

A CANTILEVER BALCONY FOR A SMALL THEATRE.

By

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A Thesis

Presented to the Faculty of the Graduate School

of the

University of Oregon

in partial fulfillment of the requirements

for the degree of

Master of Science

June, 1932

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The slope of the Balcony and height of the steppings shall first be considered, so that the steel used in the construction of the balcony may be designed accordingly.

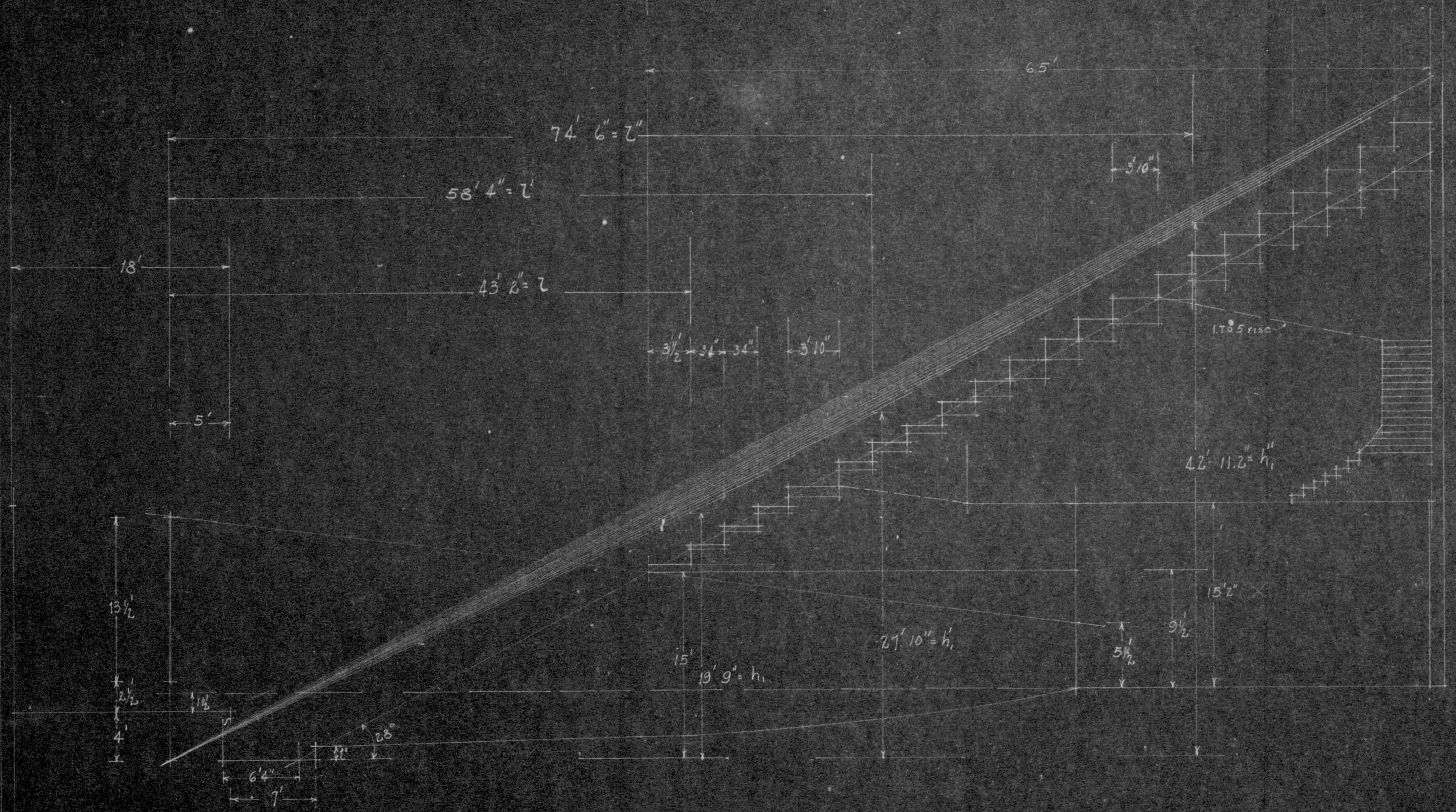
When "setting up" the sections of the Parquette circle or balcony in the theater, it is desirable to sight from the eye level of the spectator, which will be considered as 4 feet 2 inches from the floor when the spectator is seated, (and 4 feet 10 inches to 5 feet when standing). The theoretical principles used when fixing the heights of the steppings upon which the seats are placed are as follows:

A point is fixed on the curtain line 4 feet below the stage level, and from this point, after the distance from the stage, the stepping, and the floor level is placed, set up the spectator's eyes 4 feet 2 inches above the floor, vertical with the back rail of the seat. Now from the 4 feet point on the curtain line, a line should be drawn cutting through the eye of the spectator in the first row, and produced until it cuts a vertical line set up at the back of the second row. Then from the point where the vertical and radial lines intersect 3 inches is measured up and that point gives the eye level of the second row. From the point below the stage, a line is drawn through the eye level of the second row, and produced until it intersects the vertical line set up at the back of the third row, and from that point again measured up 3 inches for each row, and from each eye level, measured down 4 feet 2 inches will give the floor level for each stepping.

After getting the heights of the stepping in this way, the nosings are not tangent to a straight line, but to a concave curve, and the steppings are not equal in height but become steeper as they recede from the stage. This curve has been named "the isacoustic" or equal hearing curve, and is a refinement seldom practiced. Thus each row of spectators

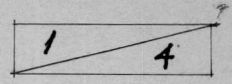
the stage. This curve has been named "the isacoustic" or equal hearing curve, and is a refinement seldom practiced. Thus each row of spectators in this balcony have a sight line 3 inches above the sight line of the row just ahead.

See next sheet for graphical representation.



GRAPHICAL METHOD OF FINDING HEIGHTS OF STEPPINGS FROM SIGHT LINES. (THE ISACOUSTIC CURVE)

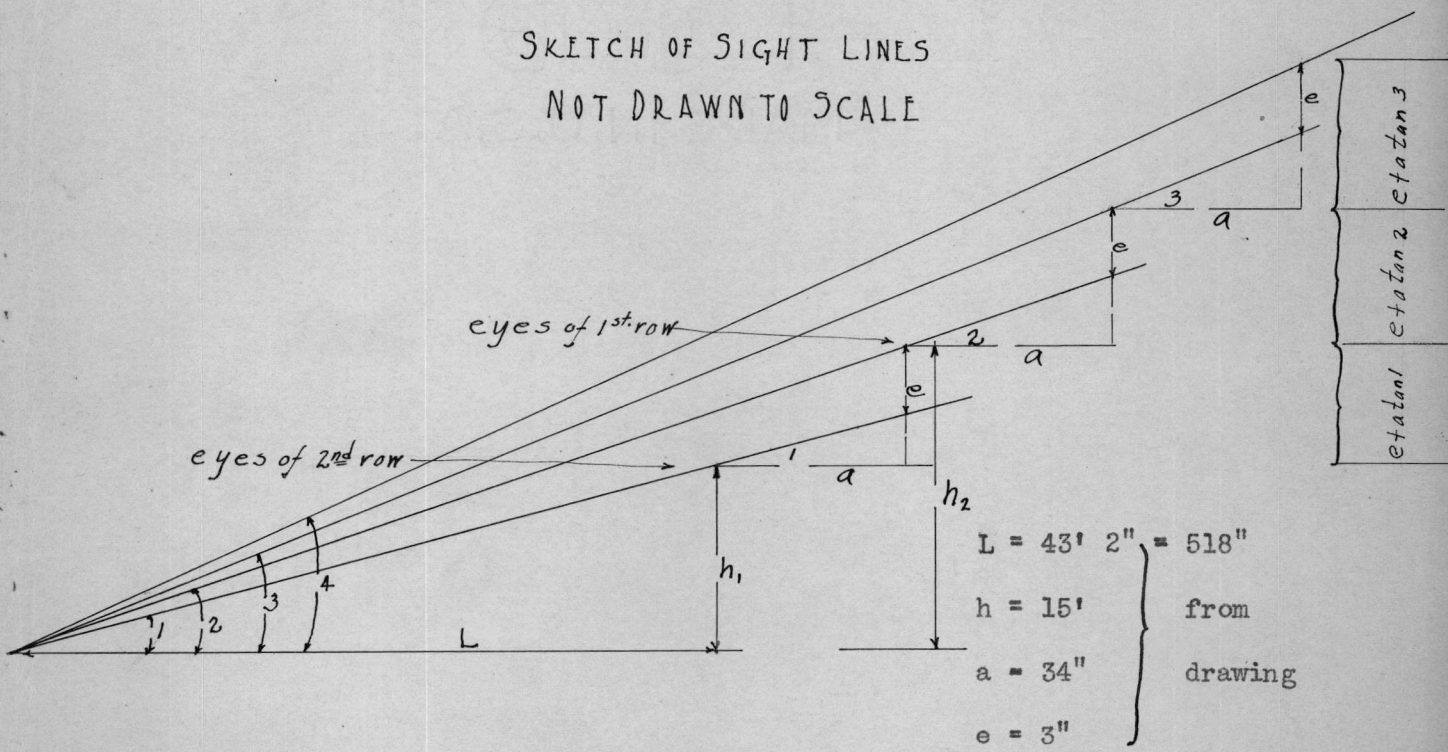
SCALE  $\frac{1}{8}'' = 1'$  C.C.M.



### Mathematical Computations of Balcony Seating

To find a mathematical equation by which the height and difference in height of the balcony steps (i.e. seating of the balcony) may be expressed exactly to conform to the sight lines as drawn. That is, the line of sight of each person is 3" above that of the person occupying the seat directly ahead.

SKETCH OF SIGHT LINES  
NOT DRAWN TO SCALE



$L = 43' 2'' = 518''$   
 $h = 15'$   
 $a = 34''$   
 $e = 3''$

from drawing

Each step (platform) is measured down a constant distance (50") from the line of sight (or eye) of the person seated. Therefore the height of each step; step 2 =  $e + a \tan 1$ ; step 3 =  $e + a \tan 2$ ; etc.

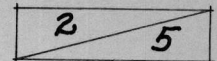
$$h_1 = 15' + 7'' + 4' 2''$$

$$h_1 = 19' 9'' = 237''$$

$$h_2 = h_1 + e + a \tan 1$$

$$h_3 = h_2 + e + a \tan 2$$

$$\tan 1 = \frac{h_1}{L} = \frac{19.75}{43.1666} = .4575$$



$$\begin{aligned} \tan 2 &= \frac{h_2}{L+a} = \frac{h_1 + e + a \tan 1}{L+a} = \frac{237 + 3 + 34(.4575)}{43.1666' + 34''} = \frac{237 + 3 + 15.55}{518'' + 34''} \\ &= \frac{255.55}{552} = .4625 \end{aligned}$$

$$\begin{aligned} \tan 3 &= \frac{h_3}{L+2a} = \frac{h_2 + e + a \tan 2}{L+2a} = \frac{255.55 + 3 + 34(.4625)}{518 + 68} = \frac{258.55 + 15.71}{586} \\ &= \frac{274.26}{586} = .468 \end{aligned}$$

$$\begin{aligned} \tan 4 &= \frac{h_4}{L+3a} = \frac{h_3 + e + a \tan 3}{L+3a} = \frac{274.26 + 3 + 34(.468)}{620} = \frac{277.26 + 15.9}{620} \\ &= \frac{293.16}{620} = .473 \end{aligned}$$

The height of the first step being taken as zero, because the first row of seats are to be placed on the floor level of the Balcony directly back of the rail, the height of the following steps up to the first aisle are given.

Height of steps up to first aisle:	Difference in Height
$e + a \tan 1 = 3 + 34(.4575) = 3 + 15.55 = 18.55''$	of steps to first aisle.
$e + a \tan 2 = 3 + 34(.4625) = 3 + 15.71 = 18.71''$	----- .160''
$e + a \tan 3 = 3 + 34(.468) = 3 + 15.9 = 18.9''$	----- .190''
$e + a \tan 4 = 3 + 34(.473) = 3 + 16.10 = 19.10''$	----- .200''

These figures give us the correct difference in height to 3 places of the first 4 rows of seats - which brings us to the first aisle. That is, each step increases by a small amount above the height of the preceding step. The increment of difference in height changes so slowly for the first few steps that for the first 3 places it remains almost the same. The total difference of the first 4 steps to be taken as .55" (counting the balcony floor level as step 1).

Now starting from the first aisle, to find the height of the steps up to the second aisle.

Using  $h_1$  for the steps above the first aisle (as shown on the graphic diagram sheet ).  $a' = \text{aisle width} = 3' 10'' = 46''$

$$h_1^1 = h_4^1 + a^1 \tan 4 + (e + a \tan 4) = 293.16 + 46 (.473) + 3 + 34(.473)$$

$$= 293.16 + 21.78 + 19.1 = 334.04''$$

$$L' = 12 \times 58.333 = 700''$$

$$\tan 1 = \frac{h_1^1}{L'} = \frac{334.04}{700} = .477$$

$$\tan 2 = \frac{h_2^1}{L' + a} = \frac{h_1^1 + e + a \tan 1}{58.333 + 34} = \frac{334.04 + 3 + 34(.477)}{700 + 34}$$

$$= \frac{337.04 + 16.22}{734} = \frac{353.26}{734} = .482$$

$$\tan 3 = \frac{h_3^1}{L' + 2a} = \frac{h_2^1 + e + a \tan 2}{L' + 2a} = \frac{352.81 + 3 + 34(.482)}{700 + 68}$$

$$= \frac{355.81 + 16.35}{768} = \frac{372.81}{768} = .4855$$

$$\tan 4 = \frac{h_4^1}{L' + 3a} = \frac{h_3^1 + e + a \tan 3}{L' + 3a} = \frac{372.61 + 3 + 34(.4855)}{700 + 102}$$

$$= \frac{375.61 + 16.5}{802} = \frac{392.11}{802} = .4895$$

$$\tan 5 = \frac{h_5^1}{L' + 4a} = \frac{h_4^1 + e + a \tan 4}{L' + 4a} = \frac{392.11 + 3 + 34(.4895)}{700 + 136}$$

$$= \frac{395.11 + 16.63}{836} = \frac{411.74}{836} = .493$$

$$\tan 6 = \frac{h_6^1}{L' + 5a} = \frac{h_5^1 + e + a \tan 5}{L' + 5a} = \frac{411.74 + 3 + 34(.493)}{700 + 170}$$

$$= \frac{414.74 + 16.75}{870} = \frac{431.49}{870} = .497$$

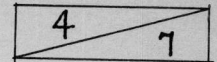
$$\tan 7 = \frac{h_7^1}{L' + 6a} = \frac{h_6^1 + e + a \tan 6}{L' + 6a} = \frac{431.49 + 3 + 34(.497)}{700 + 204}$$

$$= \frac{434.49 + 16.9}{904} = \frac{451.39}{904} = .5006$$

$$\tan 8 = \frac{h_8^1}{L' + 7a} = \frac{h_7^1 + e + a \tan 7}{L' + 7a} = \frac{451.49 + 3 + 34(.5006)}{700 + 238}$$

$$= \frac{454.49 + 17.02}{938} = \frac{471.51}{938} = .504$$





Again, considering the first step above the aisle as zero height.

Height of steps starting with the second step above the first aisle.	Difference in height of steps.
$e + a \tan 1 = 3 + 16.2 = 19.2$ inches.	----- .15"
$e + a \tan 2 = 3 + 16.35 = 19.35$ inches.	----- .15"
$e + a \tan 3 = 3 + 16.50 = 19.50$ inches.	----- .14"
$e + a \tan 4 = 3 + 16.64 = 19.64$ inches.	----- .13"
$e + a \tan 5 = 3 + 16.77 = 19.77$ inches.	----- .13"
$e + a \tan 6 = 3 + 16.9 = 19.9$ inches.	----- .12"
$e + a \tan 7 = 3 + 17.02 = 20.02$ inches.	----- .11"
$e + a \tan 8 = 3 + 17.1 = 20.13$ inches.	----- .93"

This gives the correct difference, to 3 places, of the heights of the steppings from the first to the second aisle. A total of approximately .93 inches in 8 steps.

Starting again above the second aisle: The first step taken as zero height.

Use  $h_1''$  for the steps above the second aisle.

$$h_1'' = h_8' + a \tan 8 + (e + a \tan 8) = 472 + 46(.504) + (3 + 34(.504))$$

$$= 472 + 23.15 + 20.1 = 515.25''$$

$$L'' = 74\frac{1}{2}' = 894'' \quad (\text{see graphic diagram sheet } )$$

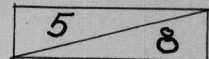
$$\tan 1 = \frac{h_1''}{L''} = \frac{515.25}{894} = .573$$

$$\tan 2 = \frac{h_2''}{L'' + a} = \frac{h_1'' + e + a \tan 1}{894 + 34} = \frac{515.25 + 3 + 34(.573)}{928} = \frac{518.25 + 19.65}{928}$$

$$= \frac{537.9}{928} = .580$$

$$\tan 3 = \frac{h_3''}{L'' + 2a} = \frac{h_2'' + e + a \tan 2}{894 + 68} = \frac{537.9 + 3 + 34(.580)}{962} = \frac{540.9 + 19.7}{962}$$

$$= \frac{560.6}{962} = .584$$



$$\tan 4 = \frac{h_4''}{L'' + 3a} = \frac{h_3'' + e + a \tan 3}{894 + 102} = \frac{560.6 + 3 + 34(.584)}{996} = \frac{563.6 + 19.84}{996}$$

$$= \frac{583.44}{996} = .586$$

$$\tan 5 = \frac{h_5''}{L'' + 4a} = \frac{h_4'' + e + a \tan 4}{894 + 136} = \frac{583.44 + 3 + 34(.586)}{1030} = \frac{586.44 + 19.9}{1030}$$

$$= \frac{606.34}{1030} = .589$$

$$\tan 6 = \frac{h_6''}{L'' + 5a} = \frac{h_5'' + e + a \tan 5}{894 + 170} = \frac{606.34 + 3 + 34(.589)}{1064} = \frac{609.34 + 20}{1064}$$

$$= \frac{629.34}{1064} = .59$$

$$\tan 7 = \frac{h_7''}{L'' + 6a} = \frac{h_6'' + e + a \tan 6}{894 + 204} = \frac{629.34 + 3 + 34(.59)}{1098} = \frac{632.34 + 20.10}{1098}$$

$$= \frac{652.44}{1098} = .595$$

Starting with the second step above the second aisle.

Height of steps above the second

Difference in heights  
of steps.

$$e + a \tan 1 = 3 + 19.65 = 22.65 \text{ inches.}$$

$$e + a \tan 2 = 3 + 19.75 = 22.75 \text{ inches.}$$

$$e + a \tan 3 = 3 + 19.84 = 22.84 \text{ inches.}$$

$$e + a \tan 4 = 3 + 19.93 = 22.93 \text{ inches.}$$

$$e + a \tan 5 = 3 + 20.1 = 23.1 \text{ inches.}$$

$$e + a \tan 6 = 3 + 20.17 = 23.17 \text{ inches.}$$

$$e + a \tan 7 = 3 + 20.221 = 23.221 \text{ inches.}$$

	.10"
	.09"
	.09"
	.08"
	.07"
	.05"
	.481"
	.481"

The height of the last step at the top of the balcony stepping  
= 23.22"

The first step after each aisle has been omitted in finding the difference in the heights of steppings in each case because of the difference in the value a (or width of aisle) for this stepping:

a' = 3' 10" = 46" Using the same angle as for the steps preceding the aisle, the height of the stepping following the first aisle

Height of step following the second aisle.

Using the same angle as for the step preceding the aisle.

$$= a' \tan 8 = 46(.504) = 23.15''$$

To find the distance of the curve, drawn tangent to the nosing of the steppings, above a straight line drawn from a point 2 ft. in front of the stage at the orchestra floor level and tangent to the nosing of the first step (see graphical diagram sheet ). All the increases in heights of steppings must be summed up.

Thus, starting at the toe of the balcony and the first step, if the additional height of the first step be denoted by (a), the additional height of the second above the first by (b), the third above the second by (c), etc. Then the total height of the last step above the previously mentioned tangent straight line will be  $a + (a + b) + (a + b + c) + \dots + (a + b + \dots + n)$ . Numerically: from my calculations.

inches

1st	step rise	= .16	= .16
2nd	step rise	= .16 + .16	= .32
3rd	step rise	= .32 + .19	= .51
4th	step rise	= .51 + .20	= .71
5th	step rise	= .71 + .20 (the same angle being used for the aisle step as for the preceding step).	= .91
6th	step rise	= .91 + .10 (change in height of step preceding and 2nd following the aisle).	= 1.01
7th	step rise	= 1.01 + .15	= 1.16
8th	step rise	= 1.16 + .15	= 1.31
9th	step rise	= 1.31 + .14	= 1.45
10th	step rise	= 1.44 + .13	= 1.58
11th	step rise	= 1.58 + .13	= 1.71
12th	step rise	= 1.71 + .12	= 1.83
13th	step rise	= 1.83 + .11	= 1.94
14th	step rise	= 1.94 + .11 (step following aisle).	= 2.05
15th	step rise	= 2.05 + 2.25 (change in height of step preceding and 2nd step following aisle).	= 4.30
16th	step rise	= 4.30 + .10	= 4.40
17th	step rise	= 4.40 + .09	= 4.49
18th	step rise	= 4.49 + .09	= 4.58
19th	step rise	= 4.58 + .08	= 4.66
20th	step rise	= 4.66 + .07	= 4.73
21st	step rise	= 4.73 + .05	= 4.78
			<u>48.59"</u>

12.66"

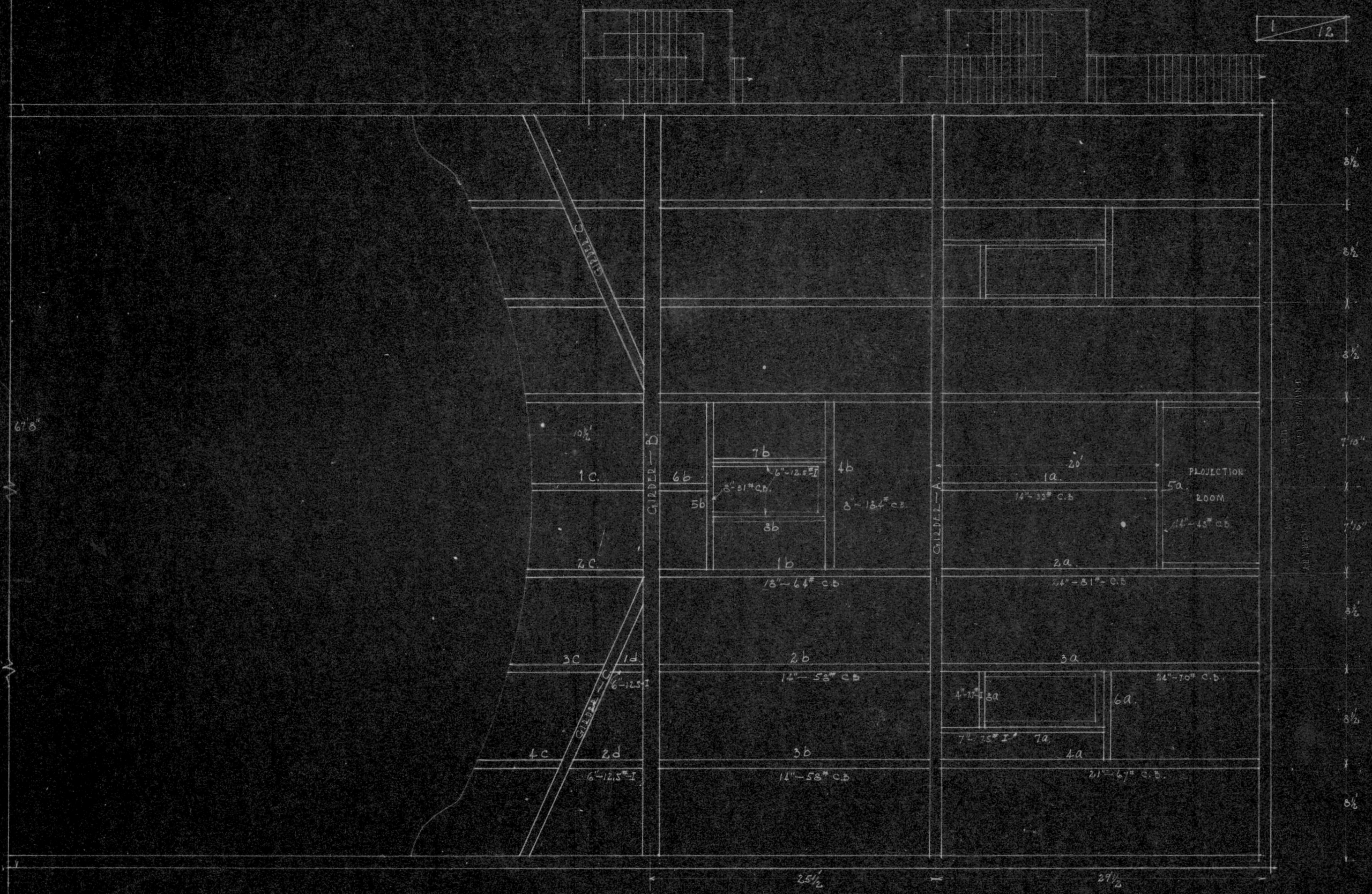
Thus, the rear of the balcony has increased in height  $48.59 = 4' 59''$  due to the isacoustic curvature. In this design 3" has been used for the rise of sight lines of one row of spectators above the other. This is less than the amount generally used in working a balcony problem on this basis, 6 inches being the usual difference used in heights of sight lines for large theaters. But in order that the rise of the rear of the balcony would not be so great as to make the height of this building out of proportion with its width, an amount of 3" has been used merely to show the calculations and procedure for a refinement of this type.

