

IDENTIFYING TOPOGRAPHIC SIGNATURES OF RIVER NETWORK
DEVELOPMENT IN VARIED TERRAINS OF THE PACIFIC NORTHWEST,
USA

by

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DISSERTATION ABSTRACT

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Title: Identifying Topographic Signatures of River Network Development in Varied Terrains of the Pacific Northwest, USA

Geomorphology leverages the competition between processes of uplift and erosion to infer geologic time-scale forcing on the earth's upper crust. While this framework has revolutionized our understanding of global mountain building, it has mostly been applied to relatively simple tectonic landscapes and few tools exist for applying such analysis to more complex volcanic terrains. Such settings, and particularly continental volcanic arcs, make up some of the earth's most geologically active landscapes and are thus of broad scientific interest (Hilley et al. (2022)). In this work we focus on varied terrains of the Pacific northwestern region of the USA to develop tools and workflows that can be applied to the study of the Cascade Volcanic arc.

In Chapter II we derive a land form classification scheme based on classical surface theory that can be used as an objective means of studying topography. We focus this analysis on the Oregon Coast range, a region studied for its assumed steady-state characteristics. We show the efficacy of our curvature-based approach in identifying process regimes that define the evolution of fluvial landscapes, and show how land surface geometry varies as a function of drainage area.

In Chapter III we apply the methods used in Chapter II to probe timescales of fluvial development in the Central Oregon Cascades. Here a well known state-shift in hydrology is driven by weathering processes in aging volcanic bedrock and so the degree of fluvial development can inform both bedrock age and hydrologic regime (Jefferson, Grant, Lewis, and Lancaster (2010)). We show that this evolution is marked by an increase in the magnitude of land surface curvatures providing an objective means of studying this state-shift that does not depend on modern catchment delineations. We show that a full transition from volcanic constructional to fluvially dissected topography occurs over ~ 1 My, which is consistent with past estimates (Jefferson et al. (2010)).

In Chapter IV we use an interdisciplinary approach to show how magmatism is expressed in mountain range scale land forms of the Columbia River Gorge where a cross section of the Cascade Arc has been exposed by persistent incision of the Columbia River. We combine modelling of fluvial knickpoints with a flexural uplift model applied to a geologic structural datum to show the relative influence of intrusive and extrusive magmatism to land form development. We also show how our model for the magmatic origin of this section of the Cascades is supported by heat flow, bouguer gravity, and surface wave tomography data.

Together the chapters of this dissertation show how through multidisciplinary approaches geomorphology can aid in disentangling the landscape evolution history of complex volcanic terrains.

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CHAPTER I

INTRODUCTION

Geomorphology is the study of how the geometry of topographic surfaces relates to the geologic processes that shape them. Much of the modern science is rooted in the observation that land is sculpted through competition between forces that uplift the landscape - such as tectonics, volcanism, and mantle dynamics - and erosional processes that modulate topographic growth by transporting uplifted material to sediment sinks where it is more gravitationally stable. Most erosion processes are gradient driven, and so all else equal higher landscape uplift rates, which generate more relief, cause faster erosion and overall steeper topography (Whipple, DiBiase, and Crosby (2013)). This basic conceptual framework has inspired generations of geomorphologists to find quantitative methods for relating topographic form to erosion rates with the goal of mapping uplift variation and thus better understanding spatial and temporal patterns of deeper forcing on the landscape (Wobus et al. (2006)). This in turn requires disentangling effects of climatic, lithologic, and biologic variation on topographic development and so most workers have focused on landscapes where some variables can be assumed constant. As a result there are few applications of geomorphology to terrains built through combinations of tectonics and magmatism.

Volcanic arcs comprise some of the earths most geologically active terrains, and have been identified as priority settings for focused research efforts in coming decades (Hilley et al. (2022)). Here repaving of topography by eruptive products makes it difficult to disentangle geologic histories using traditional mapping techniques; thus the development of geomorphology tools that could be applied to understand geologic histories of arcs could revolutionize our understanding of

such settings. One reason it is challenging to do geomorphology studies in volcanic settings is that most studies surface process models leverage geometric relationships that are directly associated with a certain process, and there are relatively few metrics of topographic form that are decoupled from process and thus can be used to quantify local variation in lithology, hydrology, and uplift rate characteristic of volcanic settings. The primary goal of this work is to develop quantitative tools and approaches of topographic analysis that can be used in settings where assumptions common to most geomorphology studies break down towards use of topography as a dataset for studying the evolution of the Cascades volcanic arc.

In Chapter II we look at a section of the well studied Oregon Coast Range and perform a rigorous calculation of topographic surface characteristics that builds directly on the canonical work by Carl Frederick Gauss (Gauss (1902)). We demonstrate how this approach provides an abundance of information not previously used in landscape evolution contexts, and show how the curvature of the landscape evolves with drainage area thus putting these novel metrics in the context of standard approaches of geomorphology. In doing so we derive a quantitative metric of landscape form that is independent of process, but can be easily linked to process transitions throughout this characteristic fluvial landscape. While this chapter has no explicit connection to the volcanic landscape of the Cascade Volcanic Arc we use the Coast Range as a test location for defining the tool and demonstrate its connections to more standard slope, area, and curvature metrics common in the literature.

In Chapter III we apply some of the tools derived in Chapter II to the Oregon Cascades, where a hydrologic state-shift has been connected to a decrease in permeability over time in basalts sourced from the arc (Tague and Grant

(2004)). While the timescale of this state-shift has already been explored using morphological approaches (Jefferson et al. (2010); Luo, Grudzinski, and Pederson (2010)), a dependence on drainage density in these approaches limits their applicability to specific regions where fluvial processes are present and hydraulic watersheds are plausibly defined by topography. Since young volcanic landscapes, such as the High Cascades, are groundwater dominated with no consistent connection between topography and hydrology (Gannett, Lite, Morgan, and Collins (2001)) we argue that our curvature based approach, which makes no assumptions about process, provides a more general approach for exploring the complex coupling of hydrology and topography in volcanic landscapes. We confirm past estimates for a timescale of ~ 1 My for this transition to take place.

Chapter IV is an example of how geomorphology can be used to probe dynamics of arc construction, and focuses on the topographic development of the Columbia River Gorge. Here we use an approach that combines the analysis of fluvial knickpoints with flexural modelling of the upper crust as constrained by deposits of a Columbia River paleo-channel. We correlate the observed deformation with heat flow, gravity, and seismic tomography data, all of which are consistent with ongoing mid-crustal magmatism as a driver of uplift in the Columbia River Gorge. The cross section provided by the Columbia allows us to partition uplift into relative contributions from intrusive and extrusive magmatism and show that the longer-wavelength intrusive component exerts a stronger control on the topographic response, likely due to the higher permeability of erupted deposits.

This dissertation serves as a template for how concepts of geomorphology can be built upon to provide a means of better understanding complex volcanic settings, however more broadly is an example of how interdisciplinary research

can be used to tackle challenging problems in the earth sciences. The technical methods used in Chapters II and III combine geomorphology with formal surface theory, while Chapter IV combines technical analysis of several different datasets, which together paint a holistic picture of volcanic mountain building that cannot be discerned from any one of the data sets alone.

CHAPTER II

THE INTRINSIC AND EXTRINSIC GEOMETRY OF LANDSCAPES

2.1 Connecting Landscape Form to Process

The majority of research on erosion dynamics has been done in fluvially dissected landscapes where sediment transport is driven by river networks. Quantitative models of erosion in these settings are inherently scale dependant with different models granting different insight. For example, at the scale of a mountain range average elevations tracks deformation of the crust and individual erosional features appear as noise about this long wavelength trend. For some land forms at this scale erosion has been modeled simply as a diffusive process governed by the heat equation

$$z_t = \kappa_s \nabla^2 z \quad (2.1)$$

where z is elevation, $\nabla^2 z = z_{xx} + z_{yy}$ is the Laplacian of the surface, and κ_s is an empirical diffusivity governing the rate of topographic relaxation (Ruh (2020); Watts (2001)) (note that in this work we use subscript notation for partial derivatives, which is consistent with the curvature literature). This approach allows for the calculation of the landscape relaxation time following orogenic events and can easily admit basic uplift forcings such as the isostatic response of the crust to erosional unloading (Watts (2001)). This is useful for understanding long timescale cycles of uplift and erosion and the connections between ancient tectonic events and the modern rock record, however does little to put modern topography in context of these orogenic cycles. Modeling erosion as purely diffusive does little to elucidate the diverse physical mechanisms at play, and therefore is limited in its predictive power.

On the other end of the spatial scale there is a rich body of literature focused on building mechanistic models of individual erosion processes at the outcrop/field scale down to the scale of individual sediment particles. For example within fluvial channels the popular *stream power* family of models (Bagnold (1966); Howard and Kerby (1983); Seidl, Dietrich, and Kirchner (1994)) show the potential of average shear stresses within rivers to mobilize sediment, while an adjacent literature focuses on connections between fluid turbulence and the development of observed bed structures (A. A. Hurst, Anderson, and Crimaldi (2021); Li, Venditti, Rennie, and Nelson (2022); MacVicar and Roy (2007); Stoessner, Kara, MacVicar, and Best (2010); Venditti (2007); Venditti et al. (2014); Wrick and Pasternack (2008)). The last decades have also seen the development of hillslope sediment transport models with both continuum mechanics (Gabet (2003); Lamb, Scheingross, Amidon, Swanson, and Limaye (2011); Roering, Kirchner, and Dietrich (1999); Roering, Perron, and Kirchner (2007)) and statistical mechanics (Furbish, Haff, Dietrich, and Heimsath (2009); S. G. Williams and Furbish (2021)) approaches, as well as models describing landsliding (Booth, Roering, and Rempel (2013)), debris flow processes (Struble, McGuire, McCoy, Barnhart, and Marc (2023)), gully retreat (Flores-Cervantes, Istanbuluoglu, and Bras (2006)), plunge-pool formation (Scheingross and Lamb (2017)) and a host of other location specific modes of sediment mobilization and transport.

Outcrop scale models and observations are useful because they can be calibrated with laboratory scale physical models (Baynes et al. (2018); Deshpande, Furbish, Arratia, and Jerolmack (2021); Gabet (2003); Lamb and Dietrich (2009); Pasternack, Ellis, Leier, Vallé, and Marr (2006)), and also be connected to locally derived erosion rates using tools like thermochronology (Nie et al. (2018);

Simoes, Braun, and Bonnet (2010)) and cosmogenic nuclide analysis (Gayer, Mukhopadhyay, and Meade (2008); Hippe et al. (n.d.)). In addition they have provided the foundation for a growing ability to quantify the dynamic stability of landscapes towards the forecasting of geologic hazards. The weakness of this approach is that individual mechanistic models are difficult to connect across process regimes. Studies tend to compartmentalize erosion into isolated regions that don't reflect the continuum nature of landscapes, or capture multiscale feedbacks that may play a large roll in topographic development.

Between the observational scales defined above is a range of scales at which fluvial landscapes are defined by dendritic networks of fluvial channels divided by ridge networks to which they are connected by hillslopes and first order channels. These are the scales at which it is easiest to connect topographic form to the temporal evolution of the landscape and deep uplift forcings, which is often done through implementation of landscape evolution equations of the basic form

$$z_t = U - E \tag{2.2}$$

where $z = z(x, y, t)$ is elevation, $U = U(x, y, t)$ is uplift rate, and $E = E(x, y, t)$ is the erosion rate Braun and Willett (2013); Whipple and Tucker (1999). In many studies erosion is broken into two terms defining fluvial and hillslope processes. The fluvial component is generally defined along a river longitudinal profile using a version of the stream power model written as $E(s) = KA(s)^m|S(s)|^n$ where $s = s(x, y)$ is an along stream coordinate, K, m , and n are empirical constants that capture effects of lithology, hydrology and climate, $A(s)$ is drainage area upstream of a point in the channel (itself a proxy for discharge), and $S(s)$ is along-channel slope (Whipple and Tucker (1999)). The hillslope flux is usually derived by defining a relationship between slope and sediment flux. While steep

hillslopes are best modeled using a non-linear relationship (Roering et al. (1999, 2007)), the simplest models assume sediment flux is a linear function of slope in which case hillslope erosion can be written as $E = \nabla \cdot \mathbf{q} = D\nabla^2 z$ where $\mathbf{q}$ is slope dependent sediment flux and D is a diffusivity that quantifies the efficiency of hillslope transport processes (Barnhart et al. (2020)). Note that in this case curvature $\nabla^2 z$ is defined as positive downwards, which differs from much of the geomorphology literature (Roering et al. (2007)) but will be consistent with the derivation of invariant curvature metrics in section 2.3.

With the above definitions of erosion equation 2.2 can be written in terms of competition between uplift and erosion as

$$z_t = U - KA^m |\nabla z|^n - D\nabla^2 z. \quad (2.3)$$

Modelling at the landscape scale has proven powerful for understanding timescales of topographic development, and factors that determine how uplift (or climate) perturbations propagate through the landscape. It has the benefit of being able to encompass transitions in process dominance between channels and hillslopes, but has the disadvantage of blurring over transitional processes that are likely important drivers of landscape adjustment. Landscape evolution governed by equation 2.3 depends strongly on parameter values that are difficult to constrain and likely not constant throughout the landscape as well as in time (Snyder, Whipple, Tucker, and Merritts (2000)). Thus landscape evolution model results are nearly always non-unique, and less mechanistic than may be desired in many cases: it is challenging to calibrate semi-empirical fluvial and hillslope erosion laws over the full range of relevant scales (Venditti, Li, Deal, Dingle, and Church (2019)).

A commonality between landscape evolution models at all scales, is that sediment transport rates are usually assumed to be recorded in metrics of land

surface geometry, such as slope and curvature. This builds on ideas established in the late 19th century when work by G.K. Gilbert and W.M. Davis suggested connections between hillslope convexity and rates and styles of denudation in mountain terrains (Davis (1892); Gilbert (1877)). These early observations, and subsequent papers (Fenneman (1908); Gilbert (1908)) showed that both slope and curvature near drainage divides is related to erosion rates. These early works noted spatial variation in the dominance of different erosion processes and began the effort, ongoing today, to partition the landscape into areas defined by particular mechanisms of erosion.

It is generally accepted that fluvial networks are organized by the accumulation of drainage area (and thus discharge), which gives rise to a rich and complex tiling of drainage basins that can appear to have sometimes fractal (Claps and Oliveto (1996)) or even periodic structure (Perron, Kirchner, and Dietrich (2009)). This quality was first defined by John Tilton Hack (Hack et al. (1957)) who showed that the lengths of stream channels in select regions of Virginia and Maryland are at any point in the channel related to upstream drainage area (A) via the power-law expression $L = cA^b$ where c and b are empirical constants (Figure 1).

This relationship has proven remarkable in its ability to describe the geometry of networks ranging from the laboratory to continental scale with an average value for the scaling exponent of $b \approx 0.6$, though the value does exhibit spatial variation (Cheraghi, Rinaldo, Sander, Perona, and Barry (2018)). Hack also noted a power-law relationship between drainage area and channel slope, which later work by J.J. Flint (Flint (1974)) would suggest is indicative of an equilibrium tendency. In modern studies this relationship - termed *Flints Law* - is often leveraged for the sake of landscape process partitioning, and the evaluation

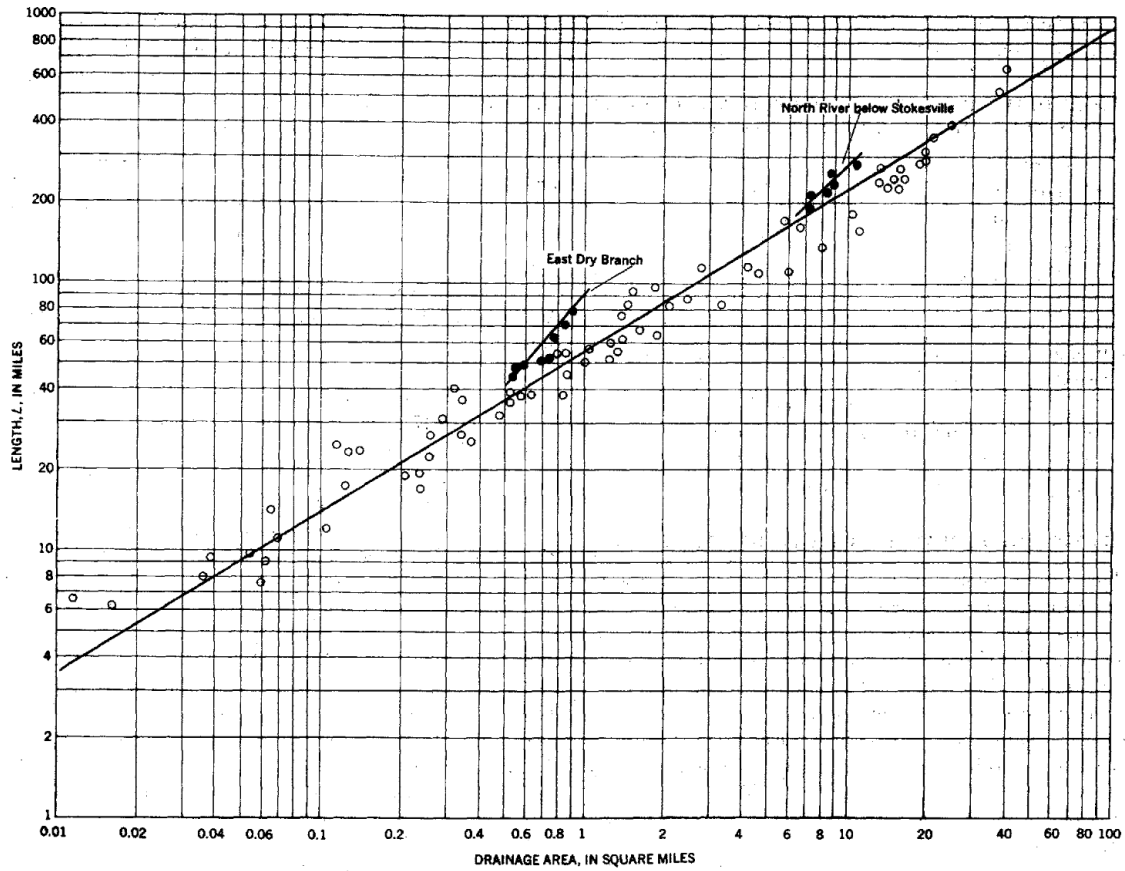


Figure 1. Figure from Hack et al. (1957) showing the power-law relationship between stream length and upstream drainage area, now known to as *Hack's Law*, in several catchments in Maryland and Virginia.

of the degree of equilibrium of fluvial networks. In particular, considerable effort has gone into identifying the locations and causes of scaling breaks in binned plots of slope-area data, which have been correlated both to process transitions, and the transmission of signals of dynamic adjustment in landscapes (Montgomery and Foufoula-Georgiou (1993); Stock and Dietrich (2003)). In this work we quantify the evolution of topographic curvature in such drainage area space, seeking better resolution on the location of process transitions and to gain insight in to the organizational structure of fluvial landscapes.

2.2 Connecting topographic curvature to classical surface theory

The term “curvature” is not associated with a single geometric entity, but is rather an entire class of mathematical operations that describe the degree to which a surface (or more generally, a manifold) deviates from being planar. It is an invariant quality of a surface, independent of chosen coordinates and unchanged under linear transformations. Measures of curvature are often classified as either extrinsic or intrinsic, where extrinsic measures depend on an externally defined reference frame (for example, a higher dimensional space in which the surface is embedded) while intrinsic measures do not. A common conceptualization of this idea states that intrinsic curvature must be built on information that would be readily available to tiny inhabitants of a surface who have no external perspective, or knowledge of any additional dimensions within which the surface is embedded (Needham (2021); Struik (1950)).

Drawing connections between extrinsic and intrinsic curvature has occupied some of the greatest scientific minds of the last several centuries (Needham (2021); Penrose (2004)) and has led to significant scientific discoveries in that time. Perhaps the most striking example is Albert Einsteins 1915 hypothesis that

gravity reflects the intrinsic curvature of 4-dimensional space-time bent by local concentrations of mass/energy (Einstein (1982)). This realization revolutionized cosmology, heralded the modern era of satellite technology, and showed that while our 3-d human perspective struggles to interpret the 4-d space we inhabit much is learned from considering the intrinsic properties of systems embedded in higher dimensional spaces. For the purpose of geomorphology, a combined extrinsic/intrinsic perspective of surfaces viewed as 2-d manifolds embedded in a 3-d space allows us to derive a rich description of topographic form. We will show that standard surface characterization from 2-d map projections lose significant information compared to a more rigorous curvature analysis.

There are a variety of methods for curvature calculation presented in the geomorphology literature. For example Shary (1995) used a similar conceptual framework to that of this chapter, but with a more explicit connection to the gravity field, to derive 12 curvature classes that they argued could be used to classify the landscape and possibly gain insight into the degree of landscape equilibrium. Passalacqua, Do Trung, Foufoula-Georgiou, Sapiro, and Dietrich (2010) used curvature of constant elevation lines in combination with drainage area thresholding to construct a method for channel extraction. Minár, Evans, and Jenčo (2020) presented an extensive list of possible land surface curvature metrics and proposed possible links to topographic equilibrium. Finally, Schmidt, Evans, and Brinkmann (2003) derived the same invariant metrics presented in this chapter for 2-d polynomial fits of topography. The goal of our work is not to present new definitions of curvature, but rather present a system of curvature metrics that are useful in landscape evolution contexts. In landscape evolution studies “curvature” usually refers to an approximation given by the Laplacian operator, either applied

directly to DEMs or 2-d polynomial fits of elevation data (M. D. Hurst, Mudd, Walcott, Attal, and Yoo (2012); Perron et al. (2009); Roering et al. (1999); Schmidt et al. (2003)). Such measures, which come directly from mechanistic derivations assuming diffusive behavior, are extrinsically defined and thus depend on the choice of coordinate system, and orientation within that reference frame. This means that such methods admit systematic error that scales with slope and deformation of the surface. In 1-dimension slope dependent error of a second partial derivative curvature (d^2z/dx^2), shown in Figure 2 can be easily quantified through comparison to the standard calculus definition of curvature (k)

$$k = \frac{d^2z/dx^2}{[(1 + dz/dx)^2]^{3/2}} \quad (2.4)$$

with the error given as (Bergbauer and Pollard (2003) and references therein)

$$error(\%) = \frac{k - d^2z/dx^2}{k} \times 100 = 1 - [1 + (dz/dx)^2]^{3/2} \times 100. \quad (2.5)$$

In 2-dimensions error is much more complicated and also depends on the curvature and orientation of the surface relative to the defined coordinates Bergbauer and Pollard (2003). Thus curvature methods that depend on externally defined global coordinate systems are limited in their utility on complex surfaces motivating our more formal definition of land surface curvatures.

Real Earth surface topography is nearly always approximated by a gridded discrete set of elevation points called a Digital Elevation Model (DEM). To understand topography in the terms of classical surface theory it is necessary to make a conceptual shift from viewing a DEM as a flat map of topography to viewing it as a set of irregularly spaced data points sampling a 2-d surface embedded in a 3-dimensional space. This is an important distinction because while map projections of DEMs assume uniform distances and angles between grid points

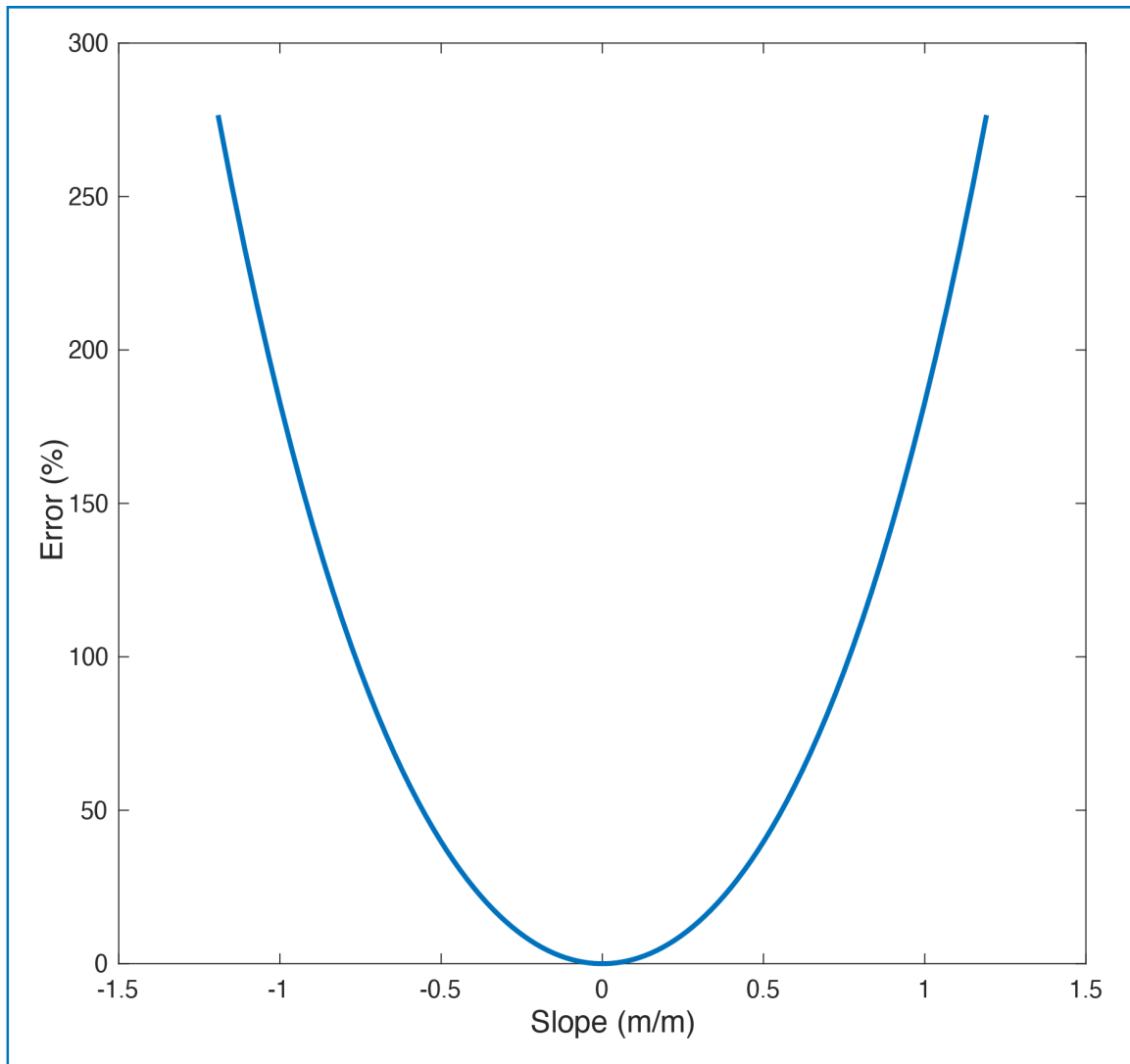


Figure 2. Error in curvature defined as second partial derivative as a function of slope along a 1-d curve.

it is mathematically impossible to map a surface onto a lower dimensional manifold without introducing distortion, a fact proven by mathematician Leonard Euler in 1775 (Needham (2021)). It follows that surface metrics calculated on gridded elevation data admit this distortion and introduce an additional source of error that is difficult to quantify.

The effects of the projection process can be seen in Figure 3, which compares the distances and angles of a map projection (Figure 3.A) to those of the same grid lines overlain on a 3-d model of the surface (Figure 3.B) . It can be seen that the E-W and N-S grid lines of the map are not in fact perpendicular on the surface nor are grid cells composed of uniform dimensions. In other words, if one were to walk from point p along vectors defined by the grid lines they would not cover the distance $d\hat{s}$ to arrive at point $\hat{q}$ as implied by the map, but would rather travel the distance ds arriving at point q .

Defining relationships between surfaces and their map representation was a goal of Carl Frederick Gauss, who roughly fifty years after Euler’s treatment of the problem would publish a revolutionary approach for extracting a combination of intrinsic and extrinsic surface information that could be applied within Cartesian coordinate systems (Gauss (1902)). One reason this is challenging is that while Euclidean spaces exhibit global parallelism (the quality that parallel lines remain parallel for ever; Figure 3.A), curved surfaces do not, and so cannot be defined by any single set of global coordinates. Gauss got around this by everywhere defining a local coordinate system comprised of arbitrary intersecting curves (the dashed lines in Figure 3.B) and using them to define variation in the surface about a point.

Because coordinate curves can themselves be defined in terms of a Cartesian reference frame, Gauss was able to relate intrinsic distortion of the surface to

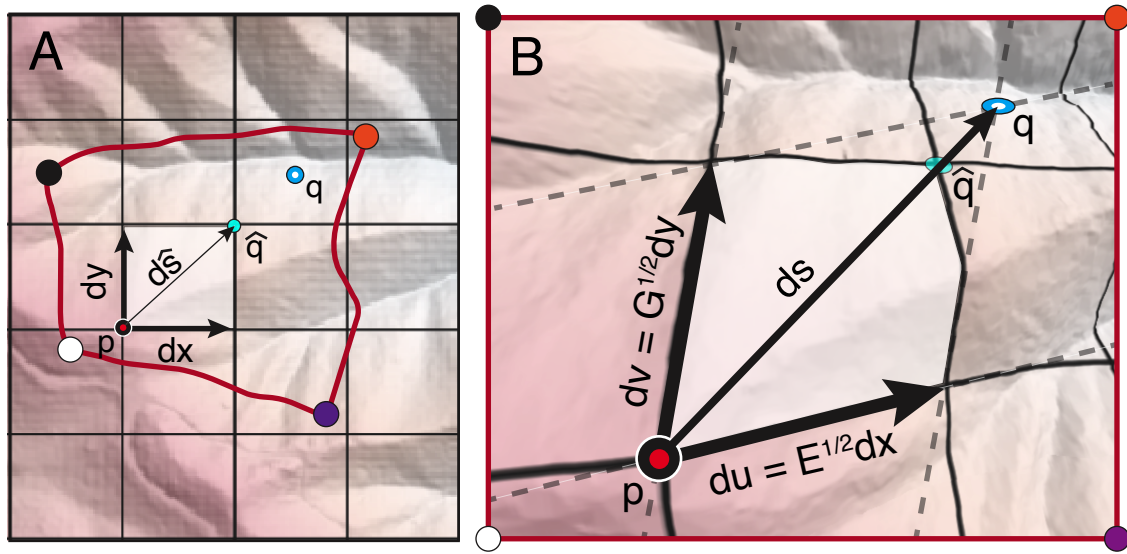


Figure 3. Difference between distances and angles measured on a map projection versus on the surface. A. Map projection of DEM including map grid defined by E-W and N-S lines with grid spacing dx . The red line corresponds to the rectangular outline of panel B. B. DEM viewed as a 2-d manifold embedded in a 3-d space. Dashed lines show a locally defined $u-v$ coordinate system that follows x and y curves on the map projection, but which are not orthogonal or of equal length due to surface distortion. E and G are coefficients of the first fundamental form, and ds is the displacement vector that results moving one grid space along each of these coordinate vectors

extrinsically defined curvatures through two quadratic partial differential equations known as the *first* and *second fundamental forms*. Through this approach Gauss was able to derive a measure of curvature that is independent of the shape of the surface in space, an observation that he highlighted in his *Theorema Egregium* (Remarkable Theorem), and which showed the ability of his method to extract truly intrinsic surface qualities. This discovery inspired an effort to derive a fully general description of surfaces, which led to the proof of the *fundamental theorem of surface theory* in 1867 by Pierre Ossian Bonnet (Cogliati and R (2022)), which states that all qualities of a surface, with the exception of its orientation in space, can be derived using the 6 coefficients of the *first* and *second fundamental forms*.

During the same time period Bernhard Riemann, a student of Gauss, set the stage for modern differential geometry by generalizing Gauss' ideas to higher dimensional spaces and showing that characterizing curvature of an n -dimensional manifold requires $\frac{1}{12}n^2(n^2 - 1)$ distinct components. Thus while a single measure of curvature can be sufficient to characterize a 2-d map projection of topography (or curvature in a temperature field as in the heat equation), fully characterizing topographic curvature in 3-d space requires six distinct components, which are the coefficients of the *first* and *second fundamental forms*. The field of differential geometry was again revolutionized by the development of tensor calculus in 1901 by Gregorio Ricci and Tullio Levi-Civita (Needham (2021)). Their methods combined with the theory of Riemannian manifolds has allowed the field to evolve well beyond the work of Gauss, however we find the original theory is sufficient to shed significant light on the structure of topography. We therefore root our analysis in the fundamental forms and a relatively simple classical definition of topographic curvature.

2.3 Oregon Coast Range

We test our method of geometric classification in the Oregon Coast Range, USA, in the Umpqua River Basin near Reedsport, Oregon. The Oregon Coast Range is one of the most heavily studied terrains in the field of geomorphology and is generally considered to be an archetypal example of a landscape where uplift and erosion are balanced over long timescales (a common definition of ‘equilibrium’ in landscape evolution studies)(Reneau and Dietrich (1991)). The Coast Range is mostly composed of a suite of accreted Eocene turbidites known as the Tyee Formation that is fairly uniform and gently sloped, mostly at angles less than 20 degrees, and so there is thought to be little variation in the lithologic control on erosion (Roering, Kirchner, and Dietrich (2005)). It has a temperate climate with precipitation rates that are fairly uniform throughout. This spatially uniform landscape forcing results in pervasive ridge-valley topography, thought to be indicative of steady fluvial erosion processes .

2.4 Calculation of curvature

In this section we derive the coefficients of Gauss’ two fundamental forms, and use them to derive curvature metrics used in our topographic analysis. While there are many different methods for deriving such measures curvature we base our derivation on that of Struik (1950), which is the same derivation used in similar work presented in Bergbauer and Pollard (2003) and Mynatt, Bergbauer, and Pollard (2007). Everything in the following sections on deriving curvature can be found in chapter 2 of Struik (1950) unless otherwise noted.

2.4.1 Deriving the first and second fundamental forms. Surface curvature at a given point can be defined by considering the 1-d curvature of paths over the surface. Infinitesimal displacements along such a path trace out small

sections of curve (ds) that lie completely within the so-called *osculating plane*, which also contains the unit tangent ($\mathbf{t}$) and normal ($\mathbf{N}$) vectors of the curve. We first define a position on the surface in 3-d Euclidean space as

$$\mathbf{r} = \mathbf{r}(u, v) = r_1\mathbf{e}_1 + r_2\mathbf{e}_2 + r_3\mathbf{e}_3, \quad (2.6)$$

where the parameters u and v represent two intersecting curves on the surface that define a local coordinate system, and $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ are basis vectors representing a Cartesian reference frame. The choice of u and v is completely arbitrary except for the requirement

$$\mathbf{r}_u \times \mathbf{r}_v \neq 0 \quad (2.7)$$

which ensures that a normal vector can be defined.

