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Broadcast Time in Communication Networks

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ABSTRACT

Broadcasting is the information dissemination process whereby a set of messages is transmitted from one member to all other members of a communication network. We model a communication network by a graph and place certain restrictions upon the broadcasting process. Upper and lower bounds on the time to broadcast m messages throughout a network of n members are determined. Other time-related issues are addressed, including the estimation of time-optimal segmentation of messages which are to be broadcast.

Introduction

Broadcasting is the information dissemination process whereby a set of messages is transmitted from one member - the originator - to all other members of a communication network. Broadcasting would appear to have frequent practical application in the control of large distributed systems (eg., business, government, military, and computer networks). Broadcasting could be used to alert members of a status change detectable at the originator's site or to communicate a general order by the originator. A maximum (logical or physical) message size, such as exists in packet-switched data networks, motivates consideration of the process of multiple message broadcasting.

We model a communication network by a graph $G=(V,E)$ consisting of a set V of vertices, or members, and a set E of edges, or communication lines. A message is transmitted by a call placed between two members of the network. We assume that information dissemination is carried out under the following restrictions:

- *(i) a call requires one unit of time, the length of which may depend upon message length;*
- *(ii) a member may be involved, as sender or receiver, in at most one call at any time;*
- *(iii) a call may only be placed between two adjacent (directly connected) members;*
- *(iv) calls may be placed concurrently between distinct member pairs at any time.*

Let us assume that messages tagged for broadcast take priority over all other pending calls at a given member. Thus, after broadcasting has been initiated, the network gradually becomes dedicated to broadcasting until the process has been completed. As such, the time required to complete broadcasting is of primary interest. Let $b_m(v)$, $v \varepsilon V$, equal the minimum number of time units required to broadcast a set of m messages from member v throughout a connected network G. A network is <u>connected</u> is there exists a path between all pairs of members in the network. A network must be connected if broadcast is to be possible. We assume G to be connected throughout this paper.

Let the broadcast time of network G, $B_m(G)$, equal $\max_{v \in V}[b_m(v)]$. Let $\overline{B}_m(n)$ be $\max[B_m(G)]$ and $\underline{B}_m(n)$ be $\min[B_m(G)]$, where n is the number of vertices in network G. $\overline{B}_m(n)$ is the maximum of the broadcast times over all connected networks of n members while $\underline{B}_m(n)$ is the corresponding minimum value. In this paper we determine values for $\overline{B}_m(n)$ and $\underline{B}_m(n)$, for m, $n \geq 1$. Other issues related to broadcasting in networks are discussed, including an estimation of the time-optimal number of packets for the broadcast of a given message.

<u>Results</u>

Our first result follows from the observation that the number of members informed of a given message can at most double during any time unit. Networks providing this performance during any time unit have been determined [1]. Therefore,

(1) $\underline{B}_1(n) = \lceil \log_2 n \rceil$, for $n \geq 1$.

Networks within which this lower bound can be realized from any member are referred to as <u>minimal broadcast networks</u>. Several classes of minimal broadcast networks have been described [1]. Initial progress toward describing the class of minimal broadcast networks with the fewest lines has also been reported [2,3].

If only one call can be placed at any time during broadcasting, the corresponding upper bound is realized.

(2) $\overline{B}_1(n) = n-1$, for $n \geq 1$.

This maximum time is required from the end member of a path network or from any member of a star network. A path network is a tree with maximum member degree of 2. A member's degree is the number of other members to which it is directly connected. A tree is an acyclic, connected network. A star network is a tree with one central member to which all others are directly connected.

Several results follow immediately from our definitions, the above two bounds, and the observation that in a network with three or fewer members at most one call can be placed at any time.

(3) $\underline{B}_m(1) = \overline{B}_m(1) = 0$, for $m \geq 1$.

(4) $\underline{B}_m(2) = \overline{B}_m(2) = m$, for $m \geq 1$.

(5) $\underline{B}_m(3) = \overline{B}_m(3) = 2m$, for $m \geq 1$.

Throughout the remainder of the paper we assume that $n \geq 4$.

In the case of $\overline{B}_m(n)$, the results obtained thus far can be extended in a straightforward manner.

(6) $\overline{B}_m(n) = m\overline{B}_1(n) = m(n-1)$.