In this case I have found that at the Girder supported by the columns, (which comes under the 12th step), the isacoustic curve is 12.66 inches above the straight line drawn parallel to the first step nosing. This will be used later in the design of the step frame work.

All computations have been made with a slide rule, therefore the figures to several places of decimals may not be entirely accurate.

A plan of the balcony framing will now be laid out and the necessary beams, girders, and columns designed. The balcony floor slab will be assumed in ~~the~~ depth but will not be actually figured. The process of floor slab design will be taken up later in calculations for the Mezzanine floor.

In designing the balcony stringer beams, their lengths have been scaled from the Balcony Plan. That is, the lengths used in the design are the projections of the actual lengths on a horizontal plane. This has been compensated for by using a live load of  $125\# / \text{sq. ft.}$  of horizontal area. Otherwise a load of  $100\# / \text{sq. ft.}$  would have been used.



67' 8"

10 1/2'

1C.

2C.

3C

4C

GIRDER - D

6" - 12.5" I

6" - 12.5" I

7b

6b

5b

3b

1b

18" - 64" C.B.

2b

14" - 53" C.B.

3b

11" - 58" C.B.

6" - 12.5" I

3" - 31" C.B.

8" - 13.4" C.B.

4b

GIRDER - A

20'

1a.

14" - 33" C.B.

2a.

24" - 81" C.B.

3a

24" - 70" C.B.

4" - 11" I

7L 7.5" I

7a

4a

21" - 67" C.B.

6a.

PROJECTION ROOM

14" - 45" C.B.

PROJECTION ROOM

8 1/2'

8 1/2'

8 1/2'

7' 10"

7' 10"

8 1/2'

8 1/2'

8 1/2'

25 1/2'

29 1/2'

PLAN SECTION - BALCONY FRAMING

SCALE 1/8" = 1'

CCM.

CALCULATIONS FOR LOADS ON STRINGER BEAMS

Loading:

Live Load of Balcony 125#/ sq. ft. for small beams or stringers.

100#/ sq. ft. for large girders.

Balcony floor is to be of concrete slab construction. Assume the weight of the floor to be 50#/ sq. ft.

Weight per sq. ft. for stringer beams = 125 + 50 = 175#/ sq. ft.

The framing of the balcony is symmetrical, therefore only half the stringer beams need to be figured.

SIZES FOR STRINGERS

Beam 1a

$$\begin{aligned} \text{Load} &= \frac{15' 10''}{2} \times 20 \times 175\#/ \text{ sq. ft.} \\ &= 158.333 \times 175 \\ &= 27,750\# = \text{load on beam.} \end{aligned}$$

$$\text{Weight / ft.} = \frac{27750}{20} = 1387.5\#/ \text{ ft.}$$

From Carnegie tables - use 14" x 6 $\frac{3}{4}$ " - 33# C.B. Allowable load / ft.  
= 1430#/ ft.

From direct computations

$$\begin{aligned} \text{Max. } M &= \frac{WL}{8} = \frac{28410 \times 20}{8} = 71025\# \times 12 = 853,000\# \\ \frac{M}{S} &= \frac{I}{C} = \frac{853000}{18000} = 47.4\text{"}^3 \\ \text{Use } 14\text{"} \times 6\frac{3}{4}\text{"} - 33\# \text{ C.B. } I/Y &= 47.8\text{"}^3 \end{aligned}$$

Assuming the wt. on the Projection Room floor to be not greater than the live load on the balcony floor.

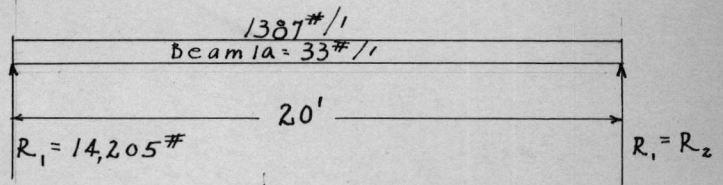
BEAM 5A

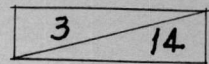
$$\text{Uniform load} = 175 \times 9\frac{1}{2} = 1660\#/ \text{ ft.}$$

$$\text{Concentrated load at center} = 14,205\#$$

$$\begin{aligned} \text{Max. } M &= \frac{1660 \times 15 \frac{5}{6} \times 15 \frac{5}{6}}{8} + 14,205 \times 15 \frac{5}{6} = 52100 + 56,400 \\ &= 108,500\# \end{aligned}$$

Use 16" - 45# C.B. I/Y = 73.8"³





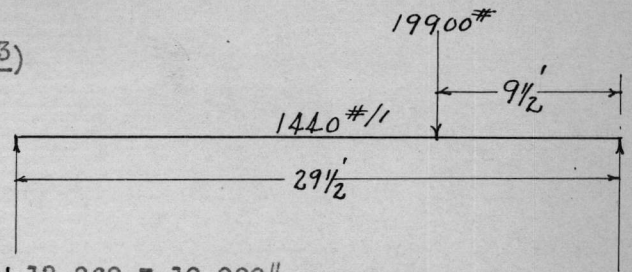
BEAM 2A

$$\text{Uniform load} = 175 \times \left( \frac{8.5}{2} + \frac{7.93}{2} \right)$$

$$= 175 \times 8.21 = 1,440 \#/\text{'}$$

Concentrated load at center

$$= \frac{14205}{2} + \frac{1660 \times 15\frac{1}{2}}{2} = 7100 + 12,860 = 19,900 \# \text{ approx.}$$



$$\sum M_{R_2} = 0$$

$$R_1 = \frac{1440 \times 29\frac{1}{2} \times 14\frac{3}{4} + 19900 \times 9\frac{1}{2}}{29\frac{1}{2}} = \frac{627500 + 189000}{29.5} = \frac{816500}{29.5} = 27,650 \#$$

$$\sum M_{R_1} = 0$$

$$R_2 = \frac{1440 \times 29\frac{1}{2} \times 14\frac{3}{4} + 19900 \times 20}{29.5} = \frac{627500 + 398000}{29.5} = \frac{1025500}{29.5} = 34750 \#$$

Max. M. occurs at  $X = \frac{27650}{1440} = 19.2'$  from left reaction.

$$M. = 27650 \times 19.2 - 1440 \times \frac{19.2^2}{2} = 531,000 - 265,000 = 266,000 \#'$$

$$12 \times \frac{266,000}{18000} = I/Y = 177.5 \text{''}^3$$

Use 24" - 81# C.B.  $I/Y = 190.1 \text{''}^3$

Checking- by taking the wt. of the beam into account.

$$\text{Uniform load} = 1440 + 81 = 1521 \#/\text{'}$$

$$R_1 = 27650 + \frac{81 \times 29.5}{2} = 27650 + 1195 = 28,845 \#$$

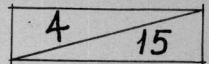
$$\text{Max. M. occurs at } X = \frac{28845}{1521} = 18.95'$$

$$M. = 28,845 \times 18.95 - 1521 \times \frac{18.95^2}{2} = 547,500 - 274,000 = 273,500 \#'$$

$$\frac{273,500 \times 12}{18000} = I/Y = 182.0 \text{''}^3$$

$\therefore$  24" - 81# C.B. is OK





BEAM 8A

Assuming 8A to support 2' of floor on each side of its center.

$$W = 4 \times 175 \times 6.5 = 4550\# \quad w = 700\#/'$$

$$M = \frac{WL}{8} = \frac{4550 \times 6.5 \times 12}{8} = 44,200\#"#$$

$$\frac{44200}{18000} = I/Y = 2.46$$

Use 4" - 7.7# I  $I/Y = 3.0$

BEAM 7A

Assume 7A to support approx. a 3 ft. width of floor slab.

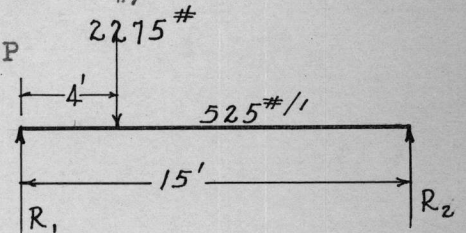
$$\text{Uniform load } W = 3 \times 175 \times 15' = 7850\# \quad w = 525\#/'$$

$$\text{Concentrated load} = \text{Reaction of 8A} = 2275\# = P$$

$$\sum M_{R_2} = 0$$

$$R_1 = \frac{2275 \times 11 + 7850 \times 7.5}{15} = \frac{25000 + 58900}{15}$$

$$= \frac{83,900\#}{15} = 5580\# \quad R_2 = 4,570\#$$



$$\text{Maximum } M. \text{ occurs at } X = \frac{5580 - 2275 - 525 \times 4}{525} = \frac{1215}{525} = 2.32$$

$$M. = 4920 \times 6.32 - 2275 \times 2.32 - 525 \times \frac{6.32^2}{2} = 15,475 \times 12 = 185,500\#"#$$

$$\frac{185500}{18000} = I/Y = 10.3\#"^3$$

Use 7" - 17.5# I - beam  $I/Y = 11.1\#"^3$

BEAM 6A

Assume a uniform load of 3 ft. of floor slab & floor load.

$$= 3 \times 175 = 525\#/'$$

$$\text{Supports 2 concentrated loads} = P_1 = 4,570\# \quad P_2 = 1950\# \text{ (Assumed)}$$

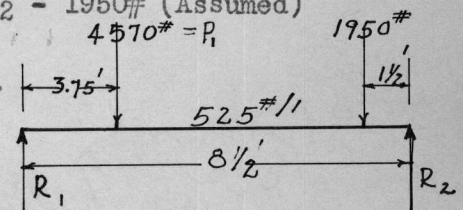
$$R_1 = \frac{4570 \times 4.75 + 1950 \times 1.5 + (525 \times 8.5) \times \frac{8.5}{2}}{8.5}$$

$$= \frac{43575}{8.5} = 5,140\# \quad R_2 = 5840\#$$

$$\text{Max. } M. \text{ occurs at } P_1. \quad M = 5140 \times 3.75 - 525 \times \frac{3.75^2}{2}$$

$$= 19280 - 3620 = 15,660\# \times 12 = 188,000\#"# \quad \frac{188000}{18000} = I/Y = 10.44\#"^3$$

Use 7" - 17.5# I - beam  $I/Y = 11.1\#"^3$



BEAM 3A

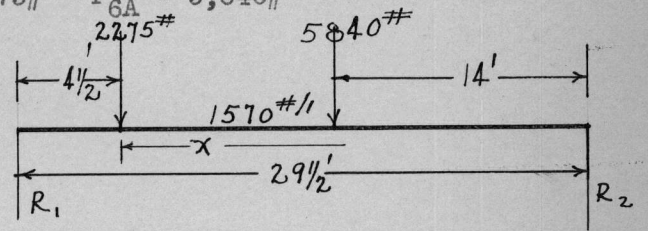
3A will be designed to take the uniform load - the entire length of the beam, also the 2 concentrated loads from 8A and 6A.

Uniform load =  $175 \times 8.5 = 1490\#/'$  +  $80\#/'$  (assumed wt. of beam)  
 =  $1570\#$

Concentrated loads  $P_{8A} = \frac{4550}{2} = 2275\#$      $P_{6A} = 5,840\#$

$\sum M_{R_2} = 0$

$$R_1 = \frac{2275 \times 25 + 5840 \times 14 + 1570 \times \frac{29.5^2}{2}}{29.5} = 28,000\#$$



$$R_2 = \frac{1570 \times 29.5 \times \frac{29.5}{2} + 5840 \times 15.5 + 2275 \times 4 \frac{1}{2}}{29.5} = \frac{684000 + 90500 + 10230}{29.5} = 26,550\#$$

Max. M. occurs at  $X = \frac{28,000 - 2275 - 1570 \times 4.5}{1570} = \frac{18665}{1570} = 11.9$

Max. M. occurs underload  $P_{6A} = 5840\#$

$$M = 26,550 \times 14 - 1570 \times \frac{14^2}{2} = 372000 - 154000 = 218000'\#$$

$$218,000'\# \times 12 = 2,620,000''\#$$

$$\frac{M}{S} = \frac{I}{Y} \quad \frac{2620000}{18000} = 145.5''^3$$

Use 24" - 70# C. B.  $I/Y = 161.6''^3$

BEAM 4A

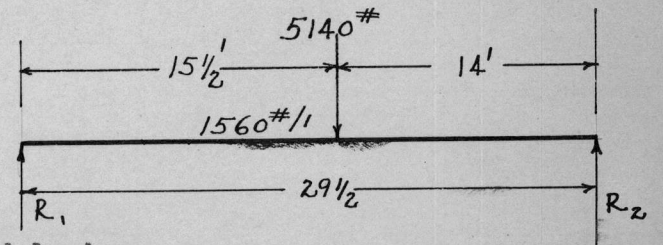
Uniform load taken as  $175 \times 8.5 = 1490\#/'$  over entire beam.

1 concentrated load  $P_{6A} = 5140\#$

Assumed wt. of beam =  $70\#/'$

Total w =  $1560\#/'$

$$R_1 = \frac{1560 \times \frac{29.5^2}{2} + 5140 \times 14}{29.5} = \frac{751900}{29.5} = 25,500\#$$



$R_2 = 25,640\#$

Max. M. occurs under the concentrated load.

$$M = 25,500 \times 15 \frac{1}{2} - 1560 \times \frac{15.5^2}{2} = 395,500 - 187500 = 208,000'\#$$

$$= 2,500,000''\#$$

$$\frac{I}{Y} = \frac{2,500,000}{18000} = 139''^3$$

Use 21" - 67# C.B.  $I/Y = 142''^3$

BEAM 3B

Carries a uniform load =  $175 \times 8.5 = 1490\#/'$

Assume the wt. of the beam =  $65\#/'$

Total load  $W = 1555\#/'$  x  $25\frac{1}{2} = 39600\#$

Use 14" - 58# C. B. Allowable load = 40.1 kips.

BEAM 2B

Use same size as 3B.

BEAM 7B = 8B

Assume a uniform load = to the weight of 2' of floor load on each side of the center.

$w = 4 \times 175 = 700\#/'$  of beam  $W = 700 \times 11' = 7700\#$

$M. = \frac{WL}{8} = \frac{7700 \times 11}{8} = 10,587\ \#'$

Use 6" - 12.5# I - Beam (American S.)

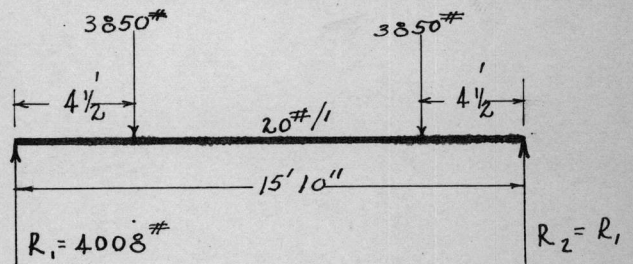
BEAM 4B

Assume 4B to carry only the concentrated loads imposed by 7B and 8B.

Assume wt. of beam to be  $20\#/'$

$M. = 4008 \times 8 - 3850 \times 3\frac{1}{2} - 20 \times \frac{8^2}{2}$   
 $= 32064 - 13500 - 640 = 17,924\ \#'$

Use 8" - 18.4# I - Beam (A. S.)



BEAM 5B

Assume 5B to support 4' of floor space plus the concentrated loads of 7B and 8B.

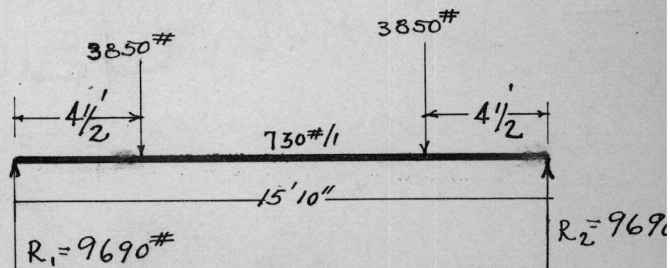
Uniform load =  $4 \times 175 = 700\#/'$  =  $w$   $W = 700 \times 15\frac{10}{12} = 11083\#$

$P_{7B} = 3850\# = P_{8B}$

Assume wt. of beam =  $30\#/'$

$M. = 9690 \times 8 - 3850 \times 3.5 - 730 \times \frac{8^2}{2}$   
 $= 40,600\ \#'$

Use 8" - 31# C.B.



BEAM 1B

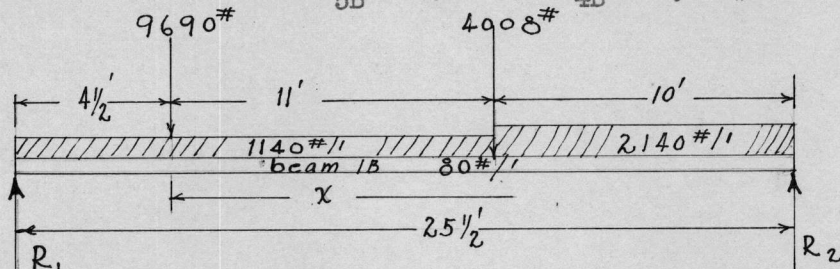
Assume uniform load 1st 10' from the right end of the beam

$$= 175 \times \left( \frac{8.5}{2} + \frac{15 \times 10''}{2} \right) = 175 \times 12.25 = 2140 \#/'$$

Assume for the remaining 15 1/2' uniform load of

$$175 \times \left( \frac{8.5}{2} + \frac{4 \frac{1}{2}}{2} \right) = 175 \times 6.5 = 1140 \#/'$$

Concentrated loads =  $P_{5B} = 9,690 \#$ ;  $P_{4B} = 4,008 \#$



$$\sum M_{R_2} = 0$$

$$25 \frac{1}{2} R_1 = 9690 \times 21 + 4008 \times 10 + 1140 \times 15 \frac{1}{2} \times 17 \frac{3}{4} + 2140 \times 10 \times 5 + 80 \times \frac{25 \frac{1}{2}^2}{2}$$

$$R_1 = \frac{203500 + 40080 + 324000 + 107000 + 26000}{25.5}$$

$$R_1 = \frac{700580}{25.5} = 27,450 \# \quad R_2 = 54840 - 27,450 = 27,390 \#$$

$$\text{Max. M. occurs at } X = \frac{27450 - 9690 - 1140 \times 4 \frac{1}{2} - 80 \times 4 \frac{1}{2}}{1220}$$

$$= \frac{27450 - 19770}{1220} = \frac{7680}{1220} = 6.3' \quad 6.3 \quad 4 \frac{1}{2} = 10.8' \text{ from left end of beam.}$$

$$\text{Max. M.} = 27450 \times 10.8 - 9690 \times 6.3 - 1140 \times \frac{10.8^2}{2} - 80 \times \frac{10.8^2}{2}$$

$$= 296500 - 132.160 = 164,340 \#'$$

Use 18" - 64# C. B.

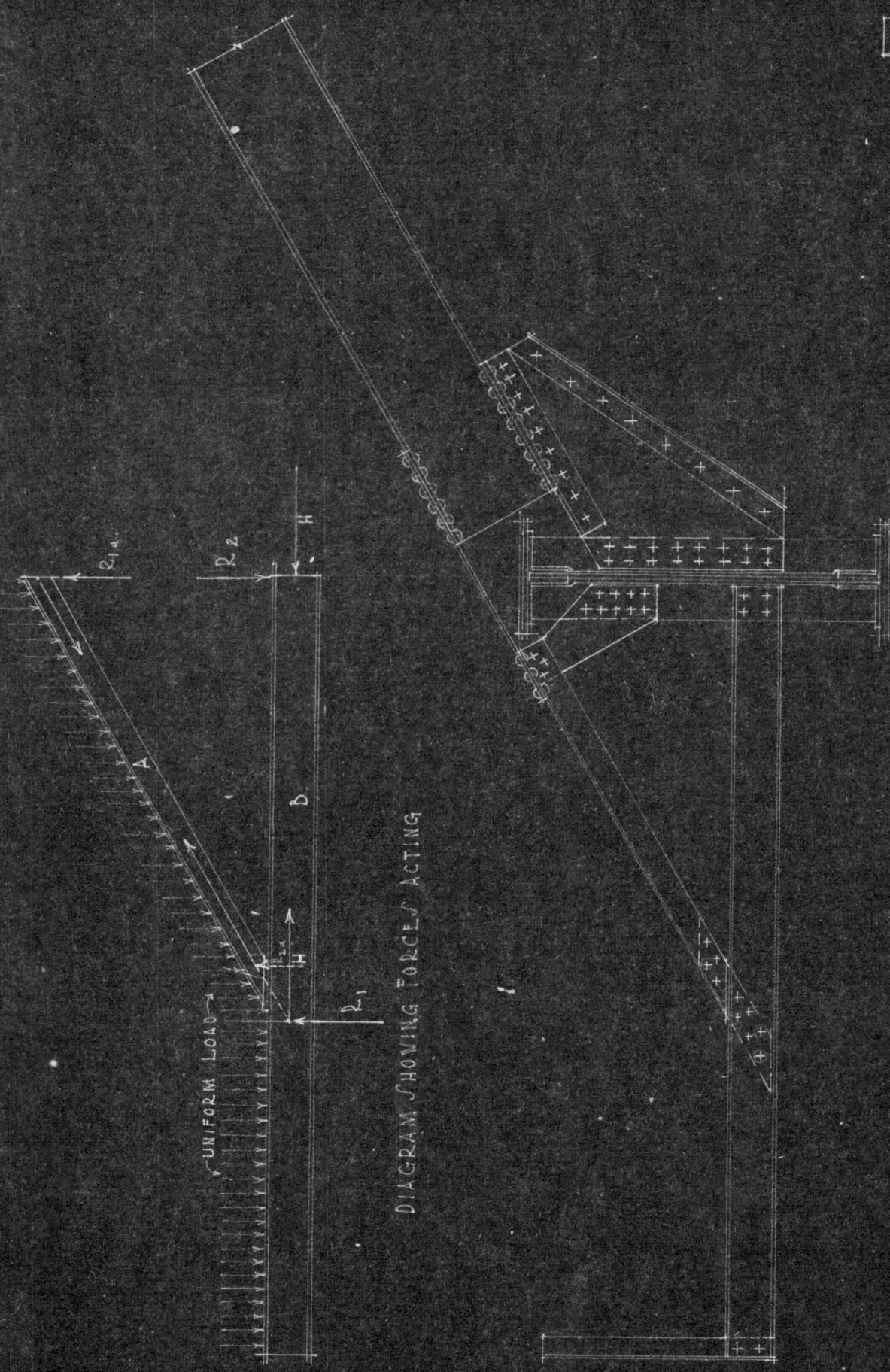


DIAGRAM SHOWING FORCES ACTING

DIAGRAM — SHOWING BALCONY CANTILEVER & CONNECTION.