A tangent vector ($\mathbf{t}$) and unit normal vector ($\mathbf{N}$) can then be defined in terms of the u, v curves as

$$\mathbf{t} = d\mathbf{r}/ds \quad (2.8)$$

and

$$\mathbf{N} = \frac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}. \quad (2.9)$$

respectively, where $d\mathbf{r} = \mathbf{r}_u du + \mathbf{r}_v dv$ is the change in position resultant of a small displacement on the surface and the magnitude of the displacement is given by the Pythagorean Theorem as

$$I = ds^2 = d\mathbf{r} \cdot d\mathbf{r} = Edu^2 + 2Fdudv + Gdv^2. \quad (2.10)$$

Equation 2.10, known as the first fundamental form (I) or surface metric formula, is a measure of distances on the surface, where $E = \mathbf{r}_u \cdot \mathbf{r}_u$, $F = \mathbf{r}_u \cdot \mathbf{r}_v$, and $G = \mathbf{r}_v \cdot \mathbf{r}_v$ (known as the metric coefficients) quantify the proportionality of real distances to distances on a 2-d map representation (Needham (2021)).

The curvature, defined as the change in the tangent $\mathbf{t}$ along ds , can be decomposed into two vector components as

$$\frac{d\mathbf{t}}{ds} = \kappa\mathbf{N} + \kappa_g\mathbf{g} \quad (2.11)$$

where κ is the normal component of curvature, found by projecting the full curvature onto the unit normal vector ($\mathbf{N}$), and κ_g is the geodesic curvature which measures meandering of the path along the surface and points in a direction orthogonal to the path within the tangent plane. Since it is always possible to define ds along a geodesic curve such that $\kappa_g = 0$ Equation 2.11 reduces to

$$\frac{d\mathbf{t}}{ds} = \kappa\mathbf{N} = \left(\frac{d\mathbf{t}}{ds} \cdot \mathbf{N}\right)\mathbf{N}. \quad (2.12)$$

Leveraging the fact that $\mathbf{N} \cdot \mathbf{t} = 0$ we can write

$$\kappa = \frac{d\mathbf{t}}{ds} \cdot \mathbf{N} = -\mathbf{t} \cdot \frac{d\mathbf{N}}{ds} \quad (2.13)$$

where $d\mathbf{N} = \mathbf{N}_u du + \mathbf{N}_v dv$. Combining Equations 2.8 and 2.13 yields the expression for normal curvature

$$\kappa = \frac{d\mathbf{r}}{ds} \cdot \frac{d\mathbf{N}}{ds} = \frac{edu^2 + 2fdudv + gdv^2}{Edu^2 + 2Fdudv + Gdv^2} = \frac{II}{I}. \quad (2.14)$$

In the above equation the expression in the numerator

$$II = d\mathbf{r} \cdot d\mathbf{N} = edu^2 + 2fdudv + gdv^2 \quad (2.15)$$

is the second fundamental form (II) with coefficients $e = \mathbf{r}_{uu} \cdot \mathbf{N}$, $f = \mathbf{r}_{uv} \cdot \mathbf{N}$, and $g = \mathbf{r}_{vv} \cdot \mathbf{N}$ that are measures of directional curvatures that are in turn scaled by the coefficients of the first fundamental form (I) to define the directional change in orientation of the surface.

2.4.2 Finding the principal curvatures. In the previous section a general expression for curvature was defined relative to a single arbitrary path over the surface. The existence of infinite paths through any given point means there are

infinite potential values for curvature, however there exist orthogonal paths along which the maximum and minimum surface curvatures, known as are found. This observation can be attributed to Leonard Euler, who showed that as the osculating plane rotates about an axis defined by the surface normal vector $\mathbf{N}$ the curvature varies systematically as

$$\kappa(\theta) = k_1 \cos^2 \theta + k_2 \sin^2 \theta \quad (2.16)$$

where k_1 and k_2 are the curvature extrema known as the *principal curvatures*. The above equation, known as *Euler's Theorem*, shows that if the principal curvatures are calculated, they can then be used to understand curvature along any path.

To find the directions of the two principal curvatures we first define $\lambda = dv/du$ allowing us to rewrite Equation 2.14 in terms of a single variable as

$$\kappa = \frac{e + 2f\lambda + g\lambda^2}{E + 2F\lambda + G\lambda^2}. \quad (2.17)$$

Since the principal curvatures represent extreme values they must correspond to directions where $d\kappa/d\lambda = 0$, and so we differentiate Equation 2.17 with respect to λ which gives

$$\frac{d\kappa}{d\lambda} = (E + 2F\lambda + G\lambda^2)(f + g\lambda) - (e + 2f\lambda + g\lambda^2)(F + G\lambda) = 0. \quad (2.18)$$

This can be put in the quadratic form

$$(Fg - Gf)\lambda^2 + (Eg - Ge)\lambda + (Ef - Fe) = 0 \quad (2.19)$$

with roots that correspond to the principal curvature directions. The magnitudes of the principal curvatures can be found through a similar approach. Given the condition $d\kappa/d\lambda = 0$ equations 2.17 and 2.18 can be combined to give a more simple expression for the curvature

$$\kappa = \frac{f + g\lambda}{F + G\lambda}. \quad (2.20)$$

Recognizing that

$$E + 2F\lambda + G\lambda^2 = (E + F\lambda) + \lambda(F + G\lambda) \quad (2.21)$$

and

$$e + 2f\lambda + g\lambda^2 = (e + f\lambda) + \lambda(f + g\lambda) \quad (2.22)$$

equation 2.18 can be rearranged to show

$$\frac{f + g\lambda}{F + G\lambda} = \frac{e + f\lambda}{E + F\lambda} \quad (2.23)$$

and so it follows that

$$\kappa = \frac{II}{I} = \frac{f + g\lambda}{F + G\lambda} = \frac{e + f\lambda}{E + F\lambda}. \quad (2.24)$$

If we rewrite the two expressions for curvature give above as

$$f - \kappa F + \lambda(g - \kappa G) = 0 \quad (2.25)$$

and

$$e - \kappa E + f - \kappa F = 0 \quad (2.26)$$

and multiply both equations through by du (remembering that $\lambda = dv/du$) then we arrive at a system of equations in terms of our original u - v coordinate system

$$\begin{bmatrix} e - \kappa E & f - \kappa F \\ f - \kappa F & g - \kappa G \end{bmatrix} \begin{bmatrix} du \\ dv \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (2.27)$$

This has a non-trivial solution if and only if the determinant of the coefficient matrix is zero. Taking the determinate gives

$$(EG - F^2)\kappa^2 - (gE - 2fF + eG)\kappa + (eg - f^2) = 0, \quad (2.28)$$

a quadratic equation in κ of which the roots are the principal curvatures of the surface. We sort these principal curvatures by magnitude, classifying the higher magnitude curvature as k_1 , and the lower magnitude curvature as k_2 .

It should be noted that in more modern literature the information contained in the fundamental forms is often presented in terms of a geometric entity known as the *Shape Operator*, which is the second order tensor given by taking the negative gradient of a normal vector taken along an arbitrary tangent. This is written mathematically as

$$S(\mathbf{t}) = -\nabla_{\mathbf{t}}\mathbf{N} \quad (2.29)$$

where $\mathbf{t}$ is the tangent vector (O'Neill (2006)). In terms of our u - v coordinate curves the first and second fundamental forms can be defined via this approach as $I(\mathbf{u}, \mathbf{v}) = \mathbf{u} \cdot \mathbf{v}$ and $II(\mathbf{u}, \mathbf{v}) = S(\mathbf{u}) \cdot \mathbf{v}$ (Needham (2021)).

In tensor form, also more common in the modern literature, the first fundamental form is described by the so-called *metric tensor*

$$I = \begin{bmatrix} E & F \\ F & G \end{bmatrix}, \quad (2.30)$$

the second fundamental form the *surface curvature tensor*

$$II = \begin{bmatrix} e & f \\ f & g \end{bmatrix}, \quad (2.31)$$

and the shape operator is given by

$$S = I^{-1}II. \quad (2.32)$$

The eigenvalues of the shape operator tensor are the principle curvatures and the eigenvectors define the directions of the paths associated with these curvatures.

The Gaussian and Mean curvatures, which we will define in the following section, are the determinate and trace of this matrix respectively.

An approach to surface curvature calculation for gridded DEM data that utilizes the shape operator can be found in Mynatt et al. (2007) and Pearce, Jones, Smith, McCaffrey, and Clegg (2006) which apply a similar approach to ours in the context of structural geology. While the mathematical simplicity of this approach is appealing, it is considerably more computationally expensive to calculate the shape operator. This in practice limits the scale over which the methods can be applied. For example when curvatures were calculated on a 1000×1000 DEM using a MATLAB code similar to that employed by Mynatt et al. (2007) the calculation took 13 seconds, while our code that instead finds the roots of the quadratic equations defined above gave identical results in 0.11 seconds, which makes the tool much more practical for landscape scale analysis.

2.4.3 Defining the Mean and Gaussian Curvatures and Surface Shape Classes. Once the principal curvatures are found using one of the approaches outlined above they can be used to calculate two useful measures of surface curvature, namely the *Mean* and *Gaussian* curvatures, which themselves measure different characteristics of the surface. The mean curvature follows directly from *Euler's Theorem*, and is the value about which the curvature oscillates in the periodic function defined by equation 2.16, and shown in figure 4.

We calculate the Mean curvature (K_M) from the principal curvatures as

$$K_M = \frac{k_1 + k_2}{2}. \quad (2.33)$$

It is an extrinsic measure of curvature, and so is an measure of the shape and orientation of the surface in reference to our defined coordinate space, but would not be directly observable to inhabitants of the surface. One reason the Mean

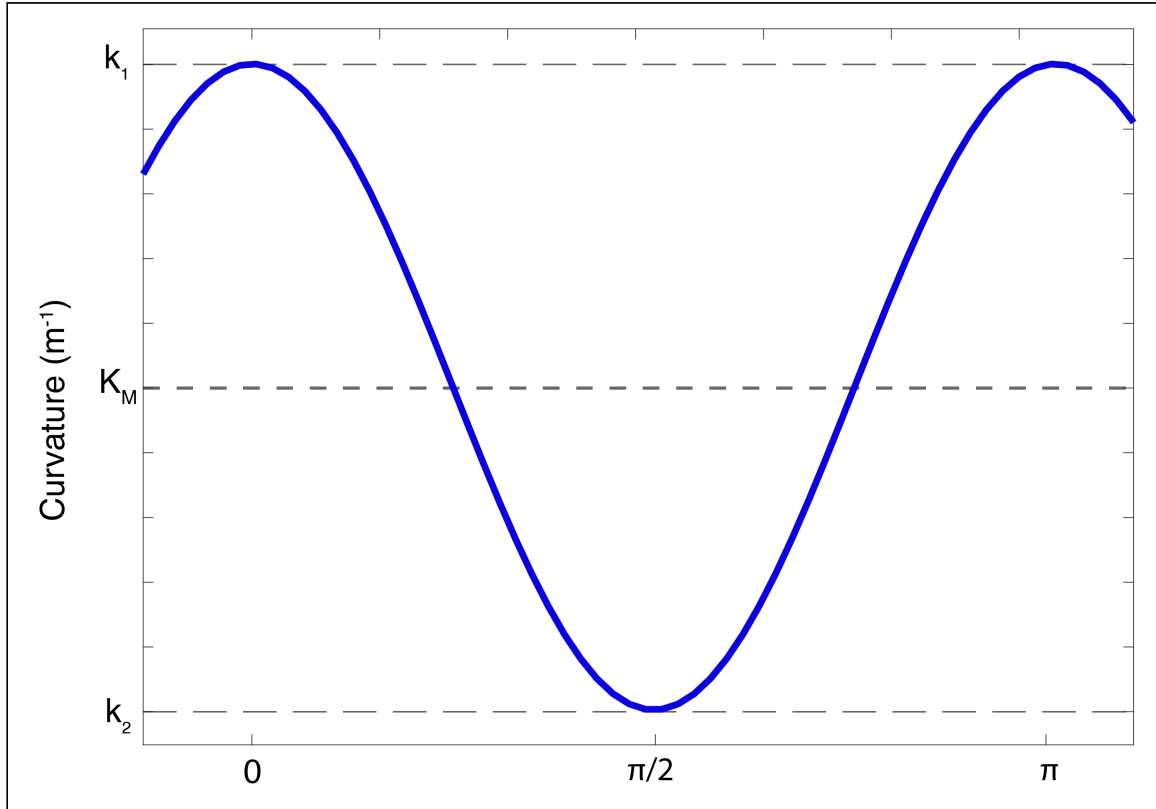


Figure 4. **Curvature as a function of the angle between the osculating plane and a reference direction.** Here the angle $\theta = 0$ is associated with the first principal curvature (k_1). k_2 is the second principal curvature and K_M is the Mean curvature about which the value oscillates.

curvature is useful is that it actually does not require any knowledge of the principal curvatures, but can be calculated as the average curvature of any two orthogonal paths through a given point. We will leverage this in section 2.7 to assess error in curvature calculated using the Laplacian operator, which should theoretically give a value twice that of the Mean curvature at any given point, as long as the coordinate paths are orthogonal and slope error (figure 2; equation 2.5) is accounted for.

The Gaussian curvature is the remarkable measure of intrinsic curvature that Gauss' *Theorema Egregium* noted as a measure of surface deformation that is invariant under isometries, which are spatial transformations that conserve distance (Needham (2021)). This means its value does not actually depend on the shape of the surface, but captures a more subtle quality, which is the degree of stretching or bending required to deform a flat plane to fit a given surface. To a tiny inhabitant of the surface the Gaussian curvature defines the distances between points, and the rate of divergence or convergence of locally parallel geodesic paths, and is thus an extremely fundamental property of surfaces. In terms of the principal curvatures it is calculated simply as

$$K_G = k_1 k_2. \tag{2.34}$$

We note that while we will consistently refer to this curvature as the Gaussian curvature, in the literature you will see it called the intrinsic curvature, total curvature, or sometimes just the curvature (Needham (2021)).

The Mean and Gaussian curvature together can be used to simply classify the geometry about a point into the 8 distinct shape classes shown in Figure 5. Since the Gaussian curvature is the product of the two principal curvatures it will only be positive in instances where k_1 and k_2 have the same sign. This tells us

that when K_G is positive a surface is either a dome or basin, though because the Gaussian curvature is isometrically invariant it does not alone determine which.

If K_G is negative then k_1 and k_2 necessarily have opposing signs and so the surface is locally a saddle, though again with an orientation in space that cannot be uniquely determined by the Gaussian curvature. If either k_1 or k_2 is equal to zero then K_G is also zero. This case itself comprises its own class of surfaces, known as *developable surfaces*, which can be formed from a plane without altering surface area and thus are intrinsically flat. Relatively developable surfaces can be extracted from the landscape by assigning a zero value to curvatures under a defined threshold (Mynatt et al. (2007)). Curvature thresholding to extract developable forms could be a promising way of quantifying the contributions of different landforms to overall curvature distributions, however in the interest of minimizing variables we do not explore this quality of surfaces here.

The sign of the mean curvature tells us the orientation of a shape in space, and so allows us to develop K_G based classifications in a landscape reference frame. K_M is positive in two cases; when either both k_1 and k_2 are positive, or the higher magnitude curvature is positive. This means that points in the landscape with $K_M > 0$ are locally either domes or antiformal saddles. Similarly if K_M is negative then the orientation of the surface must be dominantly concave up, and so the surface is either a basin or synformal saddle.

In rare cases where k_1 and k_2 are equal and opposite the surface is a perfect saddles. This too comprises its own special class of surfaces, known as *minimal surfaces*, which have been largely tied to the optimization of surface area in self-organized systems such as soap foams (Osserman (1969)). The existence of approximate perfect saddles could be shown in the landscape by rounding the

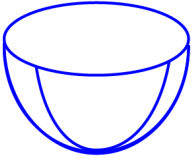
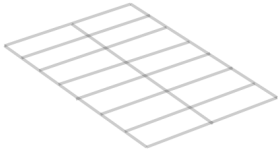
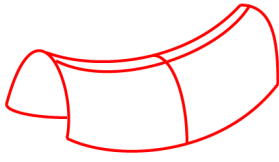
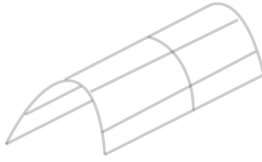
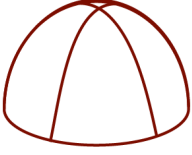
	$K_G < 0$	$K_G = 0$	$K_G > 0$
$K_M < 0$	 Synformal Saddle	 Synform	 Basin
$K_M = 0$	 Perfect Saddle	 Plane	
$K_M > 0$	 Antiformal Saddle	 Antiform	 Dome

Figure 5. Shape classes into which points on the surface can be sorted based on the signs on the Mean (K_M) and Gaussian (K_G) curvatures. In this analysis we focus on those classes that can be assigned based on raw curvature values, which are synformal saddles, antiformal saddles, basins, and domes, and do not include developable surfaces or perfect saddles.

principal curvatures to lower precision, which has the possibility of revealing process scales for which minimal surface theory could be applied, however we do not endeavour to explore this aspect of the data. Instead we focus on the 4 basic shape classes that can be assigned based on raw curvature calculations.

2.5 Calculation of Curvature on Gridded Data Sets

In order to calculate the curvatures of topography it is necessary to first select a DEM dataset, and then smooth the data in order to remove errors introduced in the gridding process. Since DEMs are themselves an interpolation of other forms of elevation data, artifacts are known to arise and are sensitive to data density, the type of raw data used, and the interpolation method used in gridding (Reuter, Hengl, Gessler, and Soille (2009)). Removing such contamination is essential for our curvature calculations since they are computed on a grid of neighboring cells, and are thus sensitive to the highest frequency content on a given surface.

While it is well established by the Shannon-Nyquist sampling theorem that discretely sampled data cannot resolve any structure with an effective wavelength less than twice the sample spacing (Stewart and Podolski (1998)), gridding artifacts stemming from data interpolation represent a less obvious constraint on the scale of structures that can be resolved in a given DEM. In the past decade, DEMs generated with LiDAR (Light Detection and Ranging) have become the preferred data source for connecting morphometric analysis to landscape evolution models due to their superior resolution and lack of obvious gridding artifacts relative to other elevation datasets.

While these factors can rightfully be taken as evidence of the superior quality of LiDAR over older data products, it is also important to recognize that

LiDAR DEMs are built through interpolation techniques that are themselves a form of low-pass filtering, and which are known to introduce errors dependent on slope, terrain roughness, and sample density, in the original point cloud (Bater and Coops (2009); Bui and Glennie (2023); Reuter et al. (2009); S. Smith, Holland, and Longley (2004)).

Within our study area there is 1-m resolution LiDAR data available through the Oregon Department of Geology and Mineral Industries (<https://gis.dogami.oregon.gov/maps/lidarviewer/>), however close examination of this data reveals interpolation artifacts that we interpret as being indicative of poor resolution in the source data. These artifacts can be seen as the triangular structures in Figure 6.B, which shows the hillshade for an unfiltered swath of LiDAR within the study area. While these artifacts, which are common in high curvature areas and persist for hundreds of meters, can easily be smoothed over it is not clear over what scale this data is actually representative of a topographic surface. While it is beyond the scope of this work to quantify how this type of error propagates into the calculation of landscape form, it does show that LiDAR grid spacing is not always reflective of the useful resolution of the data (T. Smith, Rheinwalt, and Bookhagen (2019)). Given the scale of these interpolation artifacts we opt to perform our analysis on 8.1 m resolution DEM data freely available through the National Map (<https://apps.nationalmap.gov/downloader/>).

2.5.1 Spectral Filtering. To filter the DEM we use the Discrete Fourier Transform (DFT; also a contribution of Gauss), which decomposes discretely sampled signals into sums of trigonometric functions. This allows for evaluation of spatial signals in terms of their wavenumber content, and in the context of smoothing, the ability to eliminate wavenumbers higher than a defined

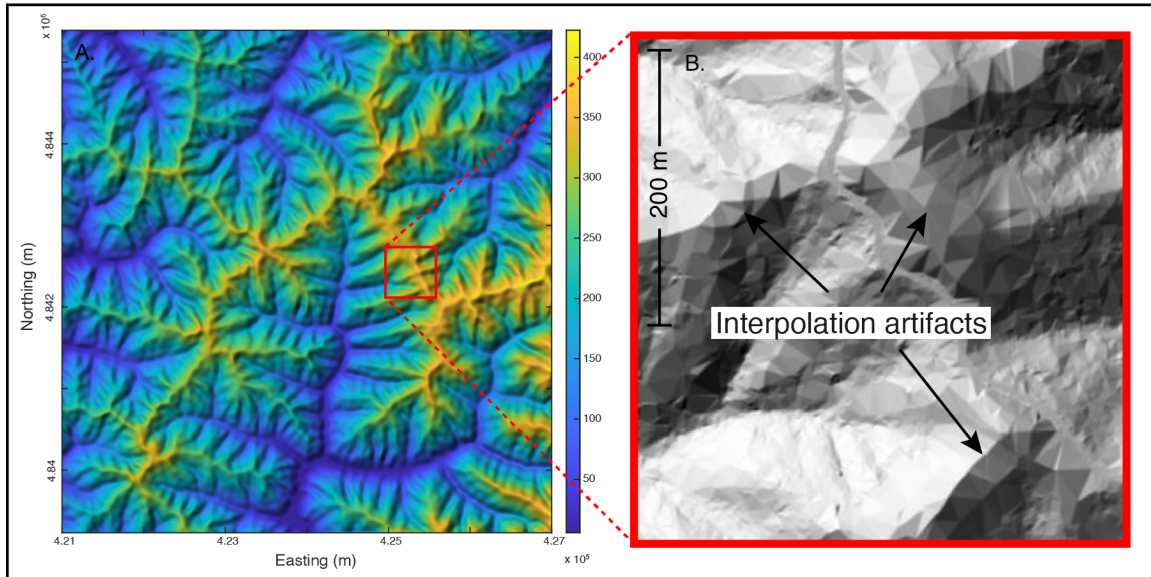


Figure 6. Interpolation artifacts in 1-m LiDAR data. A. The location of the LiDAR swath within the study area. *B.* LiDAR hillshape showing regions dominated by interpolation artifacts over length scales of hundreds of meters.

cutoff leaving behind longer wavelength structures. In this way a low-pass filter can be constructed that preserves the amplitude of signals at wavelengths longer than that of gridding noise, however this approach requires careful processing and interpretation, as calculation of the Fourier spectrum can easily generate spurious artifacts at a range of scales. Fourier methods have already been applied in geomorphology towards the identification of characteristic process scales (Perron, Kirchner, and Dietrich (2008)), landform identification (Booth, Roering, and Perron (2009)), and to assess topographic controls on the development of volcanic topography (Richardson and Karlstrom (2019)). We build on this literature to develop a low-pass filtering technique that is slightly more robust against long-wavelength contamination, and show the utility of this method for removing erroneous high wavenumber signals introduced in the process of gridding topographic data (Reuter et al. (2009)).

2.5.1.1 *Minimizing Long-Wavelength Contamination.* One challenge of applying Fourier analysis to DEMs, is that the method fits the surface with a sum of periodic functions which collectively must go to zero at the edges. Since topography at the scales of interest is not bounded by regions of zero elevation it is required to taper the data on the margins such that this boundary condition is met. Most geomorphology studies to date accomplish this by convolving the DEM grid with a 2-d raised cosine (aka Hanning window), such that the resulting topography is equal to its actual value only in center of the grid, and is elsewhere subdued by an amount that scales with distance towards the margins.

This approach not only diminishes the spectral power of relevant landscape features, but introduces artificial long-wavelength signals that are difficult to differentiate from real structures in the resulting power spectrum. These effects of windowing are generally not acknowledged in the geomorphology literature, however this issue is well described in the signal processing literature (ex. Harris (1978)), as well as the geophysical literature (where spectral filtering is common for computing gravity/topography admittance). For example McNutt (2005) showed that contamination can be minimized by first reflecting the topographic grid in each direction, then tapering the data only in the reflected portions, which are outside the limits of the original DEM. This avoids the introduction of distortion within the study area through the windowing process, and only introduces artificial signals with significant power that are at or above the scale of the original DEM extent. After removing high frequency content and performing a reverse Fourier transform the original data can then be extracted yielding a more accurate low-pass filtered representation of the surface.

We adopt this mirroring approach and apply a Tukey (Harris (1978)) window, which consists of a boxcar function convolved with a cosine taper along the margins. We apply the default Tukey window in the *window2* function in Matlab. We find this low-pass filtering approach more compelling than the cosine taper method as distortion is not introduced in the pre-processing step. Figure 7 compares our pre-processing approach with the default window used in the 2dSpecTools toolbox for Matlab Perron et al. (2008), and also shows the effects of applying the window used by (Perron et al. (2008)) to a mirrored version of the data. It can be seen that both methods employing the hanning window cause significant distortion of the data that can be expected to modify the spectral content over relevant length scales.

2.5.1.2 Calculation of amplitude spectrum. Once the data has been mirrored the Discrete Fourier Transform (DFT) is calculated as

$$Z(k_x, k_y) = \sum_{m=0}^{N_x-1} \sum_{n=0}^{N_y-1} z(m\Delta x, n\Delta y) e^{-2\pi i(\frac{k_x m}{N_x} + \frac{k_y n}{N_y})} \quad (2.35)$$

where N_x and N_y are the number of grid cells in each direction, m and n are array indices, Δx and Δy are the grid spacings in each direction, and k_x and k_y are the wavenumbers in the respective x and y directions (Perron et al. (2008)). Each value in the output array given by the above equation is associated with a frequency in each the x and y directions with magnitudes

$$f_x = \frac{k_x}{N_x \Delta x}, \quad f_y = \frac{k_y}{N_y \Delta y}. \quad (2.36)$$

These frequencies can then be used to define a radial frequency as

$$f_r = \sqrt{(f_x^2 + f_y^2)}. \quad (2.37)$$

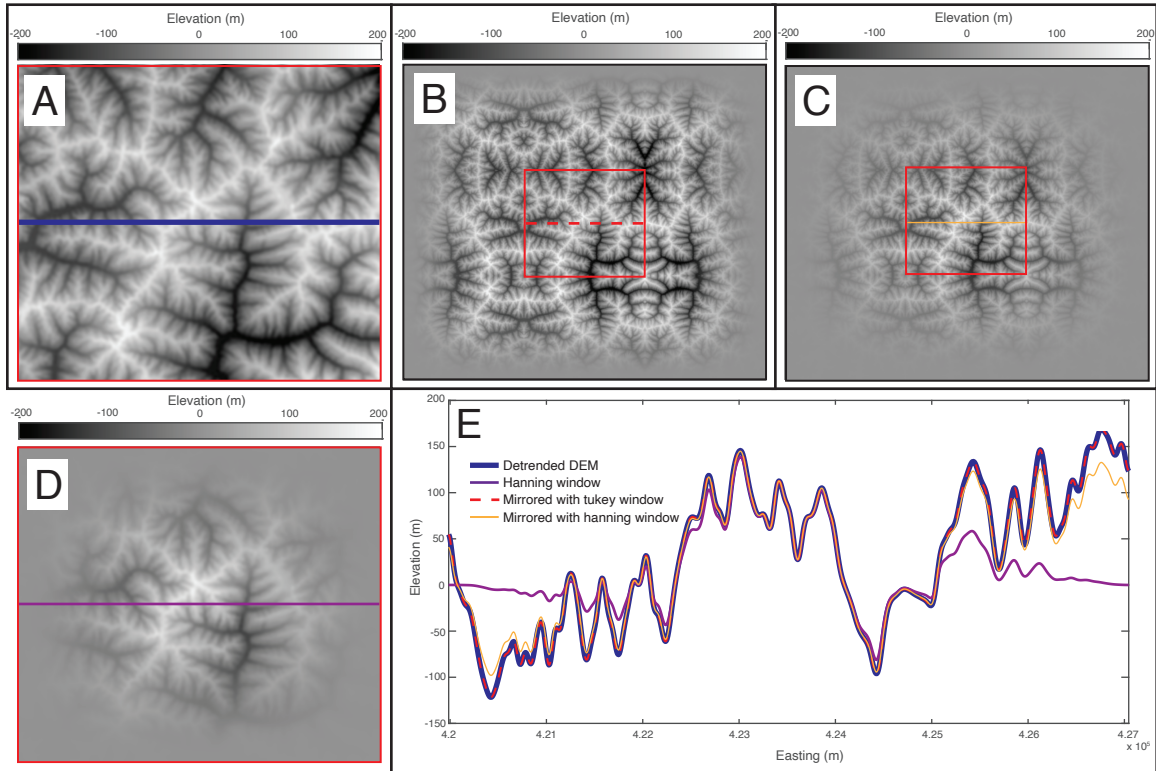


Figure 7. Comparison of windowing techniques used in preprocessing DEMs. A. Raw DEM. B. Mirrored data with Tukey window. The red box is the original DEM extent. C. Mirrored with hanning window. D. Hanning window applied to DEM without mirroring. E. Profiles of topography for each of the preprocessing methods compared to the original DEM.

We calculated the DFT periodogram as

$$P_f(k_x, k_y) = \frac{1}{N_x^2 N_y^2} |Z(k_x, k_y)|^2 \quad (2.38)$$

which can be plotted against radial frequency to provide an estimate of the landscape power spectrum as seen in figure 8.A.

2.5.1.3 Removal of High-Frequency Noise. The primary goal of the filtering process is to remove high wavenumber artifacts of the gridding process. Since DEMs are themselves interpolations of other forms of elevation data they are prone to gridding artifacts that vary between datasets (Reuter et al. (2009)). In addition to the gridding artifacts themselves it is expected that such discontinuities in the data give rise to *Gibbs phenomena*, which is high wavenumber “ringing” around discontinuities in the data that comes from trying to fit a discontinuity with layered high-wavenumber signals. This is especially of concern for the methods used in this study, as curvature is calculated on a grid of neighboring DEM cells, and so is itself a measure of the highest frequency content in a given DEM.

In the particular dataset used in this study the grid artifacts, and resultant Gibbs phenomena can easily be seen in the amplitude spectrum. In figure 8.A the farthest right bump in the power spectrum is the result of gridding artifacts as can be seen in figure 8.E, in which the raw topography (panel B) is high pass filtered to only admit wavelengths under 30 m. The stripes that comprise most of the signal at these wavelengths is characteristic of gridding artifacts (Bater and Coops (2009)). The bump at slightly lower frequencies we attribute to Gibbs phenomena that stem from the gridding artifacts. Features associated with this peak can be seen in 8.D which shows the result of bandpass filtering the raw DEM to isolate wavelengths between 30 m and 50 m. The interpretation of these features as Gibbs phenomena comes from the observation that structures at these scales follow the

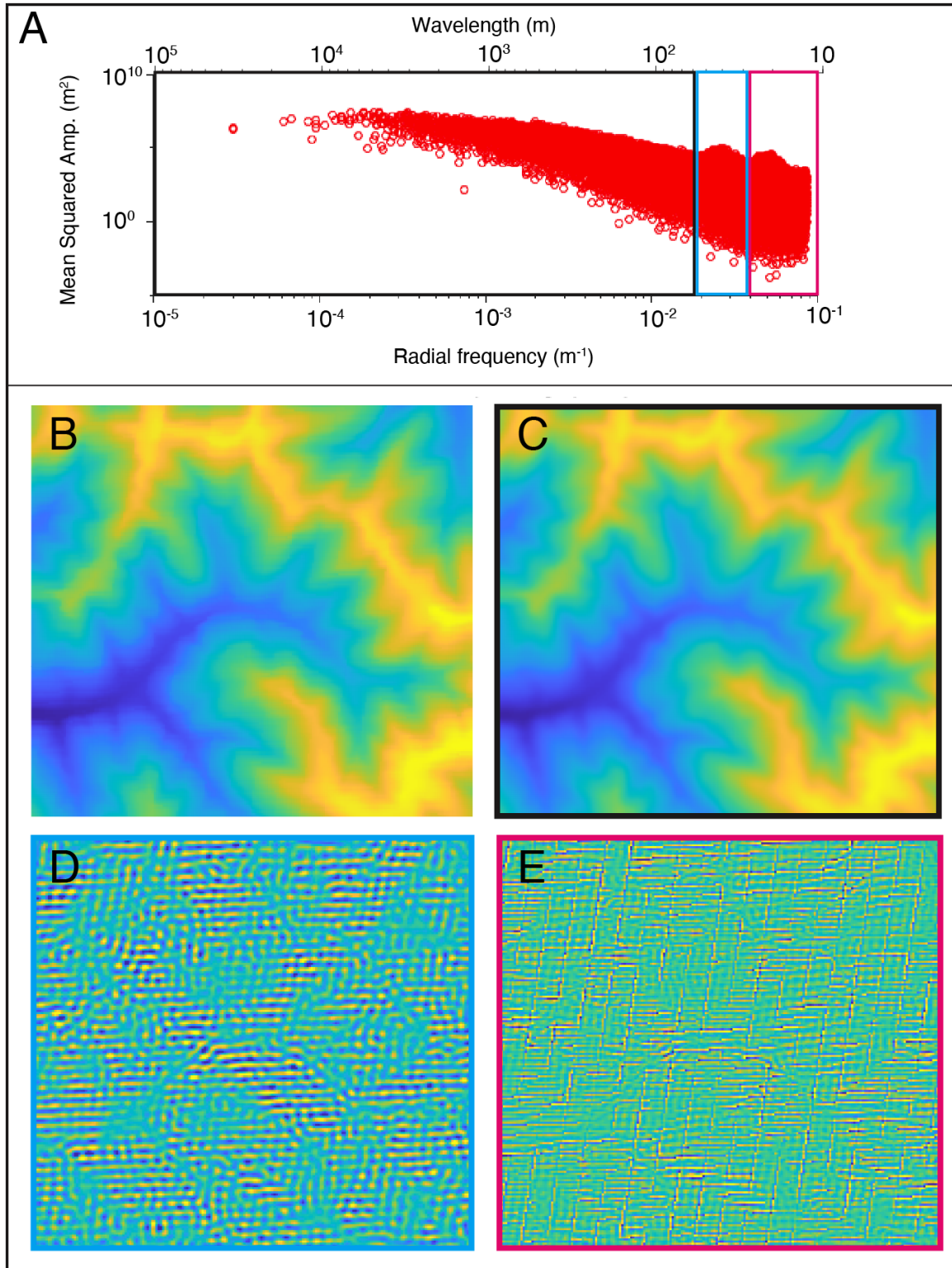


Figure 8. Identification of gridding artifacts in the 8.1 m DEM data used in this study. **A.** Mean squared amplitude vs. radial frequency of topography. Colored boxes highlight frequency ranges shown in panels C, D, and E. **B.** Raw DEM. **C.** DEM bandpass filtered to admit wavelengths between 30 and 50 m showing the gibbs phenomena thought to produce the spike in the power spectrum at these wavelengths. **E.** DEM high-pass filtered to admit wavelengths less than 30 m. Straight line are artifacts from the gridding process.

orientation of grid lines, but truly defining the source of these signals requires further analysis. This could be accomplished through testing of different windowing methods, which should conserve real structures, but alter the expression of such ringing signals (Harris (1978)). We conclude that this data is not suitable for the calculation of curvatures at wavelengths of less than 50 m and so take this as the minimum low-pass filter cut-off for accurate curvature calculation, however we will do most of our analysis on DEMs filtered to 200 m, and so do not further explore the sources of these high frequency signals. The DEM filtered to a lowpass filter cut-off of 50 m to eliminate these signals can be seen in figure 8.C.

