholesale price contract between the buyer and the supplier, and the buyer is only willing to share costs with the incumbent supplier. However, they do not consider the order quantity allocations and auditing in their study. Also, the buyer offers the wholesale price in this setting. They find that as the competition is introduced, cost-sharing is less effective as a lever. Agrawal and Lee (2017) study the effect of sourcing policies in influencing a supplier's decision to adopt sustainable practices. They also investigate the effect of competition between suppliers who invest in the cost of sustainability. Under this setting, suppliers do not have the pricing power and the buyer can choose different sourcing policies (i.e., sustainable or conventional). They find that the buyer does not find it profitable to deter a supplier in the presence of supplier competition. However, in our study the suppliers compete together to quote wholesale prices. Also, we consider the auditing as a tool that buyer can use to detect any noncompliance.

In contrast to the extant literature, we focus on understanding the efficacy of timing of contracting, and of competition between suppliers who have the pricing power, on enhancing sustainability compliance in the market. To the best of our knowledge, this issue has not been studied in the sustainability literature. Thus, with a different focus, we investigate the effect of the interplay between different timing of contracting and competition to an endogenized wholesale contract, and the buyer's auditing effort, on the supplier's compliance to the code of conduct. We solve for the subgame-perfect equilibrium and characterize the contract terms, the buyer's auditing effort, and the supplier's compliance effort at the equilibrium.

Model Setting

We consider a supply chain in which a buyer sources a product from two suppliers and then resells the product to the consumer market. It costs c_i per unit for the supplier i to manufacture the product, where i=1,2. Suppliers are not symmetric in production cost, and the second supplier is more costly; i.e., $c_1 < c_2$. The buyer orders quantity q_i to supplier i where $i = 1, 2$. The market for the product has the demand size of D and the selling price of p. For focusing on the sustainability aspect, we assume p to be exogenous and D to be deterministic. The buyer and suppliers coordinate the business by a simple wholesale price contract. This contract has a wholesale price and an order quantity. We assume the buyer's order quantity satisfies $0 \leq q_i \leq D$. Two suppliers compete by quoting wholesale prices to the buyer. The wholesale price that the supplier i offers to the buyer is w_i, where $i = 1, 2$. We assume that $c_i \leq w_i \leq p$.

The buyer uses a supplier code of conduct to set the social and environmental standards for suppliers. The buyer audits suppliers for social and environmental compliance based on the code of conduct. Consumers, the buyer, and suppliers have the same understanding of what defines a violation of this social and environmental code of conduct. The customers expect that the buyer sources from compliant suppliers. If the buyer sources from a non-compliant supplier, the market will react negatively to this buyer's practice.

We assume that suppliers are initially non-compliant and that they can exert efforts to be compliant with the code of conduct. The outcome of supplier's compliance effort is random: The supplier gets to be compliant with the probability e_i and gets to continue to be non-compliant with the probability $1 - e_i$. We can explain with this perspective: That a higher compliance effort leads to a higher probability of being compliant. This is consistent with literature, where probability e_i is used as a proxy for the supplier's compliance effort (Plambeck and Taylor, 2016). The supplier incurs the compliance effort cost $K_s(e_i)$ as soon as the supplier exerts the compliance effort e_i. The cost $K_s(e_i)$ for all $e_i \in [0, 1]$ is continuous and strictly increasing in e_i with $K_s(0) = 0$ and is differentiable.

The buyer cannot observe the suppliers' true compliance status. Therefore, the buyer has to audit suppliers to learn about suppliers' compliance status, but only imperfectly. The supplier does pass the audit successfully, if the supplier is compliant with the code of conduct. If the supplier is not compliant with the code of conduct, the auditing process will have a random outcome: With probability a_i the buyer catches the non-compliant supplier, and the supplier fails the audit. Also, with probability $1 - a_i$, the buyer is not able to find any noncompliances, and the supplier passes the audit. The buyer's probability of accurately identifying a non-compliant supplier increases with the level of auditing effort. The buyer's auditing effort has the cost of $K_b(a_i)$, which is continuous and strictly increasing in $a_i \in [0, 1]$, where $K_b(0) = 0$ and is differentiable.

After suppliers exert their compliance efforts, and the buyer's auditing is done, the buyer observes "pass" with probability $e_i + (1 - e_i)(1 - a_i)$ or "fail" with probability $(1 - e_i)a_i$. We assume that the outcome of compliance effort is not observable by the supplier, unless the supplier exerts zero or full effort. We come up with two possible explanations for this. One reason is that the supplier lacks the ability to audit internally. Perhaps another way to think about it is that a suppliers effort includes both the compliance effort and any associated internal auditing to reveal compliance status. However, the suppliers effort still only provides him with a chance of compliance, and not the true state, as it is never perfect unless one exerts full effort. We show the probability tree related to buyer-supplier interaction in Figure 10.

Furthermore, if the buyer's audit successfully catches a non-compliant supplier, the buyer will not sign a contract with the non-compliant supplier. Instead, the buyer

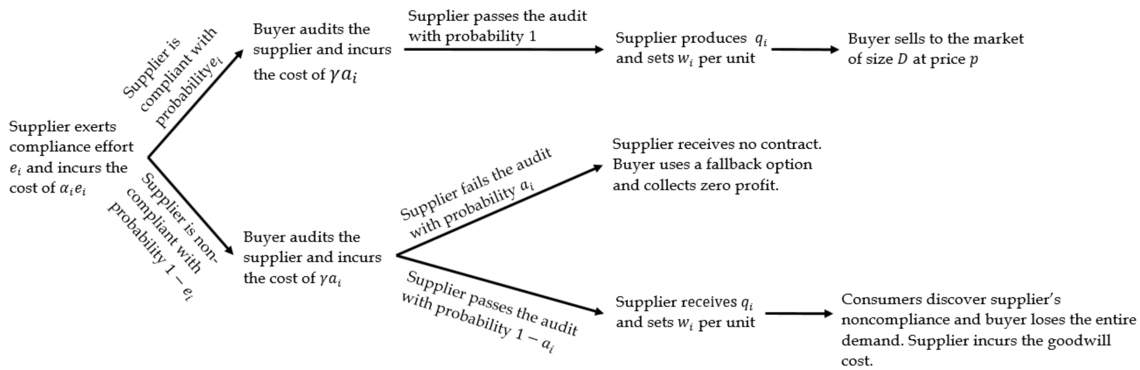


FIGURE 10. Probability Tree of Buyer-Supplier Interaction

invokes a costly fall-back option of cost C, and the buyer collects $(p - C)D$ where $c_i < C \leq p$. We assume that $C = p$, and that, therefore, the buyer's reservation profit is zero. If a non-compliant supplier passes the audit, then consumers will identify non-compliance of the supplier with the probability of 1 in a timely manner via media or NGO reports. The market disruption is the reaction of consumers to buyer's sourcing from a non-complaint supplier. Since our focus is on examining the effect of competition, we make a simplifying assumption that the buyer loses all demand D, with probability 1, due to market disruption. If the buyer sources from two suppliers, and if at least one of them is not complaint, the buyer will lose the whole market. The non-complaint supplier who gets the order incurs a goodwill cost GC.

Our model presents a one-period problem, in which the buyer and suppliers contract for a one-time supply and take a sequence of actions. We divide the sequence of actions into two stages: auditing stage and contracting stage. Auditing stage has two phases. In the first phase of the auditing stage, the buyer exerts auditing efforts a_1 and a_2 to audit each supplier. In the second phase of the auditing stage, suppliers choose their compliance efforts e_1 and e_2 simultaneously. In the contacting stage, first suppliers compete together to quote wholesale prices to the buyer. In the second phase of contracting stage, the buyer allocates order quantities q_1 and q_2 to each supplier.

For tractability, we assume that the buyer's auditing cost, $K_b(a)$ and the supplier's compliance effort cost, $K_s(e)$ are linear in $a \in [0,1]$ and in $e \in [0,1]$ respectively. We assume $K_b(a) = \gamma \times a_i$ and $K_s(e) = \alpha_i \times e_i$, where γ and α_i are cost coefficients and positive constants for $i = 1,2$. We assume that it is less costly to exert compliance effort for the supplier whose production cost is more expensive, i.e., $\alpha_1 > \alpha_2$. Also, we assume that the goodwill cost is pretty high in comparison with the supplier's effort cost, when the supplier exerts full compliance effort, i.e., $GC >> \alpha_i$. In addition the goodwill cost is bigger than maximum profit channel; and subsequently is bigger than the supplier's potential profit; i.e., $GC >> D(p - c_i) >> q_i(w_i - c_i)$.

As we mention in our Introduction section, we define four different scenarios, so that we can show the effect of competition and timing of contracting on suppliers' compliance, buyer's auditing efforts, and financial metrics. In these models, we investigate the effect of different timing of contracting (i.e., auditing stage before or after contracting stage), on the buyer's auditing and profit, and on the suppliers' compliance effort and profit. We define a benchmark model that has the same timing of events just with one supplier. We compare the benchmark model with the competition model which has two suppliers with the same sequence of events. In the next sections, first we present the benchmark model for the Auditing precedes Contracting sequence. Next, we present the competition model for this sequence of action.

Benchmark Model with a Single Supplier When Auditing Precedes Contracting

We first analyze the scenario wherein the auditing happens prior to the contracting stage. That is, the buyer decides the auditing effort, and then the supplier chooses the compliance effort. Next, in the contracting stage, the supplier sets the wholesale price, and in the second phase, the buyer sets the order quantity before the execution of the contract. Figure 13 shows the sequence of events for this model.

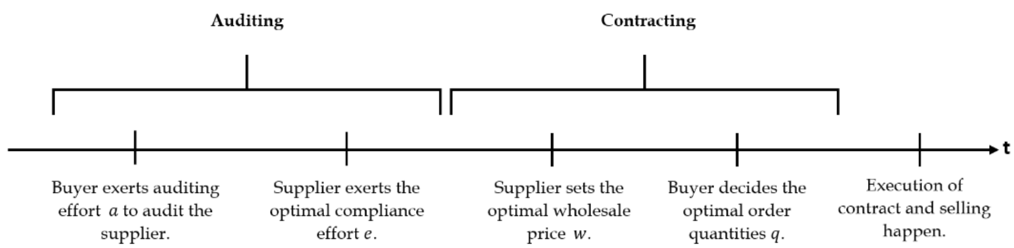


FIGURE 11. Sequence of Actions in Single Supplier for Contracting Follows Auditing.

We use the same parameters that we define in the model setup and omit the indices of two suppliers, because this model is a single supplier model, i.e., $e_i = e$, $a_i = a$, $w_i = w$, and $c_i = c$. We use this single supplier scenario as a benchmark for further analysis. Notice that in this benchmark model that considers a single supplier, one can still consider either a supplier who has a lower cost of production and higher compliance cost or vice versa. We solve this benchmark model backward. We start the analysis with the optimal order quantity decision.

Optimal Order Quantity Decision

We first analyze the buyer's ordering decision in the contracting stage, which happens after the supplier's wholesale price decision. The buyer receives the signal of "pass" or "fail" from the auditing stage. Therefore, the buyer is faced with two possible situations: when the supplier passes and when the supplier fails the audit. We define $\theta = Pr(Complaint|Pass)$. First, we present the buyer's profit function when the supplier passes the audit.

$$\pi_B^P(q) = \theta(p - w)q + (1 - \theta)(-wq) \tag{3.1}$$

The buyer's profit function in (3.1) states that if the supplier who passes the audit is complaint, the buyer makes profit of $(p - w)q$. If the supplier who passes the audit is not compliant, then the buyer incurs the cost of sourcing. The buyer chooses the order quantity q to maximize its profit:

$$\max_{0 \leq q \leq D} \quad \pi_B^P(q) \tag{3.2}$$

When the supplier fails the audit, the buyer does not order, therefore the buyer makes zero profit. We assume that the buyer uses a fall-back option to meet the demand. Lemma 6 presents the optimal order quantity decision of the buyer.

Lemma 6. *Consider that the auditing precedes the contracting and the buyer faces a single supplier. The buyer's optimal order quantity is:*

- *When the supplier passes the audit,*
  - *if $\theta p \geq w$ then the buyer's optimal order is $q^* = D$,*
  - *otherwise the buyer's optimal order is $q^* = 0$.*
- *When the supplier fails the audit, the buyer does not order.*

Proof of Lemma 6. The objective function in the optimization problem (3.2) is linear in q. We simplify it and it gives us $(\theta p - w)q$. When $\theta p - w \geq 0$ then the buyer orders $q^* = D$, otherwise $q^* = 0$. □

Lemma 6 shows that when the supplier passes the audit and the supplier offers a low wholesale price (less than the threshold), the buyer has the incentive to order and sets the order as high as demand. When the buyer observes that the supplier passes the audit and the supplier offers a high wholesale price, the buyer does not order. Also, if the supplier fails the audit, the buyer will choose the fall-back option for sourcing, and the contracting does not happen with the supplier. In the next section we analyze the supplier's optimal wholesale price decision.

Optimal Wholesale Price Decision

We go one step back in time and analyze the supplier's problem for the wholesale price. The supplier's profit function depends on whether the supplier passes or fails the audit. If the supplier passes the audit, the profit function is:

$$\pi_S^P(w) = \theta(w-c)q + (1-\theta)\begin{cases}(w-c)q - GC & ,q>0\\ 0 & ,q=0\end{cases} \tag{3.3}$$

In the supplier's profit function (3.3), if the supplier passes the audit and he is complaint, then the supplier will collect $(w-c)q$. If the supplier passes the audit while he is not complaint, then consumers discover the noncompliance with the probability one and the supplier incurs a goodwill cost in that case, where he receives a positive order. In this phase of the contracting stage, the supplier chooses the wholesale price

w to maximize its profit.

$$\begin{aligned} \max_{c \leq w \leq p} \quad & \pi_S^P(w) \\ \text{subject to} \quad & \pi_S^P(w) \geq 0, \end{aligned} \tag{3.4}$$

Lemma 7 presents the supplier's optimal wholesale price under two cases of pass and fail.

Lemma 7. *Consider that the auditing precedes the contracting and the buyer faces a supplier. The supplier's optimal wholesale price decision is:*

- *When the supplier passes the audit, if $\theta < \frac{cD+GC}{Dp+GC}$ then the supplier does not participate and there is no contracting. If $\theta \geq \frac{cD+GC}{Dp+GC}$, then the supplier participates and sets the wholesale price $w^* = p\theta$.*
- *When the supplier fails the audit, there is no contracting.*

Proof of Lemma 7. We simplify the supplier's profit function when the supplier passes the audit and write it as $(w - c)D - (1 - \theta)GC$. Based on the supplier's participation constraint, if $c + \frac{(1-\theta)GC}{D} > \theta p \Leftrightarrow \theta < \frac{cD+GC}{Dp+GC}$, then the supplier does not participate and there is no contracting. If $c + \frac{(1-\theta)GC}{D} \leq \theta p \Leftrightarrow \theta \geq \frac{cD+GC}{Dp+GC}$, then the supplier participates and sets the wholesale price $w^* = p\theta$. □

Lemma 7 shows that when the probability of being compliant for a supplier who passes the audit, is low, then the supplier has no willingness to participate in the contracting. When the probability of being compliant for a supplier who passes the audit, is high, the supplier participates in the contracting and sets the wholesale price in a way that squeezes out the buyer's profit. We can also explain the result based on demand in the market and goodwill cost. When the demand (goodwill cost) is high (low) the supplier has the willingness to participate in the contracting and quote a wholesale price. When the demand (goodwill cost) is low (high) the supplier knows that he cannot afford the cost of production and/or the goodwill cost in case he gets caught by the buyer. In the next section we analyze the supplier's optimal compliance effort.

Optimal Compliance Effort Decision

We move one step backward in time and solve for the supplier's compliance effort. In this phase of the auditing stage, we find the supplier's optimal compliance effort for a given buyer's auditing effort a. We use our finding in previous sections to form the supplier's expected profit in this phase. First, we present the supplier's profit function in this phase:

$$\Pi_S(e) = Pr(Pass)\pi_S^{P^*} + Pr(Fail)\pi_S^{F^*} - \alpha e \tag{3.5}$$

From our analysis in the contracting stage, we know that if the supplier fails the audit, the supplier will not receive any contract. Therefore, in profit function (3.5), the second term is zero. The supplier chooses e to maximize its profit. We show this maximization in optimization program (3.6) :

$$\begin{aligned} \max_{0\le e\le 1} \quad & \Pi_S(e) \\ \text{subject to} \quad & \Pi_S(e) \ge 0, \end{aligned} \tag{3.6}$$

Lemma 8 presents the supplier's optimal compliance effort in this phase.

Lemma 8. *Consider that the auditing precedes the contracting, and the buyer faces a supplier. The supplier's optimal compliance effort is:*

- *If* $D < \frac{\alpha}{p-c}$, *then* $e^* = 0$.

– *If* $D \geq \frac{\alpha}{p-c}$, *then* $e^* = 1$.

Proof of Lemma 8. We express the supplier's profit by plugging-in the optimal solutions from contracting stage. The supplier's profit is:

$$\Pi_S(e) = \begin{cases} -\alpha e & 0 \leq e < \frac{(1-a)\frac{cD+GC}{pD+GC}}{1-a\frac{cD+GC}{pD+GC}} \\ [e + (1-e)(1-a)][(p\theta - c)D - (1-\theta)GC] - \alpha e & \frac{(1-a)\frac{cD+GC}{pD+GC}}{1-a\frac{cD+GC}{pD+GC}} \leq e \leq 1 \end{cases} \tag{3.7}$$

The supplier's profit is linear in e, and it is continuous at $e = \frac{(1-a)\frac{cD+GC}{pD+GC}}{1-a\frac{cD+GC}{pD+GC}}$. The slope of the first piece of supplier's profit is always negative. Therefore, candidates for optimal solution are $e = 0$ or $e = 1$. The value of the function at zero is zero and at 1 is $D(p-c) - \alpha$. Therefore, if $D < \frac{\alpha}{p-c}$, then $e^* = 0$, otherwise $e^* = 1$. □

Lemma 8 shows that when the demand is high, the supplier has the willingness to exert full compliance effort. When the demand is lower than a threshold, the supplier is faced with a negative profit, as the cost of compliance is bigger than its profit. Therefore, the supplier has no willingness to exert the compliance effort. In the next section, we analyze the buyer's optimal auditing effort.

Optimal Auditing Effort Decision

We move one step backward in time and solve for the buyer's optimal auditing effort a^*. The buyer chooses a to maximize its profit. The outcome of auditing is one of three different situations. If the supplier is compliant, the supplier passes the audit. If the supplier is not compliant then either the supplier fails or passes the audit. We present the buyer's profit-to-go in (3.8).

$$\begin{aligned}\Pi_B(a) =& Pr(\text{compliant and pass})[(p-w^*)q^*] \\ &+Pr(\text{noncompliant and pass})(-w^*q^*) \\ &+Pr(\text{noncompliant and fail}) \times 0 - \gamma a\end{aligned} \tag{3.8}$$

The buyer's profit function in (3.8) is linear and decreasing in a, therefore the optimal decision for the buyer is to exert zero auditing effort. Proposition 8 presents the buyer's optimal auditing effort, along with the sub-game prefect equilibrium of the model.

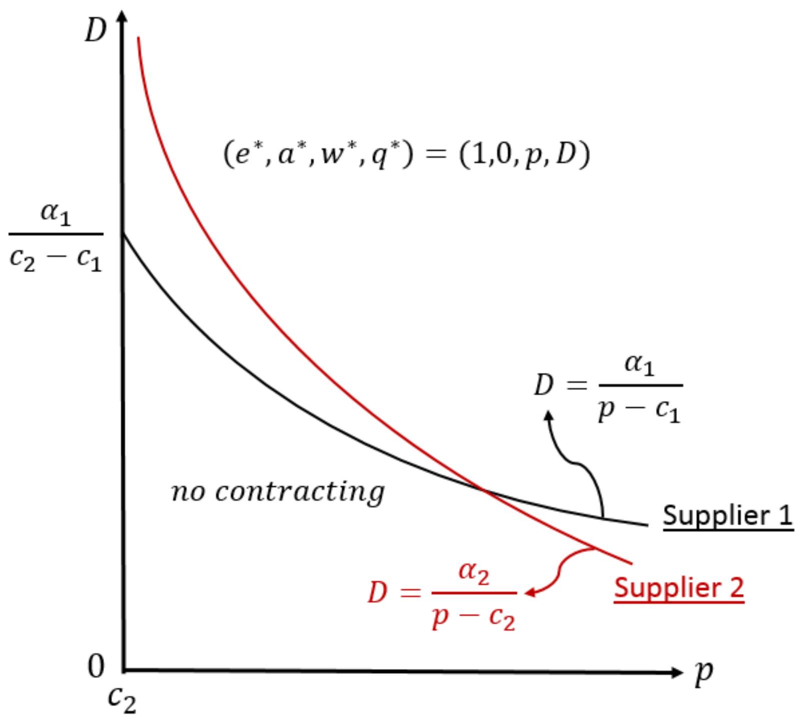


FIGURE 12. Presentation of Model Equilibrium of Single Supplier on Plane of Demand and Market Price.

Proposition 8. *Consider that the auditing precedes the contracting and the buyer faces a supplier. The buyer's optimal auditing effort for the range of all available* *parameters is $a^* = 0$. The sub-game prefect equilibrium of model for different range of parameters is:*

- *If $D < \frac{\alpha}{p-c}$, then there is no contracting.*
- *If $D \geq \frac{\alpha}{p-c}$, then $(e^*, w^*, q^*) = (1, p, D)$. Also, the supplier squeezes out the buyer's profit to zero.*

Proof of Proposition 8. We form the buyer's profit function by plugging-in the optimal solutions. The buyer's profit function for all $D < \frac{\alpha}{p-c}$ is $-\gamma a$ and for $D \geq \frac{\alpha}{p-c}$ is $(p-w)D - \gamma a$. Both functions are decreasing in a, therefore the optimal solution is $a^* = 0$. □

Proposition 8 shows that when the buyer considers a single supplier and the auditing precedes the contracting, if the demand in the market is high enough, then the supplier exerts full compliance effort, and there is no need for the buyer to exert any auditing effort. By considering the benchmark model, if the demand or market price is low, then the supplier does not participate, and the buyer does not order. Therefore, we have no contracting. When either the demand or the price is high, the supplier exerts full effort, and the buyer exerts no auditing effort. The supplier is able to squeeze out the buyer's profit to zero. In this benchmark model, when the buyer works with supplier 2 (i.e., the supplier with a higher production cost) instead of supplier 1 (i.e., the supplier with a lower production cost), we observe that the contracting opportunity increases when demand is low and price is high. We can explain this behavior by looking at the production cost and compliance cost of supplier 2. Recall that the production cost of supplier 2 is higher, and its compliance cost is lower *vis-à-vis* that for supplier 1. Therefore, with the high market price, he can collect a higher profit margin and exerts full compliance effort. We observe a similar behavior in favor of supplier 1, when the price is low and the demand is high. By switching from supplier 2 to supplier 1, the contracting opportunity increases. We present the model for supplier competition in the next section. We analyze the model backwards, similar to our process with the benchmark model.

Model with Supplier Competition When Auditing Precedes Contracting

We analyze the scenario wherein the buyer has two suppliers for sourcing the product and the auditing stage happens prior to contracting. That is, in the auditing stage, suppliers decide the compliance efforts after the buyer sets the auditing efforts. Then, in the contracting stage, suppliers first compete by quoting wholesale prices to the buyer. Subsequently, the buyer allocates the order quantities between two suppliers. Figure 13 shows the sequence of events for this model.

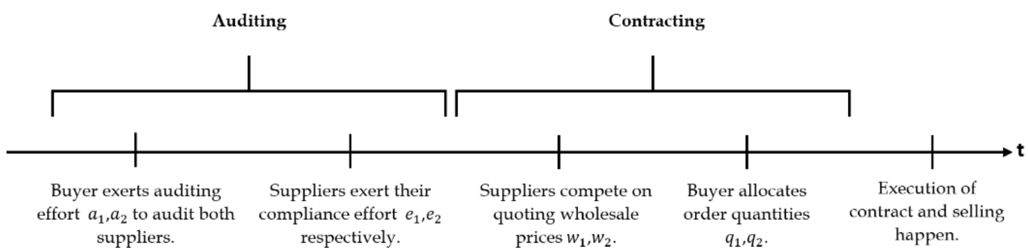


FIGURE 13. Sequence of Actions in Competition for Contracting Follows Auditing.

We analyze this model backward and solve for the sub-game perfect Nash equilibrium, starting with the buyer's order quantity decisions.

Optimal Order Quantity Decisions

We first analyze the buyer's ordering decision, which is made after auditing is concluded. Given the buyer's and the suppliers' efforts, respectively (a_1, a_2) and (e_1, e_2), the buyer allocates the order quantity q_1 to the supplier 1 and q_2 to the supplier 2. This decision depends on the outcome of auditing: Suppliers either "pass" or "fail" the audit. Therefore, we have four different scenarios, based on the outcome of auditing. First, we consider the case that both suppliers pass the audit.

When both suppliers pass the audit: We show the buyer's profit function under this case with π_B^{PP}. This profit function depends on whether the supplier who passes the audit is compliant. Based on this, we have the buyer's profit-to-go as following:

$$\begin{aligned}\pi_B^{PP}(q_1, q_2) =& \Phi_{C,C}[(p - w_1)q_1 + (p - w_2)q_2] \\ &+\Phi_{NC,C}\begin{cases}(p - w_2)q_2, & q_1 = 0, q_2 \geqslant 0 \\ -w_1q_1 - w_2q_2, & q_1 > 0, q_2 \geqslant 0\end{cases} \\ &+\Phi_{C,NC}\begin{cases}(p - w_1)q_1, & q_1 \geqslant 0, q_2 = 0 \\ -w_1q_1 - w_2q_2, & q_1 \geqslant 0, q_2 > 0\end{cases} \\ &+\Phi_{NC,NC}(-w_1q_1 - w_2q_2)\end{aligned} \tag{3.9}$$

where $\Phi_{C,C} = Pr(C, C|P, P)$, $\Phi_{NC,C} = Pr(NC, C|P, P)$, $\Phi_{C,NC} = Pr(C, NC|P, P)$, and $\Phi_{NC,NC} = Pr(NC, NC|P, P)$.

Conditional probabilities in the buyer's profit function (3.9) express the status of each supplier, given that they both pass the audit. We multiply these probabilities by the profit that the buyer collects after execution of the contract. For instance, for the second term in the buyer's profit function (3.9), the conditional probability $Pr(NC, C|P, P)$ represents the case that the first supplier who passes the audit is not compliant, and that the second supplier who passes the audit is compliant. In this case, if the buyer sources from the second supplier, the buyer will collect $(p - w_2)q_2$.

Otherwise, the buyer incurs the sourcing cost, because the market reacts to the non-compliance, and the buyer collects zero revenue.

In the case that both suppliers pass the audit, the buyer allocates order quantities of (q_1, q_2) to maximize its expected profit:

$$\begin{aligned} \max_{0\leq q_1\leq D,\, 0\leq q_2\leq D} \quad & \pi_B^{PP}(q_1, q_2) \\ \text{subject to} \quad & q_1 + q_2 \leqslant D, \end{aligned} \tag{3.10}$$

where $\pi_B^{PP}(q_1, q_2)$ is defined in (3.9). Lemma 9 summarizes the buyer's optional order quantities when both suppliers pass the audit.

Lemma 9. *Consider that the auditing precedes the contracting and the buyer faces two suppliers who both pass the audit. The buyer's optimal order allocations between suppliers are:*

- *If* $w_2 < p(\Phi_{C,C} + \Phi_{NC,C})$ *and* $w_2 \leqslant w_1 - p(\Phi_{C,NC} - \Phi_{NC,C})$*, then* $(q_1^*, q_2^*) = (0, D)$*,*
- *If* $w_1 < p(\Phi_{C,C} + \Phi_{C,NC})$ *and* $w_2 > w_1 - p(\Phi_{C,NC} - \Phi_{NC,C})$*, then* $(q_1^*, q_2^*) = (D, 0)$*,*
- *Otherwise,* $(q_1^*, q_2^*) = (0, 0)$*.*

Proof of Lemma 9. Given that the objective function and constraint in the optimization problem (3.10) are linear in q_1 and q_2, the optimal solution happens in the corner solutions. Therefore,

$$\text{if } \pi_B^{PP}(q_1, q_2)|_{(q_1,q_2)=(D,0)} > \pi_B^{PP}(q_1, q_2)|_{(q_1,q_2)=(0,0)} \text{ and}$$

$$\pi_B^{PP}(q_1, q_2)|_{(q_1,q_2)=(D,0)} > \pi_B^{PP}(q_1, q_2)|_{(q_1,q_2)=(0,D)}$$

$\Leftrightarrow w_1 < p(\Phi_{C,C} + \Phi_{C,NC})$ and $w_2 > w_1 - p(\Phi_{C,NC} - \Phi_{NC,C})$ then $(q_1^*, q_2^*) = (D, 0)$.

Similarly, we show that the buyer's optimal order quantities are $(q_1^*, q_2^*) = (0, D)$ if $\pi_B^{PP}(q_1, q_2)|_{(q_1,q_2)=(0,D)}$ is bigger than $\pi_B^{PP}(q_1, q_2)|_{(q_1,q_2)=(0,0)}$ and is also bigger than $\pi_B^{PP}(q_1, q_2)|_{(q_1,q_2)=(D,0)}$. □

We show the outcome of Lemma 9 in Figures 14 and Figure 15.

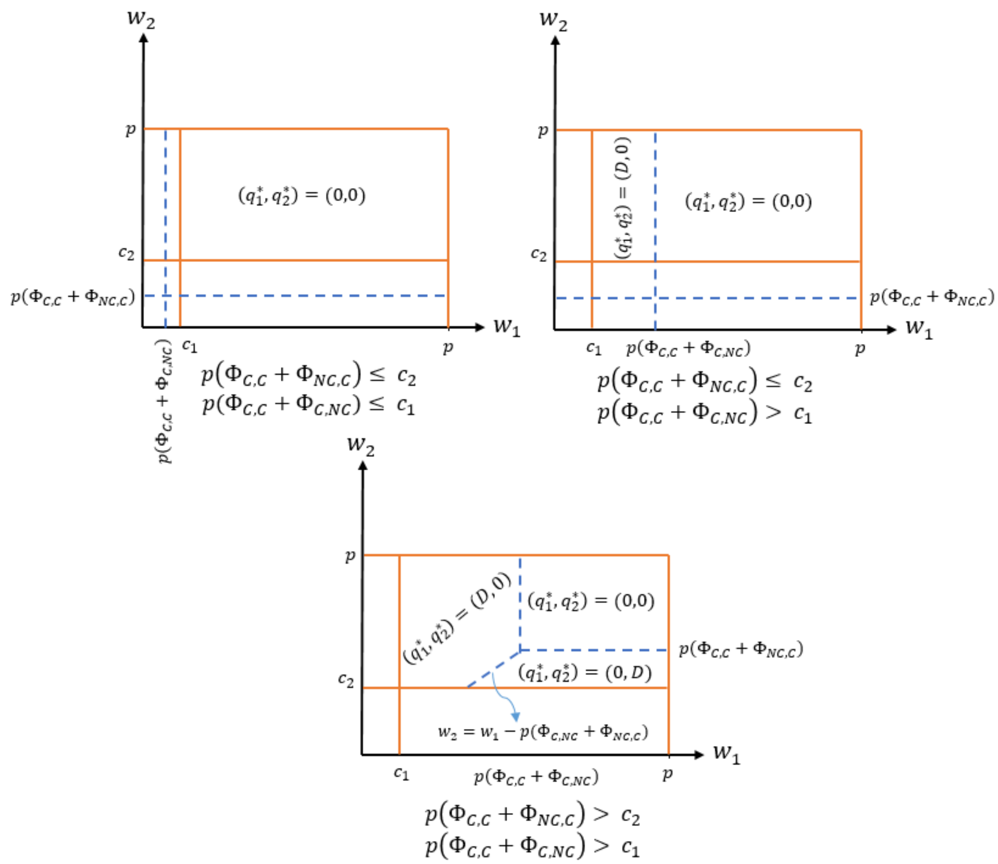


FIGURE 14. Presentation of Optimal Order Quantities on Plane of Wholesale Prices When $\Phi_{C,NC} > \Phi_{NC,C}$.

As Figure 14 shows, when the wholesale prices are high enough, the buyer has no willingness to order at all. When the wholesale price gets lower, then the buyer orders. Because we are focusing on sustainability, when both wholesale prices are equal, the buyer orders from supplier 2, whose cost of being compliant with the code of conduct is less.

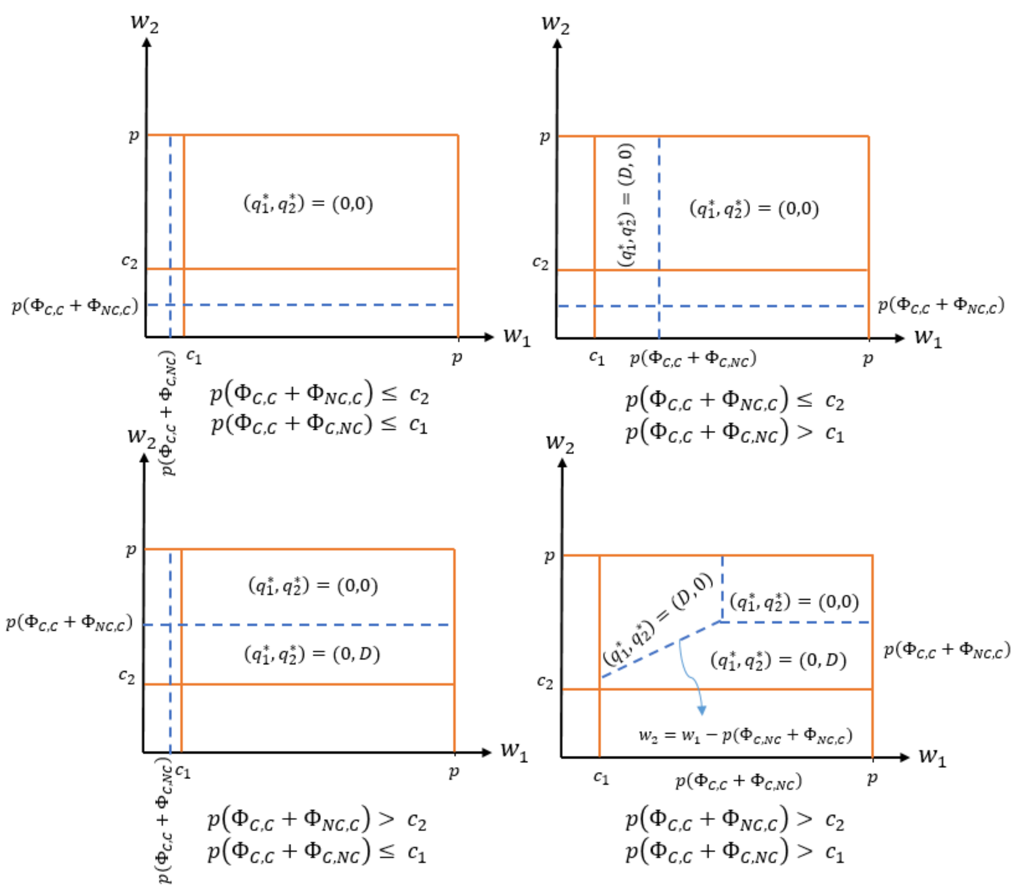


FIGURE 15. Presentation of Optimal Order Quantities on Plane of Wholesale Prices When $\Phi_{C,NC} \leqslant \Phi_{NC,C}$.

When one of suppliers fails the audit: We show the buyer's profit function under this case with π_B^{PF} or π_B^{FP}. This profit function depends on whether the supplier who passes the audit is compliant. Also, the supplier who fails the audit will not receive any contract. Based on this, we have the buyer's profit-to-go for the two cases wherein one of the suppliers fails the audit as following:

$$\pi_B^{PF} = Pr(C, NC|P, F)(p - w_1)q_1 + Pr(NC, NC|P, F)(-w_1 q_1) \tag{3.11}$$

$$\pi_B^{FP} = Pr(C, NC|F, P)(p - w_2)q_2 + Pr(NC, NC|F, F)(-w_2 q_2) \tag{3.12}$$

Since the profit functions are symmetric, we explain one of them here in detail. The first term of buyer's expected profit function (3.11) shows the profit of the buyer in case the supplier who passes the audit is compliant. In this case the buyer collects the profit from the market. The second term of profit function (3.11) shows the profit that the buyer collects when the supplier who passes the audit is not compliant. In this case, the buyer faces the market disruption and incurs the cost of sourcing.

As the two events of passing and failing the audit are independent events for each supplier, we can rewrite the conditional probabilities in the buyer's expected profit function (3.11) as following:

$$\begin{aligned}
Pr(C, NC|P, F) =& Pr(C_1|P_1) \times Pr(NC_2|F_2) \\
=& \frac{Pr(\text{Supplier 1 is Compliant } \& \ Pass)}{Pr(Pass)} \times \\
& \frac{Pr(\text{Supplier 2 is Non-Compliant } \& \ Fail)}{Pr(Fail)} \\
=& \frac{Pr(\text{Supplier 1 is Compliant } \& \ Pass)}{Pr(Pass)} \times 1 \\
=& Pr(C_1|P_1)
\end{aligned}$$

Therefore, we can rewrite the buyer's expected profit in (3.11) and (3.12) as following:

$$\pi_B^{PF}(q_1) = \theta_1(p - w_1)q_1 + (1 - \theta_1)(-w_1 q_1) \tag{3.13}$$

$$\pi_B^{FP}(q_2) = \theta_2(p - w_2)q_2 + (1 - \theta_2)(-w_2 q_2) \tag{3.14}$$

where $\theta_1 = Pr(C_1|P_1)$, $1 - \theta_1 = Pr(NC_1|P_1)$, $\theta_2 = Pr(C_2|P_2)$, and $1 - \theta_2 = Pr(NC_2|P_2)$.

In the case that one of the suppliers fails the audit, the buyer allocates the order quantity of q_i to the supplier who passes the audit to maximize its expected profit:

$$\max_{0 \leq q_i \leq D} \quad \pi_B^{PF}(q_i) \tag{3.15}$$

We define a new symbol for the conditional probabilities in $\pi_B^{PF}(q_1)$ and $\pi_B^{FP}(q_2)$. Lemma 10 presents the buyer's optimal order quantity decision when one of the suppliers fails the audit.

Lemma 10. *Consider that the auditing precedes the contracting and the buyer faces two suppliers and one of the suppliers fails the audit. The buyer's optimal order allocation decision is:*

- *When the supplier 2 fails the audit:*
  - *if* $w_1 > \theta_1 p$ *then* $(q_1^*, q_2^*) = (0, 0)$.
  - *Otherwise,* $(q_1^*, q_2^*) = (D, 0)$.
- *When the supplier 1 fails the audit:*
  - *if* $w_2 > \theta_2 p$ *then* $(q_1^*, q_2^*) = (0, 0)$.
  - *Otherwise,* $(q_1^*, q_2^*) = (0, D)$.

Proof of Lemma 10. We show the proof for one of cases, because they are symmetric. We simplify the buyer's objective function $\pi_B^{PF}(q_1)$. It gives us $(\theta_1 p - w_1)q_1$. This function is linear in q_1, therefore when the coefficient is positive $q_1^* = D$, otherwise $q_1^* = 0$. Also, the $q_2^* = 0$ because the second supplier fails the audit. □

Lemma 10 shows that when the wholesale price is higher than a threshold, then the buyer has no willingness to order at all. When the wholesale price is lower than the threshold, the buyer can make a non-negative profit, and subsequently, the buyer orders from the supplier who passes the audit.

When both suppliers fail the audit: In this case the buyer will not order at all from either supplier. Based on our assumption, in this case, the buyer selects his fall-back option and collects zero profit. In the next section, we present the optimal wholesale price decisions of suppliers.

Optimal Wholesale Price Decisions

Now, we move one step backward in time and solve for the Nash Equilibrium of suppliers' wholesale prices. In fact, both suppliers compete together for quoting the best wholesale price to the buyer. Each supplier chooses a wholesale price to maximize its profit for a given competitor's wholesale price. In this phase of the contracting stage, we find each supplier's best response for a given set of buyer's auditing efforts (a_1, a_2), the suppliers' compliance efforts (e_1, e_2), and the buyer's optimal order quantities (q_1^*, q_2^*). This decision depends on the outcome of auditing: Suppliers either "pass" or "fail" the audit. Therefore, we have four different scenarios based on the outcome of auditing. First, we consider the case that both suppliers pass the audit.

When both suppliers pass the audit: In this scenario, the supplier's profit depends on whether the supplier who passes the audit is compliant. If the supplier is compliant, then he will collect his profit. If the supplier who passes the audit is not compliant, then he has to pay the goodwill cost. The supplier incurs the goodwill cost if and only if the supplier receives a non-negative order from the buyer. Based on this scenario, each supplier's expected profit-to-go is:

$$\pi_{s_i}^{PP} = \theta_i(w_i - c_i)q_i^* + (1-\theta_i)\begin{cases}(w_i - c_i)q_i^* - GC, & q_i^* > 0 \\ 0, & q_i^* = 0\end{cases}. \tag{3.16}$$

Passing and failing the audit for each supplier is an independent event. Therefore, we can rewrite the conditional probabilities in (3.16): $Pr(C_i|P,P) = Pr(C_i|P_i) = \theta_i$ and $Pr(NC_i|P,P) = Pr(NC_i|P_i) = 1-\theta_i$. Each supplier chooses w_i to maximize its profit. We solve for the best response function for each supplier by running the following optimization:

$$\begin{aligned} \max_{c_i \leq w_i \leq p} \quad & \pi_{S_i}^{PP}(w_i) \\ \text{subject to} \quad & \pi_{S_i}^{PP}(w_i) \geqslant 0, \end{aligned} \tag{3.17}$$

We solve for the supplier's best response while we pay attention to supplier's participation constraint in optimization program (3.17). If $q_i^* = 0$, then based on supplier's objective function we have no contracting. If $q_i^* = D$, then from supplier's participation constraint we can find the minimum acceptable wholesale price that the supplier can offer. This wholesale price threshold is $c_i + \frac{(1-\theta_i)GC}{D}$. We define this threshold as $\Omega_i = c_i + \frac{(1-\theta_i)DC}{D}$. Therefore, the structure of Nash Equilibrium of suppliers' wholesale price competition has four elements. The NE shows whether each supplier participates or not, and if the supplier participates, then what wholesale price is offered by the supplier. We first solve for each supplier's best response function. Then we intersect the best response functions to find NE. Lemma 11 presents Nash Equilibrium of Suppliers' wholesale prices. Before we present NE, here we show the relationship between θ_i, $\Phi_{C,C}$, $\Phi_{C,NC}$, and $\Phi_{NC,C}$:

$$\Phi_{C,C} + \Phi_{C,NC} = Pr(C_1|P_1) = \theta_1$$
$$\Phi_{C,C} + \Phi_{NC,C} = Pr(C_2|P_2) = \theta_2$$

Lemma 11. *Consider that the auditing precedes the contracting and the buyer faces two suppliers and both of suppliers pass the audit. The suppliers' optimal decision whether to participate or not and in case of participation suppliers' optimal wholesale prices are:*

- *If* $\theta_1 < \frac{c_1 D+GC}{Dp+GC}$ *and* $\theta_2 < \frac{c_2 D+GC}{Dp+GC}$, *then both suppliers do not participate and there is no contracting.*
- *If* $\theta_1 < \frac{c_1 D+GC}{Dp+GC}$ *and* $\theta_2 \geqslant \frac{c_2 D+GC}{Dp+GC}$, *then supplier 1 does not participate and Supplier 2 participates and offers* $w_2^* = p\theta_2$.
- *If* $\theta_1 \geqslant \frac{c_1 D+GC}{Dp+GC}$ *and* $\theta_2 < \frac{c_2 D+GC}{Dp+GC}$, *then supplier 1 participates and offers* $w_1^* = p\theta_1$ *and supplier 2 does not participate.*
- *If* $\theta_1 \geqslant \frac{c_1 D+GC}{Dp+GC}$ *and* $\theta_2 \geqslant \theta_1 + \frac{D(c2-c1)}{DP+GC}$, *then both suppliers participate and optimal wholesale prices are* $(w_1^*, w_2^*) = (\Omega_1, \Omega_1 - p(\theta_1 - \theta_2))$.
- *If* $\theta_2 \geqslant \frac{c_2 D+GC}{Dp+GC}$ *and* $\theta_2 < \theta_1 + \frac{D(c2-c1)}{DP+GC}$, *then both suppliers participate and optimal wholesale prices are* $(w_1^*, w_2^*) = (\Omega_2 - p(\theta_2 - \theta_1), \Omega_2)$.

Proof of Lemma 11. We present the proof of this proposition in the Appendix. □

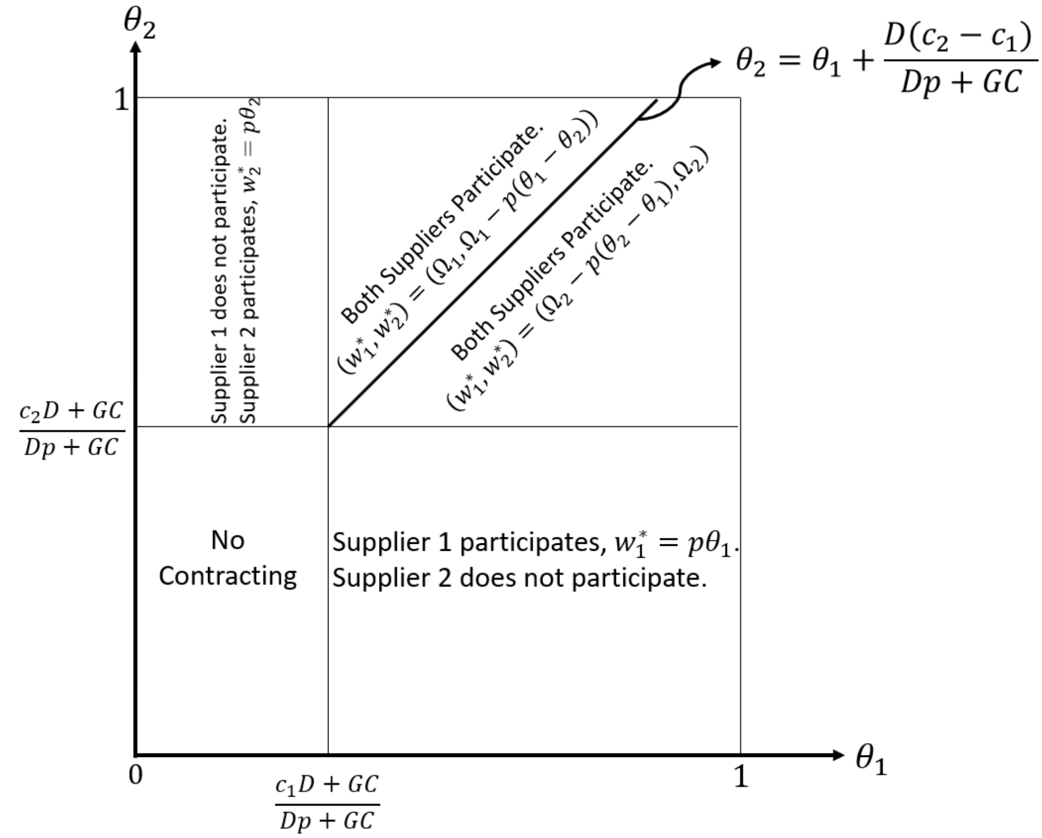


FIGURE 16. Presentation of Nash Equilibrium of Wholesale Prices on Plane of θ_1 and θ_2.

The Figure 16 is a graphical representation of Lemma 11. Given that both suppliers pass the audit we do not observe any contracting when the chance of being compliant with the code of conduct is low for both suppliers. When the chance of being compliant is low for one of the suppliers but high for the other, then the former supplier does not participate in the contracting, and the latter offers a wholesale price in a way so as to squeeze out the buyer's profit to zero.

Given that both suppliers pass the audit, when the chance of being compliant with the code of conduct is high for both suppliers, they compete to quote a lower wholesale price in order to price out the other supplier. We see that when the chance of being compliant with the code of conduct for supplier 2 is relatively higher than for supplier 1, then supplier 2 sets its wholesale price a little lower than the minimum participation price of supplier 1 to price out the first supplier. On the other hand, when the chance of being compliant with the code of conduct is higher for supplier 1, then supplier 1 sets the wholesale price a little lower than the minimum participation price of supplier 2, to price out supplier 2 and receive the entire order from the buyer.

When one of the suppliers fails the audit: In this scenario, the supplier's profit depends on whether the supplier who passes the audit is compliant. If the supplier is compliant, then he will collect his profit. If the supplier who passes the audit is not compliant, then he has to pay the goodwill cost. The supplier incurs the goodwill cost if and only if the supplier receives a non-negative order from the buyer. Also, the supplier who fails the audit does not receive any contract. We show the profit function for the case where the supplier 1 passes the audit and the supplier 2 fails the audit. The other case when supplier 1 fails the audit is similar to this case. Based on this scenario, the supplier's expected profit-to-go is:

$$\pi_{s_1}^{PF} = \theta_1(w_1 - c_1)q_1^* + (1-\theta_1)\begin{cases}(w_1 - c_1)q_1^* - GC & , q_1^* > 0 \\ 0 & , q_1^* = 0\end{cases}. \qquad (3.18)$$

The supplier chooses w_1 to maximize its profit-to-go. The optimization program below shows the supplier's profit maximization:

$$\begin{aligned} \max_{c_1 \le w_1 \le p} \quad & \pi_{S_1}^{PF}(w_1) \\ \text{subject to} \quad & \pi_{S_1}^{PF}(w_1) \geqslant 0, \end{aligned} \qquad (3.19)$$

Lemma 12 presents the optimal wholesale price for the supplier who passes the audit in this case.