This time is required in star networks, which allow at most one call during any time unit throughout a multiple message broadcast.

By analogy, the following conjecture can be posed concerning $\underline{B}_m(n)$:

(7') $\underline{B}_m(n) = m\underline{B}_1(n) = m \lceil \log_2 n \rceil$.

That the conjecture is invalid can be demonstrated by viewing broadcasting as a process of initiating and clearing the initial m-1 message followed by broadcasting the last message. Messages are sequentially placed into a "pipeline" which passes through all members. A message is considered to be initiated and cleared as soon as a new message can be placed into the pipeline such that the new message reaches another member only after it has completed processing *of the initiated and cleared message. Let $\underline{IC}_m(n)$ be the minimum time required to initiate and clear m messages in a connected network of n members. As such, we may now conjecture that*

$$(8') \quad \underline{B}_m(n) = \underline{IC}_{m-1}(n) + \underline{B}_1(n) \quad .$$

In a network with more than three members, either the originator or the first member called must forward a message during a second time unit before the message is initiated and cleared. In a cycle network, if both of these members forward a message, the message is initiated and cleared. A cycle network is a unicyclic graph with a maximum member degree of 2. Thus, we may rewrite (8'), conjecturing that

$$(9') \quad \underline{B}_m(n) = 2(m-1) + \lceil \log_2 n \rceil \ .$$

Though a more general view of broadcasting will be required to finally determine the value of $\underline{B}_m(n)$, the pipeline view does provide a basis for conjecturing bounds on broadcasting within a specific network. Given a connected network G, let d_{max} be the maximum degree of a member in G. The time to initiate and clear a message is equal to the maximum number of calls in which any member must participate during the broadcast of any one message. This number is less than or equal to d_{max}. Let D be the diameter of G, being the maximum distance between any two members of G. The distance between two members is the minimum number of edges on a path connecting them. The time to broadcast one message in G requires at least D time units. Assuming G has n members, we have

$$(10) \quad 2(m-1)+D \leq B_m(G) \leq d_{max}(m-1) + (n-1) \quad .$$

The lower bound is realized in cycle networks while the upper bound is realized in path and star networks. Since the diameter of a 4, 5, or 6 member cycle network is equal to $\lceil \log_2 n \rceil$, the bound conjectured in (9') is realized in these networks. Any cycle can out perform the bound conjectured in (7') when m is sufficiently large.

Answers to the following two questions will allow a more exact determination of $\underline{B}_m(n)$:

(i) *What is the minimum number of calls required to broadcast m messages throughout a network of n members?*

(ii) *What is the maximum number of calls that can be placed during the initial t time units of broadcasting?*

The answer to the first question, call it $C_m(n)$, is simply $m(n-1)$. Each message requires a minimum of $n-1$ calls in order to be transmitted to the $n-1$ initially uninformed members of the network. The answer to the second question, call it $C_t(n)$, follows from two observations. During the initial $\lfloor \log_2 n \rfloor$ time units the number of calls placed during each successive time unit can at most double. After that, at most $\lfloor n/2 \rfloor$ calls can be placed during each time unit. Therefore,

(11) $\quad C_m(n) = m(n-1);$

(12) $\quad C_t(n) = 2^t - 1, \text{ for } t \le \lfloor \log_2 n \rfloor;$

$$2^{\lfloor \log_2 n \rfloor} - 1 + (t - \lfloor \log_2 n \rfloor)\lfloor n/2 \rfloor, \quad \text{for } t > \lfloor \log_2 n \rfloor .$$

This suggests the following conjecture,

(16') $\quad \underline{B}_m(n) = \min_t\, [C_t(n) \ge C_m(n))]$

Since it requires at least $\lceil \log_2 n \rceil$ time units to broadcast one message, this can be rewritten as

$$(17') \quad \underline{B}_m(n) = \left\lceil \frac{m(n-1) - 2^{\lfloor \log_2 n \rfloor} + 1}{\lfloor n/2 \rfloor} \right\rceil + \lfloor \log_2 n \rfloor$$

When there is an odd number of members in G, this reduces to

(18') $\quad \underline{B}_m(n) = 2m - 1 + \lfloor \log_2 n \rfloor$, for odd n.