2.5.2 Scale Selection. Once the landscape has been filtered all of the curvature metrics outlined in sections 2.4.2 and 2.4.3 are calculated for every pixel. The values for each of the curvature metrics are binned by drainage area in order to couch our curvature analysis in a context consistent with that of the literature for identifying process transitions. We first look for dominant trend in the evolution of each metric, and iteratively smooth the landscape to greater wavelengths to get a sense of how our measures of curvature vary with scale. To eliminate the high wavenumber gridding artifacts identified in the previous section we first smooth the landscape to 50 m, then increase the low-pass filter cutoff by increments of 50 m up to 600 m.

The results can be seen in Figure 9, which shows the land surface gradient, Gaussian and Mean curvatures, as well as the two principal curvatures and *area proportionality factor* (α) binned by drainage area. The area proportionality factor (α) measures the proportionality of surface area on the landscape to its map-view representation, and can be calculated in terms of the coefficients of the first

fundamental form as $\alpha = \sqrt{EG - F^2}$. It is thus a measure of distortion inherent to the map projection.

We see that general trends in all of these curvature metrics are similar across this range of filter cutoffs, making the selection of a particular scale for further analysis somewhat arbitrary. However we note that at cutoffs greater than ~ 200 m peaks in the curves start to get pulled to longer wavelengths suggesting a shift in the structures being resolved at those scales. This is most easily seen in the main peaks of the Gaussian curvature in Figure 9.B. We take this as an upper end of the range of length scales for primary landscape features, and so do our focused analysis on a DEM filtered to 200 m. This filter scale allows us to identify certain landscape features, however inhibits our ability to see the effects of finer scale processes. For example signatures of processes, such as tree throw, cannot be resolved and would require analysis of finer datasets and/or less aggressive filters.

2.6 Alignment of Slope and Principal Curvature Vectors

Upon calculating the principal curvatures, they are sorted such that the first principal curvature (k_1) is defined as the curvature with the greatest magnitude regardless of sign as explained in section 2.4.2. This is not a unique choice (Figure 4 shows the first principal curvature as the more positive of the curvatures) but we find this way of defining the principal curvatures is slightly easier to interpret in terms of landform development.

Since we also are able to define the orientation of the principal curvature vectors via Equation 2.19 we can then look at how each principal curvature aligns with slope throughout the landscape. It is common in geomorphology studies not to define the method of slope calculation, however many studies use either the gradient operator ($\nabla z(x, y) = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$) (Roering et al. (1999); Scherler

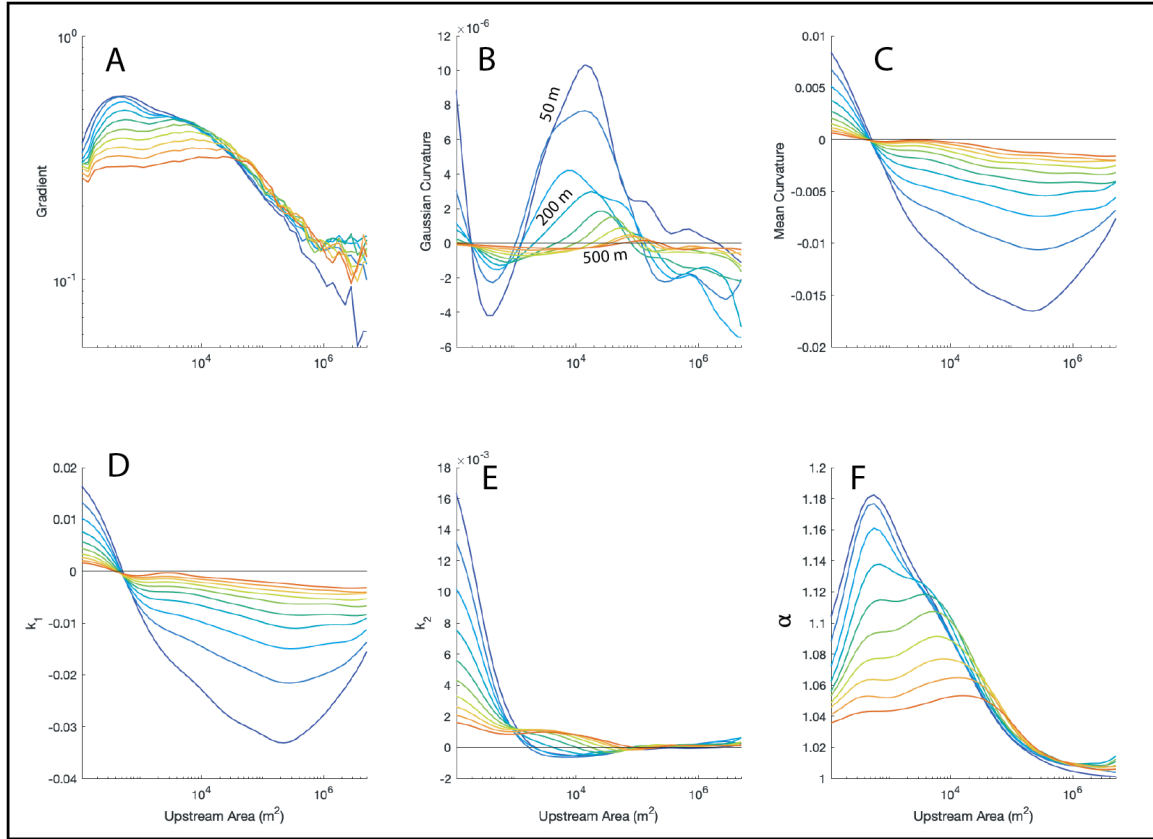


Figure 9. Surface geometry metrics binned by upstream drainage area for a range of low-pass filter cut-offs between 50 m and 500 m calculated on 50 m intervals. A. Gradient. B. Gaussian curvature (K_G). C. Mean Curvature (K_M). D. First principal curvature (k_1). E. Second principal curvature (k_2). F. Surface area proportionality factor (α)

and Schwanghart (2019)), or an 8-point neighborhood gradient which calculates the slope between a pixel and its lowest immediate neighbour (Montgomery and Foufoula-Georgiou (1993); Schwanghart and Scherler (2014)). Both these methods can only resolve slope contributions in directions defined by the DEM grid structure and so do not have sufficient aspect resolution for correlation with principal curvature vectors. For this reason we instead use the slope of the tangent plane at each pixel defined via the unit normal vector given by (Equation 2.9). If $\mathbf{N}$ is written in vector form as $\mathbf{N} = N_1\mathbf{e}_1 + N_2\mathbf{e}_2 + N_3\mathbf{e}_3$ then the magnitude of the slope is given by

$$S_T = \tan\left(\frac{\pi}{2} - \sin^{-1}(N_3)\right) \quad (2.39)$$

and the orientation of the normal vector on the projected map can be written in unit vector form as

$$\mathbf{S}_T = \frac{N_1\mathbf{e}_1 + N_2\mathbf{e}_2}{\sqrt{N_1^2 + N_2^2}}. \quad (2.40)$$

The degree of alignment between a principal curvature vector and this measure of slope can be found using the scalar product where $\mathbf{S}_T \cdot \mathbf{k}_i = 1$ if the slope and curvature are perfectly aligned and $\mathbf{S}_T \cdot \mathbf{k}_i = 0$ when they are orthogonal.

Viewing this alignment metric in map view, as seen in Figure 10, reveals intuitive trends that can help inform the location of process transitions. The second principal curvature (k_2) is the least curved path over the surface and so will always track the long axis of a given structure, which will only align with the tangent slope in areas of symmetric convergence and divergence in directions orthogonal to k_2 . Indeed Figure 10.D shows maximum alignment of $\mathbf{k}_2$ and $\mathbf{S}_T$ along ridgelines and channels, and in the hillslopes and colluvial hollows that mark the termination of those structures. The first principal curvature (k_1) is aligned

with slope in transition regions that are neither especially convergent or divergent. Sharp transitions in the alignment of the principal curvatures thus reflect transition areas.

We can see the same alignment trend in area space, though with less detail. Figure 11 shows both the average magnitude of each principal curvature, and our alignment metric binned as a function of drainage area. The sharp increase in alignment between $\mathbf{S}_T$ and $\mathbf{k}_2$ at drainage areas slightly higher than 10^3 m^2 follows the transition of both principal curvatures to being dominantly negative, itself a signature of flow convergence that we will show is associated with a major shift in landscape concavity. We will suggest that this increase in alignment is associated with colluvial hollows at the head of the drainage network, and thus is a signature of convergent processes that mark the onset of hydrologic channelization in landscapes.

Since we rely on a non-standard slope metric for this analysis it is important to compare this metric to others commonly used in the field. Figure 36 shows comparison between S_T , the gradient operator (∇z) , the 8-point neighborhood gradient calculated using the *gradient8* function in TopoToolbox, and the gradient calculated in 1-dimension along all channels above an area threshold of $(10^4) \text{ m}^2$, which are extracted using the *STREAMobj* object class in TopoToolbox. We note that while S_T and the 8-point neighborhood gradient are well aligned in the steepest parts on the landscape, the 8-point neighborhood gradient predicts much lower slopes at high drainage areas than the other metrics. This is relevant as it is the metric often used to define process transitions in slope area space (Montgomery and Foufoula-Georgiou (1993)). The gradient operator predicts systematically higher slopes than the other metrics, which is important to note as it is often used

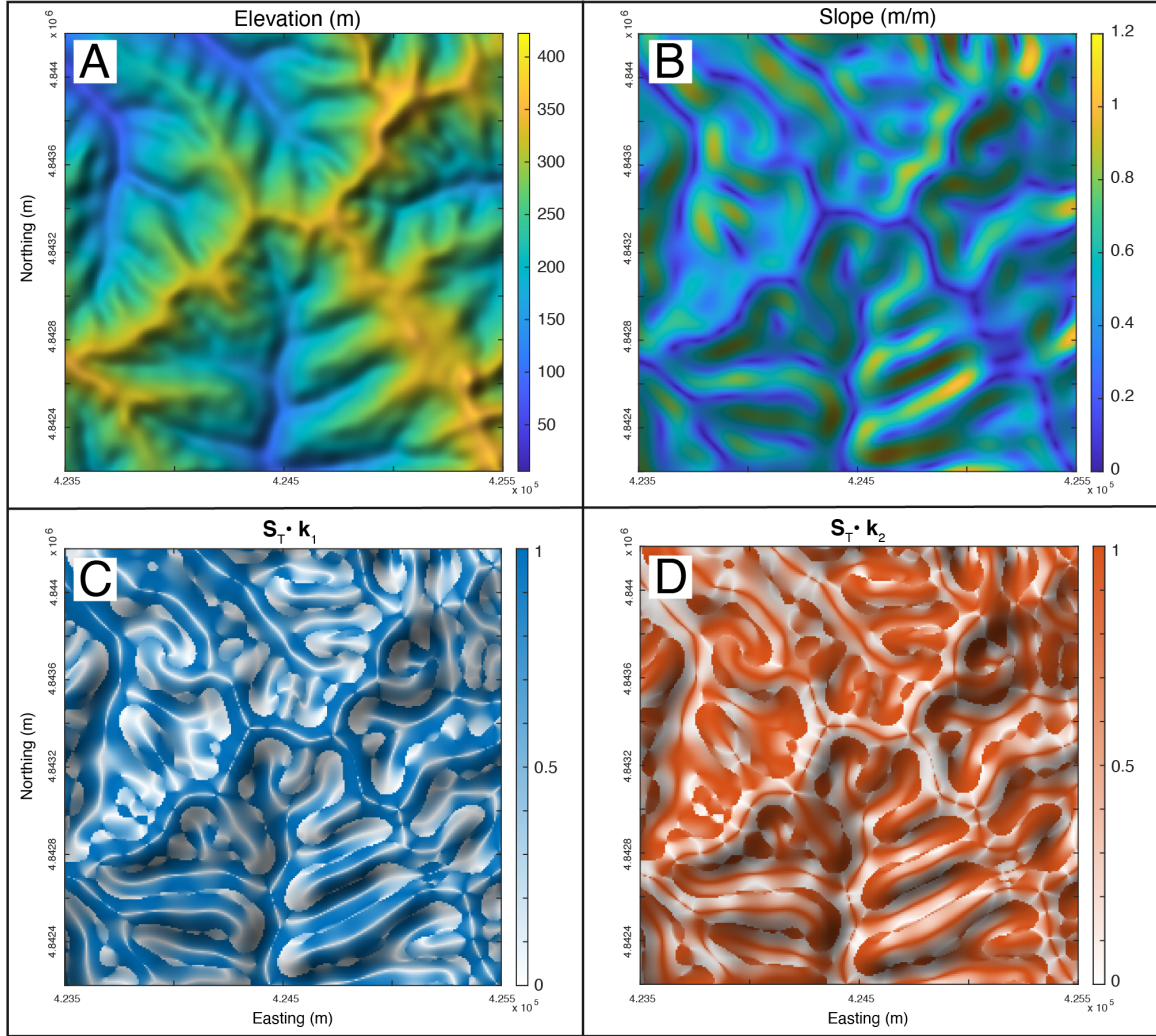


Figure 10. Alignment between principal curvatures and slope of the tangent plane measured via the surface normal N . A. DEM low-pass filtered to 200 m. B. Slope of tangent plane. C. Scalar product of the slope vector and first principal curvature vector (k_1). D. Scalar product of the slope vector second principal curvature vector (k_2).

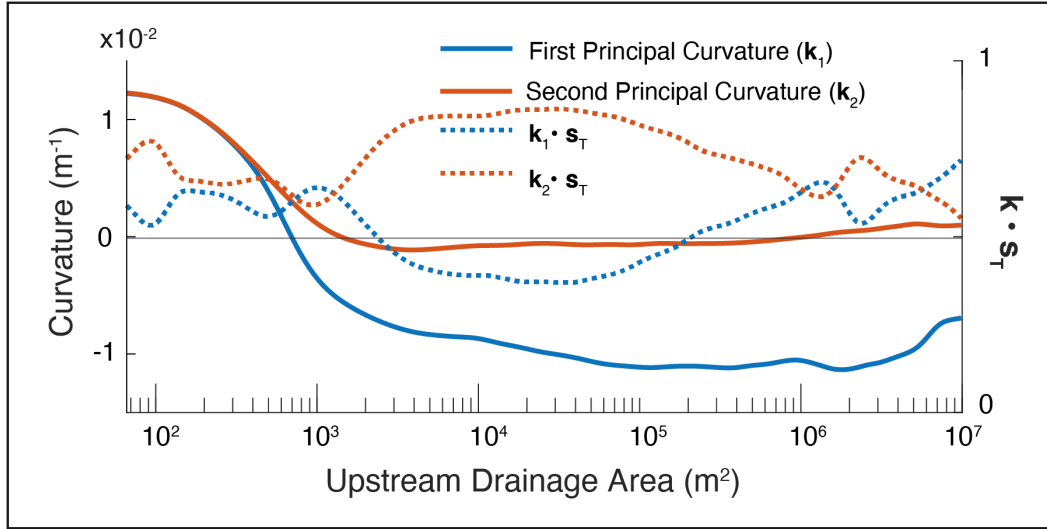


Figure 11. Principal curvatures binned by upstream drainage area for DEM filtered to 200 m. Left axis measures mean curvature values shown by solid lines. Right axis measures the scalar product between the principal curvatures and slope shown by the dotted lines.

in hillslope models (Roering et al. (2007)). The tangent slope is well aligned with the slope of channels in the upper reaches of the network, however these metrics also diverge at drainage areas that seem to correspond with a transition zone already identified in the literature as connected to debris flow processes (Stock and Dietrich (2003)). We have no reason to think any one of these slope metrics superior to the others, however do note that each way of calculating slope extracts different information from the landscape and will lead to different conclusions if used to define the area thresholds of process transitions.

2.7 Connections to the Laplacian

The distribution of both the Mean (K_M) and Gaussian (K_G) curvatures are shown in map-view (Figure 13) and in area space (Figure 14) giving different, but complementary, views of how these curvature metrics relate to structures in the landscape. As outlined in section 2.4.3, the general shape of the surface is characterized by the Gaussian curvature, which takes on positive values when the

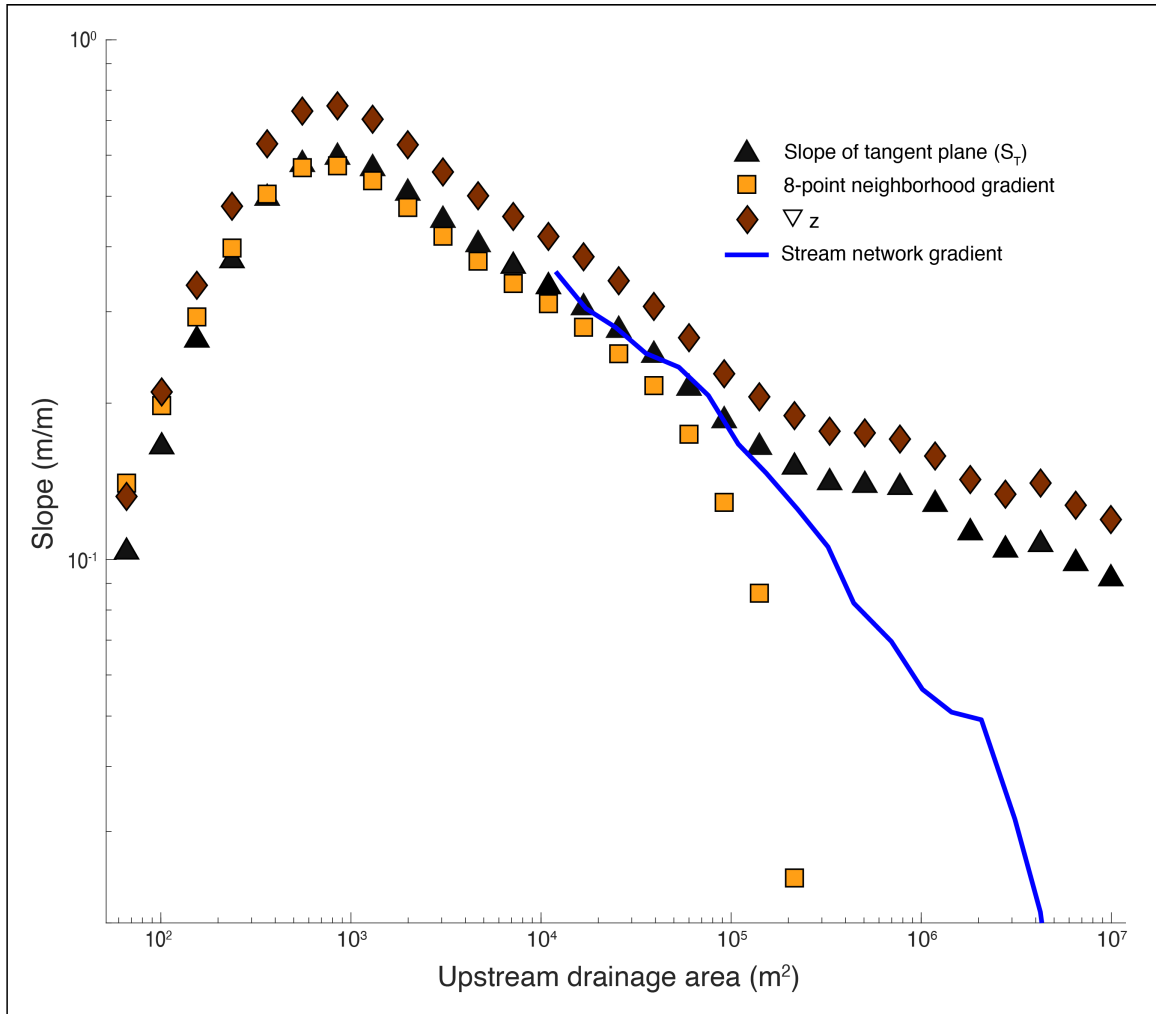


Figure 12. Comparison of different slope metrics binned by drainage area.

sign of the two principal curvatures are the same and negative values when they are not. Thus basins and domes are marked by positive Gaussian curvature and negative values indicate saddle structures, where the orientation of these shape classes is recorded in the sign of the mean curvature. In Figure 13.B we see that the landscape is composed of a somewhat tessellated pattern of oscillating Gaussian curvatures while Figure 13.D shows that the implied alternating domal/saddle structures have orientation that is dictated by their location either in the divergent (ridge/hilltop) or convergent (valley network) portion of the landscape. The area space perspective shown in Figure 14 gives a more blurred representation, and suggests that the landscape dominantly consists of sources and sinks represented by domes and basins with the exception of a transitional area around where the mean curvature switches from positive to negative. We will examine this area space trend in greater detail in section 2.9, and show that the curvature evolution shown in Figure 14 is useful in defining the evolution of process regimes in the Oregon Coast Range.

The Mean curvature also provides a way of quantifying complicated error inherent to common map view Laplacian methods. While in this study K_M is calculated at point using the two principal curvatures, *Euler's Theorem* (Equation 2.16) actually shows that it can be found using the curvatures of any two orthogonal paths. This means that if one is able to apply the Laplacian operator along coordinate vectors that are orthogonal on the surface and account for slope error, then the resulting measure of curvature relates to K_M via the relation

$$K_M = \frac{z_{xx} + z_{yy}}{2} = \frac{\nabla^2 z}{2}. \quad (2.41)$$

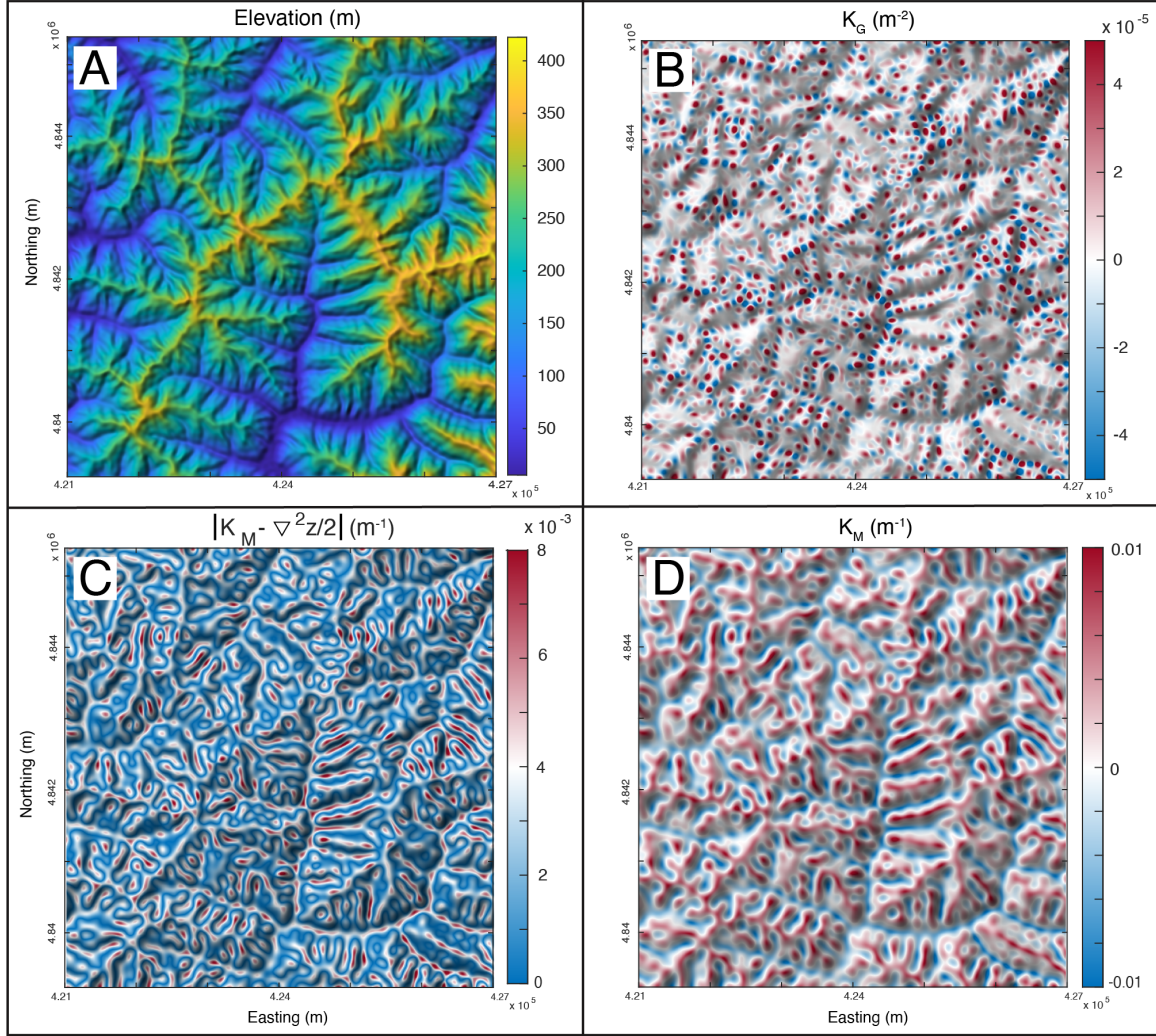


Figure 13. Map-view of surface curvature metrics. A. Topography. B. Gaussian Curvature (K_G). C. Difference between mean curvature calculated using the principal curvatures and Mean curvature calculated using the components of the laplacian operator. D. Mean curvature (K_M).

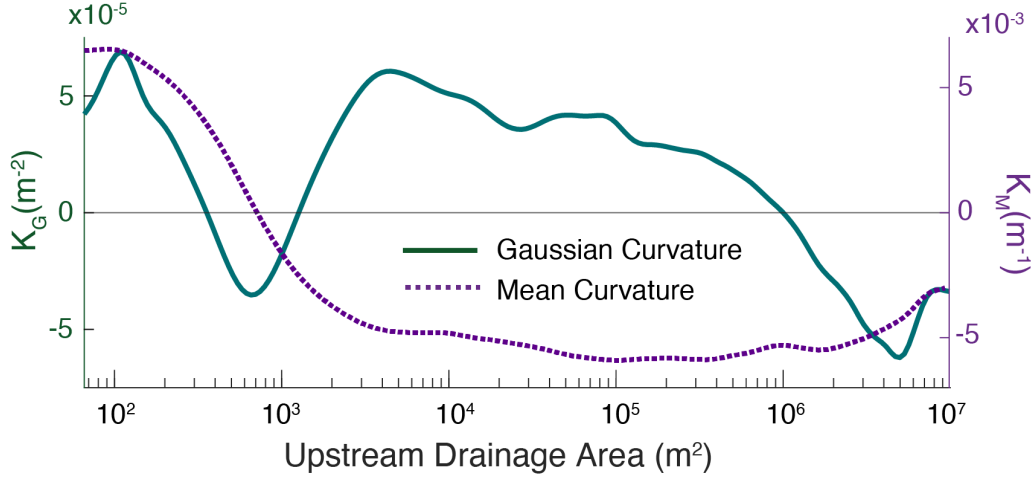


Figure 14. Surface curvatures binned by upstream drainage area.

The magnitude of the difference between the mean curvature and that given by the calculation of directional derivatives along the map grid can be then written as

$$error = |K_M - \frac{\nabla^2 z}{2}|. \quad (2.42)$$

Figure 13.C shows the distribution of error in map-view. We see that maximum error in the Laplacian calculation occurs along ridge and channel networks where the magnitudes of both the Gaussian and Mean curvatures are highest. This is not surprising since these are the regions in the landscape with the most complexity, and thus the most distortion introduced through projection onto the map grid. This is also reflected in the area-space representation given in Figure 15.A, which shows that agreement between the Mean curvature calculated using the principal curvatures and that calculated using the Laplacian are highest in areas where the magnitude of both curvatures are lowest. Perhaps non-intuitively slope is shown to have a lesser effect on the error in Laplacian curvature (Figure 15.C) and in fact error is minimized at higher slope. This is contrary to the general

expectation (Bergbauer and Pollard (2003)), however makes sense given the strong control exerted by surface curvature itself on error in the Laplacian (Figure 15.B).

We note that this analysis only pertains to the simplest possible implementation of the Laplacian operator calculated on our filtered DEM surface, and thus should not be viewed as a measure of error in curvatures calculated in geomorphology studies that use the Laplacian as most studies implement more sophisticated pre-processing steps (M. D. Hurst et al. (2012)), and often calculate curvature along 1-d profiles (Roering et al. (2007); Struble and Roering (2021)). Rather this is a simplistic measure of the relative contributions of slope and map distortion to general errors in 2-dimensional partial derivative curvature calculations. It should also be noted that, while the magnitude of the curvature is very different, the general trend in area-space is the same, and so it is likely that general inferences of connections between curvature and erosion rate in the literature (Roering et al. (2007); Struble and Roering (2021)) are not changed by this observation.

2.8 Land Surface Shape-Class Distribution

As outlined in section 2.4.3, the Mean and Gaussian curvatures can together be used to sort the landscape into four distinct shape classes. These amount to a way of partitioning the landscape that provides a more intuitive framework for discussing geomorphic structures than curvature metrics themselves. We look at the distribution of these shape classes in the landscape by generating probability density functions of each shape class as a function of contributing drainage area. The initial PDFs are shown in Figure 16.B, which shows that probabilities are strongly weighted by the distribution of drainage area (shown in Figure 16.A) and

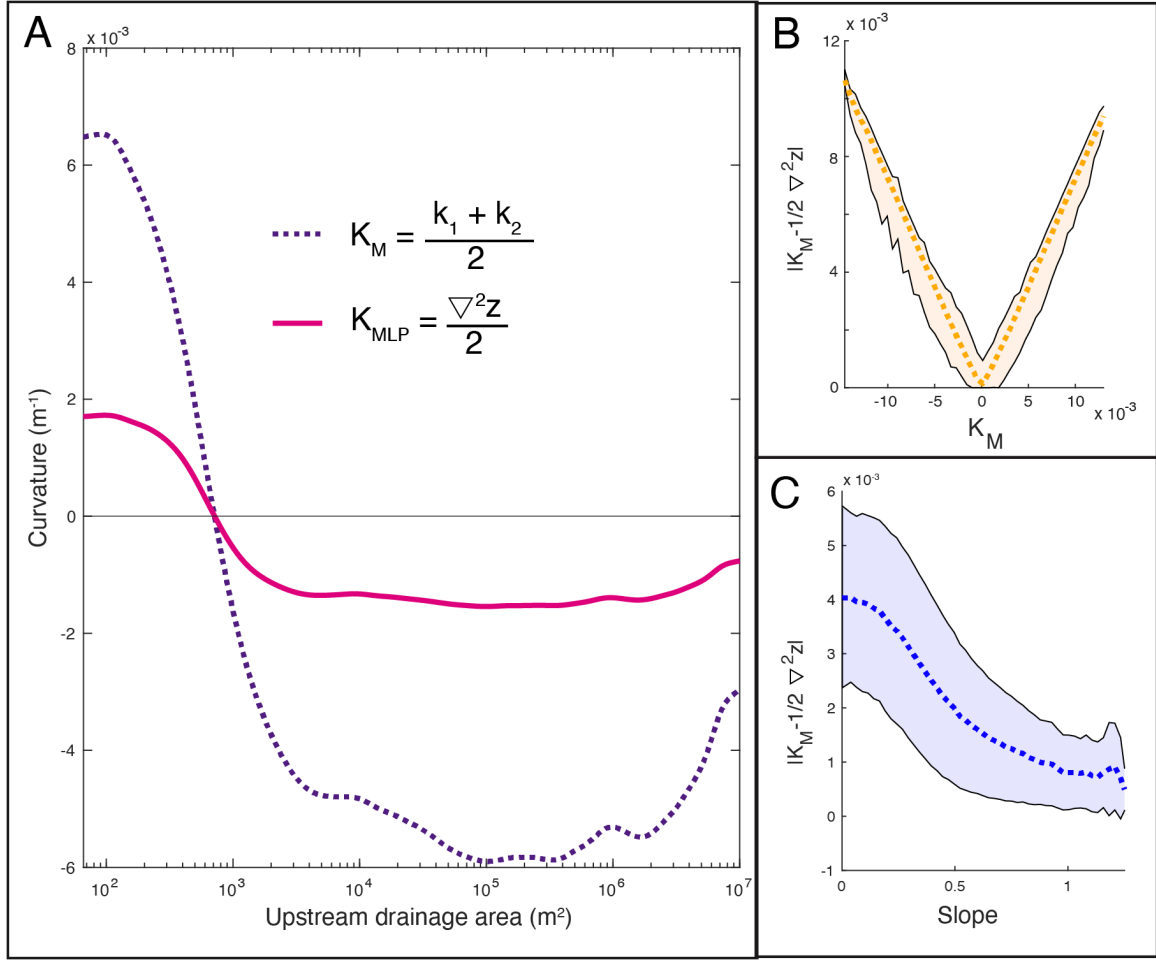


Figure 15. Comparison between mean curvature calculated using the principal curvatures (K_M), and mean curvature calculated using the components of the laplacian (K_{MLP}). A. Calculated mean curvatures binned by upstream drainage area. B. Difference between K_M and K_{MLP} binned by K_M . C. Difference between K_M and K_{MLP} binned by slope

are thus not a good reflection of shape class probabilities existing in a particular area-dependant process regime.

For this reason we calculate the conditional probability of each shape class at a given drainage area. We first define the probabilities of each shape class as a function of drainage area (Figure 16.B), which can be thought of as the probability of both the shape class and drainage area occurring simultaneously. More formally this is the intersection of the sets containing shape classes and drainage area values which is written $P(C \cap A)$. We can also define the probability of each drainage area value ($P(A)$; Figure 16.A), which allows us to calculate the conditional probability of a shape class at a given drainage area ($P(C|A)$; (Figure 16.C)) using the probability axiom (Dekking (2010))

$$P(C \cap A) = P(C|A)P(A) \rightarrow P(C|A) = \frac{P(C \cap A)}{P(A)}. \quad (2.43)$$

We find this version of the shape class metric the most useful for interrogating the connection between surface geometry and surface processes, and explicitly connect it to the other quantitative metrics of topographic form in section 2.9.