Lemma 12. *Consider that the auditing precedes the contracting and the buyer faces two suppliers and one of the suppliers fails the audit. The supplier's optimal decision whether to participate or not, and supplier's optimal wholesale prices in case of participation are:*

- *If* $\theta_i < \frac{c_i D+GC}{Dp+GC}$, *then supplier* i *does not participate and we have no contracting.*
- *If* $\theta_i \geqslant \frac{c_i D+GC}{Dp+GC}$, *then supplier* i *participates and offers the wholesale price* $w_i^* = p\theta_i$.

Proof of Lemma 12. For all $\theta_i < \frac{w_i}{p}$ the supplier does not receive any order and we do not have any contracting. When $\theta_i \geqslant \frac{w_i}{p}$ we simplify the profit function as $(w_i - c_i)D - (1-\theta_i)GC$. This profit function is increasing in wholesale price. Also, from constraint we have $w_i \geqslant \Omega_i$. Therefore, whenever $\Omega_i > p\theta_i$ we have no contracting. Otherwise the supplier chooses the wholesale price at the maximum possible level and sets $w_i^* = p\theta_i$. □

When one of the suppliers fails, the supplier who passes the audit looks at his chance of being complaint with the code of conduct. In case, this chance is high, then the supplier will set the wholesale price in a way to squeeze out the buyer's profit. Otherwise, the supplier does not participate, and we have no contracting.

When both suppliers fail the audit: In this scenario, suppliers do not receive any contracting, and the buyer uses a fall-back option to satisfy the demand and the buyer makes no profit. In the next section, we will analyze the suppliers' problem to find the optimal compliance efforts.

We move one step backward in time and solve for Nash Equilibrium of suppliers' compliance efforts . In fact, suppliers move simultaneously to choose their compliance effort to maximize their profit. In this phase of the auditing stage, we find each supplier's best response for a given set of buyer's efforts (a_1, a_2). We use our finding in previous sections to form the supplier's expected profit in this phase. First, we present supplier 1's profit function in this phase:

$$\begin{aligned}\Pi_{S_1}(e_1, e_2) =& Pr(P_1, P_2)\pi_{S_1}^{PP^*} + Pr(P_1, F_2)\pi_{S_1}^{PF^*} \\ &+ Pr(F_1, P_2)\pi_{S_1}^{FP^*} + Pr(F_1, F_2)\pi_{S_1}^{FF^*} - \alpha_1 e_1\end{aligned} \tag{3.20}$$

In profit function (3.20), we have all four combinations of passing and failing of each supplier. For example, the first term shows the profit of supplier 1 when both suppliers pass the audit. From our analysis in the contracting stage, we know that if the supplier fails the audit, the supplier will not receive any contract. Therefore, in profit function (3.20), the last two terms will be zero. Supplier 1 chooses e_1 to maximize its profit. We show this maximization in optimization program (3.21) :

$$\begin{aligned}\max_{0 \leq e_1 \leq 1} \quad & \Pi_{S_1}(e_1, e_2) \\ \text{subject to} \quad & \Pi_{S_1}(e_1, e_2) \geqslant 0,\end{aligned} \tag{3.21}$$

Lemma 13 presents the best response function of supplier 1 in this phase.

Lemma 13. *Consider that the auditing precedes the contracting and that the buyer faces two suppliers. Supplier 1's best response function e_1^* for any given e_2 is:*

- *If $0 \leqslant D \leqslant \frac{\alpha_1}{p-c_1}$, then $e_1^* = 0$ for all e_2 in $[0, 1]$.*

- *IF* $\frac{\alpha_1}{p-c_1} < D < \frac{\alpha_1}{c_2-c_1}$*, then* $e_1^* = 1$ *for all* e_2 *in* $[0, \bar{e}_2]$*, and* $e_1^* = 0$ *for all* e_2 *in* $(\bar{e}_2, 1]$.

- *If* $D \geqslant \frac{\alpha_1}{c_2-c_1}$*, then* $e_1^* = 1$ *for all* e_2 *in* $[0, 1]$.

where $\bar{e}_2 = \frac{(1-a_2)C_2D+(p-c_1)D+(1-a_2)GC-\alpha_1}{(p-a_2c_2)D+(1-a_2)GC}$.

The supplier does not exert any effort when the cost of compliance is very high. On the other hand, when the cost of compliance is very low, the supplier exerts full effort. While the cost is in the mid-range, for all small compliance efforts of the competitor, the supplier exerts full effort, and for high efforts of the competitor, it exerts no effort.

The profit function for the second supplier is very similar to the first supplier. We present the supplier 2's profit function in (3.22).

$$\begin{aligned} \Pi_{S_2}(e_1, e_2) = & Pr(P_1, P_2)\pi_{S_2}^{PP^*} + Pr(P_1, F_2)\pi_{S_2}^{PF^*} \\ & + Pr(F_1, P_2)\pi_{S_2}^{FP^*} + Pr(F_1, F_2)\pi_{S_2}^{FF^*} - \alpha_2 e_2 \end{aligned} \tag{3.22}$$

In profit function (3.22), we have all four combinations of passing and failing of each supplier. For example, the first term shows the profit of supplier 2 when both suppliers pass the audit. From our analysis in contracting stage, we know that if the supplier fails the audit, the supplier will not receive any contract. Therefore, in the profit function (3.22), the second term and the fourth term will be zero. Supplier 2 chooses e_2 to maximize its profit. We show this maximization in optimization program (3.23) :

$$\begin{aligned} \max_{0 \leq e_2 \leq 1} \quad & \Pi_{S_2}(e_1, e_2) \\ \text{subject to} \quad & \Pi_{S_2}(e_1, e_2) \geqslant 0, \end{aligned} \tag{3.23}$$

To find Nash Equilibrium of suppliers' compliance efforts, we should find the second supplier's best response for any given e_1 and then intersect both best response functions of supplier 1 and supplier 2 to find NE. Notice that supplier 1's best response is always happening at $e_1^* = 0$ or $e_1^* = 1$. Therefore, for finding Nash Equilibrium, we need to find supplier 2's best response function at $e_1^* = 0$ and $e_1^* = 1$.

Lemma 14 presents Nash Equilibrium of suppliers' compliance efforts.

Lemma 14. *Consider that the auditing precedes the contracting and the buyer faces two suppliers. Nash Equilibrium of suppliers' compliance efforts are:*

– *If* $D < \frac{\alpha_1}{p-c_1}$ *and* $D < \frac{\alpha_2}{p-c_2}$, *then* $(e_1^*, e_2^*) = (0,0)$.

– *If* $\frac{\alpha_1}{p-c_1} \leqslant D < \frac{\alpha_2}{p-c_2}$ *or* $\{D \geqslant \frac{\alpha_1}{c_2-c_1}$ *and* $D \geqslant \frac{\alpha_2}{p-c_2}\}$, *then* $(e_1^*, e_2^*) = (1,0)$.

– *If* $\frac{\alpha_2}{p-c_2} \leqslant D < \frac{\alpha_1}{p-c_1}$, *then* $(e_1^*, e_2^*) = (0,1)$.

– *If* $\frac{\alpha_2}{p-c_2} \leqslant D < \frac{\alpha_1}{c_2-c_1}$ *and* $D \geqslant \frac{\alpha_1}{p-c_1}$, *then* $(e_1^*, e_2^*) = (1,0)$ *and* $(e_1^*, e_2^*) = (0,1)$.

Figure 17 is a graphic representation of Proposition 14. When the demand or market price is low, both parties have no incentive to exert any efforts, and we have no contracting. When the market price increases and the demand is low, supplier 2, who has a higher cost of production but lower compliance cost, can afford to exert the compliance cost and exert full effort. While demand increases and it is in the mid-range (not too high and not too low), both suppliers can participate and exert efforts. When demand is very high, if the production cost advantage of supplier 1 is higher than his cost for full compliance, then supplier 1 has the advantage and exerts full effort, and supplier 2 exerts zero. In the next section, we analyze the buyer's problem to find the optimal auditing efforts.

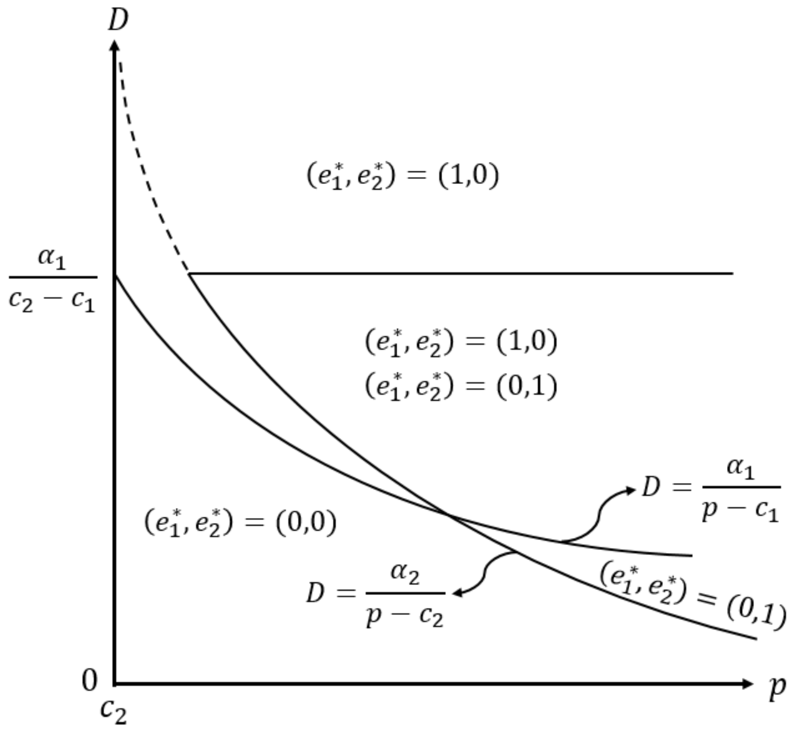


FIGURE 17. Presentation of Nash Equilibrium of Compliance Efforts on Plane of Demand and Market Price.

Optimal Auditing Efforts Decisions

We move one step backward in time and solve for the buyer's optimal auditing efforts (a_1^*, a_2^*). The buyer chooses (a_1, a_2) to maximize its profit. The outcome of auditing is one of three different situations. If the supplier is compliant, the supplier will pass the audit. If the supplier is not compliant, then either the supplier fails or passes the audit. We present the buyer's profit-to-go in (3.24). As we have two suppliers, the buyer's expected profit-to-go function in this phase of the auditing stage includes nine different terms, as a result of these three-by-three combinations. Each of these terms shows the expected profit for one of the above combinations.

$$
\begin{aligned}
\Pi_B(a_1,a_2) =& Pr(P_1C_1,P_2C_2)[(p-w_1^*)q_1^*+(p-w_2^*)q_2^*] \\
&+Pr(P_1C_1,P_2NC_2)\begin{cases}(p-w_1^*)q_1^*, & q_1^*\geqslant 0, q_2^*=0\\ -w_1^*q_1^*-w_2^*q_2^*, & q_1^*\geqslant 0, q_2^*>0\end{cases} \\
&+Pr(P_1C_1,F_2)(p-w_1^*)q_1^* \\
&+Pr(P_1NC_1,P_2C_2)\begin{cases}(p-w_2^*)q_2^*, & q_1^*=0, q_2^*\geqslant 0\\ -w_1^*q_1^*-w_2^*q_2^*, & q_1^*>0, q_2^*\geqslant 0\end{cases} \\
&+Pr(P_1NC_1,P_2NC_2)(-w_1^*q_1^*-w_2^*q_2^*) \\
&+Pr(P_1NC_1,F_2)(-w_1^*q_1^*)+Pr(F_1,P_2C_2)(p-w_2^*)q_2^* \\
&+Pr(F_1,P_2NC_2)(-w_2^*q_2^*)+Pr(F_1,F_2)\times 0-\gamma(a_1+a_2) \qquad (3.24)
\end{aligned}
$$

Proposition 9 presents the buyer's optimal auditing efforts, along with sub-game perfect Nash Equilibrium of the model.

Proposition 9. *Consider that the auditing precedes the contracting and the buyer faces two suppliers. The buyer's optimal auditing efforts for the range of all available parameters are* $(a_1^*, a_2^*) = (0,0)$*. The sub-game prefect Nash Equilibrium of model for different range of parameters are:*

- *If* $D < \frac{\alpha_1}{p-c_1}$ *and* $D < \frac{\alpha_2}{p-c_2}$*, then* $(e_1^*, e_2^*) = (0,0)$ *and there is no contracting.*
- *If* $\frac{\alpha_1}{p-c_1} \leqslant D < \frac{\alpha_2}{p-c_2}$ *or* $\{D \geqslant \frac{\alpha_1}{c_2-c_1}$ *and* $D \geqslant \frac{\alpha_2}{p-c_2}\}$*, then* $(e_1^*, e_2^*) = (1,0)$*. Also,* $(q_1^*, w_1^*) = (D,p)$ *and supplier 2 does not participate. The supplier squeezes out the buyer's profit to zero.*

– *If* $\frac{\alpha_2}{p-c_2} \leqslant D < \frac{\alpha_1}{p-c_1}$*, then* $(e_1^*, e_2^*) = (0, 1)$*. Also,* $(q_2^*, w_2^*) = (D, p)$ *and supplier 1 does not participate. The supplier squeezes out the buyer's profit to zero.*

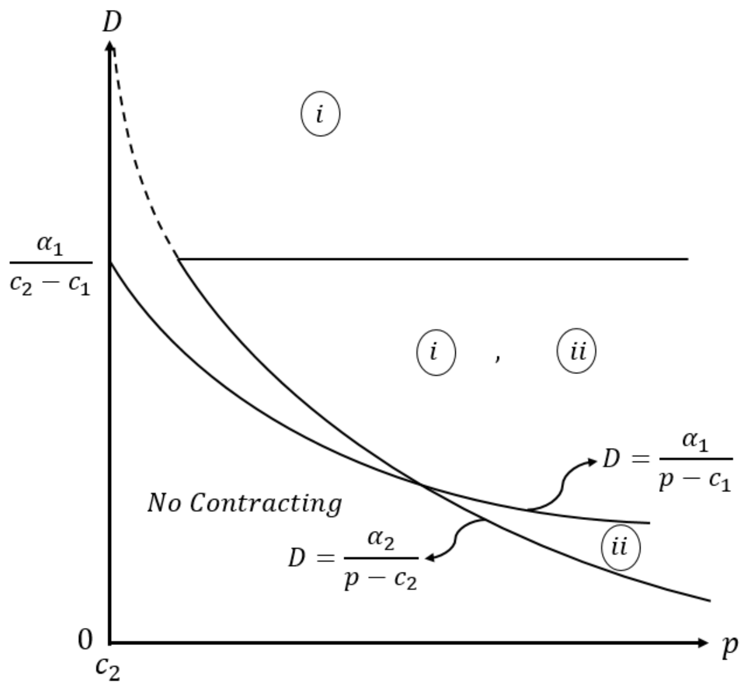
– *If* $\frac{\alpha_2}{p-c_2} \leqslant D < \frac{\alpha_1}{c_2-c_1}$ *and* $D \geqslant \frac{\alpha_1}{p-c_1}$ *, then* $(e_1^*, e_2^*) = (1, 0)$ *and* $(e_1^*, e_2^*) = (0, 1)$*. The optimal wholesale price and order quantity are identical to related cases in above.*

Proposition 9 shows that when the auditing precedes the contracting, supplier competition puts the pressure on each supplier to exert full effort, whenever the supplier finds it beneficial to participate. Therefore, the buyer's auditing does not need to be as high, because the supplier exerts full effort to be compliant with the code of conduct. We present the intuition and insights related to comparison of benchmark model and supplier competition in next section.

Effect of Supplier Competition

In this section we compare the benchmark model with the supplier competition when the auditing precedes the contracting. Based on the conclusion of the benchmark model, when demand and price are above a threshold, both suppliers participate and exert their compliance effort to get the whole order. But, when demand is high enough, we observe that if the production cost advantage of supplier 1 (i.e., $c_2 - c_1$) is more than its full effort compliance cost (i.e., α_1), then the supplier 1 is the supplier who exerts full compliance effort and participates. When the demand is in mid-range, both suppliers can participate; and, as our sub-game prefect equilibrium shows, this case exists for both. Also, when the demand is low and price is high, supplier 2 has the willingness to exert the effort, because its production cost is higher and compliance cost lower than the supplier 1. Therefore, its profit margin increases, and supplier 2 participates in the contracting alone and exerts full effort.

Our result also shows that the supplier competition leads to creating new contracting opportunity. When the demand is low and the market price is high, by switching from supplier 1 to supplier 2 the contracting opportunity increases. When the demand is high and the market price is low, the buyer can increase the contracting opportunity by switching supplier 2 with supplier 1. This is driven by the trade-off between the production cost and the compliance cost of two suppliers. When the profit of producing the order minus the cost of being fully compliant for one of suppliers beats the other supplier's, then buyer's decision to switch to the winning supplier will increase the contracting opportunity.



Equilibrium i : $(e_1^*, e_2^*, a_1^*, a_2^*, q_1^*, q_2^*, w_1^*, w_2^*) = (1,0,0,0,D,0,p,0)$

Equilibrium ii : $(e_1^*, e_2^*, a_1^*, a_2^*, q_1^*, q_2^*, w_1^*, w_2^*) = (0,1,0,0,0,D,0,p)$

FIGURE 18. Presentation of Equilibrium of Supplier Competition on Plane of Demand and Market Price.

Our analysis provides managers with another effective decision-making tool about choice of suppliers to consider to improve compliance in their supplier base structure. We find that when auditing precedes contracting, the buyer's auditing effort could be low. The supplier who participates in the contracting has this willingness to be compliant to the code of conduct; and based on its pricing power, the supplier squeezes out the buyer's profit to zero.

Conclusion and Summary

We consider supplier competition in conjunction with auditing as yet another tool for a buyer to enhance suppliers' social and environmental compliance, that has not been considered in the literature. We investigate the effect of supplier competition particularly in scenarios wherein suppliers determine the wholesale prices. We consider a game-theoretic framework to investigate the impact of timing of contracting with two possible supplier base structures (i.e., a single supplier or two asymmetric competing suppliers) on the suppliers' compliance, on the buyer's auditing effort, and on the buyer's and the suppliers' profits. We solve for the subgame-perfect equilibrium and characterize the contract terms, the buyer's auditing effort, and the suppliers' compliance efforts at the equilibrium.

Our results indicate that when the auditing precedes the contracting, the buyer does not have to exert significant auditing effort since suppliers find it beneficial to be compliant with the code of conduct in the presence of competition. In the process the supplier is also able to squeeze out the buyer's profit to zero. When the demand in the market is high, the supplier with the lower cost of production (but higher compliance cost) wins the competition. In contrast, when the market price is high while demand is low, the supplier with the advantage in compliance cost wins the competition. We also show that the supplier competition leads to increasing the contracting opportunity.

Bridge to Next Chapter

In the first essay, we investigated how buyer's policy of commitment to contract terms affects the sustainability and financial performance of the supply chain. The second essay focused on understanding the impact of supplier competition on the buyer's ability to influence suppliers' compliance when suppliers have more parity in contracting power. In the next chapter, which is the third essay, we focus on a broader research question pertaining to buyer-supplier relationship management strategies to influence supplier's sustainability performance. The primary research question considers how the specific nature of trust (i.e., calculative vs. relational trust) between the buyer and the supplier influences the impact of supplier relationship management strategies (transactional and collaborative) on supplier's sustainability performance. This research question is not easily amenable to analytical modeling, and hence addressed by developing a conceptual and empirical framework.

CHAPTER IV

A CONCEPTUAL FRAMEWORK FOR UNDERSTANDING THE ROLE OF TRUST BETWEEN BUYERS AND SUPPLIERS IN INFLUENCING SUPPLIERS' SUSTAINABILITY

Introduction

Social and environmental performance of suppliers of well-known brands has been in the spotlight over the last decade and has been documented by news media, agencies, and researchers. For instance, International Labor Organization (ILO) reports that worldwide, 218 million children are employed by different manufacturing companies, almost 73 million of whom also work in hazardous conditions (ILO, 2013). Child labor keeps the production costs of local suppliers down, and the benefit therefrom goes to brands who buy from these suppliers (Locke, 2003). As another example, a supplier's apparel factory in Bangladesh collapsed in April, 2013. During the incident, more than 1000 people were killed. The apparel factory was a supplier to Walmart and Matalan. Media reported that the building was not safe for workers (Al-Mahmood et al., 2013). In another incident, a supplier to Toyota in Alabama forced workers to work overtime at regular pay. The supplier also did not provide workers with standard workplace safety. As a consequence of this supplier's behavior, some employees got injured (Waldman, 2017). In another case, it has been revealed that BMW and Volkswagen were sourcing from suppliers that employed child labor (Bengtsen and Kelly, 2016). Such incidents are not limited to only social misconduct of suppliers. There have also been many incidents of environmental violations. For instance, it was publicized by NGOs and media that suppliers of Nike and Marks &

Spencer released toxic chemicals into the rivers and polluted the air (Hurley et al., 2017).

Suppliers' social and environmental misconduct hurt the reputation of, cause serious problems for, and have threatened the revenue stream of brands. Several studies show that brands incur huge costs because of such incidents of publicized misconduct (Guo et al., 2016; Plambeck and Taylor, 2016). Many of these well-known brands are concerned about suppliers' social and environmental misconduct (Lee et al., 2012). Some brands motivate the suppliers to be compliant with social and environmental standards by incentivizing them, such as Nike's supplier incentive program for sustainability (Nike, 2018). Other brands, such as Starbucks, penalize suppliers upon observing their social and/or environmental misconduct (Lewis et al., 2012; Porteous and Rammohan, 2013; Porteous et al., 2015). In some cases, brands choose to employ collaborative approaches to enhance suppliers' social and environmental performance. Leading brands try to improve their suppliers' social and environmental performance through joint problem-solving and supplier training (Bai and Sarkis, 2010). IKEA works closely with its suppliers to develop a mutual understanding of sustainability objectives by working on environmental and social responsibility goals and codes of conduct (Andersen and Skjoett-Larsen, 2009).

Despite above efforts many buyer firms across range of industries struggle with achieving social and environmentally responsible behavior in their supply chains because of their inadequate or ineffective supplier relationship management practices. Previous research on managing buyer-supplier relationships has examined how buyers use a specific subset of transactional and collaborative strategies to influence suppliers' operational performance (Zajac and Olsen, 1993). Few studies have extended their focus to also understanding the impact of the above subset of relationship management strategies on suppliers' sustainability performance (Klassen and McLaughlin, 1996; Plambeck et al., 2012; Bag, 2018). However, no previous research has examined the impact of a comprehensive set of transactional and collaborative relationship management strategies on enhancing suppliers' sustainability performance.

The notion of trust has been another important construct in understanding buyer-supplier relationships. Previous research has examined how trust between buyers and suppliers moderates the impact of transactional and collaborative approaches on suppliers' performance, albeit not sustainability performance (Benton and Maloni, 2005; Ireland and Webb, 2007). While both buyers and suppliers understand that they are negatively affected when customers penalize a brand for social and environmental violations in their supply chain, they may yet not be willing to behave in a responsible manner due to lack of trust and misalignment of incentives (Gualandris and Kalchschmidt, 2016). Parmigiani et al. (2011) suggest that there may emerge a growing level of trust between buyer and suppliers, because of pressure from customers regarding sustainability misconduct. Customer pressure can catalyze the buyer-supplier relationship towards social and environmental responsibility in the supply chain. Parties may try to work together by considering to develop trust as a basis for responding to this pressure. A major question arises as to what extent is trust important for catalyzing the buyer supplier relationship towards a more sustainable behavior.

To better understand the effect of buyers' approaches for enhancing the suppliers' social and environmental performance, we propose a conceptual framework to study the efficacy of a comprehensive set of both transactional and collaborative supplier relationship management approaches to enhance suppliers' sustainability performance. Specifically, we propose a framework to investigate the role of the specific nature of trust (i.e., calculative and relational trust) between buyers and suppliers in influencing the impact of their supplier relationship management strategies on suppliers' sustainability performance.

Some key research questions of interest include:

1) Do supplier relationship management strategies (transactional and collaborative) influence the supplier's social and environmental (i.e., sustainability) performance?

2) If so, how do transactional relationship management approaches compare *vis-à-vis* collaborative relationship management approaches? Are transactional (relational) approaches relatively better for enhancing suppliers' environmental (social) performance?

3) How does the specific nature of trust between the buyer and the supplier influence the impact of supplier relationship management strategies on supplier's sustainability performance?

In this chapter we propose a conceptual framework to lay a foundation for developing a theoretical lens to understand this phenomena for promoting social and environmentally responsible behavior in supply chains. After a careful examination and unification of the literature we develop a set of hypotheses to enable a formalized study that can address most of the above research questions. We engage with managers involved in sustainability roles at firms across multiple industries (e.g., footwear and apparel, aerospace, semi-conductors and metals manufacturing) to conduct a preliminary field-based face validation of the proposed framework. The actual field-based study that requires extensive data collection with a deeper engagement with participant firms and industry associations is relegated to post-doctoral work. Requisite scales for measurement of constructs in this proposed framework will be developed or adapted from the literature.

The remainder of the chapter is organized as follows. In the next section we review the relevant literature. Then, we introduce our framework and develop the proposed hypotheses. We conclude with a brief summary.

Literature Review

Our research relates to two streams of literature: (1) supplier relationship management for the purpose of sustainability; and (2) role of trust in the buyer-supplier relationship management for enhancing supplier's sustainability.

In sustainable operations management, there are not many papers which look at the effectiveness of different supplier relationship management approaches on social and environmental performance. Scholars point out that there is a need to understand the value of social and environmental practices that affect the firm's value (Servaes and Tamayo, 2013). Bowen et al. (2001) and Gimenez and Tachizawa (2012) indicate that there is a need to investigate the effect of different supplier management practices on suppliers' sustainability performance.

Cousins et al. (2004) focus on activities that a buyer can engage, to improve and manage the environmental performance of his or her supplier, while considering the available resources and losses incurred because of incidents of environmental non-compliance in the market. They suggest that although the supplier monitoring and offering incentives need more resources, they will help proactive companies who have taken these actions to gain more competitive advantage, because of improvement in suppliers' environmental performance. Zhu et al. (2012) study the effects of collaboration with suppliers and assessment of their performance on improving the sustainability performance of the firm.

Other researchers look at the green supply chain and green value chains. In these studies, the focus is on the environmental aspects of the supply chain. For instance, Bowen et al. (2001) and Corbett and Klassen (2006) characterize different aspects of practices that can help to improve the supplier's environmental performance. Most studies investigate the environmental performance of suppliers. Scholars determine that there should be additional studies on social aspects of suppliers' sustainability performance (Seuring and Müller, 2008; Wu and Pagell, 2011).

Bowen et al. (2001) study the effect of collaborative partnering with suppliers on supplier's environmental performance. They find a positive relationship between them. Locke and Romis (2007) provide a case study on two Mexican factories and find that having a code of conduct and auditing alone cannot help to improve some aspects of supplier's social performance, such as hiring child labor and work safety. Their study proposes that collaborative strategies, such as training the supplier or joint investment, can reduce the incidents of misconduct. Locke et al. (2012) study the efficiency of the supplier program of HP. They find that the collaboration of the buyer with suppliers to establish a solid national context for workplace safety is more efficient than regular audit capability building, or supply chain power. Vachon and Klassen (2006) study the relationship between buyers' environmental collaboration strategies and logistical and technological changes in a supply chain. Paulraj (2011) empirically investigates the effect of the buyer firm participating in environmental collaboration, such as providing the supplier with the design specification for sustainability as collaboration for cleaner production, on the supplier's environmental performance. He finds that this relationship is significantly positive and helps the supplier to improve the sustainability performance. Hollos et al. (2012) study the effect of collaboration on the green performance of suppliers. They find that a collaborative approach, such as providing feedback to the supplier from the buyer, can improve the environmental performance of the supplier, including demonstrable increases in recycling and in waste reduction.

Some researchers study the effect of incentives and penalties on the supplier's social and environmental performance. Klassen and McLaughlin (1996) show that there exists a positive relationship between environmental management practices, such as giving awards to suppliers to incentivize them, with suppliers' environmental performance. Lewis et al. (2012) investigate the effect of incentives on the sustainability performance of suppliers by analyzing a game theory model. They incorporate some realistic features, such as two-part, nonlinear tariff payment structure, in their analysis. Plambeck et al. (2012) study the behavior and strategies that leading companies, such as Nike, adopt for improving their suppliers' environmental performance in China. They find that these brands provide suppliers with tools and incentives to improve environmental performance. They indicate that these incentives are more useful than traditional compliance auditing and supplier's disclosure. Porteous et al. (2015) study a model that investigates the relationship between incentive and penalties with suppliers' social and environmental performance. They consider several incentive structures, such as increased business, better terms and conditions, and price premium. For penalties, they consider reduced business, fines, and termination of contract. They analyze the model based on an ordinary least squares (OLS) and PROBIT method. They find that the threat of terminating the contract, with warnings; the incentive of training the supplier; and public recognition each have a significant positive effect on reducing social and environmental violations in supplier facilities and practices. In a recent work, Bag (2018) studies supplier management and sustainable innovation in supply networks. He finds that supplier relationship management positively affects the suppliers' performance.

Another related stream of research is the importance of trust, within supplier-buyer relationships, toward improving suppliers' social and environmental performance. This segment of the literature is epitomized by couple of studies. Carter and Jennings (2002) study the effect of buyers' sustainability practices on suppliers' performance. They find that sustainable purchasing leads to buyers' enhanced trust in suppliers. Sharfman et al. (2009) study the effect of trust upon buyer-supplier uncertainty and pro-active environmental supplier management, and on buyers' cooperative approaches for supply chain environmental management. Parmigiani et al. (2011) indicate in their study that there may exist a growing level of trust between firms in the supply chain, due to social and environmental pressure from customers and buyer firms, in which collaboration and knowledge transfer will help suppliers meet the buyers' social and environmental standards. Anisul Huq et al. (2014) use a multi-case study approach to investigate the adoption and implementation of socially and environmentally sustainable practices at a supplier firm in Bangladesh that is a supplier for two brands. They find that moving from absolute use of power, to collaboration and open dialogues with trust between firms, can promote the social performance in the supply chain. These studies suggest that there could be a moderating role for trust between sustainable supplier management and supplier's performance. Gualandris and Kalchschmidt (2015) investigate how the social and environmental performance of the supply chain can be enhanced as sustainable supply chain management is implemented. They find that trust plays a moderating role between sustainability practices and social and environmental performance. To the best of our knowledge, our unifying conceptual framework lays the foundation for a study that is the first to examine the efficacy of transactional and collaborative approaches adopted by buyers in a comprehensive manner to enhance suppliers' social and environmental performance. Our conceptual framework also proposes the need to investigate for the first time with greater specificity the roles of calculative trust and relational trust between buyer and supplier in influencing the efficacy of supplier relationship management approaches adopted by the buyer on supplier's sustainability performance.

Conceptual Development

In this section we examine the theoretical underpinnings of our proposed conceptual framework to develop and present the hypotheses. First, we present the different approaches that a buyer can use to manage its relationship with the suppliers. Then, we define the social and environmental performance of suppliers. Next, we use social exchange theory to propose our hypothesis and to show the relationship between a buyer's supplier relationship management strategies and supplier's social and environmental performance. Finally, we use the literature and theories to posit our hypothesis about the moderating role of calculative and relational trust in these relationships. Figure 19 shows our conceptual framework.

Interorganizational Relationships

In recent years, scholars have been paying more attention to interorganizational relationships (Carey et al., 2011; Cao and Lumineau, 2015). In the supply chain literature, supplier relationship management is expected to be central to the

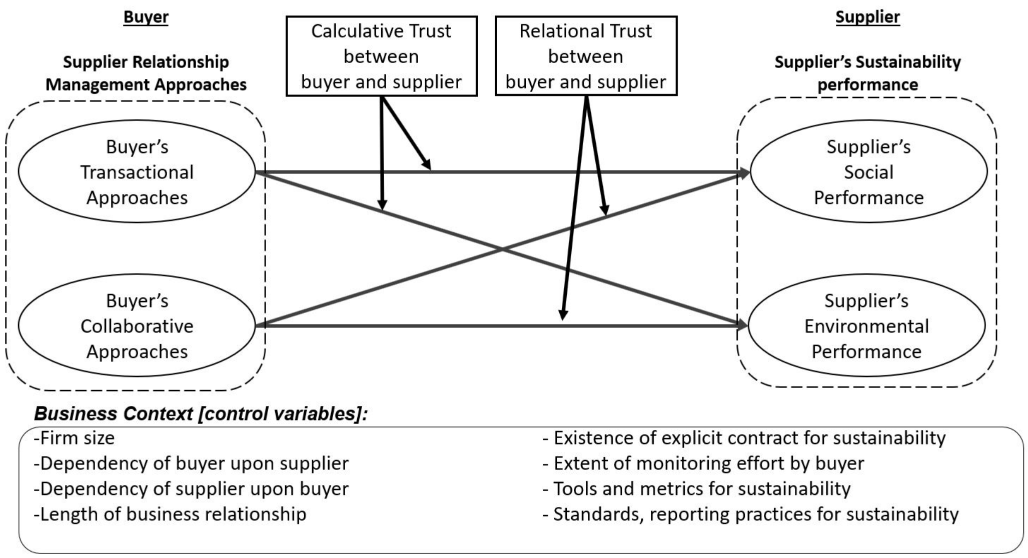


FIGURE 19. Conceptual Framework

performance of the firms (Amoako-Gyampah et al., 2019). Fynes et al. (2005) defines supplier relationship management as a purposeful practice that firms engage in to manage their interactions with suppliers. Supplier relationship management becomes an important business activity because of market competition, the need to consider sustainability, and the necessity to reduce the cost in order to be cost-competitive (Lambert and Schwieterman, 2012). There exist two dominant approaches for supplier relationship management: (1) transactional (contractual) approaches; and (2) collaborative approaches (Burt et al., 2003; Spekman and Carraway, 2006; Whipple et al., 2010).

In the Transactional approach, which is often adversarial, the buyer-supplier relationship will be categorized based on functions and tasks (Sanders et al., 2007). In this type of interaction, buyers set the conditions of contracts and are responsible to check the compliance of suppliers based on those conditions (Spekman and Carraway, 2006). In this relationship, contracts stipulate different terms and conditions based on the expectations of the buyer (Reyniers and Tapiero, 1995). When buyers use transactional approaches, suppliers have willingness to revise their processes or improve their status, because they are at risk of not winning the contracts with the buyers (Hahn et al., 1986). Buyers can count on these transactional strategies, as they engage suppliers, to improve their performance without a high effort from buyers or much investment (Krause et al., 2000). Coviello et al. (2002) explain that the transactional approaches would be more impersonal. The choice of transactional approach depends on opportunistic behavior of the supplier and the cost of implementation for the buyer (Mellewigt et al., 2007). Buyers can use different incentives and penalties as transactional approaches in their contracts with suppliers (Atkinson, 1998).

In the context of sustainability, buyers use different incentives to motivate their supplier base to be more socially and environmentally responsible. Porteous et al. (2015) mention price premiums; public recognition of the supplier, e.g., awards; preferred supplier status; increased business engagements; and better terms and conditions in the supply contract as different forms of incentives for suppliers. Some buyers, such as Walmart, Gap, and Puma, offer loans to their suppliers for improving their social and environmental conditions (Plambeck and Taylor, 2016). Researchers show that for avoiding incidents of misconduct, buyers can use penalties as another form of transactional approach (Davidson III and Worrell, 2001). Porteous et al. (2015) list penalties such as fines, reduction of business, and termination of contract as different methods of dis-incentivizing suppliers, in order to avoid incidents of social and environmental misconduct.

The other supplier relationship approach is a collaborative one. Michael Dell, founder of Dell Computers, said, "Collaboration is the new imperative" (Burt et al., 2003). In a collaborative relationship, the buyer and the supplier cooperate jointly for the long term, and on a continuous basis, to plan and modify their business practices to improve performance (Coviello et al., 2002; Spekman and Carraway, 2006; Whipple et al., 2010). Collaborative relationships are longer-term and cooperative. Coviello et al. (2002) explain that collaborative approaches are more interpersonal and are plausible to operate in a sustained manner. Collaborative relationships involve both economic and social elements (Bunduchi, 2008). In a collaborative relationship, the buyer has the willingness to share technical information, to train the supplier's personnel, and to invest in the supplier's operation if, in return, the supplier improves its performance and creates benefits for both sides (Zajac and Olsen, 1993). Zhang and Cao (2018) indicate that firms that engage in collaborative practices are likely to share resources such as technical expertise and joint training.

Many companies start collaboration with their supplier with the aim to improve social and environmental sustainability in their supplier's facility and processes; e.g. , Nike tries to collaborate with its suppliers to promote workers' rights and reduce packaging waste (Gage, 2016). Researchers and industry reports reveal multiple ways to collaborate with suppliers for promoting environmental and social sustainability practices. Buyers can offer a variety of supplier training programs (Porteous et al., 2015; Kainuma and Tawara, 2006). Leading companies try to improve their suppliers' sustainability status through joint problem-solving (Pimentel Claro et al., 2006; Canning and Hanmer-Lloyd, 2001; Rickert et al., 2000; Bai and Sarkis, 2010). In another way, buyers can provide technical know-how to their suppliers for improving their processes and the sustainability of their facilities (Pedersen and Andersen, 2006; Bai and Sarkis, 2010). In some cases, buyer and supplier jointly invest on environmentally friendly technology or improvement of social practices (Pedersen and Andersen, 2006; Simpson et al., 2007; Bai and Sarkis, 2010). Buyers such as Nike and IKEA work very closely with their suppliers to develop a mutual understanding of sustainability objectives, by working on environmental and social responsibility goals and codes of conduct (Pedersen and Andersen, 2006). In the next section, we introduce the suppliers' performance from social and environmental perspectives.

Supplier's Social and Environmental Performance

Suppliers' sustainability performance is presented from economic, social, and environmental perspectives. Scholars define environmental and social performance in different ways (Maloni and Brown, 2006; Zhu and Sarkis, 2007; Pullman et al., 2009). Valiente et al. (2012) explain that the supplier's social and environmental performance takes into account both the descriptive and the normative dimensions of corporate responsibility, as well as emphasizing everything that suppliers are achieving in the domain of environmental and social responsibility policies, practices, and results. To assess the social and environmental performance of suppliers, researchers measure the improvement in different metrics. For example, to capture the environmental performance, Zhu and Sarkis (2007) introduce improvement of reduction in reduced waste and other pollutants as a way to see environmental performance. Valiente et al. (2012) assess the occupational safety and health of workers in the supplier's firm to capture social performance.

In general, for measuring a supplier's social and environmental performance, we can look at the reduction in social and environmental violations. Porteous and Rammohan (2013) define violations as any kind of noncompliance with or breaches of national or regional law, industry regulations, and the supplier's code of conduct. If the number of these violations is reduced, we can conclude that the supplier's compliance advances, and therefore, the supplier's performance improves.

To find the above-mentioned violations, most buyers develop a sustainability code of conduct for their suppliers. The code of conduct sets expectations for suppliers' social and environmental practices. Buyers audit their suppliers for compliance with their code of conduct, utilizing formal, external third party audits or internal audits, and they strive to reduce the social and environmental violations in their supplier base (Egels-Zandén, 2007; Awaysheh and Klassen, 2010).

The social dimension of the supplier's responsibility consists of health, safety, and working conditions of employees, lawful employment practices, worker benefits (including fair wages and training), freedom of association, commitment to diversity and nondiscrimination practices, and improvement in awareness and protection of the claims and rights of people in the communities served (van der Wiele et al., 2001; Paulraj, 2011; Porteous et al., 2012; Lee et al., 2012). The environmental dimension of suppliers' responsibility consists of responsible use of resources (e.g., energy, water, materials, and hazardous substances); emissions reduction; land use; and waste management of water and materials (Hutchins and Sutherland, 2008; Porteous et al., 2012; Lee et al., 2012). In the next section, we present our hypothesis for the relationship between the buyer's supplier relationship management strategies and suppliers' sustainability performance.

Relationship between Buyer's Supplier Relationship Management Strategies and Supplier's Social and Environmental Performance

We rely on social exchange theory as our main theoretical framework to present the research model and develop our hypotheses. Social exchange theory is a useful set up to see the effect of interactions on outcomes in a dyadic relationship. Social exchange theory is composition of principles obtained from different fields, including psychology, sociology, and economics (Bandura, 1986; Blau, 2017; Griffith et al., 2006; Rickert et al., 2000). It introduces a framework in which any interactions between buyer and supplier can be considered as an exchange of resources (Cropanzano and Mitchell, 2005). These resources can be economic (e.g., money or goods), but can also be social in nature, for instance, status or collaboration (Lambe et al., 2001; Cropanzano and Mitchell, 2005).

An important principle of social exchange theory is that two parties start a relationship with the expectation that the result of the exchange will be beneficial to them (Lambe et al., 2001; Blau, 2017). Also, Lambe et al. (2001) point out that the parties will remain in business together as long as the benefits from the relationship exceed the alternative relationships.

We use the social exchange theory to propose that both transactional approaches and collaborative approaches may be effective to improve the supplier's social and environmental performance. The supplier's need to enter or keep the business relationship is stimulated by motivation to protect his economic outcomes. In the case of transactional approaches, the supplier knows that, in order to keep the contract going and to receive some incentives and avoid termination, he needs to perform well in the areas that the buyer requires. The supplier has the incentive to fix problems that can improve his performance and that help him to be competitive enough and not lose the business opportunity (Terpend and Krause, 2015). Taking advantage of the transactional approaches is an appealing choice for buyers, because the buyers can use them to improve suppliers' performance without too much investment (Krause et al., 2000).

Scholars criticize the use of transactional approaches, because under the pressure of securing the contract and competition, such approaches encourage suppliers to use tricks and short-term fixes. Rather than significant long-term improvements (Deming, 2018). In our proposed framework we consider buyer's supplier relationship strategies in a comprehensive manner and also classify them into transactional and collaborative strategies. Based on the above discussion, we first focus on buyer's transactional relationship management strategies and posit hypotheses stated below:

H1a: Buyer's use of transactional strategies has a positive relationship with supplier's social sustainability performance.

H1b: Buyer's use of transactional strategies has a positive relationship with supplier's environmental sustainability performance.

For buyer's collaborative approaches, the supplier's motivation to improve her performance is not only based on the economic benefits. When the buyer chooses to use the collaborative approach, a supplier also benefits from the social outcomes of this relationship (Lambe et al., 2001). These benefits for suppliers include a desire to identify with the buyer and to obtain knowledge from a buyer (Maloni and Benton, 2000; Terpend and Krause, 2015). Social exchange theory states that a supplier who receives economic benefits from the relationship will reciprocate it. Also, social exchange theory expects that as time goes by and the buyer-supplier relationship continues, the use of collaborative approaches will improve the efficiency of the relationship, simultaneously reducing the level of uncertainty (Terpend and Krause, 2015). Altogether, it initiates a higher level of coordination (Jap, 2001) and facilitates the information sharing between two parties (Barratt, 2004). As Jap (2001) shows these activities will benefit both parties and improve performance. Based on the above discussion we posit hypotheses associated with buyer's collaborative strategies:

H2a: Buyer's use of collaborative strategies has a positive relationship with supplier's social sustainability performance.

H2b: Buyer's use of collaborative strategies has a positive relationship with supplier's environmental sustainability performance.

In the next section, we present our hypothesis related to the moderating role of trust in the above relationship.

Trust, Calculative Trust, and Relational Trust

Rousseau et al. (1998) defines trust as "a psychological state comprising the intention to accept vulnerability based upon positive expectations of the intentions or behavior of another". Trust is among the most highly-cited dimensions of critical import in understanding buyer-supplier relationships in the supply chain literature. In interorganizational relationships, Zaheer et al. (1998a) mention that trust indicates a partner's expectation that the other party is reliable, will act as predicted, and will behave equitably. Other researchers define trust as "the firm's belief that another company will perform actions that will result in positive actions for the firm, as well as not take unexpected actions that would result in negative outcomes for the firm" (Anderson and Narus, 1990). Trust promotes recognition of stability, improves mutual coordination, and reduces the performance losses that would occur because of opportunism (Poppo et al., 2016).

There are different types of trust. Transaction cost economics and game theory researchers suggest that a system which places incentives with rewards can lead to a steady and inevitable outcome (Axelrod, 2006; Williamson, 1996). It can be referred to as a type of trust, which "delimit[s] the elusive notion of trust" (Williamson, 1993). This type of trust is related to the concept of calculative trust, which means that parties may act in a reliable manner due to commitments that they've made (Dyer and Chu, 2000; Suh and Kwon, 2006). Calculative trust enlightens expectations by intentionally assessing progressive conditions: It needs to calculate the associated value, or costs and benefits, of violations of terms, or of cheating and cooperation (Bromiley and Harris, 2006; Lewicki et al., 2006; Poppo et al., 2016). Under calculative trust, firms know possible outcomes of their actions, and if one company can compete for the transaction with others, then the transaction will be assigned to the firm which will generate the largest net gain (Williamson, 1993). Parkhe (1993) points out when calculative trust is high, firms know that failing to achieve cooperation and performance goals lead to termination of relationship or penalties. Threats and sanctions maintain collaboration, control exchanges, and reduce opportunistic behavior (Poppo et al., 2016).

Relational trust is another form of trust which shows a long-established and solid business relationship (Granovetter, 1985; Ring and Van de Ven, 1994). Repeated interactions between firms form relational trust (Rousseau et al., 1998). These interactions let parties form expectations and develop shared values to define the ways that they can work together better (Bercovitz et al., 2006). Dependability and reliability in all previous interactions will lead to the generating of positive expectations about the other party's intentions (Rousseau et al., 1998). This type of trust comes into consideration when parties can expect to behave according to the partner's priorities and preferences (Lewicki et al., 2006; Saparito et al., 2004). Successful fulfillment of expectations, and reliable interactions over the course of time, evoke a high willingness of parties to rely on each other and to expand the relationship (Rousseau et al., 1998). Lewicki et al. (1996) state that when relational trust is high, parties have a better understanding of each other, and they can behave and respond like each other. This mutual understanding improves outcomes, decreases risk, increases efficiency, and reduces opportunistic actions (Poppo et al., 2016). These two types of trust are distinct constructs with different bases (Rousseau et al., 1998; Poppo et al., 2016). Calculative trust is founded on reasonable evaluation of well-established rewards and punishments. Additionally, parties regularly make decisions as to whether to cooperate, based on their costs and benefits calculations (Saparito et al., 2004). These decisions need to be made accurately and in determined ways (Poppo et al., 2016). On the other side, relational trust evolves from repeated interactions in a long relationship. The mutual understanding and shared values help parties to consider themselves as "us", and decisions are made based on overall quality of relationships, rather than single interactions (Rousseau et al., 1998; Uzzi, 2011). Considering calculative trust, wherein interactions will be terminated or affected by conditions as soon as violations from terms happen, relational trust allows parties to cooperate and to resolve the unmet expectation via joint efforts (Rousseau et al., 1998).

Scholars indicate that trust should increase across the supply chain, in order to boost the social and environmental performance (Parmigiani et al., 2011; Anisul Huq et al., 2014). Trust in a buyer-supplier relationship, which is considered an extra safeguard against exploitation, improves the effect of a specific interaction (e.g., a collaboration) for a buyer (Artz, 1999). Doney and Cannon (1997) study the impact of supplier trust on a buyer's current supplier choice and future purchase intentions. Zaheer et al. (1998b) shows that the level of trust between buyer and supplier is a powerful influence on supplier performance. Carter and Jennings (2002) show that buyer's trust in supplier has a positive impact, both on buyer's relationship commitment and on cooperation between parties. Trust provides confidence to parties that predetermined outcomes will be achieved, and this should lead to greater tendency to cooperate (Andaleeb, 1995). Trust plays a moderating role between practices and performance in the supply chain. Fynes and Voss (2002) show that trust plays a moderating role between quality practices and performance between buyer-supplier relationships. Corsten and Felde (2005) show that when trust is high, the relationship between collaboration strategies and supplier's performance will be stronger. Trust provides a safeguard for future interactions between buyer and supplier, which provides parties with incentives and ways for developing precious capabilities (Gualandris and Kalchschmidt, 2016). Scholars show that trust can facilitate communication and information sharing, and that it leads to a higher performance (Benton and Maloni, 2005; Ireland and Webb, 2007). Therefore, trust increases the success level of interorganizational activities by bringing motivation for supplier and buyer to work closely to achieve higher performance capabilities (Gualandris and Kalchschmidt, 2016). Trust magnifies the ability of buyer and supplier to combine their resources to achieve their goal (Morgan and Hunt, 1994; Dyer and Chu, 2003). Thus, when calculative/relational trust is at a high level, we expect that the buyer's transactional and collaborative approaches work better, and that the supplier engages and participates to improve the social and environmental performance. However, the effect of the buyer's transactional and collaborative approaches on the supplier's environmental and social performance will be of little value, when the trust is low. We expect this, as the absence of a high level of trust, and the low level of coordination hinder buyer efforts to completely take advantage of supplier relationship approaches. Based on the above discussion, we propose the following hypotheses:

H3a/b: The higher the calculative trust in buyer-supplier relationship, the stronger the positive relationship between transactional approaches and supplier's social/

environmental performance.

H3c/d: The higher the relational trust in buyer-supplier relationship, the stronger the positive relationship between relational approaches and supplier's social / environmental performance.

Control variables

Lastly, we consider several control variables to allow for heterogeneity in our conceptual framework. These also enable us to have a better understanding of some boundary conditions for the posited relationships. First, we consider length of doing business (prior business history), because experience is necessary to support trust (Lewicki et al., 1996). We consider dependency of buyer on supplier and vice versa. Dependency may lead to different behavior and affect the trust based on the literature. We also consider firm size of buyer and supplier; contracting details for sustainability; and monitoring mechanism based on Lusch and Brown (1996) and Dahlstrom and Nygaard (1999). Lastly, we consider tools and metrics, standards, and practices for reporting sustainability adopted by the buyer and supplier firms.