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CANTILEVER BEAMS

CANTILEVERS 1C and 2C

Length of beam A =  $5.5 / \cos 30^\circ = \frac{5.5}{.866} = 6\frac{1}{2}'$

Uniform load on A =  $175 \times 6.5 \times \frac{15'10''}{2} = 9100\#$

Uniform load on B =  $175 \times 4\frac{1}{2} \times$

$\frac{15'10''}{2} = 6300\#$

$R_1 = \frac{6300 \times 7.75}{5.5} = 8900\#$

$R_2 = \frac{6300 \times 2.25}{5.5} = 1420\#$

The horizontal component of the

force of Beam A =  $H = R_1 \times \cot 30^\circ$

=  $8900 \times 1.732 = 15,400\#$

Max. M. in B at  $R_1$

=  $6300 \times \frac{4.5}{2} = 14,200 \times 12 = 170,000\#$

Try 2 - 7" - 9.8# channels

Moment caused by the weight of the channels

=  $19.6 \times \frac{4.5^2}{2} = 198 \# = 2380\#$

Total M. =  $172,380\#$

Allowable working stress  $S. = 18,000\# / \text{sq.in.}$

$S = \frac{MY}{I} + \frac{P}{A}$

In using 2 channels each channel is considered to take half the stress. Therefore in finding the stress S. only the elements of one channel will be used with half the maximum bending moment and half the thrust H.

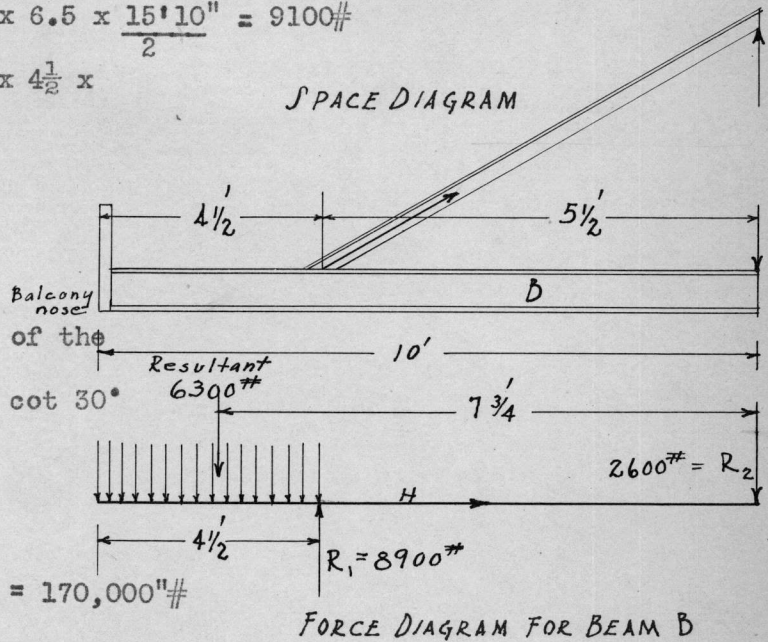
$S = \frac{86,190 \times 3.5}{21.1} + \frac{7700}{2.85} = 14350 + 2700 = 17,050\# / \text{sq. in.}$

Which is under the allowable stress and the 2 - 7" - 9.8# channels will be considered as satisfactory.

For Beam A in the diagram

Direct stress in A =  $R_1 / \sin 30^\circ = \frac{8900}{.5} = 17,800\#$

Max. M. of A =  $\frac{9100 \times 6.5}{8} = 7400 \times 12 = 88,900\#$



BEAM 3C CANTILEVER

Length of beam A =  $4.5 / \cos 30^\circ = \frac{4.5}{.866} = 5.2'$

Uniform load on A =  $5.2 \times 8.5 \times 175 = 7750\#$

Uniform load on B =  $175 \times 4\frac{1}{2} \times 8.5 = 6800\#$

$R_1 = \frac{6800 \times 6.75}{4.5} = 10,200\#$

$R_2 = \frac{6800 \times 2.25}{4.5} = 3400\#$

Thrust H =  $R_1 \times \cot 30^\circ = 10,200 \times 1.732 = 17,700\#$

Max. M. of B =  $6800 \times 2.25 = 15,300'\#$

=  $184,000''\#$

Try 2 - 7" - 9.8# channels

$S = \frac{MY}{I} + \frac{P}{A}$

Allowable stress = 18000#/sq. in.

Each channel takes half the total stress.

Therefore using half the moment and thrust for one channel

$S = \frac{92000 \times 3.5}{21.1} + \frac{8850}{2.85} = 15,250 + 3,100 = 18,300 \# / \text{sq. in.}$

This value comes very near to the allowable stress, and is therefore considered satisfactory.

CALCULATIONS FOR A

Direct pull on A =  $R_1 / \sin 30^\circ = \frac{10200}{.5} = 20,400\#$

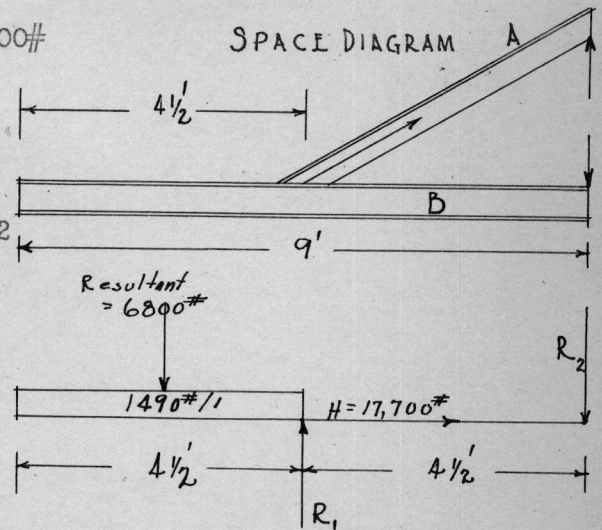
Max. M. of A =  $\frac{7750 \times 5.2}{8} = 5040'\# = 60500''\#$

Try 2-4" x 3" x 5/16" angles

Using one angle and half the Max. Moment and load to figure the stress.

$S = \frac{MY}{I} + \frac{P}{A} = \frac{30250 \times 1.26}{3.4} + \frac{10200}{2.09} = 11200 + 4880$

=  $16,080\# / \text{sq. in.}$  which is under the allowable and therefore satisfactory.



CANTILEVER 4C

The uniform load on A & B = 1490#/ft

$$\text{Length of A} = 3.5 \frac{1}{2} \cos 30^\circ = \frac{3.5}{.866} = 4.04'$$

$$R_1 \text{ of B} = \frac{6800 \times 5.75}{3.5} = 11,170\#$$

$$R_2 \text{ of B} = \frac{6800 \times 2.25}{3.5} = 4,370\#$$

The Thrust H =  $R_1 / \tan 30^\circ$

$$= \frac{11,150}{.5773} = 19,300\#$$

= Direct stress on B.

Max. M. of B = 6800 x 2.25

$$= 15,300\# = 184,000\#"#$$

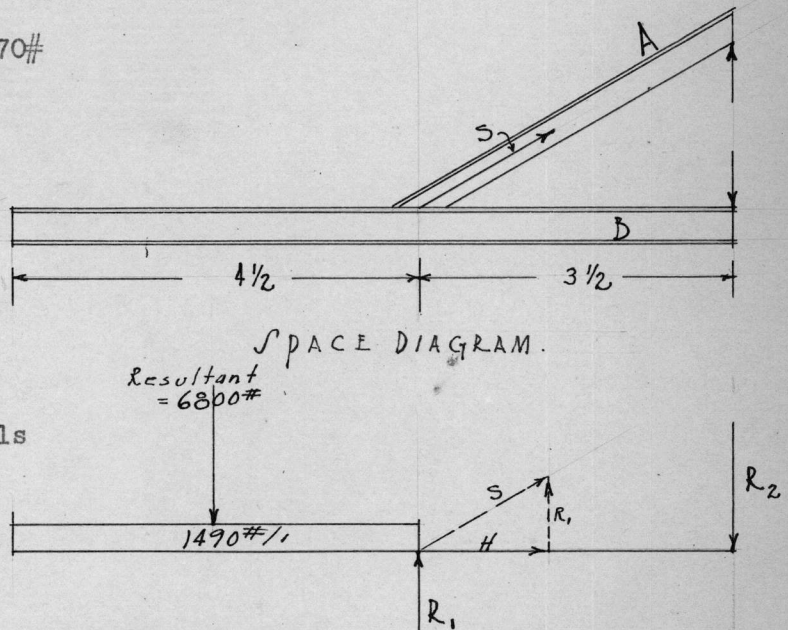
Try 2 - 7" - 12.5# channels

M. for wt. of the channels

$$\text{at } R = 25 \times \frac{4.5^2}{2} = 253\#$$

$$= 3040\#"#$$

Total M. = 187,440#"#



Allowable stress = 18,000#/sq. in. = S

$$S = \frac{MY}{I} + \frac{P}{A} \quad \text{Taking half the Max. Moment and half the load P and}$$

using the values of 1 channel

$$S = \frac{93720 \times 3.5}{24.1} + \frac{96500}{3.58} = 13600 + 2700 = 16,300\# / \text{sq. in.}$$

Which is below the allowable stress & is therefore satisfactory.

For Beam A in the diagram.

$$\text{Direct pull on A} = R_1 / \sin 30^\circ = \frac{11,150}{.5} = 22,300\#$$

$$\text{Max. M. of A} = \frac{1490 \times 4.04^2}{8} = 3040\# \times 12 = 36,500\#"#$$

Try 2 - 4" x 3" x 1/4" angles.

Using the elements of 1 angle and half the calculated Max. Moment and load.

$$S = \frac{MY}{I} + \frac{P}{A} = \frac{18,250 \times 1.26}{2.8} + \frac{11,150}{1.69} = 8,240 + 6,600 = 14,840\# / \text{sq. in.}$$

This stress is much below the allowable but the angles will be considered OK in order to have the same size member as in 3C.



Try 2 - 4" x 3" x 3/8" angles

Using the elements of one angle and half the Max. M. and direct stress to figure the stress / sq. in.

$$S = \frac{MY}{I} + \frac{P}{A}$$

$$S = \frac{44450 \times 1.26}{4} + \frac{8900}{2.48} = 14000 + 3580 = 17,580\# / \text{sq. in.}$$

This is below the allowable stress and a more economic section could be picked, but because of the fact that I am trying to pick angles with the same length legs for each cantilever construction, some of the sections chosen may be in excess of the required size. Thus the 2 - 4" x 3" x 3/8" as chosen are preferable.

These angles for each cantilever are marked A in the figures. Their weight has not been considered in computing the bending moment as it would be practically negligible.

BEAMS 1D

$$L = 3' = 36"$$

$$\text{Load} = 175 \times 3 \times 8.5 = 4460\#$$

Use a 6" - 12.5# A. S. I - beam

BEAMS 2D

$$L = 7' \quad \text{Load} = 175 \times 7 \times 8.5 = 10,400\#$$

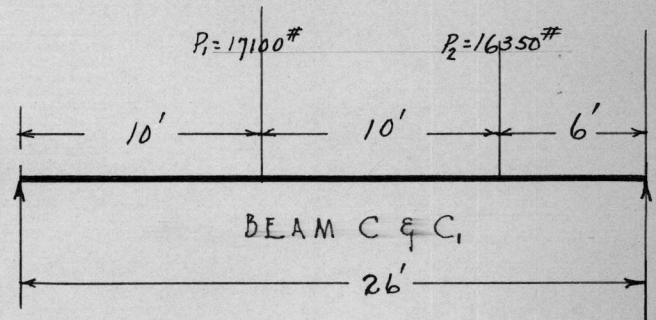
Use 6" - 12.5# A. S. I - beam.

BEAMS C & C carry 2 concentrated loads only.

The loads will be figured from the plan:- lengths of beams and loads taken to cover an area which is the horizontal projection of the balcony slope.

$$P_1 = 175(8' + 7/2') \times 8.5' = 17,100\#$$

$$P_2 = 175(9.5 + 3/2) \times 8.5 = 16,350\#$$



$$R_1 = \frac{17,100 \times 16 + 16,350 \times 6}{26} = \frac{273,500 + 98,000}{26} = 14,280\#$$

$$R_2 = (17,100 + 16,350) - 14,280 = 19,170\#$$

Assume the wt. of the beam to be 60#/sq. ft.

$$\text{Then } R_1 = 14,280 + \frac{60 \times 26}{2} = 15,060\#$$

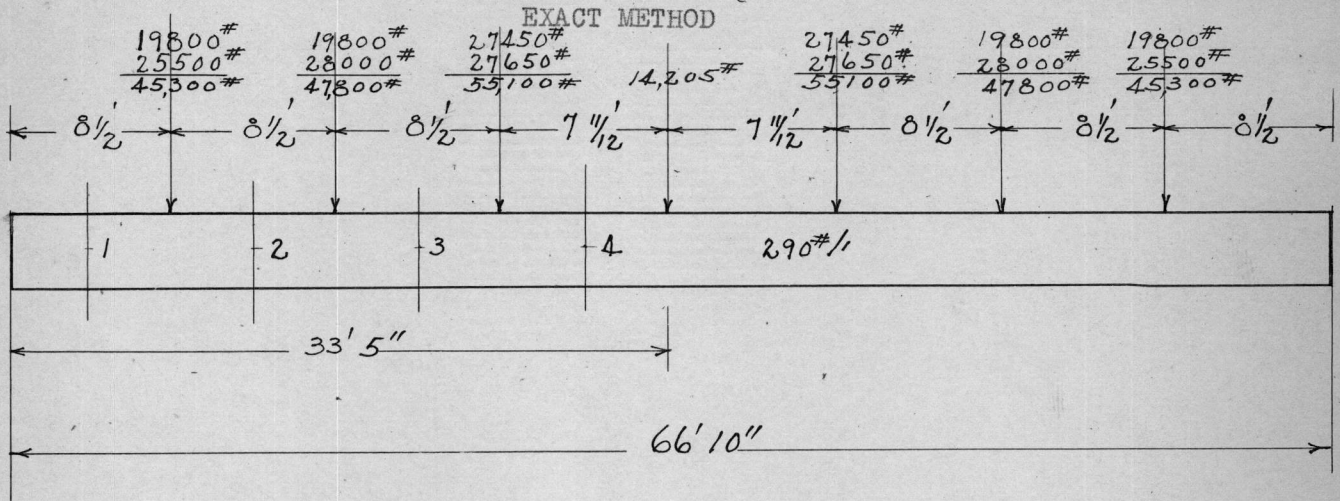
$$R_2 = 19,170 + 780 = 19,950\#$$

Max. M occurs under  $P_1$

$$M = 15,060 \times 10 - 60 \times \frac{10^2}{2} = 150,600 - 3000 = 153,600'\#$$

For Beam C use an 18" - 57# CB Allowable M = 156,450'\#

GIRDER A - COMPUTATIONS FOR DESIGN



$$\begin{aligned} \text{Max. } M &= 164,987 \times 33 \frac{5}{12} - 45,300 \times 24 \frac{11}{12} - 47,800 \times 16 \frac{5}{12} \\ &\quad - 55,100 \times 7 \frac{11}{12} - 290 \times \frac{33 \frac{5}{12}^2}{2} \\ &= 5,510,000 - 1,131,000 - 789,000 - 440,000 - 97,300 = \\ &= 3,052,000 \text{ '#} \quad = 36,640,000 \text{ ''#} \end{aligned}$$

Max. Shear = 164,987# Allowable shearing stress = 12,000 #/sq. ft.

$$\frac{164,987}{12,000} = 13.70 \text{ sq. in. necessary to resist shear.}$$

The web shall be designed to take the shear.

Choosing a thickness of web = .375 Depth of girder =  $\frac{13.7}{.375} = 36.6''$

Depth shall not be less than 1/15 of span =  $\frac{1}{15} \times 802 = 53.5''$

Thickness shall not be less than 1/160 of unsupported distance between flanges.

Use a 54" depth of web plate.

$$\frac{1}{160} \times 38.5 \text{ (assuming } 8'' \times 6'' \text{ angles to be used)} = .24''$$

Use  $\frac{3}{8}'' = .375''$  thickness

For Flange Areas:

$$M = \frac{SI}{y} \quad y = \frac{54}{2} \quad 1/4 = 27.25" \quad S = 18,000 \#/\text{sq.in.}$$

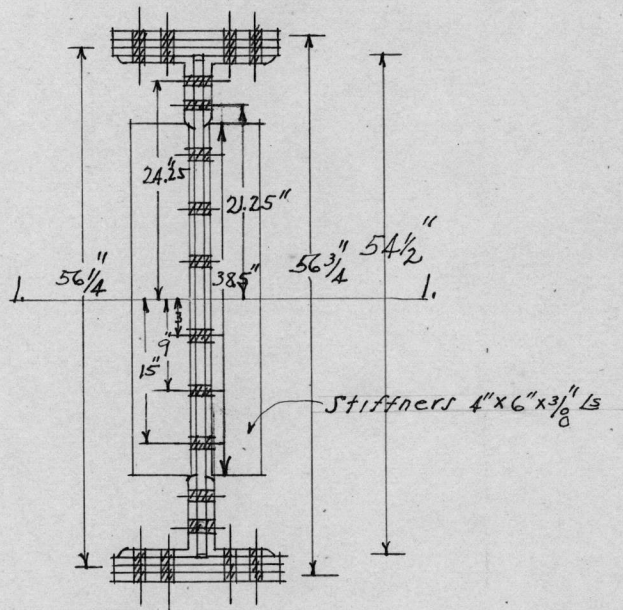
$$I = \frac{36,640,000 \times 27.25}{18,000} = 55,500 \text{"}^4 \quad \text{necessary I}$$

$$I_{1-1} \text{ of 4 - 8" x 6" x } \frac{3}{4} \text{ angles} = 26,364 \text{"}^4$$

$$I_{1-1} \text{ of 54" x } \frac{3}{8} \text{ Web plate} = 1312.2 \times \frac{3}{8} \text{"} = 4,920.8 \text{"}^4$$

$$I_{1-1} \text{ of 4 - 5/8" x 14 5/8" plates \& 2 - 1/2" x 14 3/8" plates} = 1540 \times 14 \frac{3}{8} + 1229.2 \times 14 \frac{3}{8} = 39,750 \text{"}^4$$

$$71,034.8 \text{"}^4$$



Deductions for rivet holes in angles and plates:

3/4" rivets to be used

$$2 \text{ holes} - (.05583 \times 1 \frac{7}{8}) 2 + 2 \left( \frac{105}{64} \times \frac{21.25^2}{64} \right) = 1,480.21$$

$$2 \text{ holes} - (.05583 \times 1 \frac{1}{8}) 2 + 2 \left( \frac{105}{64} \times \frac{24.25^2}{64} \right) = 1,930.21$$

$$8 \text{ holes} - (.94922 \times \frac{7}{8}) 8 + 8 \left( \frac{126}{64} \times \frac{28.125^2}{64} \right) = 12,456.65$$

$$15,867.07 \text{"}^4$$

$$\text{Net I} = 71034.8 - 15867.07 = 55,167.73 \text{"}^4$$

Reductions for Rivet Holes in stiffeners:

Assume rivets in stiffeners at center of Girder to be max. spaced.

$$\frac{38.5}{6} = 6 \text{ rivets necessary in the stiffener.}$$

$$I_{1-1} \text{ 2 holes} - 2(.05583 \times 1 \frac{1}{8}) + 2\left(\frac{63}{64} \times 3^2\right) = 18.00$$

$$I_{1-1} \text{ 2 holes} - 2(.05583 \times 1 \frac{1}{8}) + 2\left(\frac{63}{64} \times 9^2\right) = 161.00$$

$$I_{1-1} \text{ 2 holes} - 2\left(.063 + 2\left(\frac{63}{64} \times 15^2\right)\right) = \frac{448.00}{727.00''^4}$$

$$55,167.73 - 727.00 = 54,440.73 = I \text{ of net section at center of Girder.}$$

The extreme fiber stress due to bending:

$$= S = \frac{My}{I} = \frac{36,640,000 \times 28.75}{54,400.73} = 19,300 \#/\text{sq. in. which is above}$$

the 18000#/sq. in. allowable stress, but because of using 100#/' instead of 125#/' (see page 6 ) we will consider the girder o.k. because 4/5 of 19300 = 15,500#/sq. in. which is safe enough.

$$\text{Weight of Girder: 4 - 8 x 6 x 3/4 angles} = 135.2 \#/'$$

$$\text{Web and 6 plates} = \underline{154 \#/'}$$

$$289.2 \#/' = \text{Wt. of Girder.}$$

#### CALCULATIONS FOR LENGTH OF FLANGE PLATES

Moment of inertia about the neutral axis with the outside plate removed top and bottom.

$$I_{1-1} \text{ of 4 - 8" x 6" x 3/4" angles} = 26364.$$

$$I_{1-1} \text{ of 54" x 3/8" web plate} = 4920.8$$

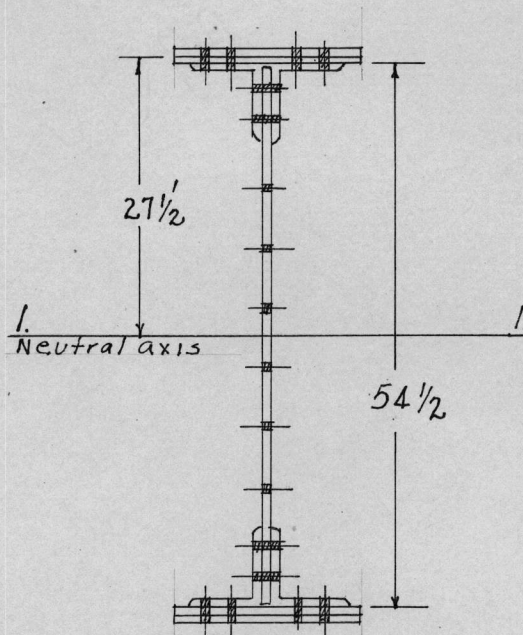
$$I_{1-1} \text{ of 4 - 5/8" x 14 3/8" Plates} = (1540.3 + 402.6)14 \frac{1}{2} = \underline{28200.}$$

$$59,484.8''^4$$

Reductions for Rivet Holes:

$$I_{1-1} \text{ for rivet holes of stiffener} = 727.00''^4 \text{ from top of page 6.}$$

$$\begin{aligned}
 I_{1-1} \text{ for rivet holes in angle legs} &= 3410.42 \\
 I_{1-1} \text{ " " " in flange plates} & \\
 = 8 (.6667 \times 7/8) + (1 \ 3/4 \times 27.5) &= \underline{10604.67} \\
 &14742.1''^4 \\
 59484.8 - 14742.1 &= 44,742.7''^4 = I \text{ for net section}
 \end{aligned}$$



The bending moment that the girder will resist with these two plates off =  $M = 18,000 \times 44,742.7 = 31,100,000''\#$

The bending moment will be figured half way between each concentrated load starting at the left end.

$$M_1 = 164,987 \times \frac{8.5}{2} - 150 \times \frac{4.25^2}{2} = 700,650''\# = 8,410,000''\#$$

$$\begin{aligned}
 M_2 &= 164,987 \times 12.75 - 45300 \times 4.25 - 150 \times \frac{12.75^2}{2} = 1,894,800''\# \\
 &= 22,700,000''\#
 \end{aligned}$$

$$\begin{aligned}
 M_3 &= 164,987 \times 21.25 - 45300 \times 12.75 - 47800 \times 4.25 - 150 \times \frac{21.25^2}{2} \\
 &= 2,795,500''\# = 33,550,000''\#
 \end{aligned}$$

$$\begin{aligned}
 M_4 &= 164,987 \times 29.5 - 45300 \times 21 - 47800 \times 12.5 - 55100 \times 4 - 150 \times \frac{29.5^2}{2} \\
 &= 3,035,500''\# = 36,400,000''\#
 \end{aligned}$$

Moment of Inertia about the neutral axis - 2nd plate top and bottom removed.