2.9 Geometric View of the Oregon Coast Range

Having defined in the previous sections several quantitative measures of land surface geometry we now look holistically at these metrics as a suite of complementary data that inform how the geometry of topographic surfaces varies as a function of catchment area in fluvially dissected landscapes. We first examine the study area as a whole, then decompose the landscape into regions defined by transition points in the invariant measures of curvature, which we argue are indicative of process transitions that define how signals of fluvial dissection propagate through connected catchments.

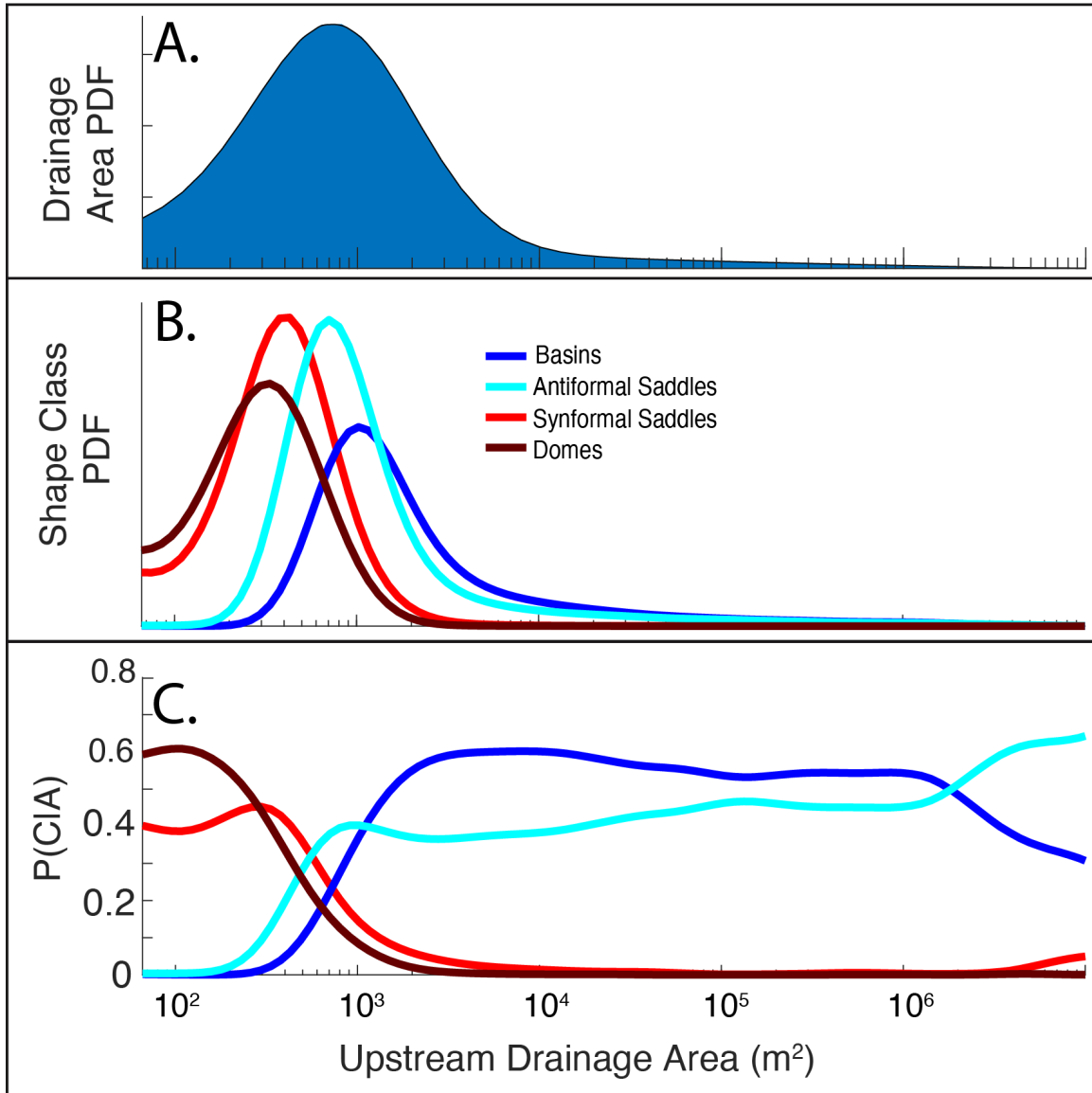


Figure 16. Shape class distributions as a function of drainage area. A. Probability density function of upstream drainage areas within the study area. *B.* Probability density functions of shape classes as a function of drainage area. *C.* Conditional PDFs of shape classes at a given drainage area.

Figure 17 shows a compilation of the geometric data and will serve as a template for the decomposition of the landscape in the following sections. Panels A -E are the area-space representations of the geometry metrics presented in the previous section and do not contain any new information. Panel F shows the map-view distribution of shape classes, while Panel H present this same information as a pie chart. We see that at this spatial scale the landscape is evenly divided between structures that are concave up and those that are concave down, which is reflective of a symmetry in mean curvature that we will show in more detail in section X. Panel G shows a probability density function of slopes and is helpful in drawing connections between our analysis and the broader geomorphology literature, in which process regions are often defined in terms of average slope (Roering et al. (2007); Stock and Dietrich (2003)). We will first look at a landscape decomposition based on the Gaussian curvature.

2.9.1 Gaussian curvature-based landscape partitioning. We take a detailed look at how the Gaussian curvature changes with increasing drainage area, and how it relates to process regimes already identified in the literature.

2.9.1.1 Region 1: Drainage areas less than $3.3 \times 10^2 \text{ m}^2$.

The first threshold we apply in our landscape analysis corresponds with the first inflection point in the Gaussian curvature, which occurs at drainage areas of $\sim 330 \text{ m}^2$. This region, highlighted in Figure 18, is comprised of the highest elevations in the landscape and is associated with the network of peaks and ridges that divide adjacent catchments and, in terms of plan-form area, make up 17% of the landscape (Panel A). In map view this network is seen to consist of almost uniformly spaced domes (peaks) connected via antiformal saddles that make up

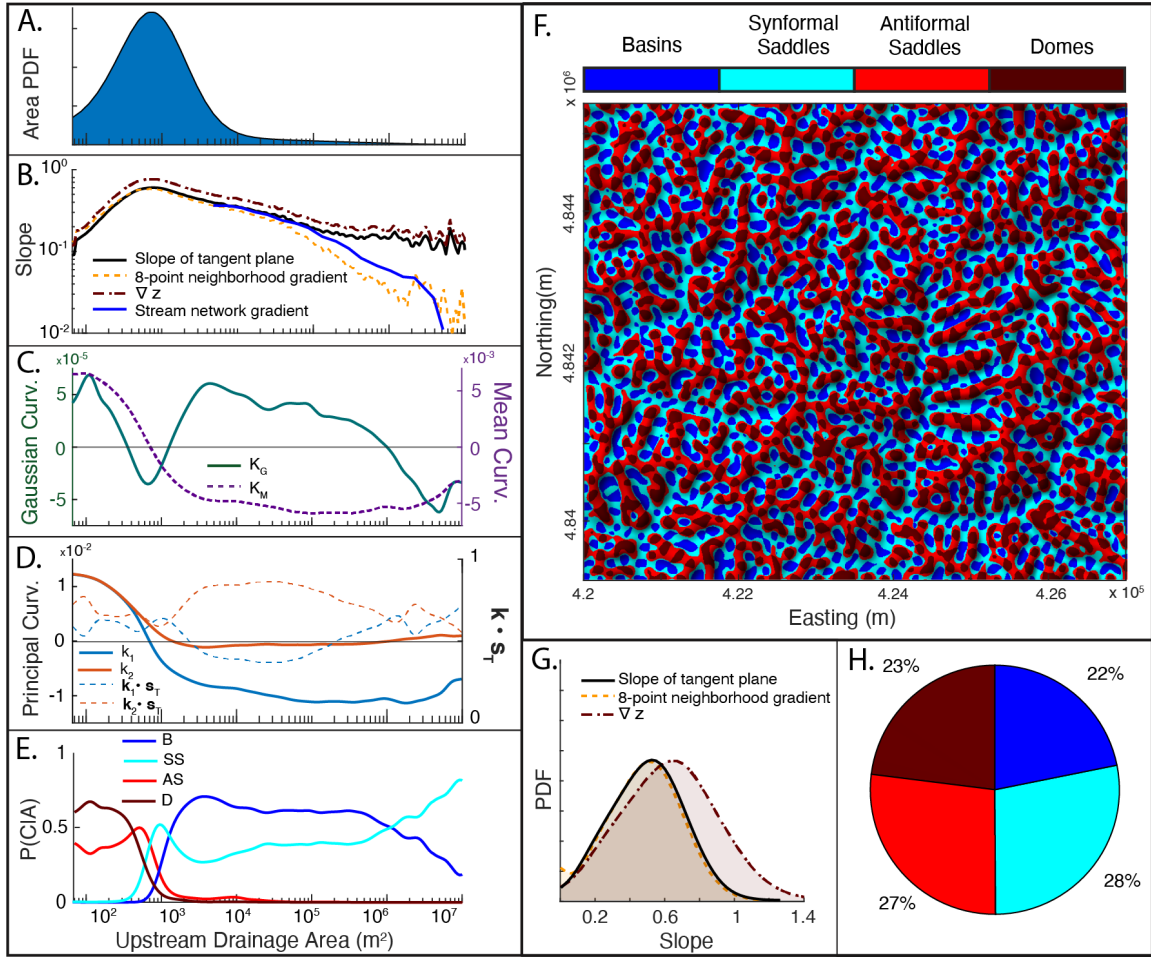


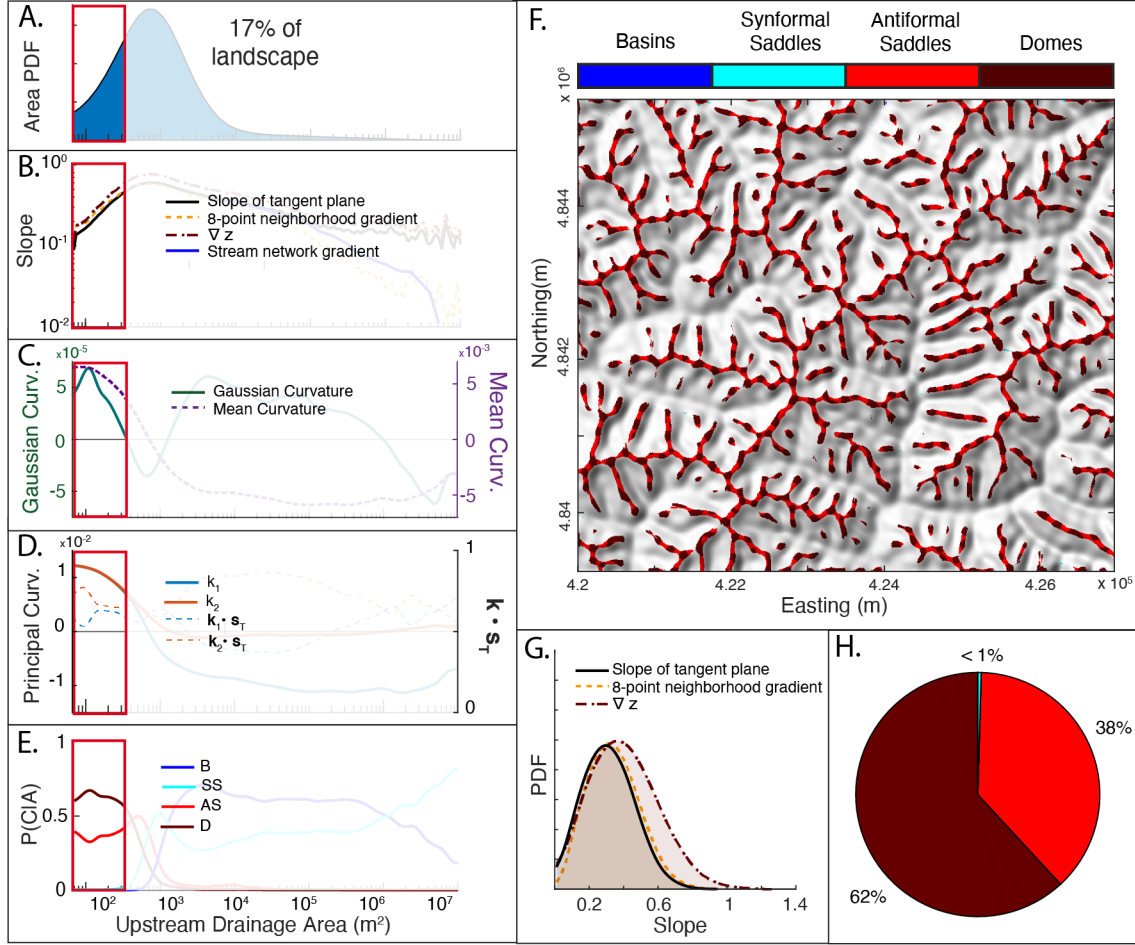
Figure 17. Compilation of surface geometry data used in this study. **A.** PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.

the ridge network (Panel F). Together these two shape classes comprise over 99% of this landscape region (Panels E and H).

Binned by area the two principal curvatures have fall on the same trend in this region (Panel D), which we find noteworthy since in surface theory areas where the two principal curvatures are the same represent special geometric entities, known as an *umbilic points*, which are source or sink regions that are deeply connected to the overall structure and topology of vector and/or scalar fields (Needham (2021)). While is is difficult to see a direct connection between these binned curvature values and the structure of individual points we find this connection to geometric theory interesting, especially in a portion of the landscape where the gradient vector field is characteristically divergent and serves as a primary boundary in the organization of fluvial catchments. In this region slope increases with drainage area, likely corresponding to convex rollovers onto hillslopes.

This is a region of the landscape where curvature is often calculated on 1-d profiles selected such that they are orthogonal to ridge-lines. The slope distribution given in Panel G can be roughly correlated with Figure 2 to get a broad sense of the potential uncertainty associated with such calculations.

2.9.1.2 Region 2: Drainage areas between $3.3 \times 10^2 \text{ m}^2$ and $1.3 \times 10^3 \text{ m}^2$. As drainage are increases beyond the 330 m^2 threshold that defined region 1 the Gaussian curvature transitions to being negative and remains so up to drainage areas of $\sim 1300 \text{ m}^2$. This region, which we correlate to side slopes, is highlighted in Figure 19, and makes up 58% of our study area. Here the landscape is strongly transitional, encompassing the only inflection point in the mean curvature (Panel C), and a zone of divergence in the two principal



*Figure 18. Surface geometry data for points in the landscape with upstream drainage areas less than $3.3 \times 10^2 \text{ m}^2$. The red box in each of the area plots highlights the region of interest. **A.** PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.*

curvatures (Panel D). Accordingly this is the most geometrically complex part of the landscape. Broadly this region encompasses a transition from the dominance of domes to that of basins, however in more detail the landscape is primarily saddles, with an equal partitioning of synforms and antiforms (Panels F and H). This is also the steepest section of the landscape and thus accounts for the majority of landscape relief and course-scale roughness (Panels B and G). The distribution of gradient values found in this region (Panel G) agrees with values found for Coast Range hillslopes in other studies (Roering et al. (2007)).

2.9.1.3 Region 3: Drainage areas between $1.3 \times 10^3 \text{ m}^2$ and $4.5 \times 10^3 \text{ m}^2$. Following a second inflection point in area space the Gaussian curvature sharply increases to a maximum at $\sim 4500 \text{ m}^2$. The location of this local maximum, which is seen in Figure 9 to be fairly consistent over a range of low-pass filter cutoffs, defines the upper bound for the next region we examine, which is shown in Figure 20. In this region the combination of dominantly positive Gaussian curvature, negative mean curvature, and a decrease in slope with added drainage area are all consistent with colluvial deposits at the base of hillslopes, where debris from steeper upper reaches collects on the margins of channel networks as slope decreases thus limiting the ability for material to carry momentum downhill. The landscape is now strongly convergent, as shown by the strong dominance of concave geometries in Panels E, F, and H. This region also aligns with a sharp increase in alignment between the second principal curvature (k_2) and the tangent slope, with a corresponding decrease in alignment between slope and the first principal curvature (k_1).

This trend, seen in Panel C, is what we expect for the onset of channelization and so we interpret this as a signal of first order channels at the

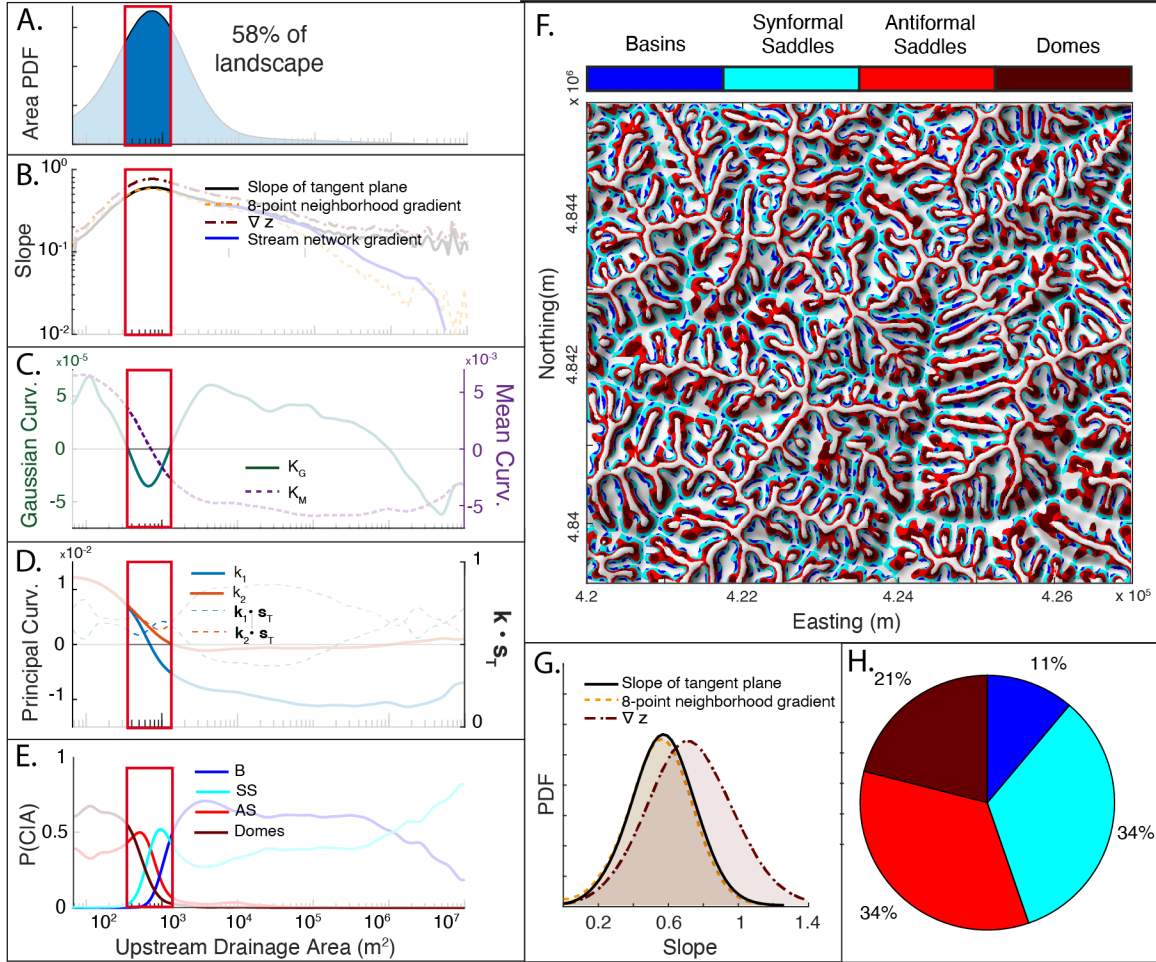
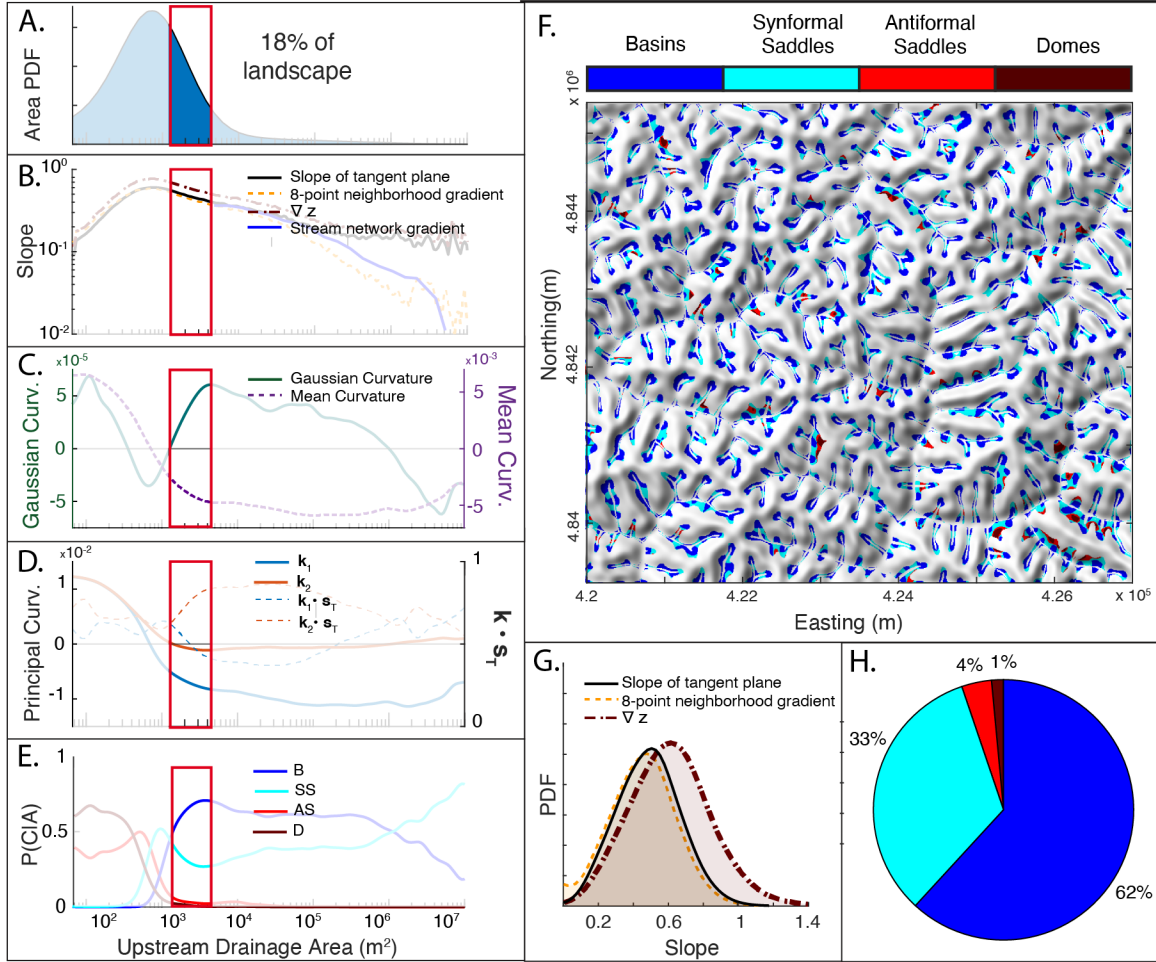


Figure 19. Surface geometry data for points in the landscape with upstream drainage areas between $3.3 \times 10^2 \text{ m}^2$ and $1.3 \times 10^3 \text{ m}^2$. The red box in each of the area plots highlights the region of interest. **A.** PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.

head of the fluvial network. This region accounts for 18% of the study area, and so has a similar contribution to the total landscape as the ridge-valley network defined by our analysis as region 1.

2.9.1.4 Region 4: Drainage areas between $4.5 \times 10^3 \text{ m}^2$ and $9.8 \times 10^5 \text{ m}^2$. The next defined region has a range of drainage areas spanning 3 orders of magnitude over which geometric change happens gradually. Figure 21 shows that the magnitude of positive Gaussian curvature decreases towards another inflection point at $A = 9.8 \times 10^5 \text{ m}^2$ which is indicative of a gradual decrease in the relative proportion of basins to synformal saddles (Panel E). This trend intuitively implies a growing influence of channels in defining landscape curvature and so we associate this region with channels at the head of the fluvial network. This is consistent with the mapview extent of this region (Panel F) which shows that these areas exist at the bottoms of valleys in an area in, or directly adjacent to, main channels.

As drainage area increases the contribution to the total composition of the landscape approaches zero with this entire region only making up 7% of the study area (Panel A). This region contains on average the highest magnitude negative mean curvatures, implying that it is extremely convergent, which taken with the fact that it underlies a region defined by decreasing slope and thus the collection of colluvium (region 3) can be taken to mean that it is a prime region for the rapid accumulation of both hillslope debris and overland flow and thus is expected to be a region where debris flow processes dominate. This is in broad agreement with other literature focused on debris flow processes (Penserini, Roering, and Streig (2017); Stock and Dietrich (2003); Struble et al. (2023)). This region also contains the major divergence in the values of the different slope metrics. We include on Panel G the average gradient calculated in 1-d along the longitudinal profiles of



*Figure 20. Surface geometry data for points in the landscape with upstream drainage areas between $1.3 \times 10^3 \text{ m}^2$ and $4.5 \times 10^3 \text{ m}^2$. The red box in each of the area plots highlights the region of interest. **A.** PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.*

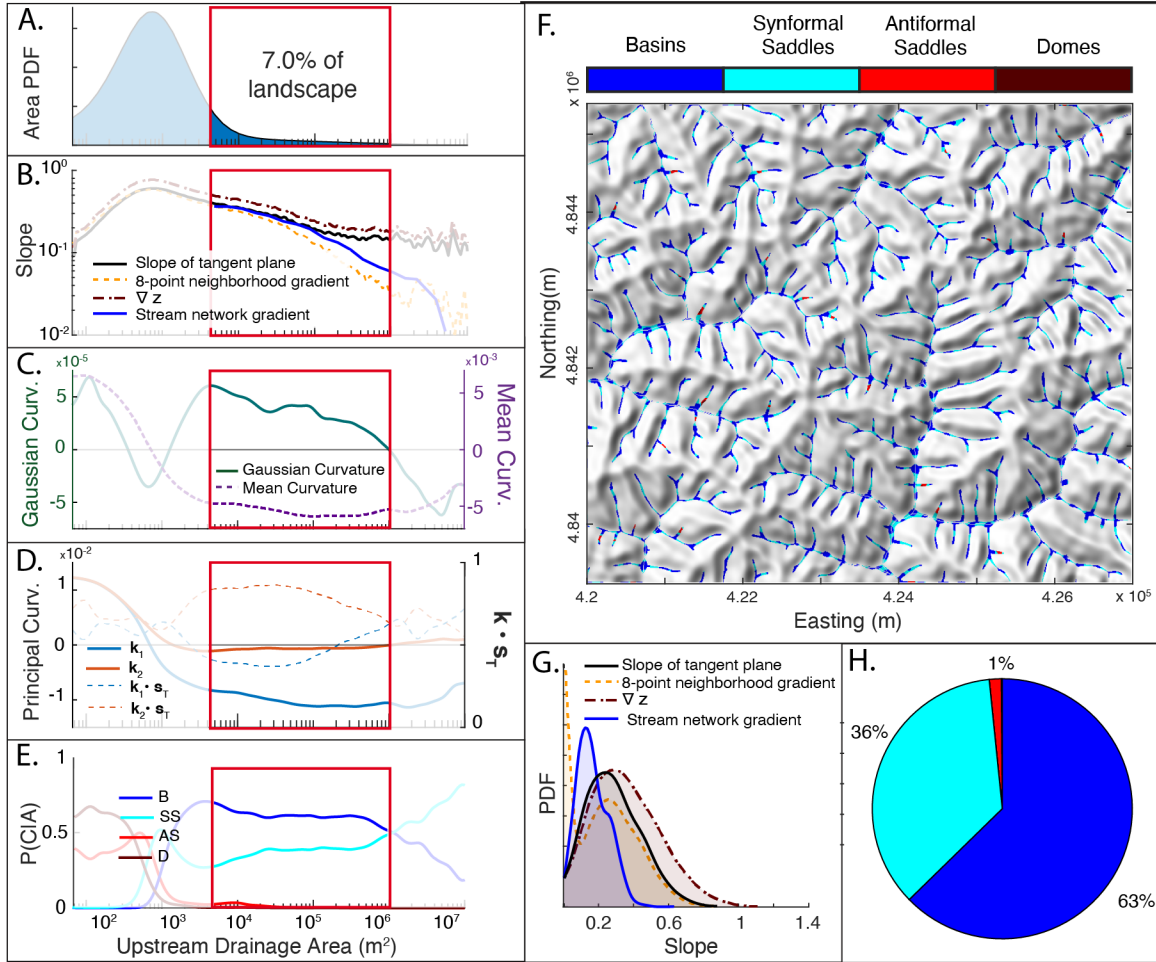
streams within the study region extracted using the STREAMObj object class from TopoToolbox using a threshold drainage area for channelization of 4500 m^2 .

2.9.1.5 Region 5: Drainage areas greater than $9.8 \times 10^5 \text{ m}^2$.

Our final point of differentiation based on the evolution of the Gaussian curvature is located at its final transition from mostly positive to mostly negative, which occurs at a drainage areas of $9.8 \times 10^5 \text{ m}^2$, or roughly 1 km^2 . In the geomorphology literature this is a commonly applied threshold area for the onset of fluvial processes, and –perhaps reassuringly– this is reflected in the continued evolution on the curvature metrics. The increasingly negative Gaussian curvature (Panel C) is an expression of a greater tendency towards synformal saddles, which is the morphology of a stereotypical channel structure.

In spite of this we see that actually the fluvial channels are almost evenly partitioned into basins and saddles (Panel H), which reflects fine-scale oscillations in the Gaussian curvature that are obscured by the binning process, but are observable in the map-view representation of the data (Panel F). This points out a major weakness in area space representations of landscape data, as binning over orders of magnitude does not allow for the observation of fine-scale structure that is likely important for understanding processes, or the interactions between different sections of the landscape.

We also note that in this region the different metrics for slope give very different distributions. While it seems clear that the 1-d channel gradient is the best for understanding the slope of channels it is less clear how to best connect this measure to the measure of tangent slope on which we depend for our correlation between slope and principal curvature vectors. This points out a major challenge for slope-area analysis, which is that it may not be possible to capture the subtlety



*Figure 21. Surface geometry data for points in the landscape with upstream drainage areas between $4.5 \times 10^3 \text{ m}^2$ and $9.8 \times 10^5 \text{ m}^2$. The red box in each of the area plots highlights the region of interest. **A.** PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.*

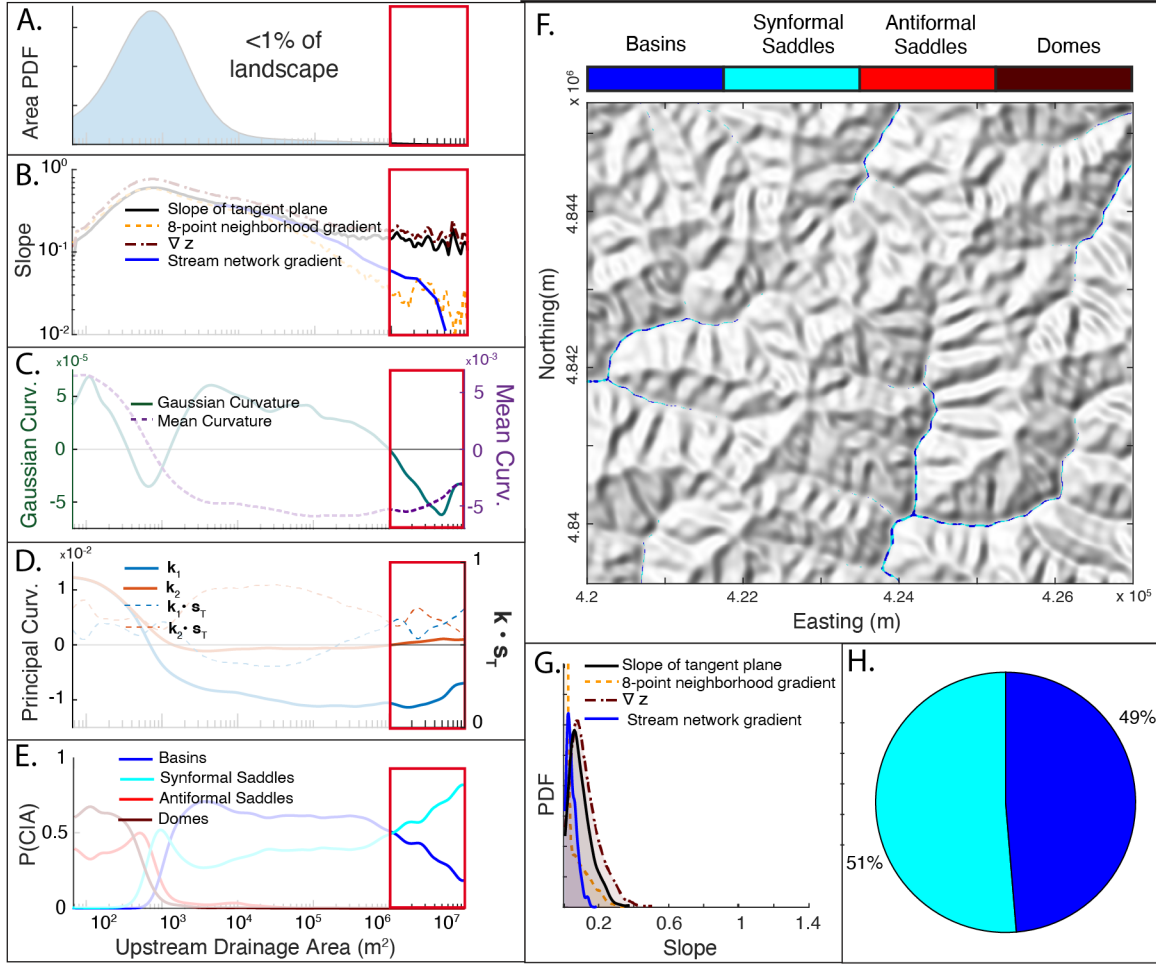
of the full range of processes using a single metric. It is thus a bit unclear exactly what is being shown by slope-area plots across process regimes with variable surface geometry. Finally we note that despite fluvial channels being the connection between the landscape and base level that likely sets the pace of erosion, fluvial channels make up less than 1% of the landscape by area.