Summary

Suppliers' social and environmental responsibility has been a major ongoing concern for leading brands, as awareness about environmental degradation, publicized suppliers' social and environmental incidents of misconduct, and climate change has increased. In addition, activities of NGOs around reporting various social and environmental issues in developing countries have pushed brands to pay more attention to their suppliers' social and environmental performance. For achieving a higher compliance to social and environmental standards, brands use different strategies. Apte and Sheth (2016) indicate that brands face challenges to improving social and environmental performance in their supply chain, because they try to do it solely through monitoring policies and compliance, a strategy that is not helpful and which fails "time and time again." Leading companies start employing different incentive, penalties, and collaboration mechanisms to enhance sustainability of their suppliers. Thus, there is a strong need to undertake research that can guide firms to chart their strategies to improve socially and environmentally responsible behavior in their supply chains.

In this chapter we propose a conceptual framework to lay a foundation for developing a theoretical lens to understand this phenomena for promoting social and environmentally responsible behavior in supply chains. After a careful examination and unification of the literature we develop a set of hypotheses to enable a formalized study. We propose a conceptual framework to study the efficacy of a comprehensive set of both transactional and collaborative supplier relationship management approaches to enhance suppliers' sustainability performance. Specifically, we propose a framework to investigate the role of the nature of trust (i.e., calculative and relational trust) between buyers and suppliers in influencing the impact of their supplier relationship management strategies on suppliers' sustainability performance.

CHAPTER V

CONCLUSION

Consumers hold brands accountable when their suppliers' sustainability violations become known in the marketplace. Brands find it difficult to reduce the likelihood of social and environmental violations in their supplier base. Also, an important conundrum for manufacturers today continues to be the challenge of balancing the economic benefits of outsourcing with the loss in their ability to control sustainability in their supply chain. This dissertation with three essays investigates important ways to alleviate this major supply chain sustainability conundrum by offering insights at both strategic and tactical levels to make suppliers more socially and environmentally responsible.

In the first essay, we examine the effect of the buyer's commitment to price and/or quantity in enhancing the supplier's compliance to the code of conduct for sustainability. While the efficacy of commitment to contract terms has been studied in the operations management literature with mixed results, it has not been hitherto examined in the literature on sustainability in operations and supply chains. We analyze and compare multi-stage game-theoretic models to investigate the effect of varying levels of commitment to contract terms on sustainability and financial metrics. We find that increasing the level of commitment improves the supplier's likelihood of compliance to the sustainability standard. Interestingly, we also find that both contracting opportunity and profit for the buyer increase monotonically with the degree of commitment. Additionally, committing to only the price or only the quantity (*vis-à-vis* no-commitment) has an asymmetric effect. We show that committing to the price and committing to the quantity are complementary strategies for the buyer and substitutes for the supplier. Our study provides managers with yet another effective tool to consider to improve compliance in their supplier base. Our results show that commitment provides the buyer with significant opportunity to advance both social good and profit in a *win-win* manner.

The second essay considers a different scenario wherein the supplier competition is possible. In this chapter, we study the model wherein the buyer and the supplier(s) have more parity in their contracting power and hence the buyer's power is limited. In contrast with the first essay, the supplier offers a wholesale price and the buyer is limited to only offering the quantity. We consider a game-theoretic framework to investigate the impact of timing of contracting with two possible supplier base structures (i.e., single supplier and supplier competition) on the suppliers' compliance, on the buyer's auditing effort, and on the buyer's and the suppliers' profits. We solve for the subgame-perfect equilibrium and characterize the contract terms, the buyer's auditing effort, and the suppliers' compliance efforts at the equilibrium. We analyze a single supplier model (benchmark) and compare it with a supplier competition model, wherein the auditing precedes the contracting. The interplay between the suppliers' compliance costs and the suppliers' production costs under two different supplier base structures provides useful insights. Our results show that when auditing precedes contracting, the buyer may need to exert low auditing effort, because the timing of contracting assures that the supplier who can participate in contracting will exert the effort to be compliant with the code of conduct. Also, we find that when demand is high enough, if the production cost advantage of cheaper supplier is greater than its full effort compliance cost, then this the cheaper supplier is the supplier who exerts the full compliance effort and participates in contracting.

In the third essay, we address a broader question that is not easily amenable to analytical modeling. Therefore, we use an empirical framework. In this chapter, we propose a conceptual framework to lay a foundation for developing a theoretical lens to understand this phenomena for promoting social and environmentally responsible behavior in supply chains. After a careful examination and unification of the literature we develop a set of hypotheses to enable a formalized study. We propose a conceptual framework to study the efficacy of a comprehensive set of both transactional and collaborative supplier relationship management approaches to enhance suppliers' sustainability performance. Specifically, we propose a framework to investigate the role of the specific nature of trust (i.e., calculative and relational trust) between buyers and suppliers in influencing the impact of their supplier relationship management strategies on suppliers' sustainability performance.

APPENDIX A

TECHNICAL PROOFS - CHAPTER II

Proof of Lemma 1. Here, we present the proof of optimal contract terms when the supplier passes the audit in the no-commitment model. We simplify the objective function of optimization problem (2.9) as follows: $(\frac{e}{e+(1-e)(1-a)})pq_p - w_p q_p = (\frac{e}{e+(1-e)(1-a)}p - w_p)q_p$. The objective function is decreasing in w_p. Therefore, when $0 \leq e < \frac{1-a}{\frac{p}{c}-a}$, the objective function is maximized by choosing $(w_p^*, q_p^*) = (c, 0)$, which means *no-contracting*. When $\frac{1-a}{\frac{p}{c}-a} \leq e \leq 1$, the objective function is maximized by choosing $(w_p^*, q_p^*) = (c, D)$. We plug the optimal contract terms into supplier's profit function and we have $\pi_S^p(w_p^*, q_p^*) = 0$. □

Proof of Lemma 2. Here, we present the proof of optimal contract terms when the supplier fails the audit in the no-commitment model. Given the objective function in optimization problem (2.10) is decreasing in w_f and increasing in q_f, the optimal solution happens when the constraint is binding. Therefore, if $0 \leqslant e < \left[1 - \frac{(p-c)D}{\beta}\right]^+$, then we have *no-contracting*. If $\left[1 - \frac{(p-c)D}{\beta}\right]^+ \leqslant e \leqslant 1$, then $(q_f^*, w_f^*) = (D, c + \frac{\beta(1-e)}{D})$. We plug the optimal contract terms into the supplier's profit function and we have $\pi_S^f(w_f^*, q_f^*, e) = 0$. □

Figure 20 summarizes results of Lemma 1 and Lemma 2. It also illustrates the parameter space for the buyer's optimal contracting decisions, where $\emptyset$ means *no-contracting*.

Proof of Proposition 1. We present the proof of NE of efforts in the no-commitment model in here. Based on Lemma 1, Lemma 2, and the fact that $e^* = 0$, for all $D < \frac{\beta}{p-c}$ (which we show in Figure 20), the buyer's profit function in (2.12)

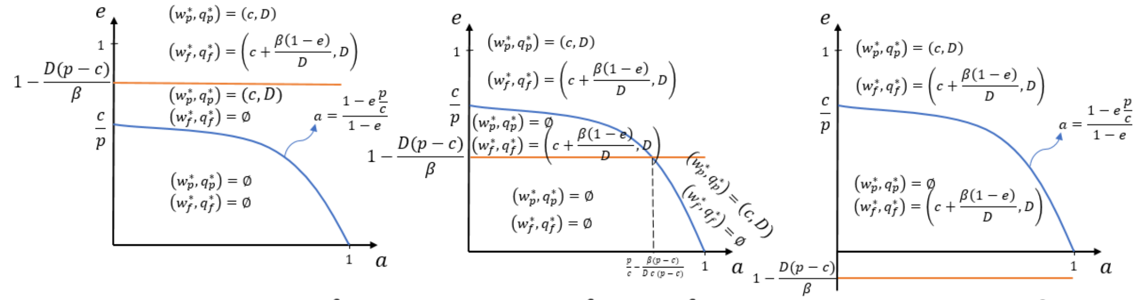


FIGURE 20. Parameter Space for the Buyer's Optimal Contracting Decision for no-commitment

is equal to $-\gamma a$, which is decreasing in a. Therefore, the buyer's best response is zero, $a^* = 0$. If $D \geqslant \frac{\beta}{p-c}$, then the buyer's profit function in optimization program (2.12) is $\pi_B(a, e) = aD(p-c) - a\beta - a\gamma$. This function is linear in a. The first order condition of this function with respect to a is $\frac{\partial \pi_B}{\partial a} = D(p-c) - \beta - \gamma$. It follows from linearity of the function that if $D(p-c) < \beta + \gamma$ then $a^* = 0$, otherwise $a^* = 1$. □

Proof of Lemma 3. The proof of optimal quantity, when the supplier passes the audit in the commitment to wholesale price model, is as follows: the buyer's objective function for the case when the supplier passes the audit is $\frac{e}{e+(1-e)(1-a)} p\, q_p - wq_p = (\frac{e}{e+(1-e)(1-a)} p - w)q_p$, which is linear in q_p. Therefore, when $\frac{1-a}{\frac{p}{w}-a} \leq e \leq 1$, then $q_p^* = D$. Also when $0 \leq e < \frac{1-a}{\frac{p}{w}-a}$ then $q_p^* = 0$. The proof of optimal quantity when the supplier fails the audit is as follows: the buyer's objective function is $(p-w)q_f$. The supplier's participation constraint is $(w-c)q_f - \beta(1-e) \geqslant 0$. Therefore, based on linearity of both objective function and the constraint: if $\left[1 - \frac{(w-c)D}{\beta}\right]^+ \leq e \leq 1$, then the buyer's optimal order quantity is $q_f^* = D$. Also, if $0 \leq e < \left[1 - \frac{(w-c)D}{\beta}\right]^+$, then the buyer does not order, i.e., $q_f^* = 0$. we use the result of Lemma 3 to show a parameter space that we use for proof of next Lemmas. We illustrate this parameter space in Figure 21. We divide our space into 3 regions based on D, p, w, c, and β. In each region, we have different patterns of optimal order quantities (q_p^*, q_f^*). To find the conditions for each region, we plot $e = \frac{1-a}{\frac{p}{w}-a}$ and $e = 1 - \frac{D(w-c)}{\beta}$ on a plane of (a, e). When $a = 0$, we find that $e = \frac{1-a}{\frac{p}{w}-a}$ intercept with axis e at $\frac{w}{p}$. Then, we compare the $\frac{w}{p}$ and $1 - \frac{D(w-c)}{\beta}$ to derive conditions for each regions in Figure 21. When we are forming the profit functions for the buyer and the supplier in next Lemmas, we have different regions in which we have multiple piecewise profit functions based on the different buyer's ordering pattern in each regions of Figure 21. □

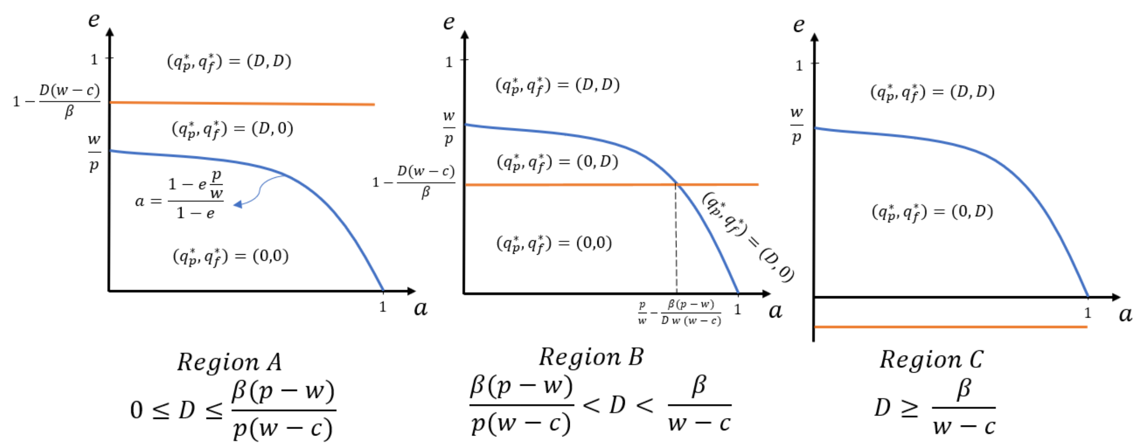


FIGURE 21. Parameter Space based on the Buyer's Optimal Quantity Decision for model of commitment to price

We present the buyer's best response function for commitment to wholesale price in Lemma 15. Next, we present the supplier's best response function for commitment to wholesale price in Lemma 16. We solve for the Nash equilibrium using the best response functions and subsequently present the proof of Lemma 4.

Lemma 15. *When the buyer commits to the wholesale price, for any given supplier's compliance effort e, the buyer's best response $a^*(e)$ is:*

- *Region 1)$a^* = 0$ for any e in $[0, 1]$, the buyer's best response function form 1 (B-BRF 1),*

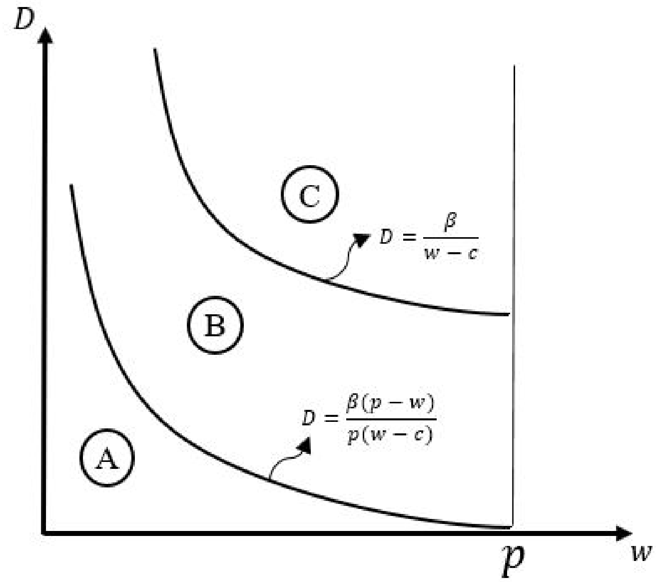


FIGURE 22. Graphical Presentation of Regions of Lemma 3 on plane of (w,D)

$$- \textit{Region 2)} a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e < e_2 \\ \textit{any value in } [0,1] & e = e_2 \\ 0 & e_2 < e \leqslant 1 \end{cases}, \textit{ the buyer's best response}$$

function form 2 (B-BRF 2),

– Region 3) $a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e < e_2 \\ \textit{any value in } [0,1] & e = e_2 \\ 0 & e_2 < e \leqslant e_4 \\ 1 & e_4 < e < e_3 \\ \textit{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}$, *the buyer's best response function form 3 (B-BRF 3),*

– Region 4) $a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e < e_3 \\ \textit{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}$, *the buyer's best response function form 4 (B-BRF 4),*

– Region 5) $a^* = \begin{cases} 0 & 0 \leqslant e < e_4 \\ 1 & e_4 \leqslant e < e_3 \\ \textit{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}$, *the buyer's best response function form 5 (B-BRF 5),*

– Region 6) $a^* = \begin{cases} 1 & 0 \leqslant e < e_3 \\ \textit{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}$, *the buyer's best response function form 6 (B-BRF 6),* *where* $e_1 = \frac{\gamma}{D(p-w)}$, $e_2 = 1 - \frac{\gamma}{Dw}$, $e_3 = 1 - \frac{\gamma}{Dp}$, $e_4 = 1 - \frac{D(w-c)}{\beta}$. *We illustrate regions in Figure 23 and region for each best response function is defined based on following conditions:*

- *Region 1:* $\frac{\beta(p-w)}{p(w-c)} < D < \frac{\gamma}{p-w}$ *or* $\{D < \frac{p\gamma}{(p-w)w}$ *and* $D < \sqrt{\frac{\gamma\beta}{p(w-c)}}$ *and* $D \leqslant \frac{\beta(p-w)}{p(w-c)}\}$.
- *Region 2:* $\frac{p\gamma}{(p-w)w} \leqslant D < \sqrt{\frac{\gamma\beta}{p(w-c)}}$.
- *Region 3:* $\sqrt{\frac{\gamma\beta}{p(w-c)}} \leqslant D < \sqrt{\frac{\gamma\beta}{w(w-c)}}$ *and* $D \geqslant \frac{p\gamma}{(p-w)w}$.
- *Region 4:* $D \geqslant \sqrt{\frac{\gamma\beta}{w(w-c)}}$ *and* $D > \dot{D}(p,w,c,\beta)$, *where* $\dot{D}(p,w,c,\beta)$ *is the solution of* $D(p-w)[1 - \frac{D(w-c)}{\beta}] - \gamma = 0$.
- *Region 5:* $\{\frac{\gamma}{p-w} \leqslant D < \frac{\beta}{w-c}$ *and* $\frac{\beta(p-w)}{p(w-c)} \leqslant D \leqslant \dot{D}(p,w,c,\beta)\}$ *or* $\{\sqrt{\frac{\gamma\beta}{p(w-c)}} \leqslant D < \frac{\beta(p-w)}{p(w-c)}$ *and* $D < \frac{p\gamma}{(p-w)w}\}$.
- *Region 6:* $D \geqslant \frac{\beta}{w-c}$ *and* $D \geqslant \frac{\gamma}{p-w}$.

Also we have the following orders of parameters in each region:

- *In Region 2 and Region 3 (B-BRF 2 and B-BRF 3):* $e_1 < \frac{w}{p} < e_2$.
- *In Region 4 (B-BRF 4):* $e_1 < \frac{w}{p} < e_3$.
- *In Region 5 (B-BRF 5):* $\frac{w}{p} < e_4$.

Proof of Lemma 15. For each region, in which we show in Figure 21, we plug the associated optimal order quantity in optimization $\max_{0\leq a\leq 1} \Pi_B(w_p, q_p^*, w_f, q_f^*, e, a)$ where $w_f = w_p = w$. Then we solve for the buyer's optimal auditing effort a.

Region A: $(0 \leqslant D \leqslant \frac{\beta(p-w)}{p(w-c)})$

For region A, as we show in Figure 21, we have three subregions for a given e:

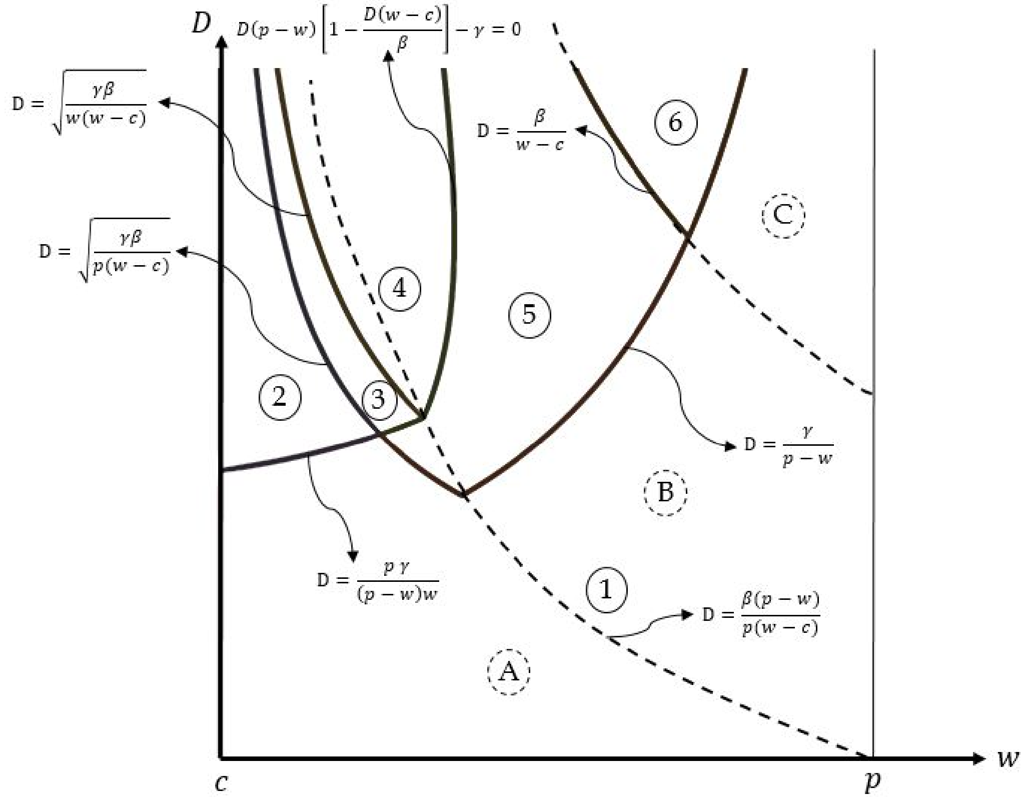


FIGURE 23. The Buyer's Best Response Function in Relation to (D,w)

$0 \leqslant e \leqslant \frac{w}{p}, \frac{w}{p} < e < 1-\frac{D(w-c)}{\beta}$, and $1-\frac{D(w-c)}{\beta} \leqslant e \leqslant 1$. We write the buyer's objective function for each of sub-cases of e .

<u>*Subregion A-i:*</u> $0 \leqslant e \leqslant \frac{w}{p}$, We define $\hat{a}(e,p,w) \stackrel{\text{def}}{=} \frac{1-e\frac{p}{w}}{1-e}$. The buyer's profit in this region is:

$$\Pi_B^{A-i}(a,e) = \begin{cases} -\gamma a & 0 \leqslant a \leqslant \hat{a} \\ eD(p-w)+(1-e)(1-a)(-wD)-\gamma a & \hat{a} < a \leqslant 1 \end{cases}.$$

Whole function is piecewise linear, continuous, and convex in a. The buyer's profit is differentiable in each piece, and its FOC with respect to a is decreasing in a until

$\hat{a}$. Also the FOC of the first piece is smaller than the second piece.

$$\frac{\partial \Pi_B^{A-i}(a,e)}{\partial a} = \begin{cases} -\gamma & 0 \leqslant a \leqslant \hat{a} \\ Dw(1-e)-\gamma & \hat{a} < a \leqslant 1 \end{cases}$$

Following the convexity of function $\Pi_B^{A-i}(a,e)$ in a, by comparing the value of $\Pi_B^{A-i}(a,e)$ at zero, $\Pi_B^{A-i}(a,e)|_{a=0} = 0$ and one, $\Pi_B^{A-i}(a,e)|_{a=1} = De(p-w)-\gamma$, we find that if $e \geqslant \frac{\gamma}{D(p-w)} \Rightarrow a^* = 1$, and if $e < \frac{\gamma}{D(p-w)} \Rightarrow a^* = 0$. We define $e_1 \stackrel{\text{def}}{=} \frac{\gamma}{D(p-w)}$. The optimal a^* depends whether e_1 is less or greater than $\frac{w}{p}$. Therefore, for any given $0 \leqslant e \leqslant \frac{w}{p}$:

$$If \quad e_1 \leqslant \frac{w}{p}\ , \textit{ or equivalently } \ \gamma \leqslant \frac{Dw(p-w)}{p} \quad \textit{then} \quad a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e \leqslant \frac{w}{p} \end{cases}.$$

$$If \quad e_1 > \frac{w}{p}\ , \textit{ or equivalently } \ \gamma > \frac{Dw(p-w)}{p} \quad \textit{then} \quad a^* = 0 \quad \textit{for all} \quad 0 \leqslant e \leqslant \frac{w}{p}.$$

Note that $a^*(e)$ can be discontinuous in e (jump up or jump down in e). For example, the buyer's best response function is not continuous at e_1 in *subregion A-i*. We can observe it when we plug e_1 into $\Pi_B^{A-i}(a,e_1)$:

$$\Pi_B^{A-i}(a,e_1) = \begin{cases} -a\gamma & 0 \leqslant a \leqslant \hat{a}(e_1) \\ (1-a)(\frac{p\gamma}{p-w} - Dw) & \hat{a}(e_1) < a \leqslant 1 \end{cases}.$$

From above function we can see for any value between $(0,1)$, value of $\Pi_B^{A-i}(a,e_1)$ is less than $\Pi_B^{A-i}(a,e_1)|_{a=0}$ and $\Pi_B^{A-i}(a,e_1)|_{a=1}$.

<u>*Subregion A-ii:*</u> $\frac{w}{p} < e < 1 - \frac{D(w-c)}{\beta}$, we define $e_2 \stackrel{\text{def}}{=} 1 - \frac{\gamma}{Dw}$ and $e_4 \stackrel{\text{def}}{=} 1 - \frac{D(w-c)}{\beta}$.

The buyer's profit in this region is:

$$\Pi_B^{A-ii}(a,e) = eD(p-w) + (1-e)(1-a)(-wD) - \gamma a.$$

As $\Pi_B^{A-ii}(a,e)$ is linear in a we compare the values of profit function at zero and one. $\Pi_B^{A-ii}(a,e)|_{a=0} = D(ep-w)$ and $\Pi_B^{A-ii}(a,e)|_{a=1} = De(p-w)-\gamma$. If $e \geqslant 1-\frac{\gamma}{Dw} \Rightarrow$ $\Pi_B^{A-ii}(a,e)|_{a=0} > \Pi_B^{A-ii}(a,e)|_{a=1}$ then $a^*=0$, Otherwise $a^*=1$. Note that, $a^*(e)$ depends on the order of $\frac{w}{p}$, e_2, and e_4. Then for any given $\frac{w}{p} \leqslant e < e_4$:

$$If \quad \frac{w}{p} < e_2 < e_4 \text{ , } or\ equivalently \ \frac{D^2w(w-c)}{\beta} \leqslant \gamma \leqslant \frac{Dw(p-w)}{p} \quad then$$

$$a^* = \begin{cases} 1 & \frac{w}{p} < e < e_2 \\ \text{any value in } [0,1] & e = e_2 \\ 0 & e_2 < e < e_4 \end{cases} .$$

$$If \quad e_2 < \frac{w}{p} < e_4 \text{ , } or\ equivalently \ \gamma > \frac{Dw(p-w)}{p} \quad then \quad a^*=0 \quad for\ all \quad \frac{w}{p} < e < e_4.$$

$$If \quad \frac{w}{p} < e_4 < e_2 \text{ , } or\ equivalently \ 0 < \gamma < \frac{D^2w(w-c)}{\beta} \quad then$$

$$a^* = 1 \quad for\ all \quad \frac{w}{p} < e < e_4.$$

At e_2 in *subregion A-ii*, $\pi_B^{A-ii}(a,e_2)$= $D(p-w) - \frac{p\gamma}{w}$. From this function, we can see for any value of $a \in [0,1]$, value of $\Pi_B^{A-ii}(a,e_2)$ is constant in a. Therefore, $a^*(e_2)$ is all values of $a \in [0,1]$.

Subregion A-iii: $1-\frac{D(w-c)}{\beta} \leqslant e \leqslant 1$, Note that we define $e_4 \overset{\text{def}}{=} 1-\frac{D(w-c)}{\beta}$. The buyer's profit in this region is:

$$\Pi_B^{A-iii}(a,e) = eD(p-w) + (1-e)(1-a)(-wD) + (1-e)aD(p-w) - \gamma a.$$

The function $\Pi_B^{A-iii}(a,e)$ is linear in a. We compare the values of profit function at zero and one. $\Pi_B^{A-iii}(a,e)|_{a=0} = D(ep-w)$ and $\Pi_B^{A-iii}(a,e)|_{a=1} = D(p-w)-\gamma$. If $e \geqslant 1-\frac{\gamma}{Dp}$ then $\Pi_B^{A-iii}(a,e)|_{a=0} > \Pi_B^{A-iii}(a,e)|_{a=1}$ then $a^*=0$, otherwise $a^*=1$. We define $e_3 \stackrel{\text{def}}{=} 1-\frac{\gamma}{Dp}$. The optimal a^* depends on the order of e_4 and e_3.

$$If \quad e_4 \leqslant e_3 \text{ , } or\ equivalently \ \ \gamma > \frac{D^2p(w-c)}{\beta} \quad then$$

$$a^* = \begin{cases} 1 & e_4 \leqslant e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases} .$$

$$If \quad e_3 < e_4 \text{ , } or\ equivalently \ \ 0 < \gamma < \frac{D^2p(w-c)}{\beta} \quad then \quad a^* = 0 \quad for\ all \quad e_4 \leqslant e \leqslant 1.$$

Before we assemble the above conditions to find the buyer's best response function, we show the buyer's best response function is discontinuous at e_4 in the boundary of *subregion A-iii.* We plug e_4 into $\Pi_B^{A-iii}(a,e_4) = [\frac{D^2p(w-c)}{\beta} - \gamma]a + D(p-w) - \frac{D^2p(w-c)}{\beta}$. Therefore, we see for any value between $(0,1)$, value of $\Pi_B^{A-iii}(a,e_4)$ is less than $\Pi_B^{A-iii}(a,e_4)|_{a=0}$ and $\Pi_B^{A-iii}(a,e_4)|_{a=1}$.

We now assemble the results in subregions A-i, A-ii, and A-iii to find the optimal a. The order of e_1, e_2, e_3, and e_4 depends on the value of γ vs. the three thresholds: $\frac{D^2(w-c)w}{\beta}$, $\frac{D^2p(w-c)}{\beta}$, and $\frac{D(p-w)w}{p}$. Note that we always have $\frac{D^2(w-c)w}{\beta} < \frac{D^2p(w-c)}{\beta}$. Recall that *region A* is defined by $D \leqslant \frac{\beta(p-w)}{p(w-c)}$. Under this inequality we have

$\frac{D^2(w-c)w}{\beta} \leqslant \frac{D(p-w)w}{p}$. Then, it remains to compare $\frac{D^2p(w-c)}{\beta}$ vs. $\frac{D(p-w)w}{p}$. We find $\frac{D^2p(w-c)}{\beta} \leqslant \frac{D(p-w)w}{p}$ if and only if $D \leqslant \frac{w\beta(p-w)}{p^2(w-c)}$. Therefore, under $D \leqslant \frac{w\beta(p-w)}{p^2(w-c)}$, we have $\frac{D^2(w-c)w}{\beta} \leqslant \frac{D^2p(w-c)}{\beta} \leqslant \frac{D(p-w)w}{p}$. Otherwise, $\frac{D^2(w-c)w}{\beta} \leqslant \frac{D(p-w)w}{p} < \frac{D^2p(w-c)}{\beta}$. So we can assemble the buyer's best response function in Region A by the value γ vs. the three thresholds.

A-1: *when* $0 \leqslant D < \frac{w\beta(p-w)}{p^2(w-c)}$:

A-1-a: A-1 and $0 < \gamma \leqslant \frac{D^2(w-c)w}{\beta}$.

$$\textit{We have}\ \ e_1 < \frac{w}{p} < e_3 \ \textit{and}\ e_4 < e_2 \ \textit{and}\ e_4 < e_3.$$

$$\textit{Therefore,}\ \ a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}.$$

A-1-b: A-1 and $\frac{D^2(w-c)w}{\beta} < \gamma \leqslant \frac{D^2p(w-c)}{\beta}$.

$$\textit{We have}\ \ e_1 < \frac{w}{p} < e_2 < e_4 < e_3.$$

$$\textit{Therefore,}\quad a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e < e_2 \\ \text{any value in } [0,1] & e = e_2 \\ 0 & e_2 < e \leqslant e_4 \\ 1 & e_4 < e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases} .$$

A-1-c: A-1 and $\frac{D^2 p(w-c)}{\beta} < \gamma \leqslant \frac{D(p-w)w}{p}$.

$$\textit{We have}\quad e_1 < \frac{w}{p} < e_2 < e_4 \textit{ and } e_3 < e_4.$$

$$\textit{Therefore,}\quad a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e < e_2 \\ \text{any value in } [0,1] & e = e_2 \\ 0 & e_2 < e \leqslant 1 \end{cases} .$$

A-1-d: A-1 and $\frac{D(p-w)w}{p} < \gamma$.

$$\textit{We have}\quad e_2 < \frac{w}{p} < e_1 \textit{ and } e_3 < e_4. \textit{ Therefore,}\quad a^* = 0.$$

A-2: *when* $\frac{w\beta(p-w)}{p^2(w-c)} \leqslant D \leqslant \frac{\beta(p-w)}{p(w-c)}$:

A-2-a: A-2 and $0 < \gamma \leqslant \frac{D^2(w-c)w}{\beta}$.

$$\textit{We have} \;\; e_1 < \frac{w}{p} < e_3 \textit{ and } e_4 < e_2 \textit{ and } e_4 < e_3.$$

$$\textit{Therefore,} \;\; a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}.$$

A-2-b: A-2 and $\frac{D^2(w-c)w}{\beta} < \gamma \leqslant \frac{D(p-w)w}{p}$.

$$\textit{We have} \;\; e_1 < \frac{w}{p} < e_2 < e_4 < e_3.$$

$$\textit{Therefore,} \;\; a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e < e_2 \\ \text{any value in } [0,1] & e = e_2 \\ 0 & e_2 < e \leqslant e_4 \\ 1 & e_4 < e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}.$$

A-2-c: A-2 and $\frac{D(p-w)w}{p} < \gamma \leqslant \frac{D^2p(w-c)}{\beta}$.

$$We\ have\ \ e_2 < \frac{w}{p} < e_1\ and\ \frac{w}{p} < e_4 < e_3.$$

$$Therefore,\ \ a^* = \begin{cases} 0 & 0 \leqslant e < e_4 \\ 1 & e_4 \leqslant e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}.$$

A-2-d: A-2 and $\frac{D^2p(w-c)}{\beta} < \gamma$.

$$We\ have\ \ e_2 < \frac{w}{p} < e_1\ and\ e_3 < e_4.\ Therefore,\ \ a^* = 0.$$

Region B: $(\frac{\beta(p-w)}{p(w-c)} < D < \frac{\beta}{w-c})$

For region B, as we show in Figure 21, we have three subregions for a given e: $0 \leqslant e \leqslant 1 - \frac{D(w-c)}{\beta}, 1 - \frac{D(w-c)}{\beta} < e < \frac{w}{p}$, and $\frac{w}{p} \leqslant e \leqslant 1$. We write the buyer's objective function for each of sub-cases of e . Note that we previously defined $e_4 = 1 - \frac{D(w-c)}{\beta}$.

Subregion B-i: $0 \leqslant e \leqslant e_4$, the buyer's profit is:

$$\Pi_B^{B-i}(a,e) = \begin{cases} -\gamma a & 0 \leqslant a \leqslant \hat{a} \\ eD(p-w) + (1-e)(1-a)(-wD) - \gamma a & \hat{a} < a \leqslant 1 \end{cases}.$$

Whole function is piecewise linear, continuous, and convex in a. The buyer's profit is differentiable in each piece and its FOC with respect to a is decreasing in a until $\hat{a}$ and it may be increasing or decreasing in a.

$$\frac{\partial \Pi_B^{B-i}(a,e)}{\partial a} = \begin{cases} -\gamma & 0 \leqslant a \leqslant \hat{a} \\ Dw(1-e)-\gamma & \hat{a} < a \leqslant 1 \end{cases}$$

We compare the value of function at zero and one. $\Pi_B^{B-i}(a,e)|_{a=0} = 0$ and $\Pi_B^{B-i}(a,e)|_{a=1} = De(p-w)-\gamma$. We find that if $e \leqslant \frac{\gamma}{D(p-w)} \Rightarrow a^* = 1$ and if $e > \frac{\gamma}{D(p-w)} \Rightarrow a^* = 0$. Note that we previously defined $e_1 = \frac{\gamma}{D(p-w)}$; then for any given $0 \leqslant e \leqslant e_4$:

$$If \quad 0 < e_1 < e_4 \text{ , } \textit{or equivalently } \; 0 < \gamma \leqslant D(p-w)[1-\frac{D(w-c)}{\beta}] \quad then$$

$$a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e \leqslant e_4 \end{cases}.$$

$$If \quad e_1 \geqslant e_4 \text{ , } \textit{or equivalently } \; \gamma > D(p-w)[1-\frac{D(w-c)}{\beta}] \quad then$$

$$a^* = 0 \quad \textit{for all} \quad 0 \leqslant e \leqslant e_4.$$

Note that $a^*(e)$ may jump up or jump down in e. For example, the buyer's best response function is not continuous at e_1 in *subregion B-i*, simply we can observe it when we plug e_1 into $\Pi_B^{B-i}(a,e_1)$:

$$\Pi_B^{B-i}(a,e_1) = \begin{cases} -a\gamma & 0 \leqslant a \leqslant \hat{a}(e_1) \\ (1-a)(\frac{p\gamma}{p-w}-Dw) & \hat{a}(e_1) < a \leqslant 1 \end{cases}.$$

From above function we can see for any value between $(0,1)$, value of $\Pi_B^{B-i}(a,e_1)$ is less than $\Pi_B^{B-i}(a,e_1)|_{a=0}$ and $\Pi_B^{B-i}(a,e_1)|_{a=1}$. Similarly, we can show that the buyer's best response function discontinuity at e_4 in subregion B-i with plug e_4 into $\Pi_B^{B-i}(a,e_4)$:

$$\Pi_B^{B-i}(a,e_4) = \begin{cases} -a\gamma & 0 \leqslant a \leqslant \hat{a}(e_4) \\ D(p-w) - \frac{D^2(w-c)(p-aw)}{\beta} - \gamma a & \hat{a}(e_4) < a \leqslant 1 \end{cases}.$$

From above function we can see for any value between $(0,1)$, value of $\Pi_B^{B-i}(a,e_4)$ is less than $\Pi_B^{B-i}(a,e_4)|_{a=0}$ and $\Pi_B^{B-i}(a,e_4)|_{a=1}$.

Subregion B-ii: $e_4 < e < \frac{w}{p}$, the buyer's profit is:

$$\Pi_B^{B-ii}(a,e) = \begin{cases} (1-e)aD(p-w) - \gamma a & 0 \leqslant a \leqslant \hat{a} \\ eD(p-w) + (1-e)(1-a)(-wD) + (1-e)aD(p-w) - \gamma a & \hat{a} < a \leqslant 1 \end{cases}.$$

Whole function is piecewise linear and continuous in a. The buyer's profit is differentiable in each piece and slope of the second piece is greater than slope of the first piece .

$$\frac{\partial \Pi_B^{B-ii}(a,e)}{\partial a} = \begin{cases} D(1-e)(p-w) - \gamma & 0 \leqslant a \leqslant \hat{a} \\ Dp(1-e) - \gamma & \hat{a} < a \leqslant 1 \end{cases}$$

Similar to above regions, we compare the value $\Pi_B^{B-ii}(a,e)|_{a=0} = 0$ and $\Pi_B^{B-ii}(a,e)|_{a=1} = D(p-w) - \gamma$. By comparing these two values we find for any given e while $e_4 < e < \frac{w}{p}$:

$$If\ \gamma \leqslant D(p-w) \quad then \quad a^* = 1 \quad for\ all \quad e_4 < e < \frac{w}{p}.$$

$$If\ \gamma > D(p-w) \quad then \quad a^* = 0 \quad for\ all \quad e_4 < e < \frac{w}{p}.$$

Subregion B-iii: $\frac{w}{p} \leqslant e \leqslant 1$, the buyer's profit is:

$$\Pi_B^{B-iii}(a,e) = eD(p-w) + (1-e)(1-a)(-wD) + (1-e)aD(p-w) - \gamma a,$$
$$for\ all \quad 0 \leqslant a \leqslant 1.$$

This function is linear in a and $\frac{\partial \Pi_B^{B-iii}(a,e)}{\partial a} = Dp(1-e) - \gamma$. Similar to above subregions we compare the value of function at zero and one. $\Pi_B^{B-iii}(a,e)|_{a=0} = D(ep-w)$ and $\Pi_B^{B-iii}(a,e)|_{a=1} = D(p-w) - \gamma$. By comparing these two values we find that if $e \leqslant 1 - \frac{\gamma}{Dp} \Rightarrow a^* = 1$ and if $e > 1 - \frac{\gamma}{Dp} \Rightarrow a^* = 0$. As we previously defined $e_3 = 1 - \frac{\gamma}{Dp}$ then for any given $\frac{w}{p} \leqslant e \leqslant 1$:

$$If \quad \frac{w}{p} \leqslant e_3 \leqslant 1\ ,\ or\ equivalently\ \ 0 < \gamma \leqslant D(p-w)$$

$$then \quad a^* = \begin{cases} 1 & \frac{w}{p} \leqslant e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}.$$

$$If \quad e_3 < \frac{w}{p} < 1\ ,\ or\ equivalently\ \ \gamma > D(p-w) \quad then \quad a^* = 0 \quad for\ all \quad \frac{w}{p} \leqslant e \leqslant 1.$$

We assemble above subregions to form the general buyer's best response function for *Region B*. Recall that *Region B* is defined by $\frac{\beta(p-w)}{p(w-c)} < D < \frac{\beta}{w-c}$. Therefore, the γ

thresholds that we find for B-i, B-ii, and B-iii follow this order: $D(p-w)[1-\frac{D(w-c)}{\beta}] < D(p-w)$. We assemble the above cases based on this characteristic and we label them by $B-1$ to $B-3$:

$$B-1: \quad 0 < \gamma \leqslant D(p-w)[1-\frac{D(w-c)}{\beta}], \ we\ have \ \ e_1 < e_4$$

$$and\ e_1 < \frac{w}{p} < e_3.$$

$$Therefore, \ \ a^* = \begin{cases} 0 & 0 \leqslant e < e_1 \\ 1 & e_1 \leqslant e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}.$$

$$B-2: \quad D(p-w)[1-\frac{D(w-c)}{\beta}] < \gamma \leqslant D(p-w), \ we\ have \ \ e_4 < e_1$$

$$and\ e_4 < \frac{w}{p} < e_3.$$

$$Therefore, \ \ a^* = \begin{cases} 0 & 0 \leqslant e < e_4 \\ 1 & e_4 \leqslant e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases}.$$

$$B-3: \quad \gamma > D(p-w), \ we\ have \ \ e_4 < e_1 \ \ and\ e_3 < \frac{w}{p}.$$

$$Therefore, \ \ a^* = 0 \quad for\ all \quad 0 \leqslant e \leqslant 1.$$

Region C: $(D \geqslant \frac{\beta}{w-c})$

For region C, as we show in Figure 21 we have two subregions for a given e: $0 \leqslant e \leqslant \frac{w}{p}$, and $\frac{w}{p} < e \leqslant 1$. We write the buyer's objective function for each of sub-cases of

e .

<u>*Subregion C-i:*</u> $0 \leqslant e \leqslant \frac{w}{p}$, the buyer's profit is:

$$\Pi_B^{C-i}(a,e) = \begin{cases} (1-e)aD(p-w) - \gamma a & 0 \leqslant a \leqslant \hat{a} \\ eD(p-w) + \lambda - \gamma a & \hat{a} < a \leqslant 1 \end{cases}.$$

where, $\lambda = (1-e)(1-a)(-wD) + (1-e)aD(p-w)$ .Whole function is piecewise linear and continuous in a. The buyer's profit is differentiable in each piece and slope of the second piece is greater than slope of the first piece.

$$\frac{\partial \Pi_B^{C-i}(a,e)}{\partial a} = \begin{cases} D(1-e)(p-w) - \gamma & 0 \leqslant a \leqslant \hat{a} \\ Dp(1-e) - \gamma & \hat{a} < a \leqslant 1 \end{cases}$$

We compare the value of $\Pi_B^{C-i}(a,e)$ at zero and one. $\Pi_B^{C-i}(a,e)|_{a=0} = 0$ and $\Pi_B^{C-i}(a,e)|_{a=1} = D(p-w) - \gamma$. By comparing these two values we find for any given e while $0 \leqslant e \leqslant \frac{w}{p}$:

$$If\ \gamma \leqslant D(p-w) \quad then \quad a^* = 1 \quad for\ all \quad 0 \leqslant e \leqslant \frac{w}{p}.$$

$$If\ \gamma > D(p-w) \quad then \quad a^* = 0 \quad for\ all \quad 0 \leqslant e \leqslant \frac{w}{p}.$$

<u>*Subregion C-ii:*</u> $\frac{w}{p} < e \leqslant 1$, the buyer's profit is:

$$\Pi_B^{C-ii}(a,e) = eD(p-w) + (1-e)(1-a)(-wD) + (1-e)aD(p-w) - \gamma a \quad for\ all \quad 0 \leqslant a \leqslant 1.$$

This function is linear in a and $\frac{\partial \Pi_B^{C-ii}(a,e)}{\partial a} = Dp(1-e) - \gamma$. We compare the value of $\Pi_B^{C-ii}(a,e)$ at zero and one. $\Pi_B^{C-ii}(a,e)|_{a=0} = D(ep-w)$ and $\Pi_B^{C-ii}(a,e)|_{a=1} =$

$D(p-w)-\gamma$. By comparing these two values we find that if $e \leqslant 1-\frac{\gamma}{Dp} \Rightarrow a^*=1$ and if $e > 1-\frac{\gamma}{Dp} \Rightarrow a^*=0$. As we previously set $1-\frac{\gamma}{Dp}$ equal to e_3, then for any given $\frac{w}{p} \leqslant e \leqslant 1$:

$$If \quad \frac{w}{p} \leqslant e_3 \leqslant 1 \text{ , } or\ equivalently \;\; 0<\gamma \leqslant D(p-w) \quad then$$

$$a^* = \begin{cases} 1 & \frac{w}{p} < e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases} .$$

$$If \quad e_3 < \frac{w}{p} < 1 \text{ , } or\ equivalently \;\; \gamma > D(p-w) \quad then \quad a^*=0 \quad for\ all \quad \frac{w}{p} \leqslant e \leqslant 1.$$

We can assemble above subregions to form the general buyer's best response function for *Region C*. We label these regions by $C-1$ and $C-2$:

$$C-1: \quad if \quad 0<\gamma \leqslant D(p-w) \;\; then\ a^* = \begin{cases} 1 & 0 \leqslant e < e_3 \\ \text{any value in } [0,1] & e = e_3 \\ 0 & e_3 < e \leqslant 1 \end{cases} .$$

$$C-2: \quad if \quad \gamma > D(p-w) \;\; then\ a^*=0 \quad \text{for all} \quad e \in [0,1].$$

We put together final conditions of *region A,region B, and region C* and we draw them on the plane of D and w as we show in Figure 23.

One can see that the buyer's best response structure is the same in the following regions. Therefore, we combine these regions into regions 1 through 6 using the following process:

- Region 1: A-1-d ∪ A-2-d ∪ B-3 ∪ C-2

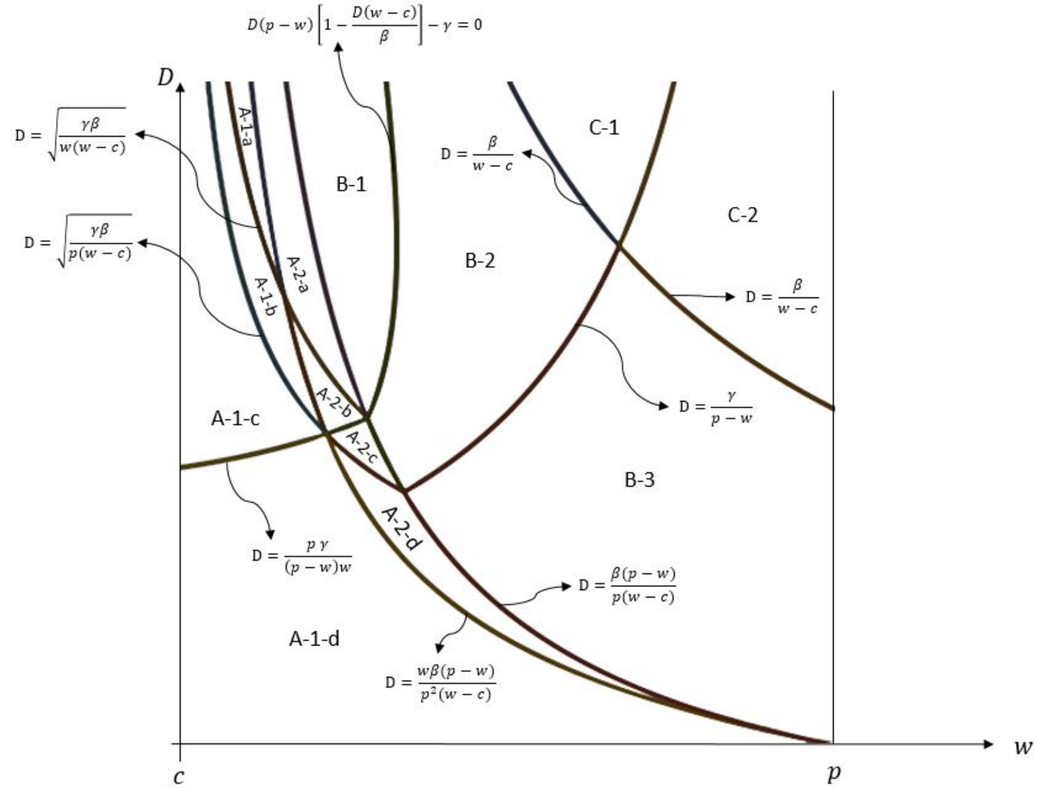


FIGURE 24. The Buyer's Best Response Function Parameter Space Based on Demand and Wholesale Price

- Region 2: A-1-c
- Region 3: A-1-b ∪ A-2-b
- Region 4: A-1-a ∪ A-2-a ∪ B-1
- Region 5: A-2-c ∪ B-2
- Region 6: C-1

We show these new regions in Figure 23. Hence, we prove the Lemma. □

Lemma 16. *For any given buyer's auditing effort a, the supplier's best response (S-BRF),* $e^*(a)$ *is as follows for each of the regions in Figure 25:*

- *In region A-i, $e^*(a)$ is in one of the following forms: S-BRF 1-1 or S-BRF 1-2;*
- *In region A-ii, $e^*(a)$ has a unique form of S-BRF 2-2. Note that when $c > \frac{p(2\beta-\alpha)^2}{4\beta^2}$, then region A-ii vanishes;*
- *In region B-i, $e^*(a)$ is in one of the following forms: S-BRF 1-1, S-BRF 1-2, S-BRF 1-3, S-BRF 1-4, or S-BRF 1-5;*
- *In region B-ii, $e^*(a)$ is in one of the following forms: S-BRF 2-1, S-BRF 2-3, S-BRF 2-4, or S-BRF 2-5;*
- *In region C, $e^*(a)$ is S-BRF 2-1,*

where the possible forms of S-BRF $e^*(a)$ are:

S-BRF 1-1) $e^* = 0$ *for all* a *in* $[0, 1]$.