I <sub>1-1</sub> of 4 angles	= 26364.
I <sub>1-1</sub> of Web plate	= 4920.8
I <sub>1-1</sub> of 2 remaining flange plates	= <u>1375.</u>
	32,659.8" <sup>4</sup>

Deductions for rivet holes:

I <sub>1-1</sub> for rivet holes of stiffeners	= 727.00
I <sub>1-1</sub> " " " of angle legs	= 3410.42
I <sub>1-1</sub> " " " of flange plates	
= 8 (.21663 x 7/8) + (77 x $\frac{2}{27.19}$ )	= <u>7130.00</u>
	11,267.42" <sup>4</sup>

32659.8 - 11,267.42 = 21,392.38"<sup>4</sup> I of net section.

The bending moment which the girder will resist with the two outer plates off of top and bottom of girder

= M =  $\frac{18,000 \times 21,392.38}{27.875} = 13,800,000$ "#

The outer plates shall be cut off 14' from the ends of the girder. The second plates shall be cut off 5' from the ends. The third plates shall extend the entire length.

SPACING OF STIFFENERS:

$$S = 85t \sqrt{\frac{18,000 (A/V) - 1}{164,987}} = 85 \times .375 \sqrt{\frac{18,000 \times 20.25 - 1}{164,987}}$$

$$= 31.85 \times 1.205 = 38.4"$$

Stiffeners shall be placed under each concentrated load.

V at first concentrated load = 164,987 - 45,300 - 290 x 8.5 = 117,227"#

$$S = 85t \sqrt{\frac{18,000 \times 20.25 - 1}{177,227}} = 31.85 \times 2.1 = 67"$$

Between support and 1st concentrated load 2 stiffeners will be used.

Between loads 1 stiffener is sufficient.

Sizes of stiffeners under loads =  $\frac{55,100}{15,000}$  (largest load) = 3.7 sq. in.  
 (considered as short column) Use 2 - 3 1/2" x 3 1/2" x 3/8" angles on each side of web.

Number of rivets necessary in stiffener angles under loads:

Safe stress of 3/4" rivet = 6750# (bearing of 3/8" web governs)

$$\frac{45300}{6750} = 7 \text{ rivets}$$

$$\frac{47800}{6750} = 8 \text{ rivets}$$

$$\frac{55100}{6750} = 9 \text{ rivets}$$

$$\frac{14205}{6750} = 3 \text{ rivets}$$

#### END STIFFENERS

Rivets in End Stiffeners must resist 164,987#

$$\frac{164,987}{6750} = 25 - 3/4" \text{ rivets necessary.}$$

Rivets - Number and spacing to fasten flange angles to web.

The longitudinal shear at supports must be figured

Plates and angles only considered.

$M_s$  figured at K, with 2 outer plates removed (top and bottom)

$$M_s \text{ of 2 angles} = 19.88 \times 25.69" = 511.0$$

$$M_s \text{ of plate} = 14.5 \times 5/8 \times 27.562 = \underline{249.5}$$

$$\text{TOTAL STATICAL M} \quad 760.5$$

$$\text{Longitudinal shear} = \frac{VM_s}{It}$$

$$= \frac{164,987 \times 760}{21392.38 \times .375} = 15600\#/\text{sq. in.} \times 3/8 = 5850\#/"$$

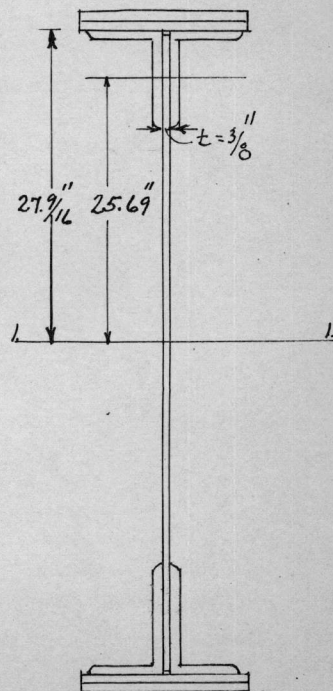
Rivet spacing between support and first

stiffener:

$$= \frac{5850 \times 4 \times 1/4 \times 12}{6750} = 45 - 3/4" \text{ rivets}$$

necessary, placed in 2 rows

$$\frac{4.25 \times 12}{2.3} = 2.22" \text{ spacing.}$$





$M_s$  with 1st outer plate removed:

$$M_s \text{ of 1st plate} = 14.5 \times 5/8 \times 28.187 = 249.5$$

$$M_s \text{ of other plates and angles} = \underline{760.5}$$

1010.0 Statical M.

Longitudinal shear at first stiffener, 4.25' from support

$$= \frac{164,987 - 290 \times 4.25 \times 1010}{44742.7} = \frac{163,757 \times 1010}{44742.7} = 3690 \#/'$$

Number of Rivets to center of distance between next 2 concentrated loads

$$= \frac{3690 \times 8.5 \times 12}{6750} = 56 \text{ rivets in two rows.}$$

$$\text{Spacing} = \frac{8.5 \times 12}{28} = 3.6'' \quad \text{Use 29 rivets / row, spaced 3.5'' o.c.}$$

$M_s$  at a point midway between 1st and 2nd concentrated loads:

$$= 12.75' \text{ from end of the beam, all plates on.}$$

$$M_s \text{ of first plate} = 14.5 \times 1/2 \times 28.747 = 208.5$$

$$M_s \text{ of 2 plates and angles} = \underline{1010.0}$$

1218.5 Total  $M_s$

$$\text{Longitudinal Shear} = \frac{68,187 \times 1218.5}{55167.73} = 1500 \#/' \text{ of length.}$$

$$\frac{1500 \times 8.5 \times 12}{6750} = 23 \text{ rivets necessary in the next } 8 \frac{1}{2}'$$

$$\text{Spacing} = \frac{102}{12} = 8 \frac{1}{2}'' \quad \text{Use rivets in two rows max. spacing of 6'' to center of beam. to}$$

Longitudinal Shear between the plate and angles at the support:

$$M_s \text{ of Plate} = 249.5 \text{ from page}$$

$$S = \frac{164,987 \times 249.5}{21392.38} = 1920 \#/' \text{ of length.}$$

for the first 4.25'

$$\frac{1920 \times 4.25 \times 12}{10603} = 9.3 \text{ rivets necessary in the plates 1st } 4 \frac{1}{4}'$$

Spacing can be adjusted for convenience of riveting and punching

spaced not greater than 6''

This Girder will not be used in the Balcony Design as the two column supports under Girder A have been omitted in the above calculations.

I have gone through the calculations for the purpose of showing the method used and as a comparison with the same Girder when the two column supports are figured in the later design. In this Girder,  $7/8$ " rivets should be used in the flange angles so that the proper spacing can be obtained.

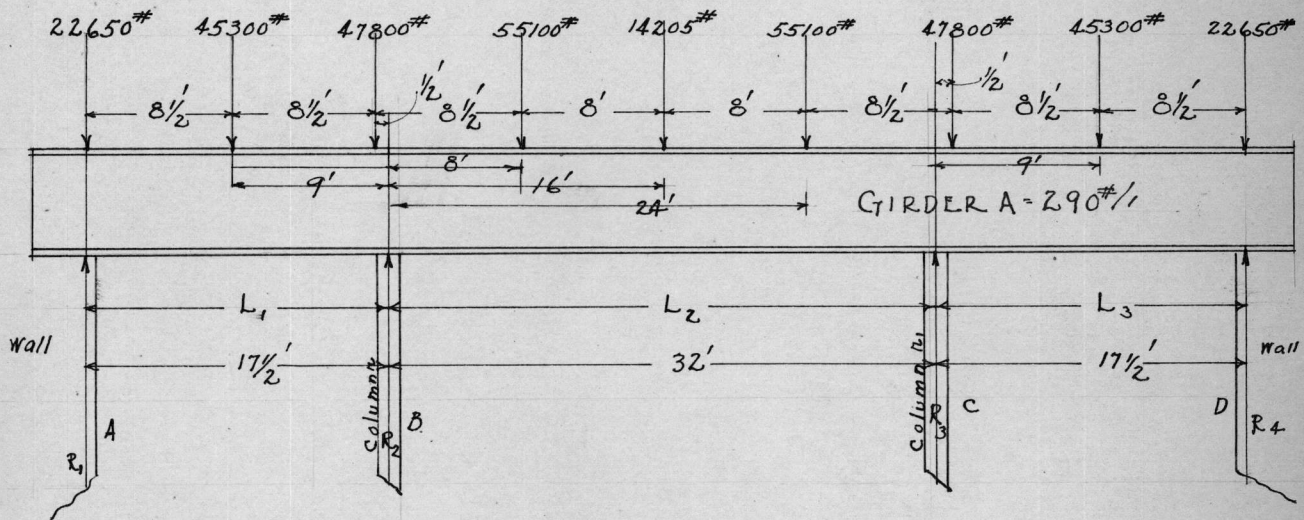
GIRDER A DESIGN WITH COLUMN SUPPORTS

METHOD -- THEOREM OF THREE MOMENTS

Girder is considered continuous over four supports.

Assumed weight of Girder = 290#/'

This is the correct design for Girder A to be used in the construction of the balcony.



Using the THEOREM OF THREE MOMENTS for a continuous beam with unequal spans, loads concentrated and not equally spaced, the equation becomes:

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{P_1(L_1 - a)}{L_1} + \frac{P_1'(L_1 - a')}{L_1} + \frac{P_1''(L_1 - a'')}{L_1} +$$

$$----- + \frac{P_2(L_2 - b)}{L_2} + \frac{P_2'(L_2 - b')}{L_2} + \frac{P_2''(L_2 - b'')}{L_2} + -----$$

$$- P_1 L_1(L_1 - a) - P_1' L_1(L_1 - a') - P_1'' L_1(L_1 - a'') - ----- - P_2 L_2(L_2 - b)$$

$$- P_2' L_2(L_2 - b') - P_2'' L_2(L_2 - b'') - -----.$$

$M_A = 0 \quad M_B = 0$  Beam is not fixed at the ends

The Moments and Reactions for the wt. of the Girder will be figured

later.

Using only the concentrated loads, the equations are, starting at the support B:

$$\begin{aligned}
 0 + 2M_B(17 \frac{1}{2} + 32) + M_C 32 &= \frac{47800(17 \frac{1}{2} - 1/2)}{17.5} + \frac{45300(17 \frac{1}{2} - 9)}{17.5} \\
 + \frac{55,100(32 - 8)}{32} + \frac{14205(32 - 16)}{32} + \frac{55,100(32 - 24)}{32} \\
 - 47800 \times 17.5(17 \frac{1}{2} - 1/2) - 45300 \times 17.5(17 \frac{1}{2} - 9) \\
 - 55100 \times 32(32 - 8) - 14205 \times 32(32 - 16) - 55100 \times 32(32 - 24) \\
 99M_B + 32M_C &= 46500 + 22620 + 41400 + 7102 + 13800 - 14,250,000 \\
 - 6750,000 - 28,200,000 - 3640,000 - 14,120,000
 \end{aligned}$$

$$(1) \quad 99M_B + 32M_C = 131,402 - 66,960,000 = -66,828,600$$

Setting up the equation from the support C (or r')

$$\begin{aligned}
 M_B L_2 + 2M_C(L_2 + L_3) + M_D L_3 &= \frac{P_2(L_2 - a)}{L_2} + \frac{P_2'(L_2 - a')}{L_2} + \text{-----} \\
 + \frac{P_3(L_3 - b)}{L_3} + \frac{P_3'(L_3 - b')}{L_3} + \text{-----} - P_2 L_2(L_2 - a) - P_2' L_2(L_2 - a') \\
 - \text{-----} - P_3' L_3(L_3 - b') - \text{-----}.
 \end{aligned}$$

Substituting values:

$$\begin{aligned}
 32M_B + 99M_C + 0 &= \frac{55,100(32 - 8)}{32} + \frac{14,205(32 - 16)}{32} + \frac{55,100(32 - 24)}{32} \\
 + \frac{47,800(17 \frac{1}{2} - 1/2)}{17.5} + \frac{45,300(17 \frac{1}{2} - 9)}{17.5} - 55,100 \times 32(32 - 8) \\
 - 14205 \times 32(32 - 16) - 55,100 \times 32(32 - 24) - 47,800 \times 17 \frac{1}{2}(17 \frac{1}{2} - \frac{1}{2}) \\
 - 45300 \times 17 \frac{1}{2}(17 \frac{1}{2} - 9). \\
 32M_B + 99M_C &= 41400 + 7102 + 13800 + 46500 + 22620 - 28,200,000 \\
 - 3,640,000 - 14,120,000 - 14,250,000 - 6,750,000 = 131,402 \\
 - 66,960,000
 \end{aligned}$$

$$(2) \quad 32M_B + 99M_C = -66,828,600$$

Solving simultaneously (1) and (2)

$$\text{Mult. (1) by 99} \quad 9801M_B + 3168M_C = -6,610,000,000$$

$$\text{Mult. (2) by 32} \quad \underline{1024M_B + 3168M_C = -2,140,000,000}$$

$$8777M_B = -4,470,000,000$$

$$M_B = -510,000' \# = -6,120,000'' \#$$

$$M_C = -\frac{66,828,600 + 510,000 \times 32}{99} = -\frac{50,528,600}{99}$$

$$M_C = -510,000' \# = -6,120,000'' \#$$

To Determine the Reactions, knowing the moments at supports.

Since the Girder is symmetrical

$$R_1 = R_4;$$

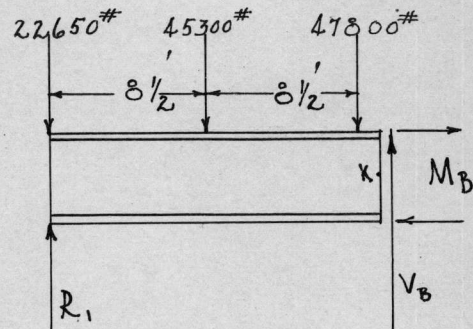
$$R_2 = R_3$$

A section is taken just to the left

of  $R_2$

$V_B$  is unknown

$$M_B \text{ is known} = -510,000' \#$$



M around any pt. must = 0.

SECTION TO LEFT OF  $R_2$

With center of Moments at K.

$$17 \frac{1}{2} R_1 + 510,000 = 22,650 \times 17 \frac{1}{2} + 45,300 \times 9 + 47,800 \times \frac{1}{2}$$

$$17.5 R_1 = 396,500 + 407,500 + 23,900 - 510,000 = 827,900 - 510,000$$

$$R_1 = \frac{317,900}{17.5} = 18,150' \# = R_4$$

$$R_2 + R_3 = 310,544 - 18,150 \times 2 = 310,544 - 36,300 = 274,244' \#$$

$$R_2 = 137,122' \# = R_3$$

THE MOMENTS & REACTIONS FOR THE WEIGHT OF THE GIRDER

Assumed to be 290#/'

Three moment equation for a uniform load.

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = -w_1 L_1^3 - w_2 L_2^3 \quad M_A = 0, M_D = 0$$

Starting at B.

$$0 + 2M_B(17\ 1/2 + 32) + M_C \cdot 32 = - \frac{200 \times 17.5^3}{4} - \frac{200 \times 32^3}{4}$$

(1)  $99M_B + 32M_C = -356,000 - 2376,000 = -2,732,000$

From C.

$$M_B \cdot 1/2 + 2M_C(L_2 + L_3) + M_D \cdot 1/3 = - \frac{M_2 L_2^3}{6} - \frac{M_3 L_3^3}{6}$$

(2)  $32M_B + 99M_C + 0 = -2,376,000 - 356,000 = -2,732,000$

Solving simultaneously

Mult. (1) by 99       $9801 M_B + 3168M_C = -270,000,000$

Mult. (2) by 32       $1024 M_B + 3168M_C = -87,400,000$

$8777M_B = -182,600,000$

$M_B = -20,800 \#'$

$M_C = - \frac{2,732,000 + 32 \times 20,800}{99} = \frac{2,065,000}{99} = -20,800 \#'$

Reactions:

$17\ 1/2 R_1 + 20,800 = 200 \times \frac{17.5^2}{2} = 44,400$

$R_1 = \frac{44,400 - 20,800}{17.5} = \frac{23,600}{17.5} = 1350 \# = R_4$

$R_2 + R_3 = 200 \times 67 - 1350 \times 2 = 19,400 - 2700 = 16,700$

$R_2 = \frac{16,700}{2} = 8350 \#$

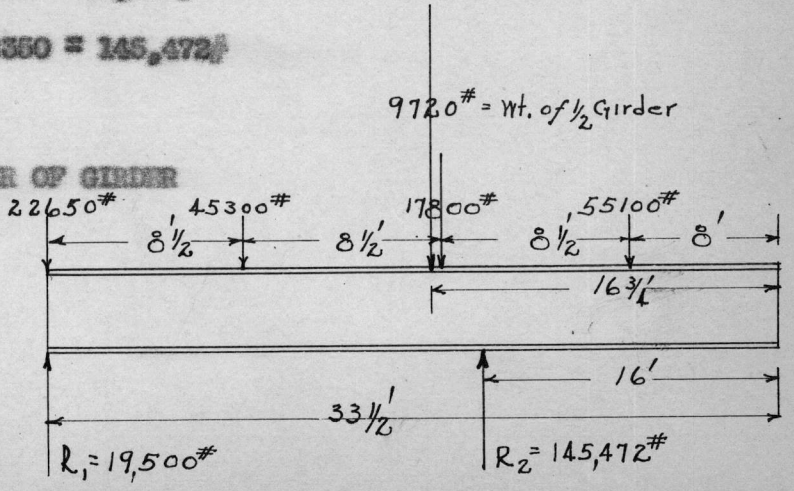
Total  $M_B = M_C = -510,000 - 20,800 = 530,800 \#'$  =  $6,380,000 \#'$

Total  $R_1 = R_4 = 19,150 + 1350 = 19,500 \#$

Total  $R_2 = R_3 = 137,122 + 8350 = 145,472 \#$

BENDING MOMENT AT THE CENTER OF GIRDER

SECTION - 1/2 GIRDER



$$\begin{aligned}M_2 &= 19,500 \times 33.5 + 145,472 \times 16 - 22,650 \times 33.5 - 45,300 \times 25 - \\ &- 47,800 \times 16.5 - 55,100 \times 8 - 9,720 \times 10.75 \\ &= 654,000 + 2,328,000 - 760,000 - 1,130,000 - 766,000 - 441,000 \\ &- 163,000 = 2,984,000 - 3,260,000 = 276,000' \# = - 3,310,000' \#\end{aligned}$$

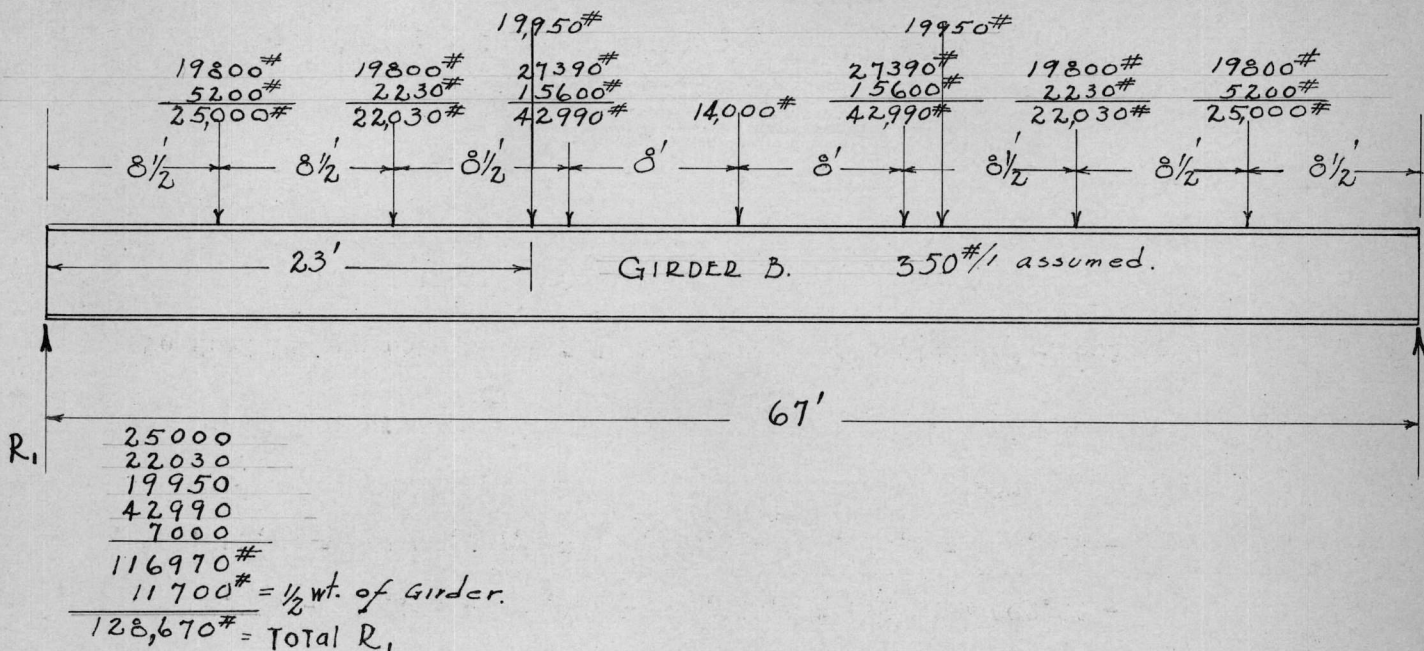
The Moment over the supports  $R_2$  and  $R_3$  is the controlling moment  
 $= - 530,800' \#$

FOR GIRDER A -- USE A 33" 125# C.B. Section

$$\text{Allowable M} = 577,060' \#$$

which is a much lighter section than that assumed in calculations.

COMPUTATIONS FOR DESIGN OF GIRDER B



The Girder supports all concentrated loads brought on by the stringer beams, cantilevers and beams C. and C.

Max. Moment Occurs at the Center/

$$\begin{aligned}
 M &= 128,670 \times 33 \frac{1}{2} - 25,000 \times 25 - 22,030 \times 16 \frac{1}{2} - 42,990 \times 8 \\
 &- 19,950 \times 10 \frac{1}{2} = 4,310,000 - 625,000 - 364,000 - 344,000 \\
 &- 210,000 = 4,310,000 - 1,543,000 = 2,767,000 \frac{1}{2} = 33,200,000 \frac{1}{2}
 \end{aligned}$$

GIRD STRESS METHOD -- OR APPROX. METHOD

The Girder is Limited to a Depth of 5' = 60" Because of lack of head room below.

From Carnegie p. 229

$$2A = \frac{2M}{fd} \quad 2A = \frac{2 \times 33,200,000}{10,000 \times 60} = 61.5 \text{ sq. in.} = \text{both flanges.}$$



From tables:

4 flange angles 8" x 6" x 7/16" A = 27 sq. in.