2.9.2 Mean curvature based landscape partitioning.

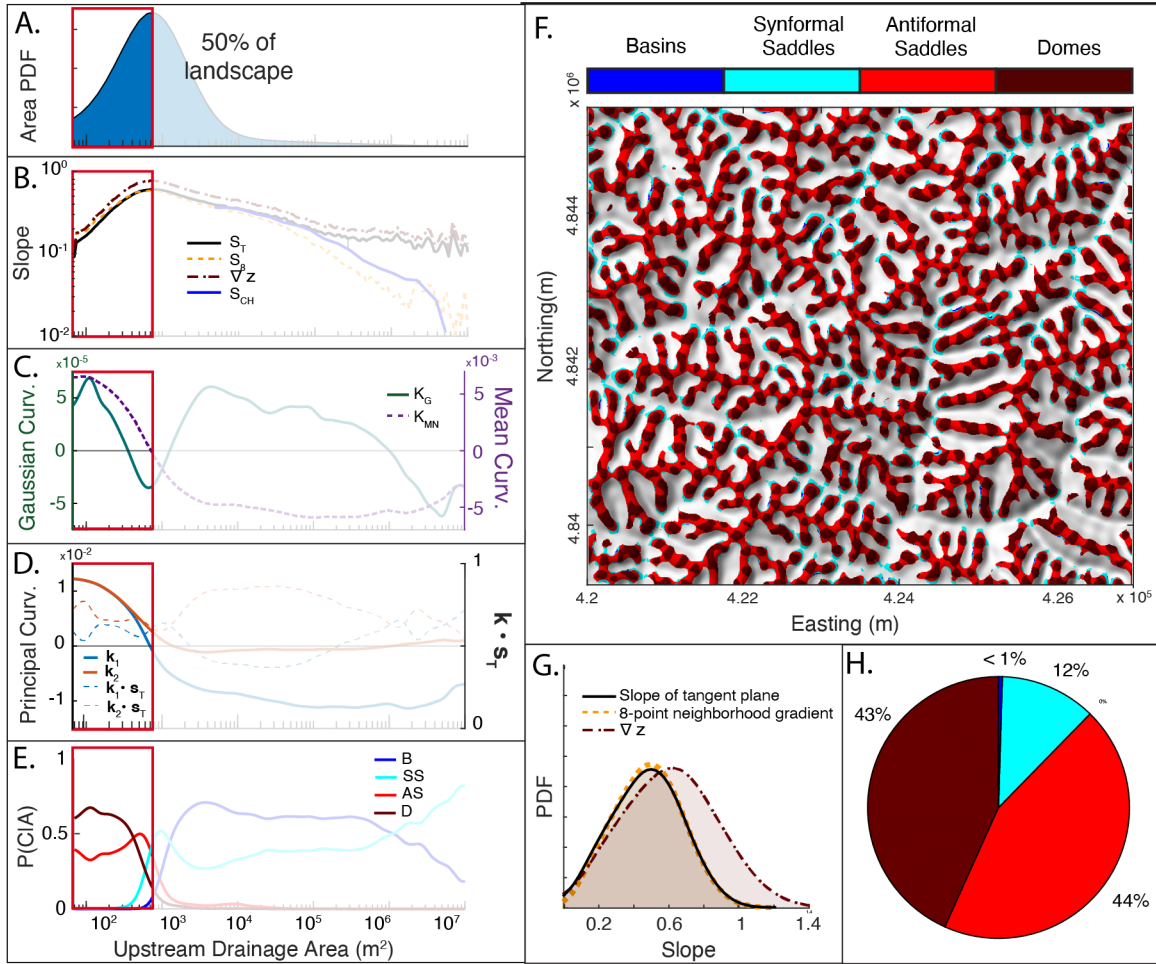
2.9.2.1 Mean curvature inflection point. In the previous section we laid out one possible approach for classifying landscape process transitions in terms of inflection points and extrema of the Gaussian curvature. The distribution of Mean curvatures in area space is much less complex, but can also provide interesting insight into the organizational structure of the landscape, though only with resolution allowed by averaging into area bins.

The Mean curvature has a single inflection point, which occurs at drainage areas of $\sim 750 \text{ m}^2$ within the transitional hillslope regime classified above as region 2. We decompose the landscape into two regions separated at this point with results shown in Figures 24 and 23. The existence of linear hillslope regions is well documented in the literature, and their relationship to hilltop curvatures have convincingly been tied to rates and style of erosion (Roering et al. (1999, 2007)). In the context of our work we see that in fact an inflection point, and thus region of linearity, is required for maintaining continuity between dominantly convex and concave regions of the landscape outlined above.

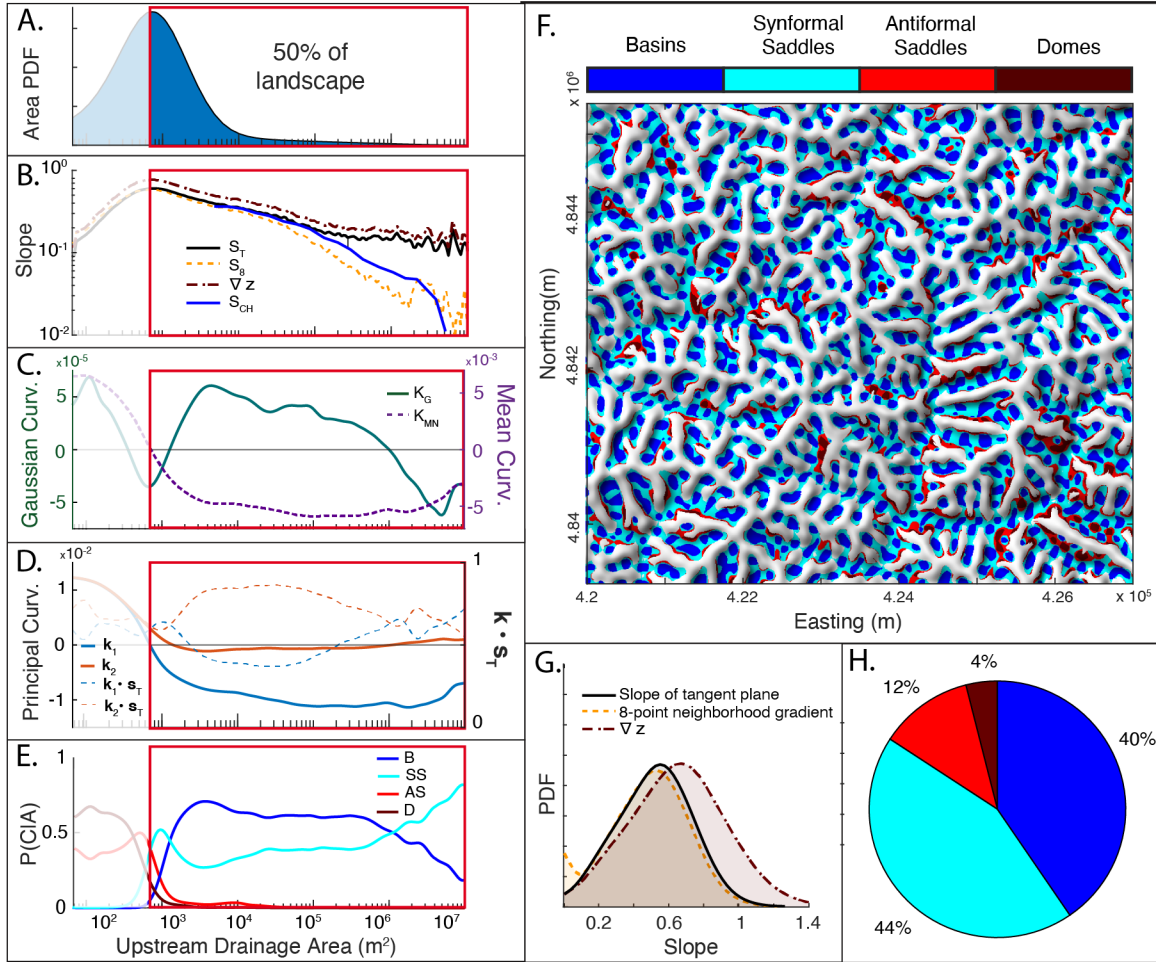
Not surprisingly this inflection point is aligned with the highest slopes in the landscape, which fall at the same drainage area in all three of the slope metrics evaluated in this study. The slope at this point in the landscape likely represents an average angle of repose, above which slope are gravitationally unstable, and



*Figure 22. Surface geometry data for points in the landscape with upstream drainage areas greater than $9.8 \times 10^5 \text{ m}^2$. The red box in each of the area plots highlights the region of interest. **A.** PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.*



*Figure 23. Surface geometry data for points in the landscape where the Mean curvature (K_M) is majority positive. The red box in each of the area plots highlights the region of interest. **A.** PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.*



*Figure 24. Surface geometry data for points in the landscape where the Mean curvature (K_M) is majority negative. The red box in each of the area plots highlights the region of interest. **A.** PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.*

below which material will tend to collect as colluvium. This is consistent with the curvature data which shows that at length scales represented by our analysis the average geometries upslope of this demarcation line are convex (Figure 23) and thus steepen downhill, while below this threshold geometry tends to be concave (Figure 24) with decreasing slope downhill. This can be seen in Panel E, F, and H of Figures 23 and 24 where domes and antiformal saddles give way to basins and synformal saddles roughly at this point.

This inflection point also aligns with the most probable drainage area in the study area and, though area is not normally distributed, the landscape is equally partitioned about this inflection point such that 50% of points in the landscape sit both above and below this threshold. This remarkable landscape symmetry is further explored in Figure 25 which shows a map-view representation of the landscape divided into concave and convex regions as defined by this area threshold and shows the distributions of tangential slope (S_T), Mean curvature (K_M), and Gaussian curvature (K_G) in these respective portions of the landscape. We see that there are nearly identical distributions of slope above and below this area threshold, and that the Gaussian curvature distributions are also nearly aligned while the distributions of mean curvatures roughly mirror each other.

This work is still too preliminary to make any sort of statement as to the meaning of this inflection point, but this strong signal of symmetry and geometric balance in the landscape seems profound enough to in itself warrant further study that explores how this inflection point varies between landscapes.

2.9.2.2 Mean curvature based channel threshold: Drainage areas greater than $1 \times 10^5 \text{ m}^2$. Other than the inflection point discussed in the previous section the Mean curvature (K_M) only has one major transition

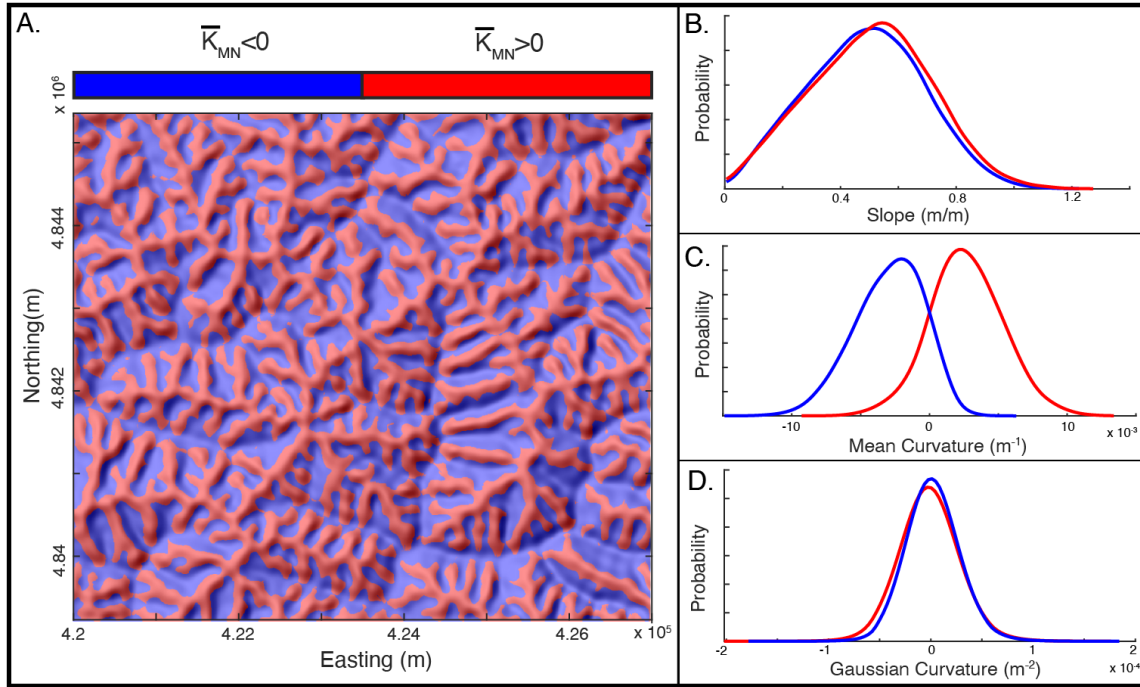


Figure 25. Distribution of surface geometry metrics for regions defined by the Mean curvature inflection point. A. Map-view of landscape partitioned about the point of inflection in K_M . *B.* Distribution of tangent slope in regions of both negative and positive average K_M . *C.* Distribution of Mean curvature in regions of both negative and positive average K_M . *D.* Distribution of gaussian curvatures in regions of both negative and positive average K_M

point, which is a minimum at $\sim 1 \times 10^5 \text{ m}^2$. While this turning point is not very pronounced in topography filtered to 200 m, it can be seen in figure 9 that the existence of an extreme value at approximately these drainage areas is a robust feature of the data at a range of scales. Figure 26 shows the results of imposing a threshold at this drainage area. As would be expected this region is composed of synformal saddles and basins consistent with the channel network, and with a relative distribution that is between that of regions 4 and 5 defined by the Gaussian curvature.

While partitioning the landscape this way does not stand out from the other channel network thresholds in terms of curvatures, it is notable that it aligns with a scaling break in the slope-area data, which has often been associated with process transitions (Montgomery and Foufoula-Georgiou (1993)), and so this point may represent the onset of fluvial processes. Conceptually this makes sense as we would expect narrow channels at this point in the landscape, which would correspond with high Mean curvature values.

2.10 Other Potential Applications

2.10.1 Identifying debris-flow source regions. One way to define the heads of channel networks is through the identification of colluvial hollows that make up the transition regions between hillslopes and first order debris flow channels. Colluvial hollows are of broad interest since they often are areas where hillslope sediments collect that are ultimately mobilized in debris flows (Stock and Dietrich (2003); Struble et al. (2023)), and so they have direct connection to debris flow hazards. Morphologically colluvial hollows tend to be regions of omnidirectional upward concavity, and so we expect them to present in our shape classification as basins, and more specifically the basins containing the smallest

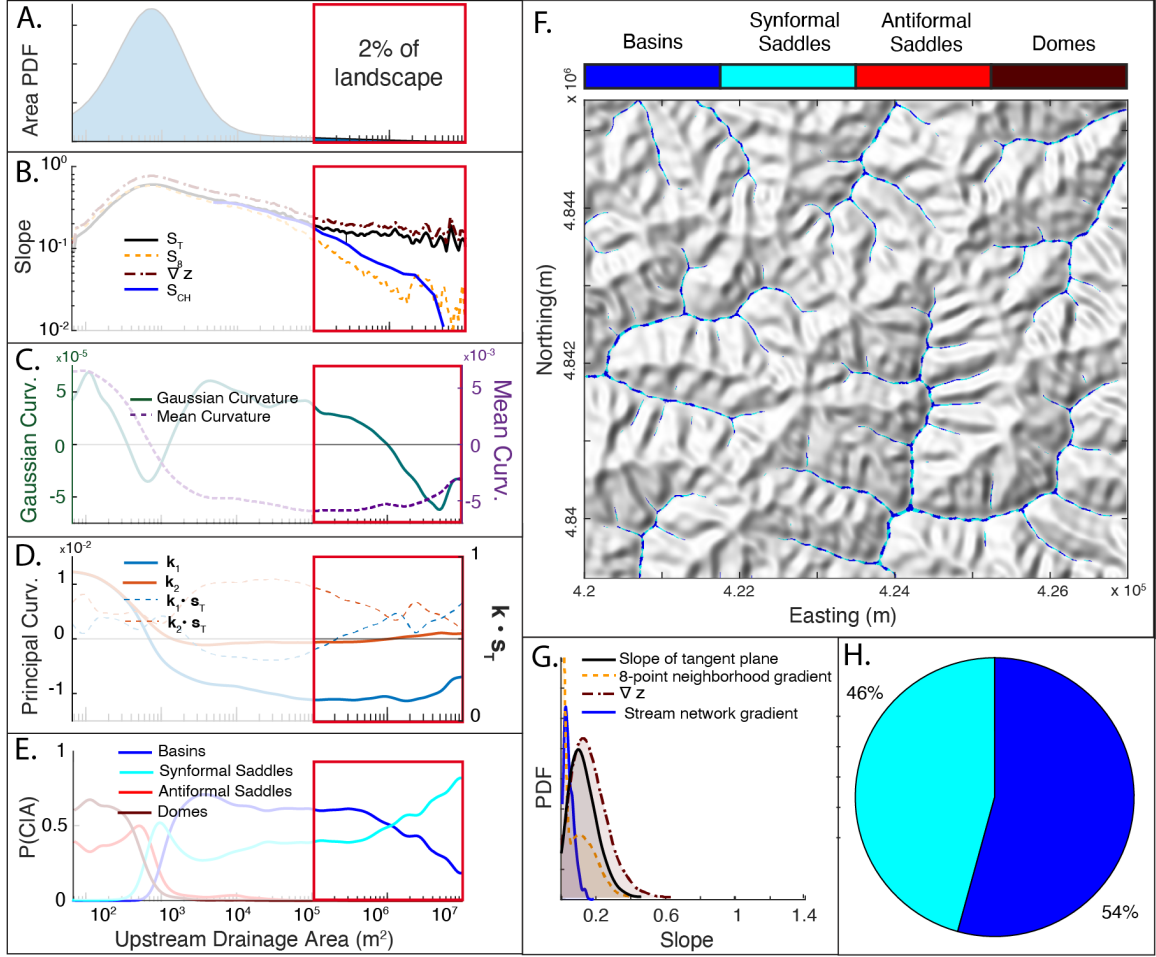


Figure 26. Surface geometry data for points in the landscape with upstream drainage area greater than $1 \times 10^5 \text{ m}^2$ as determined by the global minimum in the mean curvature. The red box in each of the area plots highlights the region of interest. **A.** PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.

accumulated area. We can identify potential colluvial hollows by selecting basin structures on the basis of accumulated area. Since channelization is found in section 2.9 to become established at drainage areas between $4.5 \times 10^3 \text{ m}^2$ and $9.8 \times 10^5 \text{ m}^2$ we assign a threshold drainage area of $1 \times 10^5 \text{ m}^2$ and extract all basin structures that do not include points with drainage areas greater than the threshold. The results of this exercise are shown in Figure 27.

2.11 Conclusions

In this study we have applied our method of surface classification to the simplest possible landscape in the hopes of identifying structures that define steady-state fluvial topography. It is certain that the magnitudes and distributions of each of the metrics defined here vary between landscapes, and depend on a host of variables including erosion rate, climate, lithology, ecology, etc. A natural next step in developing these tools is to explore how variation in these factors is expressed in curvature, and how signals of topographic disequilibrium are recorded in the system of land surface curvatures.

In addition we have compared both curvature and slope calculated using formal surface theory with approximations common in the geomorphology literature. We see that slope and curvature dependent errors in these metrics can be avoided with our more rigorous approach to classi

2.11.1 Bridge. In Chapter II we derived a system of land surface curvatures and showed how they are expressed in a landscape defined by fluvial equilibrium. In Chapter III we show the applicability of this methodology to more complicated landscapes by applying it to a bedrock chronosequence in the Oregon Cascades, where time dependent hydrology in volcanic bedrock is recorded in the

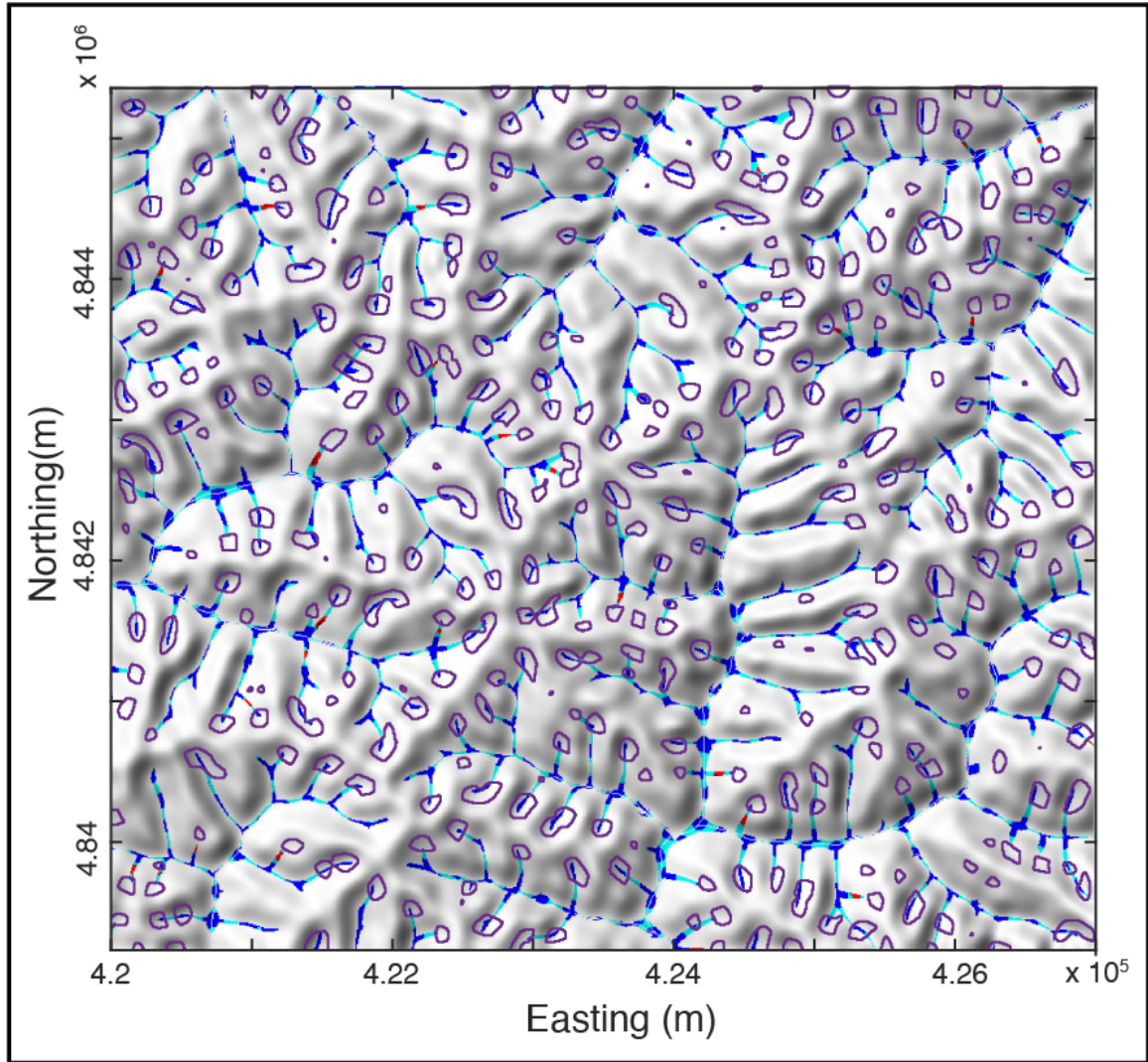


Figure 27. Map of colluvial hollows inferred from our analysis. Purple outlines mark basin structures that only containing points with areas less than an assigned threshold of $1 \times 10^5 \text{ m}^2$. These are overlain on a shape-class map of region 4, which we interpret to represent the uppermost reaches of the channel network

degree of surface dissection by fluvial channels. We show that this hydrologic and topographic transition is reflected in curvature metrics derived in Chapter II.

CHAPTER III

CURVATURE SIGNATURES OF VOLCANIC BEDROCK AGING IN THE CENTRAL OREGON CASCADES

Along most continental subduction margins the highest and most geologically active topography is located along the volcanic arc front, however there exist few tools for studying the long-term dynamics of such terrains. As a result the potential of geomorphology as a framework for studying volcanic landscape evolution has not been fully realized. However a growing body of work suggests that with the proper tools and approaches geomorphology has the potential to identify signatures of volcanic forcing on the landscape and thus could revolutionize our understanding of the dynamics of volcanic landscape evolution and thus magmatic processes (O'Hara, Karlstrom, and Roering (2019); O'Hara, Klema, and Karlstrom (2021); Singer et al. (2018)). One reason this is challenging is that basaltic bedrock has a non-trivial time-dependent permeability structure and so general approaches in geomorphology that correlate erosional efficiency to upstream drainage area are not easily applied. In this chapter we show how curvature metrics derived in Chapter II can be applied to a basaltic bedrock chronosequence in the central Oregon Cascades to infer timescales of a well-documented state-shift from groundwater to surface water dominated hydrology, which is correlated with the evolution of volcanic constructional topography to a fluvially dissected landscape with signatures of equilibrium. Since this approach does not require assumptions regarding the partitioning of meteoric water between the surface and subsurface it provides an objective way of quantifying volcanic topography towards better understanding the mechanisms and timescales of volcanic landscape evolution.

3.1 The Cascades Chronosequence

In the central Oregon Cascades bedrock age increases westward from the arc axis (Figure 28) reflecting an eastward migration of the main locus of volcanism over timescales of tens of Myr (du Bray and John (2011)). Arc front migration relative to the plate boundary is common in subduction zones (Dickenson and Snyder (1979); Karlstrom, Lee, and Manga (2014)), and in the context of the Cascades is likely due to the clockwise rotation of the Pacific Northwest fore-arc block relative to North America (R. E. Wells and McCaffrey (2013)) combined with potential climatic and magmatic drivers (Hildreth and Wilson (2007); Karlstrom et al. (2014); V. A. Muller, Sternai, Sue, Simon-Labric, and Valla (2022)). As a result the Oregon Cascades are often referenced in terms of two geologic subprovinces; the older Western Cascades, and the younger High Cascades where volcanic bedrock is associated with the modern arc (du Bray and John (2011)). While this bedrock chronosequence is defined on the basis of detailed field mapping (Sherrod and Smith (2000)) fundamental differences between the Western and High Cascades are also reflected in both hydrology and topography, which themselves are tightly coupled. Thus the Cascades chronosequence is an ideal natural laboratory for the study of time dependent hydrologic processes in volcanic landscapes, and how they are recorded in the evolution of topography.

3.1.0.1 Groundwater Dominated High Cascades. Young basaltic bedrock has high bulk permeability meaning that meteoric water in the High Cascades is mostly absorbed into shallow aquifers and there is little surface runoff (Saar and Manga (2004)). As a result the High Cascades contain few fluvial channels and topography is mostly defined by primary volcanic structures and glacial valleys carved during the LGM (Deligne et al. (2017); Sweeney and Roering

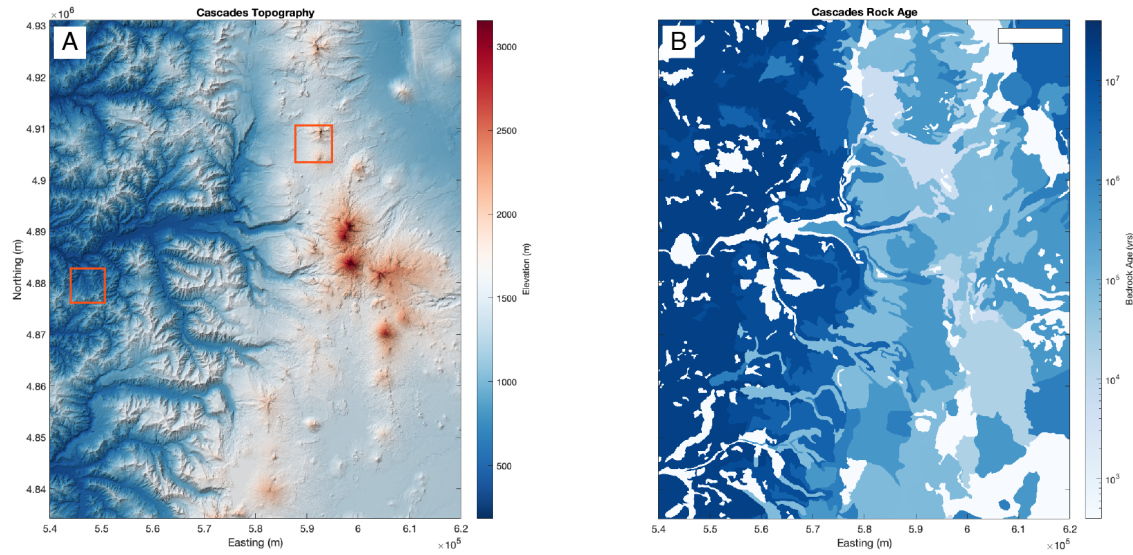


Figure 28. Map of the central Oregon Cascades. A. Topography. Red boxes indicate areas of focused analysis in Figs. 30 and 33. B. Age of volcanic bedrock units (from Sherrod and Smith (2000))

(2017)). Where stream channels exist in the High Cascades they are mostly spring fed with damped hydraulic responses to precipitation indicating the dominance of stable groundwater systems in modulating their discharge (Manga (1999); Tague and Grant (2004)). This can be seen in Figure 29, which is taken from Deligne et al. (2017) and compares an annual hydrograph of the McKenzie River to that of the Little North Santiam River. The more muted response of the McKenzie to precipitation and run-off events is characteristic of spring fed channels (Tague and Grant (2004)). In addition there also exist in the High Cascades ephemeral channels that have rapidly incised into lava flows and show the relevance of standard geomorphic processes in the development of volcanic terrains (Sweeney and Roering (2017)), however these channels are only locally similar to those of non-volcanic landscapes, possibly representing an intermediate stage of fluvial landscape development.

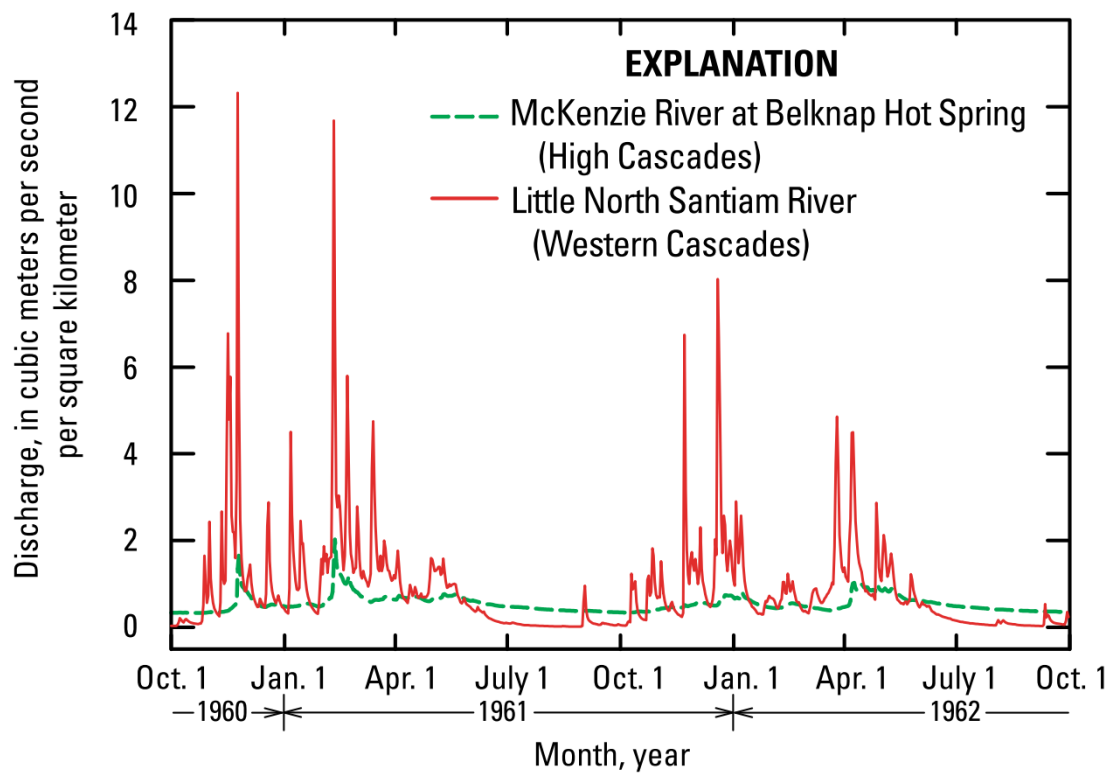


Figure 29. Characteristic hydrographs of the spring dominated High Cascades, and the surface run-off dominates Western Cascades (from Deligne et al. (2017) which is a modified figure from Tague and Grant (2004))

3.1.0.2 Surface Water Dominated Western Cascades. Over time there is a decrease in the bulk permeability of basaltic bedrock as micro-pores are clogged with fine sediment such as glacial till (Sweeney and Roering (2017)), volcanic ash (Jefferson et al. (2010)), and soil particulate formed through en situ chemical weathering (Lohse and Dietrich (2005)). The diminishing infiltration capacity corresponds with an increase in overland flow and a correlated increase in the ability of fluvial channels to mobilize sediment (Jefferson et al. (2010)). As a result the aging landscape is increasingly defined by channel networks with slope-area scaling relationships and curvature distributions similar to those defined in Chapter II for the Oregon Coast Range. Figure 30, shows the same presentation of surface metrics given in Chapter II, but calculated on a section of the Western Cascades in which bedrock has an age of 20 ± 2.9 My (Sherrod and Smith (2000)). It can be seen that while the magnitudes of these metrics - and the exact locations of inflection points and extrema - are slightly different than those of the Coast Range, the landscape has similar area-space trends and geometric distributions. This implies the Western Cascades can be considered an equilibrium landscape despite its complex initial conditions and evolutionary trajectory.

3.2 Signatures of Topographic Development

The idea of leveraging the delayed development of fluvial channels in the Cascades as a means of understanding the hydraulic state-shift is not new and builds directly on the work of Jefferson et al. (2010), who used a combination of drainage density, bedrock age, and stream base flows to show that processes driving the hydraulic state-shift occur over 100 ky timescales, with the full transition to fluvial topography taking ~ 1 Ma. Figure 31 shows this result in the form of a basin averaged drainage density plotted against basin averaged bedrock age.