S-BRF 1-2) $e^* = \hat{e}$ *for all* a *in* $[0, 1]$.

$$\textit{S-BRF 1-3)} \quad e^* = \begin{cases} 0 & 0 \leqslant a < a_2^{B-I} \\ \hat{e} & a_2^{B-I} \leqslant a < a_1^{B-I} \\ 0 & a_1^{B-II} \leqslant a \leqslant 1 \end{cases}.$$

$$\textit{S-BRF 1-4)} \quad e^* = \begin{cases} 0 & 0 \leqslant a < a_2^{B-I} \\ \hat{e} & a_2^{B-I} \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a < \frac{\alpha^2}{4\beta(\alpha-D(w-c))} \\ 0 & \frac{\alpha^2}{4\beta(\alpha-D(w-c))} \leqslant a \leqslant 1 \end{cases}.$$

$$\textit{S-BRF 1-5)} \quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a < a_2^{B-II} \\ \hat{e} & a_2^{B-II} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}1)\quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}2)\quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < \dot{a} \\ \bar{e} & \dot{a} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}3)\quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < a_1^{B-II} \\ \bar{e} & a_1^{B-II} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}4)\quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a < a_2^{B-II} \\ \hat{e} & a_2^{B-II} \leqslant a < a_1^{B-II} \\ \bar{e} & a_1^{B-II} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}5)\quad e^* = \begin{cases} 0 & 0 \leqslant a < a_2^{B-I} \\ \hat{e} & a_2^{B-I} \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant 1 \end{cases}.$$

Also, the above-mentioned regions are illustrated in Figure 25 and defined as follows:

- *Region A-i:* $\{c < w \leqslant \frac{p(2\beta-\alpha)^2}{4\beta^2}$ *and* $D \leqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}\}$ *or* $\{\frac{p(2\beta-\alpha)^2}{4\beta^2} < w < p$ *and* $D \leqslant \frac{\beta(p-w)}{p(w-c)}\}$.
- *Region A-ii:* $\{c < w \leqslant \frac{p(2\beta-\alpha)^2}{4\beta^2}$ *and* $\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < D \leqslant \frac{\beta(p-w)}{p(w-c)}\}$.
- *Region B-i:* $\frac{p(2\beta-\alpha)^2}{4\beta^2} < w < p$ *and* $\frac{\beta(p-w)}{p(w-c)} < D \leqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$.

– *Region B-ii:* $\{c < w \leqslant \frac{p(2\beta-\alpha)^2}{4\beta^2}$ *and* $\frac{\beta(p-w)}{p(w-c)} < D < \frac{\beta}{w-c}\}$ *or* $\{\frac{p(2\beta-\alpha)^2}{4\beta^2} < w < p$ *and* $\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < D < \frac{\beta}{w-c}\}$.

– *Region C:* $D \geqslant \frac{\beta}{w-c}$.

In addition, $\hat{e} = \frac{1-a}{\frac{p}{w}-a}$, $\bar{e} = 1 - \frac{\alpha}{2a\beta}$,
$\dot{a} = \frac{\alpha(4\beta(p-w)+w\alpha+\sqrt{w^2\alpha^2+8(p-w)(w\alpha-2Dp(w-c))\beta+16(p-w)^2\beta^2})}{8D(p-w)(w-c)\beta}$,
$a_1^{B-I} = \frac{-(w^2\alpha-pw(2D(w-c)-\alpha)-\beta(p-w)^2)-\sqrt{\phi}}{2(w^2(D(w-c)-\alpha))}$,
$a_2^{B-I} = \frac{-(w^2\alpha-pw(2D(w-c)-\alpha)-\beta(p-w)^2)+\sqrt{\phi}}{2(w^2(D(w-c)-\alpha))}$,
$a_1^{B-II} = \frac{-(-[4\alpha\beta(p-w)+w\alpha^2])+\sqrt{[(-[4\alpha\beta(p-w)+w\alpha^2])^2-4(4D\beta(p-w)(w-c))(p\alpha^2)]}}{2(4D\beta(p-w)(w-c))}$,
$a_2^{B-II} = \frac{-(-[4\alpha\beta(p-w)+w\alpha^2])-\sqrt{[(-[4\alpha\beta(p-w)+w\alpha^2])^2-4(4D\beta(p-w)(w-c))(p\alpha^2)]}}{2(4D\beta(p-w)(w-c))}$, *where* $\phi = ((w^2\alpha - pw(2D(w-c)-\alpha)-\beta(p-w)^2)^2 - 4(w^2(D(w-c)-\alpha))(Dp^2(w-c)-pw\alpha))$.
Also, the supplier's best response function is continuous at $a = \frac{p\alpha}{2\beta(p-w)+w\alpha}$ *and is not continuous at* $\frac{\alpha^2}{4\beta(\alpha-D(w-c))}$, $\dot{a}$, a_2^{B-I}, a_2^{B-II}, *and* a_1^{B-II}.

Proof of Lemma 16. Similar to the buyer's best response function, for each region of Figure 21, we plug the associated optimal order quantity in optimization $\max_{0\leq e\leq 1}\Pi_S(w_p, q_p^*, w_f, q_f^*, e, a)$, where $w_f = w_p = w$. Then, solve for the supplier's optimal compliance effort e. We start with region A, then region B, and finally we find the e^* in region C.

Region A: $0 \leqslant D \leqslant \frac{\beta(p-w)}{p(w-c)}$

For region A of Figure 21, the supplier's profit $\Pi_S^A(a,e)$ is as following. Note that we define $\hat{e} \stackrel{\text{def}}{=} \frac{1-a}{\frac{p}{w}-a}$. Note that $\hat{e}$ is the inverse function of $\hat{a}$ in Lemma 3.

$$\Pi_S^A(a,e) = \begin{cases} -\alpha e & 0 \leqslant e < \hat{e} \\ [e+(1-e)(1-a)]D(w-c) - \alpha e & \hat{e} \leqslant e < e_4 \\ [e+(1-e)(1-a)]D(w-c) + \Lambda - \alpha e & e_4 \leqslant e \leqslant 1 \end{cases} \tag{A.1}$$

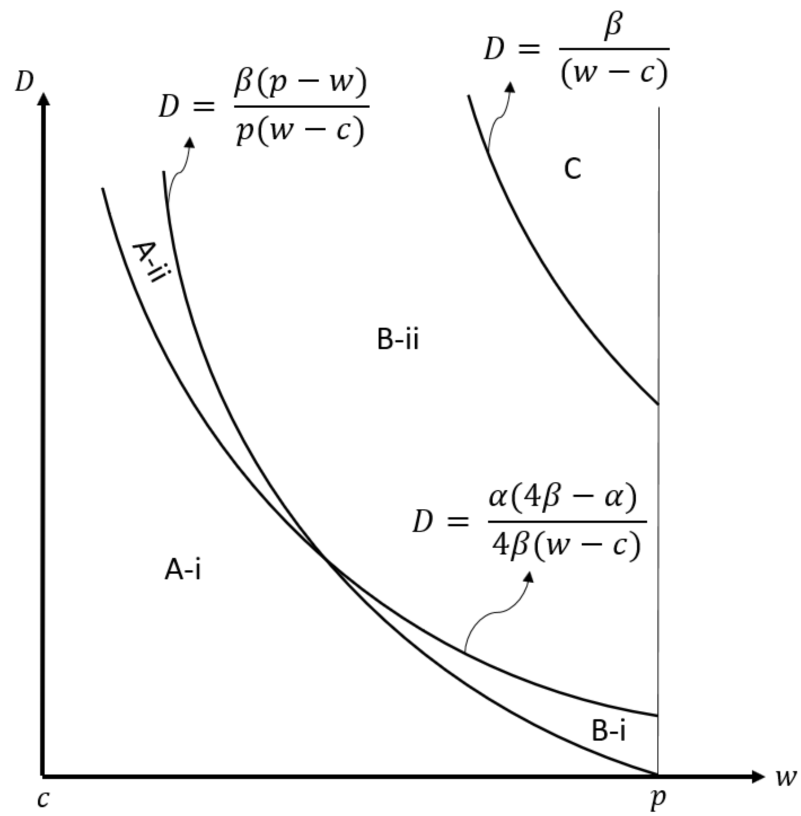


FIGURE 25. Different Cases for the Supplier's Best Response Function Based on plane of (w,D)

where $\Lambda = (1-e)a[D(w-c) - \beta(1-e)]$. The function $\Pi_S^A(a,e)$ is not continuous at $\hat{e}$, because $\lim_{e \to \hat{e}^-} \Pi_S^A = -\dfrac{(1-a)w\alpha}{p-aw}$ and $\Pi_S^A(a,e)|_{e=\hat{e}} = -\frac{(1-a)(w\alpha - Dp(w-c))}{p-aw}$, while $\Pi_S^A(a,e)|_{e=\hat{e}} > \lim_{e \to \hat{e}^-} \Pi_S^A$. Also, the supplier's profit function which is shown in (A.1) is continuous at e_4. Derivative of the supplier's profit with respect to e is:

$$\frac{\partial \Pi_S^A(a,e)}{\partial e} = \begin{cases} -\alpha & 0 \leqslant e < \hat{e} \\ Da(w-c) - \alpha & \hat{e} \leqslant e < e_4 \\ 2a(1-e)\beta - \alpha & e_4 \leqslant e \leqslant 1 \end{cases} \tag{A.2}$$

From (A.2) we find that at $e = e_4$ between the 2nd and 3rd intervals, the derivative increasing across the boundary, or equivalently, $\frac{\partial \Pi_S^A}{\partial e}|_{e=(e_4)^-} < \frac{\partial \Pi_S^A}{\partial e}|_{e=e_4}$ also $\frac{\partial \Pi_S^A}{\partial e}|_{e=1} = -\alpha < 0$. Also, the third piece of function (A.1) is concave in e. Therefore, there are three candidate optimal solutions: $e = 0$, $e = \hat{e}$, and a local optimal in the third interval ($e = \bar{e} = 1 - \frac{\alpha}{2a\beta}$).

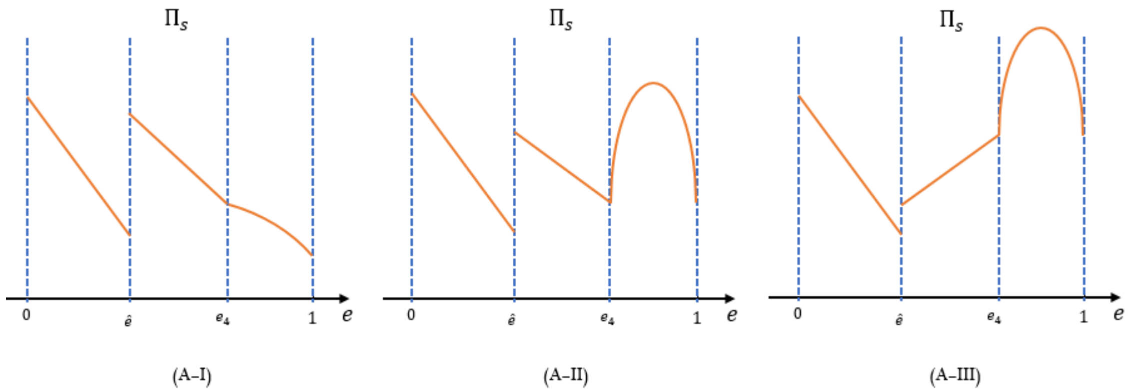


FIGURE 26. Possible Forms of the Supplier's Profit Function in Region A

Now we can divide the *Region A* of the supplier's problem to three sub regions based on order of zero, $\frac{\partial \Pi_S^A}{\partial e}|_{e=(e_4)^-}$, and $\frac{\partial \Pi_S^A}{\partial e}|_{e=e_4}$. We show graphically these three cases in Figure 26. Three cases are:

I) $\frac{\partial \Pi_S^A}{\partial e}|_{e=(e_4)^-} \leqslant \frac{\partial \Pi_S^A}{\partial e}|_{e=e_4} \leqslant 0$,

II) $\frac{\partial \Pi_S^A}{\partial e}|_{e=(e_4)^-} \leqslant 0 < \frac{\partial \Pi_S^A}{\partial e}|_{e=e_4}$, and

III) $0 < \frac{\partial \Pi_S^A}{\partial e}|_{e=(e_4)^-} \leqslant \frac{\partial \Pi_S^A}{\partial e}|_{e=e_4}$.

Before we analyze A-I, A-II, and A-III, we present all sub-regions in them and their associated candidate solutions in Table 1.

Sub-Region	Candidate Solutions
A-I	0, and $\hat{e}$
A-II	0, $\hat{e}$, and $\bar{e}$
A-III	0, and $\bar{e}$

TABLE 1. Different Subregions in Region A and Associated Candidate Solutions

In our analysis, we assume that at all levels of compliance effort e, sum of the compliance effort cost and the associated correction cost to subsequently become compliant (after the fact) is greater than the cost of being fully complaint upfront. This means $\alpha e + \beta(1-e) > \alpha$, therefore $\alpha < \beta$.

<u>*Subregion A-I:*</u>

$$\frac{\partial \Pi_S^A}{\partial e}|_{e=(e_4)^-} \leqslant \frac{\partial \Pi_S^A}{\partial e}|_{e=e_4} \leqslant 0 \tag{A.3}$$

or equivalently, $Da(w-c) - \alpha \leqslant 2Da(w-c) - \alpha \leqslant 0$.

In this case as we show in Figure 26, Π_S^A is decreasing in both intervals of $e \in (0, \hat{e})$ and $e \in (\hat{e}, 1)$. So, we compare the value of function at 0 and $\hat{e}$. $\Pi_S^A|_{e=0} = 0$ and $\Pi_S^A|_{e=\hat{e}} = \frac{(1-a)(Dp(w-c)-w\alpha)}{p-aw}$. From $\Pi_S^A|_{e=\hat{e}} < \pi_S^A|_{e=0}$ we have:

$$\frac{(1-a)(Dp(w-c)-w\alpha)}{p-aw} < 0 \Leftrightarrow Dp(w-c) - w\alpha < 0 \Leftrightarrow p < \frac{w\alpha}{D(w-c)}.$$

We rewrite the above condition using the definition of e_4:

$$p < \frac{w\alpha}{D(w-c)} \Leftrightarrow p < \frac{w\alpha}{\beta(1-(1-\frac{D(w-c)}{\beta}))} \Leftrightarrow p < \frac{w\alpha}{\beta(1-e_4)}. \tag{A.4}$$

The main condition of Region A is $D \leqslant \frac{\beta(p-w)}{p(w-c)}$, we rewrite it this condition as:

$$\frac{w}{1-\frac{D(w-c)}{\beta}} \leqslant p \Leftrightarrow \frac{w}{e_4} \leqslant p. \tag{A.5}$$

Therefore, from (A.4) and (A.5) we conclude that in Region A, the supplier's profit at $e = 0$ is greater the supplier's profit at $e = \hat{e}$ if and only if $\frac{w}{e_4} \leqslant p < \frac{w\alpha}{\beta(1-e_4)}$. For this condition to be valid we must have $\frac{w}{e_4} \leqslant \frac{w\alpha}{\beta(1-e_4)}$:

$$\frac{w}{e_4} \leqslant \frac{w\alpha}{\beta(1-e_4)} \Leftrightarrow D \leqslant \frac{\alpha\beta}{(w-c)(\alpha+\beta)}. \tag{A.6}$$

Now we consider the condition in Region A-I, if condition (A.3) holds then $a \leqslant \frac{\alpha}{2D(w-c)}$. Note that $\frac{\alpha}{2D(w-c)}$ can be less or greater than 1 and $0 \leqslant a \leqslant 1$. We find the condition for order of $\frac{\alpha}{2D(w-c)}$ vs. 1:

$$\text{If } \frac{\alpha}{2D(w-c)} \leqslant 1, \text{ then } D \geqslant \frac{\alpha}{2(w-c)}. \tag{A.7}$$

$$\text{If } \frac{\alpha}{2D(w-c)} > 1, \text{ then } D < \frac{\alpha}{2(w-c)}. \tag{A.8}$$

Also, by definition of our parameters, we have this relationship between boundaries of (A.7) and (A.6): $\frac{\alpha}{2(w-c)} < \frac{\alpha\beta}{(w-c)(\alpha+\beta)}$. Therefore, if (A.7) holds, then for all a in $[0, \frac{\alpha}{2D(w-c)}]$ $e^* = 0$. If (A.8) holds, then for all a in $[0,1]$ $e^* = 0$. A-1 and A-2 summarize conditions of $\pi_S^A|_{e=0} > \Pi_S^A|_{e=\hat{e}}$:

$$\left.\begin{array}{l} A-1: \quad if \quad \dfrac{w}{e_4} \leqslant p \leqslant \dfrac{w\alpha}{\beta(1-e_4)} \quad \text{and} \\ \qquad\qquad 0 \leqslant D \leqslant \dfrac{\alpha}{2(w-c)} \\ \qquad\qquad \text{for all} \quad 0 \leqslant a \leqslant 1 \\ A-2: \quad if \quad \dfrac{w}{e_4} \leqslant p \leqslant \dfrac{w\alpha}{\beta(1-e_4)} \quad \text{and} \\ \qquad\qquad \dfrac{\alpha}{2(w-c)} \leqslant D \leqslant \dfrac{\alpha\beta}{(w-c)(\alpha+\beta)} \\ \qquad\qquad \text{for all} \quad 0 \leqslant a \leqslant \dfrac{\alpha}{2D(w-c)} \end{array}\right\} \Longrightarrow e^* = 0 \; .$$

If the condition (A.3) holds and $\pi_S^A|_{e=0} \leqslant \Pi_S^A|_{e=\hat{e}}$ then $e^* = \hat{e}$. Similar to the above case, we derive the conditions based on our model parameters. A-3 to A-5 present condition for $\pi_S^A|_{e=0} \leqslant \pi_S^A|_{e=\hat{e}}$:

$$\left.\begin{array}{llll} A-3: & if & \frac{w}{e_4} \leqslant p \quad \text{and} \quad \frac{\alpha\beta}{(w-c)(\alpha+\beta)} \leqslant D \leqslant \frac{\beta}{w-c} \\ & & \text{for all} \quad 0 \leqslant a \leqslant \frac{\alpha}{2D(w-c)} \\ A-4: & if & \frac{w\alpha}{\beta(1-e_4)} \leqslant p \quad \text{and} \quad \frac{\alpha}{2(w-c)} \leqslant D \leqslant \frac{\alpha\beta}{(w-c)(\alpha+\beta)} \\ & & \text{for all} \quad 0 \leqslant a \leqslant \frac{\alpha}{2D(w-c)} \\ A-5: & if & \frac{w\alpha}{\beta(1-e_4)} \leqslant p \quad \text{and} \quad 0 \leqslant D \leqslant \frac{\alpha}{2(w-c)} \\ & & \text{for all} \quad 0 \leqslant a \leqslant 1 \end{array}\right\} \implies e^* = \hat{e}.$$

We can re-write the condition A-3 and label them as:

A-3-i) $if \ \frac{w}{e_4} \leqslant p$ and $\frac{\alpha\beta}{(w-c)(\alpha+\beta)} \leqslant D \leqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$ for all $0 \leqslant a \leqslant \frac{\alpha}{2D(w-c)}$.

A-3-ii) $if \ \frac{w}{e_4} \leqslant p$ and $\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \leqslant D \leqslant \frac{\beta}{w-c}$ for all $0 \leqslant a \leqslant \frac{\alpha}{2D(w-c)}$.

Subregion A-II:

$$\frac{\partial \Pi_S^A}{\partial e}|_{e=(e_4)^-} \leqslant 0 < \frac{\partial \Pi_S^A}{\partial e}|_{e=e_4} \tag{A.9}$$

or equivalently, $Da(w-c) - \alpha \leqslant 0 < 2Da(w-c) - \alpha$.

In this case, as $\frac{\partial \pi_S^A}{\partial e}|_{e=e_4} > 0$ and $\frac{\partial \pi_S^A}{\partial e}|_{e=1} = -\alpha < 0$, thus we can use concavity characterization of third piece of $\pi_S^A(a, e)$ which implies one local maximum($\bar{e} = 1 - \frac{\alpha}{2a\beta}$) for function (A.1). In addition we have two more local maxima; we know that as the second piece of function (A.1) is decreasing in e based on condition (A.9) then $\hat{e}$ is another candidate for optimal solution. Likewise as the first piece of function (A.1) is decreasing in e so $e = 0$ is the third candidate. Therefore, we can compare the value of the supplier's profit function for these three candidates to find the optimal solution under the condition of *Region A* and condition (A.9).

$$
\begin{aligned}
\Pi_S^A|_{e=0} =& 0 \\
\Pi_S^A|_{e=\bar{e}} =& D(w-c) - \frac{\alpha(4a\beta-\alpha)}{4a\beta} \\
\Pi_S^A|_{e=\hat{e}} =& \frac{(1-a)(Dp(w-c)-w\alpha)}{p-aw}
\end{aligned}
$$

If the condition (A.9) holds and $\Pi_S^A|_{e=0} > \Pi_S^A|_{e=\hat{e}}$ and $\Pi_S^A|_{e=0} > \Pi_S^A|_{e=\bar{e}}$ then $e^* = 0$. Similar to case A-I, we derive the condition based on our model parameters:

$$
A-10: \quad if \quad \frac{w}{e_4} < p < \frac{w\alpha}{\beta(1-e_4)} \quad \text{and} \quad \frac{\alpha}{2(w-c)} < D < \frac{\alpha\beta}{(w-c)(\alpha+\beta)}
$$

$$
\text{for all} \quad \frac{\alpha}{2D(w-c)} \leqslant a \leqslant 1 \quad \Longrightarrow \quad e^* = 0.
$$

If the condition (A.9) holds and $\Pi_S^A|_{e=\hat{e}} > \Pi_S^A|_{e=0}$ and $\Pi_S^A|_{e=\hat{e}} > \Pi_S^A|_{e=\bar{e}}$ then $e^* = \hat{e}$. We derive the conditions based on our model parameters as follows:

$$\left.\begin{aligned}
&A-8: \quad if\ \frac{w}{e_4} < p \text{ and } \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < D < \frac{\beta}{w-c}\\
&\qquad\qquad \text{for all } \frac{\alpha}{2D(w-c)} \leqslant a \leqslant \dot{a}\\
&A-9: \quad if\ \frac{w}{e_4} < p \text{ and } \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} < D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}\\
&\qquad\qquad \text{for all } \frac{\alpha}{2D(w-c)} \leqslant a \leqslant \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)}\\
&A-11: \quad if\ \frac{w\alpha}{\beta(1-e_4)} < p \text{ and } \frac{\alpha}{2(w-c)} < D < \frac{\alpha\beta}{(w-c)(\alpha+\beta)}\\
&\qquad\qquad \text{for all } \frac{\alpha}{2D(w-c)} \leqslant a \leqslant 1\\
&A-12: \quad if\ \frac{w}{e_4} < p \text{ and } \frac{\alpha\beta}{(w-c)(\alpha+\beta)} < D < \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)}\\
&\qquad\qquad \text{for all } \frac{\alpha}{2D(w-c)} \leqslant a \leqslant 1\\
&A-13: \quad if\ \frac{w}{e_4} < p \text{ and } \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} < D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}\\
&\qquad\qquad \text{for all } \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} \leqslant a \leqslant 1
\end{aligned}\right\} \implies e^* = \hat{e},$$

where $\dot{a} = \frac{\alpha(4\beta(p-w)+w\alpha+\sqrt{w^2\alpha^2+8(p-w)(w\alpha-2Dp(w-c))\beta+16(p-w)^2\beta^2})}{8D(p-w)(w-c)\beta}$.

If the condition (A.9) holds and $\Pi_S^A|_{e=\bar{e}} > \Pi_S^A|_{e=0}$ and $\Pi_S^A|_{e=\bar{e}} > \Pi_S^A|_{e=\hat{e}}$ then $e^* = \bar{e}$. We derive the conditions based on our model parameters as follows:

$$\left.\begin{aligned}
&A-6: \quad if\ \frac{w}{e_4} < p \text{ and } \frac{\alpha}{(w-c)} < D < \frac{\beta}{w-c}\\
&\quad \text{for all } \dot{a} \leqslant a \leqslant \frac{\alpha}{D(w-c)}\\
&A-7: \quad if\ \frac{w}{e_4} < p \text{ and } \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < D < \frac{\alpha}{w-c}\\
&\quad \text{for all } \dot{a} \leqslant a \leqslant 1
\end{aligned}\right\} \implies e^* = \bar{e}.$$

Subregion A-III:

$$0 < \frac{\partial \Pi_S^A}{\partial e}|_{e=(e_4)^-} \leqslant \frac{\partial \Pi_S^A}{\partial e}|_{e=e_4} \tag{A.10}$$

or equivalently, $0 < Da(w-c) - \alpha \leqslant 2Da(w-c) - \alpha$.

In this subregion, second piece of function (A.1) is increasing in e as $\frac{\partial \Pi_S^A}{\partial e}|_{e=(e_4)^-} > 0$. In addition we know that $\frac{\partial \Pi_S^A}{\partial e}|_{e=e_4} > 0$ and $\frac{\partial \Pi_S^A}{\partial e}|_{e=1} = -\alpha < 0$ thus we can use concavity characterization of third piece of $\Pi_S^A(a,e)$ which implies one local maximum($\bar{e} = 1 - \frac{\alpha}{2a\beta}$) for function (A.1) for all $e \in [e_4, 1]$. Likewise as the first piece of function (A.1) is decreasing in e so $e = 0$ is another candidate for global maximum. Therefore for finding the global maximum of function (A.1) based on condition of this subregion, we need to compare the value of function at two points:

$$\begin{aligned}\Pi_S^A|_{e=0} =& 0,\\ \Pi_S^A|_{e=\bar{e}} =& D(w-c) - \frac{\alpha(4a\beta-\alpha)}{4a\beta}.\end{aligned}$$

Holding condition (A.10), we find that $D(w-c) - \frac{\alpha(4a\beta-\alpha)}{4a\beta} > 0$. Therefore,:

$$\begin{aligned}A-14: \text{ for all } \quad \frac{\alpha}{D(w-c)} \leqslant a \leqslant 1 \ \ if \quad \frac{w}{e_4} < p\\ \text{and } \quad \frac{\alpha}{w-c} < D < \frac{\beta}{w-c} \quad \implies e^* = \bar{e}.\end{aligned}$$

We assemble cases A-1 through A-14 to form general behavior of the supplier's best response function with associated conditions in *Region A*. First, we take the union of the cases to complete the interval of the a from zero to one. Then, we take the union of them based on fixed intervals of D and p. We show these steps for each case as follows.

– Combine A-2 with A-10, then combine the outcome with A-1. It gives us: $D < \frac{w\alpha}{p(w-c)}$ and $D < \frac{\alpha\beta}{(w-c)(\alpha+\beta)}$ then $e^* = 0$ for all $0 \leqslant a \leqslant 1$.

– Combine A-4 with A-11, after that combine the outcome with A-11. It gives us: if $\frac{w\alpha}{p(w-c)} < D < \frac{\alpha\beta}{(w-c)(\alpha+\beta)}$ then $e^* = \hat{e}$ for all $0 \leqslant a \leqslant 1$.

– First, combine A-9 with A-13. Then combine the outcome with A-12. Finally, combine the result of previous combination with A-3-i. It gives us: if $\frac{\alpha\beta}{(w-c)(\alpha+\beta)} < D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$ then $e^* = \hat{e}$ for all $0 \leqslant a \leqslant 1$.

– First, combine A-6 with A-14. Combine the outcome with A-7. Then, again combine the result with A-8, and after that with A-3-ii. It gives us: if $\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < D < \frac{\beta(p-w)}{p(w-c)}$ then $e^* = \begin{cases} \hat{e} & 0 \leqslant a < \dot{a} \\ \bar{e} & \dot{a} \leqslant a \leqslant 1 \end{cases}$.

By looking at the above thresholds, we prove that in region A-i of statement of Lemma 16, $e^*(a)$ is in one of the following forms: S-BRF 1-1 or S-BRF 1-2; and in region A-ii, $e^*(a)$ has a unique form of S-BRF 2-2. To provide a general structure for S-BRF in Region A, we compare the four thresholds $\frac{w\alpha}{p(w-c)}$, $\frac{\alpha\beta}{(w-c)(\alpha+\beta)}$, $\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$, and $\frac{\beta(p-w)}{p(w-c)}$, and we derive the optimal supplier's best response function for *Region A* as following:

$$if \;\; w < p \leqslant \frac{w(\alpha+\beta)}{\beta} \quad \text{and} \quad 0 \leqslant D \leqslant \frac{\beta(p-w)}{p(w-c)} \implies e^* = 0 \quad \text{for all} \quad a \in [0,1]$$

$$if \;\; \frac{w(\alpha+\beta)}{\beta} < p < \frac{4w\beta^2}{(2\beta-\alpha)^2}$$

$$\begin{cases} \text{and} \quad 0 \leqslant D < \frac{w\alpha}{p(w-c)} & \implies e^* = 0 \quad \text{for all} \quad a \in [0,1] \\ \text{and} \quad \frac{w\alpha}{p(w-c)} \leqslant D < \frac{\beta(p-w)}{p(w-c)} & \implies e^* = \hat{e} \quad \text{for all} \quad a \in [0,1] \end{cases}$$

$$if \quad \frac{4w\beta^2}{(2\beta-\alpha)^2} \leqslant p \quad \begin{cases} \text{and} \quad 0 \leqslant D < \frac{w\alpha}{p(w-c)} & \Longrightarrow e^* = 0 \quad \text{for all} \quad a \in [0,1] \\ \text{and} \quad \frac{w\alpha}{p(w-c)} \leqslant D < \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)} & \Longrightarrow e^* = \hat{e} \quad \text{for all} \quad a \in [0,1] \\ \text{and} \quad \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)} \leqslant D \leqslant \frac{\beta(p-w)}{p(w-c)} & \Longrightarrow e^* = \begin{cases} \hat{e} & 0 \leqslant a < \dot{a} \\ \bar{e} & \dot{a} \leqslant a \leqslant 1 \end{cases} \end{cases}$$

We prove that in region A-i of statement of Lemma 16, $e^*(a)$ is in one of the following forms: S-BRF 1-1 or S-BRF 1-2; and in region A-ii, $e^*(a)$ has a unique form of S-BRF 2-2. Before we start analysis of Region B, we present some characteristics of $e^* = \hat{e}$ and $e^* = \bar{e}$. The intersection of $e^* = \hat{e}$ and $e^* = \bar{e}$ happens at $a = \frac{p\alpha}{2\beta(p-w)+w\alpha}$:

$$\bar{e} = \hat{e} \Rightarrow 1 - \frac{\alpha}{2a\beta} = \frac{1-a}{\frac{p}{w} - a} \Rightarrow a = \frac{p\alpha}{2\beta(p-w)+w\alpha}.$$

In addition to this, we know that $e^* = \hat{e}$ is decreasing and concave in a and $e^* = \bar{e}$ is increasing and concave in a for all $a \in (0,1)$:

$$\frac{\partial \hat{e}}{\partial a} = -\frac{w(p-w)}{(p-aw)^2} < 0 \quad , \frac{\partial^2 \hat{e}}{\partial a^2} = -\frac{2w^2(p-w)}{(p-aw)^3} < 0,$$
$$\frac{\partial \bar{e}}{\partial a} = \frac{\alpha}{2a^2\beta} > 0 \quad , \quad \frac{\partial^2 \bar{e}}{\partial a^2} = -\frac{\alpha}{a^3\beta} < 0.$$

In $e^* = \begin{cases} \hat{e} & 0 \leqslant a < \dot{a} \\ \bar{e} & \dot{a} \leqslant a \leqslant 1 \end{cases}$ as intersection happen in another point (i.e., $a = \dot{a}$) function is not continuous in a. Also we observe a jump in the function at $a = \dot{a}$ based on above-mentioned characteristics of $\hat{e}$ and $\bar{e}$ as $\dot{a} > \frac{p\alpha}{2\beta(p-w)+w\alpha} \Leftrightarrow \frac{\alpha(4\beta(p-w)+w\alpha+\sqrt{w^2\alpha^2+8(p-w)(w\alpha-2Dp(w-c))\beta+16(p-w)^2\beta^2)}}{8D(p-w)(w-c)\beta} - \frac{p\alpha}{2\beta(p-w)+w\alpha} > 0 \Leftrightarrow 8(p-w)\beta[w\alpha + 2\beta(p-w) - 2Dp(w-c)] > 0$. In the last inequality if $[w\alpha + 2\beta(p-w) - 2Dp(w-c)] > 0$ then we prove our claim: $[w\alpha + 2\beta(p-w) - 2Dp(w-c)] >$

$0 \overset{0 \leqslant D \leqslant \frac{\beta(p-w)}{p(w-c)}}{\Longleftrightarrow} 0 < D < \frac{2\beta(p-w)+w\alpha}{2p(w-c)}$.

Region B: $\frac{\beta(p-w)}{p(w-c)} < D < \frac{\beta}{w-c}$

For region B of Figure 21, we define two subregions and the supplier's profit are $\Pi_S^{B-I}(a,e)$ and $\Pi_S^{B-II}(a,e)$ in each region respectively. In B-I when $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$ we see three different cases for optimal order quantities including $(q_p^*, q_f^*) = (0,0)$, $(q_p^*, q_f^*) = (0,D)$, and $(q_p^*, q_f^*) = (D,D)$. In B-II when $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$ we see three different cases for optimal order quantities including $(q_p^*, q_f^*) = (0,0)$, $(q_p^*, q_f^*) = (D,0)$, and $(q_p^*, q_f^*) = (D,D)$.

Sub-region B-I: for all $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$, $\Pi_S^{B-I}(a,e) =$

$$\begin{cases} -\alpha e & 0 \leqslant e \leqslant e_4 \\ a(1-e)[D(w-c) - \beta(1-e)] - \alpha e & e_4 < e < \hat{e} \\ [e + (1-e)(1-a)]D(w-c) + a(1-e)[D(w-c) - \beta(1-e)] - \alpha e & \hat{e} \leqslant e \leqslant 1 \end{cases} \cdot \tag{B - I}$$

Sub-region B-II: for all $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$, $\Pi_S^{B-II}(a,e) =$

$$\begin{cases} -\alpha e & 0 \leqslant e < \hat{e} \\ [e + (1-e)(1-a)]D(w-c) - \alpha e & \hat{e} \leqslant e < e_4 \\ [e + (1-e)(1-a)]D(w-c) + a(1-e)[D(w-c) - \beta(1-e)] - \alpha e & e_4 \leqslant e \leqslant 1 \end{cases} \cdot \tag{B - II}$$

Before we analyze B-I and B-II, we present all sub-regions in them and their associated candidate solutions in Table 2.

Subregion B-I: The derivative of the supplier's profit function with respect to e in

Region	Sub-regions	Candidate Solutions
B-I	1	0 and $\hat{e}$
	2-i	0 and $\hat{e}$
	2-ii	0, $\tilde{e}$, and $\bar{e}$
B-II	1	0 and $\hat{e}$
	2	0, $\hat{e}$, and $\bar{e}$
	3	0 and $\hat{e}$

TABLE 2. Different Subregions in Region B and Associated Candidate Solutions

subregion (B - I) is:

$$\frac{\partial \Pi_S^{B-I}(a,e)}{\partial e} = \begin{cases} -\alpha & 0 \leqslant e \leqslant e_4 \\ a[2\beta(1-e) - D(w-c)] - \alpha & e_4 < e < \hat{e} \\ 2a(1-e)\beta - \alpha & \hat{e} \leqslant e \leqslant 1 \end{cases} \cdot \tag{A.11}$$

the supplier's profit function in region (B - I) is continuous at $e = e_4$, because $\lim_{e \to e_4} \Pi_S^{B-I}(a,e) = \Pi_S^{B-I}(a,e)|_{e=e_4} = -e_4\alpha$. The function (B - I) is not continuous at $e = \hat{e}$ as $\lim_{e \to \hat{e}^-} \Pi_S^{B-I}(a,e) < \Pi_S^{B-I}|_{e=\hat{e}}$. For showing this we can find the value of $\Pi_S^{B-I}|_{e=\hat{e}} - \lim_{e \to \hat{e}^-} \Pi_S^{B-I}(a,e) = \dfrac{D(1-a)p(w-c)}{p-aw} > 0$ where:

$$\begin{aligned} \Pi_S^{B-I}|_{e=\hat{e}} =& D(w-c) - \frac{a\beta(p-w)^2}{(p-aw)^2} - \frac{(1-a)w\alpha}{p-aw}, \\ \lim_{e \to \hat{e}^-} \Pi_S^{B-I}(a,e) =& \frac{(p-aw)[Da(p-w)(w-c) - (1-a)w\alpha] - a\beta(p-w)^2}{(p-aw)^2}. \end{aligned}$$

For each pair of e and a in $[0,1]$, the third piece of the supplier's profit function in (B - I) (defined over interval of $\hat{e} \leqslant e \leqslant 1$) is always greater than second piece of the supplier's profit function in (B - I) (defined over interval of $e_4 < e < \hat{e}$) . Also, $\Pi_S^{B-I}|_{e=0} = 0$ and $\Pi_S^{B-I}|_{e=e_4} < 0$. As we show in (A.11), $\Pi_S^{B-I}(a,e)$ for all $e \in [0, e_4]$ is linearly decreasing in e. Also second and third pieces of $\Pi_S^{B-I}(a,e)$ are concave in e because $\frac{\partial^2 \Pi_S^{B-I}(a,e)}{\partial e_2} = -2a\beta < 0$. From function (A.11) we verify that $\frac{\partial \Pi_S^{B-I}(a,e)}{\partial e}|_{e=1} < 0$. We can show that $\frac{\partial \Pi_S^{B-I}(a,e)}{\partial e}|_{e=\hat{e}^-} < \frac{\partial \Pi_S^{B-I}(a,e)}{\partial e}|_{e=\hat{e}}$ where:

$$\frac{\partial \pi_S^{B-I}(a,e)}{\partial e}|_{e=\hat{e}^-} = \frac{2a(p-w)\beta}{p-aw} - \alpha - Da(w-c),$$
$$\frac{\partial \pi_S^{B-I}(a,e)}{\partial e}|_{e=\hat{e}} = \frac{2a(p-w)\beta}{p-aw} - \alpha.$$

We define the the local maximum of third piece of function (B - I) as $\bar{e} \stackrel{\text{def}}{=} 1 - \frac{\alpha}{2a\beta}$. Then we compare the position of $\bar{e}$ with $\hat{e}$ to create two sub-regions for B-I as following. Subregions *B-I-1*: $\bar{e} \geqslant \hat{e}$, and in sub-region *B-I-2*: $\bar{e} < \hat{e}$.

B-I-1) when maximum of the third piece of piecewise function (B - I) happens at any $e \in [\hat{e}, 1]$:

Combining the conditions that define Region B, B-I, and *subregion B-I-1* ($\bar{e} \geqslant \hat{e}$), then we have:

$$\frac{2\beta(p-w)+w\alpha}{2p(w-c)} \leqslant D < \frac{\beta}{w-c} \quad \text{and} \quad \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}. \tag{A.12}$$

We show different possible scenarios of the supplier's profit function under condition of B-I-1 ($\bar{e} \geqslant \hat{e}$) in Figure 27.

Recall that for any pair of e and a in $[0,1]$, the third piece of the supplier's profit function in (B - I) (defined over interval of $\hat{e} \leqslant e \leqslant 1$) is always greater than second piece of the supplier's profit function in (B - I) (defined over interval of $e_4 < e < \hat{e}$). Therefore, if $\bar{e}$ exists in $[\hat{e}, 1]$, then it is always the local optimal solution for all e in $[e_4, 1]$. Thus, under the condition of case B-I-1 to find the optimal solution, we compare $\Pi_S^{B-I}|_{e=0} = 0$ with $\Pi_S^{B-I}|_{e=\bar{e}} = D(w-c) - \frac{\alpha(4a\beta-\alpha)}{4a\beta}$.

Under condition (A.12), if $\Pi_S^{B-I}|_{e=0} < \Pi_S^{B-I}|_{e=\bar{e}}$ then $e^* = \bar{e}$. The procedure that

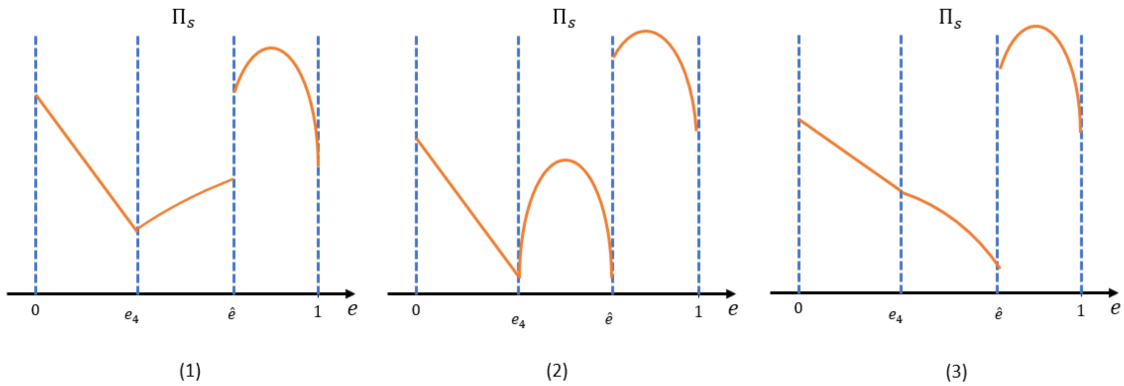


FIGURE 27. Possible Forms of the Supplier's Profit Function in Region B-I-1

we follow to derive the e^* is similar to that for Region A-I. Namely, we first derive the critical condition for the two candidate solutions $e = 0$ and $e = \bar{e}$, which is $D(w-c) - \frac{\alpha(4a\beta-\alpha)}{4a\beta} > 0$. Next, we identify the condition for this critical condition to hold in region B-I-1, which means comparing the critical condition with definition of Region B-I-1, $\frac{2\beta(p-w)+w\alpha}{2p(w-c)} \leqslant D < \frac{\beta}{w-c}$ and $\frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$. Finally, we combine these two conditions to generate various scenarios of optimal best response function. We skip the details and present the best response function.

$$
\left.\begin{aligned}
&(P1):\ if\ \ \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \leqslant D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \\
&\qquad\quad \text{and}\ \ \frac{w\alpha(2\beta-\alpha)}{2\beta[2D(w-c)-\alpha]} \leqslant p < \hat{p} \\
&\qquad\quad\ \ \text{for all}\ \ \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} \\
&(P2):\ if\ \ \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \leqslant D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \\
&\qquad\quad \text{and}\ \ \hat{p} \leqslant p < \frac{w(2\beta-\alpha)}{2\beta e_4} \\
&\qquad\quad\ \ \text{for all}\ \ \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} \\
&(P3):\ if\ \ \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \leqslant D < \frac{\beta}{w-c} \\
&\qquad\quad \text{and}\ \ w \leqslant p < \frac{w(2\beta-\alpha)}{2\beta e_4} \\
&\qquad\quad\ \ \text{for all}\ \ \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}
\end{aligned}\right\} \implies e^* = \bar{e}, \tag{A.13}
$$

where $\hat{p} \overset{\text{def}}{=} \frac{w[\frac{D(w-c)\alpha^2}{D(w-c)-\alpha}+4\beta^2]}{4\beta[\beta-D(w-c)]}$.

If the condition (A.12) holds and $\Pi_S^{B-I}|_{e=0} \geqslant \Pi_S^{B-I}|_{e=\bar{e}}$ then $e^* = 0$. We derive the conditions based on our model parameters similar to preceding case as follows:

$$\left.\begin{array}{l}(P4):\ if\ \dfrac{\alpha}{2(w-c)}\leqslant D<\dfrac{2\alpha\beta}{(w-c)(2\beta+\alpha)}\\ \qquad\text{and}\quad w\leqslant p<\dfrac{w(2\beta-\alpha)}{2\beta e_4}\\ \qquad\text{for all}\quad \dfrac{p\alpha}{2\beta(p-w)+w\alpha}\leqslant a\leqslant\dfrac{p}{w}-\dfrac{\beta(p-w)}{Dw(w-c)}\\ (P5):\ if\ \dfrac{2\alpha\beta}{(w-c)(2\beta+\alpha)}\leqslant D<\dfrac{\alpha(4\beta-\alpha)}{4\beta(w-c)}\\ \qquad\text{and}\quad \dfrac{w\alpha(2\beta-\alpha)}{2\beta[2D(w-c)-\alpha]}\leqslant p<\hat{p}\\ \qquad\text{for all}\quad \dfrac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)}\leqslant a\leqslant\dfrac{p}{w}-\dfrac{\beta(p-w)}{Dw(w-c)}\\ (P6):\ if\ \dfrac{2\alpha\beta}{(w-c)(2\beta+\alpha)}\leqslant D<\dfrac{\alpha(4\beta-\alpha)}{4\beta(w-c)}\\ \qquad\text{and}\quad w\leqslant p<\dfrac{w\alpha(2\beta-\alpha)}{2\beta[2D(w-c)-\alpha]}\\ \qquad\text{for all}\quad \dfrac{p\alpha}{2\beta(p-w)+w\alpha}\leqslant a\leqslant\dfrac{p}{w}-\dfrac{\beta(p-w)}{Dw(w-c)}\end{array}\right\}\implies e^*=0. \tag{A.14}$$

B-I-2) when maximum of the third piece of function (B - I) happens at any $e \in (-\infty, \hat{e})$.

When condition B-I holds, the condition of *subregion B-I-2* ($\bar{e} < \hat{e}$) is equivalent to:

$$\bar{e}<\hat{e}\Leftrightarrow\begin{cases}(1)\frac{2\beta(p-w)+w\alpha}{2p(w-c)}\leqslant D<\frac{\beta}{w-c}\quad\text{and}\quad 0\leqslant a\leqslant\frac{p\alpha}{2\beta(p-w)+w\alpha}\\ \text{or}\\ (2)\frac{\beta(p-w)}{p(w-c)}\leqslant D<\frac{2\beta(p-w)+w\alpha}{2p(w-c)}\quad\text{and}\quad 0\leqslant a\leqslant\frac{p}{w}-\frac{\beta(p-w)}{Dw(w-c)}\end{cases}. \tag{B-I-2}$$

We define the local maximum of the second piece of function (B - I) as $\tilde{e} \stackrel{\text{def}}{=} 1 - \frac{1}{2\beta}D(w-c) - \frac{\alpha}{2a\beta}$. In B-I-2 $\frac{\partial \pi_S^{B-I}(a,e)}{\partial e}|_{e=\hat{e}} < 0$ and recall that one of the characteristics of function (B - I) is that $\frac{\partial \pi_S^{B-I}(a,e)}{\partial e}|_{e=\hat{e}^-} < \frac{\partial \pi_S^{B-I}(a,e)}{\partial e}|_{e=\hat{e}}$. Therefore, $\frac{\partial \pi_S^{B-I}(a,e)}{\partial e}|_{e=\hat{e}^-} < 0$, and $\tilde{e}$ can be either in $[e_4, \hat{e}]$ or in $(-\infty, e_4)$. So we divide the region B-I-2 into two sub-regions based on position of $\tilde{e}$ vs. e_4. We illustrate both sub-regions B-I-2-i and B-I-2-ii in Figure 28. In subregion *B-I-2-i*, $\tilde{e} < e_4$ and in sub-region *B-I-2-ii*, $\tilde{e} \geqslant e_4$.

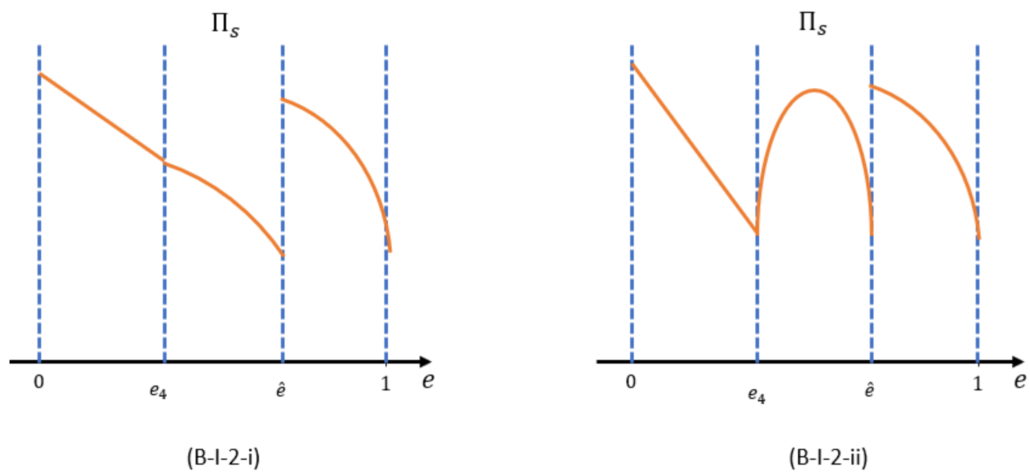


FIGURE 28. Possible Forms of the Supplier's Profit Function in Region B-I-2

Suppose that the condition B-I-2 holds and in addition the condition of B-I-2-i ($\tilde{e} < e_4$) holds too, we simplify these conditions and summarize it as follows: The procedure that we follow to derive the e^* is similar to that for Region A-I. Namely, we first derive the critical condition for the two candidate solutions $e = 0$ and $e = \bar{e}$, which is $D(w-c) - \frac{\alpha(4a\beta-\alpha)}{4a\beta} > 0$. Next, we identify the condition for this critical condition to hold in region B-I-1, which means comparing the critical condition with definition of Region B-I-2, $\{\frac{2\beta(p-w)+w\alpha}{2p(w-c)} \leqslant D < \frac{\beta}{w-c}$ and $0 \leqslant a \leqslant \frac{p\alpha}{2\beta(p-w)+w\alpha}\}$ or $\{\frac{\beta(p-w)}{p(w-c)} \leqslant D < \frac{2\beta(p-w)+w\alpha}{2p(w-c)}$ and $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}\}$. Finally, we combine these two conditions to generate various scenarios of optimal best response function. We skip the details and present the best response function.

$$\left.\begin{aligned}
&(P1): \{\frac{2\beta(p-w)+w\alpha}{2p(w-c)} \leqslant D < \frac{2\beta(p-w)+w\alpha}{p(w-c)} \\
&\qquad \text{and} \quad w \leqslant p < w(2-\frac{\alpha}{\beta}) \quad \text{and} \quad 0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha}\} \text{ or} \\
&(P2): \{\frac{2\beta(p-w)+w\alpha}{2p(w-c)} \leqslant D < \frac{\beta}{w-c} \\
&\qquad \text{and} \quad w(2-\frac{\alpha}{\beta}) \leqslant p \quad \text{and} \quad 0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha}\} \text{ or} \\
&(P3): \{\frac{2\beta(p-w)+w\alpha}{p(w-c)} \leqslant D < \frac{\beta}{w-c} \\
&\qquad \text{and} \quad w \leqslant p < w(2-\frac{\alpha}{\beta}) \quad \text{and} \quad 0 \leqslant a < \frac{\alpha}{D(w-c)}\} \text{ or} \\
&(P4): \{\frac{\beta(p-w)}{p(w-c)} \leqslant D < \frac{2\beta(p-w)+w\alpha}{2p(w-c)} \\
&\qquad \text{and} \quad w \leqslant p \quad \text{and} \quad 0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}\}.
\end{aligned}\right\} (B\text{-}I\text{-}2\text{-}i)$$

In subregion *B-I-2-i* the supplier's function (B - I) is decreasing in e for all $e \in [0, \hat{e})$. We show this in Figure 28. We have two candidates for optimal solutions. To find the maximum of function (B - I) we compare $\Pi_S^{B-I}|_{e=\hat{e}} = D(w-c) - \frac{(1-a)w\alpha}{p-aw} - \frac{a\beta(p-w)^2}{(p-aw)^2}$ with $\Pi_S^{B-I}|_{e=0} = 0$.