4 flange plates 18" x 1/2" A = 36 sq. in.

Max. Shear = 128,670

Thickness of Web plate =  $\frac{128,670}{12000} = .107$ "

It shall not be less than 1/160 of h

$\frac{1}{160} \times 46 \frac{1}{2} = .29$ "

Use 3/8" thickness = .375"

For a more exact computation, with proper reduction for rivet holes.

$$I = Md = \frac{55,200,000 \times 60}{2 \times 18000} = 55,333 \text{ in}^4$$

= Required I

4 Flange angles 8" x 6" x 7/16"

$d_1 = 58 \frac{1}{2}$ "  $20,976 \text{ in}^4$

4 Flange plates 18" x 1/2"

$d_1 = 58 \frac{1}{2}$ "  $38,000 \text{ in}^4$

1 Web plate 57 1/2" x 3/8"

$d = 58$ "  $\frac{6,000}{59,036} \text{ in}^4$

4 Flange Rivets or holes 1" x 1 1/2"  $B = 29 \frac{3}{4} = 5312$

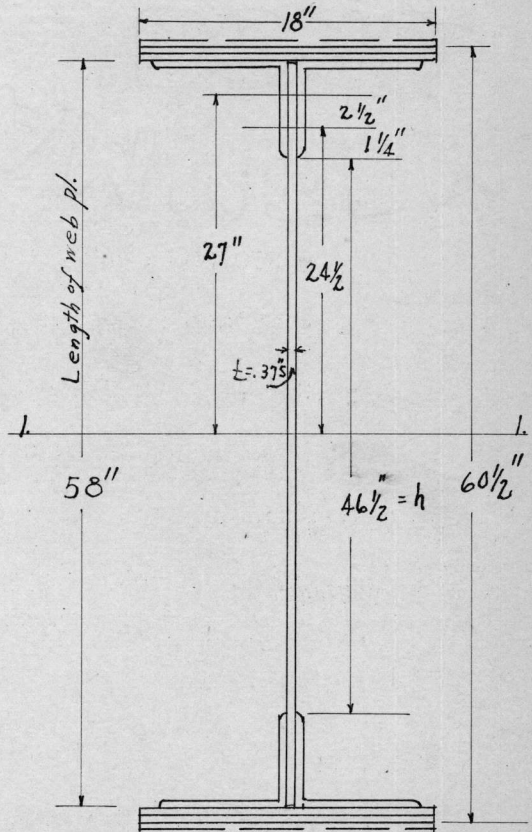
2 Web Holes 1" x 1 3/8  $L_1 = 27 = 2004$

2 Web Holes 1" x 1 3/8  $L = 24 \frac{1}{2} = \frac{1650}{8966} \text{ in}^4$

$59036 - 8966 = 50,102 \text{ in}^4$  Net I < required I

An extra 3/8" plate will be placed on top and bottom of center section.

1 - 2 - Flange plates - 18" x 3/8" -  $d_1 = 60 \frac{1}{2}$ " =  $12,500 \text{ in}^4$



$50,102 + 12,600 = 62,702 \approx 1$

Actual Wt. of Girder 4 Flange angles =  $8 \times 6 \times 17/16 = 80.9 \text{ #}$

4 " plates =  $18 \times 1/2 = 122.4$

2 " " =  $18 \times 3/8 = 45.0$

1 Web plate  $58 \times 3/8 = 74.0$

Girder Weight = 323.1 #

BENDING MOMENT DIAGRAM FOR CUTTING OFF FLANGE PLATES

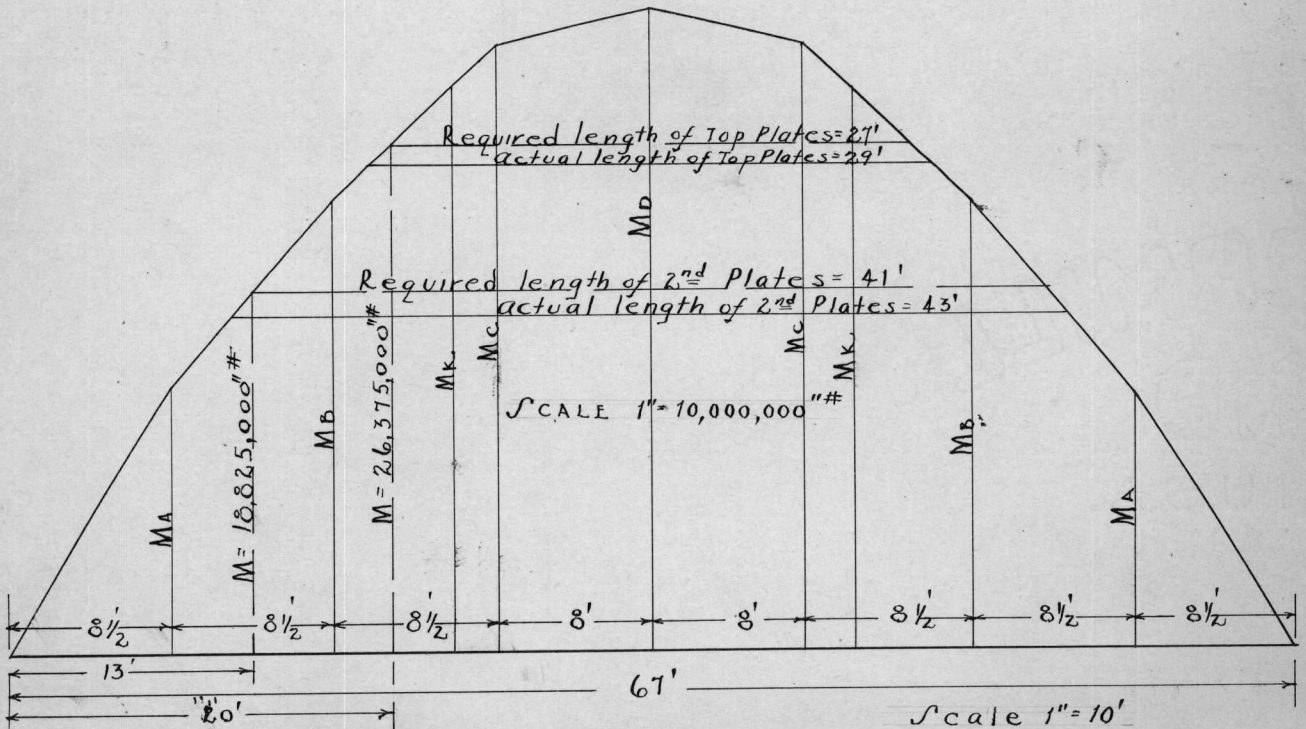
$M_A = 128,670 \times 8 \frac{1}{2} - 150 \times \frac{8.5^2}{2} = 1,094,590 \text{ #} = 13,140,000 \text{ #}^2$

$M_D = 128,670 \times 17 - 25,000 \times 8.5 - 150 \times \frac{17^2}{2} = 1,950,900 \text{ #} = 23,420,000 \text{ #}^2$

$M_C = 128,670 \times 25.5 - 25,000 \times 17 - 22,080 \times 8.5 - 150 \times \frac{25.5^2}{2}$   
 $= 2,623,200 \text{ #} = 31,560,000 \text{ #}^2$

$M_D = 128,670 \times 33.5 - 25,000 \times 25.5 - 22,080 \times 17 - 19,960 \times 10.5$   
 $- 42,990 \times 8.5 - 150 \times \frac{33.5^2}{2} = 2,767,000 \text{ #} = 33,200,000 \text{ #}^2$

$M_E = 128,670 \times 23 - 25,000 \times 14 \frac{1}{2} - 22,080 \times 6 - 150 \times \frac{23^2}{2}$   
 $= 2,423,100 \text{ #} = 29,100,000 \text{ #}^2$



BENDING MOMENT DIAGRAM

Resisting Moment of Girder Top and Bottom 1st outer Flange plates off.

Using for the safe stress  $f$ , the ratio  $\frac{h}{d} \times 18000 = \frac{46.5}{60.5} \times 18000$

$$2A = \frac{2M}{fd} \quad 2M = 2Afd = \frac{63 \times 46.5 \times 18000 \times 60.5}{60.5} = 52,750,000$$

$$M = \frac{52,750,000}{2} = 26,375,000 \text{ in}^2$$

Resisting Moment of Girder 2nd plate top and bottom off.

$$2A = 27 + 18 = 45 \text{ sq. in.}$$

$$M = \frac{45 \times 46.5 \times 18000 \times 60.5}{60.5 \times 2} = \frac{37,650,000}{2} = 18,825,000 \text{ in}^2$$

From the Moment diagram, the top and bottom outside plates shall be 29' long. The 2nd plates top and bottom shall be 43' long - 1' has been added to each end to take care of shear. The plates next to angles shall extend full length of beam.

The Max. Moment That The Girder Will Resist At The Center, With All the Plates in the Section and Using an Allowable Stress  $f$

$$\begin{aligned} &= \frac{h}{d} \times 18000 \quad 2A = 63 + 15.5 = 78.5 \text{ sq. in.} + 1/8 \text{ of web area} \\ &= 78.5 + \frac{21.75}{8} = 79.22 \end{aligned}$$

$$M = \frac{2Afd}{2} = \frac{79.22 \times 46.5 \times 18000 \times 61.25}{61.25 \times 2} = 33,500,000 \text{ in}^2$$

Using 1/8 the Area of the web and  $\frac{h}{d} \times 18000$ /sq. in. The Girder is o.k.

#### SPACING OF STIFFENERS

$$s = 88t \sqrt{\frac{18000 A}{V} - 1} = 88 \times .375 \sqrt{\frac{18000 \times 21.75}{126,670} - 1} = 31.85 \sqrt{3.04 - 1}$$

$$= 31.85 \times 1.43 = 45.6''$$

From p. 11 A.I.C.C. Handbook:

$A$  = Gross Area of web.  $V$  = vert. shear at the pt.

Stiffeners shall be placed under all concentrated loads with a driving fit.

V at first concentrated load = 102,305<sup>3</sup>

Spacing after the first concentrated load.

$$s = 85t \sqrt{\frac{18000 \times 21.75 - 1}{102,305}} = 31.85 \sqrt{\frac{385.5 - 1}{102,305}} = 31.85 \times 1.98 = 63.0''$$

V at the second concentrated load = 77,300<sup>3</sup>

$$s = 85t \sqrt{\frac{18000 \times 21.75 - 1}{77,300}} = 31.85 \sqrt{\frac{385.5 - 1}{77,300}} = 31.85 \times 2.02 = 64.4''$$

There shall be 2 intermediate stiffeners between the reaction and the first concentrated load. Between concentrated loads after the first only 1 stiffener is needed.

Number of rivets necessary in the stiffener under the first concentrated load. Bearing of 7/8" rivet on 5/8" of steel = 7875<sup>3</sup>

Under 1st concentrated load	=	$\frac{26000}{7875}$	=	3.31	or	4 rivets
" 2nd	"	"	=	$\frac{22030}{7875}$	=	2.8 or 3 rivets
" 3rd	"	"	=	$\frac{19950}{7875}$	=	2.5 or 3 rivets
" 4th	"	"	=	$\frac{42000}{7875}$	=	5.4 or 6 rivets
" 5th	"	"	=	$\frac{14000}{7875}$	=	2 rivets

\*. rivets under all concentrated loads can be maximum spaced.

Necessary Rivets in the End Stiffeners:

$$\frac{\text{Reaction}}{\text{Strength of one rivet}} = \frac{123,670}{7875} = 17 \text{ rivets necessary.}$$

Required rivets in flange angles

Horizontal shear taken at the bottom of the angle legs.

$$t = 3/8 = .375'' \quad \text{Longitudinal shear} = \frac{VH_s}{I}$$

H<sub>s</sub> at supports 2 plates off.

$$H_s \text{ of plate} = 1/2 \times 18 \times 29.5 = 265 \quad \text{''}^3$$

$$H_s \text{ of angles} = 11.86 \times 28 = 332 \quad \text{''}^3$$

H<sub>s</sub> of web above the point

$$\text{taken} = 5 \ 3/4 \times 3/8 \times 26.12 = 56.4 \quad \text{''}^3 \quad \text{Total} = 653.4 \quad \text{''}^3$$

I of section =  $20,978 + 16,000 + 6000 = 43,000 \text{ in}^2 - 8966 = 34,102 \text{ in}^2$   
 (approx. from page ...)

$$L.S./\text{in} = \frac{128,670 \times 653.4}{34,102} = 2400/\text{in}$$

2 rows of rivets:- spacing =  $\frac{2 \times 7875}{2400} = 6.56 \text{ in}$

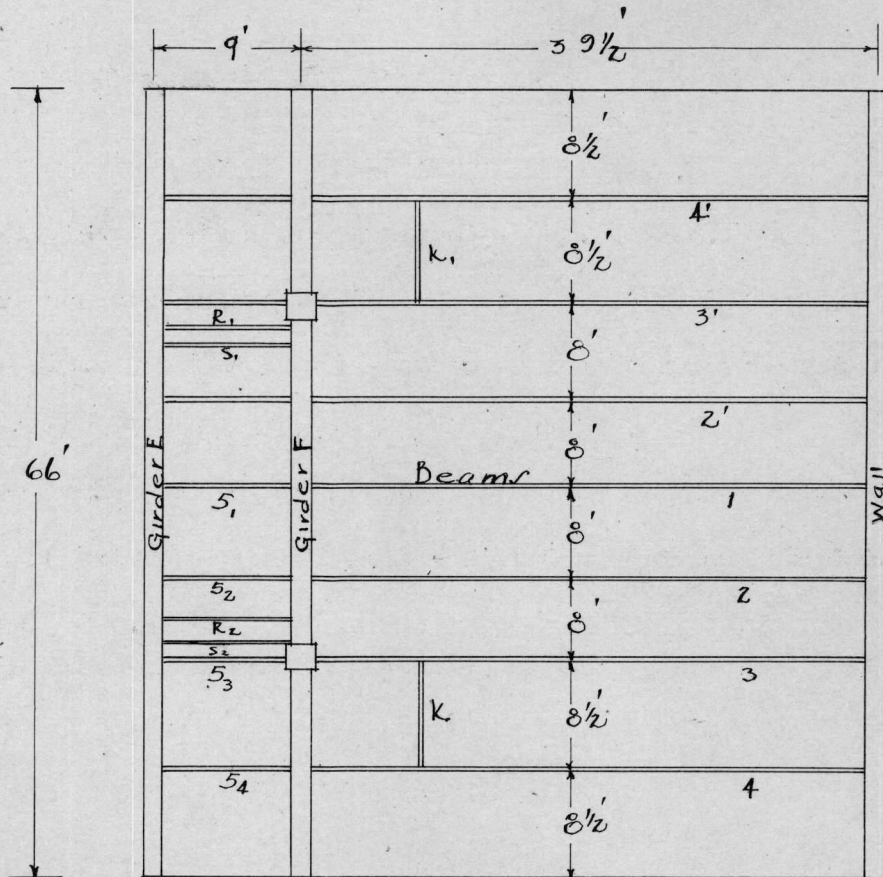
∴ rivets can be maximum spaced in flange angles.

rivets in flange plates may also be max. spaced.

Stiffeners under concentrated loads shall be  $6 \text{ in} \times 3 \frac{1}{2} \text{ in} \times \frac{3}{8} \text{ in}$  angles milled to fit. The large angles are made necessary because the stringer beams will connect to the six inch leg.

Stiffeners between concentrated loads shall be  $3 \frac{1}{2} \text{ in} \times 3 \frac{1}{2} \text{ in} \times \frac{3}{8} \text{ in}$  angles. End stiffeners shall be 2 -  $6 \text{ in} \times 4 \text{ in} \times \frac{3}{8} \text{ in}$  angles placed one on each side.

MEZANINE FLOOR CONSTRUCTION



PLAN - BEAMS & GIRDERS

Using a floor load = 100 / sq. ft. (live load)

Concrete floor slab construction will be used with triangle mesh reinforcing 2000 / sq. in. concrete will be used.

SOLID SLAB CONSTRUCTION

A strip of slab 1'-12" wide will be used and treated as a beam. Assume supporting beams to be 8" wide. Reinforcing to run perpendicular to the beams. A clear span of 8 1/2' will be used (center to center) One span only, taking a section 1' wide. Assume for weight calculations that the slab is 4 1/2" in thickness.

Weight of slab per sq. ft. = 54#  
 Live Load per sq. ft. = 100#  
 Total dead and live load = 154# per ft. of length.

Positive Moment at center:

$$M = \frac{wL^2}{8} = \frac{154 \times 8.27 \times 8.27 \times 12}{8} = 15,800 \text{ ft}\cdot\text{lb}$$

In diagram 1 (Reinforced Concrete p. 49) for  $f_c = 20,000$  #/sq.in.

$$f_c = 800$$

$$K = 131.3$$

$$p = 0.75\% = 0.0075; j = .875$$

$$bd^2 = \frac{M}{K} = \frac{15,800}{131.3} = 120$$

$$b = 12" \text{ (assumed section)}$$

$$d^2 = \frac{120}{12} = 10. \quad d = 3.16"$$

Use  $d = 3.25"$  (mesh to be protected by 1.25" of concrete including thickness of the mesh.)

$$A_s = \frac{M}{f_c j d} = \frac{15,800}{20,000 \times .875 \times 3.25} = 0.278 \text{ sq. in.}$$

This will be supplied by (from Carnegie p. 275) Triangle Mesh  
 Style no. 207; 3 strands. Net area per ft. width = .281 sq. in.  
 ∴ original assumptions for weight calculation purposes that a  
 4 1/2" slab would be used, is correct.

Negative Moment at the interior supports =  $M = \frac{wL^2}{12}$

$$M = \frac{154 \times 8.27 \times 8.27 \times 12}{12} = 10,500 \text{ ft}\cdot\text{lb}$$

$$A_s = \frac{M}{f_c j d} = \frac{10,500}{20,000 \times .875 \times 3.25} = 0.185 \text{ sq. in.}$$

From Carnegie p. 275

Use style no. 208. Mesh in the upper part of the slab over each support.  
 This mesh shall extend 6" past the pts. of inflection on each side  
 of the supporting beams.

Bond: The Triangle Mesh gives perfect bond. Therefore the bond stress need not be figured.

Shear: (assuming width of support 8") and using the slab as a simple beam.

$$V = \frac{154 \times 8.27}{2} = 636\#$$

$$v = \frac{V}{b \cdot d} = \frac{636}{12 \times .875 \times 8.25} = 10.65\# / \text{sq. in. O.K.}$$

Temperature Reinforcement will be taken care of by the cross wires of the mesh.

#### BEAMS FOR MECHANICAL FLOOR

For the three center beams 1, 2 & 3' the load is uniform and

$$= \text{to } 8' \times 154 \times 29 \frac{1}{2} = 36,500\#$$

$$M = \frac{WL}{8} = \frac{36,500 \times 29 \frac{1}{2}}{8} = 134,000'\#$$

Use 16" 53# C.B. Allowable M = 145,650'# (this will take care of beam weight)

#### Beams 3 & 3'

Support in addition to the uniform floor a section of partition.

Height of Partition = approx. 15'

Weight of Partition

Metal lath and plaster = 8#/sq. ft. 2 sides = 16# x 15' = 240#/run.

Joists spaced 16" o.c. 2 by 4's used.

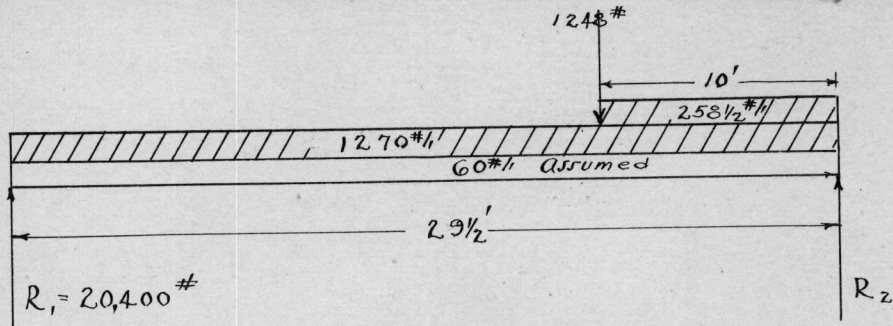
$$\text{Weight per joist} = \frac{2 \times 4 \times 15 \times 2.5}{12} = 25\#$$

$$25 \times \frac{12}{16} = 18 \frac{1}{2}\# / \text{run.}$$



Total weight of Partitions/run =  $240 + 18 \frac{1}{2} = 258.5$

Uniform floor load on the beam =  $154 \times 8 + 8.5 = 1270 \frac{1}{2}$



This beam must also take the concentrated load of the cross partition to be carried by a cross beam. Concentrated load =  $\frac{(9-2)}{2}$  (taking 2' of full height for opening loss)  $\times 258$

$$= \frac{7 \times 258}{2} = 1,113 + \frac{30 \times 9}{2} \text{ (wt. of beam)} = 1113 + 135 = 1248$$

$$29.5 R_1 = 1270 \times \frac{29.5^2}{2} + 1248 \times 10 + 258.5 \times \frac{10^2}{2} + 60 \times \frac{29.5^2}{2}$$

$$R_1 = \frac{552,000 + 12,480 + 12,925 + 23,750}{29.5} = \frac{601,155}{29.5} = 20,400$$

$$\text{Pt. of Max. } M = 20,400 - 1,330x = 0$$

$$x = \frac{20,400}{1330} = 15.3' \text{ from left support.}$$

$$\text{Max. } M = 20,400 \times 15.3 - 1330 \times 15.3 = 312,000 - 166,000 = 166,000'$$

Use 16" 63# C.B. Allowable  $M = 168,650'$

Beams 4 & 4"

Have partition loads extending 12' from left end. Stairway out out extends 9'

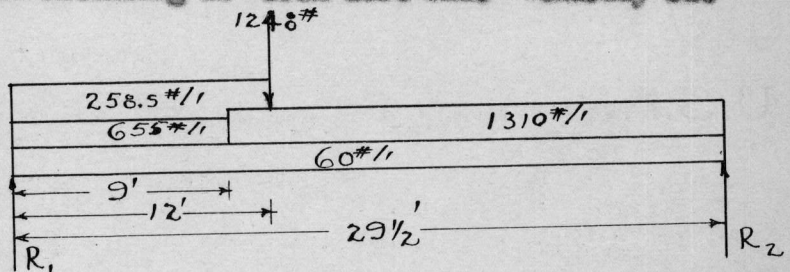
Uniform loads on the

beam =  $154 \times 8.5 =$

$$1310 \frac{1}{2}$$

Wt. along stairway cut

$$\text{out} = 154 \times \frac{8 \frac{1}{2}}{2} = 655 \frac{1}{2}$$



$$29.5 R_1 = 258.5 \times 12 \times 23.5 + 666 \times 9 \times 25 + 1510 \times \frac{29.5^2}{2} + 60 \times \frac{29.5^2}{8}$$

$$1248 \times 17.5$$

$$R_1 = \frac{73,000 + 147,500 + 275,500 + 26,100 + 21320}{29.5} = \frac{543,920}{29.5}$$

$$= 18,400\#$$

$$\text{Max. H occurs at } 18,400 - 60 \times 12 - 666 \times 9 - 258.5 \times 12$$

$$- 1510 \times 3 = 1370 = 0$$

$$x = \frac{18,400 - 14,898}{1370} = \frac{3502}{1370} = 2.56'$$

= 14.56' from the left support.

$$R_2 = (14898 + 1370 \times 17 \frac{1}{2}) - 18400 = 38,898 - 18,400 = 20,498\#$$

$$\text{Max. H} = 20,498 \times 14.94 - 1370 = \frac{14.94^2}{2} = 306,000 - 163,990$$

$$= 163,000\#$$

Use 16" 63# C.B. Allowable H = 168,560#

Beams 5, 5<sub>1</sub>, 5<sub>2</sub>, 5<sub>3</sub>, & 5<sub>4</sub>:

$$\text{Uniform load on Beam} = 154 \times 8 \frac{1}{2} \times 9 = 11800\#$$

$$\text{Max. H} = \frac{11,800 \times 4 \frac{1}{2}}{8} = 6,650\#$$

$$R_1 \approx R_2 = 5,000\#$$

6" 20# H beams will be used to give proper support to  
the slab.