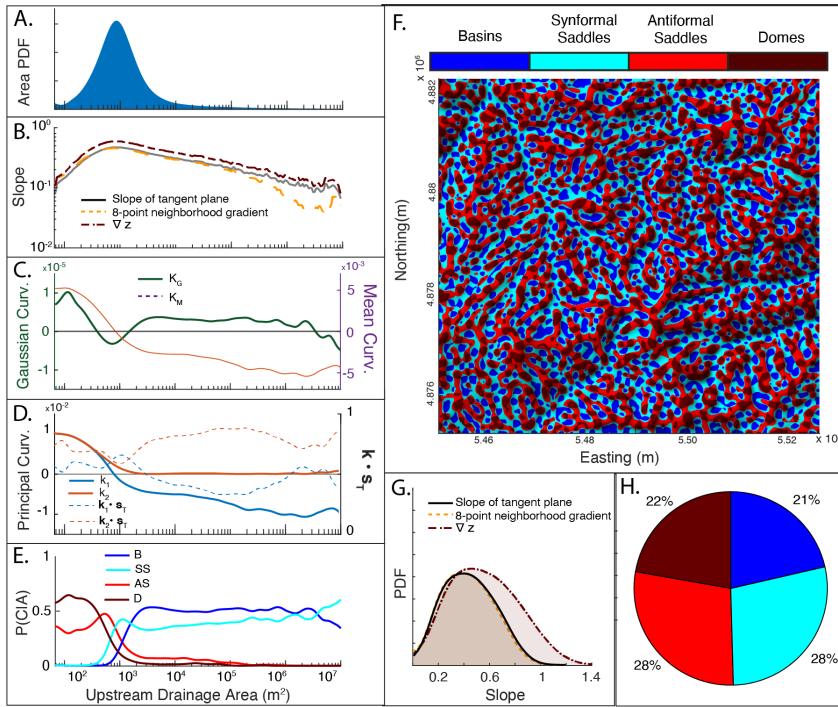


Figure 30. Compilation of surface geometry data used in Chapter II applied to the Western Cascades. A. PDF of upstream drainage area. **B.** Slope metrics binned by upstream drainage area. **C.** Surface curvatures binned by upstream drainage area. **D.** Principal curvature values and slope alignment binned by upstream drainage area. **E.** Conditional PDFs of shape classes as a function of upstream drainage area. **F.** Map-view of shape class distribution. **G.** PDFs of slope metrics. **H.** Pie chart distribution of shape classes.

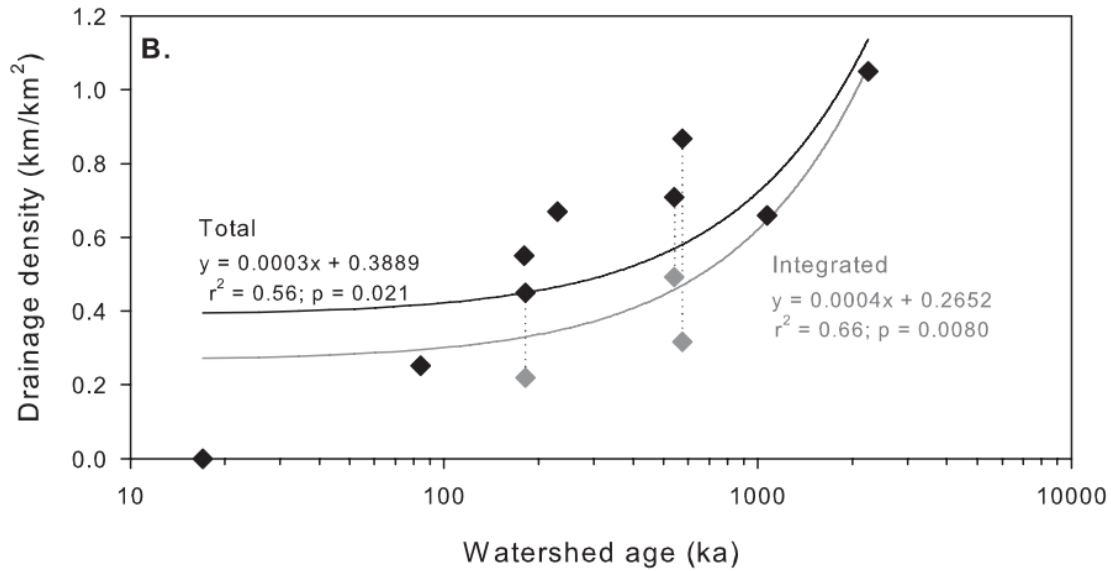


Figure 31. Plot of drainage density versus bedrock age in the upper McKenzie River Basin (from Jefferson et al. (2010))

A spatial correlation between bedrock age and drainage density can be observed throughout the Cascades, and is seen in Figure 32 which shows map-view representations of bedrock age and drainage density (calculated off of USGS BlueLines) averaged over several spatial scales using a simple moving window filter. The problem with this method for inferring bedrock age and hydrologic characteristics is that it requires making a choice as to what defines a channel, which is challenging in the complex topography of volcanic landscapes. This was demonstrated by Luo and Stepinski (2008), who compared different methods of drainage network extraction in the Cascades and showed that drainage density is highly dependent on classification method. They also found that a curvature based algorithm that does not depend on any drainage area assumptions (Molloy and Stepinski (2007)) most accurately recreates the channel network as defined by field mapping. Still it does not provide a tool for more detailed morphometric analysis in un-channelized portions of the landscape along the arc axis. This

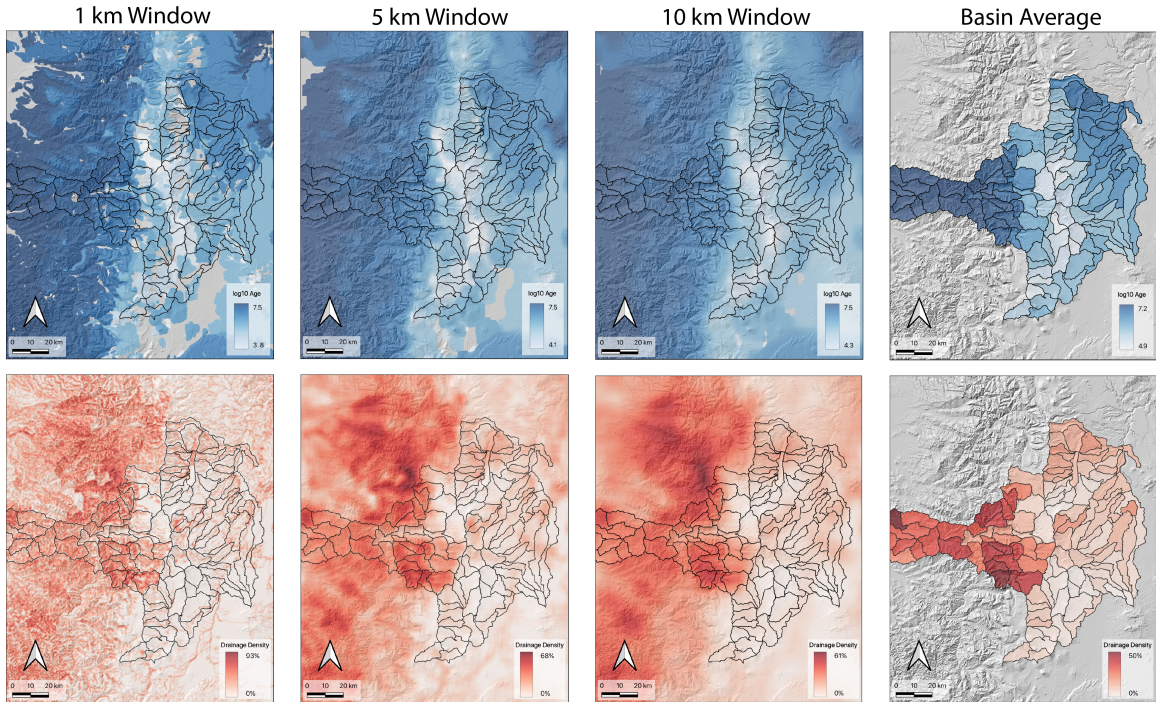


Figure 32. Maps of surface bedrock age in the Oregon Cascades averaged over different spatial scales.

relates to another challenge in applying the methods of Jefferson et al. (2010) more generally, which is that both drainage density and bedrock age are averaged within topographic watersheds, which may not always align with hydraulic divides in volcanic landscapes where relict topography is overprinted with permeable surface lithologies (Gannett et al. (2001)).

3.2.1 Curvature based surface classification. The surface classification method derived in Chapter II carries no inherent assumptions about process and so can be used as an objective measure of topographic form across the entire Cascades chronosequence. Area-space representations are not appropriate in the High Cascades due to disconnects between topographic and hydrologic watersheds (Gannett et al. (2001)), and so we focus on the surface evolution relative to bedrock age. We first explore the fundamental differences in topographic

form by comparing the distributions of surface geometry metrics in the High and Western Cascades to those of the Oregon Coast Range. In this section we focus on the Mean curvature (K_M), Gaussian curvature (K_G), the tangent slope of the surface (S_T), and the surface area proportionality constant (α). α is a measure of the change in area between the surface and its map projection that follows directly from the first fundamental form, and which we interpret to be a local measure of topographic relief.

Figure 33 shows PDFs for each of these metrics calculated on 10 m DEMs filtered to 200 m for direct comparison with the analysis in Chapter II. Even this very general presentation of the geometric data gives insight into the fundamental differences between fluvial and volcanic-constructional landscapes. While curvatures are normally distributed in all of the landscapes the narrow distributions in the High Cascades indicate much lower magnitude curvatures than the Western Cascades, which have wider distributions that are more similar to those of the Coast Range. This possibly suggests that the widening of curvature distributions is a temporal signature of landscapes trending towards fluvial equilibrium. A similar trend is seen in the distributions of both slope and α , with overall lower magnitude values in the High Cascades, and higher magnitude values in the Western Cascades and Coast Range.

3.2.2 Timescales of Fluvial Development. We now apply our analysis to the full Cascades chronosequence to give a more detailed view of how the hydrologic state-shift is recorded in land surface geometry. Surface metrics are calculate throughout the full study area and are binned both by UTM Easting and by age with the results shown in Figures 34-35. We first note that this analysis confirms the result from Jefferson et al. (2010) that the topographic complexity

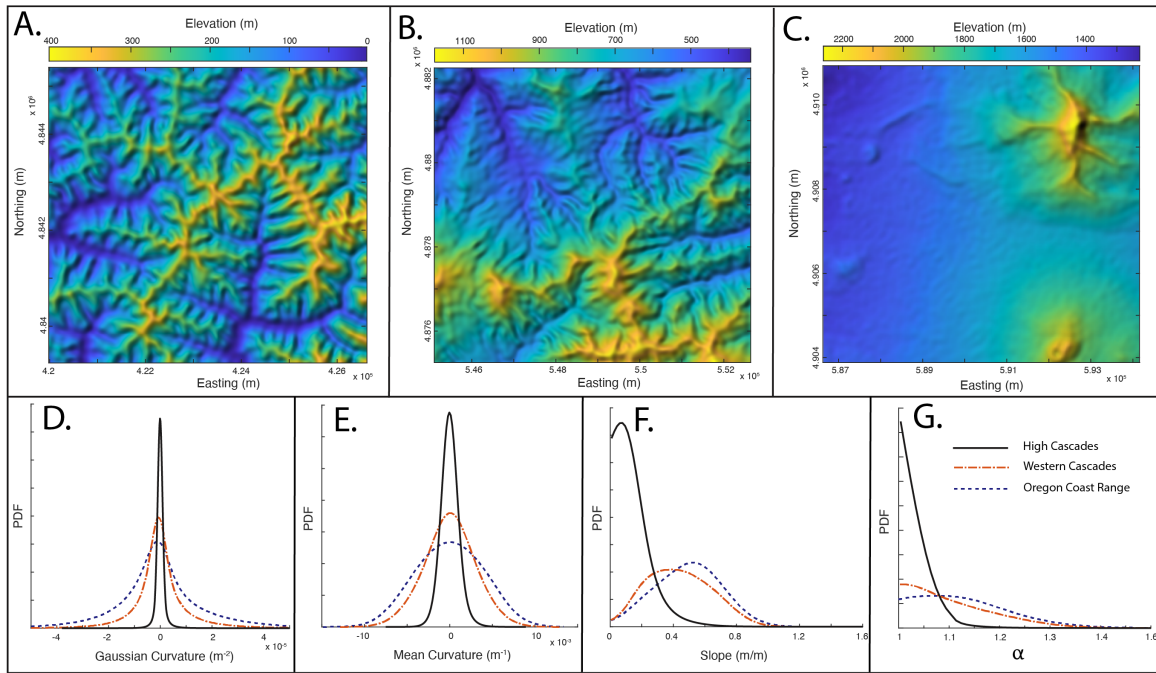


Figure 33. Surface metric distributions for sections of the Oregon Coast Range, Western Cascades, and High Cascades. A. Oregon Coast Range topography. B. Western Cascades topography. C. High Cascades topography. D. Gaussian curvature PDFs. E. Mean curvature PDFs. F. Slope PDFs. G. α PDFs.

associated with fluvial dissection emerges in the Cascades when volcanic bedrock ages reach about 1 Ma. This same timescale is reflected in the increased magnitude of all geometric parameters around that time. In terms of easting the transition is more gradual, but we note a sharp increase coincident with major arc bounding normal faults (R. M. Conrey, Taylor, Donnelly-Nolan, and Sherrod (2002)). This possibly indicates some structural control on the state-shift, which could be due to the build up of young basaltic lavas along the margins of the arc (Sherrod and Smith (2000)). The bell shape of the surface metrics binned by easting is not easily explained, and could possibly be connected to subsidence of the Willamette Valley (Mitchell, Vincent, Weldon, and Richards (1994)) or orographic precipitation. However there is also similarity in the shape and wavelength of these trends to hyaloclastite deformation in the Columbia River Gorge Tolan and Beeson (1984), which we explore in depth in Chapter IV. We note that flexural deformation of bedrock has been identified in limited sections of the Western Cascades (Lopez and Meigs (2016); Pesek, Perez, Meigs, Rowden, and Giles (2020)), and so there is a possibility that these surface geometry trends record signatures of structural deformation that are not widely recognized.

3.3 Further Directions

In this chapter we have demonstrated a simple way in which surface geometry can be used to study the evolutionary trajectory of arc topography in the Oregon Cascades. This analysis was conducted over large spatial scales in an attempt to extract statistical characteristics of topography, however geometry is inherently scale dependent and so there is much to be learned by such analysis at a range scales. For example it is likely possible to identify signatures of topographic forms not easily described by standard geomorphology tools such as stratocones,

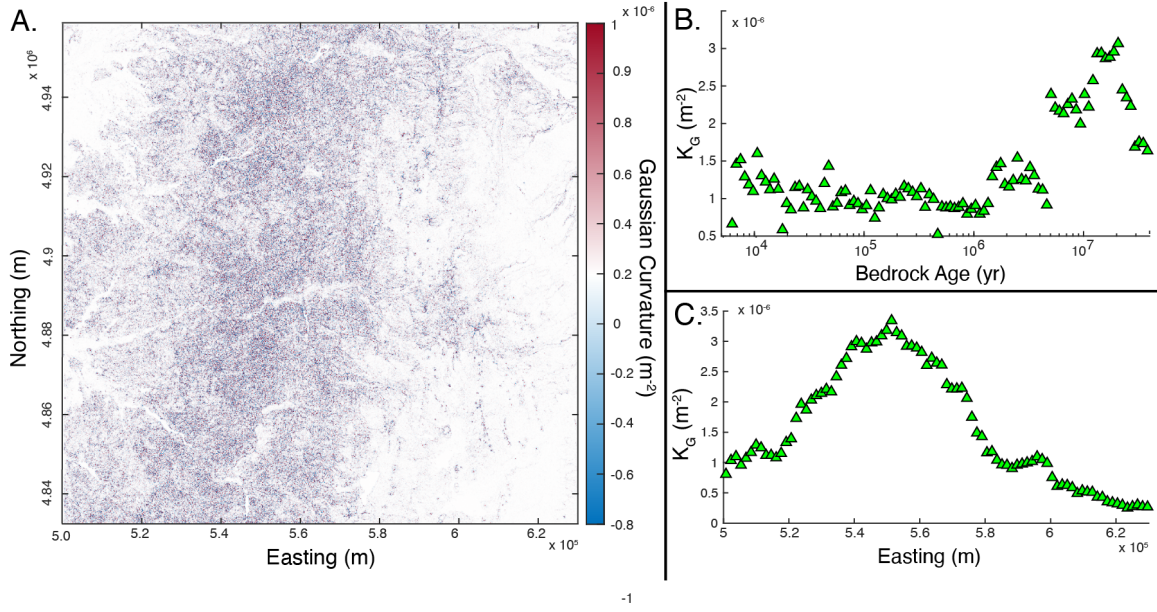


Figure 34. **Gaussian curvature of the Cascades chronosequence.** A. Map-view of calculated Gaussian curvature. B. Gaussian curvature binned by bedrock age. C. Gaussian curvature binned by UTM easting.

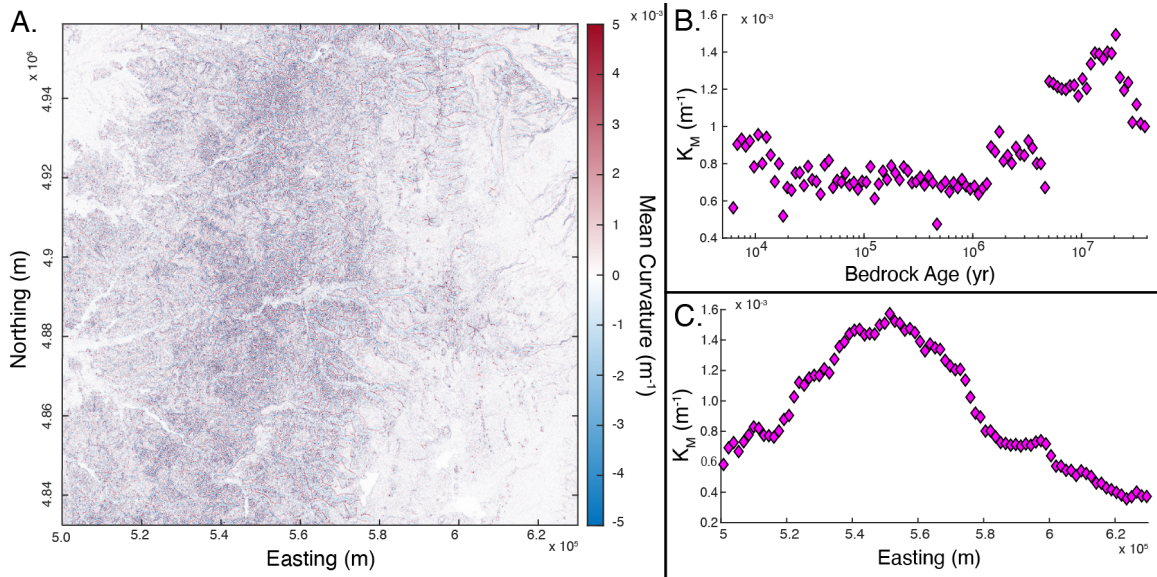


Figure 35. **Mean curvature of the Cascades chronosequence.** A. Map-view of calculated Mean curvature. B. Mean curvature binned by bedrock age. C. Mean curvature binned by UTM easting.

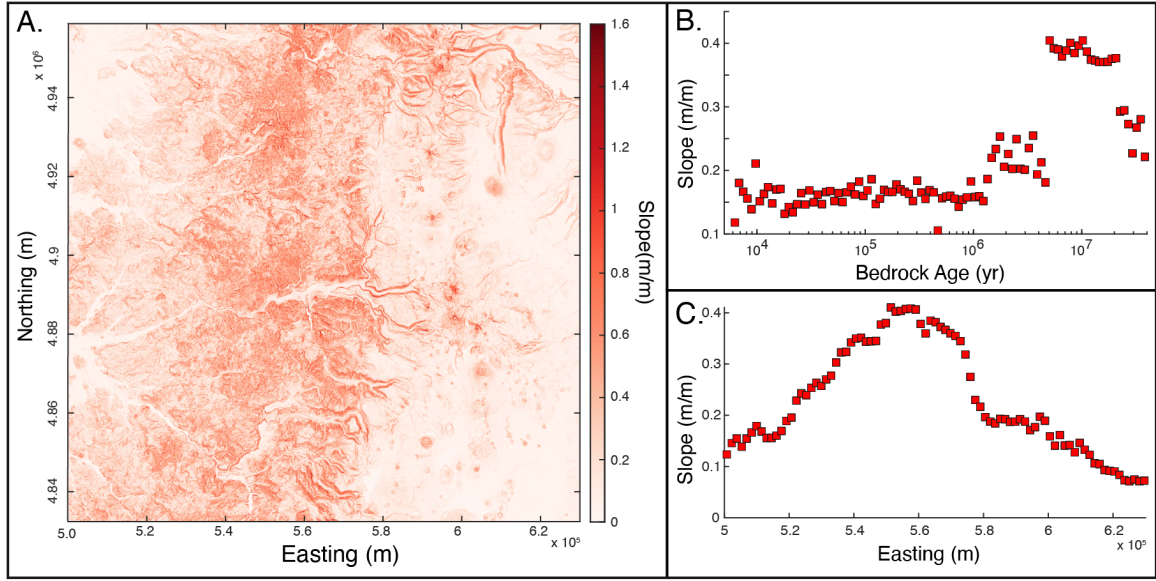


Figure 36. Tangent slope of the Cascades chronosequence. A. Map-view of calculated tangent slope. *B.* Tangent slope binned by bedrock age. *C.* Tangent slope binned by UTM easting.

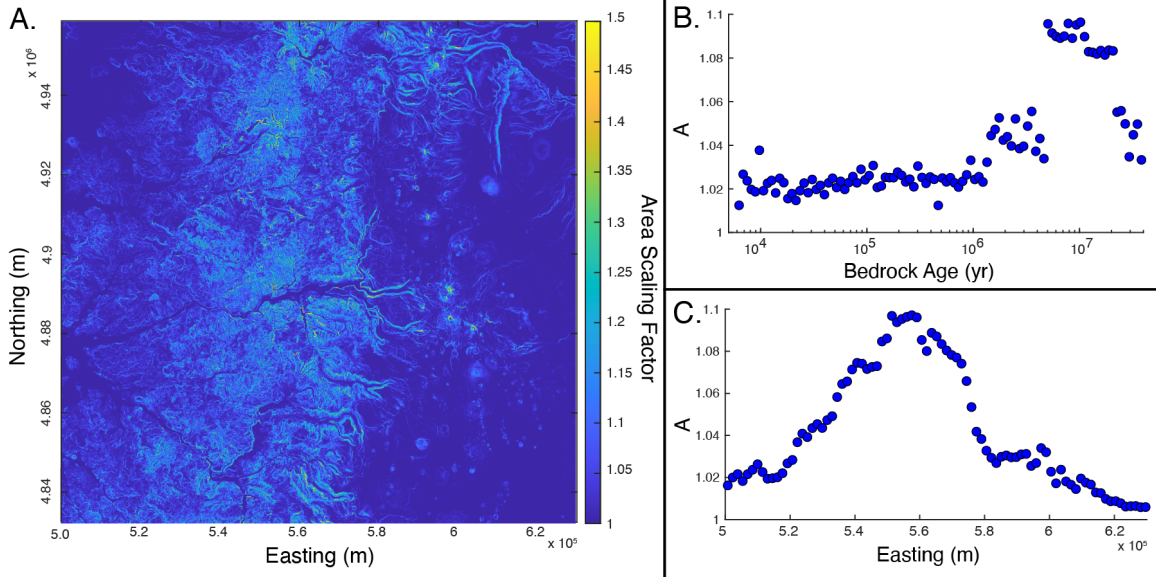


Figure 37. α of the Cascades chronosequence. A. Map-view of calculated area proportionality factor (α). *B.* Area proportionality factor (α) binned by bedrock age. *C.* Area proportionality factor (α) binned by UTM easting.

lava flows, and glacial landforms. We have also stopped short of describing the evolution of the Cascades in terms of the shape classes derived in Chapter II, or looked in detail at how the distributions and orientations of the principal curvatures evolve in aging volcanic bedrock. The different shape in the curvature distributions between the Western and High Cascades also make this an ideal setting for the application of curvature thresholds in defining surface shape classes, which could likely be used to more rigorously define the geometric evolution of the Cascade Arc.

3.4 Conclusion

In this chapter we have shown how the transition from a volcanic constructional to a fluvially dissected landscape in the Oregon Cascades is recorded in land surface geometry. Our approach confirms past estimates of a state-shift from ground to surface water dominated hydrology occurring over ~ 1 My with a correlated development of ridge-valley topography. Next steps in this work include more sophisticated classification including an analysis of shape class distributions as was done for the Oregon Coast Range in Chapter II.

3.5 Bridge

In Chapters II and III, we explored a new method for land surface curvature classification based on classical surface theory. We now use a more traditional geomorphology approach of knickpoint analysis to disentangle the volcanic history of the Cascade Arc seen in cross-section in the Columbia River Gorge. While this setting is complicated in the context of landscapes in which knickpoint analysis is generally applied, we show that an interdisciplinary approach leveraging geologic and geophysical data gives considerable insight into the evolution of mountain range scale topography in this setting allowing for unique insights into long-

term trends in magmatic processes and how magmatism contributes to landscape evolution in arc settings. This work is taken from a manuscript in review, and so is comprised of main body text in addition to supporting information contained in Appendix A.

CHAPTER IV

THE MAGMATIC ORIGIN OF THE COLUMBIA RIVER GORGE, USA

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4.1 Introduction

Volcanic arcs represent the highest, steepest, and the most rapidly changing topography at subduction zones. Localized and unsteady uplift driven by magmatism, superimposed on background tectonic deformation, makes these terrains an outstanding laboratory for the study of volcanic mountain building (Gregory-Wodzicki (2000); Lee, Thurner, Paterson, and Cao (2015)). For over 100 years the Columbia River Gorge (hereafter the Gorge), an enigmatic high relief landscape that has shaped human society in the region (Lyman (1963)), has been recognized as an ideal setting for the study of arc processes (Bretz (1917); I. A. Williams (1923)). As the largest river on Earth to cross an active volcanic arc, the Columbia River has dissected the Cascades for at least 17 Myr (Evarts, Conrey, Fleck, and Hagstrum (2009)), exposing a cross section of the Cascade Range and serving as a near-sea-level base level driving erosion of the surrounding landscape.

4.2 The Columbia River Gorge

As the regional topographic low, the Columbia River corridor has received, conveyed, and been diverted by lava flows from Columbia River Flood Basalt volcanism to the east and locally from the Cascade volcanic arc that it bisects (Reidel, Camp, Tolan, Kauffman, and Garwood (2013)). Remnants of these volcanic deposits record subsequent deformation of the arc by their position relative to the modern Columbia River (O'Connor et al. (2021)). An especially prominent

structural datum is marked by the most recent Columbia River diversion in the Gorge at 3.5–3.0 Ma, when arc-derived lava flows and associated hyaloclastite filled the existing valley and displaced the river northward to its modern course (Beeson and Tolan (1990)). Hyaloclastite deposits filling this “Bridal Veil” (BV) paleo-channel (Figure 38) record as much as ~ 900 m of subsequent uplift in a broad, asymmetric anticline spanning ~ 80 km of the arc (Beeson and Tolan (1990)). A superposition of this uplift and overlying erupted deposits forms the iconic high-relief topography of the Gorge, which locally rises to 1512 m at the shield volcano Mount Defiance.

Clockwise rotation of the forearc relative to North America (Figure 38) since at least the early Miocene (R. E. Wells and McCaffrey (2013)) provides a framework to analyze the underlying drivers of this uplift. North of the Columbia River, tectonic rotation produces compression, mainly manifest by east-west oriented folds and thrust faults (Reidel et al. (2013)). South of the Columbia River the regime is mostly extensional as evidenced by north-south oriented normal faults. Rotational shear is accommodated by right lateral displacements on north to north-northwest strike-slip faults (R. M. Conrey et al. (2002); R. E. Wells (1998); R. E. Wells and McCaffrey (2013)). The Columbia River thus sits at a transition point where tectonic strain is dominantly north-south, making crustal shortening an unlikely source of the broad uplift centered on the Gorge. Instead, we hypothesize that the uplift is due primarily to magmatic crustal inflation. We will show that late Pliocene and Quaternary deformation of BV channel deposits is a flexural response to magmatic intrusions below an elastic upper crust. Magmatic forcing is active at present, as inferred from gravity, heat flow, and seismic tomography coincident with uplift, and patterns of Quaternary volcanism. Current Columbia

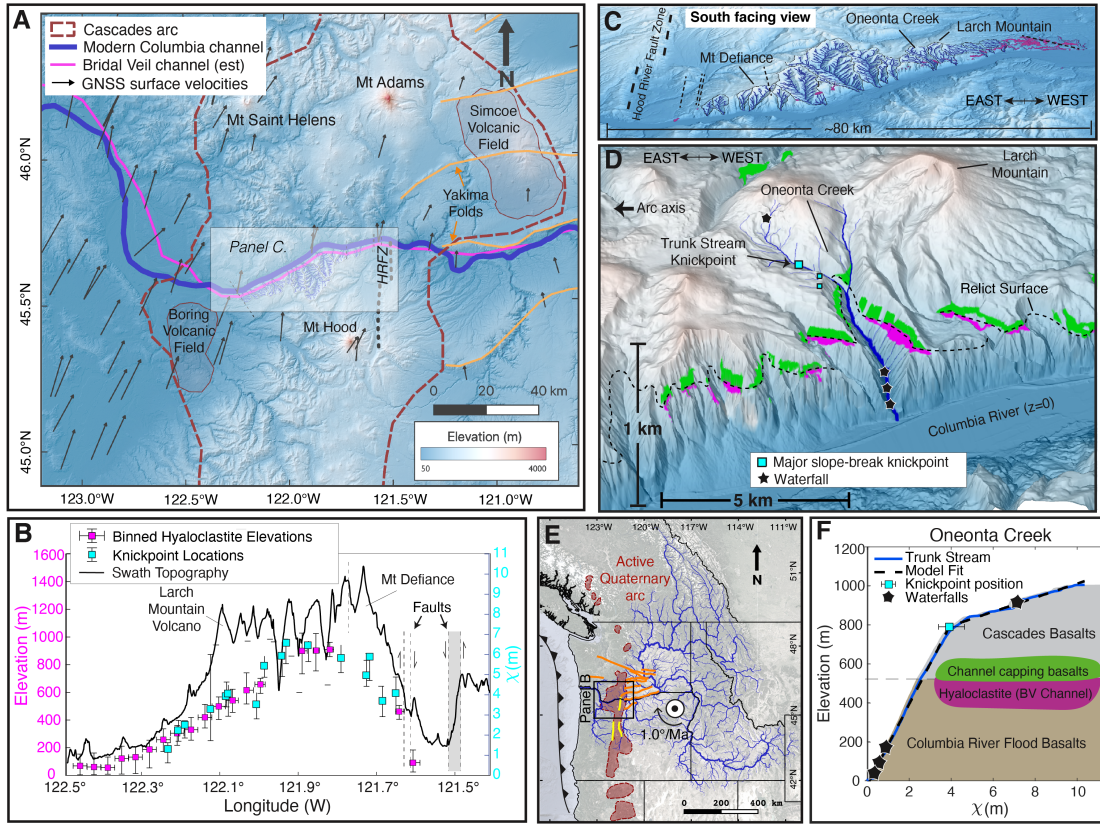


Figure 38. Overview of the Columbia River Gorge. **A.** Structural setting, showing GNSS velocities with respect to stable North American reference frame (McCaffrey et al. (2013)) (black arrows), Yakima Folds (orange lines), the modern and paleo ('Bridal Veil') Columbia River (blue and pink lines), and extent of the Cascades arc (red dashes). **B.** Symbols are Bridal Veil Channel hyaloclastite deposits and best-fitting knickpoint locations in Columbia tributaries. Maximum topographic elevations within 10 km south of the Columbia (black line) with prominent faults (dashed lines) (McLaughry et al. (2012)). **C.** Southward orthoview of fluvial catchments, hyaloclastite deposits, and major faults. **D.** Orthoview of Oneonta Creek showing the location of the trunk stream knickpoint used in this study (large blue square), analogous knickpoints in smaller tributary channels, and waterfalls associated with layered basalt stratigraphy (black stars). **E.** Regional map of the Columbia river drainage network, showing the Euler pole for Cascadia forearc block relative to North America. Orange lines are compressional structures while yellow lines mark arc-marginal normal faults. **F.** Longitudinal profile using area-normalized upstream distance χ for Oneonta creek showing the location of the best fitting slope-break knickpoint (Supplement). Lithologic units are labeled.

tributary drainages record transient topographic adjustment to this spatially variable magmatic uplift field in the form of fluvial knickpoints that mirror uplift of the BV paleo-channel.

4.3 Transcrustal magmatic structure

The mechanical response of Earth’s crust to an applied load depends on the magnitude and spatial wavelengths of forcing as well as rheological properties that are strongly time and temperature dependent (Audet, Jellinek, and Uno (2007); Burov and Diament (1995)). In the case of an elastic response to vertical loading the lateral wavelength of deformation is set by a flexural rigidity that scales with the effective elastic thickness of the crust (T_e) cubed. In a maturing arc setting, heat released from generations of magmatic intrusions ascending through a transcrustal transport network (Sparks and Cashman (2017)) likely induces a shallowing of T_e as crustal heating promotes ductile mechanical response at depth (Karlstrom, Paterson, and Jellinek (2017)). Intrusions dilate overlying elastic crust, producing a mechanical response similar to a thin elastic plate with thickness T_e set by this rheologic transition (O’Hara et al. (2021); Wilson and Russell (2020)).

The magnitude and pattern of uplift recorded in BV channel hyaloclastites is well-modeled with $T_e = 4 - 8$ km subject to a net upward force of $\sim 34 \times 10^6$ N distributed over a wavelength of 110 – 130 km. Removal of crust by erosion by the Columbia River and its tributaries contributes to this force imbalance, but not enough for isostatic rebound to be a primary driver of uplift (section A.0.5). We estimate the removal of 270 km³ of sediment by erosion in tributaries of the Columbia that enter within the Gorge. For values of T_e that fit the deformation wavelength of hyaloclastite deposits the implied removal of mass would contribute ~ 120 m of flexural uplift at most. This unloading, however, is offset by a

comparable volume of erupted material deposited on top of the paleo-surface by Cascade Range volcanoes. We therefore neglect erosional unloading relative to magmatic inflation in forcing hyaloclastite uplift.