To compare these two values, we write the supplier's indifference equation, $\Pi_S^{B-I}|_{e=\hat{e}} = D(w-c) - \frac{(1-a)w\alpha}{p-aw} - \frac{a\beta(p-w)^2}{(p-aw)^2} - \Pi_S^{B-I}|_{e=0} = 0$, based on a like $\eta a^2 + \nu a + \xi = 0$ where $\eta \overset{\text{def}}{=} w^2(D(w-c)-\alpha)$, where $\nu \overset{\text{def}}{=} w^2\alpha - pw(2D(w-c)-\alpha) - \beta(p-w)^2$ and $\xi \overset{\text{def}}{=} Dp^2(w-c) - pw\alpha$. Solutions of this equation are: $a_1^{B-I} = \frac{-\nu-\sqrt{\Delta}}{2\eta}$ and $a_2^{B-I} = \frac{-\nu+\sqrt{\Delta}}{2\eta}$, where $\Delta = \nu^2 - 4\eta\xi$. Now we look at each component of *subregion B-I-2-i* and use the above indifference equation's characteristic to find the supplier's best response function in this subregion. For *B-I-2-i part (1)* [2]:

$$If\ \ \Delta < 0 \Rightarrow \pi_S^{B-I}|_{e=\hat{e}} < \pi_S^{B-I}|_{e=0} \implies e^* = 0 \quad \text{for all } a \in [0, \frac{p\alpha}{2\beta(p-w)+w\alpha}). \qquad (B\text{-}I\text{-}2\text{-}i\text{-}P1S1)$$

In this case there is no need to check the positivity or negativity of η because $\Delta < 0$ implies that η will be negative. We prove it by showing $\Delta < 0$ enforces $\frac{2\beta(p-w)+w\alpha}{2p(w-c)} < D < \frac{p^2\beta^2+w^2(\beta-\alpha)^2-2pw\beta(\beta+\alpha)}{-4pw\beta(w-c)}$ and $\eta < 0$ implies that $D < \frac{\alpha}{w-c}$. If we show that $\frac{p^2\beta^2+w^2(\beta-\alpha)^2-2pw\beta(\beta+\alpha)}{-4pw\beta(w-c)} < \frac{\alpha}{w-c}$ then we have the proof:

$$\frac{p^2\beta^2+w^2(\beta-\alpha)^2-2pw\beta(\beta+\alpha)}{-4pw\beta(w-c)} < \frac{\alpha}{w-c}$$

$$\Leftrightarrow p^2\beta^2+w^2(\beta-\alpha)^2-2pw\beta(\beta+\alpha) > -4pw\alpha\beta$$

$$\Leftrightarrow p^2\beta^2+w^2(\beta-\alpha)^2-2pw\beta(\beta+\alpha)+4pw\alpha\beta > 0$$

$$\Leftrightarrow [\beta p - w(\beta-\alpha)]^2 > 0.$$

When $\Delta \geqslant 0$ and we have feasible solutions for the supplier's indifference equation, if $\eta > 0$ then $a_1^{B-I} < a_2^{B-I}$. Proof of that is straight forward: $\frac{-\nu-\sqrt{\Delta}}{2\eta} < \frac{-\nu+\sqrt{\Delta}}{2\eta} \overset{\eta>0}{\iff} a_1^{B-I} < a_2^{B-I}$. Similarly, $\frac{-\nu-\sqrt{\Delta}}{2\eta} > \frac{-\nu+\sqrt{\Delta}}{2\eta} \overset{\eta<0}{\iff} a_1^{B-I} > a_2^{B-I}$.

Consider the case $\Delta \geqslant 0$ and $\eta > 0$ then based on condition *B-I-2-i part (1)* we can show that $\frac{p\alpha}{2\beta(p-w)+w\alpha} < a_1^{B-I}$. In this case as $\eta > 0$ the value of the supplier's indifference equation will be positive for all $a \in (-\infty, a_1^{B-I})$ which implies $\pi_S^{B-I}|_{e=\hat{e}} >$

[2] *B-I-2-i-P1S1 represents subregion B-I-2-i part (1) sub-case(1)*

$\pi_S^{B-I}|_{e=0}$, so

$$if\ \ \Delta \geqslant 0 \quad \text{and} \quad \eta > 0 \Rightarrow \frac{p\alpha}{2\beta(p-w)+w\alpha} < a_1^{B-I} < a_2^{B-I}$$
$$\Longrightarrow\ e^* = \hat{e} \quad \text{for all } a \in [0, \frac{p\alpha}{2\beta(p-w)+w\alpha}). \qquad (B\text{-}I\text{-}2\text{-}i\text{-}P1S2)$$

We prove the result *B-I-2-S2* by simplifying the condition; $\Delta \geqslant 0$ and $\eta > 0$ implies that:

$$\frac{\alpha}{w-c} < D < \frac{2\beta(p-w)+w\alpha}{p(w-c)} \quad \text{for all} \tag{A.15}$$
$$(\alpha \geqslant \frac{\beta}{2} \text{ and } p < w(2-\frac{\alpha}{\beta})) \quad \text{or} \quad (\alpha < \frac{\beta}{2} \text{ and } p \leqslant \frac{w(2\beta-\alpha)}{2(\beta-\alpha)})$$

or

$$\frac{2\beta(p-w)+w\alpha}{2p(w-c)} < D < \frac{2\beta(p-w)+w\alpha}{p(w-c)} \quad \text{for all} \tag{A.16}$$
$$\alpha < \frac{\beta}{2} \text{ and } p < w(2-\frac{\alpha}{\beta}) \text{ and } p > \frac{w(2\beta-\alpha)}{2(\beta-\alpha)}.$$

Now if we assume that $\frac{p\alpha}{2\beta(p-w)+w\alpha} > a_1^{B-I}$, it implies that: $D < \frac{\alpha[2\beta(p-w)-w\alpha]}{4p\beta(w-c)}$. Using condition (A.15), $\frac{\alpha[2\beta(p-w)-w\alpha]}{4p\beta(w-c)} < \frac{\alpha}{w-c}$. Using condition (A.16), $\frac{\alpha[2\beta(p-w)-w\alpha]}{4p\beta(w-c)} < \frac{2\beta(p-w)+w\alpha}{2p(w-c)}$ therefore under all conditions of *B-I-2-i-S2* we cannot have $\frac{p\alpha}{2\beta(p-w)+w\alpha} > a_1^{B-I}$.

Similarly, while $\eta < 0$:

$$If \;\; \Delta \geqslant 0 \quad \text{and} \quad \eta < 0 \quad \text{and} \quad 0 < a_2^{B-I} < \frac{p\alpha}{2\beta(p-w)+w\alpha} < a_1^{B-I}$$
$$\implies e^* = \begin{cases} 0 & 0 \leqslant a < a_2^{B-I} \\ \hat{e} & a_2^{B-I} \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \end{cases}.$$
$$(\textit{B-I-2-i-P1S3})$$

$$If \;\; \Delta \geqslant 0 \quad \text{and} \quad \eta < 0 \quad \text{and} \quad 0 < a_2^{B-I} < a_1^{B-I} < \frac{p\alpha}{2\beta(p-w)+w\alpha}$$
$$\implies e^* = \begin{cases} 0 & 0 \leqslant a < a_2^{B-I} \\ \hat{e} & a_2^{B-I} \leqslant a < a_1^{B-I} \\ 0 & a_1^{B-I} \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \end{cases}.$$
$$(\textit{B-I-2-i-P1S4})$$

$$If \;\; \Delta \geqslant 0 \quad \text{and} \quad \eta < 0 \quad \text{and} \quad a_2^{B-I} < 0 < \frac{p\alpha}{2\beta(p-w)+w\alpha} < a_1^{B-I}$$
$$\implies e^* = \hat{e} \quad \text{for all } a \in [0, \frac{p\alpha}{2\beta(p-w)+w\alpha}).$$
$$(\textit{B-I-2-i-P1S5})$$

For *B-I-2-i-(P2)* as $\frac{2\beta(p-w)+w\alpha}{2p(w-c)} \leqslant D < \frac{\beta}{w-c}$ and rest of parameter conditions are identical to *(P1)* we can derive the optimal supplier's response like *part (1)*. Conditions for Δ *and* η are similar to *(P1)* therefore we will have subregions *B-I-2-i-P2S1* to *B-I-2-i-P2S5* . For *B-I-2-i-(P3)*, condition implies that $\Delta \geqslant 0 \;\; \eta > 0$. We show this by simplifying $\Delta \geqslant 0$:

$$\begin{aligned}\Delta =&(p-w)^2[p^2\beta^2 - 2pw\beta(2cD+\alpha+\beta) + w^2((\beta-\alpha)^2 + 4Dp\beta)] \geqslant 0\\ \Leftrightarrow D \geqslant& \frac{2pw\beta(\beta+\alpha) - p^2\beta^2 - w^2(\beta-\alpha)^2}{4pw\beta(w-c)}.\end{aligned}$$

and under model assumptions showing $\frac{2pw\beta(\beta+\alpha)-p^2\beta^2-w^2(\beta-\alpha)^2}{4pw\beta(w-c)} < \frac{2\beta(p-w)+w\alpha}{p(w-c)}$ is straightforward where $\frac{2\beta(p-w)+w\alpha}{p(w-c)}$ is the lower bound of *B-I-2-i part (3)*. for $\eta > 0$ we show that:

$$\eta = Dw^2(w-c) - w^2\alpha > 0 \Leftrightarrow D > \frac{\alpha}{(w-c)}.$$

In *B-I-2-i-(P3)*, $\frac{2\beta(p-w)+w\alpha}{p(w-c)} \leqslant D < \frac{\beta}{w-c}$; based on above inequality and using $\frac{\alpha}{(w-c)} < \frac{\beta}{(w-c)}$ we can conclude that $\eta > 0$ under this condition. Similar to previous parts of *B-I-2-i* we can show that with this characteristic $a_1^{B-I} < a_2^{B-I}$ and consequently $0 < \frac{\alpha}{D(w-c)} < a_1^{B-I} < a_2^{B-I}$ therefore:

$$\begin{aligned}&if\ \ \frac{2\beta(p-w)+w\alpha}{p(w-c)} \leqslant D < \frac{\beta}{w-c} \quad \text{and} \quad w \leqslant p < w(2-\frac{\alpha}{\beta})\\ &\Longrightarrow\ e^* = \hat{e} \quad \text{for all} \quad 0 \leqslant a < \frac{\alpha}{D(w-c)}.\end{aligned}$$

Last part of *B-I-2-i* is *part (4)* which we can divide it to four cases based on the value of Δ and η. Similar to proof of *part(1)*, if $\Delta < 0$ then $\eta < 0$ and subsequently $\pi_S^{B-I}|_{e=\hat{e}} < \pi_S^{B-I}|_{e=0}$ then $e^* = 0$. We can rewrite this as following:

$$if\ \ \Delta < 0 \Rightarrow \pi_S^{B-I}|_{e=\hat{e}} < \pi_S^{B-I}|_{e=0} \implies e^* = 0 \quad \text{for all } a \in [0, \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}]. \qquad (B\text{-}I\text{-}2\text{-}i\text{-}P4S1)$$

If we have the case when $\Delta \geqslant 0$ and $\eta > 0$ then based on condition *B-I-2-i part (4)* we can show that $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a_1^{B-I}$. In this case as $\eta > 0$ the value of the supplier's indifference equation will be positive for all $a \in (-\infty, a_1^{B-I})$ which implies $\pi_S^{B-I}|_{e=\hat{e}} > \pi_S^{B-I}|_{e=0}$ so:

$$if \;\; \Delta \geqslant 0 \quad \text{and} \quad \eta > 0 \Rightarrow 0 < \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a_1^{B-I} < a_2^{B-I}$$
$$\Longrightarrow \; e^* = \hat{e} \quad \text{for all } a \in [0, \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}]. \qquad (\textit{B-I-2-i-P4S2})$$

Similar to *part(1)*, we have the following sub-cases while $\eta < 0$:

$$if \;\; \Delta \geqslant 0 \quad \text{and} \quad \eta < 0 \quad \text{and} \quad 0 < a_2^{B-I} < a_1^{B-I} \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$$
$$\Longrightarrow \; e^* = \begin{cases} 0 & 0 \leqslant a < a_2^{B-I} \\ \hat{e} & a_2^{B-I} \leqslant a < a_1^{B-I} \\ 0 & a_1^{B-I} \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} \end{cases}, \quad (\textit{B-I-2-i-P4S3})$$

$$if \;\; \Delta \geqslant 0 \quad \text{and} \quad \eta < 0 \quad \text{and} \quad a_2^{B-I} < 0 < \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a_1^{B-I}$$
$$\Longrightarrow \; e^* = \hat{e} \quad \text{for all } a \in [0, \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}]. \quad (\textit{B-I-2-i-P4S4})$$

Next, consider sub-region *B-I-2-ii* which is when $\tilde{e} \geqslant e_4$. Suppose that the condition B-I-2 holds and in addition the condition of B-I-2-ii ($\tilde{e} \geqslant e_4$) holds too. We simplify these conditions similar to Region A and summarize it as follows:

$$\begin{aligned}&\frac{2\beta(p-w)+w\alpha}{p(w-c)} \leqslant D < \frac{\beta}{w-c} \quad \text{and} \quad w \leqslant p < w(2-\frac{\alpha}{\beta})\\ &\text{and} \quad \frac{\alpha}{D(w-c)} \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha}. \end{aligned} \tag{\textit{B-I-2-ii}}$$

As we illustrate in Figure 28 and based on characteristics of the supplier's function in (B - I), we have three candidate solutions. To find the optimal solution, we compare the value of function (B - I) at $e = 0$, $e = \tilde{e}$, and $e = \hat{e}$ where:

$$\begin{aligned}\Pi_S^{B-I}|_{e=0} =&0,\\ \Pi_S^{B-I}|_{e=\hat{e}} =&D(w-c) - \frac{a\beta(p-w)^2}{(p-aw)^2} - \frac{(1-a)w\alpha}{p-aw},\\ \Pi_S^{B-I}|_{e=\tilde{e}} =&\frac{(Da(w-c)+\alpha)^2}{4a\beta} - \alpha.\end{aligned}$$

Under condition *B-I-2-ii*, we prove below that $\pi_S^{B-I}|_{e=\hat{e}} > \pi_S^{B-I}|_{e=0}$ and $\pi_S^{B-I}|_{e=\hat{e}} > \pi_S^{B-I}|_{e=\tilde{e}}$. Therefore $e^* = \hat{e}$ for all $a \in [\frac{\alpha}{D(w-c)}, \frac{p\alpha}{2\beta(p-w)+w\alpha}]$. Now we show that $\pi_S^{B-I}|_{e=\hat{e}} > \pi_S^{B-I}|_{e=0}$:

$$\begin{aligned}&\pi_S^{B-I}|_{e=\hat{e}} > \pi_S^{B-I}|_{e=0}\\ \Leftrightarrow &D(w-c) - \frac{a\beta(p-w)^2}{(p-aw)^2} - \frac{(1-a)w\alpha}{p-aw} > 0\\ \Leftrightarrow &D > \frac{(1-a)w(p-aw)\alpha + a(p-w)^2\beta}{(w-c)(p-aw)^2}.\end{aligned} \tag{A.17}$$

Based on lower bound of condition B-I-2-ii for D, it suffices to show that $\frac{(1-a)w(p-aw)\alpha+a(p-w)^2\beta}{(w-c)(p-aw)^2} < \frac{2\beta(p-w)+w\alpha}{p(w-c)}$:

$$\begin{aligned}
&\frac{(1-a)w(p-aw)\alpha+a(p-w)^2\beta}{(w-c)(p-aw)^2} < \frac{2\beta(p-w)+w\alpha}{p(w-c)} \\
\Leftrightarrow\ & a < \frac{p\left[w(3\beta-\alpha)+p\beta-\sqrt{w^2\alpha^2-2w(p-w)\alpha\beta+(p-w)(p+7w)\beta^2}\right]}{2w^2(2\beta-\alpha)} \\
\text{or } & a > \frac{p\left[w(3\beta-\alpha)+p\beta+\sqrt{w^2\alpha^2-2w(p-w)\alpha\beta+(p-w)(p+7w)\beta^2}\right]}{2w^2(2\beta-\alpha)}.
\end{aligned}$$

Now if we show that the lower bound of a in condition B-I-2-ii is less than $\frac{p\left[w(3\beta-\alpha)+p\beta-\sqrt{w^2\alpha^2-2w(p-w)\alpha\beta+(p-w)(p+7w)\beta^2}\right]}{2w^2(2\beta-\alpha)}$ then the proof is complete.

$$\begin{aligned}
\frac{p\alpha}{2\beta(p-w)+w\alpha} &< \frac{p\left[w(3\beta-\alpha)+p\beta-\sqrt{w^2\alpha^2-2w(p-w)\alpha\beta+(p-w)(p+7w)\beta^2}\right]}{2w^2(2\beta-\alpha)} \\
&\Leftrightarrow 0 < 4(p-w)w^2(2\beta-\alpha)\beta[2p(4\beta-\alpha)\beta - w(2\beta-a\alpha)(\alpha+4\beta)] \\
&\Leftrightarrow 0 < 2p(4\beta-\alpha)\beta - w(2\beta-a\alpha)(\alpha+4\beta) \\
&\Leftrightarrow\ w - \frac{w\alpha^2}{2\beta(4\beta-\alpha)} < p \qquad \text{(A.18)}
\end{aligned}$$

Left-hand side of inequality A.18 is less than w, hence the proof is complete. Similarly, we can prove that $\pi_S^{B-I}|_{e=\hat{e}} > \pi_S^{B-I}|_{e=\tilde{e}}$.

Subregion B-II: when $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$, the supplier's profit function in region (B - II) is continuous at $e = e_4$, because $\lim_{e \to e_4} \Pi_S^{B-II}(a,e) = \Pi_S^{B-II}(a,e_4) = \frac{D(w-c)[\alpha+\beta-Da(w-c)]}{\beta} - \alpha$. The function (B - II) is not continuous at $e = \hat{e}$ as $\lim_{e\to\hat{e}^-} \Pi_S^{B-II}(a,e) < \Pi_S^{B-II}(a,e)|_{e=\hat{e}}$. For showing this we can find the value of

$\Pi_S^{B-II}(a,e)|_{e=\hat{e}} - \lim_{e\to\hat{e}^-} \Pi_S^{B-II}(a,e) = Dp(w-c) > 0$ where:

$$\Pi_S^{B-II}(a,e)|_{e=\hat{e}} = \frac{(1-a)[Dp(w-c)-w\alpha]}{p-aw},$$
$$\lim_{e\to\hat{e}^-} \Pi_S^{B-II}(a,e) = -\frac{(1-a)w\alpha}{p-aw}.$$

The derivative of the supplier's function in with respect to 2 subregion (B - II) is:

$$\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e} = \begin{cases} -\alpha & 0 \leqslant e < \hat{e} \\ Da(w-c)-\alpha & \hat{e} \leqslant e < e_4 \\ 2a(1-e)\beta - \alpha & e_4 \leqslant e \leqslant 1 \end{cases} \cdot \tag{A.19}$$

Note that $\Pi_S^{B-II}|_{e=0} = 0$. As we can observe from (A.19), $\Pi_S^{B-II}(a,e)$ for all $e \in [0,\hat{e})$ is linearly decreasing in e. Also second piece is linear in e and third pieces of $\Pi_S^{B-II}(a,e)$ is concave in e because $\frac{\partial^2 \Pi_S^{B-II}(a,e)}{\partial e_2} = -2a\beta < 0$. From function (A.19) we verify that $\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=1} < 0$. We can show that $\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} < \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4}$ where:

$$\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} = Da(w-c)-\alpha,$$
$$\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4} = 2Da(w-c)-\alpha.$$

Now we can divide the *Region B-II* of the supplier's problem to three sub regions based on order of zero, $\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-}$, and $< \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4}$. We show graphically these three cases in Figure 29. Three cases are:

Sub-region B-II-1) $\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} \leqslant \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4} \leqslant 0$,

Sub-region B-II-2) $\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} \leqslant 0 < \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4}$, and

Sub-region B-II-3) $0 < \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} \leqslant \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4}$.

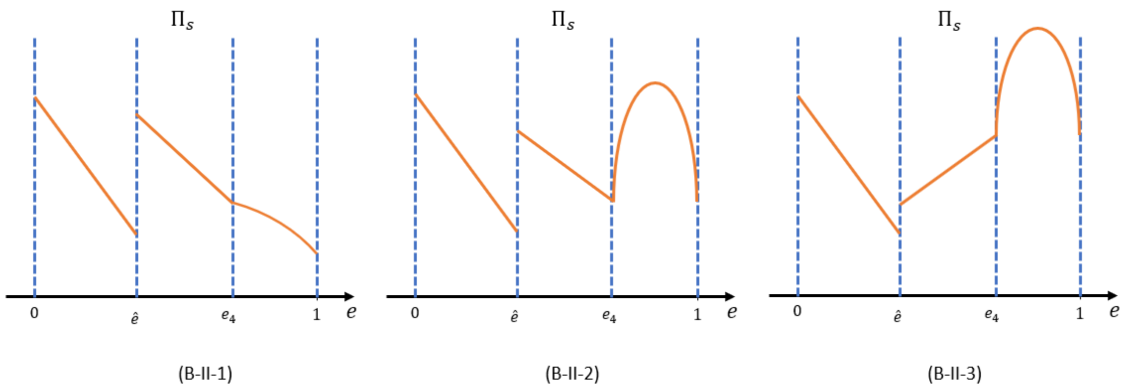


FIGURE 29. Possible Forms of the Supplier's Profit Function in Region B-II

B-II-1) when $\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} \leqslant \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4} \leqslant 0$.

Under the condition of B-II we simplify the condition of $\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} \leqslant \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4} \leqslant 0$:

$$\begin{cases} 0 \leqslant D < \frac{\alpha}{2(w-c)} \quad \text{and} \quad w \leqslant p < \frac{w}{e_4} \quad \text{and} \quad \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1 \\ \text{or} \\ \frac{\alpha}{2(w-c)} \leqslant D < \frac{\beta}{w-c} \quad \text{and} \quad \frac{w\beta(2\beta-\alpha)}{2e_4} < p \quad \text{and} \quad \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a < \frac{\alpha}{2D(w-c)} \end{cases} . \tag{B-II-1}$$

As we illustrate in Figure 29 and as a result of concavity of third piece of function (B - II) and linearity of other pieces, we have two candidates for optimal solution: $e = 0$ and $e = \hat{e}$. So under the condition *B-II-1*, we compare $\Pi_S^{B-II}(a,e)|_{e=0} = 0$ with $\Pi_S^{B-II}(a,e)|_{e=\hat{e}} = \frac{(1-a)[Dp(w-c)-w\alpha]}{p-aw}$ to find the optimal solution. If the condition B-II-1 holds and $\Pi_S^{B-II}|_{e=\hat{e}} \leqslant \Pi_S^{B-II}|_{e=0}$ then $e^* = 0$. The procedure that we follow to derive the e^* is similar to that for Region A-I. Namely, we first derive the critical condition for the two candidate solutions $e = 0$ and $e = \hat{e}$, which is

$\frac{(1-a)[Dp(w-c)-w\alpha]}{p-aw} > 0$. Next, we identify the condition for this critical condition to hold in region B-I-1, which means comparing the critical condition with definition of Region B-II-1, $\{0 \leqslant D < \frac{\alpha}{2(w-c)}$ and $w \leqslant p < \frac{w}{e_4}$ and $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1\}$ or $\{\frac{\alpha}{2(w-c)} \leqslant D < \frac{\beta}{w-c}$ and $\frac{w\beta(2\beta-\alpha)}{2e_4} < p$ and $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a < \frac{\alpha}{2D(w-c)}\}$. Finally, we combine these two conditions to generate various scenarios of optimal best response function. We skip the details and present the best response function as follows.

$$\left.\begin{array}{l}
(P1):\ if\ \ 0 \leqslant D < \dfrac{\alpha}{2(w-c)} \quad \text{and} \quad w \leqslant p < \dfrac{w}{e_4} \\
\qquad\qquad \text{for all} \ \ \dfrac{p}{w} - \dfrac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1 \\
(P2):\ if\ \ \dfrac{\alpha}{2(w-c)} \leqslant D < \dfrac{\alpha\beta}{(w-c)(\alpha+\beta)} \\
\qquad\qquad \text{and} \ \ \dfrac{w\beta(2\beta-\alpha)}{2e_4} \leqslant p < \dfrac{w}{e_4} \\
\qquad\qquad \text{for all} \ \ \dfrac{p}{w} - \dfrac{\beta(p-w)}{Dw(w-c)} < a \leqslant \dfrac{\alpha}{2D(w-c)} \\
(P3):\ if\ \ \dfrac{\alpha\beta}{(w-c)(\alpha+\beta)} \leqslant D < \dfrac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \\
\qquad\qquad \text{and} \ \ \dfrac{w\beta(2\beta-\alpha)}{2e_4} \leqslant p < \dfrac{w\alpha}{\beta(1-e_4)} \\
\qquad\qquad \text{for all} \ \ \dfrac{p}{w} - \dfrac{\beta(p-w)}{Dw(w-c)} < a \leqslant \dfrac{\alpha}{2D(w-c)}
\end{array}\right\} \implies e^* = 0.$$

If the condition B-II-1 holds and $\Pi_S^{B-II}|_{e=\hat{e}} > \Pi_S^{B-II}|_{e=0}$ then $e^* = \hat{e}$. We derive the conditions based on our model parameters similar to preceding case as follows:

$$\left.\begin{aligned}
&(P4):\ if\ \ \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \leqslant D < \frac{\beta}{w-c}\\
&\qquad\quad \text{and}\ \ \frac{w\beta(2\beta-\alpha)}{2e_4} \leqslant p < \frac{w}{e_4}\\
&\qquad\quad \text{for all}\ \ \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant \frac{\alpha}{2D(w-c)}\\
&(P5):\ if\ \ \frac{\alpha\beta}{(w-c)(\alpha+\beta)} \leqslant D < \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)}\\
&\qquad\quad \text{and}\ \ \frac{w\alpha}{\beta(1-e_4)} \leqslant p < \frac{w}{e_4}\\
&\qquad\quad \text{for all}\ \ \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant \frac{\alpha}{2D(w-c)}
\end{aligned}\right\} \implies e^* = \hat{e}.$$

B-II-2) when $\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} \leqslant 0 < \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4}$.

Under the condition of B-II we simplify the condition of

$\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} \leqslant 0 < \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4}$:

$$\left.\begin{aligned}
&(a): \{\frac{\alpha}{2(w-c)} < D \leqslant \frac{\alpha}{w-c} \quad \text{and} \quad \frac{w\beta(2\beta-\alpha)}{2e_4} < p < \frac{w}{e_4} \\
&\qquad \text{and} \quad \frac{\alpha}{2D(w-c)} < a \leqslant 1\} \\
&\qquad \text{or} \\
&(b): \{\frac{\alpha}{2(w-c)} < D \leqslant \frac{\alpha}{w-c} \quad \text{and} \quad w < p < \frac{w\beta(2\beta-\alpha)}{2e_4} \\
&\qquad \text{and} \quad \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1\} \\
&\qquad \text{or} \\
&(c): \{\frac{\alpha}{w-c} < D \leqslant \frac{\beta}{w-c} \quad \text{and} \quad \frac{w\beta(2\beta-\alpha)}{2e_4} < p < \frac{w}{e_4} \\
&\qquad \text{and} \quad \frac{\alpha}{2D(w-c)} < a \leqslant \frac{\alpha}{D(w-c)}\} \\
&\qquad \text{or} \\
&(d): \{\frac{\alpha}{w-c} < D \leqslant \frac{\beta}{w-c} \quad \text{and} \quad \frac{w(\beta-\alpha)}{\beta e_4} \leqslant p < \frac{w\beta(2\beta-\alpha)}{2e_4} \\
&\qquad \text{and} \quad \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant \frac{\alpha}{D(w-c)}\}.
\end{aligned}\right\} (B\text{-}II\text{-}2)$$

We illustrate subregion B-II-2 in Figure B - II. Under condition *B-II-2*, the supplier's function (B - II) is linearly decreasing in e for all $e \in [0, \hat{e})$ and it's not continuous at $e = \hat{e}$ (as we proved it previously). Also function is decreasing linearly in e for all $e \in [\hat{e}, e_4)$ and it is concave with one local maximum ($\bar{e} \stackrel{\text{def}}{=} 1 - \frac{\alpha}{2a\beta}$). Given these characteristics of function under this condition, in each of B-II-2 sub-conditions we compare the value of function (B - II) at $e = 0$, $e = \hat{e}$ and $e = \bar{e}$ to find the optimal solution. Thus, we compare $\Pi_S^{B-II}(a,e)|_{e=0} = 0$, with $\Pi_S^{B-II}(a,e)|_{e=\hat{e}} = \frac{(1-a)[Dp(w-c)-w\alpha]}{p-aw}$, and with $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} = D(w-c) - \frac{\alpha(4a\beta-\alpha)}{4a\beta}$ under all sub-conditions of *B-II-2*.

Given that one of the values is zero, we can write the indifference equation with two other profit functions as follows: $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} - \Pi_S^{B-II}(a,e)|_{e=\hat{e}} = 0$, we sort this equation based on a and write it as $\rho a^2 + \tau a + \varrho = 0$ where $\rho \stackrel{\text{def}}{=} 4D\beta(p-w)(w-c)$ and $\tau \stackrel{\text{def}}{=} -[4\alpha\beta(p-w) + w\alpha^2]$ and $\varrho \stackrel{\text{def}}{=} p\alpha^2$. Following the definitions, ρ which is the coefficient of a^2 is positive and τ is negative for all defined parameters. Solving this equation for a gives us two solutions $a_1^{B-II} = \frac{-\tau+\sqrt{\delta}}{2\rho}$ and $a_2^{B-II} = \frac{-\tau-\sqrt{\delta}}{2\rho}$ where $\delta = \tau^2 - 4\rho\varrho$. when $\delta \geqslant 0$ and we have feasible solutions for the supplier's indifference equation,then $a_1^{B-II} > a_2^{B-II}$. Proof of that is straight forward: $\frac{-\tau-\sqrt{\delta}}{2\rho} < \frac{-\tau+\sqrt{\delta}}{2\rho} \overset{\rho>0}{\iff} a_2^{B-II} < a_1^{B-II}$. Now we need to consider all possible cases for the supplier's profit value in each sub-case. Given that we start with *B-II-2-part (a)*, if $\pi_S^{B-II}(a,e)|_{e=\bar{e}} \geqslant 0$ then we name it condition *B-II-2-a-1* and rewrite this condition as:

$$\begin{cases} (i): \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < D < \frac{\alpha}{w-c} \text{ and } \frac{w\beta(2\beta-\alpha)}{2e_4} < p < \frac{w}{e_4} \text{ and } \frac{\alpha}{2D(w-c)} < a < 1 \\ (ii): \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} < D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \text{ and } \frac{w\beta(2\beta-\alpha)}{2e_4} < p < \frac{w}{e_4} \\ \text{ and } \frac{\alpha}{2D(w-c)} < a < \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)}. \end{cases} \tag{\textit{B-II-2-a-1}}$$

If condition B-II-2-a-1-i holds and $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} - \Pi_S^{B-II}(a,e)|_{e=\hat{e}} > 0$ then $e^* = \bar{e}$, otherwise $e^* = \hat{e}$. When B-II-2-a-1-i holds, similar to previous cases we can show that $a_2^{B-II} < \frac{\alpha}{2D(w-c)} < a_1^{B-II} < 1$ which implies:

$$e^* = \begin{cases} \hat{e} & \frac{\alpha}{2D(w-c)} \leqslant a \leqslant a_1^{B-II} \\ \bar{e} & a_1^{B-II} < a \leqslant 1. \end{cases} \tag{\textit{B-II-2-a-1-i}}$$

Likewise, if condition B-II-2-a-1-ii holds, then

$$a_2^{B-II} < \frac{\alpha}{2D(w-c)} < \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} < 1 < a_1^{B-II},$$

$$\text{which implies } e^* = \hat{e} \text{ for all } a \text{ in } (\frac{\alpha}{2D(w-c)}, \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)}). \quad (B\text{-}II\text{-}2\text{-}a\text{-}1\text{-}ii)$$

If condition B-II-2-a holds and $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} < 0$, then we name this condition B-II-2-a-2 and rewrite it as follows:

$$\begin{cases} (i): \frac{\alpha}{2(w-c)} < D < \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \text{ and } \frac{w\beta(2\beta-\alpha)}{2e_4} < p < \frac{w}{e_4} \text{ and } \frac{\alpha}{2D(w-c)} < a < 1 \\ (ii): \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} < D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \text{ and } \frac{w\beta(2\beta-\alpha)}{2e_4} < p < \frac{w}{e_4} \text{ and } \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} < a < 1. \end{cases}$$

$$(B\text{-}II\text{-}2\text{-}a\text{-}2)$$

In this case if $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} - \Pi_S^{B-II}(a,e)|_{e=\hat{e}} > 0$ then $e^* = 0$ otherwise we must find whether $\Pi_S^{B-II}(a,e)|_{e=\hat{e}}$ is positive or not. If $\Pi_S^{B-II}(a,e)|_{e=\hat{e}} \geqslant 0$ then $e^* = \hat{e}$ otherwise $e^* = 0$. Solving $\Pi_S^{B-II}(a,e)|_{e=\hat{e}} = 0$ for a gives us $a = 1$, if $w\alpha - Dp(w-c) > 0$ (which is the coefficient of a in this equation)then for all $a \leqslant 1, \Pi_S^{B-II}(a,e)|_{e=\hat{e}} < 0$. Also, if $w\alpha - Dp(w-c) \leqslant 0$ then for all $a \leqslant 1, \Pi_S^{B-II}(a,e)|_{e=\hat{e}} \geqslant 0$. Considering condition *B-II-2-a-2-i* we can show that $a_2^{B-II} < \frac{\alpha}{2D(w-c)} < 1 < a_1^{B-II}$. Therefore:

$$\left.\begin{array}{l} (P1): \; if \;\; \frac{\alpha\beta}{(w-c)(\alpha+\beta)} < D < \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \\ \qquad\quad \text{and } \frac{w\beta(2\beta-\alpha)}{2e_4} < p < \frac{w\alpha}{\beta(1-e_4)} \\ (P2): \; if \;\; \frac{\alpha}{2(w-c)} < D < \frac{\alpha\beta}{(w-c)(\alpha+\beta)} \\ \qquad\quad \text{and } \frac{w\beta(2\beta-\alpha)}{2e_4} < p < \frac{w}{e_4} \end{array}\right\} \Rightarrow e^* = 0 \quad \text{for all} \quad a \in (\frac{\alpha}{2D(w-c)}, 1].$$

$$(P3): \; if \; \frac{\alpha\beta}{(w-c)(\alpha+\beta)} < D < \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \text{ and}$$
$$\frac{w\alpha}{\beta(1-e_4)} < p < \frac{w}{e_4} \Rightarrow e^* = \hat{e} \quad \text{for all} \quad a \in (\frac{\alpha}{2D(w-c)}, 1]$$

If condition B-II-2-a-2-ii holds, then $a_2^{B-II} < \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} < 1 < a_1^{B-II}$,

$$\text{if } w\alpha - Dp(w-c) \leqslant 0 \text{ then } e^* = \hat{e} \text{ for all } a \text{ in } (\frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)}, 1].$$

(B-II-2-a-2-ii)

Next case is *B-II-2-part (b)*, similar to part(a) if $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} \geqslant 0$ then we name it condition *B-II-2-b-1* and rewrite this condition as following where $\tilde{p} \stackrel{\text{def}}{=} \frac{w[\frac{\alpha^2(w-c)}{D(w-c)-\alpha}+4\beta^2]}{4\beta(\beta-D(w-c))}$:

$$\begin{cases} (i): \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < D < \frac{\alpha}{w-c} \text{ and } w < p < \frac{w\beta(2\beta-\alpha)}{2e_4} \text{ and } \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1 \\ (ii): \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} < D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \text{ and } \tilde{p} < p < \frac{w\beta(2\beta-\alpha)}{2e_4} \\ \text{ and } \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a < \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)}. \end{cases}$$

(B-II-2-b-1)

In *B-II-2-b-1* if $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} - \Pi_S^{B-II}(a,e)|_{e=\hat{e}} > 0$ then $e^* = \bar{e}$ otherwise $e^* = \hat{e}$. Considering condition *B-II-2-a-1-i* we prove similar to preceding case that if $\delta < 0$ and as $\rho > 0$ then based on above mentioned characteristics $e^* = \bar{e}$, we name this case *B-II-2-b-1-i-P3*. On the other hand, if $\delta \geqslant 0$ we can have two sub-conditions for this case:

$$(P1): \ if \ \ \delta \geqslant 0 \quad \text{and} \quad \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < 1 < a_2^{B-II} < a_1^{B-II}$$

$$\Longrightarrow \ e^* = \bar{e} \text{ for all } a \text{ in } (\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}, 1].$$

$$(P2): \ if \ \ \delta \geqslant 0 \quad \text{and} \quad \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a_2^{B-II} < a_1^{B-II} < 1$$

$$\Longrightarrow \ e^* = \begin{cases} \bar{e} & \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant a_2^{B-II} \\ \hat{e} & a_2^{B-II} < a < a_1^{B-II} \\ \bar{e} & a_1^{B-II} \leqslant a \leqslant 1 \end{cases}.$$

Under the condition of case *B-II-2-b-1-ii*, $\delta \geqslant 0$ and we have:

$$(P1): \ if \ \ 0 < \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} < a_2^{B-II} < 1 < a_1^{B-II}$$

$$\Longrightarrow \ e^* = \bar{e} \text{ for all } a \text{ in } (\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}, \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)}).$$

$$(P2): \ if \ \ 0 < \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a_2^{B-II} < \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} < 1 < a_1^{B-II}$$

$$\Longrightarrow \ e^* = \begin{cases} \bar{e} & \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant a_2^{B-II} \\ \hat{e} & a_2^{B-II} < a < \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} \end{cases}.$$

Under the condition *B-II-2-part (b)*, if $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} < 0$ then we name this condition *B-II-2-b-2* and rewrite it as follows :

$$\begin{cases} (i) : \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} < D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \text{ and } w \leqslant p < \tilde{p} \text{ and } \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1 \\ (ii) : \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} < D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \text{ and } \tilde{p} < p < \frac{w\beta(2\beta-\alpha)}{2e_4} \text{ and } \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} \leqslant a \leqslant 1 \\ (iii) : \frac{\alpha}{2(w-c)} < D < \frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \text{ and } w \leqslant p < \frac{w\beta(2\beta-\alpha)}{2e_4} \text{ and } \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1 \end{cases} .$$

(B-II-2-b-2)

If *B-II-2-b-2* holds, then we need to do the similar analysis that we have done in case *B-II-2-a-2*. Under the condition *B-II-2-b-2*, $\delta \geqslant 0$. Now considering condition *B-II-2-b-2-i* similar to preceding case, one can show that $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a_2^{B-II} < 1 < a_1^{B-II}$ and $w\alpha - Dp(w-c) \leqslant 0$ then $e^* = 0$ for all $a \in (\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}, 1]$. In *B-II-2-b-2-ii*,

$$\text{if } a_2^{B-II} < \frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} < 1 < a_1^{B-II} \text{ then } w\alpha - Dp(w-c) \leqslant 0$$
$$\text{which implies } e^* = \hat{e} \quad \text{for all} \quad a \in (\frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)}, 1]$$

(B-II-2-b-2-ii-P1)

Also if $\frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)} < a_2^{B-II} < 1 < a_1^{B-II}$,

$$\text{then } w\alpha - Dp(w-c) > 0 \text{ which implies } e^* = 0 \quad \text{for all} \quad a \in (\frac{\alpha^2}{4\beta(\beta(e_4-1)+\alpha)}, 1].$$

(B-II-2-b-2-ii-P2)

Given condition *B-II-2-b-2-iii*, $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a_2^{B-II} < 1 < a_1^{B-II}$ then $w\alpha - Dp(w-c) > 0$ which implies $e^* = 0$ for all $a \in (\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}, 1]$.

Next part is *B-II-2-(c)*, under this we can show straightforward that $\pi_S^{B-II}(a,e)|_{e=\bar{e}} \geqslant 0$ and $\pi_S^{B-II}(a,e)|_{e=\hat{e}} \geqslant 0$ and $\delta \geqslant 0$, then $a_2^{B-II} < \frac{\alpha}{2D(w-c)} < a_1^{B-II} < \frac{\alpha}{D(w-c)} < 1$

which implies:

$$e^* = \begin{cases} \hat{e} & \frac{\alpha}{2D(w-c)} < a \leqslant a_1^{B-II} \\ \bar{e} & a_1^{B-II} < a \leqslant \frac{\alpha}{D(w-c)} \end{cases}.$$

Under the condition *B-II-2-(d)*, it is straightforward to show that $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} \geqslant 0$ and $\Pi_S^{B-II}(a,e)|_{e=\hat{e}} \geqslant 0$. If $\delta < 0$ as $\rho > 0$, $e^* = \bar{e}$ for all $a \in (\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}, \frac{\alpha}{D(w-c)}]$, which we label it as *B-II-2-d-P3*. In this case if $p < \frac{w(1-\alpha)}{e_4}$ we name it *B-II-2-d-P3-i*, otherwise *B-II-2-d-P3-ii*. If $\delta \geqslant 0$, we will have two cases:

$$(P1): \; if \;\; 0 < \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < \frac{\alpha}{D(w-c)} < 1 < a_2^{B-II} < a_1^{B-II}$$

$$\Longrightarrow \; e^* = \bar{e} \;\; \text{for all } a \in (\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}, \frac{\alpha}{D(w-c)}).$$

$$(P2): \; if \;\; 0 < \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a_2^{B-II} < a_1^{B-II} < \frac{\alpha}{D(w-c)} < 1$$

$$\Longrightarrow \; e^* = \begin{cases} \bar{e} & \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant a_2^{B-II} \\ \hat{e} & a_2^{B-II} < a < a_1^{B-II} \\ \bar{e} & a_1^{B-II} \leqslant a < \frac{\alpha}{D(w-c)} \end{cases}.$$

B-II-3) when $0 < \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} < \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4}$.

Under the condition of B-II, we simplify the condition of $0 < \frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=(e_4)^-} <$

$\frac{\partial \Pi_S^{B-II}(a,e)}{\partial e}|_{e=e_4}$:

$$\begin{cases} (i): \frac{\alpha}{w-c} \leqslant D < \frac{\beta}{(w-c)} \quad \text{and} \quad \frac{w(\beta-\alpha)}{\beta e_4} \leqslant p < \frac{w}{e_4} \quad \text{and} \quad \frac{\alpha}{D(w-c)} < a \leqslant 1 \\ \text{or} \\ (ii): \frac{\alpha}{w-c} \leqslant D < \frac{\beta}{(w-c)} \quad \text{and} \quad w \leqslant p \leqslant \frac{w(\beta-\alpha)}{\beta e_4} \quad \text{and} \quad \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1. \end{cases} \tag{B-II-3}$$

As we illustrate it in B - II, under condition *B-II-3*, the supplier's function *(B - II)* is linearly decreasing in e for all $e \in [0, \hat{e})$ and it's not continuous at $e = \hat{e}$ (as we proved it previously). Also function is linearly increasing in e for all $e \in [\hat{e}, e_4)$ and it is concave with one local maximum ($\bar{e} \stackrel{\text{def}}{=} 1 - \frac{\alpha}{2a\beta}$) for all $e \in [e_4, 1]$. Therefore, under this condition,in each of *B-II-3* sub-conditions, we have two candidate solutions. To find the optimal solution, we compare the value of function *(B - II)* at $e = 0$ and $e = \bar{e}$. We compare $\Pi_S^{B-II}(a,e)|_{e=0} = 0$ with $\Pi_S^{B-II}(a,e)|_{e=\bar{e}} = D(w-c) - \frac{\alpha(4a\beta-\alpha)}{4a\beta}$ under all sub-regions of *B-II-3*. Similar to all above sub-regions of *(B-II)*, we can show that under condition *(B-II-3)* always $\Pi_S^{B-II}(a,e)|_{e=0} < \Pi_S^{B-II}(a,e)|_{e=\bar{e}}$ which implies $e^* = \bar{e}$,. Therefore,

$$\text{if} \quad \frac{\alpha}{w-c} \leqslant D < \frac{\beta}{(w-c)} \text{ and } \quad \frac{w(\beta-\alpha)}{\beta e_4} \leqslant p < \frac{w}{e_4}$$
$$\text{then } e^* = \bar{e} \quad \text{for all } a \in (\frac{\alpha}{D(w-c)}, 1], \tag{B-II-3-i}$$

$$\textit{if} \quad \frac{\alpha}{w-c} \leqslant D < \frac{\beta}{(w-c)} \quad \text{and} \quad w \leqslant p \leqslant \frac{w(\beta-\alpha)}{\beta e_4}$$
$$\text{then } e^* = \bar{e} \quad \text{for all } a \in (\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}, 1]. \tag{B-II-3-ii}$$

Recall that Region B-I is defined as $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$ and Region B-II defined as $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$. Now, after we presented the analysis of each case in the above, we want to assemble Region B-I with Region B-II to form the supplier's best response function over the range of $a \in [0,1]$. To do that, we combine each sub-region of B-I with each single region of B-II.

Note that in each of the subregion of B-I, the interval of a may be a sub-interval of $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$. In following we list all regions of B-I based on the range of a:

- *when* $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$, *we have 4 regions as follows:*
  - *B-I-2-i-P4S1 through B-I-2-i-P4S4*
- *when* $0 \leqslant a \leqslant \frac{p\alpha}{2\beta(p-w)+w\alpha}$, *we have 11 regions as follows:*
  - *Combination of B-I-2-i-P3 with B-I-2-ii*
  - *B-I-2-i-P1S1 through B-I-2-i-P1S5*
  - *B-I-2-i-P2S1 through B-I-2-i-P2S5*
- *when* $\frac{p\alpha}{2\beta(p-w)+w\alpha} < a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$, *we have 5 regions as follows:*
  - *Combination of B-I-1-P1 with B-I-1-P5*
  - *B-I-1-P2, B-I-1-P3, B-I-1-P4, and B-I-1-P6*

From the above list, we combine regions defined over $0 \leqslant a \leqslant \frac{p\alpha}{2\beta(p-w)+w\alpha}$ with regions defined over $\frac{p\alpha}{2\beta(p-w)+w\alpha} < a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$ to form the range of a for $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$. We combine each region in the above list for $0 \leqslant a \leqslant \frac{p\alpha}{2\beta(p-w)+w\alpha}$ with each region in the above list for $\frac{p\alpha}{2\beta(p-w)+w\alpha} < a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$. There are total 55 possible combinations. We find that 34 out of 55 combinations are empty sets, because they do not have any intersection in term of all other variables but a.

Therefore, we have 21 regions that cover the range of $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$ after we consider the combinations. We delegate the details of analysis to Mathematica. Also, by adding sub-regions B-I-2-i-P4S1 through B-I-2-i-P4S4 from the above to this number, in total we have 25 regions in B-I for the range of $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$ which we find them based on an exhaustive search over range of $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$. We provide the list of these 25 regions in Table 3. We label them from B-I-R1 to B-I-R25. Also in the same table, we provide info that shows which regions are combined together to create each of B-I-R1 to B-I-R25 .