Beams R & S

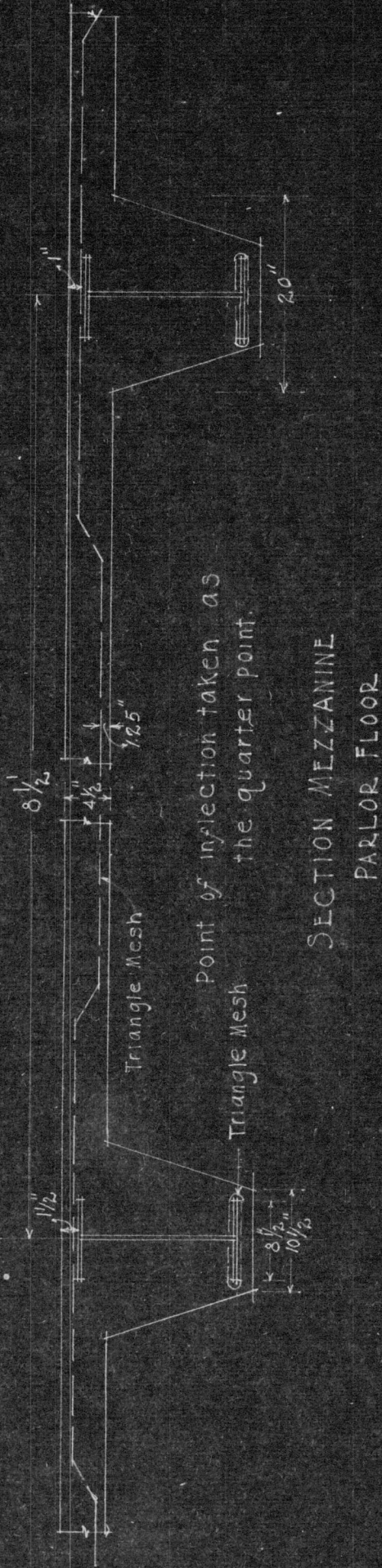
2 - 4" 7.7# (C.B. sections) beams will be used on each side of  
the mezzanine floor landing. 1 above & 1 below the steps to  
support the slab.

Beams K & K'

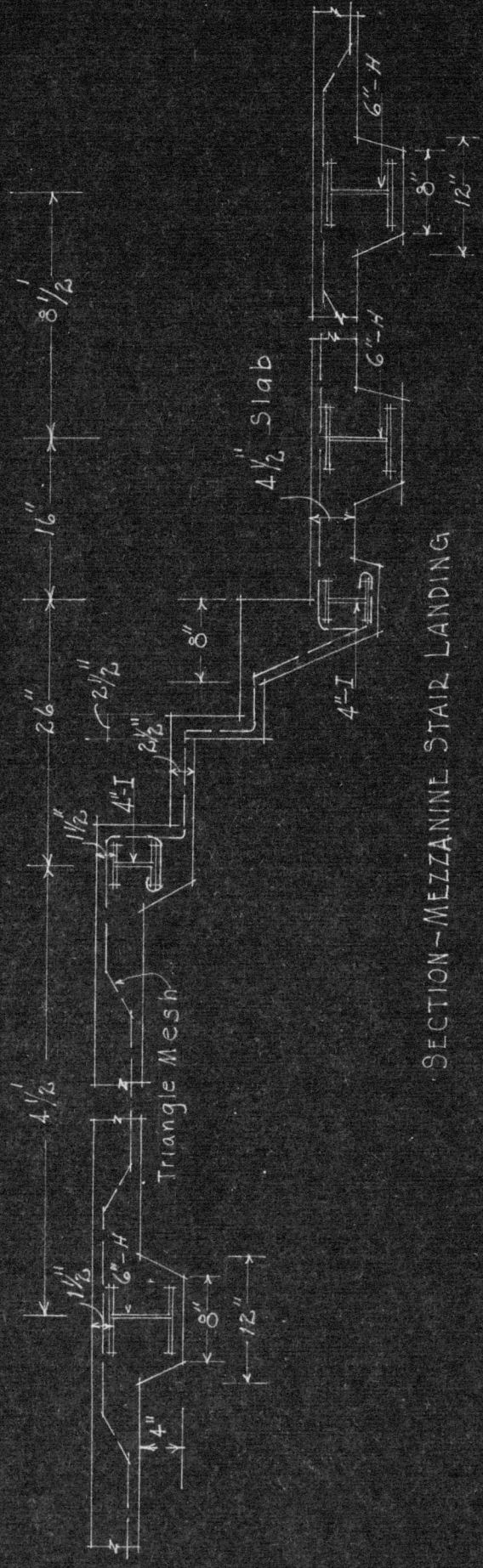
Use a 4" 7.7# beam to support the slab at each stairway cut out.

Standard connections shall be used on all stringer beams and cross beams.

Either abutting into the girder or spanning the space at stairway cut outs.



SECTION MEZZANINE  
PARLOR FLOOR



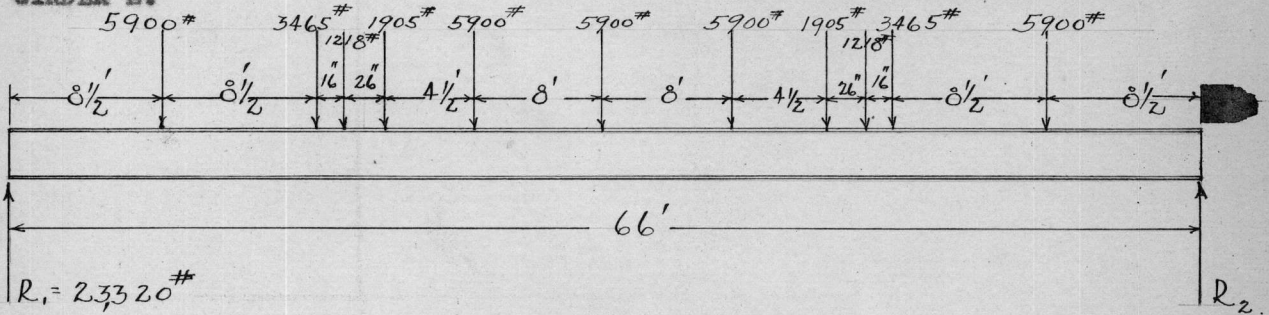
SECTION - MEZZANINE STAIR LANDING

Scale  $\frac{3}{4}'' = 1'$

ARCHITECT  
C.C.M.

**GIRDERS TO SUPPORT MEZZANINE FLOOR BEAMS**

**GIRDER E.**



**LOADING FOR GIRDER E.**

Loading is symmetrical

$$R_1 = R_2 = \frac{1}{2} \text{ total load}$$

$$R_1 = 24,640\#$$

Max. Moment will occur at the center of the beam under the center load.

$$M = 24,640 \times 33 - 5,900 \times 24.5 - 3,465 \times 16 - 1,218 \times 14 \frac{2}{3} - 1,905 \times 11 \frac{1}{2} - 5,900 \times 8 - 100 \times \frac{33^2}{2} = 815,000 - 144,500 - 55,400 - 17,900 - 21,900 - 47,200 - 54,500$$

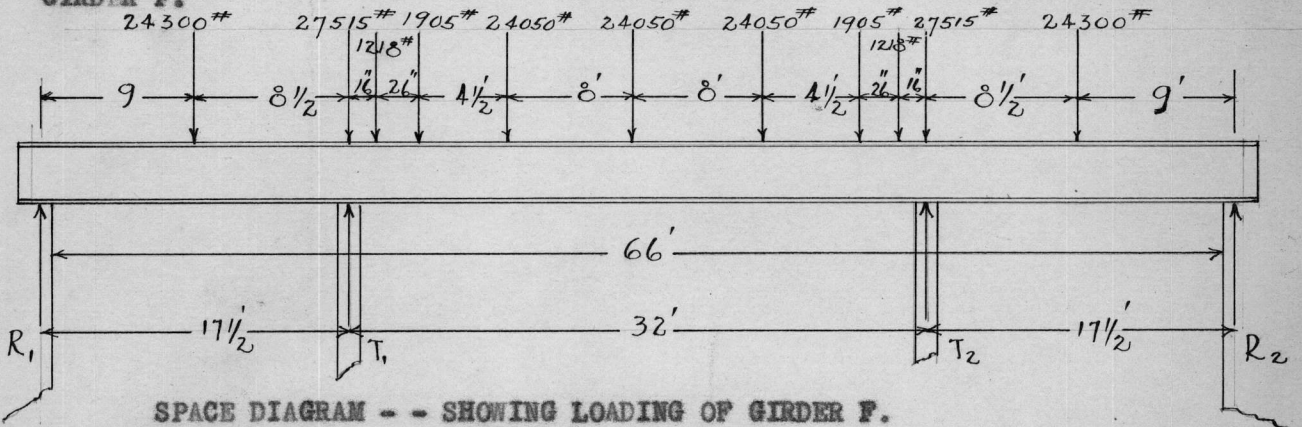
$$M = 815,000 - 319,600 = 495,400\#$$

Use a 30" - 115# C. B.

$$\text{Allowable } M = 489,600\#$$

**FIGURED BY THE METHOD OF LEAST WORK**

**GIRDER F.**



**SPACE DIAGRAM - - SHOWING LOADING OF GIRDER F.**

To express  $R_1$  &  $R_2$  in terms of the Redundants  $T_1$  &  $T_2$

Treating  $T_1$  &  $T_2$  as the Redundant Reactions.

$R_1$  is equal to  $\frac{1}{2}$  the total load minus the reaction  $T_1$

The loads (27,515# & 1,218#) will be considered as directly over the column. Therefore they will not be taken into account in the Moment equations but will be added to the value  $T_1$  after it is found for total reaction.

$R_1$  in terms of the redundant  $T_1$  using half the girder.

The girder being symmetrically loaded only half will be considered.

$$R_1 = 24,300 + 1,905 + 24,050 - \frac{24,050}{2} + 250 \times 67/2 - T_1 \text{ (the two loads over the support being neglected).}$$

$$R_1 = \underline{70,660 - T_1}$$

Setting up the Moment equations.

From  $R_1$  to  $P_1$  (first load).

$$M = R_1 X - 250 \frac{X^2}{2}$$

$$\text{Sub. for } R_1 \text{ its value} \quad \underline{M = 70,660X - TX - 125X^2}$$

From  $P_1$  to  $T_1$

$$\begin{aligned} M &= R_1 (9 + X) - (250 \times 9) (4\frac{1}{2} + X) - 24,300X - 250 \frac{X^2}{2} \\ &= \underline{625,820 - 9T - TX + 44,110X - 125X^2} \end{aligned}$$

From  $T_1$  to  $P_2$

$$\begin{aligned} M &= R_1 (17\frac{1}{2} + X) - 24,300 (8\frac{1}{2} + X) - 250 \times 17\frac{1}{2} (8\frac{1}{2} + X) + T_1 X - 250 \frac{X^2}{2} \\ &= \underline{993,250 + 41,935X - 17.5T - 125X^2} \end{aligned}$$

From  $P_2$  to  $P_3$

$$\begin{aligned} M &= R_1 (21 + X) - 24,300 (12 + X) - (250 \times 21) (10\frac{1}{2} + X) + T_1 (3\frac{1}{2} + X) \\ &\quad - 1,905X - \frac{250X^2}{2} = \underline{1,137,800 - 17.5T + 39,205X - 125X^2} \end{aligned}$$

From  $P_3$  to  $P_4$

$$\begin{aligned} M &= R_1 (25\frac{1}{2} + X) - 24,300 (16\frac{1}{2} + X) - (250 \times 25\frac{1}{2}) (12\frac{1}{2} + X) + T_1 (9 + X) \\ &\quad - 1,905 (4\frac{1}{2} + X) - 24,050X - \frac{250X^2}{2} \end{aligned}$$

$$\text{Sub. for } R_1 \text{ its value} \quad \underline{M = 1,276,925 + 14,025X - 17.5T - 125X^2}$$

The deflection or settling of the columns will not be taken into account, assuming that their value  $\delta T_1 \frac{dT_1}{dT_1}$  will be negligible and would not change the equation appreciably. Therefore the value of  $T_1$  would change very little. Thus only the work of the Bending Moments will be used to find the redundant  $T_1$ .

(See next sheet for table of Moments and  
Work equation)

LIMITS	M	dw/dt	$\int Mdw/dt dx$	X	$1/EI \int Mdw/dt dx$
R <sub>1</sub> to P <sub>1</sub>	$70,660x - Tx - 125x^2$	-x	$-70,660x^2 + Tx^2 + 125x^3 dx$	0 to 9	$1/EI [243T - 16,985,000]$
P <sub>1</sub> to T <sub>1</sub>	$625,620 - 9T - Tx + 44,110x - 125x^2$	-9 - x	$-5,632,380 - 1,022,810x - 42,985x^2 + 125x^3 + 81T + 18Tx + Tx^2 dx$	0 to 8.5	$1/EI [1,554T - 93,536,80]$
T <sub>1</sub> to P <sub>2</sub>	$993,250 + 41985x - 17.5T - 125x^2$	-17.5	$-17,390,000 - 735,000x - 306T + 2188x^2 dx$	0 to 3.5	$1/EI [1070T - 65,368,80]$
P <sub>2</sub> to P <sub>3</sub>	$1,137,800 - 17.5T + 39,205x - 125x^2$	-17.5	$-19,900,000 + 306T - 686,000x + 2,188x^2 dx$	0 to 4.5	$1/EI [1,378T - 96,388,500]$
P <sub>3</sub> to P <sub>4</sub>	$1,278,925 + 14,025x - 17.5T - 125x^2$	-17.5	$-22,380,000 - 245,500x + 306T + 2,188x^2 dx$	0 to 8	$1/EI [1,848T - 186,476,500]$

$1/EI [6,093T - 458,765,600]$

$1/EI [6,093T - 458,765,600] = 0$

$T = \frac{458,765,600 - 75,300f}{6093} \quad R_1 = 70,660 - 75,300 = -4,640f$

$T_1 = 75,300 + 27,515 + 1218 = 104,033f$

Finding the Maximum Moment of the Girder.

$M$  (at center) =  $75,300 \times 16 - 4,640 \times 33.5 - 24,300 \times 24.5 - 1,905 \times 12.5 - 24,050 \times 8 - 250 \times \frac{55.5^2}{2}$

$M = 1,205,000 - 155,500 - 590,000 - 23,800 - 192,600 - 140,400 = 102,700'f$

$M$  (at support T<sub>1</sub>) =  $4,640 \times 17.5 + 24,300 \times 8.5 + 250 \times \frac{17.5^2}{2} = 158,500 + 590,000 + 37,200 = 782,700'f$

M. is a Max. at the support.

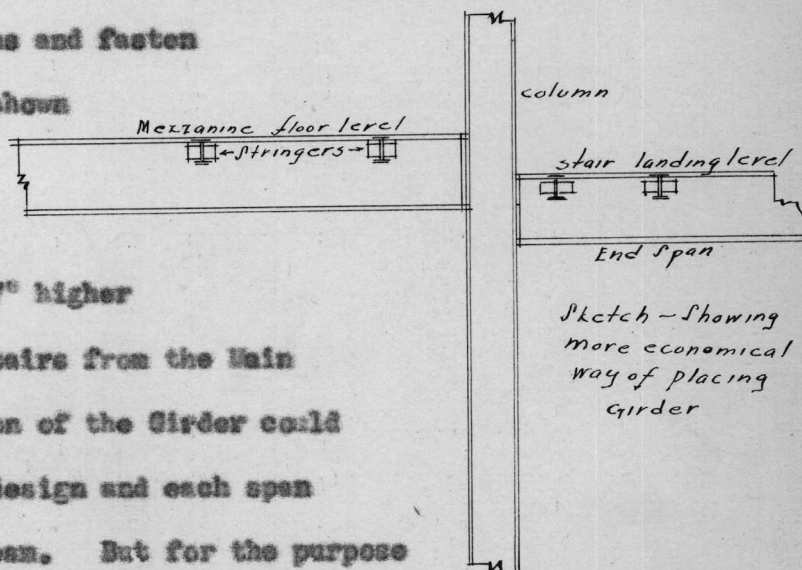
SIZE OF GIRDER F.

Using the Negative Bending Moment at the supports T<sub>1</sub> & T<sub>2</sub> as the Maximum = 782,700'

Use a 36" - 150# C. B. for the Girder. (from abridged Carnegie p. 122).  
 The assumed wt. of the Girder was 250#/'. Therefore the safety factor is increased and the Moments will not be recalculated.

This Girder has been considered as continuous and thus figured:

For purposes of design it would have been more practical to divide the Girder into three spans and fasten the column and Girder as shown in the sketch.

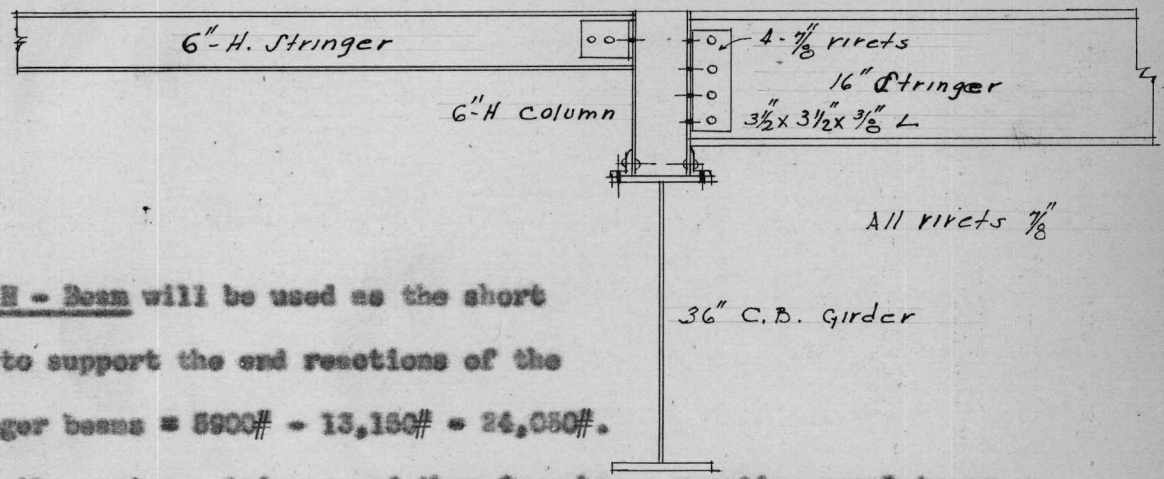


The center section of the Mezzanine floor being 20.7' higher than the landing of the stairs from the Main Foyer. The center section of the Girder could be raised for economical design and each span figured as for a simple beam. But for the purpose of showing the least work Method, the Girder will be taken as calculated above and placed below the level of the stair landing for all three spans. The stringers of the Mezzanine floor shall be raised above the Girder to their required floor level.

The Method of support of the stringer beams will be by short columns, extending up from the Girder. These short columns will be sections of Carnegie I - beams fastened to the flange of the Girder by small angles.



DETAIL OF SHORT COLUMN & CONNECTIONS TO SUPPORT STRINGERS



A 6" - H - Beam will be used as the short column to support the end reactions of the 2 stringer beams = 8900# - 13,150# = 24,050#.

This is the center stringer and therefore has a reaction equal to or larger than the others. Thus the 6" - H - beam can be used for each set of stringers, including the small beams R & S which support the slab at the top and bottom of the three steps leading from the main stair landing to the mezzanine floor level.

Rivets necessary to support the 16" stringer:

Using 7/8" rivets      web thickness of 16" - 50# C B = 3/8"

Allowable value of 7/8" rivet in bearing = 7,875# (for 3/8" bearing)

$$\frac{18150}{7875} = 2 \text{ rivets necessary } \underline{\text{use 4 rivets}} \text{ spaced } 3" \text{ o.c.}$$

Use 2 - 3 1/2" x 3 1/2" x 3/8" angles 12" long. 1 on each side of the stringer web.

For the support of the 6" - H stringer the no. of rivets necessary =

$$\frac{5,900}{7,875} = 1 \text{ rivet.} \quad \underline{2 - 7/8 \text{ rivets}} \text{ will be used.}$$

Use 2 - 3 1/2" x 3 1/2" x 3/8" angles 5" long. Rivets placed 3" apart. 1 angle will be placed on each side of stringer web.

For the angles connecting the short column and Girder flange use

2 - 3 1/2" x 3 1/2" x 3/8" angles 5" long. 1 placed on each side. 2 - 7/8 rivets will be placed in each leg of the angles, spaced 3" o. c.

This design will hold for all other stringer supports that are raised

above the Girder with the exception of the small beams of the foot and head of the three steps mentioned previously.

There will be very little tendency for the short column to tip or twist and therefore will be entirely safe with the no. of rivets and angles used at its base connection.

DESIGN OF COLUMNS TO CARRY BALCONY & MEZZANINE FLOOR.

The two columns supporting Girder A & F are considered as laterally braced at the Mezzanine floor. Therefore the columns will be designed in two separate parts. First the column consisting of the length from the Mezzanine floor (Girder F) to the bottom of the Balcony, (Girder A) which it supports, will be designed.

Distance of bottom of Balcony steps above the Mezzanine floor Girder = 15' 9" approx. Depth of Girder A = 33".

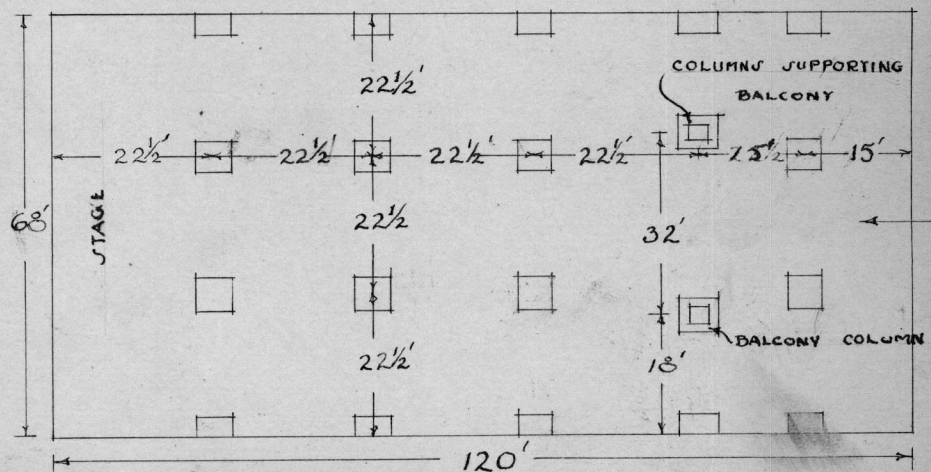
Unsupported length of column is therefore 15'.

Load on column = 145,472 $\frac{1}{2}$  from sheet

Use a 10" x 8" - 41 $\frac{1}{2}$  C. P. Section. (from p. 253 Carnegie Abridged Add.)

The column extending from the Mezzanine floor to the footing. The column will extend five feet below the orchestra floor to the footing. Thus giving ample room for possible heating and ventilation pipes. The extra load transmitted to the columns from the main floor will be:

Assuming the load per sq. ft. including floor slab as 165 $\frac{1}{2}$



Sketch shows the approximate placement of footings.