Inference of intrusive magmatic forcing as a mechanism for bedrock uplift is then augmented by a broad suite of indirect observations. Heat flow and gravity data implicate elevated temperatures and silicate melt under the area of maximum hyaloclastite uplift both at present and likely quasi-continuously since the onset of hyaloclastite deformation (Figure 39). Bouguer gravity data (Bonvalot et al. (2012)), although not corrected for locally steep topography or laterally heterogeneous subsurface structures, shows a negative anomaly over the arc consistent with flexural uplift due to a buried load (section A.0.3). Similarly, regionally interpolated heat flow measurements (Ingebritsen and Mariner (2010)) exhibit elevated near-surface temperatures consistent with a long-lived magmatic heat source centered on the arc.

High resolution seismic tomography using Rayleigh and Love waves (Jiang, Schmandt, Abers, Kiser, and Miller (2023)) shows a region of low shear-wave velocities (V_s) at depths > 20 km beneath the arc with magnitudes that likely require both partial melt and elevated temperatures (Supplement). The seismic anomaly is aligned with hyaloclastite deformation in the Gorge and extends northwest and southeast connecting Mt. St. Helens and Mt. Hood at 20-30 km depth (Figure 39; Jiang et al. (2023)).

4.4 Long-term balance of intrusions versus eruptions

Broad covariance along the Cascade Range between crustal geophysical anomalies and high topography associated with Quaternary volcanic vents (O'Hara, Karlstrom, and Ramsey (2020)) suggests that magmatism contributes to uplift

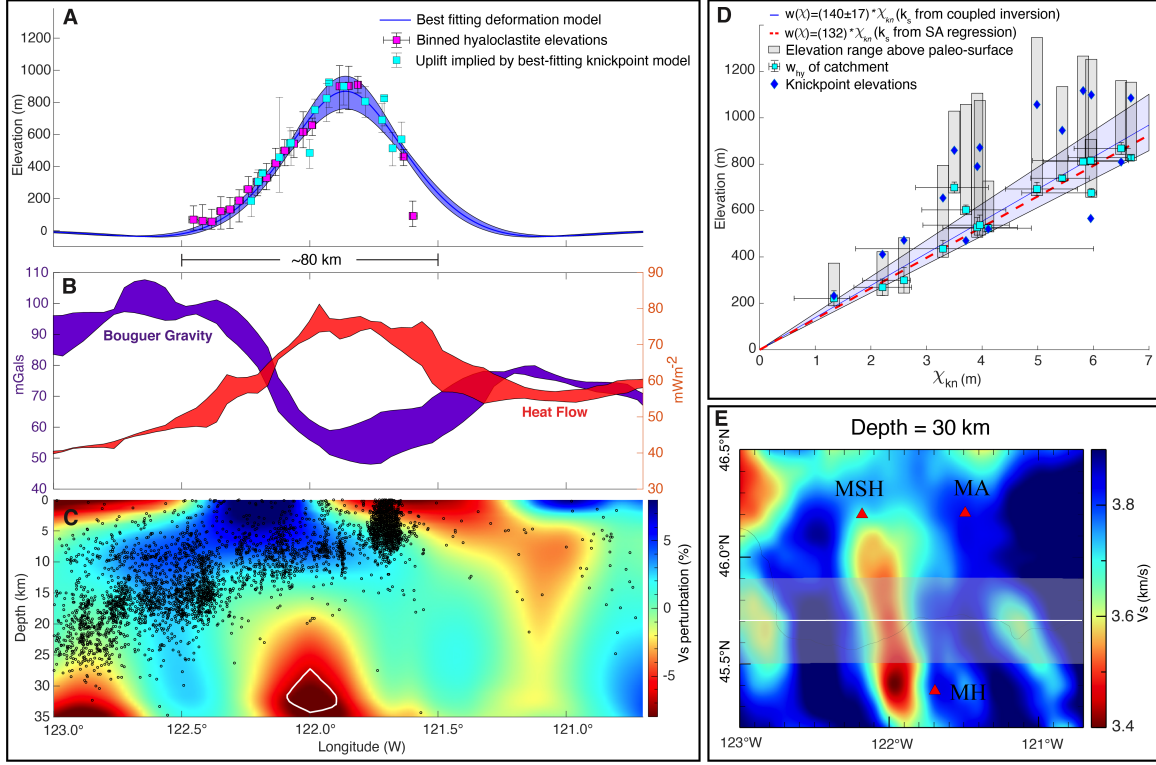


Figure 39. Geophysical anomalies across the Gorge and their relationships to BV channel deformation. **A.** Best-fitting uplift model from maximum likelihood estimation with 95% confidence model envelope. Light blue squares show knickpoint elevations inferred from a transient landscape evolution model forced by flexural uplift constrained by hyaloclastite deformation (pink squares). **B.** Interpolated Bouguer gravity and surface heat flow spanning the Gorge. **C.** Average surface wave velocity anomaly. The white polygon marks inferred 2.5 – 4% partial melt (Supplement). Black dots are earthquake hypocenters for all events underlying the Gorge in the Pacific Northwest Seismic Network catalog since 1970. **D.** Hyaloclastite elevation (squares) and knickpoint elevation (diamonds) vs knickpoint location χ_{kn} . Red dashed line shows uplift predicted by fluvial model using slope-area regression, while shaded swath shows model predictions from a coupled flexural uplift/landscape evolution model (Supplement). Mismatch between χ_{kn} and knickpoint elevation likely implies time-evolving hydraulic conductivity/erodibility of young lava flows. **E.** Regional map of shear wave seismic velocity at 30 km depth showing swath in panel C.

throughout the arc. However understanding the partitioning between intrusive and extrusive (i.e., eruptions) modes of magmatism is challenging (White, Crisp, and Spera (2006)), so rates of topographic growth from intrusive magmatic processes are difficult to quantify. The cross-section of the Cascade Range created by the Columbia River provides a unique opportunity to calculate the ratio of intrusive to extrusive magmatism (I:E) on million-year timescales, and assess the relative contribution of each to arc mountain building.

A lower bound estimate for the volume of intruded magma can be calculated as the amount of accommodation space created by hyaloclastite deflection between the western margin of uplift and the fault-bounded eastern margin of uplift (Figure 40). Total extruded volcanic output is then the difference between elevation of the paleosurface and the modern topography. If we assume an onset time between 3.0 – 3.5 Ma (section A.0.1) this gives average intrusive and extrusive magma flux estimates of 10 – 13 km³/My/km and 3 – 4 km³/My/km, respectively. Accounting for spatially variable topography within a 10 km swath south of the Gorge (assuming intrusive uplift is unchanged) gives a distribution with 95% confidence interval of I:E = $2.9^{+3.6}_{-2.4}$ (Figure 40).

We can compare this to previous estimates of magmatic flux required to produce heat flow measured in the Cascades (Ingebritsen, Mariner, and Sherrod (1994)) of 9 – 33 km³/My/km. Taken with eruptive flux estimates of 3 – 6 km³/My/km (Sherrod and Smith (2000)) this gives I:E= 1.5 – 11. Topographically derived I:E values for the Gorge are a lower bound, because they do not account for deep intrusions which generate longer wavelength deformation near the surface (O’Hara et al. (2021)), and only account for magma underlying the deformed paleo-surface.

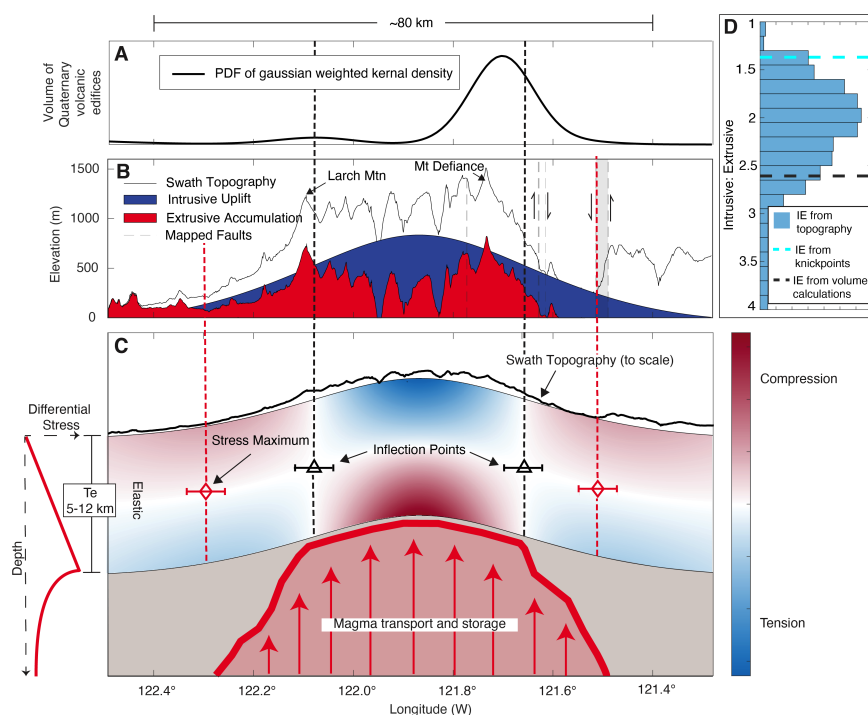


Figure 40. Inference of transcrustal magmatic structure. **A.** Volume weighted Gaussian kernel density of Quaternary edifices within 10 km of the study area. **B.** Intrusive and extrusive magmatic topography, using best fitting flexural uplift model. Dashed lines indicate major fault systems that bound the Hood River Valley (Mcclaughry et al. (2012)). **C.** Best fitting plate model defining elastic thickness of the upper crust. Alternating tensile and compressional deviatoric stresses in the plate are inferred to direct shallow magma ascent and faulting. **D.** Ratios of intrusive:extrusive (I:E) magmatism calculated using topography (assuming north-south continuity in flexural uplift), knickpoint positions, and total calculated volumes of erupted and intruded magma.

Seismic tomography of present day crustal structure suggests ongoing intrusion and provides modern context for these flux estimates. The volume of stored partial melt implied by seismic anomalies depends on lower crustal compositions, temperature, and grain boundary melt geometry (Takei (2002)). Petrologic constraints on storage temperatures for Mt. St. Helens magmas suggest lower crustal temperatures in the range of 750 – 950 C (Wanke (2019)). A magma storage zone with this temperature range is consistent with observed heat flow at the arc front, which also suggests sustained heat from partial melt at depth (section A.0.6). Assuming textural equilibrium of melt in the deep crust, seismic tomography suggests $\sim 4 \text{ km}^3$ magma per kilometer of arc currently underneath the Gorge.

4.5 Topographic response to magmatism

We can gain additional insight into the time-dependent rates of magmatism by leveraging the stable, near-sea-level base level imposed by the Columbia River as it transects the Cascade Range. Actively eroding landscapes tend towards dynamic equilibrium conditions wherein surface erosion and uplift are balanced and fluvial channels maintain power-law scaling between channel slope and upstream drainage area (Flint (1974)). An increase in uplift rate generates an upstream-propagating kinematic wave of incision that manifests as a step-change in this slope-area relationship often referred to as a ‘slope-break knickpoint’ (Whipple et al. (2013)). Thus we expect that a past increase in the rate of magmatic uplift should be recorded in the slope-area relations of tributaries entering the Columbia River.

We focus on channels that enter from the south side of the Gorge where southward tilting bedrock limits deep-seated landsliding that is prevalent on the

north side of the river (Pierson, Evarts, and Bard (2016)) (section A.0.2). Here 16 prominent tributary streams entering the Columbia River provide a natural landscape evolution experiment in which adjacent streams are forced by a known uplift perturbation beginning at the same time (deposition of BV deposits), but with a magnitude that varies significantly between tributaries. In all catchments, channels cut through similar layered basalt stratigraphy consisting of Cascades arc volcanic rock overlying Columbia River Flood Basalts and lenses of hyaloclastite (Beeson and Tolan (1990)). Variability between basalt layers causes sharp localized knickpoints in stream profiles (the iconic waterfalls of the Columbia Gorge), however at the catchment scale most tributaries contain a single slope-break knickpoint that separates each longitudinal profile into two segments with constant concavity but different slope-area scaling (Figure 38).

The upstream position of these knickpoints tracks hyaloclastite deformation (Figure 1), implicating a geomorphic adjustment to magmatic uplift. Knickpoint propagation speed depends on upstream drainage area (A) (Berlin and Anderson (2007)), and so to compare different sized catchments directly we normalize upstream distance along each channel with respect to drainage area and examine knickpoint positions relative to the non-dimensional upstream coordinate χ (Royden and Taylor Perron (2013)). The ‘stream power’ model for fluvial bedrock erosion in its simplest form predicts that a knickpoint associated with an increase in uplift rate occurring at time τ_o is given by $\chi_{kn} = U\tau_o/k_s$ (section A.0.2) where U is the average uplift rate and k_s is an empirical constant that scales the power law relationship between upstream drainage area and channel slope, $dz/dx = k_s A(x)^{-\theta}$ (Flint (1974)). Maintaining continuity in the elastically deformed upper crust requires that τ_o (assumed to coincide with diversion of the Columbia at 3.0-3.5

Ma) is the same in all catchments. The stream power prediction for the position of knickpoints associated with the hyaloclastite uplift (w_{hy}) can then be written as $\chi_{kn} = w_{hy}/k_s$, which matches the data with $k_s = 140 \pm 17$ for all catchments (Figure 39).

The lateral position of knickpoints is less well correlated to knickpoint elevations in channels (Figure 39.D), which reflect variable Pliocene-Quaternary arc lavas overlying the paleosurface. This observation is consistent with episodic deposition of volcanic bedrock, which leads to strong hydromechanical layering and hydraulic anisotropy that impacts bedrock erosion long after primary emplacement (Jefferson et al. (2010); Sweeney and Roering (2017)). We infer that young lava flows in the Gorge produce topographic relief but have negligible influence on time-averaged rates of channel incision in underlying strata.

Inference of constant k_s across Gorge tributaries is consistent with fluvial erosion patterns in other high-relief landscapes. Although this parameter is sometimes assumed to covary with uplift rate (Whipple et al. (2013)), a global compilation of fluvial catchments in active tectonic settings (Hilley et al. (2019)) suggests a threshold in channel steepness that creates an upper limit on k_s in rapidly uplifting landscapes. Increase of coarse sediment supply with uplift, resulting in transport limited conditions that modulate the efficiency of fluvial bedrock erosion (Lai, Roering, Finnegan, Dorsey, and Yen (2021)), is one way limits on k_s might be achieved.

The inferred relationship between knickpoint locations and hyaloclastite uplift shows that fluvial profiles record the relative contribution of intrusion versus eruption to topographic development. With thickness of erupted deposits as the difference between observed channel elevations and uplift associated with

the knickpoints, the ratio of intrusive to extrusive magmatic uplift can then be calculated directly from fluvial channels as $I:E = k_s \chi_{kn} / z_{kn} = 1.4 \pm 0.5$. The alignment of this value with I:E calculated through other means (Figure 3D) suggests that long-term magma dynamics can be encoded in river networks, as has been inferred for tectonic and/or climatic forcing in non-magmatic landscapes (Schoenbohm, Whipple, Burchfiel, and Chen (2004); Wobus et al. (2006)).

4.6 Entwined upper crustal deformation, arc mountain building, and magma ascent

Direct constraints on the mechanical response of the upper crust to intrusive magmatism can explain several other primary features of the central Cascades arc. In particular, plate-like flexure of the elastic upper crust generates alternating tensile and compressional stresses (Hieronymus and Bercovici (2001)) that may explain prominent normal faults along the Hood River Fault Zone (Figure 38). These extensional features are consistent with north-south maximum horizontal compressive stress, and align with peaks in deviatoric stresses from our best-fitting flexural model (Figure 40). To the west of the arc front, subsidence of the Portland basin occurs coincident with the implied flexural bulge of our model (R. Wells et al. (2020)). While flexure cannot itself explain the magnitude of subsidence there (or in the Hood River Valley to the east), the resulting topography would route fluvial sediments and erupted material to arc margins, causing further loading of the plate and increased subsidence (Garcia-Castellanos (2002)).

Unusual spatial patterns of magmatism in the vicinity of the Columbia river, the widest point of the Quaternary Cascades arc (Hildreth (2007)), may also reflect the influence of shallow crustal stresses implied by our model. Within the Gorge, local maxima in both the volume-weighted Gaussian kernel density

of volcanic vents (O’Hara et al. (2020)) and the thickness of associated erupted deposits are located east and west of maximum hyaloclastite layer deformation (Figure 40). This is consistent with compressional flexural stresses in the lower half of an elastic upper crust that route ascending magma towards off-axis minima in differential stresses (Hieronymus and Bercovici (2001)). The most active Holocene Cascades volcano, Mount St. Helens, sits notably off-axis in the forearc northwest of the Gorge. Arc-centered flexure can contribute to observed lateral migration of ascending magma around inherited upper crustal structures (Bedrosian, Peacock, Bowles-Martinez, Schultz, and Hill (2018)).

Outside the gorge, local widening of the Quaternary arc is associated with the Boring and Simcoe distributed vent fields (Figure 38) (Hildreth (2007)). The Columbia River itself could play a role in this local widening of arc magmatic surface expression, as surface unloading from persistent fluvial incision generates topographic stresses known to influence magma ascent pathways in other settings (J. R. Muller, Ito, and Martel (2001)). Current topography represents a ~ 25 MPa topographic load difference between the Gorge and adjacent arc (Garcia, Sandwell, and Luttrell (2015)). Quaternary average erosion/sediment production rates implied by projecting the hyaloclastite layer over the Gorge are $90 \text{ km}^3/\text{Myr}$. This missing volume represents total magmatic topographic construction, at least 270 km^3 or $\sim 6 - 9 \text{ km}^3/\text{My}/\text{km}$ based on $\sim 10 - 15 \text{ km}$ north-south extent of the Gorge. This roughly matches inferred deep magmatic influx rates.

Finally, transcrustal evidence ranging from geomorphology of river channels to seismic tomography of the crust suggest persistent, long-lived magmatism that is not directly associated with any major stratovolcano. Seismic and geodetic data show that arc magma reservoirs are commonly offset from their associated

volcanoes (Lerner et al. (2020)), suggesting that sustained magma storage between adjacent volcanoes (as inferred here) is more common than previously recognized (Bedrosian et al. (2018)). In general, this implies that arc magma transport networks are not always strongly aligned with volcanic edifices. Our work suggests that intrusive magmatism has a broad organizing influence (Karlstrom, Dufek, and Manga (2009)) spanning surface topographic development, crustal stress state, and magma ascent pathways.

APPENDIX

APPENDIX A - CHAPTER IV SUPPLEMENTARY MATERIAL

A.0.1 Constraints on structure and timing of Gorge

deformation. The Columbia River has crossed the Cascades arc for at least 17 Myr though its exact location has shifted several times as the result of volcanic infilling of the channel. In the Gorge laharic breccias of the Eagle Creek Formation unconformably underlie early Columbia River Basalt (CRB) flows, suggesting a dominance of deposition by fluvial tributary systems draining local volcanic terrain in the early Miocene (Korosec (1987)). The first known avulsion of the main Columbia channel occurred through emplacement of the Grande Ronde Basalts between 16.0 and 15.5 Ma (Reidel and Tolan (2013a)). The $\sim 150,400 \text{ km}^3$ of material associated with this period of CRB volcanism erupted from fissures in eastern Oregon and inundated the established Columbia River network. This displaced the river and forced it on a more southern course across the Cascades along a developing fold structure known as the Mosier-Bull Run syncline (Reidel and Tolan (2013b)). Subsequent emplacement of the Wanapum basalts forced the river back north into the Bridal Veil Channel by 12 Ma, where it persisted until it was pushed to its modern course during a widespread pulse of mafic volcanism in the northern Oregon Cascades between ~ 4.4 and 2.1 Ma (R. Conrey, Sherrod, Uto, and Uchiumi (1996); Tolan and Beeson (1984)).

In the mid Pliocene a series of voluminous lavas, mostly with low potassium tholeiitic basalt (LKT) compositions, were erupted from the area south of the modern Columbia River. Some of these lavas flowed into the ancestral channel of the Columbia River and were rapidly chilled and fragmented during interaction with water generating hyaloclastite (Mcclaughry et al. (2012); Swanson (1986)).

The Bridal Veil channel was eventually filled by hyaloclastite and associated LKT lava flows (Figure A.41) and the river was diverted north to its present course. $^{40}\text{Ar}/^{39}\text{Ar}$ age determinations for LKT lava flows inter-bedded with, and overlying, the hyaloclastite (Fleck, Hagstrum, Calvert, Evarts, and Conrey (2014); Mcclaughry et al. (2012)) suggest diversion of the channel between 3.5 and 3.0 Ma (Table S1) thus determining our age brackets for the beginning of the current phase of uplift. During this time arc-marginal normal faulting associated with the northward propagation of an intra-arc rift began in the Hood River Fault Zone (R. Conrey, Sherrod, Hooper, and Swanson (1997); Mcclaughry et al. (2012)).

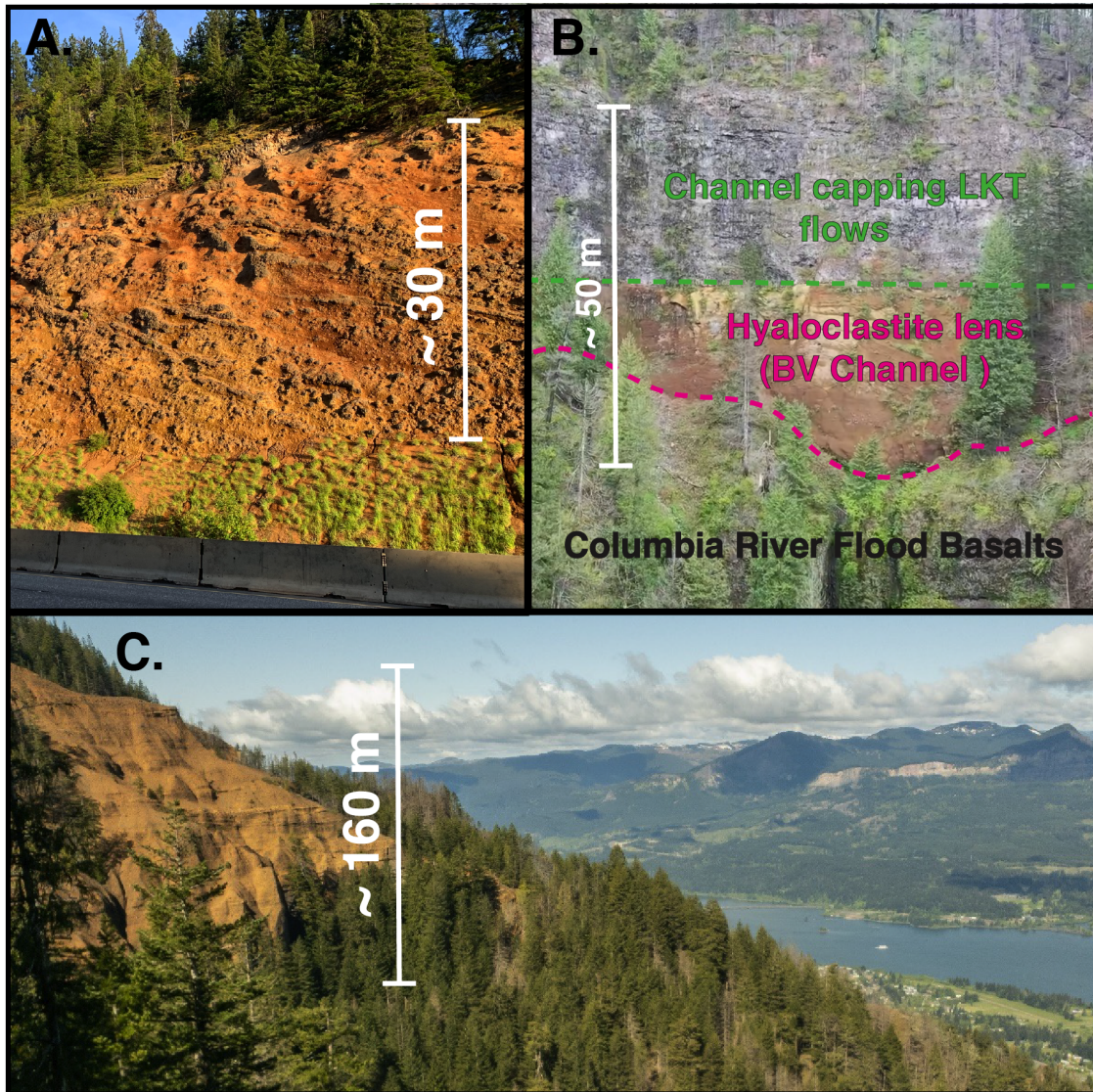


Figure A.41. Photos of hyaloclastite deposits associated with the BV channel. A. Hyaloclastite (orange/yellow) with interbedded pillow complexes seen from I-84 about 4 km west of Hood River, OR. near Ruthton Point **B.** Lens of hyaloclastite material with channel capping LKT flows in Ainsworth State Park near Dodson, OR. **C.** Exposures of hyaloclastite material uplifted nearly 1000 m above the modern Columbia River (visible on right side of panel) seen from the Pacific Crest Trail near Cascades Locks, OR.

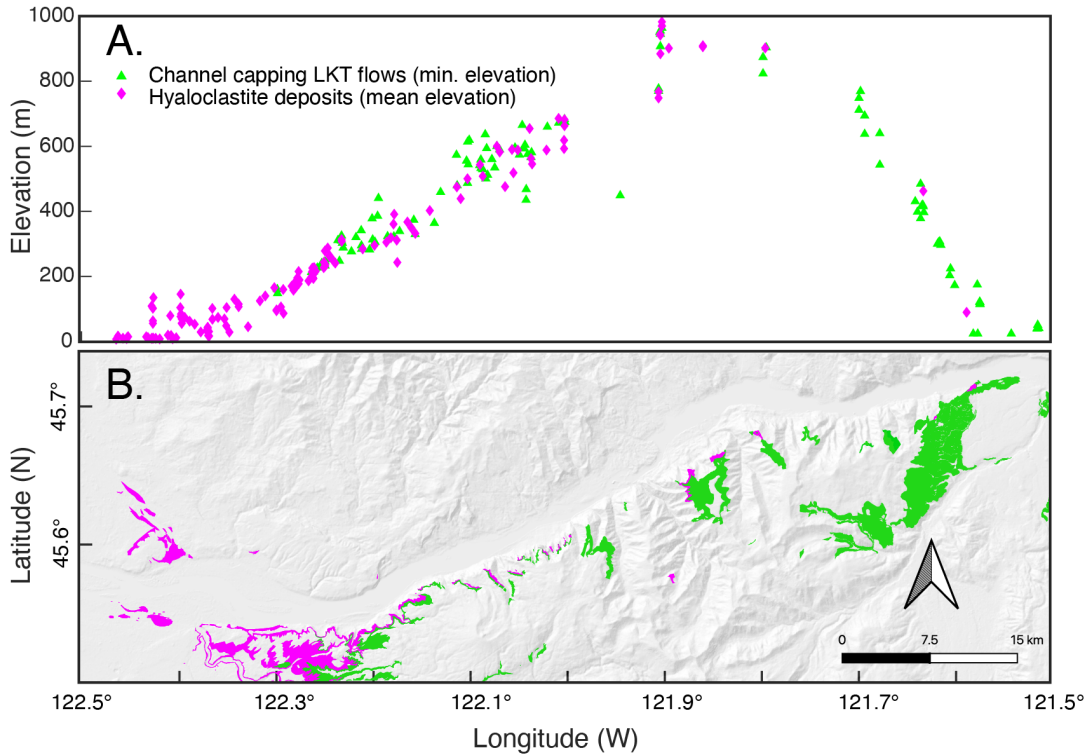


Figure A.42. Raw Bridal-Veil Channel elevation data. A. Mean elevations of hyaloclastite deposits and minimum elevations of LKT flows that overlie the BV Channel. *B.* Map view of hyaloclastite and LKT polygons.

A.0.2 Topographic analysis and fluvial erosion. Today the anticlinal uplift of the hyaloclastites (Figure A.42) is superimposed on a slight ($2 - 3^\circ$) north-south downward tilt observable in CRB deposits and underlying volcanoclastic sediments. As a result, the north side of the Columbia River is prone to deep-seated landslides that tend to fail along slip surfaces at unconformable contacts at the base of the CRB and move into the accommodation space created by Columbia incision (Pierson et al. (2016)). On the south side of the Columbia, these slip surfaces are buttressed by southward tilt so deep-seated mass wasting events are not common. For this reason we focus our topographic analysis on the south side of the Columbia River. Of the seventeen largest catchments on the south side of the Columbia one (Tanner Creek) is eliminated as its longitudinal profile

has a large knickpoint near its confluence with the Columbia that is inconsistent with the dominant pattern of transient adjustment. This knickpoint is likely related to partial filling of the Tanner Creek channel by basalt flows observed near the channel mouth, but compositional affinity and age are poorly resolved in current maps limiting our ability to confirm this explanation.

It has been documented that mature fluvial networks exhibit inverse power-law scaling between channel slope and upstream drainage area at a given point in the channel that is often interpreted as indicative of a steady-state balance between uplift and erosion (Flint (1974); Whipple et al. (2013)). This is expressed as

$$\frac{dz}{dx} = k_s A(x)^{-\theta}, \quad (\text{A.1})$$

where x is the along-channel spatial coordinate (increases upstream), θ is a constant that sets longitudinal curvature of the channel, $A(x)$ is drainage area upstream of position x , and k_s is an empirical constant. Changes in uplift rate generate knickpoints that travel up through the channel network until the entire system re-equilibrates to the new uplift conditions. These “slope-break” knickpoints (Whipple et al. (2013)) manifest as discontinuities in the relationship between channel slope and drainage area given by Eq. A.1.

We identify knickpoints in the trunk streams of sixteen Columbia tributaries using 3 m resolution LiDAR data compiled by the Oregon Department of Geology and Mineral Industries (<https://www.oregongeology.org/lidar/>) using the default flow routing algorithm in MATLAB-based Topotoolbox (Schwanghart and Scherler (2014)). Each profile is smoothed using a 100m moving window to eliminate channel features associated with lithologic variation (waterfalls), as well as DEM error inherent to gridded elevation data sets in steep terrain. We use a coordinate transformation to remove the effects of drainage area (and concavity) from

longitudinal profiles in Eq. A.1 which allows us to write

$$z(x) = k_s \chi(x), \quad (\text{A.2})$$

where $z(x)$ is elevation relative to base level and $\chi(x) = \int_{s_o}^x \frac{A_o^\theta}{A(x)^\theta} dx$ is an area-normalized upstream coordinate (Perron and Royden (2013)). A_o is an arbitrary constant introduced to give χ units of length. In these coordinates longitudinal profiles ($z(x)$ versus $\chi(x)$) of equilibrium channels plot as straight lines with slope $dz/d\chi = k_s$, and slope-break knickpoints manifest as changes to linear channel slopes (Royden and Taylor Perron (2013)). While it is common in the literature to assign A_o a value of $1 \times 10^6 \text{ m}^2$ (Perron and Royden (2013)) we instead choose $A_o = 1 \text{ m}^2$ in which case the slope of the χ -space longitudinal profile is equivalent to k_s used in slope-area analysis (Whipple et al. (2013)), allowing us to directly compare two common approaches for classifying fluvial profiles (Figure A.44). To constrain θ we perform a grid search over concavity values ranging from 0.3 to 0.6 where for each θ value we find a piece-wise least squares linear fit of all profiles assuming a single knickpoint in each. This gives knickpoint locations for all catchments as well as the value of θ that best linearizes the stream profiles (Figure A.43). We find $\theta = 0.46_{-0.02}^{+0.01}$ which is consistent with expectation for fluvial erosion (Whipple (2004)).

We assume a simple ‘stream power’ type erosion law in which fluvial erosion is assumed proportional to channel slope raised an exponent (n) (Whipple and Tucker (1999)). Although a range of values have been proposed, it has been suggested from hydraulic geometry relationships that $n \approx 1$ in bedrock channels (Venditti et al. (2019)). Assuming this linear form of the stream power model, knickpoint upstream positions in χ -space scale with total cumulative uplift since the onset of increased uplift rate that initiated the knickpoint (Goren, Fox, and

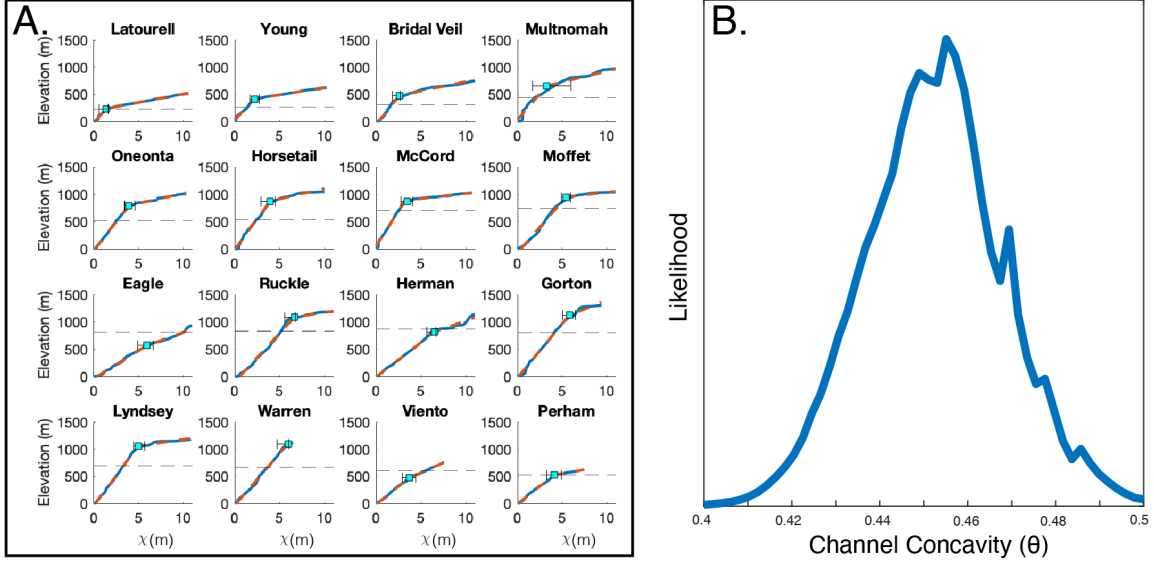


Figure A.43. 16 Columbia River tributaries used in this study. A. Blue lines are longitudinal profiles extracted from 3m LIDAR data plotted against an area-normalized χ -scale calculated using best fitting concavity value of $\theta = 0.455$ and $A_o = 1 \text{ m}^2$. Red dashed lines are piece-wise linear fits that best capture slope-break knickpoints with best fitting knickpoint locations marked by blue squares on each profile. *B.* PDF of θ values calculated using best-fitting knickpoint decompositions for each value.