B-I	Combination of Sub-Regions
B-I-R1	B-I-2-i-P4S1
B-I-R2	B-I-2-i-P4S2
B-I-R3	B-I-2-i-P4S3
B-I-R4	B-I-2-i-P4S4
B-I-R5	B-I-2-i-P3 and B-I-2-ii, and B-I-1-P3
B-I-R6	B-I-2-i-P1S1, and B-I-1-P4
B-I-R7	B-I-2-i-P1S1, and B-I-1-P6
B-I-R8	B-I-2-i-P1S2, and B-I-1-P3
B-I-R9	B-I-2-i-P1S3 and B-I-1-P1, and B-I-1-P5
B-I-R10	B-I-2-i-P1S3, and B-I-1-P2
B-I-R11	B-I-2-i-P1S3, and B-I-1-P3
B-I-R12	B-I-2-i-P1S4, and B-I-1-P4
B-I-R13	B-I-2-i-P1S4, and B-I-1-P6
B-I-R14	B-I-2-i-P1S5, and B-I-1-P2
B-I-R15	B-I-2-i-P1S5, and B-I-1-P3
B-I-R16	B-I-2-i-P2S1, and B-I-1-P4
B-I-R17	B-I-2-i-P2S1, and B-I-1-P6
B-I-R18	B-I-2-i-P2S2, and B-I-1-P3
B-I-R19	B-I-2-i-P2S3 and B-I-1-P1, and B-I-1-P5
B-I-R20	B-I-2-i-P2S3, and B-I-1-P2
B-I-R21	B-I-2-i-P2S3, and B-I-1-P3
B-I-R22	B-I-2-i-P2S4, and B-I-1-P4
B-I-R23	B-I-2-i-P2S4, and B-I-1-P6
B-I-R24	B-I-2-i-P2S5, and B-I-1-P2
B-I-R25	B-I-2-i-P2S5, and B-I-1-P3

TABLE 3. List of 25 sub-regions of B-I for the range of $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$

Next, we list all regions of B-II based on the range of a. Note that in each of the subregion of B-II, the interval of a may be a sub-interval of $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$.

– *when* $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$, *we have 13 regions as follows:*

- *B-II-1*
- *B-II-3-ii*
- *B-II-2-b-1-i-P1 through B-II-2-b-1-i-P3*
- *Combination of B-II-2-b-2-ii-P2 with B-II-2-b-1-ii-P1*
- *Combination of B-II-2-b-2-ii-P1 with B-II-2-b-1-ii-P2*
- *B-II-2-b-2-i and B-II-2-b-2-iii*
- *Combination of B-II-2-d-P1 with B-II-3-i*
- *Combination of B-II-2-d-P2 with B-II-3-i*
- *Combination of B-II-2-d-P3-i with B-II-3-i*
- *Combination of B-II-2-d-P3-ii with B-II-3-i*

– *when* $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant \frac{\alpha}{2D(w-c)}$, *we have 4 regions as follows:*

- *B-II-1-P2 through B-II-1-P5*

– *when* $\frac{\alpha}{2D(w-c)} < a \leqslant 1$, *we have 6 regions as follows:*

- *B-II-2-a-1-i*
- *Combination of B-II-2-a-1-ii with B-II-2-a-2-ii*
- *B-II-2-a-2-i-P1 through B-II-2-a-2-i-P3*
- *Combination of B-II-3-i-P2 with B-II-2-c*

From the above list, we combine regions defined over $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant \frac{\alpha}{2D(w-c)}$ with regions defined over $\frac{\alpha}{2D(w-c)} < a \leqslant 1$ to form the range of a for $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$. We pick each region in the above list for $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant \frac{\alpha}{2D(w-c)}$ and combine it with each region in the above list for $\frac{\alpha}{2D(w-c)} < a \leqslant 1$. There are total 24 possible combinations.

We find that 18 out of 24 combinations are empty sets, because they do not have any intersection in term of all other variables but a. Therefore, after we consider the combinations, we have 6 regions that cover the range of $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$. We delegate the details of analysis to Mathematica. Also, by adding 13 sub-regions that listed in the above in which cover the range of $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$ without any combination, in total we have 19 regions in B-II for the range of $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$ which we find them based on an exhaustive search over range of $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$. We provide the list of these 19 regions in Table 4.

We label them from B-II-R1 to B-II-R19. Also in the same table, we provide info that shows which regions are combined together to create each of B-II-R1 to B-II-R19 . Now, we pick one region from 25 regions in B-I which is defined over $0 \leqslant a \leqslant \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$ and combine it with one region from 19 regions in B-II which is defined over $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$ to create $e^*(a)$ over the full support of $0 \leqslant a \leqslant 1$. We do the same for all combinations.

There are total 475 possible combinations. After going through each combination with the aid of Mathematica, we find that: firstly, only 41 combinations are not empty sets. Also, we find that there exist only 9 distinct forms of the supplier's best response function in Region B as we list them in Table 5 by labels of S-BRF 1-1 to S-BRF 1-5 and S-BRF 2-1, SBRF 2-3 to S-BRF 2-5 and also show the structure of each of them as follows.

B-II	Combination of Sub-Regions
B-II-R1	B-II-1
B-II-R2	B-II-3-ii
B-II-R3	B-II-2-b-1-i-P1
B-II-R4	B-II-2-b-1-i-P2
B-II-R5	B-II-2-b-1-i-P3
B-II-R6	B-II-2-b-ii-P2 and B-II-2-b-1-ii-P1
B-II-R7	B-II-2-b-2-ii-P1 and B-II2-b-1-ii-P2
B-II-R8	B-II-2-b-2-i
B-II-R9	B-II-2-b-2-iii
B-II-R10	B-II-2-d-P1 and B-II-3-i
B-II-R11	B-II-2-d-P2 and B-II-3-i
B-II-R12	B-II-3-d-P3-i and B-II-3-i
B-II-R13	B-II-3-d-P3-ii and B-II-3-i
B-II-R14	B-II-1-P2 and B-II-2-a-2-i-P3
B-II-R15	B-II-1-P3 and B-II-2-a-2-i-P2
B-II-R16	B-II-1-P4 and B-II-2-a-1-i
B-II-R17	B-II-1-P4 and B-II-2-a-1-ii and B-II-2-a-2-ii
B-II-R18	B-II-1-P4 and B-II-3-i and B-II-2-c
B-II-R19	B-II-1-P5 and B-II-2-a-2-i-P1

TABLE 4. List of 19 sub-regions of B-II for the range of $\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)} < a \leqslant 1$

Specifically, S-BRF 1-1 has 7 regions, S-BRF 1-2 has 2 regions, S-BRF 1-3 has 5 regions, S-BRF 1-4 has 4 regions, S-BRF 1-5 has 2 regions, S-BRF 2-1 has 14 regions, S-BRF 2-3 has 2 regions, S-BRF 2-4 has 4 regions, and S-BRF 2-5 has 2 regions.

	Derived from Sub-regions of		Location in Figure 17	
S-BRF Form	B-I	B-II	B-i	B-ii
S-BRF 1-1	B-I-R1	B-II-R1	✓	
	B-I-R1	B-II-R14	✓	
	B-I-R1	B-II-R15	✓	
	B-I-R6	B-II-R9	✓	
	B-I-R7	B-II-R8	✓	
	B-I-R16	B-II-R9	✓	
	B-I-R17	B-II-R8	✓	
S-BRF 1-2	B-I-R4	B-II-R17	✓	
	B-I-R4	B-II-R19	✓	
S-BRF 1-3	B-I-R3	B-II-R15	✓	
	B-I-R12	B-II-R9	✓	
	B-I-R23	B-II-R8	✓	
	B-I-R22	B-II-R9	✓	
	B-I-R13	B-II-R8	✓	
S-BRF 1-4	B-I-R9	B-II-R8	✓	
	B-I-R20	B-II-R6	✓	
	B-I-R19	B-II-R8	✓	
	B-I-R10	B-II-R6	✓	
S-BRF 1-5	B-I-R14	B-II-R7	✓	
	B-I-R24	B-II-R7	✓	
S-BRF 2-1	B-I-R5	B-II-R2		✓
	B-I-R8	B-II-R2		✓
	B-I-R8	B-II-R10		✓
	B-I-R8	B-II-R12		✓
	B-I-R8	B-II-R13		✓
	B-I-R15	B-II-R3		✓
	B-I-R15	B-II-R5		✓
	B-I-R18	B-II-R2		✓
	B-I-R18	B-II-R10		✓
	B-I-R18	B-II-R12		✓
	B-I-R18	B-II-R13		✓
	B-I-R25	B-II-R3		✓
	B-I-R25	B-II-R4		✓
	B-I-R25	B-II-R5		✓
S-BRF 2-3	B-I-R2	B-II-R18		✓
	B-I-R4	B-II-R16		✓
S-BRF 2-4	B-I-R8	B-II-R11		✓
	B-I-R15	B-II-R4		✓
	B-I-R18	B-II-R11		✓
S-BRF 2-5	B-I-R11	B-II-R3		✓
	B-I-R21	B-II-R3		✓

TABLE 5. List of the Supplier's Best Response Function forms in Region B

S-BRF 1-1) $e^* = 0$ for all a in $[0,1]$.

S-BRF 1-2) $e^* = \hat{e}$ for all a in $[0,1]$.

$$\text{S-BRF 1-3)} \quad e^* = \begin{cases} 0 & 0 \leqslant a < a_2^{B-I} \\ \hat{e} & a_2^{B-I} \leqslant a < a_1^{B-I} \\ 0 & a_1^{B-II} \leqslant a \leqslant 1 \end{cases}.$$

$$\text{S-BRF 1-4)} \quad e^* = \begin{cases} 0 & 0 \leqslant a < a_2^{B-I} \\ \hat{e} & a_2^{B-I} \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a < \frac{\alpha^2}{4\beta(\alpha-D(w-c))} \\ 0 & \frac{\alpha^2}{4\beta(\alpha-D(w-c))} \leqslant a \leqslant 1 \end{cases}.$$

$$\text{S-BRF 1-5)} \quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a < a_2^{B-II} \\ \hat{e} & a_2^{B-II} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}1)\quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}2)\quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < \dot{a} \\ \bar{e} & \dot{a} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}3)\quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < a_1^{B-II} \\ \bar{e} & a_1^{B-II} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}4)\quad e^* = \begin{cases} \hat{e} & 0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a < a_2^{B-II} \\ \hat{e} & a_2^{B-II} \leqslant a < a_1^{B-II} \\ \bar{e} & a_1^{B-II} \leqslant a \leqslant 1 \end{cases}.$$

$$S\text{-}BRF\ 2\text{-}5)\quad e^* = \begin{cases} 0 & 0 \leqslant a < a_2^{B-I} \\ \hat{e} & a_2^{B-I} \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant 1 \end{cases}.$$

Also, $\hat{e} = \frac{1-a}{\frac{p}{w}-a}$, $\bar{e} = 1 - \frac{\alpha}{2a\beta}$, $\dot{a} = \frac{\alpha(4\beta(p-w)+w\alpha+\sqrt{w^2\alpha^2+8(p-w)(w\alpha-2Dp(w-c))\beta+16(p-w)^2\beta^2})}{8D(p-w)(w-c)\beta}$,
$a_1^{B-I} = \frac{-(w^2\alpha-pw(2D(w-c)-\alpha)-\beta(p-w)^2)-\sqrt{\phi}}{2(w^2(D(w-c)-\alpha))}$,
$a_2^{B-I} = \frac{-(w^2\alpha-pw(2D(w-c)-\alpha)-\beta(p-w)^2)+\sqrt{\phi}}{2(w^2(D(w-c)-\alpha))}$,
$a_1^{B-II} = \frac{-(-[4\alpha\beta(p-w)+w\alpha^2])+\sqrt{[(-[4\alpha\beta(p-w)+w\alpha^2])^2-4(4D\beta(p-w)(w-c))(p\alpha^2)]}}{2(4D\beta(p-w)(w-c))}$,
$a_2^{B-II} = \frac{-(-[4\alpha\beta(p-w)+w\alpha^2])-\sqrt{[(-[4\alpha\beta(p-w)+w\alpha^2])^2-4(4D\beta(p-w)(w-c))(p\alpha^2)]}}{2(4D\beta(p-w)(w-c))}$ where $\phi = ((w^2\alpha - pw(2D(w-c)-\alpha)-\beta(p-w)^2)^2 - 4(w^2(D(w-c)-\alpha))(Dp^2(w-c)-pw\alpha))$.

As we show in Table 5, S-BRF 1-1 to S-BRF 1-5 are in Region B-i of Figure 25 in

Lemma 16. Also, S-BRF 2-1 to S-BRF 2-5 are in Region B-ii of Figure 25 in Lemma 16. To prove this characteristic, we take each region in Table 5 and if it is in region B-i, then it satisfies the condition $D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. Also, if it is in region B-ii, then it satisfies $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. For example, choose S-BRF 1-1 from Region B-i of Figure 25 in Lemma 16. Also, from Table 5 we choose one of regions that belongs to S-BRF 1-1. For instance, we pick B-I-R7 and B-II-R8. We follow the following procedure to show that this region satisfies condition $D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$.

Region B-I-R7 includes the combination of B-I-2-i-P1S1 and B-I-1-P6. Also, B-II-R8 includes B-II-2-b-2-i.

- In Region B-I-2-i-P1S1 under:

$$\frac{2\beta(p-w)+w\alpha}{2p(w-c)} \leqslant D < \frac{2\beta(p-w)+w\alpha}{p(w-c)} \quad \text{and} \quad w \leqslant p < w(2-\frac{\alpha}{\beta}) \quad \text{and} \tag{A.20}$$

$$\begin{aligned}&[(w^2\alpha - pw(2D(w-c)-\alpha) - \beta(p-w)^2)^2 \\ &- 4(w^2(D(w-c)-\alpha))(Dp^2(w-c)-pw\alpha)] < 0,\end{aligned} \tag{A.21}$$

  we have $e^* = 0$ for all a in $[0, \frac{p\alpha}{2\beta(p-w)+w\alpha})$.

- In Region B-I-1-P6, under :

$$\frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \leqslant D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \quad \text{and} \quad w \leqslant p < \frac{w\alpha(2\beta-\alpha)}{2\beta[2D(w-c)-\alpha]}, \tag{A.22}$$

  we have $e^* = 0$ for all a in $[\frac{p\alpha}{2\beta(p-w)+w\alpha}, \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}]$.

– In Region B-II-2-b-2-i, under:

$$\frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} < D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \text{ and } w \leqslant p < \frac{w[\frac{\alpha^2(w-c)}{D(w-c)-\alpha}+4\beta^2]}{4\beta(\beta-D(w-c))}, \quad \text{(A.23)}$$

we have $e^* = 0$ for all a in $(\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}, 1]$.

Note that the range of a defined in the above cases are complementary. Next, we identify the intersection of three regions defined by (A.20), (A.22), and (A.23) to create $e^*(a)$ for all a in $[0, 1]$. Their intersection based on D is:

$$\frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} < D < \frac{2pw\beta(\beta+\alpha)-p^2\beta^2-w^2(\beta-\alpha)^2}{4pw\beta(w-c)}. \quad \text{(A.24)}$$

Finally, we prove that the region defined by (A.24) is sub-region of Region B-i of Figure 25 in Lemma 16 by showing that $\frac{2pw\beta(\beta+\alpha)-p^2\beta^2-w^2(\beta-\alpha)^2}{4pw\beta(w-c)} < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$:

$$\begin{aligned} &\frac{2pw\beta(\beta+\alpha)-p^2\beta^2-w^2(\beta-\alpha)^2}{4pw\beta(w-c)} < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} \\ \Leftrightarrow\ & 2pw\beta(\beta+\alpha)-p^2\beta^2-w^2(\beta-\alpha)^2 < \alpha(4\beta-\alpha) \\ \Leftrightarrow\ & 0 < (p-w)[p\beta^2 - w(\beta-\alpha)^2] \\ \Leftrightarrow\ & \frac{(\beta-\alpha)^2}{\beta^2} < 1 < \frac{p}{w}. \end{aligned}$$

In a similar procedure, we prove that all the supplier's best response function forms S-BRF 1-1 to S-BRF 1-5 satisfy the condition $D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. The proof with more details is available on software upon request.

To prove one of regions from region B-ii in Figure 25 of Lemma 16, we consider S-BRF 2-3. Also, from Table 5, as an example we pick B-I-R2 and B-II-R18. The next step is to prove that this region satisfies the condition $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. The region B-I-R2 of Table 5 includes *B-I-2-i-P4S2*. Also, region B-II-R18 includes *B-II-1-P4*, *B-II-2-c*, and *B-II-3-i*.

– In Region B-I-2-i-P4S2 under

$$\begin{aligned} &(p-w)^2[p^2\beta^2 - 2pw\beta(2cD+\alpha+\beta) + w^2((\beta-\alpha)^2 + 4Dp\beta)] \geqslant 0 \quad \text{and} \\ &Dw^2(w-c) - w^2\alpha > 0, \end{aligned} \tag{A.25}$$

we have $e^* = \hat{e}$ for all $a \in [0, \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}]$.

– In Region B-II-1-P4 under

$$\frac{2\alpha\beta}{(w-c)(2\beta+\alpha)} \leqslant D < \frac{\beta}{w-c} \quad \text{and} \quad \frac{w\beta(2\beta-\alpha)}{2e_4} \leqslant p < \frac{w}{e_4}, \tag{A.26}$$

we have $e^* = \hat{e}$ for all $a \in [\frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}, \frac{\alpha}{2D(w-c)}]$.

– In Region B-II-2-c under

$$\frac{\alpha}{w-c} < D \leqslant \frac{\beta}{w-c} \quad \text{and} \quad \frac{w\beta(2\beta-\alpha)}{2e_4} < p < \frac{w}{e_4}\ , \tag{A.27}$$

we have $e^* = \begin{cases} \hat{e} & \frac{\alpha}{2D(w-c)} < a \leqslant a_1^{B-II} \\ \bar{e} & a_1^{B-II} < a \leqslant \frac{\alpha}{D(w-c)} \end{cases}$.

– In Region B-II-3-i under

$$\frac{\alpha}{w-c} \leqslant D < \frac{\beta}{(w-c)} \text{ and } \frac{w(\beta-\alpha)}{\beta e_4} \leqslant p < \frac{w}{e_4}, \tag{A.28}$$

we have $e^* = \bar{e}$ for all $a \in (\frac{\alpha}{D(w-c)}, 1]$.

We identify the intersection of four regions defined by (A.25), (A.26), (A.27), and (A.28) to create $e^*(a)$ for all a in $[0, 1]$. Their intersection is:

$$\begin{aligned}
&\{\frac{\beta(p-w)}{p(w-c)} < D < \frac{2\beta(p-w)+w\alpha}{2p(w-c)} \text{ and } \frac{w\beta}{\beta-\alpha} < p\} \text{ or}\\
&\{\frac{\alpha}{w-c} < D < \frac{2\beta(p-w)+w\alpha}{2p(w-c)} \text{ and } \frac{w(2\beta-\alpha)}{2(\beta-\alpha)} < p < \frac{w\beta}{\beta-\alpha}\}, \text{ or}\\
&\{\frac{\beta(2p-w(2\beta-\alpha)\beta)}{2p(w-c)} < D < \frac{2\beta(p-w)+w\alpha}{2p(w-c)} \quad \text{and} \quad \frac{w(2\beta-\alpha)\beta^2}{2(\beta-\alpha)} < p\}.
\end{aligned}$$

We prove that S-BRF 2-3 is always in Region B-ii of Figure 25 in Lemma 16 by showing that the lower bound of D in above condition is always bigger than $\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. So if $\frac{\beta(p-w)}{p(w-c)} < D < \frac{2\beta(p-w)+w\alpha}{2p(w-c)}$ and $\frac{w\beta}{\beta-\alpha} < p$ then:

$$\begin{aligned}
&\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < \frac{\beta(p-w)}{p(w-c)}\\
\Leftrightarrow\ &\frac{\alpha(4\beta-\alpha)}{4\beta} < \frac{\beta(p-w)}{p}\\
\Leftrightarrow\ &\alpha(1-\frac{\alpha}{4\beta}) < \beta(1-\frac{w}{p})\\
\Leftrightarrow\ &\alpha(1-\frac{\alpha}{4\beta}) < \alpha < \beta(1-\frac{w}{p}).
\end{aligned} \tag{A.29}$$

The inequality (A.29) holds because in this case we have $\frac{w\beta}{\beta-\alpha} < p \Leftrightarrow \alpha < \beta(1-\frac{w}{p})$. For the second part of condition which we $\frac{\alpha}{w-c} < D < \frac{2\beta(p-w)+w\alpha}{2p(w-c)}$ and $\frac{w(2\beta-\alpha)}{2(\beta-\alpha)} < p < \frac{w\beta}{\beta-\alpha}$, it suffices to show that:

$$\begin{aligned}
&\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < \frac{\alpha}{w-c}\\
\Leftrightarrow\ &\frac{4\beta-\alpha}{4\beta} < 1\\
\Leftrightarrow\ &1-\frac{\alpha}{4\beta} < 1.
\end{aligned}$$

Also for the last part which we have $\frac{\beta(2p-w(2\beta-\alpha)\beta)}{2p(w-c)} < D < \frac{2\beta(p-w)+w\alpha}{2p(w-c)}$ and $\frac{w(2\beta-\alpha)\beta^2}{2(\beta-\alpha)} < p$, it suffices to show that: $\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < \frac{\beta(2p-w(2\beta-\alpha)\beta)}{2p(w-c)} \Leftrightarrow \frac{2w\beta^3}{2\beta-\alpha} < p$.

$$\begin{aligned} & \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < \frac{\beta(2p-w(2\beta-\alpha)\beta)}{2p(w-c)} \\ \Leftrightarrow\ & \frac{2w\beta^3}{2\beta-\alpha} < p \end{aligned}$$

If we show that $\frac{2w\beta^3}{2\beta-\alpha} < \frac{w(2\beta-\alpha)\beta^2}{2(\beta-\alpha)}$ then the proof is complete:

$$\begin{aligned} & \frac{2w\beta^3}{2\beta-\alpha} < \frac{w(2\beta-\alpha)\beta^2}{2(\beta-\alpha)} \\ \Leftrightarrow\ & 4\beta(\beta-\alpha) < (2\beta-\alpha)^2 \\ \Leftrightarrow\ & 0 < \alpha^2. \end{aligned}$$

In a similar procedure, we prove that all the supplier's best response function forms S-BRF 2-1 to S-BRF 2-5 satisfy the condition $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. The proof with more details is available in Mathematica codes in Appendix.

Therefore, we prove that in Region B-i of Lemma 16, possible S-BRF forms are: S-BRF 1-1, S-BRF 1-2, S-BRF 1-3, S-BRF 1-4, and S-BRF 1-5. Also, we prove that in Region B-II of Lemma 16, possible S-BRF forms are: S-BRF 2-1, S-BRF 2-3, S-BRF 2-4, and S-BRF 2-5.

Now, we discuss couple of characteristics of the supplier's best response functions in *region B* which help us in finding NE in the next Lemma. S-BRF is continuous in a only at point $a = \frac{p\alpha}{2\beta(p-w)+w\alpha}$ and it is not continuous in other start and end of domain interval points (e.g. a_1^{B-II}, a_2^{B-II} and etc.) We should know the behavior of

$\hat{e}$ and $\bar{e}$. Firstly, the intersection of $e^* = \hat{e}$ and $e^* = \bar{e}$ happens at $a = \frac{p\alpha}{2\beta(p-w)+w\alpha}$:

$$\bar{e} = \hat{e} \Rightarrow 1 - \frac{\alpha}{2a\beta} = \frac{1-a}{\frac{p}{w} - ea} \Rightarrow a = \frac{p\alpha}{2\beta(p-w)+w\alpha}.$$

In addition to this, we know that $e^* = \hat{e}$ is decreasing and concave in a and $e^* = \bar{e}$ is increasing and concave in a for all $a \in (0,1)$:

$$\frac{\partial \hat{e}}{\partial a} = -\frac{w(p-w)}{(p-aw)^2} < 0 \quad , \frac{\partial^2 \hat{e}}{\partial a^2} = -\frac{2w^2(p-w)}{(p-aw)^3} < 0,$$
$$\frac{\partial \bar{e}}{\partial a} = \frac{\alpha}{2a^2\beta} > 0 \quad , \quad \frac{\partial^2 \bar{e}}{\partial a^2} = -\frac{\alpha}{a^3\beta} < 0.$$

In S-BRF 2-3 as intersection happens in another point (i.e., $a = a_1^{B-II}$) function is not continuous in a. Also we observe a jump in the function at $a = a_1^{B-II}$ based on above-mentioned characteristics of $\hat{e}$ and $\bar{e}$ as $\frac{p\alpha}{2\beta(p-w)+w\alpha} < a_1^{B-II}$, where $a_1^{B-II} = \frac{4\alpha\beta(p-w)+w\alpha^2+\sqrt{\alpha^2[(4p\beta-w(4\beta-\alpha))^2-16Dp\beta(w-c)(p-w)]}}{8D\beta(p-w)(w-c)}$:

$$a_1^{B-II} > \frac{p\alpha}{2\beta(p-w)+w\alpha}$$

$$\Leftrightarrow \frac{4\alpha\beta(p-w)+w\alpha^2+\sqrt{\alpha^2[(4p\beta-w(4\beta-\alpha))^2-16Dp\beta(w-c)(p-w)]}}{8D\beta(p-w)(w-c)}$$
$$-\frac{p\alpha}{2\beta(p-w)+w\alpha} > 0$$
$$\Leftrightarrow \frac{p\alpha(w\alpha^2+\sqrt{\alpha^2[(4p\beta-w(4\beta-\alpha))^2-16Dp\beta(w-c)(p-w)]})}{[2\beta(p-w)+w\alpha]\Phi} > 0$$

where $\Phi = (4\alpha\beta(p-w)+w\alpha^2-\sqrt{\alpha^2[(4p\beta-w(4\beta-\alpha))^2-16Dp\beta(w-c)(p-w)]})$ in the last inequality, the nominator is positive, and in denominator $[2\beta(p-w)+w\alpha] > 0$, if $(4\alpha\beta(p-w)+w\alpha^2-\sqrt{\alpha^2[(4p\beta-w(4\beta-\alpha))^2-16Dp\beta(w-c)(p-w)]}) > 0$

then we prove our claim:

$$
\begin{aligned}
&(4\alpha\beta(p-w)+w\alpha^2-\sqrt{\alpha^2[(4p\beta-w(4\beta-\alpha))^2-16Dp\beta(w-c)(p-w)]})>0\\
&\Leftrightarrow [4\alpha\beta(p-w)+w\alpha^2]^2-[\alpha^2[(4p\beta-w(4\beta-\alpha))^2-16Dp\beta(w-c)(p-w)]]>0\\
&\Leftrightarrow 16Dp(p-w)(w-c)\alpha^2\beta>0\ .
\end{aligned}
$$

Region C: ($\frac{\beta}{w-c}\leqslant D$)

Following the structure of optimal order quantities in Figure 21, we form the supplier's profit function for *Region C*:

$$
\Pi_S^C(a,e)=\begin{cases}(1-e)a[D(w-c)-\beta(1-e)]-\alpha e & 0\leqslant e<\hat{e}\\ [e+(1-e)(1-a)]D(w-c)+(1-e)a[D(w-c)-\beta(1-e)]-\alpha e & \hat{e}\leqslant e\leqslant 1\end{cases}. \tag{A.30}
$$

From (A.30), we verify that the supplier's function in region C is not continuous at $\hat{e}$ because $\Pi_S^C(a,e)|_{e=\hat{e}}-\lim_{e\to\hat{e}^-}\Pi_S^C(a,e)=\dfrac{D(1-a)p(w-c)}{p-aw}>0$ where:

$$
\begin{aligned}
\lim_{e\to\hat{e}_r^-}\Pi_S^C(a,e)=&\frac{(p-aw)[Da(w-c)(p-w)-(1-a)w\alpha]-a\beta(p-w)^2}{(p-aw)^2},\\
\Pi_S^C(a,e)|_{e=\hat{e}}=&D(w-c)-\frac{(1-a)w(p-aw)+a(p-w)^2\beta}{(p-aw)^2}.
\end{aligned}
$$

Also, we can verify that $\Pi_S^C(a,e)|_{e=\hat{e}}>\Pi_S^C(a,e)|_{e=\hat{e}^-}$. And in general we prove that for all $e\in(0,1)$ the second piece of the supplier's function is greater than the first piece:

$$
\begin{aligned}
& D(w-c) - \frac{(1-a)w(p-aw) + a(p-w)^2\beta}{(p-aw)^2} \\
& - \frac{(p-aw)[Da(w-c)(p-w) - (1-a)w\alpha] - a\beta(p-w)^2}{(p-aw)^2} > 0 \\
\Leftrightarrow & \frac{Dp(1-a)(w-c)}{p-aw} > 0.
\end{aligned}
$$

Derivative of the supplier's profit with respect to e is:

$$
\frac{\partial \Pi_S^C(a,e)}{\partial e} = \begin{cases} a[2\beta(1-e) - D(w-c)] - \alpha & 0 \leqslant e < \hat{e} \\ 2a(1-e)\beta - \alpha & \hat{e} \leqslant e \leqslant 1 \end{cases}. \tag{A.31}
$$

From (A.31), we verify that $\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=1} < 0$. In addition we can show that $\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=(\hat{e})^-} < \frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=\hat{e}}$ where:

$$
\begin{aligned}
\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{\hat{e}^-} &= -Da(w-c) - \alpha + \frac{2a(p-w)\beta}{p-aw} \\
\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{\hat{e}} &= -\alpha + \frac{2a(p-w)\beta}{p-aw}.
\end{aligned}
$$

Second order derivative of the supplier's profit function (A.30) with respect to e is $-2a\beta$ for all $e \in (0,1)$ which shows that both pieces of the function (A.30) are concave in e.

We divide the *region* C into two sub-regions based on order of $\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=(\hat{e})^-}$, $\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=\hat{e}}$, and zero.

- Sub-region C-I: $\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=(\hat{e})^-} < \frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=\hat{e}} \leqslant 0$, and
- Sub-region C-II: $\{\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=(\hat{e})^-} < 0 < \frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=\hat{e}}\}$ or $\{0 \leqslant \frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=(\hat{e})^-} < \frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=\hat{e}}\}$.

C-I) when $\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=(\hat{e})^-} < \frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=\hat{e}} \leqslant 0.$

If condition of Region C holds, then simplifying the condition of C-I gives us:

$$0 \leqslant a \leqslant \frac{p\alpha}{2\beta(p-w)+w\alpha} \quad \text{and} \quad \frac{\beta}{w-c} \leqslant D. \tag{A.32}$$

Under condition (A.32), the second piece of function (A.30) is decreasing concave in e and first piece is concave in e. Now if $\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=0} < 0$, then we name this new sub-region *C-I-i* which is defined by: $a[2\beta - D(w-c)] - \alpha < 0$. We illustrate this sub-region in Figure 30. To find optimal solution when condition *C-I-i* holds,

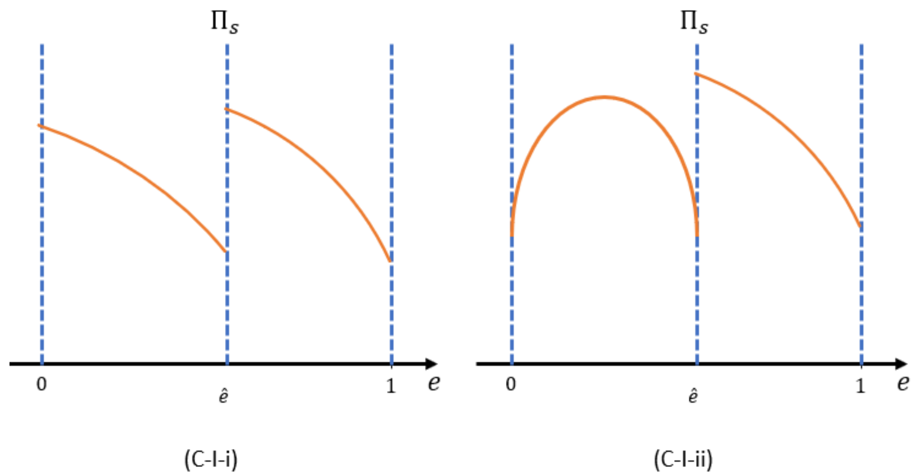


FIGURE 30. Possible Forms of the Supplier's Profit Function in Region C-I

we compare $\Pi_S^C(a,e)|_{e=0}$ with $\Pi_S^C(a,e)|_{e=\hat{e}}$. As we proved similarly in *Region B-I*, condition of *C-I-i* implies that $\Pi_S^C(a,e)|_{e=0} < \Pi_S^C(a,e)|_{e=\hat{e}}$ which means $e^* = \hat{e}$. Next sub-region is *C-I-ii*, which is defined when $\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=0} \geqslant 0$. In this situation we have two candidate solution as we illustrate in Figure 30. Thus, to find optimal solution, we compare $\Pi_S^C(a,e)|_{e=\tilde{e}}$ with $\Pi_S^C(a,e)|_{e=\hat{e}}$ where $\tilde{e}$ is the global maximum of first piece $\tilde{e} \stackrel{\text{def}}{=} 1 - \frac{1}{2\beta}D(w-c) - \frac{\alpha}{2a\beta}$. As we proved it in *Region B-I*, similarly here condition of $a[2\beta - D(w-c)] - \alpha \geqslant 0$ implies that $\Pi_S^C(a,e)|_{e=\tilde{e}} < \Pi_S^C(a,e)|_{e=\hat{e}}$

which means $e^* = \hat{e}$. Therefore for all sub-region *C-I* we proved that:

$$if \ \frac{\beta}{w-c} \leqslant D \implies e^* = \hat{e} \quad \text{for all} \quad a \in [0, \frac{p\alpha}{2\beta(p-w)+w\alpha}].$$

C-II) when $\{\frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=(\hat{e})^-} < 0 < \frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=\hat{e}}\}$ or $\{0 \leqslant \frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=(\hat{e})^-} < \frac{\partial \Pi_S^C(a,e)}{\partial e}|_{e=\hat{e}}\}$.

If condition of Region C holds, then simplifying the condition of C-II gives us:

$$\frac{p\alpha}{2\beta(p-w)+w\alpha} < a \leqslant 1 \quad \text{and} \quad \frac{\beta}{w-c} \leqslant D. \tag{A.33}$$

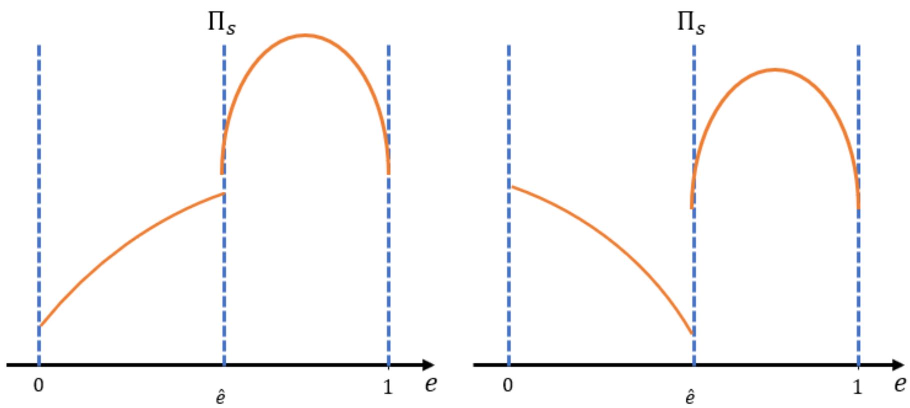


FIGURE 31. Possible Forms of the Supplier's Profit Function in Region C-II

We show possible cases of this sub-region in Figure 31. When condition *C-II* holds, the local optimal of second piece ($\bar{e} \stackrel{\text{def}}{=} 1 - \frac{\alpha}{2a\beta}$) occurs at e in $[\hat{e}, 1]$. As we proved in characteristics of function (A.30), if the $e = \bar{e}$ exist in interval $[\hat{e}, 1]$, then it is the global maximum of function (A.30). Therefore for all sub-regions of *C-II*, we proved that:

$$if \ \frac{\beta}{w-c} \leqslant D \implies e^* = \bar{e} \quad \text{for all} \quad a \in (\frac{p\alpha}{2\beta(p-w)+w\alpha}, 1].$$

For *Region C*, combining sub-region C-I and sub-region C-II provides us the final best response function for this region:

$$\text{S-BRF 2-1: } e^* = \begin{cases} \hat{e} & 0 \leqslant a \leqslant \frac{p\alpha}{2\beta(p-w)+w\alpha} \\ \bar{e} & \frac{p\alpha}{2\beta(p-w)+w\alpha} < a \leqslant 1 \end{cases}.$$

This S-BRF belongs to Region B-ii of Figure 25 in Lemma 16. We prove it by comparing the lower bound of D in definition of Region C with $D = \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$:

$$\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < \frac{\beta}{w-c} \Leftrightarrow (2\beta-\alpha)^2 > 0.$$

Hence, we complete the proof of Lemma 16. □

Proof of Lemma 4. We derive the Nash Equilibrium (NE) using the buyer's best response function (B-BRF) in Lemma 15 and the supplier's best response function (S-BRF) in Lemma 16. Recall that in Lemma 2 we divide the plane of (D,w) into Regions A, B and C and use them to organize the proof of the B-BFR. B-BFR can be obtained from Lemma 15 and is illustrated for regions 1 through 6 of the plane of (D,w) in Figure 22. In Lemma 16, we divide Region A into two subregions A-i and A-ii, and Region B into B-i and B-ii, in which the S-BRF has a unique form, as illustrated in Figure 25. We proceed in the order of regions A-i, A-ii, B-i, B-ii, and C in this proof.

Region A-i: We want to prove that the NE is one of the following three forms:

– NE form III: $(e^*, a^*) = (\frac{w}{p}, 0)$, and

– NE form IV: $(e^*, a^*) = (0, 0)$ or no Nash Equilibrium.

Recall from Lemma 16 that region A-i is defined by $0 \leqslant D \leqslant \frac{\beta(p-w)}{p(w-c)}$ and $D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. In this region we have two possible forms of the supplier's best response function: S-BRF 1-1 and S-BRF 1-2, given in Lemma 16. In the same region, there are at most 5 possible forms of the buyer's best response function, depending on the position of the intersection of boundary $\frac{\beta(p-w)}{p(w-c)}$ and boundary $\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$: B-BRF 1 through B-BRF 5, given in Lemma 15. The proof does not depend on how many forms of B-BFR are selected. We prove with the example of all 5 forms of B-BRF.

First, consider the S-BRF 1-1, which is $e^* = 0$ for all a in $[0, 1]$. The intersection of S-BRF 1-1 with all forms of B-BRF creates NE $(e^*, a^*) = (0, 0)$, because $a^*(0) = 0$ in all forms of the B-BRF. $(e^*, a^*) = (0, 0)$ is NE form IV. When $(e^*, a^*) = (0, 0)$, the buyer's expected profit Π_B and the supplier's expected profit Π_S are zero.

Next, consider S-BRF 1-2, which is $e^* = \hat{e}(a)$ for all a in $[0, 1]$, and intersect it with all 5 forms of B-BRF.

– Intersect S-BRF 1-2 with B-BRF 1 and B-BRF 5, respectively. Recall that $e^* = \hat{e}(a)$ is a decreasing concave continuous function in a with $e^* = \hat{e}(1) = 0$ and $e^* = \hat{e}(0) = \frac{w}{p}$. In both B-BRF 1 and B-BRF 5, $e^*(0) = \frac{w}{p}$ and $a^*(\frac{w}{p}) = 0$. Therefore, the NE is $(e^*, a^*) = (\frac{w}{p}, 0)$, which is NE form III.

– Intersect S-BRF 1-2 with B-BRF 2. Note that from Lemma 15, $e_1 < \frac{w}{p} < e_2$. There is no intercept between the B-BRF and S-BRF. So, there is no NE. We illustrate this situation in Figure 32.

– Similar to the above case, there is no NE when we intercept S-BRF 1-2 with the B-BRF 3 and B-BRF 4, respectively. We illustrate this situation in Figure 32.

In the situations without NE, we abuse the notation and treat them as if there exist the NE of $(e^*, a^*) = (0, 0)$, which is NE form IV, for expositional convenience. Such treatment does not affect the equilibrium results. No NE means that the buyer and the supplier take no action, which is $(e^*, a^*) = (0, 0)$, and that there is no continuation game, that is, $(q_f, q_p) = (0, 0)$. At the NE of $(e^*, a^*) = (0, 0)$, as Lemma 2 shows, the equilibrium of the continuation game is $(q_f, q_p) = (0, 0)$. So, technically, no NE is identical to NE of $(e^*, a^*) = (0, 0)$.

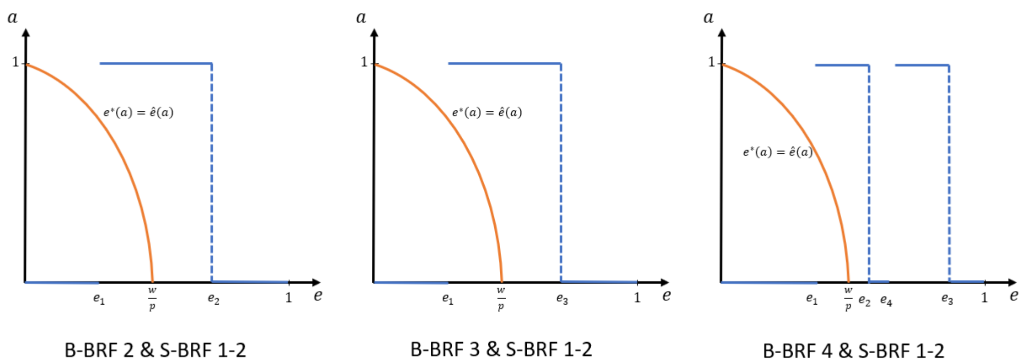


FIGURE 32. Illustration of No NE When Intersecting S-BRF 1-2 with B-BRF 2, B-BRF 3, and B-BRF 4

Region A-ii: We want to prove that the NE is one of the following four forms:

- NE form I: $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$,
- NE form II: $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$,
- NE form III: $(e^*, a^*) = (\frac{w}{p}, 0)$, and
- No Nash Equilibrium (equivalent to NE form IV).

Note that region A-ii is defined by $0 \leqslant D \leqslant \frac{\beta(p-w)}{p(w-c)}$ and $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. In this region there is one possible form of the supplier's best response function: S-BRF 2-2, given in Lemma 16. In same region, there are at most 5 possible forms of the buyer's best response function: B-BRF 1 through B-BRF 5, given in Lemma 15. The proof does not depend on how many forms of B-BFR are selected. We prove with the example of all 5 forms of B-BRF. Recall that S-BRF 2-2 is defined as follows: $e^* = \hat{e}(a)$ for all $0 \leqslant a < \dot{a}$ and $e^* = \bar{e}(a)$ for all $\dot{a} \leqslant a \leqslant 1$. S-BRF 2-2 is not continuous at $a = \dot{a}$. For $0 \leqslant a < \dot{a}$, the S-BRF 2-2 is defined as $\hat{e}(a)$ which is a decreasing continuous concave function in a . Also, $\hat{e}(1) = 0$ and $\hat{e}(0) = \frac{w}{p}$. The second piece of S-BRF 2-2, $\bar{e}(a)$, is an increasing continuous concave function in a for $\dot{a} \leqslant a \leqslant 1$.

- Intersect S-BRF 2-2 with B-BRF 1. B-BRF 1 is defined as follows: $a^* = 0$ for $0 \leqslant e \leqslant 1$. $e^*(0) = \frac{w}{p}$ and $a^*(\frac{w}{p}) = 0$. So, NE is $(e^*, a^*) = (\frac{w}{p}, 0)$, which is NE form III.

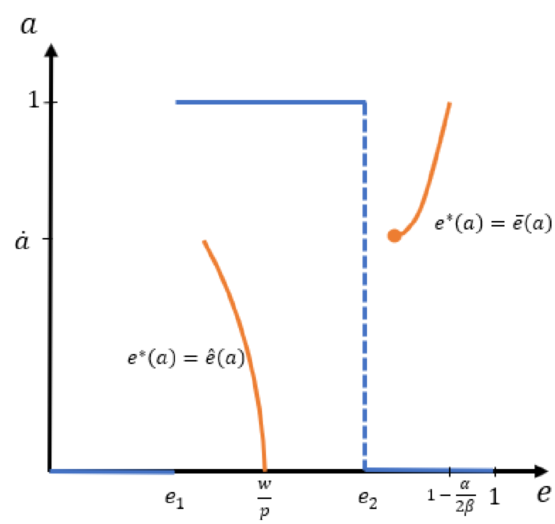


FIGURE 33. Illustration of No NE When Intersecting S-BRF 2-2 with B-BRF 2

- Intersect S-BRF 2-2 with B-BRF 2. We illustrate both the S-BFR 2-2 and B-BRF 2 in Figure 33. The B-BRF 2 is defined as follows: if $e_1 < e < e_2$ then $a^* = 1$, if $e = e_2$ (right edge) a^* can take any value in $[0, 1]$, and otherwise $a^* = 0$. Note that from Lemma 15: $e_1 < \frac{w}{p} < e_2$, then there is no intersection between S-BFR 2-2 and B-BRF 2 for all $0 \leqslant a < \dot{a}$. If $e_2 < \bar{e}(\dot{a})$, then there is no intercept between S-BFR 2-2 and B-BRF 2 for $\dot{a} \leqslant a \leqslant 1$. Therefore, there is no NE. Next we prove $\bar{e}(\dot{a}) > e_2$ for B-BRF 2. Using the expression of $\bar{e}(\dot{a})$ and e_2 we have:

$$1 - \frac{\alpha}{2\beta\dot{a}} > 1 - \frac{\gamma}{Dw} \Leftrightarrow \gamma > \frac{Dw\alpha}{2\beta\dot{a}} \tag{A.34}$$

Recall that one of the defined conditions of B-BRF 2 is $D < \sqrt{\frac{\gamma\beta}{p(w-c)}}$ which is equivalent to $\gamma > \frac{D^2 p(w-c)}{\beta}$. To complete the proof, it suffices that $\frac{D^2 p(w-c)}{\beta} > \frac{Dw\alpha}{2\beta\dot{a}}$. We plug-in $\dot{a}$ and we have:

$$\sqrt{\frac{\alpha^2(w^2\alpha^2 + 8(p-w)(w\alpha - 2Dp(w-c))\beta + 16(p-w)^2\beta^2)}{D^2(w-c)^2(p-w^2)\beta^2}} > \\ -\frac{\alpha(pw\alpha + 4(p-w)^2\beta)}{Dp(p-w)(w-c)\beta}. \tag{A.35}$$

The right-hand side of inequality (A.35) is negative and the left-hand side is positive. Hence, we completed the proof.

– Intersect S-BRF 2-2 with B-BRF 3. We illustrate both S-BRF 2-2 and B-BRF 3 in Figure 34. The B-BRF 3 is defined as follows: if $e_1 < e < e_2$ or $e_4 < e < e_3$ then $a^* = 1$; if $e = e_2$ (middle edge) or $e = e_3$ (right edge) then a^* can take any value in $[0, 1]$; and otherwise $a^* = 0$. In this case and from Lemma 15, $e_1 < \frac{w}{p} < e_2$, thus there is no intersection between S-BRF 2-2 and B-BRF 3 when $0 \leqslant a < \dot{a}$. Also, in (A.34) we prove that $e_2 < \bar{e}(\dot{a})$. Thereby, there is no intersection between S-BRF 2-2 and B-BRF 3 for all $0 \leqslant e \leqslant e_2$. We illustrate this in Figure 34. There may exist intersection between S-BRF 2-2 and B-BRF 3 when $\dot{a} \leqslant a \leqslant 1$ and $e_4 < e \leqslant 1$. Note that $e^* = \bar{e}(a)$ is increasing continuous function in a, $\bar{e}(a)|_{a=1} = 1 - \frac{\alpha}{2\beta}$. For an NE to exists, we need to prove that

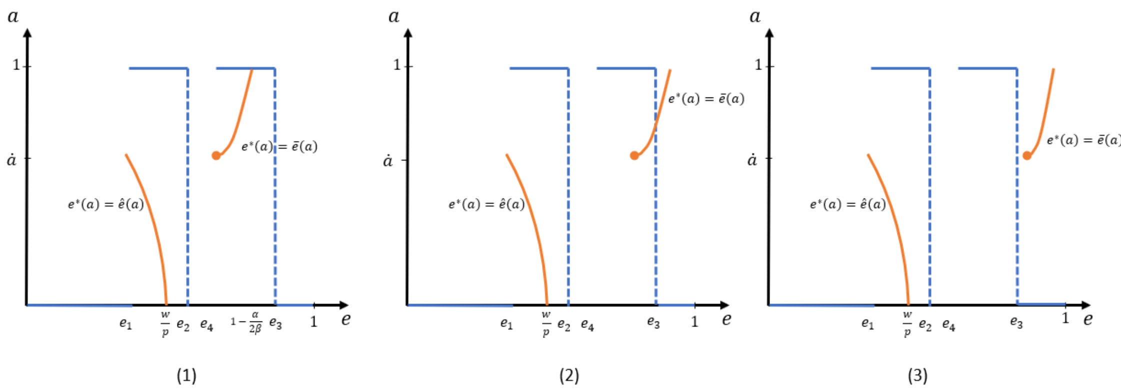


FIGURE 34. Illustration of Different Forms of NE and No NE Cases When Intersecting S-BRF 2-2 with B-BRF 3

$\bar{e}(a)|_{a=1} > e_4$:

$$\begin{aligned} \bar{e}(a)|_{a=1} >& e_4 \\ \Leftrightarrow 1 - \frac{\alpha}{2\beta} >& 1 - \frac{D(w-c)}{\beta} \\ \Leftrightarrow D >& \frac{\alpha}{2(w-c)}. \end{aligned} \tag{A.36}$$

In Region A-ii: $\frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < D$. Also, $\frac{\alpha}{2(w-c)} < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. Therefore, inequality (A.36) holds, because: $\frac{\alpha}{2(w-c)} < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)} < D$. To find NE, we compare the position of $\bar{e}(\dot{a})$ and $\bar{e}(a)|_{a=1}$ vs. e_3. We derive possible NE and associated conditions in this case as follows and we illustrate them in Figure 34:

- When $\bar{e}(\dot{a}) \leqslant e_3$ (or equivalently $w \geqslant \hat{w}(D,p,c,\alpha,\beta,\gamma)$):
  - under $\bar{e}(a)|_{a=1} < e_3$ (equivalent to $D > \frac{2\beta\gamma}{p\alpha}$), it creates NE $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$, which is NE form I. We illustrate this in Figure 34-1.
  - under $\bar{e}(a)|_{a=1} \geqslant e_3$ (equivalent to $D \leqslant \frac{2\beta\gamma}{p\alpha}$), it creates NE $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$ which is NE form II. We illustrate this in Figure 34-2.