To verify conjecture (18') it remains to define a class of graphs and a calling method which realizes the conjectured bound in those graphs. A calling scheme is a specification of a calling method which indicates how many calls are to transmit each message during each time unit without actually assigning pairs of senders and receivers. A legal calling scheme is a calling scheme for which there exists an assignment of senders and receivers satisfying the assumed restrictions on information dissemination within a complete network. A complete network is one in which every member is directly connected to all other members.

The following calling scheme realizes the conjectured lower bound for an odd number of members. Let the messages be numbered from 1 to m in order of their initiation during broadcasting. There are three phases to the calling scheme, as follows:

Phase 1 - During the first $\lfloor \log_2 n \rfloor$ time units let each informed member transmit a message. The broadcast originator initiates a new message during each time unit if one exists, otherwise it continues to transmit the last message originated.

After Phase 1, the first message originated will have been transmitted to (at least) $2^{\lfloor \log_2 n \rfloor - 1}$ *members. The remaining calls will have been distributed among another* $\lfloor \log_2 n \rfloor - 1$ *messages in numbers equal to decreasing powers of 2. If there are fewer than* $\lfloor \log_2 n \rfloor$ *messages being broadcast, the last message acquires the remaining calls.*

Phase 2 - Phase 2 is required when more than one message is being broadcast. The following process is repeated for messages 1 to $m-1$ as current message. Let the current message be transmitted to $\lfloor n/2 \rfloor - 2^{\lfloor \log_2 n \rfloor - 1}$ members. Let all members informed of a higher numbered message transmit that message. The broadcast originator initiates a new message or continues to transmit the last message, as above.

After Phase 2, the initial $m-1$ messages will have been transmitted to $\lfloor n/2 \rfloor$ members. The last message will have been transmitted to a $2^{\lfloor \log_2 n \rfloor} -1$ members. Phase 2 requires $m-1$ time units. During each time unit $\lfloor n/2 \rfloor$ calls are placed.

Phase 3 - Repeat the following process for each of the initial $m-1$ messages as current message. Let $\lfloor n/2 \rfloor$ members informed of the current message call the remaining $\lfloor n/2 \rfloor$ uninformed members. Finally, let the last message be transmitted to the $n-2^{\lfloor \log_2 n \rfloor} -2$ remaining uninformed members.

Phase 3 requires m time units. The overall scheme requires $2m-1 + \lfloor \log_2 n \rfloor$ time units. Each message has been transmitted to $n-1$ members after completion of the scheme. Table 1 presents a profile of broadcasting progress for 7 messages being broadcast through a network of 21 members. The ith column represents the number of members to which the ith message has been transmitted. The kth row indicates the state of broadcasting after k time units.

The conjecture of (18') will be verified if the above calling scheme is legal. Phases 3 and 1 are obviously legal, as the number of members informed of the messages transmitted during each time unit is less than or equal to $n-1$ (not including the broadcast originator) after the unit. Phase 2 is also legal. The sets of members informed of the messages to be transmitted during a given time unit do not intersect. Therefore, those members not required to transmit the current message can be called without redundancy or conflict by members transmitting other messages during the time unit.

The time conjectured in (18') can be realized for networks with even n, also. During Phase 2, only $(n/2) - 1$ calls need be placed during each time unit. During Phase 3, the message originator joins the $(n/2)-1$ other informed members in completing broadcasts of each of the first $(m-1)$ messages. The resultant time is at most $(2m/n)+1$ time units more than the time conjectured by (17').

The calling scheme described above can be modified to produce the time predicted by (17'). A profile of broadcasting 10 messages through 14 members, as presented in Table 2, illustrates the modification. During Phase 3 both the current and next message are transmitted. A time unit can be saved when both of these complete broadcasting during the same time unit (see starred row of Table 2). The legality of this calling scheme has yet to be established.

Discussion

We have determined exact values for $\overline{B}_m(n)$ for all n, and for $\underline{B}_m(n)$ when n is odd. In the case of $\underline{B}_m(n)$ when n is even, a legal calling scheme has been described which approximates the conjectured lower bound. A calling scheme which actually produces the optimal time for even n has been described, but is not known to be legal.

Recall that one message cannot be broadcast in less than $\lceil \log_2 n \rceil$ time units. However, if more than $\lfloor \log_2 n \rfloor$ messages are broadcast, the time per message is less than 3 time units for any n. These results suggest consideration of the following two questions: Given a message to be broadcast, can time be saved by breaking it into a number of equal length submessages and broadcasting the resultant set of messages? If so, how many submessages will yield the minimum broadcast time?