PLAN BELOW FLOOR LEVEL

The area of the floor supported by one column (from sketch preceding page) =  $\frac{(22.5 + 15/2)(32/2 + 16/2)}{2} = 18.75' \times 26' = 469$  sq. ft.

$$\text{Load} = 469 \times 165 \# = \underline{77,500 \#}$$

This is only an approximation as the problem does not cover the design of the orchestra floor and placement of footings. So a fairly close approximation is sufficient for my calculations.

Considering the columns as also laterally braced at the orchestra floor level, the design length of the column will be taken as 10' (i.e. the distance from the bottom of the mezzanine floor Girder to the main floor beams).

The loads coming on to this length of column are  $145,742 \# - 104,033 \# = 249,775 \#$

A 12" x 10" = 50# C. B. Section could be used. (p. 250 abridged Carnegie)

The total load on the column beneath the orchestra floor =

$$249,775 + 77,500 = 327,275 \#$$

This can be taken by a 14" x 12" = 78# C. B. Section (p. 244.)

For economy this size column will be used to avoid the splice at the orchestra floor. This column will extend up past the mezzanine floor Girder and spliced at this point to the smaller column supporting the Balcony Girder.

DESIGN OF FOOTINGS FOR COLUMNS SUPPORTING BALCONY GIRDER

Sloped Top Footings will be used.

Load on footing:

Load brought down by column under Balcony Girder	= 145,742#
Load on the column from Mezzanine Girder	= 104,033#
Orchestra floor load on the column	= <u>77,500#</u>
<b>Total Column Load</b>	<b>= 327,275#</b>

The Footing will be designed of 3000# concrete, reinforcing bars to be hooked.

Allowable soil pressure to be taken as 4,000#/sq. ft.

Assume wt. of footing as 24,000#

$$\text{Footing base area} = \frac{327,275 + 24,000}{4,000} = \underline{87.82 \text{ sq. Ft.}}$$

Base of footing will be taken as 9 1/2 ft. square.

$$w = \frac{351,275#}{9.5 \times 9.5} \text{ (total wt.)} = \underline{3,890# / \text{sq. Ft.}}$$

Take  $a = 2' = 24'' =$  (side of the column) i.e. width of plate beneath column base.

$$c = 1/2 (9 1/2 - 2) = 3 1/2$$

$$M = \frac{w}{2} (a + 1.2c) c^2 = \frac{3890}{2} (2 + 4.2) 12.25 = 148,000' \# = 1,770,000'' \#$$

Assume  $d = 23''$  Total depth = 32''

$$\text{Shear, } V = w L^2 - (a + 2d)^2 = 90.25 - (2 + 4 2/3)^2 \quad 3890 =$$

$$45.75 \times 3890 = \underline{178,000' \#}$$

$$v = \frac{V}{4(a + 2d) d} \quad \text{Allowable unit shear} = 40' \# \text{ per sq. in.}$$

$$40' \# = \frac{178,000}{4(8-2/3) \cdot .875 d_1} \quad d_1 = \frac{178,000}{40 \times .875 \times 24-2/3} = 19''$$

This is the depth  $d_1$  from the center of the reinforcing steel to the face of the slope directly above a point where a 45° line from the face

of the column hits the steel.

$$A_s = \frac{M}{F_s j d} = \frac{1,770,000}{16,000 \times .875 \times 24} = 4.68 \text{ sq. in. necessary steel area.}$$

in each direction.

Use 16 - 5/8" round bars in each band  $A_s = 4.9 \text{ sq. in.}$

Effective width of footing

$$= a + 2d + \frac{1}{2} (L - a - 2d) = 6 - 2/3 + \frac{1}{2} (2 - 5/6) = 8 - 1/12' = 8.08' = 97''$$

Space bars equally over the 97"  $= \frac{97}{16} \approx 6.1''$  o.c. approx.

1 extra bar will be placed outside of the effective width spaced 6 1/2" from the other bars.

The unit bond stress on each set of bars is

$$u = \frac{w (ac + c^2)}{\sum o_j d} = \frac{3890 (7 + 12.25)}{31.4 \times .875 \times 2.4} = \frac{75,000}{600} = 114 // \text{ sq. in.}$$

60 // sq. in. = Allowable bond stress for plain bars, therefore

Use 24 - 1/2" round bars spaced 4" o.c. with extras on each side as before.

$$u = \frac{3890 (7 + 12.25)}{26 \times 1.571 \times .875 \times 2.4} = \frac{75,000}{860} = 87 // \text{ sq. in.}$$

Deformed bars may be used so that special anchorage is not necessary, but bars will all be hooked at the end with full semi-circular hooks with 3 1/2" radius. The top of the footing will be 3' square. The column will be set on a plate 2' square.

The permissible working stress over the loaded area would be

$$F_a = 0.25 F_c \sqrt{\frac{A}{A_1}} = .25 \times 5000 \sqrt{\frac{9}{4}} = 750 \times 1.5 = 975 // \text{ sq. in.}$$

Actual bearing stress = 327,275 = 336 // sq. in. Therefore ok.

Formulas have all been taken from (Reinforced concrete Const.) by Hool

Vol. 1, p. 292.

COLUMN FOOTINGS -- CONTINUED.

The same Footing will now be designed from the "Hand Book of Reinforced Concrete Building Design" by Arthur E. Lord.

2000# Concrete to be used

Total column load = 327,275#

Allowable soil pressure = 4000#/ sq. ft.

from diagram 114, page 154.

Footing shall be  $9\frac{1}{2}'$  square.

Total thickness to be = 25"

Necessary steel area  $A_s = 6$  sq. in.

Minimum thickness = 10" at the edges

From diagram 109, page 149. Use  $\frac{1}{2}"$  round plain bars.

Use 26 -  $\frac{1}{2}"$  round plain bars spaced 4-1/8" o.c.

From Fig. 17, page 148

The plate on which the column rests shall be  $2\frac{1}{2}'$  square.

The top of the footing shall be 3' square.

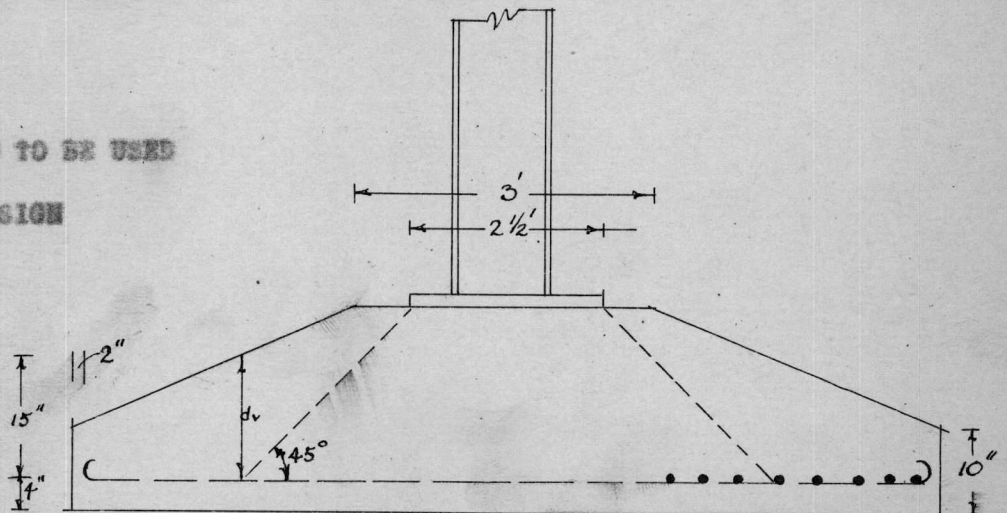
$$d_v = (b) \left( \frac{d}{B} \right) \frac{0.491 - \frac{d}{B}}{0.488} = 114 \left( \frac{25}{114} \right) \frac{.491 - .174}{.488} = 19.8 \times .68 = 13\frac{1}{2}"$$

Total length of rods =  $9' 6" + 20 \times .5" = 10' 4"$  as a minimum.

All bars to be hooked and 20 diameters past the length of one side.

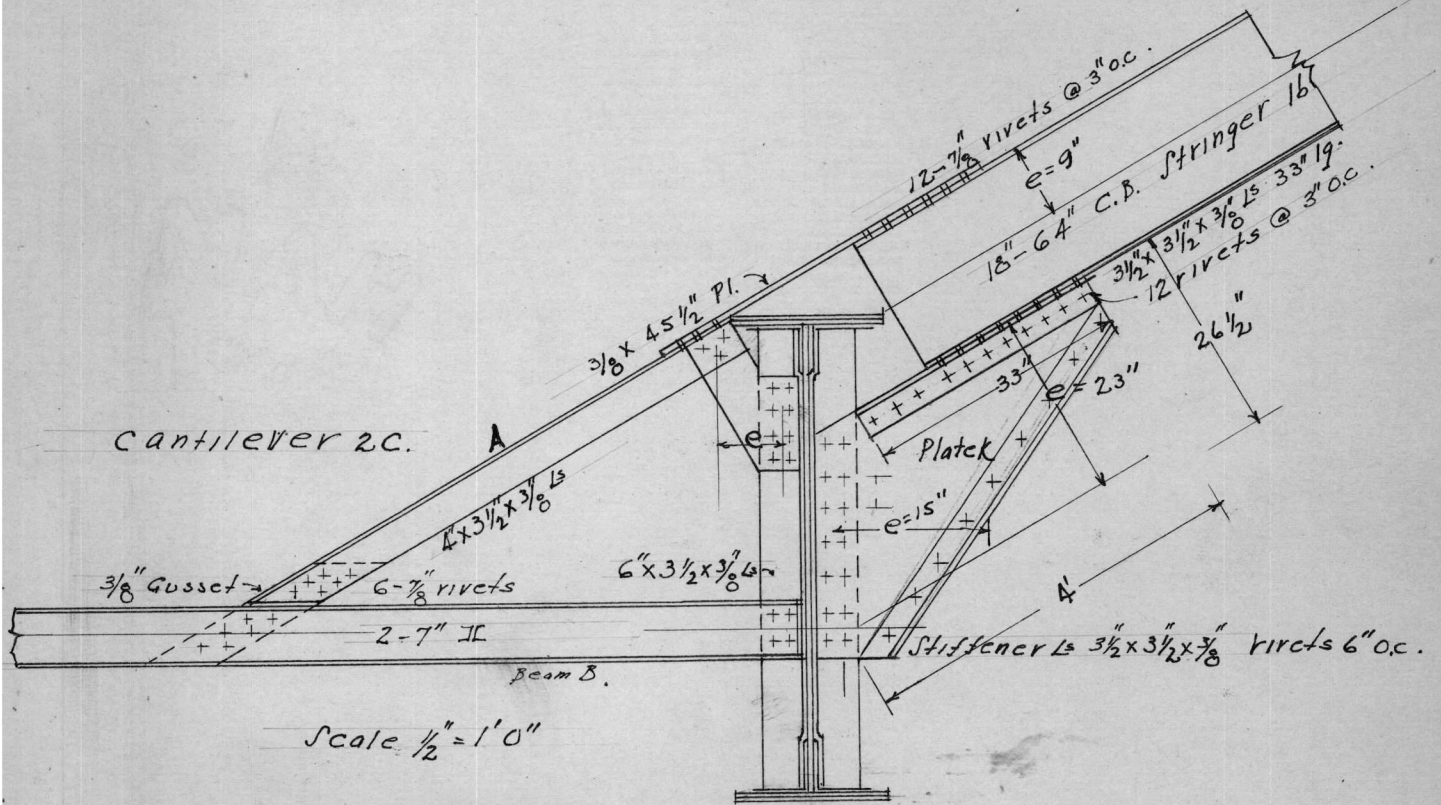
THIS FOOTING TO BE USED

IN MY DESIGN



SECTION - SLOPED FOOTING

DESIGN OF CANTILEVER BEAM CONNECTIONS TO GIRDER B.



Size of top plate and no. of rivets necessary to resist the pull of A.

Direct Stress on A =  $17,800 \frac{P}{A} = P$

1 sq. in. of steel in tension will take this stress.

Use a  $\frac{3}{8}$ " plate to give ample bearing to rivets.

$\frac{7}{8}$ " rivets will be used.

Single shear controls for the rivets.

Shearing value per rivet =  $7216 \frac{P}{A}$

Because the center of gravity of the rivets does not coincide with the neutral axis of the angle (A). There will be an eccentric force besides the direct force that the rivets (in the connection of the plate and angles) must resist.



Using a 3/8" gusset plate, single shear on the rivets govern.

Using 7/8" rivets, safe stress of a rivet in single shear = 7,216#.

Vertical Reaction of (A) at Girder:

$$4,550 = \frac{1}{2} \text{ uniform load}$$

$$\underline{8,900 = R_1}$$

Total Vertical Reaction for beam (A) = 13,450#

Rivets to connect angles to gusset plate =  $\frac{13,450}{7,216} = 2$  rivets. Use 3 rivets.

Eccentricity of the Vertical Reaction with respect to the rivets connecting the gusset plate to the girder = 10" = e.

Moment of the couple = 13,450 x 10 = 134,500#"

Try using 8 rivets spaced 3" o.c.

$$d_1^2 = 20.25 + 1.89 = 22.14 = d_2^2 = d_3^2 = d_4^2$$

$$d_1 = 4.6"$$

$$d_5^2 = 2.25 + 1.89 = 4.14 = d_6^2 = d_7^2 = d_8^2 \quad d_5 = 2.05"$$

$$a = \frac{134,500}{4(22.14) + 4(4.14)} = \frac{134,500}{105.12} = 1278\#$$

(a = resultant shear due to bending Moment at a unit distance from the center of gravity of the rivets =  $\frac{M}{\sum d^2}$ )

Stress on the outside rivet due to bending Moment = ad = 1,278 x 4.6 =

$$5,875\#$$

$$\text{Direct stress} = \frac{13,450}{8} = 1,681\#$$

Resultant Total Stress on outside rivet

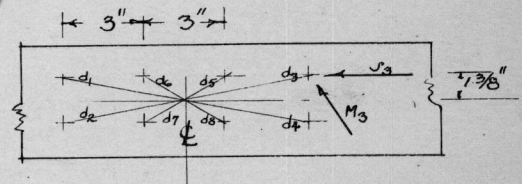
(from diagram) = 6,500# which is less than

the allowable stress per rivet; therefore 8 - 7/8" rivets are satisfactory.

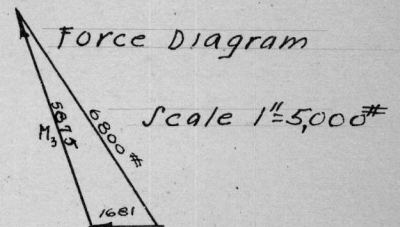
Rivets required for connecting top plate to 18" - stringer (1b).

Direct pull on the plate = 17,800# (from sheet ).

Distance from the center of gravity of the rivets to the neutral axis of the beam = 9" = e.



Rivets & Center of Gravity

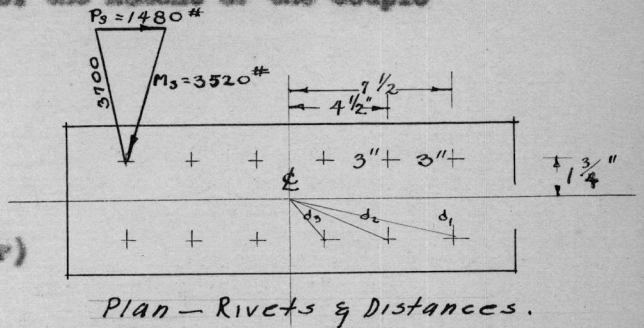


Therefore this joint also has an eccentric force acting. Replacing the direct force by a force and a couple; the Moment of the couple  $= 17,800 \times 9" = 160,200" \#$ .

The direct force  $P = 17,800 \#$

Try 12 -  $7/8"$  rivets spaced 3" o.c.

Allowable stress per rivet (single shear)  $= 7,216 \#$ .



Let  $s$  = stress on a rivet a unit distance from the center of gravity of the rivets due to bending Moment; then  $s \leq d^2 = M$ .  $s = \frac{M}{\sum d^2}$

$$d_1^2 = 7.5^2 + 1.75^2 = 59.26$$

$$d_1 = 7.7"$$

$$d_2^2 = 4.5^2 + 1.75^2 = 23.51$$

$$d_3^2 = 1.5^2 + 1.75^2 = 5.51$$

$$\frac{5.51}{87.85} \times 4 = 351.92 = \sum d^2 \quad s = \frac{160,200}{351.92} = 456 \#$$

The stress on the farthest rivet due to the bending Moment  $= 456 \times 7.7 = 3,520 \# = M_3$

Direct stress per rivet  $= \frac{17,800}{12} = 1,480 \#$

The stress due to the Moment always acts perpendicular to a line from the rivet to the center of gravity of the rivets. Total shearing stress on the outside rivets  $= 3,700 \#$  (from diagram).

This is much less than the allowable stress, but because of a possible twist from the joint below which has not been taken into account 12 -  $7/8"$  rivets will be used.

To take care of the thrust put into the girder by the channels (B) a plate will be used on the opposite side of the girder to carry this force up into the stringer beam (1b).

2-  $3 \frac{1}{2}" \times 3 \frac{1}{2}" \times 3/8"$  angles will extend the full length of the top of the plate by which the plate will be fastened to the stringer. The direct

stress in the rivets will be the thrust  $H/\cos 30^\circ =$  the direct stress in  $A = 17,800\#$ .

There is also eccentricity on the joint of the plate to the stringer.

The eccentricity  $= e = 23"$ . Moment caused by the eccentricity  $=$

$$17,800 \times 23" = 410,000\#"$$

Assume 16 rivets to be used, 8 in each angle, space 3" o.c.

Let  $a =$  the stress on a rivet a unit distance from the center of gravity of the rivets due to bending Moment:  $a = \frac{M}{\sum d^2}$

From Sec. 5 (a) A.I.S.C. Hand book p. 9

13,500#/ sq. in. rivets may be used in this case.

Safe stress (single shear 7/8" rivets  $= 8118\#$ )

$$d_1^2 = 10.5^2 + 1.75^2 = 113.46 \quad d_1 = 10.65"$$

$$d_2^2 = 7.5^2 + 1.75^2 = 59.26$$

$$d_3^2 = 4.5^2 + 1.75^2 = 23.31$$

$$d_4^2 = 1.5^2 + 1.75^2 = 5.31$$

$$201.34 \times 4 = 805.36 = \sum d^2$$

$$a = \frac{410,000}{805.36} = 509\#$$

Stress on the farthest rivet from the neutral axis  $= 509 \times 10.65 = 5,420\#$

Direct stress per rivet  $= \frac{17,800}{16} = 1,115\#$

Total stress on the outside rivets  $= 5,900\#$  (from diagram)

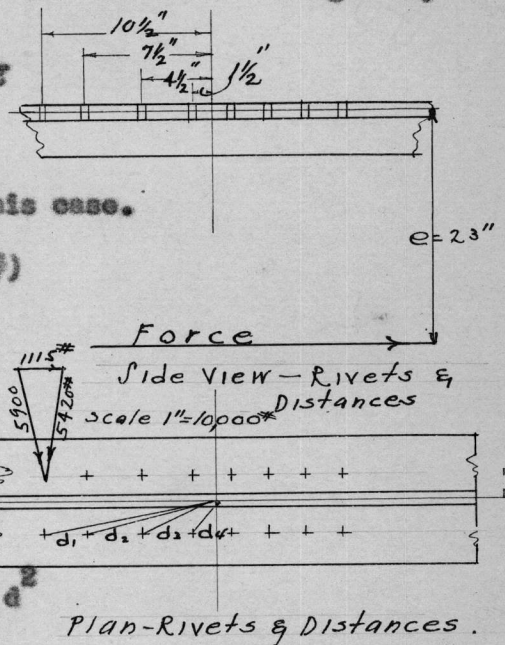
16 - 7/8" rivets spaced 3" is satisfactory.

The vertical Reaction of the stringer (1b) will now be considered.

Its Reaction perpendicular to the face of the beam  $= 27,450\#$ , but from

sheet all loads on the stringer are vertical; therefore the reactions

are vertical, and 27,450# is the true vertical reaction. This reaction



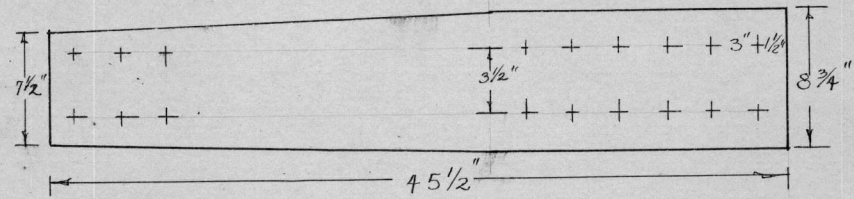
will be considered to be carried into the plate K by the group of rivets which fasten the angles to the top of the plate and will act through the center of gravity of this group. From this point a vertical line is dropped and will be considered as the line of action of the reaction. This produces a couple about the center of gravity of the group of rivets which connect the supporting plate K to the girder.

The Moment of the couple =  $15'' \times 27,450 = 410,000''/$ . This is equal in magnitude to the Moment resisted by the rivets connecting the plate to the stringer (1b). Therefore the same number of rivets will be ample as they were under stressed in the former joint. Use 16 -  $7/8''$  rivets.

The four rivets in the plate to resist the thrust of the channels (B) will be figured in the necessary 16 rivets, as the shearing stress produced by this couple tends to lessen their shearing stress. Therefore they will have very little stress because of the equalizing of the two forces.

The number of rivets fastening the beam (A) to beam (B) will be the same in number as <sup>at</sup> the opposite end of (A) = 8 -  $7/8''$  rivets in each connection.

The top plate connecting (A) & (1b) shall be as shown in the sketch and is  $3/8''$  in thickness.



BALCONY STRINGERS -- CONNECTIONS TO GIRDER A AT THE COLUMN

From Sheet:

The last step at the rear of the balcony is 4' .59" above a straight line drawn from a point 2' ahead of the stage line and at the orchestra pit level, and tangent to the nosing of the first step. At the supporting Girder A the step is 12.66" above this line.

The connection of the cantilever and stringer at Girder B was designed in a straight line and parallel to the tangent line above mentioned. Therefore the end of stringer (2b) which connects with Girder A will connect at a point 12.66" below the top of the Girder (considering the top of the girder as flush with the line denoting the step).

The end of the stringer beam (3a) will be placed a distance of 4" below the top of Girder A for its connection making the other end of the beam which frames into the wall 3' 8.59", - say 3' 8 $\frac{1}{2}$ " below the level of the last step. The Vertical Reaction of stringer (3a) = 28,000# (from sheet), using 7/8" rivets in single shear. The number of rivets necessary to support the load =  $\frac{28,000}{7,216} = 4$  rivets.

2- 4" x 4" x 3/8" angles 18" long will be used for the connection, with 6 rivets in the web of the beam and 5 rivets in other legs, (see detail). The Vertical Reaction of stringer (2b) = 19,800# Necessary number of 7/8" rivets to support the load =  $\frac{19,800}{7,216} = 3$ . Use 2- 4" x 6" x 3/8" angles 8 $\frac{1}{2}$ " long which is a standard H connection, page (299) abridged Carnegie.

CONNECTION CALCULATIONS -- NECESSARY RIVETS

The Girder A shall be connected to the column by 2 angles placed one on each side of the column web and 2 angles placed one on each outer side of the column flange. All angles riveted to the flange of the Girder.

4" x 4" x 3/8" angles shall be used.

Angles on the column web shall be 9" long with 3 rivets in each leg spaced 3" o.c.

Angles on the column flange shall be 7" long with 2 rivets in each leg spaced 3" o. c.

17 rivets in all shall make up the connection.

There has been no side thrust figured on the girder, so the number of rivets may seem in excess, but it is desired to have a firm joint in order to do away with any possible vibration.

Beam (3) has a reaction at the column = 20,400# (from sheet).