Note that $\hat{w}(D,p,c,\alpha,\beta,\gamma) = \frac{D\gamma(4\beta-\alpha)+2D^3p(p+c)-\sqrt{(D\gamma(4\beta-\alpha)+2D^3p(p+c))^2-16D^3p((D^3p^2C)+(2Dp-\gamma)\beta\gamma)}}{4D^3p}$.

- When $\bar{e}(\dot{a}) > e_3$ (or equivalently $w < \hat{w}(D,p,c,\alpha,\beta,\gamma)$), then there is no intersection at all between S-BRF 2-2 and B-BRF 3. Therefore, there is no NE. We illustrate this case in Figure 34-3.

– Intersect S-BRF 2-2 with B-BRF 4. B-BRF 4 is defined as follows: if $e_1 < e < e_3$ then $a^* = 1$, if $e = e_3$ (right edge) a^* can take any value in $[0,1]$, and otherwise $a^* = 0$. From Lemma 15, $e_1 < \frac{w}{p} < e_3$. Therefore, we have no intersection between S-BRF 2-2 and B-BRF 4 when $0 \leqslant a < \dot{a}$. Similar to steps that we show in (A.36), one can show that $\bar{e}(a)|_{a=1} > e_1$. Based on these characteristics of B-BRF 4, To find NE we compare the position of $\bar{e}(\dot{a})$ and $\bar{e}(a)|_{a=1}$ vs. e_3. Similar to the preceding case, there exist NE form I when $\bar{e}(\dot{a}) \leqslant e_3$ and $\bar{e}(a)|_{a=1} < e_3$. Also, there exist NE form II when $\bar{e}(\dot{a}) \leqslant e_3$ and $\bar{e}(a)|_{a=1} \geqslant e_3$. There is no NE if $\bar{e}(\dot{a}) > e_3$.

– Intersect S-BRF 2-2 with B-BRF 5. B-BRF 5 is defined as follows: if $e_4 < e < e_3$ then $a^* = 1$, if $e = e_3$ (right edge) a^* can take any value in $[0,1]$, and otherwise $a^* = 0$. As we proved in (A.36), $\bar{e}(a)|_{a=1} > e_4$, therefore we compare position of $\bar{e}(\dot{a})$ and $\bar{e}(a)|_{a=1}$ vs. e_3. Similar to both above cases, there exist NE form I when $\bar{e}(\dot{a}) \leqslant e_3$ and $1 - \frac{\alpha}{2\beta} < e_3$. Also, there exist NE form II when $\bar{e}(\dot{a}) \leqslant e_3$ and $1 - \frac{\alpha}{2\beta} \geqslant e_3$. There is no NE if $\bar{e}(\dot{a}) > e_3$. This case differs from both above cases in position of $\frac{w}{p}$ vs. left edge of B-BRF (e_4). Based on Lemma 15, $\frac{w}{p} < e_4$, which leads to NE form III $(e^*, a^*) = (\frac{w}{p}, 0)$ in this case. NE form III occurs simultaneously with one of above-mentioned NE form I or NE form II, no NE (equivalent to NE form IV). When $(e^*, a^*) = (\frac{w}{p}, 0)$, the supplier's profit Π_S is negative, thus it is Pareto dominated by one of NE I or NE II, or NE IV.

In Region A (both A-i and A-ii), we find that when $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$ and $w \geqslant \hat{w}(D,p,c,\alpha,\beta,\gamma)$ we have NE forms I and II with the boundary of $D = \frac{2\beta\gamma}{p\alpha}$ which divides the regions of two forms. Otherwise we have NE forms III and IV.

Region B-i: We want to prove that the NE is one of the following three forms:

- NE form III: $(e^*, a^*) = (\frac{w}{p}, 0)$, and
- NE form IV: $(e^*, a^*) = (0, 0)$ or no Nash Equilibrium.

Note that region B-i is defined by $\frac{\beta(p-w)}{p(w-c)} < D < \frac{\beta}{w-c}$ and $D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. In this region there are 5 possible forms of the supplier's best response function: S-BRF 1-1 through 1-5, given in Lemma 16. In the same region, there are 3 possible forms of the buyer's best response function: B-BRF 1, B-BRF 4, and B-BRF 5, given in Lemma 15. Consider B-BRF 1 and intersect it with all 5 forms of S-BRF:

- Intersect B-BRF 1 with S-BRF 1-1, 1-3, and 1-4, respectively. As $e^*(0) = 0$ and $a^*(0) = 0$, NE is $(e^*, a^*) = (0, 0)$, which is NE form IV.
- Intersect B-BRF 1 with S-BRF 1-2 and 1-5, respectively. As $e^*(0) = \frac{w}{p}$ and $e^*(\frac{w}{p}) = 0$, then $(e^*, a^*) = (\frac{w}{p}, 0)$, which is NE form III.

Consider B-BRF 4 and intersect it with all 5 forms of S-BRF:

- Intersect B-BRF 4 with S-BRF 1-1. The S-BRF 1-1 is defined as follows: $e^*(a) = 0$ for all $0 \leqslant a \leqslant 1$. As $a^*(0) = 0$, then $(e^*, a^*) = (0, 0)$, which is NE form IV.

- Intersect B-BRF 4 with S-BRF 1-2. The S-BRF 1-2 is defined as follows: $e^*(a) = \hat{e}$ for all $0 \leqslant a \leqslant 1$. Note that $e_1 < \frac{w}{p} < e_3$ from Lemma 15. There is no intercept between the B-BRF 4 and S-BRF 1-2. So, there is no NE.

- Intersect B-BRF 4 with S-BRF 1-3. The S-BRF 1-3 is defined as follows: $e^*(a) = 0$ for all $0 \leqslant a \leqslant a_2^{B-I}$; $e^*(a) = \hat{e}$ for all $a_2^{B-I} < a < a_1^{B-I}$; and $e^*(a) = 0$ for all $a_1^{B-I} \leqslant a \leqslant 1$. As $a^*(0) = 0$, $(e^*, a^*) = (0, 0)$, which is NE form IV.

- Intersect B-BRF 4 with S-BRF 1-4. We illustrate both BRFs in Figure 35. The S-BRF 1-4 is defined by $e^*(a) = \hat{e}$ for all $a_2^{B-I} < a < \frac{p\alpha}{2\beta(p-w)+w\alpha}$; $e^*(a) = \bar{e}$ for all $\frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant \frac{\alpha^2}{4\beta(\alpha-D(w-c))}$, otherwise $e^*(a) = 0$. From Lemma 16 and S-BRF 1-4 we know, $a_2^{B-I} < \frac{p\alpha}{2\beta(p-w)+w\alpha} < \frac{\alpha^2}{4\beta(\alpha-D(w-c))} < 1$, $a_2^{B-I} < \frac{p}{w} - \frac{\beta(p-w)}{Dw(w-c)}$. Also, similar to (A.34), one can show that $\bar{e}(a)\big|_{a=\frac{\alpha^2}{4\beta(\alpha-D(w-c))}} < e_3$. In addition, $e^*(0) = 0$ and $a^*(0) = 0$. In conclusion, NE is $(e^*, a^*) = (0, 0)$, which is NE form IV. We illustrate this case in Figure 35-1.

- Intersect B-BRF 4 with S-BRF 1-5. We illustrate both BRFs in Figure 35. The S-BRF 1-5 is defined by $e^*(a) = \hat{e}$ for all $0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha}$ and $a_2^{B-II} \leqslant a \leqslant 1$; otherwise $e^*(a) = \bar{e}$. In this case, similar to (A.34) we can show $\bar{e}(a)\big|_{a=a_2^{B-II}} < e_3$. In addition, from Lemma 15, $e_1 < \frac{w}{p} < e_3$. Therefore there is no intersection between B-BRF 4 and S-BRF 1-5. It means there is no NE. We illustrate this case in Figure 35.

Similar to last 5 cases, intersecting B-BRF 5 with S-BRF 1-1 through S-BRF 1-5 creates NE form III or NE form IV.

Region B-ii: We want to prove that if there exists any NE then the NE should be one of the following four forms:

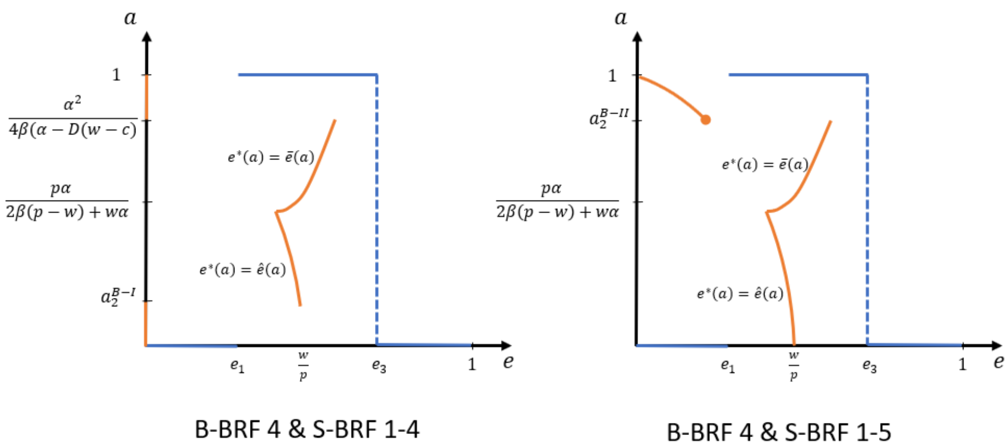


FIGURE 35. Illustration of NE and No NE Cases When Intersecting B-BRF 4 with S-BRF 1-4 and S-BRF 1-5

- NE form I: $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$,
- NE form II: $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$,
- NE form III: $(e^*, a^*) = (\frac{w}{p}, 0)$, and
- NE form IV: $(e^*, a^*) = (0, 0)$.

Note that region B-ii is defined by $\frac{\beta(p-w)}{p(w-c)} < D < \frac{\beta}{w-c}$ and $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. In this region there are 4 possible forms of the supplier's best response function: S-BRF 2-1, S-BRF 2-3, S-BRF 2-4, and S-BRF 2-5, given in Lemma 16. In the same region, there are 3 possible forms of the buyer's best response function: B-BRF 1, B-BRF 4, and B-BRF 5, given in Lemma 15. Consider B-BRF 1 and intersect it with the S-BRF 2-1, S-BRF 2-3, and S-BRF 2-4. In these cases $e^*(0) = \frac{w}{p}$, then NE is $(e^*, a^*) = (\frac{w}{p}, 0)$, which is NE form III. Intersect B-BRF 1 with the S-BRF 2-5. In this case $e^*(0) = 0$, the NE is $(e^*, a^*) = (0, 0)$, which is NE form IV. Next, we consider intersecting B-BRF 4 and B-BRF 5 with 4 forms of S-BRF:

- Intersect B-BRF 4 with S-BRF 2-1. The S-BRF 2-1 is defined as follows: $e^* = \hat{e}(a)$ for all $0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha}$; $e^* = \bar{e}(a)$ for all $\frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant 1$. Note that from Lemma 15, $e_1 < \frac{w}{p} < e_3$. There is no intersection between B-BRF and S-BRF when $0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha}$. We illustrate this in Figure 36. However, there exist intersection between B-BRF 4 and S-BRF 2-1 when $\frac{p\alpha}{2\beta(p-w)+w\alpha} \leqslant a \leqslant 1$. Note that $e^* = \bar{e}(a)$ is increasing continuous function in a, $\bar{e}(a)|_{a=1} = 1 - \frac{\alpha}{2\beta}$. Also similar to (A.36), we can show $\bar{e}(a)|_{a=1} > e_1$. Therefore, based on position of $\bar{e}(a)|_{a=1}$ vs. e_3, we derive NE in this case as follows and we illustrate them in Figure 36:

- under $\bar{e}(a)|_{a=1} < e_3$ (equivalent to $D > \frac{2\beta\gamma}{p\alpha}$), it creates NE $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$, which is NE form I. We illustrate this in Figure 36-1.
- under $\bar{e}(a)|_{a=1} \geqslant e_3$ (equivalent to $D \leqslant \frac{2\beta\gamma}{p\alpha}$), it creates NE $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$, which is NE form II. We illustrate this in Figure 36-2.

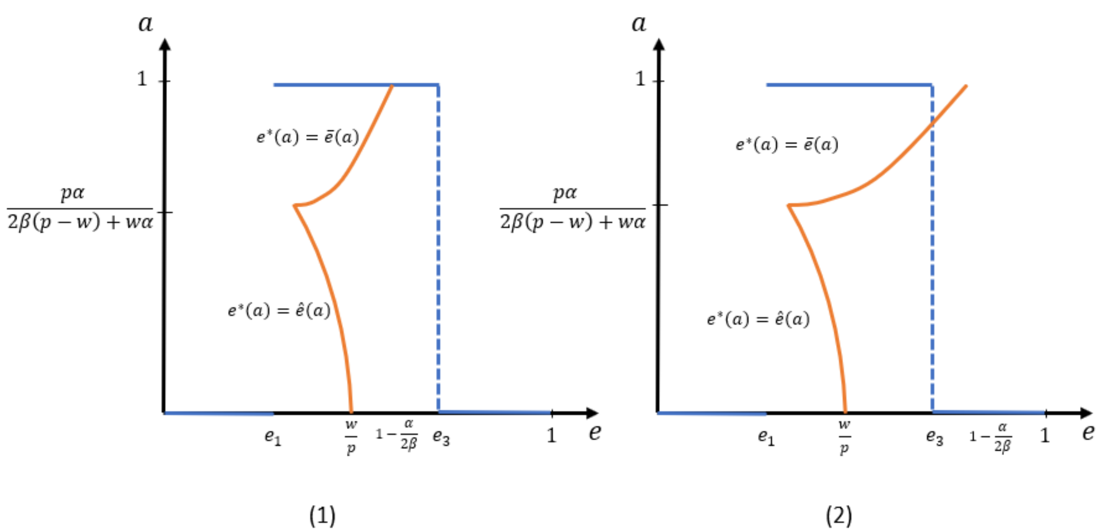


FIGURE 36. Illustration of NE form 1 and Form 2 When Intersecting B-BRF 4 and S-BRF 2-1

– Similar to the above case, intersecting of B-BRF 4 with S-BRF 2-5 creates NE form I or form II with same conditions. The S-BRF 2-5 is very similar to S-BRF 2-1, except for all $0 \leqslant a < a_2^{B-I}$, $e^* = 0$. This difference leads to

NE $(e^*, a^*) = (0, 0)$, which simultaneously occurs with one of NE form I or NE form II. However, NE $(e^*, a^*) = (0, 0)$ is Pareto dominated by one of NE form I or NE form II.

– Intersect BRF-B 5 with S-BRF 2-1 and S-BRF 2-5, respectively. The B-BRF 5 has a jump similar to B-BRF 4 with the same right edge. Therefore, similar to above cases (B-BRF 4 with S-BRF 2-1 and S-BRF 2-5), it creates NE form I, II.

– Intersect B-BRF 4 with S-BRF 2-4. The S-BRF 2-4 is defined as follows: $e^* = \hat{e}(a)$ for all $0 \leqslant a < \frac{p\alpha}{2\beta(p-w)+w\alpha}$ and $a_2^{B-II} \leqslant a < a_1^{B-II}$; and otherwise $e^* = \bar{e}(a)$. In this case from Lemma 15, $e_1 < \frac{w}{p} < e_3$. Similar to previous cases, we can show that $\bar{e}(a)|_{a=1} > e_1$ and $\bar{e}(a)|_{a=a_2^{B-II}} < e_3$. Then, we need to compare $\bar{e}(a_1^{B-II})$ vs. e_3:

  - When $\bar{e}(a_1^{B-II}) \leqslant e_3$ (or equivalently $w \geqslant \hat{w}(D, p, c, \alpha, \beta, \gamma)$):
    - under $\bar{e}(a)|_{a=1} < e_3$ (equivalent to $D > \frac{2\beta\gamma}{p\alpha}$), it creates NE $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$, which is NE form I. We illustrate this case in Figure 37-1.
    - under $\bar{e}(a)|_{a=1} \geqslant e_3$ (equivalent to $D \leqslant \frac{2\beta\gamma}{p\alpha}$), it creates NE $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$ which is NE form II. We illustrate this case in Figure 37-2.
  - When $\bar{e}(a_1^{B-II}) > e_3$ (or equivalently $w < \hat{w}(D, p, c, \alpha, \beta, \gamma)$), then there is no intersection at all between B-BRF and S-BRF. Therefore, there is no NE. We illustrate this case in Figure 37-3.

– Similar to above case, intersecting of B-BRF 4 with S-BRF 2-3 creates NE form I or form II with same conditions.We illustrate NE form I, NE form II, and no NE for this cases in Figure 38.

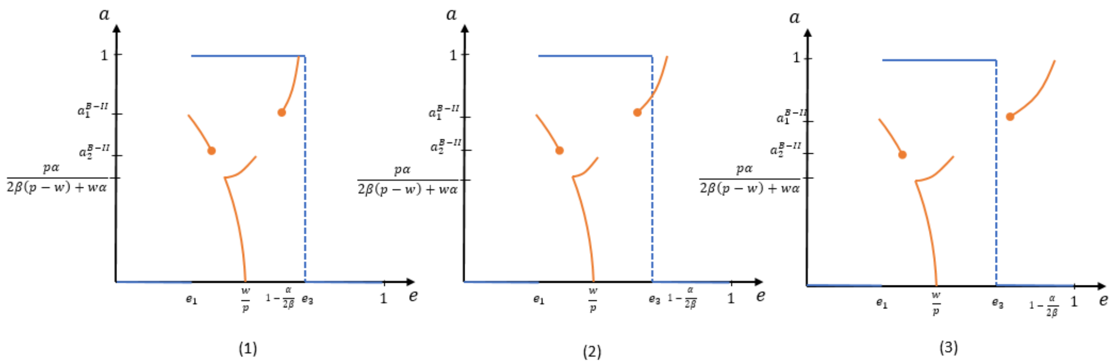


FIGURE 37. Illustration of possible NE cases when intersecting B-BRF 4 with S-BRF 2-4

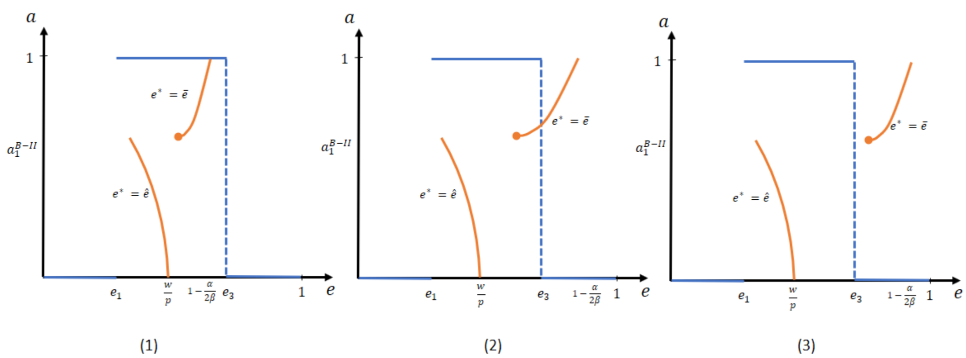


FIGURE 38. Illustration of possible NE cases when intersecting B-BRF 4 with S-BRF 2-3

– Intersect B-BRF 5 with S-BRF 2-4. In this case from Lemma15, $e_4 < \frac{w}{p} < e_3$. We also show in (A.36) $\bar{e}(a)|_{a=1} > e_4$. Similar to (A.36) we can show $\bar{e}(a)|_{a=a_2^{B-II}} < e_3$. Then, we need to compare $\bar{e}(a_1^{B-II})$ vs. e_3:

- When $\bar{e}(a_1^{B-II}) \leqslant e_3$ (or equivalently $w \geqslant \hat{w}(D, p, c, \alpha, \beta, \gamma)$):
  - under $\bar{e}(a)|_{a=1} < e_3$ (equivalent to $D > \frac{2\beta\gamma}{p\alpha}$), it creates NE $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$, which is NE form I.
  - under $\bar{e}(a)|_{a=1} \geqslant e_3$ (equivalent to $D \leqslant \frac{2\beta\gamma}{p\alpha}$), it creates NE $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$ which is NE form II.

- When $\bar{e}(a_1^{B-II}) > e_3$ (or equivalently $w < \hat{w}(D,p,c,\alpha,\beta,\gamma)$), then there is no intersection at all between B-BRF and S-BRF. Therefore, there is no NE.

– Similar to above case, intersecting of B-BRF 5 with S-BRF 2-3 creates NE form I or form II with same conditions.

In Region B (both B-i and B-ii), we find that when $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$ and $w \geqslant \hat{w}(D,p,c,\alpha,\beta,\gamma)$ we have NE forms I and II with the boundary of $D = \frac{2\beta\gamma}{p\alpha}$ which divides the regions of two forms. Otherwise we have NE forms III and IV.

Region C: We want to prove that if there exists any NE then the NE should be one of the following three forms:

– NE form I: $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$,

– NE form II: $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$, and

– NE form III: $(e^*, a^*) = (\frac{w}{p}, 0)$.

Note that region C is defined by $D \geqslant \frac{\beta}{w-c}$. In this region there is 1 possible form of the supplier's best response function: S-BRF 2-1, given in Lemma 16. There are 2 possible forms of the buyer's best response function: B-BRF 1 and B-BRF 6, given in Lemma 15. Intersect B-BRF 1 with S-BRF 2-1. As $a^*(0) = \frac{w}{p}$, then NE is $(e^*, a^*) = (\frac{w}{p}, 0)$, which is NE form IV. Next, intersect B-BRF 6 with S-BRF 2-1. We know that $\bar{e}(a)|_{a=1}$ is $1 - \frac{\alpha}{2\beta}$ and S-BRF 2-1 is continuous function in a. If $\bar{e}(a)|_{a=1} < e_3$ (equivalent to $D \geqslant \frac{2\beta\gamma}{p\alpha}$), then NE is $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$, which is NE form I. Otherwise, NE is $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$, which is NE form II. Now, we find the buyer's and the supplier's profit based on outcome of NE by plugging the value of NE into the optimal order quantities, the buyer's profit and the supplier's profit function in (2.6) and (2.8). The associated optimal order quantities, the buyer's profit, and supplier's profit are:

– If NE form I holds, which is $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$, then

$$q_f^* = \begin{cases} 0 & c \leqslant w < c + \frac{\alpha}{2D} \\ D & c + \frac{\alpha}{2D} \leqslant w \leqslant p \end{cases}, \quad q_p^* = D,$$

$$\Pi_B = \begin{cases} (1 - \frac{\alpha}{2\beta})D(p - w) - \gamma & c \leqslant w < c + \frac{\alpha}{2D} \\ D(p - w) - \gamma & c + \frac{\alpha}{2D} \leqslant w \leqslant p \end{cases}, \tag{cor. 1}$$

$$\Pi_S = \begin{cases} (1 - \frac{\alpha}{2\beta})D(w - c) - \alpha(1 - \frac{\alpha}{2\beta}) & c \leqslant w < c + \frac{\alpha}{2D} \\ D(w - c) - \frac{\alpha(4\beta - \alpha)}{4\beta} & c + \frac{\alpha}{2D} \leqslant w \leqslant p \end{cases}. \tag{cor. 2}$$

– If NE form II holds, which is $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$, then

$$q_f^* = \begin{cases} 0 & c \leqslant w < c + \frac{\beta\gamma}{D^2 p} \\ D & c + \frac{\beta\gamma}{D^2 p} \leqslant w \leqslant p \end{cases}, \quad q_p^* = D,$$

$$\Pi_B = \begin{cases} \frac{1}{2\beta}D(p - w)(2\beta - \alpha) - \gamma & c \leqslant w < c + \frac{\beta\gamma}{D^2 p} \\ D(p - w) - \gamma & c + \frac{\beta\gamma}{D^2 p} \leqslant w \leqslant p \end{cases}, \tag{cor. 3}$$

$$\Pi_S = \begin{cases} \frac{1}{2\beta}D(w - c)(2\beta - \alpha) - (\alpha - \frac{\alpha\gamma}{Dp}) & c \leqslant w < c + \frac{\beta\gamma}{D^2 p} \\ D(w - c) - (\alpha - \frac{\alpha\gamma}{2Dp}) & c + \frac{\beta\gamma}{D^2 p} \leqslant w \leqslant p \end{cases}. \tag{cor. 4}$$

– If NE form III holds, which is $(e^*, a^*) = (\frac{w}{p}, 0)$, then $(q_f^*, q_p^*) = (0, 0)$ and $(\Pi_S, \Pi_B) = (-\gamma\frac{w}{p}, 0)$.

– If NE form IV holds, which is $(e^*, a^*) = (0, 0)$, then $(q_f^*, q_p^*) = (0, 0)$ and $(\Pi_S, \Pi_B) = (0, 0)$.

Now we assemble the results of A-i, A-ii, B-i, B-ii, and C to prove Lemma 4. In Region A-i and B-i (these two regions are defined when $D < \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$) the possible NE are NE form III and Form IV. Also, we prove that the buyer's profit is always zero and the supplier's profit is either zero or negative for both NE form III and NE form IV. Therefore, we omit these two forms of NE from rest of analysis to find sub-game prefect equilibrium.

In the rest of regions including Region A-ii, Region B-ii, and Region C (these regions are defined when $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$), when $D < \frac{\gamma}{p-w}$ (where B-BRF is always $a^* = 0$ for all $0 \leqslant e \leqslant 1$), NE form III or NE form IV occur, which we exclude them from our analysis of sub-game prefect equilibrium with the same reason we stated above. When $D \geqslant \frac{\gamma}{p-w}$, we have three possible forms of NE including NE form I, NE form II, and NE form IV. As we proved NE form I and NE form II only occur when $w \geqslant \hat{w}(D, p, c, \alpha, \beta, \gamma)$, otherwise NE form IV occurs. We showed when $w \geqslant \hat{w}(D, p, c, \alpha, \beta, \gamma)$, if $D > \frac{2\beta\gamma}{p\alpha}$ then NE form I happens, otherwise NE form II occurs. NE form I and NE form II are considered to find sub-game prefect equilibrium in the first stage.

To form the relationship between regions of NE form and NE form II in plane of (D, w), we find intersections of following lines with each others: $D = \frac{2\beta\gamma}{p\alpha}$, $w = \hat{w}(D, p, c, \alpha, \beta, \gamma)$, $D = \frac{\gamma}{p-w}$, and $D = \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$. Intercept of $D = \frac{\gamma}{p-w}$ and $w = \hat{w}(D, p, c, \alpha, \beta, \gamma)$ leads to $\hat{D}(p, c, \alpha, \beta, \gamma)$ (minimum point of region of NE form II in Figure 2-b) where $\hat{D}(p, c, \alpha, \beta, \gamma) = \frac{\sqrt{p}(\alpha+2\gamma)+\sqrt{p((\alpha-2\gamma)^2+16\gamma)-8c\gamma(2\beta-\alpha)}}{4\sqrt{p}(p-c)}$. Also, $D = \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$ is the intercept of $D = \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)}$ and $D = \frac{\gamma}{p-w}$. Further, $w = \hat{w}(D, p, c, \alpha, \beta, \gamma)$ and $D = \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)}$ intersect at $D = \frac{2\beta\gamma}{p\alpha}$. Note that $D = \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$

is equivalent to $w = c + \frac{(4\beta-\alpha)\alpha}{4\beta D}$. We illustrate two possible positions of above

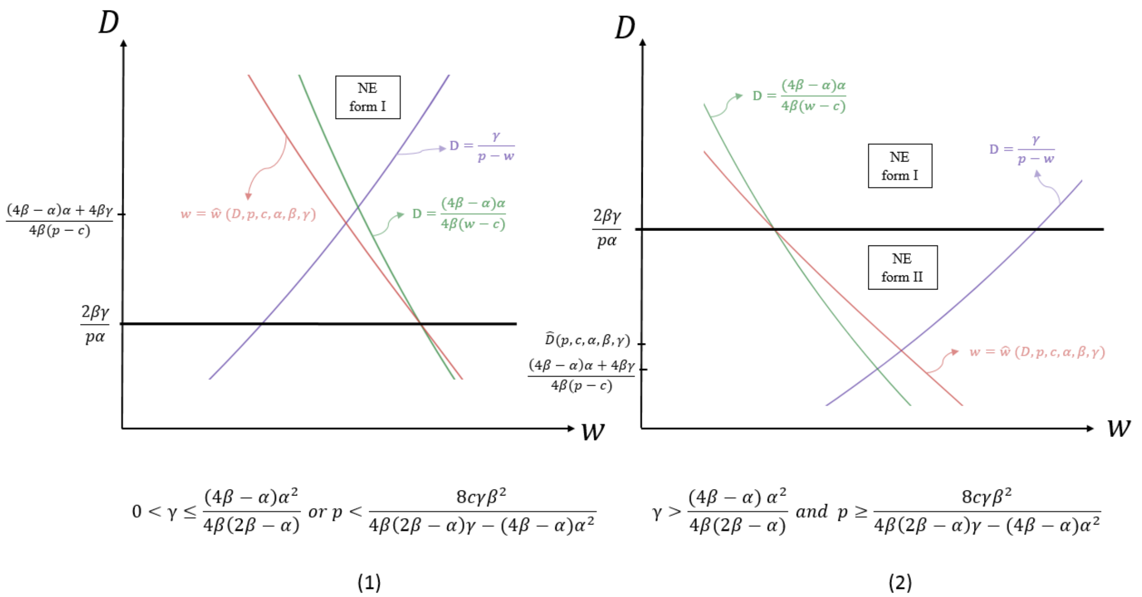


FIGURE 39. Division of plane of (D,w) for NE form I and form II

lines in Figure 39. $\frac{2\beta\gamma}{p\alpha} \geqslant \hat{D}(p,c,\alpha,\beta,\gamma)$ if and only if $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$, therefore it generates the NE parameter space which we shows in Figure 39-2. In this case, when $D > \frac{2\beta\gamma}{p\alpha}$ then we prove that $c + \frac{(4\beta-\alpha)\alpha}{4\beta D} > \hat{w}(D,p,c,\alpha,\beta,\gamma) \Leftrightarrow D^3p(Dp\alpha-2\beta\gamma)[(4\beta-\alpha)(4D\beta(p-c)-(4\beta-\alpha)\alpha)-8\gamma\beta^2] > 0$ given that $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p > \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$ and $D > \frac{2\gamma\beta}{p\alpha}$, expression in bracket is always positive. As we illustrate in Figure 2-b, when $D \geqslant \frac{\gamma}{p-w}$, $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$ and $w \geqslant \hat{w}(D,p,c,\alpha,\beta,\gamma)$, if $D \geqslant \frac{2\beta\gamma}{p\alpha}$ then NE I occurs, and if $D < \frac{2\beta\gamma}{p\alpha}$ NE II occurs. Also when $D \geqslant \frac{\gamma}{p-w}$, $D \geqslant \frac{\alpha(4\beta-\alpha)}{4\beta(w-c)}$, and $w < \hat{w}(D,p,c,\alpha,\beta,\gamma)$, then NE IV occurs. Consequently under the same condition, for all $\hat{D}(p,c,\alpha,\beta,\gamma) < D < \frac{2\gamma\beta}{p\alpha}$, we have $c + \frac{(4\beta-\alpha)\alpha}{4\beta D} \geqslant \hat{w}(D,p,c,\alpha,\beta,\gamma)$.

Also $\frac{2\beta\gamma}{p\alpha} \geqslant \hat{D}(p,c,\alpha,\beta,\gamma)$ if and only if $\gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ or $p < \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$. In this case, NE form II vanishes. We illustrate this case in Figure 39-1. Hence, we complete the proof. □

Proof of Proposition 2. Recall that the associated optimal buyer's profit and the supplier's profit for each forms of NE in Lemma 4 are:

– If NE form I holds, which is $(e^*, a^*) = (1 - \frac{\alpha}{2\beta}, 1)$, then

$$q_f^* = \begin{cases} 0 & c \leqslant w < c + \frac{\alpha}{2D} \\ D & c + \frac{\alpha}{2D} \leqslant w \leqslant p \end{cases}, \quad q_p^* = D,$$

$$\Pi_B = \begin{cases} (1 - \frac{\alpha}{2\beta})D(p - w) - \gamma & c \leqslant w < c + \frac{\alpha}{2D} \\ D(p - w) - \gamma & c + \frac{\alpha}{2D} \leqslant w \leqslant p \end{cases}, \tag{cor. 1}$$

$$\Pi_S = \begin{cases} (1 - \frac{\alpha}{2\beta})D(w - c) - \alpha(1 - \frac{\alpha}{2\beta}) & c \leqslant w < c + \frac{\alpha}{2D} \\ D(w - c) - \frac{\alpha(4\beta - \alpha)}{4\beta} & c + \frac{\alpha}{2D} \leqslant w \leqslant p \end{cases}. \tag{cor. 2}$$

– If NE form II holds, which is $(e^*, a^*) = (1 - \frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$, then

$$q_f^* = \begin{cases} 0 & c \leqslant w < c + \frac{\beta\gamma}{D^2 p} \\ D & c + \frac{\beta\gamma}{D^2 p} \leqslant w \leqslant p \end{cases}, \quad q_p^* = D,$$

$$\Pi_B = \begin{cases} \frac{1}{2\beta}D(p - w)(2\beta - \alpha) - \gamma & c \leqslant w < c + \frac{\beta\gamma}{D^2 p} \\ D(p - w) - \gamma & c + \frac{\beta\gamma}{D^2 p} \leqslant w \leqslant p \end{cases}, \tag{cor. 3}$$

$$\Pi_S = \begin{cases} \frac{1}{2\beta}D(w - c)(2\beta - \alpha) - (\alpha - \frac{\alpha\gamma}{Dp}) & c \leqslant w < c + \frac{\beta\gamma}{D^2 p} \\ D(w - c) - (\alpha - \frac{\alpha\gamma}{2Dp}) & c + \frac{\beta\gamma}{D^2 p} \leqslant w \leqslant p \end{cases}. \tag{cor. 4}$$

– If NE form III holds, which is $(e^*, a^*) = (\frac{w}{p}, 0)$, then $(q_f^*, q_p^*) = (0, 0)$ and $(\Pi_S, \Pi_B) = (-\gamma\frac{w}{p}, 0)$.

– If NE form IV holds, which is $(e^*, a^*) = (0,0)$, then $(q_f^*, q_p^*) = (0,0)$ and $(\Pi_S, \Pi_B) = (0,0)$.

From the above result, the buyer's profit is zero when NE form III and IV exists. Also, the supplier's profit is negative in region III and it's zero in region IV. Therefore, we verify that we do not have any contracting for NE regions where $(e^*, a^*) = (0,0)$ and $(e^*, a^*) = (\frac{w}{p}, 0)$. Thus, we exclude them from feasible region of this stage.

As we show above in (cor.1) and (cor.3), the buyer's profit function in NE form I and NE form II is decreasing in wholesale price. Therefore, the buyer's would like to choose the lowest feasible w to maximize his profit. On the other hand, in (cor.2) and (cor.4) the supplier's profit function in NE form I and II is increasing in wholesale price. Thus, if we find the lower bound of w from the supplier's participation constraint in optimization problem 2.13 and check its position with our feasible region then we can find the optimal w. We analyze the optimization problem 2.13 for each condition of Lemma 5 as following.

• When $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$:

As we illustrated in Figure 2-b, region associated with NE form I is located between $D = \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)}$ (left boundary), $D = \frac{\gamma}{p-w}$ (right boundary), and above the $D = \frac{2\gamma\beta}{p\alpha}$. The buyer's profit function for Ne form I is shown in (cor.1). The defined threshold for piecewise function in (cor.1) is $w = c + \frac{\alpha}{2D}$. We rewrite the defined threshold for piecewise function in (cor.1) as $D = \frac{\alpha}{2(w-c)}$. We prove that $D = \frac{\alpha}{2(w-c)}$ is less than the left boundary ($\frac{(4\beta-\alpha)\alpha}{4\beta(w-c)}$):

$$\frac{\alpha}{2(w-c)} < \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)} \Leftrightarrow 1 < \frac{(4\beta-\alpha)}{2\beta} \Leftrightarrow \alpha < 2\beta.$$

Therefore, the first piece of the buyer's profit function in (cor.1) is out side of problem's feasible region and only second piece is in the feasible region for the maximization program 2.13. The second piece of function (cor.1) which is $(p - w)D - \gamma$, is always positive, because in region NE form I we are above the curve $D = \frac{\gamma}{p-w}$.

Now consider the supplier's profit function in (cor.2). $D = \frac{\alpha}{w-c}$ is the indifference curve for the first piece of the supplier's piecewise function in (cor.2). Also, the defined threshold for the first piece is $w < c + \frac{\alpha}{2D}$ and we rewrite it as $D = \frac{\alpha}{2(w-c)}$. By showing that $\frac{\alpha}{2(w-c)} < \frac{\alpha}{w-c}$, we verify that the first piece of function (cor.2) cannot be in the feasible region. The indifference function of the second piece of the supplier's function in (cor.2) is $D = \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)}$, which is exactly the left boundary of NE form I in this case. As we proved in the above $\frac{\alpha}{2(w-c)} < \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)}$. Therefore, the second piece of the supplier's function is feasible and lies on the left boundary of region NE form I. So, the supplier's participation constraint is binding for all region NE form I. The buyer chooses the lowest w (because the buyer's profit is decreasing in w), which in this case is left boundary of region NE form I thus $w^* = c + \frac{(4\beta-\alpha)\alpha}{4\beta D}$.

Next, we consider region of NE form II. As we illustrated in Figure 2-a, region associated with NE form II is located between $w = \hat{w}(D, p, c, \alpha, \beta, \gamma)$ (left boundary), $D = \frac{\gamma}{p-w}$ (right boundary), and bellow the $D = \frac{2\gamma\beta}{p\alpha}$.

In region of NE form II, the buyer's profit function in (cor.3) is a piecewise function and its defined threshold for pieces is $w = c + \frac{\beta\gamma}{D^2 p}$. We prove that $w = c + \frac{\beta\gamma}{D^2 p}$ is less than the left boundary of region NE form (II) $w = \hat{w}(D, p, c, \alpha, \beta, \gamma)$ which means first piece of the buyer's profit function (cor.1) does not cover the region NE form II and thus only the second piece of function (cor.3) completely covers region of NE form II.

$$\hat{D}(p,c,\alpha,\beta,\gamma) > \frac{p\gamma\beta + \sqrt{p\gamma\beta[p\beta(4\beta-2\alpha+\gamma)-c(2\beta-\alpha)^2]}}{p[2\beta(p-c)+c\alpha]}$$
$$\Leftrightarrow p(\alpha-\gamma)^2[8c\alpha\gamma+p(\gamma-2\alpha)]^2+[p(-2\alpha^2+\alpha\gamma+\gamma(8+\gamma))-8c\gamma]^2>0$$

Therefore, we need to use the second piece of the buyer's profit function for the optimization program 2.13.

For the supplier's participation constraint, we consider the supplier's profit piecewise function in (cor.4). From the first piece we obtain indifference curve $\frac{1}{2\beta}D(w-c)(2\beta-\alpha)-(\alpha-\frac{\alpha\gamma}{Dp})=0$ and from the second piece we obtain indifference curve $D(w-c)-(\alpha-\frac{\alpha\gamma}{2Dp})=0$. Also, the defined threshold of pieces is $w=c+\frac{\beta\gamma}{D^2p}$. when we solve simultaneously these three equations for D and w, we find that they intersect at $(w,D)=(c+\frac{4p\alpha^2\beta}{(2\beta+\alpha)^2\gamma},\frac{(2\beta+\alpha)\gamma}{2p\alpha})$. In addition, we prove that indifference curve of the first piece is greater than indifference curve of the second piece if and only if $w>c+\frac{\beta\gamma}{D^2p}$:

$$w>c+\frac{\beta\gamma}{D^2p} \Longleftrightarrow \frac{1}{2\beta}D(w-c)(2\beta-\alpha)-(\alpha-\frac{\alpha\gamma}{Dp})>D(w-c)-(\alpha-\frac{\alpha\gamma}{2Dp}).$$

To prove this, we subtract indifference functions and obtain the condition :

$$\frac{1}{2\beta}D(w-c)(2\beta-\alpha)-(\alpha-\frac{\alpha\gamma}{Dp})-[D(w-c)-(\alpha-\frac{\alpha\gamma}{2Dp})]>0$$
$$\Leftrightarrow \frac{\alpha[D^2p(w-c)-\beta\gamma]}{2Dp\beta}>0 \Leftrightarrow D^2p(w-c)-\gamma\beta>0 \Leftrightarrow w>c+\frac{\beta\gamma}{D^2p}.$$

We illustrate the positions of the pieces of the supplier's profit function in NE form II and boundaries of region of NE II in Figure 40. For finding the feasible region based on the supplier's constraint, we consider the first piece of function (cor.4) to

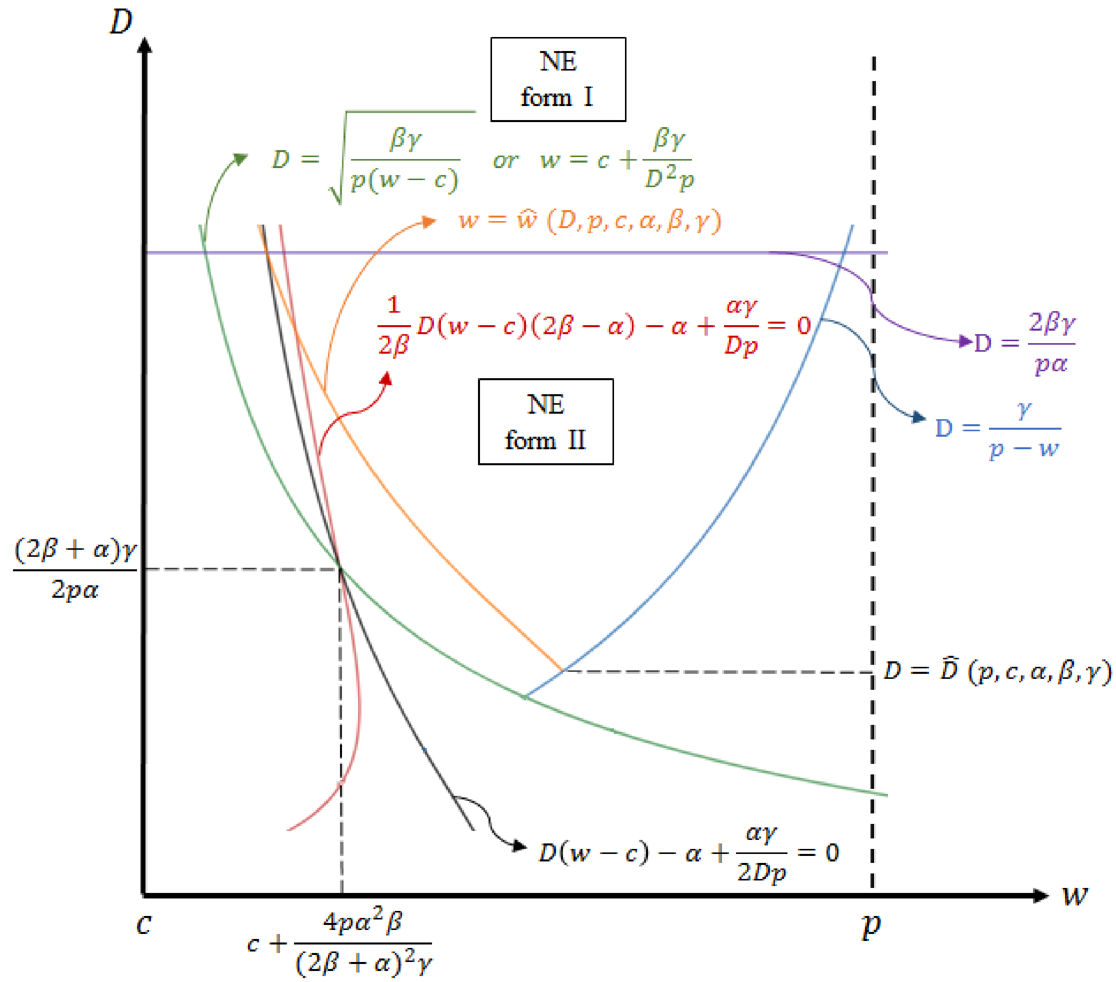


FIGURE 40. Illustration of the supplier's participation constraint for *NE region (II)* of Commitment to Wholesale Price

be positive $\frac{1}{2\beta}D(w-c)(2\beta-\alpha)-(\alpha-\frac{\alpha\gamma}{Dp})>0$ while $w<c+\frac{\beta\gamma}{D^2p}$ which gives us the region between green line and red line in Figure 40 for all $0\leqslant D<\frac{(2\beta+\alpha)\gamma}{2p\alpha}$. Also, the second piece of function (cor.4) is positive $D(w-c)-\alpha+\frac{\alpha\gamma}{2Dp}>0$, while $w\geqslant c+\frac{\beta\gamma}{D^2p}$ which gives us a region above black line for $D\geqslant\frac{(2\beta+\alpha)\gamma}{2p\alpha}$ and above green line for all $0\leqslant D<\frac{(2\beta+\alpha)\gamma}{2p\alpha}$ in Figure 40. Therefore, the feasible region derived by the supplier's participation constraint for region of NE form II is:

$$\begin{cases} \text{If } 0 \leqslant D < \frac{(2\beta+\alpha)\gamma}{2p\alpha} \text{ then } \frac{1}{2\beta}D(w-c)(2\beta-\alpha)-(\alpha-\frac{\alpha\gamma}{Dp}) \geqslant 0 \\ \text{If } \frac{(2\beta+\alpha)\gamma}{2p\alpha} \leqslant D < \frac{2\gamma\beta}{p\alpha} \text{ then } D(w-c)-\alpha+\frac{\alpha\gamma}{2Dp} \geqslant 0 \end{cases} . \qquad \text{(A.37)}$$

Given above-mentioned characteristics of the indifference curves of the supplier's profit in (cor.4), we prove that indifference curve of second piece of (cor.4) is always (less than) bellow of left boundary of region NE form II for all $\hat{D}(p,c,\alpha,\beta,\gamma) \leqslant D \leqslant \frac{2\gamma\beta}{p\alpha}$. To prove this, we rewrite the indifference curve of second piece in form of $w(D,p,c,\alpha.\gamma)$ and then compare it with $\hat{w}(D,p,c,\alpha,\beta,\gamma)$:

$$\hat{w}(D,p,c,\alpha,\beta,\gamma) > c+\frac{\alpha(2Dp-\gamma)}{2D^2p} \Leftrightarrow 8D^2(Dp\alpha-2\beta\gamma)[2Dp(\alpha-D(p-c))+(Dp-\alpha)\gamma] > 0,$$

and based on condition of this case ($\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$) the above inequality always holds. Therefore, we conclude that all region of NE form II is feasible for our optimization problem, because the supplier's participation constraint is not binding. So in region of NE form II, the buyer's optimal solution happens on left boundary of the region, which is $w^* = \hat{w}(D,p,c,\alpha,\beta,\gamma)$.

Therefore, we proved that the optimal wholesale price when $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$ is as follows:

- If $0 \leqslant D \leqslant \hat{D}(p,c,\alpha,\beta,\gamma)$, then we have no-contracting.

- If $D \geqslant \frac{2\gamma\beta}{p\alpha}$, then $w^* = c + \frac{(4\beta-\alpha)\alpha}{4\beta D}$.

- If $\hat{D}(p,c,\alpha,\beta,\gamma) < D < \frac{2\gamma\beta}{p\alpha}$, then $w^* = \hat{w}(D,p,c,\alpha,\beta,\gamma)$.

As we proved in the above when $D \geqslant \frac{2\gamma\beta}{p\alpha}$, the supplier's participation constraint is binding, thus the supplier's optimal profit does not make any profit. Also, the buyer's profit is positive, because $D(p-w)-\gamma>0$. The optimal order quantities associated with this equilibrium are $(q_p^*, q_f^*) = (D, D)$. When $\hat{D}(p,c,\alpha,\beta,\gamma) < D < \frac{2\gamma\beta}{p\alpha}$, the supplier's participation constraint is not binding, therefore the supplier's optimal profit is positive. Also the buyer makes a positive profit in the equilibrium, because $D(p-w)-\gamma>0$.

- When $\gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ or $p < \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$:

As we illustrated in Figure 2-a, region associated with NE form I is located between $D = \frac{(4\beta-\alpha)\alpha}{4\beta(w-c)}$ (left boundary) and $D = \frac{\gamma}{p-w}$ (right boundary). These two left and right boundaries intersect in $D = \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$. The buyer's profit function and the supplier's profit function are identical to preceding case. Therefore, we take the same steps to prove that the supplier's participation constraint is binding and the optimal wholesale price is $w^* = c + \frac{(4\beta-\alpha)\alpha}{4\beta D}$. Thus, when $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$ the optimal wholesale price is as follows:

- If $0 \leqslant D < \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$, then we have no-contracting.
- If $\frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)} \leqslant D$, then $w^* = c + \frac{(4\beta-\alpha)\alpha}{4\beta D}$.