Assume that the time to broadcast a message is the sum of two time components: C - a fixed-time factor which represents constant overhead for each call (ie., time to create a header and establish a channel), and M - a variable-time factor which is directly related to the length of the message. Assume that n is not a power of 2. This simplifies the following analysis, as $\lfloor \log_2 n \rfloor$ is then equal to $\lceil \log_2 n \rceil -1$.

If the message is broadcast as a single unit, broadcast time is equal to $(M+C) \lceil \log_2 n \rceil$. If the message is broadcast as K equal length submessages, according to the legal calling scheme described above, broadcast time is equal to $\left(\frac{M}{K} + C\right)\left(2K-2 + \lceil \log_2 n \rceil\right)$. The first question can be answered in the affirmative. The second time will be less than or equal to the first when C is less than or equal to $M\left(\left(\lceil \log_2 n \rceil - 2\right)/2K\right)$. Furthermore, broadcast time is minimized when $K \approx \frac{\left(\lceil \log_2 n \rceil - 2\right) M}{2C}$.

For example, assume a network with 40 members. In order to minimize broadcast time of a message with length such that C is equal to (M/8), K should be 4. Broadcasting the message as one unit would require (27M/4) time units while broadcasting it as 4 submessages would require (9M/2) time units.

In closing, several issues of interest remain. Given a known time for the constant overhead factor and a known rate for the variable-time factor, the results above can be used to determine maximum length for messages to be broadcast as one unit. Determination of networks which allow minimum time broadcasts of multiple messages but require fewer lines than complete networks is also an open problem. Problems encountered in broadcasting in a network with dynamic routing have been recently discussed [4] and can be further explored. Initial results as to the maximum number of members throughout which m messages can be broadcast in t time units have been reported [5] and can be extended.

References

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Table 2

MESSAGE

		1	2	3	4	5	6	7	8	9	10	
	0	0	0	0	0	0	0	0	0	0	0	
	1	1	0	0	0	0	0	0	0	0	0	Phase 1
	2	2	1	0	0	0	0	0	0	0	0	
	3	4	2	1	0	0	0	0	0	0	0	
	4	7	4	2	1	0	0	0	0	0	0	
	5	7	7	4	2	1	0	0	0	0	0	
	6	7	7	7	4	2	1	0	0	0	0	
	7	7	7	7	7	4	2	1	0	0	0	
Time	8	7	7	7	7	7	4	2	1	0	0	Phase 2
Unit	9	7	7	7	7	7	7	4	2	1	0	
	10	7	7	7	7	7	7	7	4	2	1	
	11	7	7	7	7	7	7	7	7	4	3	
	12	7	7	7	7	7	7	7	7	7	7	
	13	13	8	7	7	7	7	7	7	7	7	
	14	13	13	9	7	7	7	7	7	7	7	
	15	13	13	13	10	7	7	7	7	7	7	
	16	13	13	13	13	11	7	7	7	7	7	Phase 3
	17	13	13	13	13	13	12	7	7	7	7	
	* 18	13	13	13	13	13	13	13	7	7	7	*
	20	13	13	13	13	13	13	13	13	8	7	
	21	13	13	13	13	13	13	13	13	13	9	
	22	13	13	13	13	13	13	13	13	13	13	

Table 1

MESSAGE

Time Unit	1	2	3	4	5	6	7	
0	0	0	0	0	0	0	0	Phase 1
1	1	0	0	0	0	0	0	
2	2	1	0	0	0	0	0	
3	4	2	1	0	0	0	0	
4	8	4	2	1	0	0	0	
5	10	8	4	2	1	0	0	Phase 2
6	10	10	8	4	2	1	0	
7	10	10	10	8	4	2	1	
8	10	10	10	10	8	4	3	
9	10	10	10	10	10	8	7	
10	10	10	10	10	10	10	15	
11	20	10	10	10	10	10	15	Phase 3
12	20	20	10	10	10	10	15	
13	20	20	20	10	10	10	15	
14	20	20	20	20	10	10	15	
15	20	20	20	20	20	10	15	
16	20	20	20	20	20	20	15	
17	20	20	20	20	20	20	20	