Shearing value of a 7/8" rivet (single shear) = 7,216#

Necessary rivets =  $\frac{20,400}{7,216} = 3$  rivets.

A standard connection will be used consisting of 2 angles 4" x 3 1/2" x 3/8" (from p. 298 Abridged Carnegie).

(See detail sheet)

Beam (5) has a reaction at the column = 5,900#

$\frac{5,900}{7,216} = 1$  rivet necessary to support the beam.

A standard connection will be used (p. 298 Abridged Carnegie) for, from 5" to 7" beams.

Girder F shall be connected to the column at the top and bottom by a connection the same as was used for Girder A. (see preceding page)

The connection of the column to the base plate shall be composed of 2- 4" x 4" x 3/8" angles 12" long, fastened to the web of the column, 1 on each side, with 4-7/8" rivets in each leg, spaced 3" o.c. 2- 4" x 4" x 3/8" angles 7" long, placed one on each column flange, with 2 rivets in each leg spaced 4" c. c.

The base plate (bearing plate) shall be anchored to the reinforced concrete footing by 4- 1 1/4" dowl pins, one at each corner of plate, 4" in from the sides.

SUMMARY

The nature of this problem was to design a cantilever balcony for a small theatre. It has been my desire to present entire plans for a theatre building. These have been arranged and proportioned with considerable time and thought, bearing in mind structural feasibility and economy. However, my chief interest has been in the Cantilever Balcony Framing and in overcoming difficulties arising from demands of balcony architecture.

The design has been carried out in accordance with the Portland Building Code, with the exception of the heights of balcony steppings. In order to obtain the desired balcony length and proper sighting, the last few rows of steps are a little in excess of the limiting height specified. This, I do not believe to be objectionable.

Plans of the orchestra floor, mezzanine floor and balcony, together with a center section of the building, have been drawn to scale. Complete details of the balcony framework and connections, including supporting columns, footings and mezzanine floor, have been worked out and sections shown.

The height and extension of the balcony have been chosen in such a way that a man of average height, standing at the rear of the orchestra floor, may see the top of the screen; also all spectators in the balcony can see the entire orchestra pit. Great pains have been taken in the perfection of sighting in order to conform to exigencies of a theatre where objects or speakers are continually shifting their positions. Therefore, the steppings of the balcony have been worked out in such a way that the sight line of each spectator is three inches above the sight line of the person seated directly ahead. The nosings of the steppings are not tangent to a straight



line, but to a concave curve so that instead of being equal in height the steps become steeper as they recede from the stage. This curve is called "the isacoustic", or equal-hearing-curve, and is a refinement seldom practiced in small theatres.

A graphical solution of the balcony steppings has been presented, and an arithmetical method derived and checked with the graphical. The framing of the balcony consists of Beam and Girder construction, since cantilever truss work proved to be too deep to get the desired ceiling heights for rooms on the mezzanine floor.

Balcony and mezzanine floor girders were designed to span the entire auditorium, a distance of sixty-seven feet center to center of outside walls. The plate girders were designed first by the Exact Method; secondly by the Chord Stress, or Approximate Method. The Chord Stress Method proves to be a saving of labor, and its results are close enough to the results of the Exact Method so as not to make any material change in the design. Girders over the supporting columns were considered as continuous over four supports. To find proper sizes their reactions and maximum bending moments were figured by two methods and compared. The Theorem of Three Moments first being used, and later the Method of Least Work. The deflection of the columns was not taken into consideration (the columns being fairly short) in the "Method of Least Work", and therefore the results of the two methods should be identical. The same loading being used and omitting the column supports, the girder was again designed and compared with the continuous girder. The results showed a considerable increase in required size and material. Sloping stringers of the balcony all connect either to the girder webs or stiffener angles. An ingenious design for the connection of the cantilever beams to the girder was worked out in such a manner as

to eliminate all twist and possible overturning of the girder, which was designed for vertical loads only. Loads on all stringers were considered vertical and without thrust. To accomodate the steps from the main stair landing to the mezzanine floor level, short columns have been placed above the girder to support the stringer beams of the mezzanine floor.

Two thousand pound per square inch concrete was used for floor slabs and footings. The allowable working stresses of structural steel and rivets as given by the Carnegie Steel handbook was used in the design.

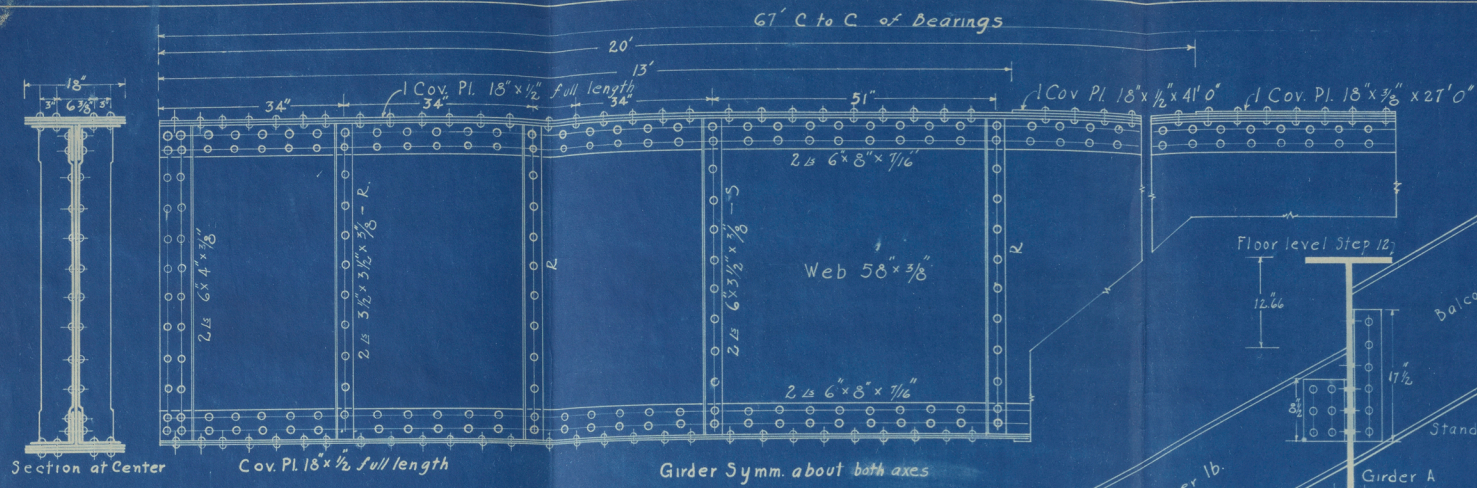
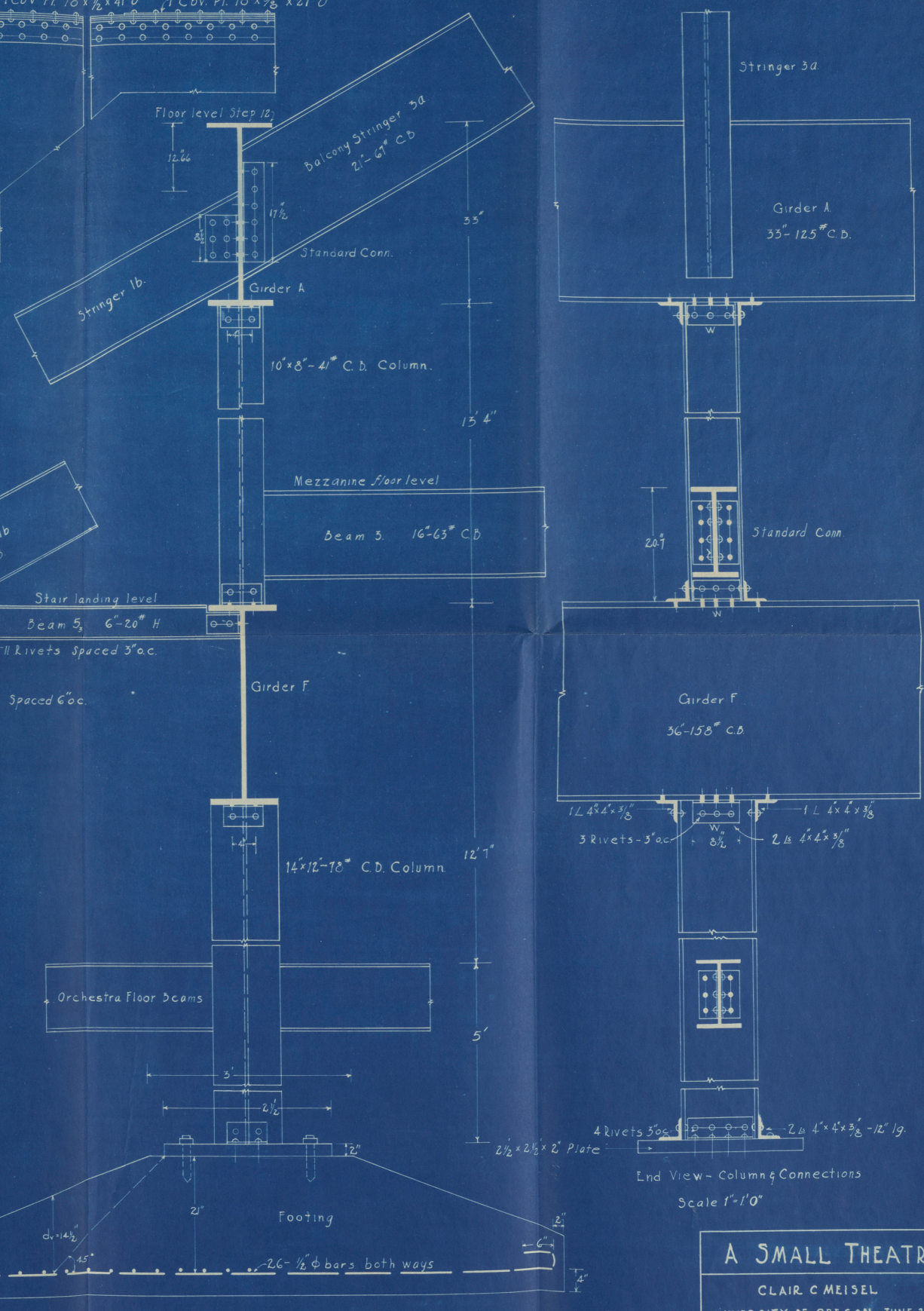
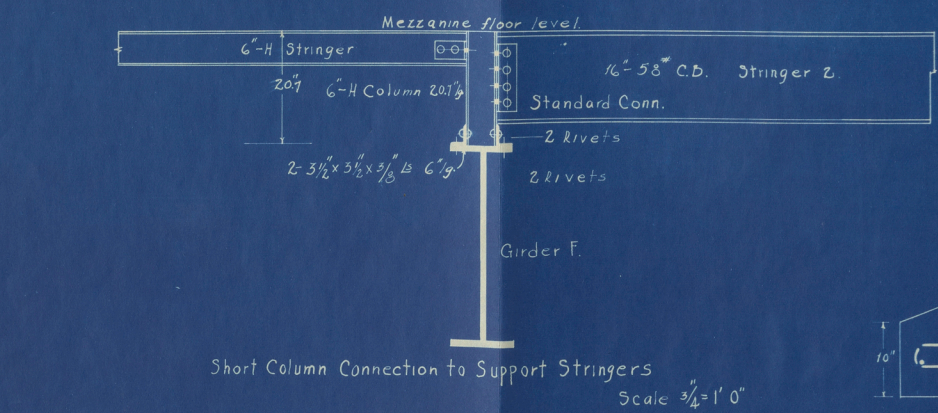
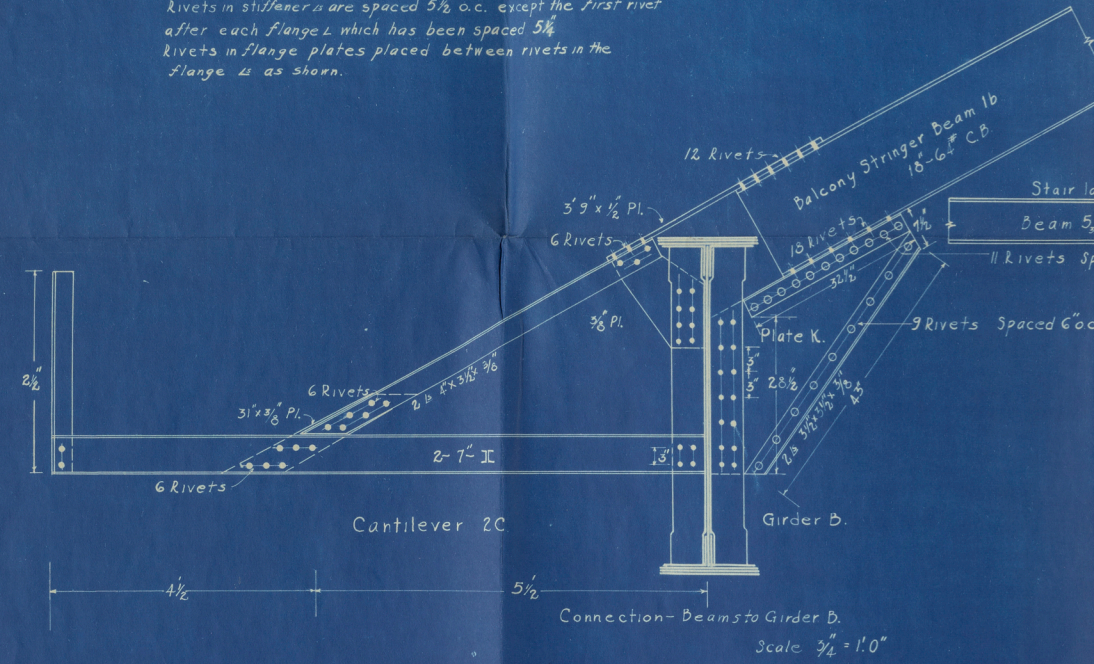
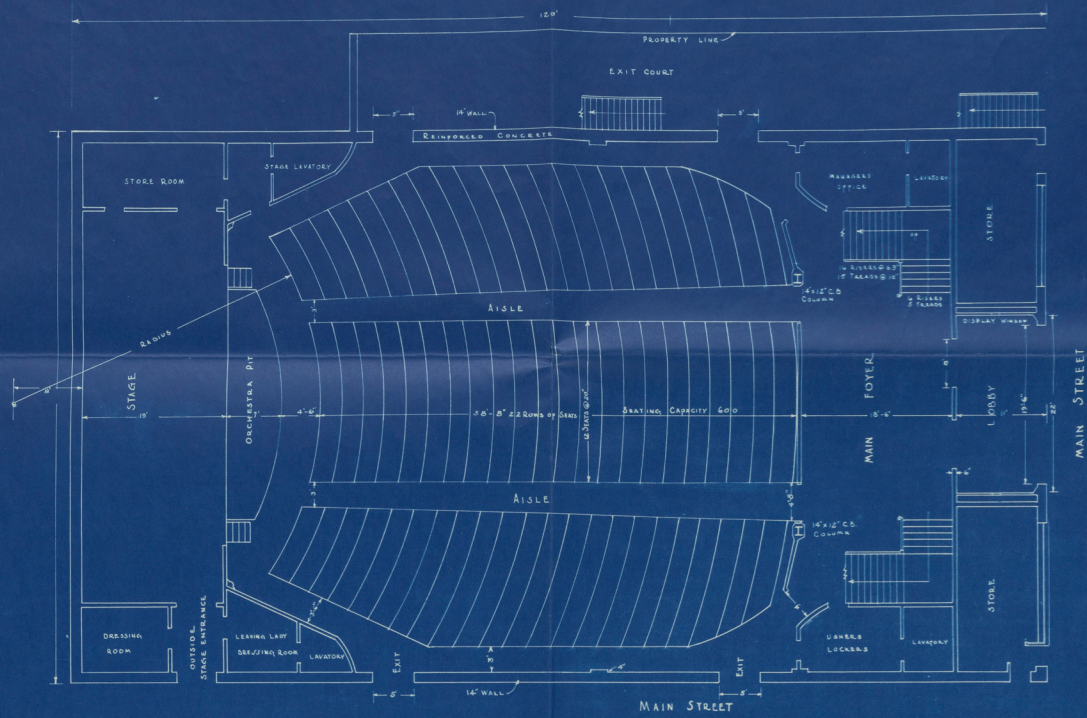


Plate Girder B

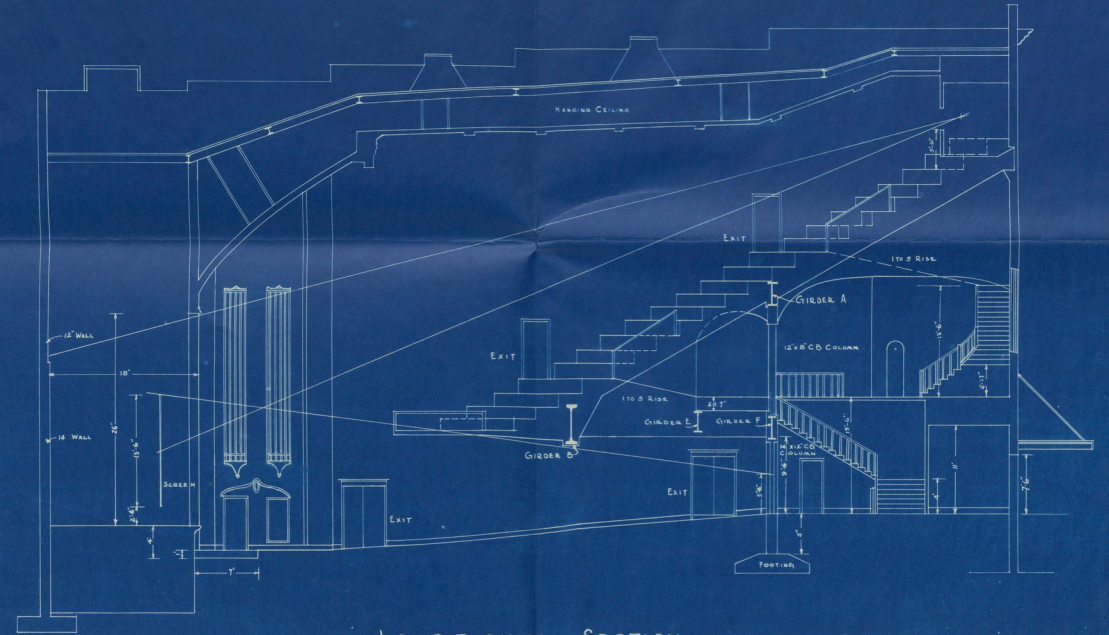
All Angles to be crimped & placed with a driving fit  
 12000 7/8" rivets used throughout  
 2 Stiffeners placed between the end Stiffener & first concentrated load. 1 Stiffener between all concentrated loads. Rivets in flange to be spaced 5" o.c. except the first spacing on either side of stiffeners which is 5 1/2". Rivets in stiffener are spaced 5 1/2" o.c. except the first rivet after each flange L which has been spaced 5 1/2". Rivets in flange plates placed between rivets in the flange L as shown.



A SMALL THEATRE  
 CLAIR C MEISEL  
 UNIVERSITY OF OREGON JUNE 20 1932

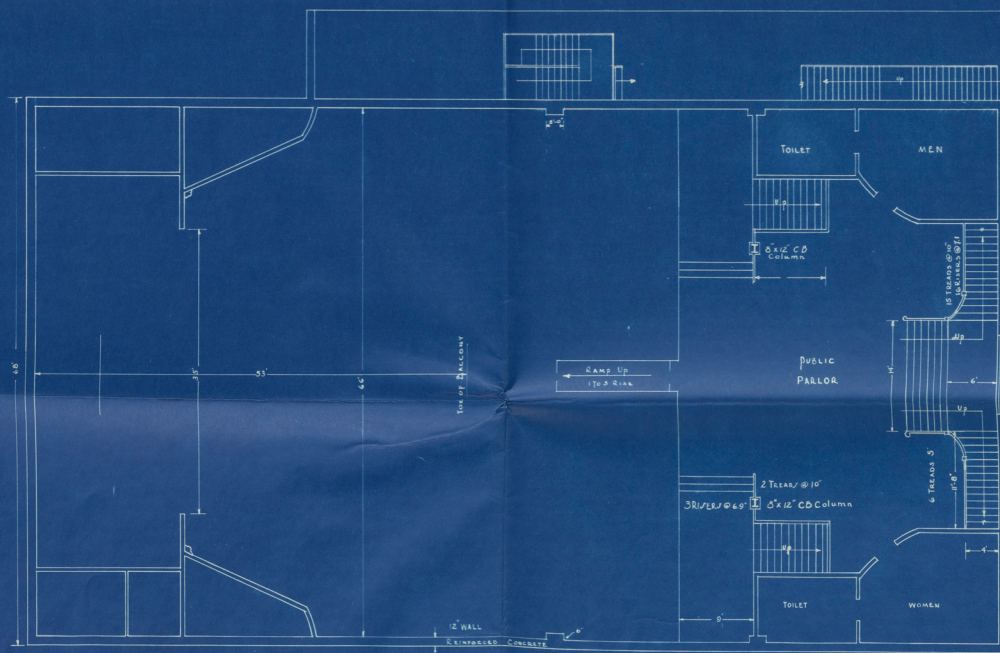


FIRST FLOOR PLAN

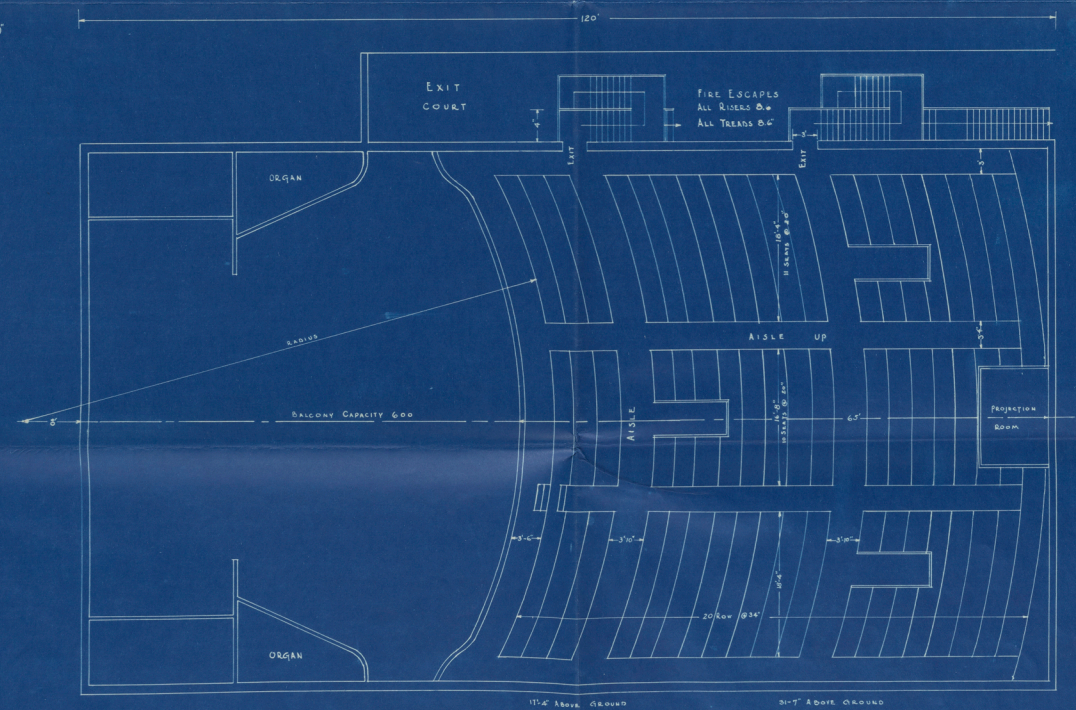


LONGITUDINAL SECTION

SCALE 1/8"=1'-0"



MEZZANINE FLOOR PLAN



BALCONY PLAN

A SMALL THEATRE

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