In this case, similar to region of NE form I in preceding case, as the supplier's participation constraint is binding the supplier's does not make any profit. Also, the buyer's profit is positive, because $D(p-w)-\gamma>0$. The optimal order quantities associated with this equilibrium are $(q_p^*, q_f^*) = (D, D)$. We illustrate optimal wholesale price along with rest of optimal contract terms and efforts in Figure 3. Hence, we complete the proof. □

Proof of Proposition 3. Using Proposition 1 and Proposition 2 and comparing the change of each mentioned metrics give us the result. $D = \frac{\beta+\gamma}{p-c}$ is the only threshold (line) for γ in *no-commitment*. We compare this condition with all curves in Figure 3 to see whether there are any possible intersections. The first curve is $D = \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$ which is shown in Figure 3-a. We have $\frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)} < \frac{\beta+\gamma}{p-c} \Leftrightarrow (1-\frac{\alpha}{4\beta})\alpha < \beta$, therefore these two curves never intersects. For *commitment to wholesale price*, we have two other curves in Figure 3-b. First look at $D = \frac{2\gamma\beta}{p\alpha}$. We find conditions for intersection of this curve with the other one as $\frac{2\gamma\beta}{p\alpha} < \frac{\beta+\gamma}{p-c} \Leftrightarrow \{0 < \gamma < \frac{\alpha\beta}{2\beta-\alpha}$ or $\{\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$ and $p < \frac{2c\beta\gamma}{(2\beta-\alpha)\gamma-\alpha\beta}\}\}$, also $\frac{2\gamma\beta}{p\alpha} \geqslant \frac{\beta+\gamma}{p-c} \Leftrightarrow \gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$ and $p \geqslant \frac{2c\beta\gamma}{(2\beta-\alpha)\gamma-\alpha\beta}$, thus if $\gamma < \frac{\alpha\beta}{2\beta-\alpha}$ then $D = \frac{2\gamma\beta}{p\alpha}$ does not intersects with $D = \frac{\beta+\gamma}{p-c}$, otherwise intersection would happen at $p = \frac{2c\beta\gamma}{(2\beta-\alpha)\gamma-\alpha\beta}$. This intersection is greater than $p = \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$ because $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$, also showing $\frac{\beta+\gamma}{p-c} > \hat{D}(p,c,\alpha,\beta,\gamma)$ is straightforward and it means that there is no more intersection between curves except the ones which we proved . Therefore, comparison of *no-commitment* policy with *commitment to wholesale price* policy for varying levels of auditing cost coefficient γ generates three cases: $0 \leqslant \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$, $\frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)} < \gamma < \frac{\alpha\beta}{2\beta-\alpha}$, and $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$. In the next part we show the changes of indexes in transition from "*no-commitment*" to "*commitment to price*", we start with the case $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$ because it is a super-set of regions in other cases and we show it in Figure 4. When $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$, regions A, B and E are in *no-contracting* zone in *no-commitment* case and all index values are zero, also region A is located in *no-contracting* zone for *commitment to wholesale price.* Therefore, in region A metrics do not change after transition. In region B because of moving from *no-contracting* to contracting zone, all indexes increases and the change in the values of metrics are equal to values of metrics in *commitment to wholesale price* model which are positive values, except the supplier's profit which experiences no change. The supplier's profit does not change as the value of it in both models is zero. In region E because of moving from *no-contracting* to contracting zone, all indexes increases and the change in the values of metrics are equal to values of metrics in *commitment to wholesale price* model which are positive values. In this region, the supplier's profit is positive in *commitment to wholesale price*, therefore by this transition the supplier's profit increases. In region C and D, we have contracting in both policies and we calculate the change. For region C, $\Delta e = (1 - \frac{\alpha}{2\beta}) - 0 > 0$, $\Delta a = 1 - 1 = 0$ and $\Delta O_{sc} = [1 - \frac{\alpha}{2\beta} + (1 - (1 - \frac{\alpha}{2\beta})) \times 1] - [0 + (1 - 0)1] = 0$ and $\Delta \pi_B = [D(p-c) - \frac{(4\beta-\alpha)\alpha}{4\beta} - \gamma] - [D(p-c) - \gamma - \beta] = \beta - \frac{(4\beta-\alpha)\alpha}{4\beta} > 0$, also $\Delta \pi_S = 0$. For region D, $\Delta e = (1 - \frac{\gamma}{Dp}) - 0 = 1 - \frac{\gamma}{Dp} > 0$, $\Delta a = \frac{\alpha Dp}{2\gamma\beta} - 1 < 0$ and $\Delta O_{sc} = [1 - \frac{\gamma}{Dp} + (1 - (1 - \frac{\gamma}{Dp}))\frac{\alpha Dp}{2\gamma\beta}] - [0 + (1 - 0)1] = \frac{\alpha Dp - 2\beta\gamma}{2Dp\beta} < 0$ and $\Delta \pi_B = [(p - \hat{w}(D, p, c, \alpha, \beta, \gamma))D - \gamma] - [D(p-c) - \gamma - \beta] > 0$. Also $\pi_S > 0$ in this region in *commitment to wholesale price* and $\pi_S = 0$ at this region in *no-commitment*. Therefore, $\Delta \pi_S > 0$.

Next case is when $0 \leqslant \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ which we show it in Figure 5. All regions under this case including regions A, B, and C have identical behavior to the regions A, B, and C of $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$ case for all metrics. Therefore, we do not discuss them again. Next case is when $\frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)} < \gamma < \frac{\alpha\beta}{2\beta-\alpha}$ which we show it in Figure 6. Regions under this case including regions A, B, and C have identical behavior to the regions A, B, and C of $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$ for all metrics. Therefore, we do not discuss them again. Region D in this case has identical behavior to region E of $\gamma \geqslant \frac{\alpha\beta}{2\beta-\alpha}$. Therefore, we do not discuss it again. □

Proof of Lemma 5. First, note that in this case $w_p = w_f = w$ and $q_p = q_f = q$. To find NE we first find the buyer's best response function and the supplier's best response function. Then we solve for NE. *The buyer's Best Response Function:* Objective function (2.6) is linear in a and its F.O.C with respect to a is $pq(1-e)-\gamma$. Therefore, for a given the supplier's compliance effort e, the buyer's best response is one of the following cases: if $pq < \gamma$ then $a^* = 0$. If $pq \geqslant \gamma$ then for all $0 \leqslant e \leqslant 1 - \frac{\gamma}{pq}$,

$a^* = 1$ and for all $1 - \frac{\gamma}{pq} < e \leqslant 1$, $a^* = 0$. *The supplier's Best Response:* Objective function (2.8) is concave in e as second order condition of this function with respect to e is $-2a\beta$. Given F.O.C with respect to e is $2a(1-e)\beta - \alpha$, and $\frac{\partial \pi_S}{\partial e}|_{e=1} = -\alpha < 0$, and following the concavity characteristic of the function: if $2a(1-e)\beta - \alpha < 0 \Rightarrow a < \frac{\alpha}{2\beta} \Rightarrow e^* = 0$ and if $2a(1-e)\beta - \alpha \geqslant 0 \Rightarrow a \geqslant \frac{\alpha}{2\beta} \Rightarrow e^* = 1 - \frac{\alpha}{2a\beta}$. Using above-mentioned the buyer's and the supplier's best response functions, if $0 \leqslant q < \frac{\gamma}{p}$ then $(a^*, e^*) = (0, 0)$. If $q \geqslant \frac{\gamma}{p}$ then we need to compare two values: $1 - \frac{\alpha}{2\beta}$ vs. $1 - \frac{\gamma}{pq}$. if $1 - \frac{\alpha}{2\beta} < 1 - \frac{\gamma}{pq} \Rightarrow q > \frac{2\gamma\beta}{p\alpha}$ then $(a^*, e^*) = (1, 1 - \frac{\alpha}{2\beta})$. Otherwise, $1 - \frac{\alpha}{2\beta} \geqslant 1 - \frac{\gamma}{pq} \Rightarrow \frac{\gamma}{p} \leqslant q \leqslant \frac{2\gamma\beta}{p\alpha}$, then $(a^*, e^*) = (\frac{\alpha pq}{2\gamma\beta}, 1 - \frac{\gamma}{pq})$. □

Proof of Proposition 4. Plugging each pair of optimal efforts into optimization problem (2.14), gives us following outcome:

If $(a^*, e^*) = (0, 0)$ then optimization problem would be :

$$\begin{aligned} \max_{q,w} \quad & -wq \\ \text{subject to} \quad & q(w-c) \geqslant 0. \end{aligned} \tag{A.38}$$

If $(a^*, e^*) = (\frac{\alpha pq}{2\gamma\beta}, 1 - \frac{\gamma}{pq})$ then optimization problem would be :

$$\begin{aligned} \max_{q,w} \quad & (p-w)q - \gamma \\ \text{subject to} \quad & q(w-c) - (\alpha - \frac{\alpha\gamma}{2pq}) \geqslant 0. \end{aligned} \tag{A.39}$$

If $(a^*, e^*) = (1, 1 - \frac{\alpha}{2\beta})$ then optimization problem would be :

$$\begin{aligned} \max_{q,w} \quad & (p-w)q - \gamma \\ \text{subject to} \quad & q(w-c) - \frac{\alpha(4\beta - \alpha)}{4\beta} \geqslant 0. \end{aligned} \tag{A.40}$$

For optimization (A.38) as objective function is decreasing in w and q and value of function is negative, the buyer chooses $q = 0$ to obtain zero profit. Therefore, for all $0 \leqslant D \leqslant \frac{\gamma}{p}$, we have *no-contracting*. For simplifying the proof process, we draw the decision space (w, q) in Figure 41. The *green* curve is objective function for both optimizations (A.39) and (A.40), the *blue* curve and *orange* curve are respectively the participation constraint in optimizations (A.39) and (A.40), and the *red* line is $q = \frac{2\gamma\beta}{p\alpha}$. From axis q we can find the NE regions across the plane, where if $0 \leqslant q < \frac{\gamma}{p}$ then $(a^*, e^*) = (0, 0)$ and if $\frac{\gamma}{p} \leqslant q \leqslant \frac{2\gamma\beta}{p\alpha}$ then $(a^*, e^*) = (\frac{\alpha pq}{2\gamma\beta}, 1 - \frac{\gamma}{pq})$ and if $q > \frac{2\gamma\beta}{p\alpha}$ then $(a^*, e^*) = (1, 1 - \frac{\alpha}{2\beta})$, therefore if we were above $q = \frac{2\gamma\beta}{p\alpha}$ then optimization (A.40) would be in place, if we were in $\frac{\gamma}{p} \leqslant q \leqslant \frac{2\gamma\beta}{p\alpha}$ then optimization (A.39) would be in place, and finally for all $0 \leqslant q < \frac{\gamma}{p}$ optimization (A.38) would be in place. The intersection of two supplier's participation in optimizations (A.39) and (A.40) happens at $(w, q) = (c + \frac{p(4\beta-\alpha)\alpha^2}{8\gamma\beta^2}, \frac{2\gamma\beta}{p\alpha})$ and if $q(w-c)-(\alpha-\frac{\alpha\gamma}{2pq}) \geqslant q(w-c)-\frac{\alpha(4\beta-\alpha)}{4\beta} \Leftrightarrow q \geqslant \frac{2\gamma\beta}{p\alpha}$. The buyer wants to make a positive profit, so he would choose any (w, q) above the *green* curve (the buyer's objective function) and for satisfying the supplier's participation constraint, if we are above the *red* line he needs to choose the combination of contract terms above the *orange* curve (the supplier's participation constraint in (A.40)), and if we are bellow *red* line he needs to choose the combination of contract terms above the *blue* curve (the supplier's participation constraint in (A.39)). The *green* curve (the buyer's objective function) intersects with $q = \frac{2\gamma\beta}{p\alpha}$ at $(w, q) = (p - \frac{p\alpha}{2\beta}, \frac{2\gamma\beta}{p\alpha})$, whenever $p - \frac{p\alpha}{2\beta} < c + \frac{p(4\beta-\alpha)\alpha^2}{8\gamma\beta^2} \Leftrightarrow 0 < \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ or $\{\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p \leqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}\}$ creates the condition of Figure 41-1. In this case, for region above the *red* line the optimization (A.40) is in place and feasible region for the problem is region above both *green* and *orange* curves. As objective function in optimization problem (A.40) is decreasing in w

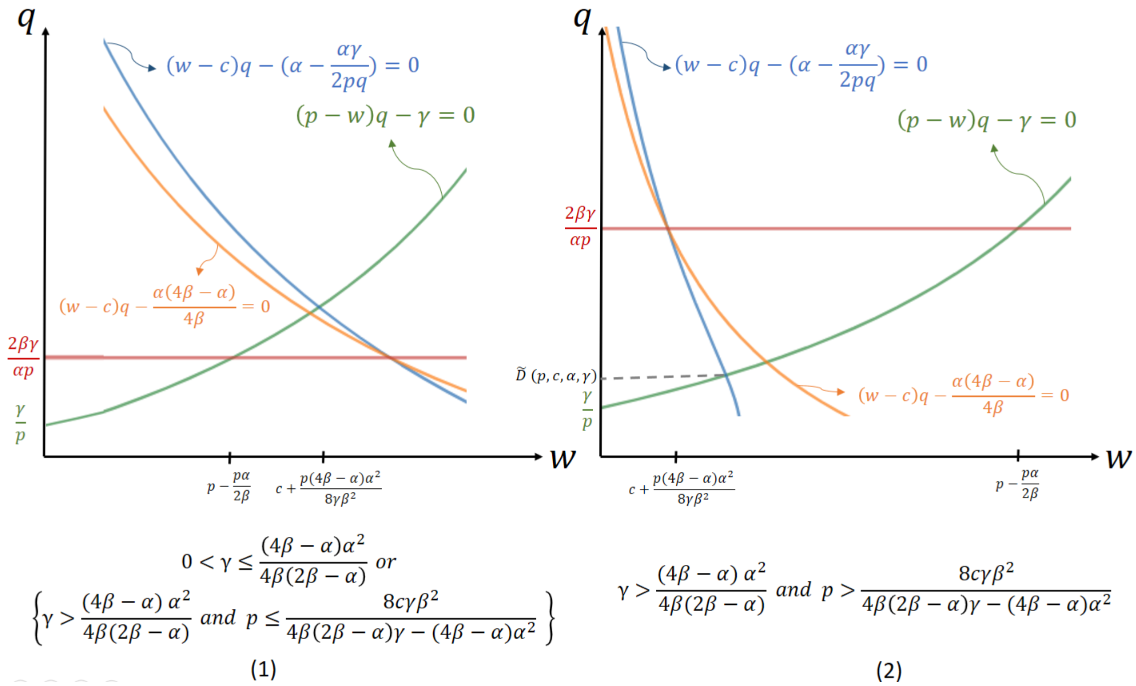


FIGURE 41. Decision space of contracting stage for full commitment model

and increasing in q then optimal order quantity is D and optimal wholesale price is left boundary of feasible region which is *orange* line (the supplier's participation indifference curve for optimization problem A.40), so $(w^*, q^*) = (c + \frac{(4\beta-\alpha)\alpha}{4\beta D}, D)$, in addition as intersection of *green* and *orange* curve happens at $q = \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$ then for all $0 \leqslant D < \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$ we have *no-contracting* as either objective function or the supplier's participation constraint would obtain a negative value. In the other case, if $p - \frac{p\alpha}{2\beta} \geqslant c + \frac{p(4\beta-\alpha)\alpha^2}{8\gamma\beta^2} \Leftrightarrow \gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p > \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$ creates the condition of Figure 41-2. In this case, above the *red* line ($D \geqslant \frac{2\gamma\beta}{p\alpha}$) we have the same optimal solutions identical to above case where $(w^*, q^*) = (c+\frac{(4\beta-\alpha)\alpha}{4\beta D}, D)$. Bellow the *red* line optimization (A.39) is in place and the supplier's participation constraint is *blue* curve. Similar to above case, objective function at (A.39) is increasing in q and decreasing in w. the intersection of *blue* curve (the supplier's participation constraint indifference curve) with *green* curve (the buyer's objective function indifference curve) happens at $(w,q) = (\frac{\sqrt{p(2c\alpha\gamma+p(\alpha^2+\gamma^2))}-p\gamma}{\alpha}, \frac{p(\alpha+\gamma)+\sqrt{p(2c\alpha\gamma+p(\alpha^2+\gamma^2))}}{2p(p-c)})$ and we define $\tilde{D} \stackrel{\text{def}}{=} \frac{p(\alpha+\gamma)+\sqrt{p(2c\alpha\gamma+p(\alpha^2+\gamma^2))}}{2p(p-c)}$, then for all $0 \leqslant D \leqslant \tilde{D}(p,c,\alpha,\beta,\gamma)$ we have *no-contracting* and for all $\tilde{D}(p,c,\alpha,\beta,\gamma) < D < \frac{2\gamma\beta}{p\alpha}$ then optimal order quantity is D and optimal wholesale price lies over the supplier's participation constraint indifference curve where $w^* = c + \frac{\alpha}{D} - \frac{\alpha\gamma}{2pD^2}$. Therefore, we prove different forms of equilibrium with their associated conditions and we summarize the equilibrium forms as follows:

Form (i): no-contracting.

Form (ii): $(w^*,q^*) = (c + \frac{(4\beta-\alpha)\alpha}{4\beta D}, D)$, $(e^*,a^*) = (1-\frac{\alpha}{2\beta}, 1)$, and $\pi_B^* > 0$, $\pi_S^* = 0$.

Form (iii): $(w^*,q^*) = (c + \frac{\alpha}{D} - \frac{\alpha\gamma}{2pD^2}, D)$, $(e^*,a^*) = (1-\frac{\gamma}{Dp}, \frac{\alpha Dp}{2\gamma\beta})$, and $\pi_B^* > 0, \pi_S^* = 0$.

Above forms are defined based on following conditions:

– $0 < \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ or $p < \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$:

  - If $0 \leqslant D < \frac{(4\beta-\alpha)\alpha+4\beta\gamma}{4\beta(p-c)}$, then form (i) occurs.
  - Otherwise, form (ii) occurs.

– When $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$:

  - If $0 \leqslant D \leqslant \tilde{D}(p,c,\alpha,\beta,\gamma)$, then form (i) occurs.
  - If $D \geqslant \frac{2\gamma\beta}{p\alpha}$, then form (ii) occurs.
  - If $\tilde{D}(p,c,\alpha,\beta,\gamma) < D < \frac{2\gamma\beta}{p\alpha}$, then form (iii) occurs.

□

Proof of Proposition 5. Using Proposition 2 and Proposition 4 and comparing the change of indexes give us the results that we show in tables next to Figures of the Proposition 5 . The equilibrium space of both models are very similar and conditions of cases based on γ are identical. We start with when $0 < \gamma \leqslant \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$. Under this condition, all indexes in both models are the same so transition from one to another does not lead to any change in indexes. In the other case, when $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$, the supplier's profit in region A of *commitment to wholesale price* model is positive while transitioning to *full commitment* model leads to have a zero profit for the supplier, thus changes in the supplier's profit is negative and the supplier's profit decreases in this transition. Subsequently, the buyer's profit increases. The rest of metrics are identical, therefore by this transition they do not change. Also, when $\gamma > \frac{(4\beta-\alpha)\alpha^2}{4\beta(2\beta-\alpha)}$ and $p \geqslant \frac{8c\gamma\beta^2}{4\beta(2\beta-\alpha)\gamma-(4\beta-\alpha)\alpha^2}$ then we prove that $\tilde{D}(p,c,\alpha,\gamma) < \hat{D}(p,c,\alpha,\beta,\gamma)$:

$$\frac{p(\alpha+\gamma)+\sqrt{p(2c\alpha\gamma+p(\alpha^2+\gamma^2))}}{2p(p-c)} < \frac{\sqrt{p}(\alpha+2\gamma)+\sqrt{p((\alpha-2\gamma)^2+16\gamma)-8c\gamma(2\beta-\alpha)}}{4\sqrt{p}(p-c)}$$

$$\Leftrightarrow 0 < 4p[2c\alpha\gamma+p(\alpha^2+\gamma^2)]+(p\alpha-\sqrt{p}\sqrt{p((\alpha-2\gamma)^2+16\gamma)-8c\gamma(2\beta-\alpha)})^2.$$

This leads to creation of region B, in which we have *no-contracting* in *commitment to wholesale price* and we have contracting in *full commitment*. Therefore, in region B all metrics improve, except the supplier's profit which still remain zero. In regions C and D all metrics do not change by transitioning from *commitment to wholesale price* to *full commitment*. □

Proof of Proposition 6. We take the derivative of metrics w.r.t. d and evaluate them:

I) We take the derivative of compliance effort with respect to d and set it greater than zero. $\frac{\partial e(\alpha(d),\beta(d))}{\partial d} = \frac{-\beta(d)\alpha'(d)+\alpha(d)\beta'(d)}{2\beta(d)^2} > 0 \Leftrightarrow \frac{\beta'(d)}{\beta(d)} > \frac{\alpha'(d)}{\alpha(d)}$.

II) We take the derivative of auditing effort with respect to d and set it greater than zero. $\frac{\partial a(D,p,\alpha(d),\beta(d),\gamma(d))}{\partial d} = \frac{pD\left[\beta(d)[\gamma(d)\alpha'(d)-\alpha(d)\gamma'(d)]-\alpha(d)\gamma(d)\beta'(d)\right]}{2\beta(d)^2\gamma(d)^2} > 0 \Leftrightarrow$ $\beta(d)[\gamma(d)\alpha'(d) - \alpha(d)\gamma'(d)] - \alpha(d)\gamma(d)\beta'(d) > 0 \Leftrightarrow \frac{\alpha(d)}{\alpha'(d)} - \frac{\beta(d)}{\beta'(d)} > \frac{\gamma(d)}{\gamma'(d)} > 0$. Also, $\frac{\partial e(D,p,\gamma(d))}{\partial d} = -\frac{\gamma'(d)}{DP} < 0$.

III) We take the derivative of overall sustainability of supply chain with respect to d and set it greater than zero. $\frac{\partial I_{oc}(D,p,\alpha(d),\beta(d),\gamma(d)))}{\partial d} = \frac{\beta(d)\alpha'(d)-\alpha(d)\beta'(d)}{2\beta(d)^2} - \frac{\gamma'(d)}{pD} > 0 \Leftrightarrow$ $Dp > \frac{2\beta(d)\gamma'(d)}{\alpha(d)(\frac{\alpha'(d)}{\alpha(d)} - \frac{\beta'(d)}{\beta(d)})}$ and $\frac{\alpha'(d)}{\alpha(d)} > \frac{\beta'(d)}{\beta(d)}$. □

Proof of Proposition 7. We consider the area above and very close to the line of $D = \frac{2\gamma(d)\beta(d)}{p\alpha(d)}$, in Figure 3. If this line moves up a little then those area above the line will drop bellow it. Therefore, if we take the derivative of the function $\frac{2\beta(d)\gamma(d)}{p\alpha(d)}$ with respect to d, and set it greater than zero, then it gives us the condition for this case: $\frac{\partial \frac{2\beta(d)\gamma(d)}{p\alpha(d)}}{\partial d} = \frac{2\left[\beta(d)[\gamma'(d)\alpha(d)-\alpha'(d)\gamma(d)]+\alpha(d)\gamma(d)\beta'(d)\right]}{p\alpha(d)^2} > 0 \Leftrightarrow 0 < \frac{\alpha'(d)}{\alpha(d)} - \frac{\beta'(d)}{\beta(d)} <$ $\frac{\gamma'(d)}{\gamma(d)}$ or $\frac{\alpha'(d)}{\alpha(d)} < \frac{\beta'(d)}{\beta(d)}$. We combine these two conditions and rewrite it as follows: $\frac{\alpha'(d)}{\alpha(d)} - \frac{\beta'(d)}{\beta(d)} < \frac{\gamma'(d)}{\gamma(d)}$. □

APPENDIX B

TECHNICAL PROOFS - CHAPTER III

Proof of Lemma 11. As we show in Figure 14 and Figure 15, there exists three cases for $\Phi_{C,NC} > \Phi_{NC,C}$ and four cases for $\Phi_{C,NC} \leqslant \Phi_{NC,C}$. We name the former Case A and the latter Case B. We find the best response function for each supplier and then intersect those best response functions to find the NE. First, look at the Case A.

In Case A, we have three sub-cases:

- A-1) $p\theta_2 \leqslant c_2$ and $p\theta_1 \leqslant c_1$.
- A-2) $p\theta_2 \leqslant c_2$ and $p\theta_1 > c_1$.
- A-3) $p\theta_2 > c_2$ and $p\theta_1 > c_1$.

In Case B, we have four sub-cases:

- B-1) $p\theta_2 \leqslant c_2$ and $p\theta_1 \leqslant c_1$.
- B-2) $p\theta_2 \leqslant c_2$ and $p\theta_1 > c_1$.
- B-3) $p\theta_2 > c_2$ and $p\theta_1 \leqslant c_1$.
- B-4) $p\theta_2 > c_2$ and $p\theta_1 > c_1$.

Based on the constraint in optimization program 3.17, if $\Omega_i > p$ then we have no feasible region. If $\Omega_i \leqslant p$, then we have two cases: 1) for all $c_i \leqslant w_i \leqslant \Omega_i$ the supplier i does not participate; 2) for all $\Omega_i \leqslant w_i \leqslant p$ the supplier i participates. When, $q_i^* = 0$, the supplier has no incentive to participate and there is no contracting. If $q_i^* = D$, then the supplier's profit function is $(w_i - c_i)D - (1 - \theta_i)GC$. This profit function is increasing in w_i, therefore the supplier chooses the maximum level of w_i which is possible. Now, we look at each sub-cases to find the best response function.

A-1) In this case, $(q_1^*, q_2^*) = (0, 0)$, therefore there is no contracting.

A-2) In this case, $(q_1^*, q_2^*) = (D, 0)$ for $w1 < p\theta_1$ and $(q_1^*, q_2^*) = (0, 0)$ for $w1 \geqslant p\theta_1$. We show this on Figure 14. Therefore, supplier 2 does not participate as he receives zero order. For supplier 1, we need to check the participation constraint. If $\Omega_1 \leqslant p\theta_1$ then, the supplier 1's profit will be $(w_i - c_i)D - (1 - \theta_i)GC$ and the supplier chooses the upper bound of feasible region. Therefore, the optimal wholesale price will be $w_1^* = p\theta_1$. Therefore, if $\Omega_1 \leqslant p\theta_1$, then NE is supplier 1 participates and set wholesale price $w_1^* = p\theta_1$ and supplier 2 does not participate. If $\Omega_1 > p\theta_1$ then $(q_1^*, q_2^*) = (0, 0)$ and we have no contracting.

A-3) Similar to above case, in this case also we need to compare the Ω_1 vs. $p\theta_1$ and Ω_2 vs. $p\theta_2$. There exist 4 combinations of this comparisons as follows:

A-3-i) When $\Omega_1 > p\theta_1$ and $\Omega_2 > p\theta_2$, then none of suppliers participate in the contracting, because the $(q_1^*, q_2^*) = (0, 0)$.

A-3-ii) When $\Omega_1 > p\theta_1$ and $\Omega_2 \leqslant p\theta_2$, then as supplier 1 does not participate. Supplier 2's profit is $(w_2 - c_2)D - (1 - \theta_2)GC$, therefore he sets the wholesale price $w_2^* = p\theta_2$.

A-3-iii) When $\Omega_1 \leqslant p\theta_1$ and $\Omega_2 > p\theta_2$, then supplier 2 does not participate as his participation constraint is not satisfied. Supplier 1 participates and as his profit function is increasing wholesale price he sets $w_1^* = p\theta_1$.

A-3-iv) When $\Omega_1 \leqslant p\theta_1$ and $\Omega_2 \leqslant p\theta_2$, then both suppliers participate as they receive positive order quantity in some regions. In this situation we need to compare the intersection of two participation constraint lines vs. $w_2 = w_1 - p(\theta_2 - \theta_1)$. It creates two sub-cases:

A-3-iv-a) $\Omega_1 \leqslant \Omega_2 - p(\theta_2 - \theta_1)$: First we solve for supplier 1's best response function. For all $w_2 \in [c_2, \Omega_2)$, the supplier 2 does not participate, therefore supplier 1 sets the wholesale price $w_1^* = p\theta_1$. For all $w_2 \in [\Omega_2, p\theta_2)$, the supplier 1 sets the price at $w_1^* = \Omega_2 - p(\theta_2 - \theta_1) - \epsilon$. For all $w_2 \in [p\theta_2, 1]$, supplier 1 sets the price at $w_1^* = p\theta_1 - \epsilon$.

Now we solve for supplier 2's best response function. For all $w_1 \in [c_1, \Omega_1]$, then supplier 1 does not participate and supplier 2 sets the wholesale price $w_2^* = p\theta_2$. For all $w_1 \in [\Omega_1, \Omega_2 - p(\theta_2 - \theta_1) - \epsilon]$, the supplier 2 can set any price which is $w_2^* \in [\Omega_2, p]$. For all $w_1 \in [\Omega_2 - p(\theta_2 - \theta_1), p\theta_1)$, the supplier 2 sets wholesale price $w_2^* = w_1 - p(\theta_2 - \theta_1)$. For all $w_1 \in [p\theta_1, 1]$, the supplier 2 sets wholesale price $w_2^* = p\theta_2 - \epsilon$. When we intersect these two best response functions, we find the NE as $(w_1^*, w_2^*) = (\Omega_2 - p(\theta_2 - \theta_1), \Omega_2)$. We show this NE in Figure 42.

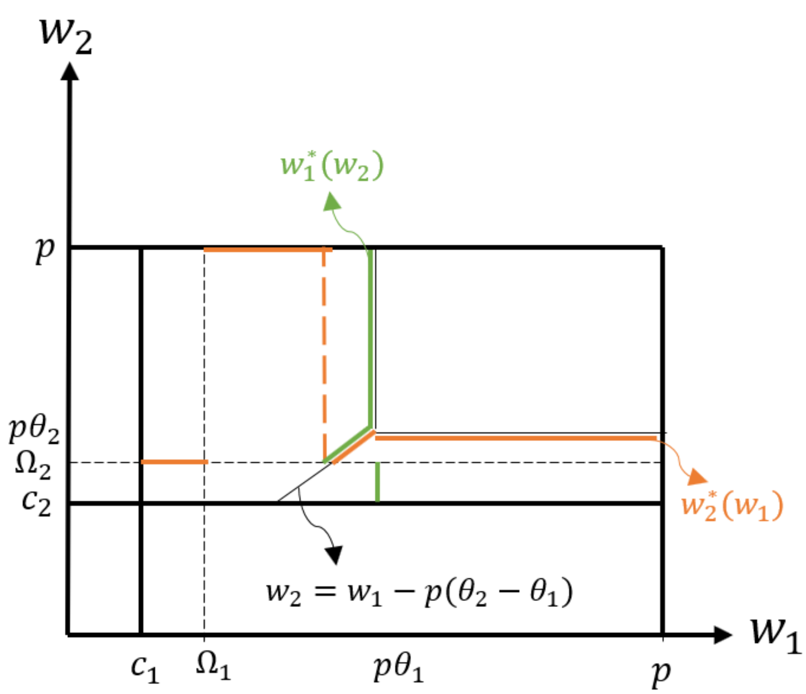


FIGURE 42. Graphic Representation of NE $(w_1^*, w_2^*) = (\Omega_2 - p(\theta_2 - \theta_1), \Omega_2)$.

A-3-iv-b) $\Omega_1 > \Omega_2 - p(\theta_2 - \theta_1)$: First we solve for supplier 1's best response function.

For all $w_2 \in [c_2, \Omega_2]$, then supplier 2 does not participate and supplier 1 sets the wholesale price $w_1^* = p\theta_1$. For all $w_2 \in [\Omega_2, \Omega_1 p(\theta_1 - \theta_2)]$, the supplier 1 can set any price which is $w_1^* \in [\Omega_1, p]$. For all $w_2 \in [\Omega_1 - p(\theta_1 - \theta_2), p\theta_2)$, the supplier 1 sets wholesale price $w_1^* = w_2 - p(\theta_2 - \theta_1)$. For all $w_2 \in [p\theta_2, 1]$, the supplier 2 sets wholesale price $w_1^* = p\theta_1 - \epsilon$.

Now we solve for supplier 2's best response function. For all $w_1 \in [c_1, \Omega_1)$, the supplier 1 does not participate, therefore supplier 2 sets the wholesale price $w_2^* = p\theta_2$. For all $w_1 \in [\Omega_1, p\theta_1)$, the supplier 2 sets the price at $w_2^* = \Omega_2 - p(\theta_2 - \theta_1)$. For all $w_1 \in [p\theta_1, 1]$, supplier 2 sets the price at $w_2^* = p\theta_2 - \epsilon$. When we intersect these two best response functions, we find the NE as $(w_1^*, w_2^*) = (\Omega_1, \Omega_1 - p(\theta_1 - \theta_2))$. We show this NE in Figure 43.

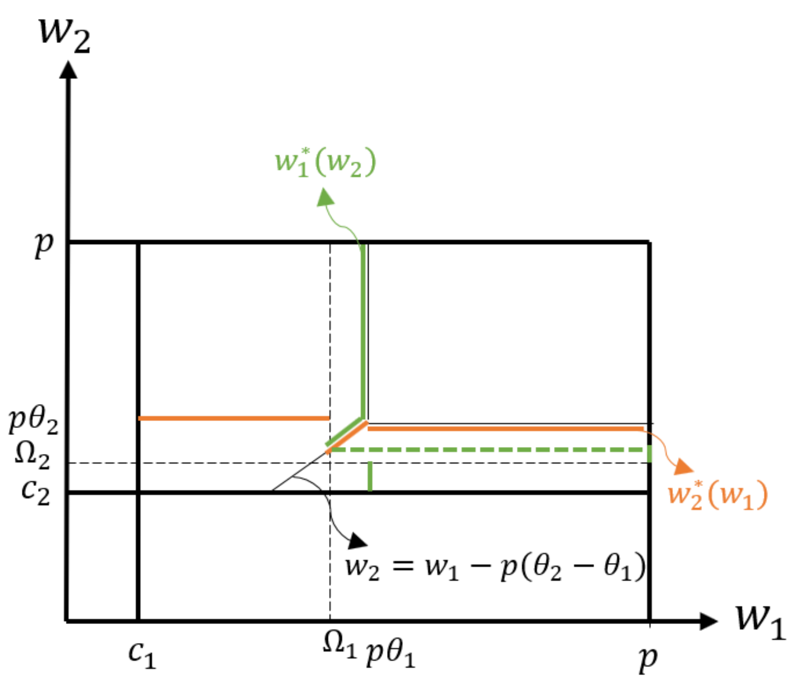


FIGURE 43. Graphic Representation of NE $(w_1^*, w_2^*) = (\Omega_1, \Omega_1 - p(\theta_1 - \theta_2))$.

B-1) In this case, $(q_1^*, q_2^*) = (0, 0)$, therefore there is no contracting.

B-2) In this case, $(q_1^*, q_2^*) = (D, 0)$ for $w1 < p\theta_1$ and $(q_1^*, q_2^*) = (0, 0)$ for $w1 \geqslant p\theta_1$.

We show this on Figure 15. Therefore, supplier 2 does not participate as he receives zero order. For supplier 1, we need to check the participation constraint. If $\Omega_1 \leqslant p\theta_1$ then, the supplier 1's profit will be $(w_i - c_i)D - (1-\theta_i)GC$ and the supplier chooses the upper bound of feasible region. Therefore, the optimal wholesale price will be $w_1^* = p\theta_1$. Therefore, if $\Omega_1 \leqslant p\theta_1$, then NE is supplier 1 participates and set wholesale price $w_1^* = p\theta_1$ and supplier 2 does not participate. If $\Omega_1 > p\theta_1$ then $(q_1^*, q_2^*) = (0,0)$ and we have no contracting.

B-3) In this case, $(q_1^*, q_2^*) = (D,0)$ for $w1 < p\theta_1$ and $(q_1^*, q_2^*) = (0,0)$ for $w1 \geqslant p\theta_1$. The optimal solution and NE will be symmetric with respect to previous case for supplier 2.

B-4) Similar to above case, in this case also we need to compare the Ω_1 vs. $p\theta_1$ and Ω_2 vs. $p\theta_2$. There exist 4 combinations of this comparisons as follows:

B-4-i) When $\Omega_1 > p\theta_1$ and $\Omega_2 > p\theta_2$, then none of suppliers participate in the contracting, because the $(q_1^*, q_2^*) = (0,0)$.

B-4-ii) When $\Omega_1 > p\theta_1$ and $\Omega_2 \leqslant p\theta_2$, then as supplier 1 does not participate. Supplier 2's profit is $(w_2 - c_2)D - (1-\theta_2)GC$, therefore he sets the wholesale price $w_2^* = p\theta_2$.

B-4-iii) When $\Omega_1 \leqslant p\theta_1$ and $\Omega_2 > p\theta_2$, then supplier 2 does not participate as his participation constraint is not satisfied. Supplier 1 participates and as his profit function is increasing wholesale price he sets $w_1^* = p\theta_1$.

B-4-iv) When $\Omega_1 \leqslant p\theta_1$ and $\Omega_2 \leqslant p\theta_2$, then both suppliers participate as they receive positive order quantity in some regions. In this situation we need to compare the intersection of two participation constraint lines vs. $w_2 = w_1 - p(\theta_2 - \theta_1)$. It creates two sub-cases:

B-4-iv-a) $\Omega_1 \leqslant \Omega_2 - p(\theta_2 - \theta_1)$: First we solve for supplier 1's best response function.

For all $w_2 \in [c_2, \Omega_2)$, the supplier 2 does not participate, therefore supplier 1 sets the wholesale price $w_1^* = p\theta_1$. For all $w_2 \in [\Omega_2, p\theta_2)$, the supplier 1 sets the price at $w_1^* = \Omega_2 - p(\theta_2 - \theta_1) - \epsilon$. For all $w_2 \in [p\theta_2, 1]$, supplier 1 sets the price at $w_1^* = p\theta_1 - \epsilon$.

Now we solve for supplier 2's best response function. For all $w_1 \in [c_1, \Omega_1]$, then supplier 1 does not participate and supplier 2 sets the wholesale price $w_2^* = p\theta_2$. For all $w_1 \in [\Omega_1, \Omega_2 - p(\theta_2 - \theta_1) - \epsilon]$, the supplier 2 can set any price which is $w_2^* \in [\Omega_2, p]$. For all $w_1 \in [\Omega_2 - p(\theta_2 - \theta_1), p\theta_1)$, the supplier 2 sets wholesale price $w_2^* = w_1 - p(\theta_2 - \theta_1)$. For all $w_1 \in [p\theta_1, 1]$, the supplier 2 sets wholesale price $w_2^* = p\theta_2 - \epsilon$. When we intersect these two best response functions, we find the NE as $(w_1^*, w_2^*) = (\Omega_2 - p(\theta_2 - \theta_1), \Omega_2)$.

B-4-iv-b) $\Omega_1 > \Omega_2 - p(\theta_2 - \theta_1)$: First we solve for supplier 1's best response function. For all $w_2 \in [c_2, \Omega_2]$, then supplier 2 does not participate and supplier 1 sets the wholesale price $w_1^* = p\theta_1$. For all $w_2 \in [\Omega_2, \Omega_1 p(\theta_1 - \theta_2)]$, the supplier 1 can set any price which is $w_1^* \in [\Omega_1, p]$. For all $w_2 \in [\Omega_1 - p(\theta_1 - \theta_2), p\theta_2)$, the supplier 1 sets wholesale price $w_1^* = w_2 - p(\theta_2 - \theta_1)$. For all $w_2 \in [p\theta_2, 1]$, the supplier 2 sets wholesale price $w_1^* = p\theta_1 - \epsilon$.

Now we solve for supplier 2's best response function. For all $w_1 \in [c_1, \Omega_1)$, the supplier 1 does not participate, therefore supplier 2 sets the wholesale price $w_2^* = p\theta_2$. For all $w_1 \in [\Omega_1, p\theta_1)$, the supplier 2 sets the price at $w_2^* = \Omega_2 - p(\theta_2 - \theta_1)$. For all $w_1 \in [p\theta_1, 1]$, supplier 2 sets the price at $w_2^* = p\theta_2 - \epsilon$. When we intersect these two best response functions, we find the NE as $(w_1^*, w_2^*) = (\Omega_1, \Omega_1 - p(\theta_1 - \theta_2))$. We abuse the notation and do not report ϵ in our solution space. We plot the condition of all above cases with their respective NE on the Plane of θ_1 and θ_2 in Figure 44.

□

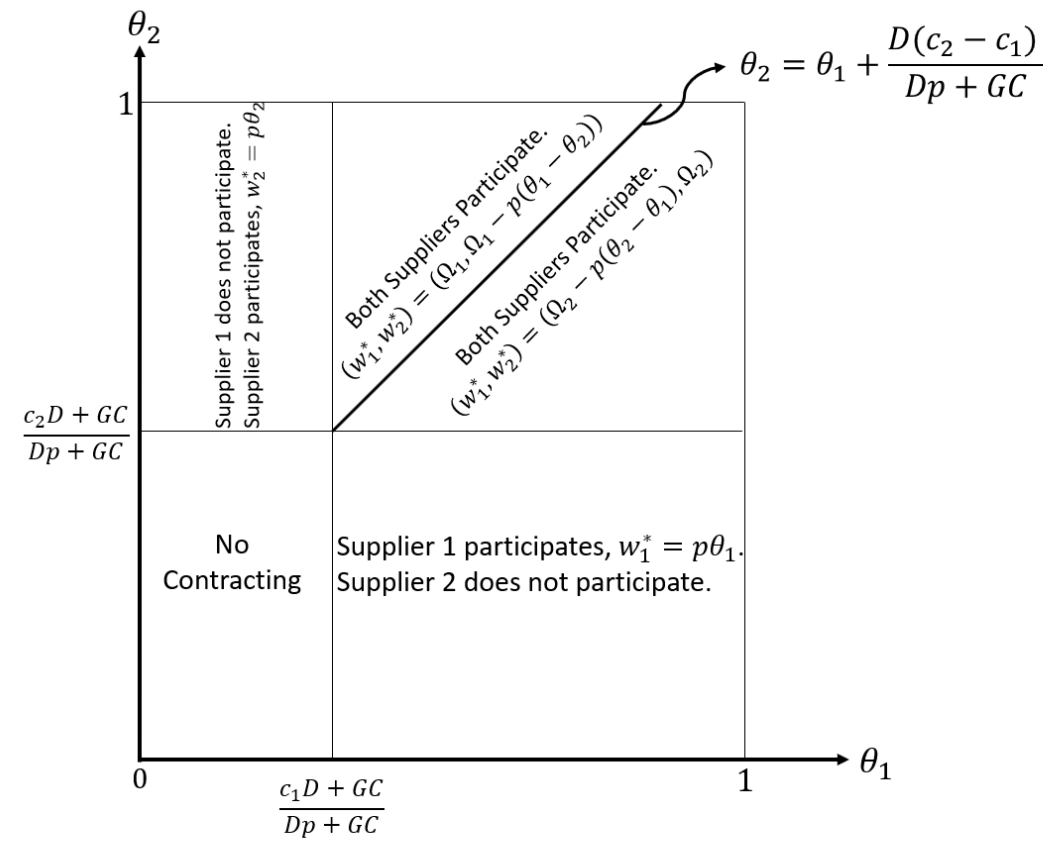


FIGURE 44. Presentation of Nash Equilibrium of wholesale Prices on Plane of θ_1 and θ_2.

Proof of Lemma 13. By using the parameter space from contracting stage in Figure 44, we form the supplier's profit function. Before presenting the function, we define new symbol. We define $\tilde{\theta}_1 = \frac{c_1 D+GC}{pD+GC}$, $\tilde{\theta}_2 = \frac{c_2 D+GC}{pD+GC}$, $\hat{e}_1 = \frac{(1-a_1)\hat{\theta}_1}{1-a_1\hat{\theta}_1}$, and $\hat{e}_2 = \frac{(1-a_2)\hat{\theta}_2}{1-a_2\hat{\theta}_2}$.

For all $\theta_2 < \hat{\theta}_2 \Leftrightarrow e_2 < \hat{e}_2$:

$$\Pi_S(e_1, e_2) = \begin{cases} -\alpha_1 e_1 & 0 \leqslant e_1 < \hat{e}_1 \\ (e_1 + (1-e_1)(1-a_1))[(p\theta_1 - c_1)D - (1-\theta_1)GC] - \alpha_1 e_1 & \hat{e}_1 \leqslant e \leqslant 1 \end{cases} \tag{B.1}$$

This function is piecewise linear in e_1 and continuous at $e_1 = \hat{e}_1$. The first piece is decreasing in e_1. Therefore, we have two solution candidates. When $0 < D < \frac{\alpha_1}{p-c_1}$ then $e_1^* = 0$ for all $e_2 \in [0, \hat{e}_2]$. Otherwise, $e_1^* = 1$ for all $e_2 \in [0, \hat{e}_2]$.

For all $\theta_2 \geqslant \hat{\theta}_2 \Leftrightarrow e_2 \geqslant \hat{e}_2$:

$$\Pi_S(e_1, e_2) = \begin{cases} -\alpha_1 e_1 & 0 \leqslant e_1 < \hat{e}_1 \\ Pr(P_1, F_2)[(p\theta_1 - c_1)D - (1-\theta_1)GC] - \alpha_1 e_1 & \hat{e}_1 \leqslant e < \bar{e}_1 \\ Pr(P_1, P_2)[(w_1^* - c_1)D - (1-\theta_1)GC] + & \\ Pr(P1, F_2)[(p\theta_1 - c_1)D - (1-\theta_1)GC] - \alpha_1 e_1 & \bar{e}_1 \leqslant e_1 \leqslant 1 \end{cases} \tag{B.2}$$

where $w_1^* = \Omega_2 - p(\theta_2 - \theta_1)$, $Pr(P_1, F_2) = [e_1 + (1-e_1)(1-a_1)][(1-e_2)(a_2)]$,$Pr(P_1, P_2) = [e_1 + (1-e_1)(1-a_1)][(1-e_2)(1-a_2)]$. This profit function is piecewise linear in e_1. The function is contentious at $e_1 = \hat{e}_1$ and $e_1 = \bar{e}_1$. The slope of first piece is always negative. The slope of second piece is always less than the slope of third piece. Therefore, the candidate solutions are $e_1 = 0$ and $e_1 = 1$. By comparing the value of function at zero and one we have: If $0 < D \leqslant \frac{\alpha_1}{p-c_1}$ then $e_1^* = 0$ for all $e_2 \in [\hat{e}_2, 1]$. If $\frac{\alpha_1}{p-c_1} \leqslant D \leqslant \frac{\alpha_1}{c_2-c_1}$ then $e_1^* = 0$ for all $e_2 \in [\bar{e}_2, 1]$. If $\frac{\alpha_1}{c_2-c_1} < D$ then $e_1^* = 1$ for all $e_2 \in [\hat{e}_2, 1]$. If $\frac{\alpha_1}{p-c_1} < D < \frac{\alpha_1}{c_2-c_1}$ then $e_1^* = 1$ for all $e_2 \in [\hat{e}_2, \bar{e}_2]$.

By merging the results of case $\theta_2 < \hat{\theta}_2 \Leftrightarrow e_2 < \hat{e}_2$ and $\theta_2 \geqslant \hat{\theta}_2 \Leftrightarrow e_2 \geqslant \hat{e}_2$:

Supplier 1's best response function e_1^* for any given e_2 is:

- If $0 \leqslant D \leqslant \frac{\alpha_1}{p-c_1}$, then $e_1^* = 0$ for all e_2 in $[0, 1]$.
- IF $\frac{\alpha_1}{p-c_1} < D < \frac{\alpha_1}{c_2-c_1}$, then $e_1^* = 1$ for all e_2 in $[0, \bar{e}_2]$, and $e_1^* = 0$ for all e_2 in $(\bar{e}_2, 1]$.
- If $D \geqslant \frac{\alpha_1}{c_2-c_1}$, then $e_1^* = 1$ for all e_2 in $[0, 1]$.

where $\bar{e}_2 = \frac{(1-a_2)C_2D+(p-c_1)D+(1-a_2)GC-\alpha_1}{(p-a_2c_2)D+(1-a_2)GC}$. □

Proof of Lemma 14. The best response function of supplier 1 is always zero or one for any given e_2. Thus, we can find the Nash equilibrium by finding the best response function of supplier 2 at $e_1^* = 0$ and $e_1^* = 1$. First we analyze the case when $e_1^* = 0$. In this case, supplier 2's profit function is :

$$\Pi_{S_2}(e_1 = 0, e_2) = \begin{cases} -\alpha_2 e_2 & 0 \leqslant e_2 < \hat{e}_2 \\ (e_2 + (1-e_2)(1-a_2))[(p\theta_2 - c_2)D - (1-\theta_2)GC] - \alpha_2 e_2 & \hat{e}_2 \leqslant e \leqslant 1 \end{cases} \tag{B.3}$$

This function is piecewise linear and continuous at e_2. the first piece is decreasing in e_2. Therefore, we can find the optimal solution by comparing the value of function at zero and one. If $D \geqslant \frac{\alpha_2}{p-c_2}$, then $e_2^* = 1$. Otherwise, $e_2^* = 0$. Next case is when $e_1^* = 1$, in this case the profit function of supplier 2 is always $-\alpha_2 e_2$. Then, for all parameters, $e_2^*(e_1^* = 1) = 0$. Therefore, the Nash equilibrium of compliance efforts is:

- If $D < \frac{\alpha_1}{p-c_1}$ and $D < \frac{\alpha_2}{p-c_2}$, then $(e_1^*, e_2^*) = (0, 0)$.
- If $\frac{\alpha_1}{p-c_1} \leqslant D < \frac{\alpha_2}{p-c_2}$ or $\{D \geqslant \frac{\alpha_1}{c_2-c_1}$ and $D \geqslant \frac{\alpha_2}{p-c_2}\}$, then $(e_1^*, e_2^*) = (1, 0)$.
- If $\frac{\alpha_2}{p-c_2} \leqslant D < \frac{\alpha_1}{p-c_1}$, then $(e_1^*, e_2^*) = (0, 1)$.
- If $\frac{\alpha_2}{p-c_2} \leqslant D < \frac{\alpha_1}{c_2-c_1}$ and $D \geqslant \frac{\alpha_1}{p-c_1}$, then $(e_1^*, e_2^*) = (1, 0)$ and $(e_1^*, e_2^*) = (0, 1)$.

□

Proof of Proposition 9. We plug-in the optimal compliance efforts and related optimal contracting decisions to find the optimal auditing effort. When $(e_1^*, e_2^*) =$

$(0,0)$, there is no contracting. When $(e_1^*, e_2^*) = (1,0)$ or $(e_1^*, e_2^*) = (0,1)$ then the buyer's profit is $-\gamma(a_1 + a_2)$, therefore the $(a_1^*, a_2^*) = (0,0)$. $\square$

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