Willett (2014)). Thus for a given uplift rate (U) and onset time (τ) Eq. A.2 can be written as $U\tau = k_s\chi$.

For a given catchment we then expect the χ -position of a transient signal associated with the onset of hyaloclastite uplift to be $\chi = w/k_s$ where w is the magnitude of hyaloclastite uplift. By computing hyaloclastite displacement for each catchment from our flexural model we can find k_s values that best map this uplift onto knickpoint positions, thus building a coupled model between hyaloclastite uplift and the transient fluvial response. We find that knickpoint positions can be fit via this model with $k_s = 140 \pm 17$ for all catchments and that error is minimized when the component of uplift associated with erupted material is not included in uplift term of the knickpoint model. In other active tectonic settings,

numerous studies (Adams, Whipple, Forte, Heimsath, and Hodges (2020); Gallen and Fernández-Blanco (2021)) vary the slope exponent n to help explain the fluvial response to uplift gradients. Here, strong physical constraints on spatially varying uplift across the Gorge reveal that it is possible to fit 16 tributary channel profiles with fewer parameters using the simpler linear stream power model.

Slope-Area Analysis

The assumption that k_s is constant across strongly (factor of 4 – 5) varying uplift requires justification, as uplift rate is commonly linked to k_s in studies of transient landscape evolution (Kirby and Whipple (2012)). We can test the robustness of constant k_s and θ between tributaries by independently constraining their values through slope-area analysis (Wobus et al. (2006)). We first calculate slope and upstream drainage area for every pixel in the LiDAR dataset using algorithms in Topotoolbox (Schwanghart and Scherler (2014)). We posit the likelihood of spatially (but not temporally) constant uplift in each channel, which means that only sections of channel below the knickpoint reflect the fluvial response to modern uplift rates. We clip off channel reaches above the elevations implied by the hyaloclastite uplift model for a given channel. We further filter this data to only include points in the landscape with upstream drainage areas greater than 2×10^6 km².

In this case we do not limit our analysis to knickpoint positions in the larger trunk streams, but include all channel points within the study area which yields a much larger data set. The resulting slope-area distribution of fluvial channels below the lithologic horizon of the hyaloclastite band is fit using the robustfit function in MATLAB. This gives best fitting values of $k_s = 132$ and $\theta = 0.45$, consistent within

uncertainty with our values derived from the regression of longitudinal profiles outlined above (Figure A.44).

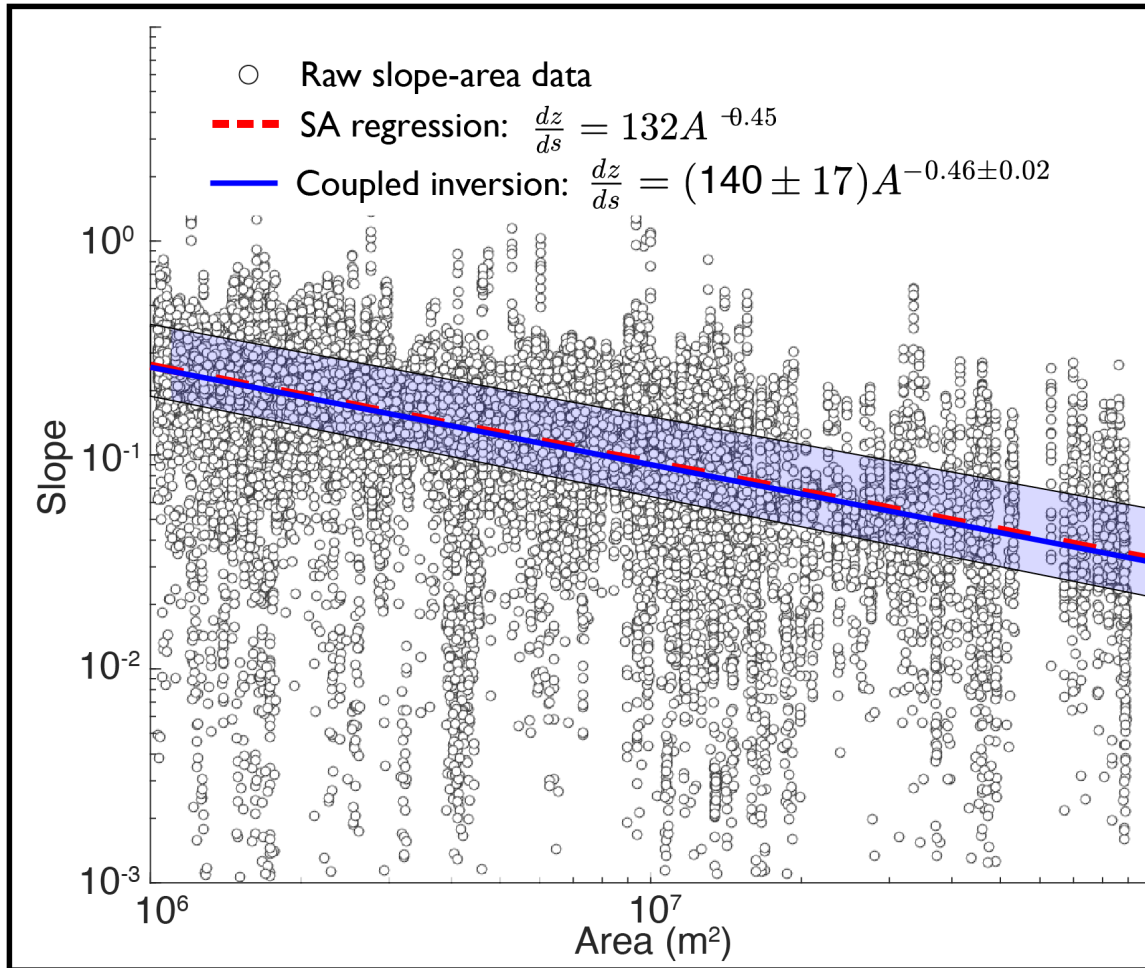


Figure A.44. Slope-area plot for fluvial channels downstream of the knickpoint. Points represent raw slope-area data for all points within fluvial channels downstream of paleosurface elevations implied by the flexure model. The blue line and shaded region represents a model fit to equation A.1 using the range of k_s values from the coupled grid-search inversion and concavity values derived in the knickpoint fitting process. The red dashed line is a fit to the raw slope-area data using the robustfit function in Matlab.

A.0.3 Interpretation of Bouguer gravity anomaly. The low in the Bouguer gravity anomaly collocated with hyaloclastite uplift (Figure A.45A and Figure 2B of main text) implies a flexural response to a buried load. To show

this we interpolate detrended Bouguer gravity data (Bonvalot et al. (2012)) onto hyaloclastite outcrop positions and calculate the admittance between gravity and hyaloclastite deformation as the slope of a linear fit between the two datasets (FigureA.46A). Similar Bouguer gravity lows in other volcanic settings have been interpreted as indicative of isostatic compensation of a magmatically thickened crust (Perkins et al. (2016)), and so we compare this value to expectation for isostatic equilibrium. FigureA.46B shows that the calculated admittance in the Columbia Gorge is indicative of a flexural response to buried loading (Forsyth (1985)). While this data informs the development of our structural model inherent parameter trade-offs in gravity models limit the extent to which we can constrain the depth and composition of the buried load (Blakely (1994)).

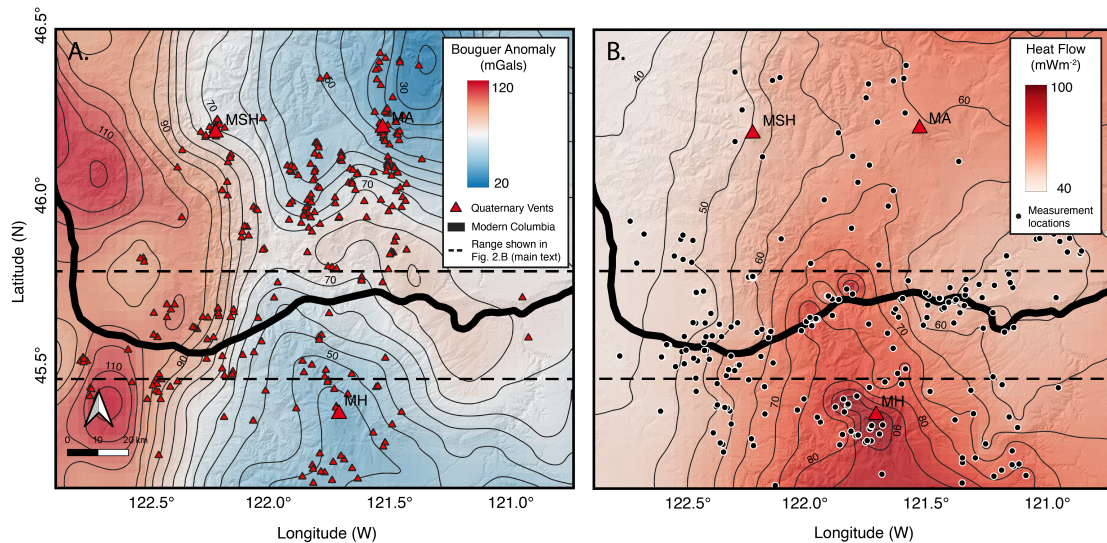


Figure A.45. Bouguer gravity and heat flow data. **A.** Bouguer gravity anomaly (Bonvalot et al. (2012)) with locations of quaternary volcanic vents, the modern Columbia River, and the extent used to generate the cross-arc gravity profile shown in Figure 39.B. **B.** Heat flow data from Williams and DeAngelo (2008) (C. F. Williams and DeAngelo (2008)) interpolated onto a 2-minute grid showing measurement stations.

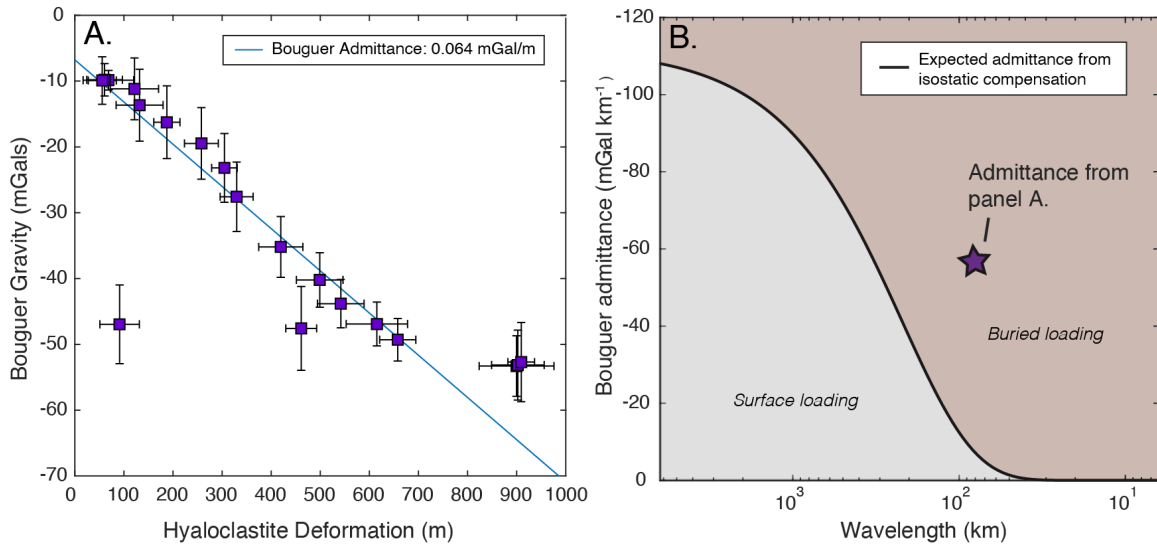


Figure A.46. Interpretation of admittance between gravity and hyaloclastite deformation. **A.** Detrended Bouguer gravity plotted versus hyaloclastite deformation where the slope of the linear fit between the datasets is the gravitational admittance over a wavelength of ~ 80 km. **B.** Regime diagram showing expected ranges of admittance from different loading scenarios. The black line represents isostatic compensation at the base of the crust. Points above the line represent a flexural response to buried loading while points below the line represent a flexural response to surface loading.

A.0.4 Elastic deformation/knickpoint response model. We build

a forward model that assumes east-west uplift of hyaloclastite deposits through the Columbia River Gorge can be approximated as deformation of a thin elastic plate subject to basal pressure from magmatic intrusions. Assuming no east-west lateral forcing on the plate (as implied by the tectonic field) the thin-plate flexure equation in 1-d is

$$D \frac{d^4 w}{dx^4} + \rho_l w g = q(x). \quad (\text{A.3})$$

Here w is deflection of the plate, the second term on the left is a restoring force that scales with uplift, $q(x)$ is a pressure distribution from magmatic intrusion at the base of the plate, and D is flexural rigidity defined as

$$D = \frac{ET_e^3}{12(1 - \nu^2)}. \quad (\text{A.4})$$

where E is the Young's Modulus, ν is Poisson's Ratio, and T_e is the elastic thickness of the plate. The pressure distribution acting on the plate $q(x)$ is modeled as a Gaussian function

$$q(x) = \frac{F_{net}}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_c)^2}{2\sigma^2}}, \quad (\text{A.5})$$

where σ^2 is the variance and x_c is the center of the pressure distribution. Since plates deforming within their flexural waveband are insensitive to the shape of the load (Watts (2001)) we assign a constant variance of $\sigma^2 = 25$ km and only vary the total force F_{net} acting on the plate. We solve Eq. A.3 in the Fourier domain (Sandwell (1984)) defining the Fourier transform pair

$$f(x) = \int_{-\infty}^{\infty} F(s) e^{2\pi i s x} ds \quad (\text{A.6})$$

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i s x} dx. \quad (\text{A.7})$$

Taking the Fourier transform gives an expression for displacement of the plate as a function of wavenumber for the pressure distribution $Q(s)$

$$W(s) = \frac{Q(s)}{(2\pi s)^4 D + \rho_l g}, \quad (\text{A.8})$$

which can be inverted to find $w(x)$.

The mean hyaloclastite uplift value within each catchment is then mapped onto knickpoint locations using equation A.2, rearranged such that $\chi_i = w_i/k_s$ for each basin (where $i = 1 : 16$).

With this forward model we perform a grid search over plausible values of flexural rigidity ($5 \times 10^{20} \text{Nm} \leq D \leq 12 \times 10^{20} \text{Nm}$), total force on the plate ($31 \times 10^6 \text{N} \leq F_{net} \leq 38 \times 10^6 \text{N}$), and the fluvial proportionality constant ($110 \text{m} \leq k_s \leq 170 \text{m}$). This we pose as a Bayesian inverse problem with a solution of the form

$$\sigma(\mathbf{m}) = k\alpha(\mathbf{m})L(\mathbf{m}) \quad (\text{A.9})$$

which states that the posterior probability density function $\sigma(\mathbf{m})$ for a parameter distribution $\mathbf{m}$ is the product of the a priori probability distribution ($\alpha(\mathbf{m})$), a likelihood function $L(\mathbf{m})$ and a normalization constant k . We assume uniform priors and so the posterior probability distribution is proportional to the likelihood function defined as

$$L(\mathbf{m}) = \prod_{i=1}^n \exp \left[-\frac{1}{2}(\mathbf{g}_i(\mathbf{m}) - \mathbf{d}_i)^T \Sigma_i^{-1} (\mathbf{g}_i(\mathbf{m}) - \mathbf{d}_i) \right] k \quad (\text{A.10})$$

where $\mathbf{d}$ is our data vector, $\mathbf{g}(\mathbf{m})$ is the output of a forward model for a given input parameter set $\mathbf{m}$, Σ^{-1} is the determinate of the covariance matrix storing data uncertainty values (Mosegaard and Tarantola (1995); Tarantola (2004)). In our model $\mathbf{d} = [\mathbf{d}_{hy}, \mathbf{d}_{kn}]^T$ where $\mathbf{d}_{hy}$ consists of mean hyaloclastite elevations in 20 km bins, and $\mathbf{d}_{kn}$ contains the best-fitting knickpoint locations for the 16

fluvial tributaries. The covariance matrix is a diagonal matrix in which uncertainty in each data point in the data vector $\mathbf{d}$ is stored along the diagonal. For $\mathbf{d}_{hy}$ the covariance is approximated as the full range about the mean in each of the bins, while for $\mathbf{d}_{kn}$ we calculate a 95% confidence interval for knickpoint locations given our best fitting value of the area constant ($\theta = 0.46$). The PDFs generated through this inversion approach (FigureA.47) define a family of models capable of explaining hyaloclastite uplift in combination with the transient topographic response observed in fluvial tributaries.

With flexural rigidity (D) constrained by the above maximum likelihood estimation we calculate effective elastic thickness T_e of the deformed upper crust by rearranging equation A.4 to

$$T_e = \left(\frac{12D(1 - \nu^2)}{E} \right)^{\frac{1}{3}}. \quad (\text{A.11})$$

FigureA.48 shows the results of varying E from $(1 - 8) \times 10^{20}\text{Pa}$ for possible lithologies underlying Gorge topography Turcotte and Schubert (2014) and vary Poisson's Ratio ν between $0.1 - 0.3$.

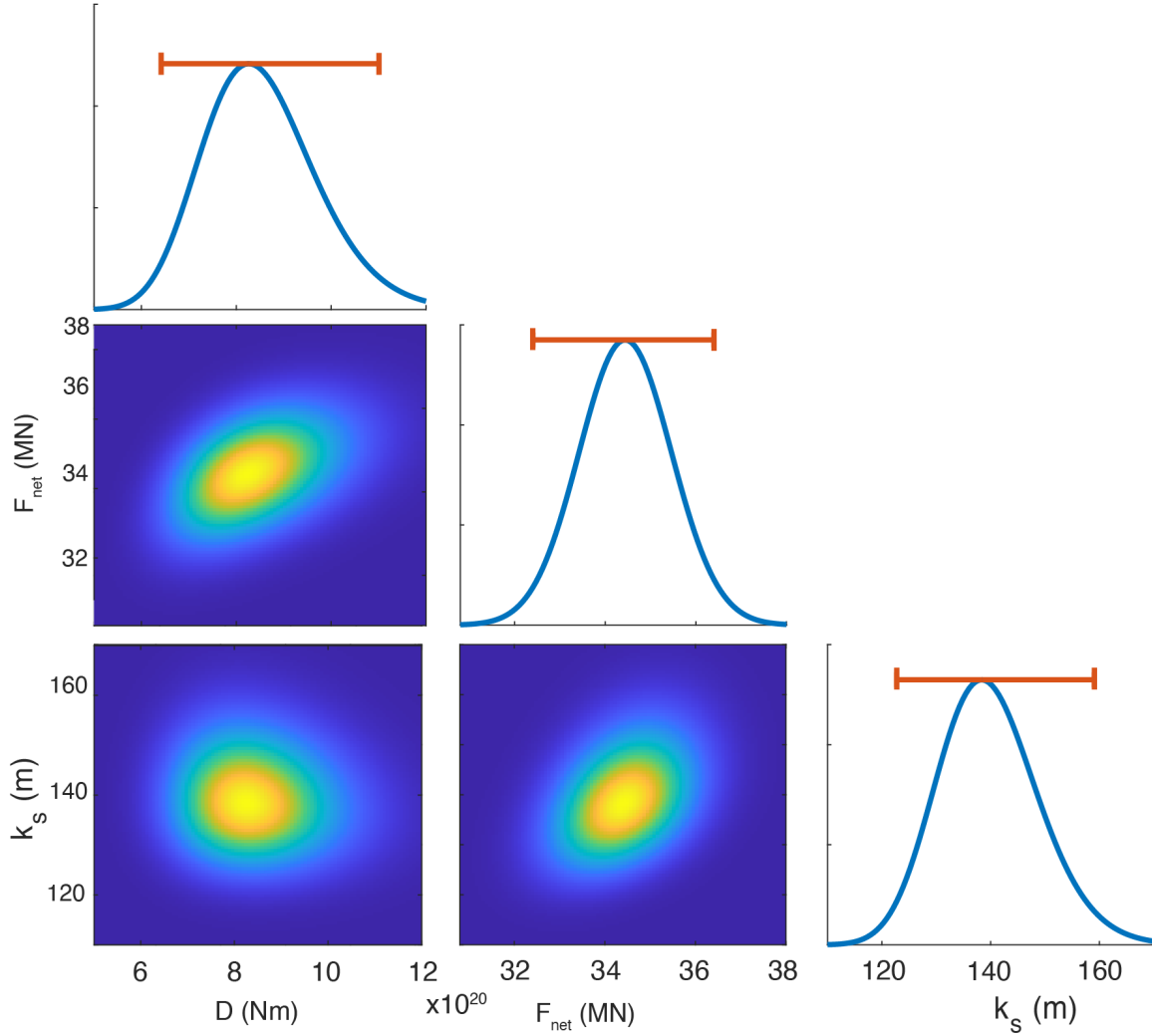
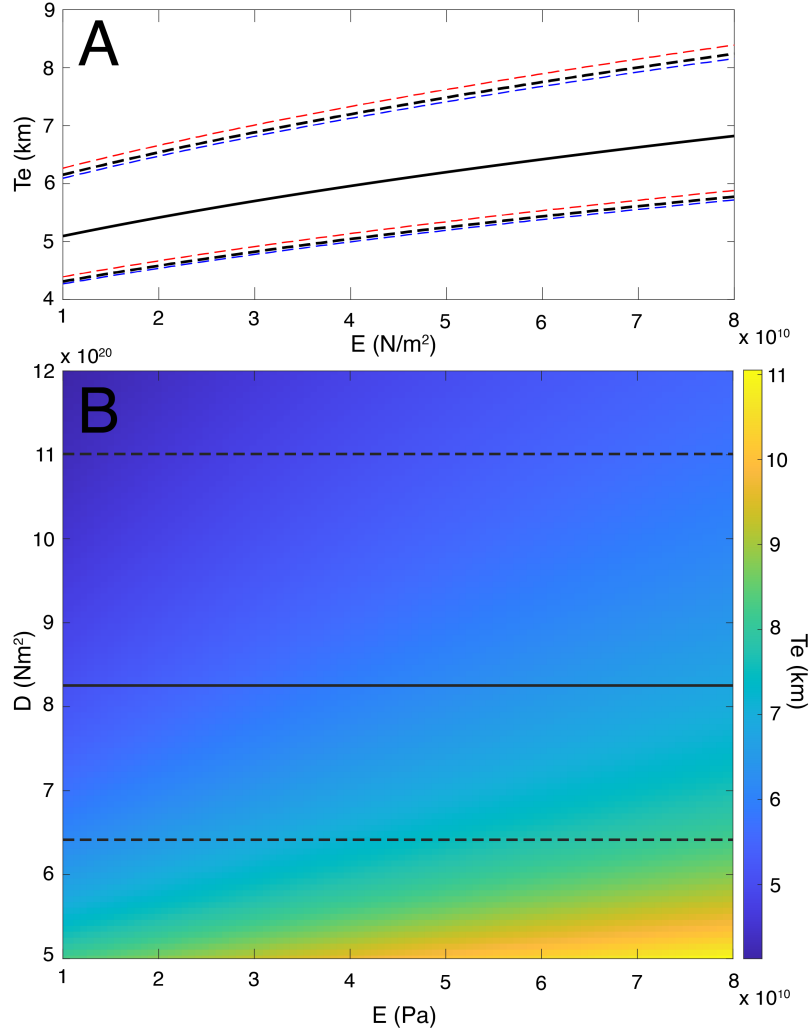


Figure A.47. Posterior PDFs and covariance plots from maximum likelihood estimation. D is flexural rigidity, F_{net} is the total force acting on the plate, and k_s is the fluvial proportionality constant that maps hyaloclastite uplift onto knickpoint positions. Orange bars represent 95% confidence intervals for parameter values.



*Figure A.48. Possible values for effective elastic thickness (Te) as a function of flexural rigidity (D), Young's Modulus (E) and Poisson's Ratio (ν). **A.** Variation in Te as a function of E . The solid black line shows Te for our best-fitting value of flexural rigidity (D) and the dashed black lines represent the 95% confidence interval. The blue and red dashed lines show variation in Te that result from varying Poisson's Ratio where blue and red represent $\nu = 0.3$ and $\nu = 0.1$ respectively. **B.** Variation in Te for all values of D explored in the maximum likelihood estimation, where the solid black line represents the best-fitting model and the dashed lines are 95% confidence intervals.*

A.0.5 Isostatic response to erosion by the Columbia River. We estimate the volume of material removed by erosion since the start of hyaloclastite uplift by projecting our best fitting flexure model north and south across the

gorge and differencing with modern topography (FigureA.49A). This gives a rough estimate for the full isostatic load where negative values represent the amount of material removed by erosion (FigureA.49B). We calculate the amount of uplift caused by removal of this material (FigureA.49C and FigureA.49D) using a 2D thin-plate flexure equation (Watts (2001)). There are additional loads pushing back down on the plate (erupted material) that will offset some of this uplift, however since topography overlying the projected flexure model includes structures not related to magmatism we do not attempt to account for this type of loading in our force balance and instead take this to be an upper bound on isostatic uplift since deposition of the hyaloclastites.

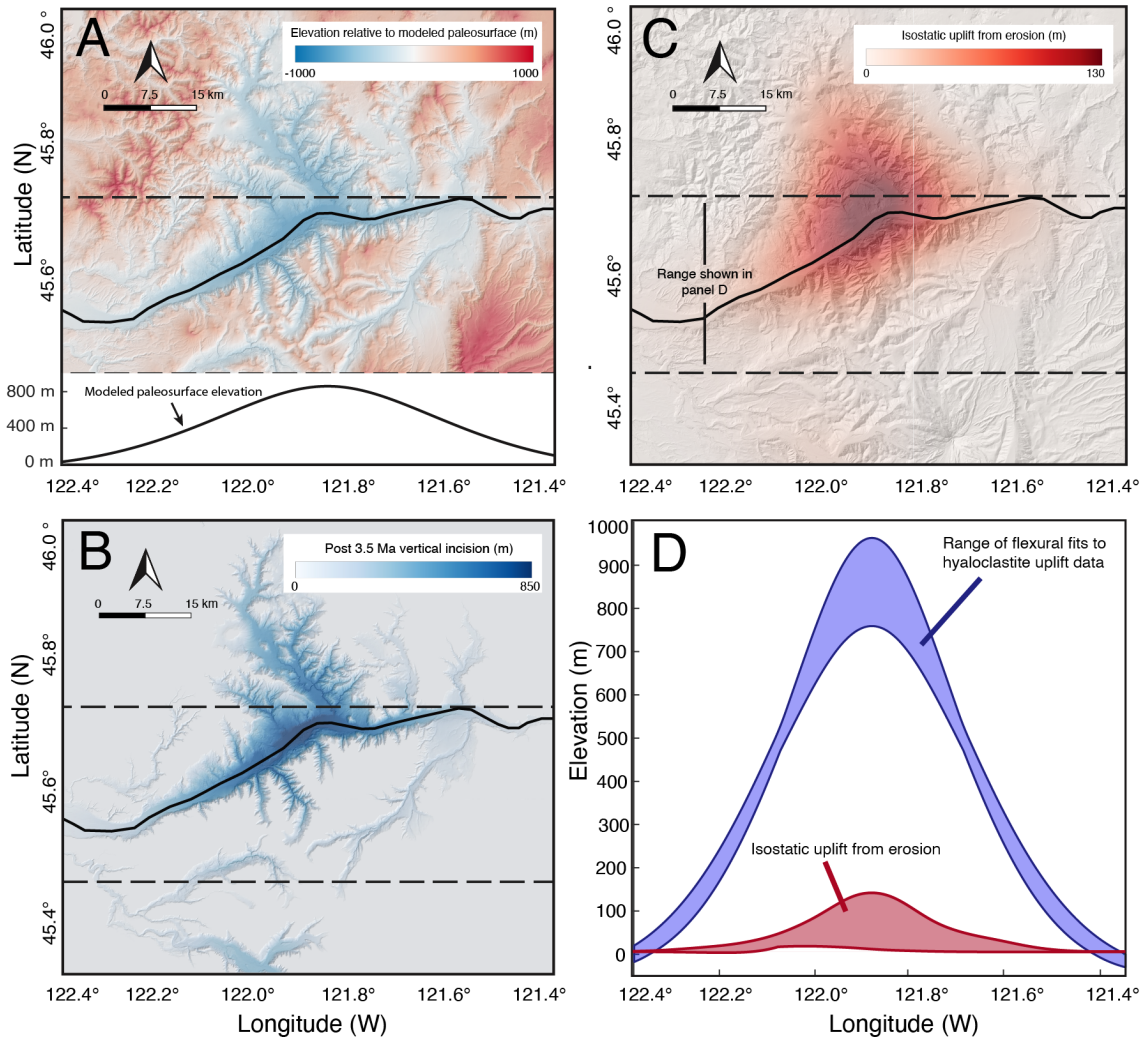


Figure A.49. Isostatic uplift from post 3.5 Ma incision implied by our uplift model. A. Elevation difference between topography and best fitting flexure model projected north and south along the arc. *B.* Erosion implied by north-south model projection. *C.* Maximum flexural uplift from erosional unloading shown in panel B. *D.* Range of model fits to hyaloclastite data and range of values for uplift caused by erosion within the north-south range of hyaloclastite deposits (shown by dashed black lines).

Seismic tomography

To derive the relationship between V_s and melt fraction, we approximate the mid-lower crustal magmatic system as a complex of crystals and melt, and apply

the effective-medium theory (Berryman (1980)) to estimate its elastic properties, including compressional wave speed (Vp), shear wave speed (Vs) and density (ρ). This follows a two-step approach adopted by a previous study (Maguire et al. (2022)).

The first step is to estimate the elastic properties of the solid and melt phase separately. For the solid phase, we use the PerpleX package (Connolly (2009)) to compute Vp , Vs , and ρ considering both basaltic and dacitic bulk compositions, which represent the two major types of magmas erupted at the Mount St. Helens (Wanke, Clynne, von Quadt, Vennemann, and Bachmann (2019); Wanke, Karakas, and Bachmann (2019)). We use the oxides reported by Blatter et al., (2017) (Blatter et al. (2017)) for the dacitic composition and those from Leeman et al., (2005) (Leeman, Lewis, Evarts, Conrey, and Streck (2005)) for the basaltic composition. Table A.1 summarises the compositions of major oxides used in this study. For the PerpleX calculations, we account for a relatively large range of temperature conditions between 750°C – 1000°C; while we vary the pressure condition in a normal range of 700 MPa – 900 MPa for such depths.

Though some higher melting temperatures are reported from petrological analysis in the mid-lower crust of the region, such as Mount St. Helens (Wanke, Clynne, et al. (2019)), we are considering the average temperature condition for a 15 km thick column. Figure A.50 shows the variations of Vp , Vs , and ρ relative to the change of pressure (P) and temperature (T) condition, respectively, which outlines several regimes where slightly different mineral assemblages are stable (see Table A.2 for details). The boundaries between the domains are associated with mineral change and they together define a complex relationship between the elastic parameters and the P-T condition.

To approximate the range of each elastic property of the solid phase, we use the corresponding values of each elastic parameter at the conditions where the minimum and maximum values of the sum of Vp , Vs , and ρ are achieved. The preference for using the sum of the three parameters over the individual parameter space is to consider their inter-connected nature. The red and yellow filled stars in Figure A.50 mark the maximum and minimum values of each parameter, respectively. For the melt phase, we follow the approach of Ueki and Iwamori (2016) (Ueki and Iwamori (2016)) to estimate the properties of Vp and ρ (with its Vs being zero) at the same P-T conditions where maximum and minimum values of elastic parameters are achieved (marked by the red and yellow stars in Figure A.50).

The method of Ueki and Iwamori (2016) is effective in deriving the relationships between pressures, temperatures, and H_2O concentrations of various hydrous melts from ultramafic to felsic compositions at pressures of 0–4.29 GPa. To account for the generally hydrous conditions of the erupted magmas (Blatter et al. (2017)), 5wt% H_2O is added to the reported oxidates before they are normalized for the melt phase calculation. Increasing the H_2O will generally make the melt phase more hydrous, thus reducing the Vp and density of the melt phase, the Vs of the two-phase aggregate as well as the volume of melt. However, the resulted change is minor. The melt-phase calculation is made for both basaltic and dacitic compositions.

In the second step, we use the estimated elastic properties of the solid and melt phase to compute the velocity of a two-phase aggregate by initially assuming air-filled pore spaces. We then correct the velocity by substituting it in the liquid phase following Gassmann's equation (Gassmann (1951)). The calculations are

made via the open-source package of ElasticC (Paulatto et al. (2022)), which is available at <http://github.com/michpaulatto/ElasticC>. Note that our approach in this step yields the low frequency or relaxed seismic velocity of the partially molten aggregate.

In the above procedures, one key parameter that will significantly influence the melt volume estimation later is the aspect ratio, which describes the geometrical organization of melt relative to a usually assumed elliptical pore space along grain boundaries (Holness (2006)). Smaller aspect ratios mean less melt is required to wet grain boundaries compared to more equant pockets isolated near grain boundary junctions. Maguire et al. (2022) report that the predicted melt volume for the shallow magmatic reservoir beneath Yellowstone can be up to 4 times more if using an aspect ratio of 1 compared to that from a value of 0.05. In our calculation, we also vary the aspect ratios between 0.05 and 1 and found that a similar conclusion holds for our case (Figure A.51). However, theoretical work (Takei (2002)) indicates that texturally equilibrated partially molten rocks have aspect ratios usually in the range between 0.1 – 0.15, which is equivalent to a dihedral angle range of 20° – 40° . We think this could be the case for mid-lower crustal reservoirs beneath the Gorge, where stored magma is likely not directly linked to active edifices. We note however that uncertainties exist on this assumption, and it is also difficult to ascribe one aspect ratio for the system considering the relatively large geographic distribution of the mid-lower crustal reservoir (Figure A.52) and potentially complex melt distribution in general.

To estimate the volume of active melts present in the mid-lower crust currently beneath the Gorge, we use the 3.55 km/s velocity contour as a diagnostic of the presence of partial melt. This contour corresponds to a melt fraction of 3 –

6% for basaltic composition, and slightly lower, 1.5 – 4% for dacitic compositions. To construct a velocity image for the melt estimation, we average the Vs along an E-W corridor centered around the Gorge area but with an extension of 0.2° to the north and south, a width that is close to the resolution of our seismic tomography model at these depths. The Vs distributions in the mid-lower crust (e.g., 20-35 km) show generally symmetric features within this corridor around the Gorge area (Figure A.52), justifying the process of averaging. Assuming an average melt fraction of 6% implies a total volume of active melts of $\sim 3.7 \text{ km}^3$ per km arc distance while an average melt fraction of 1.5% melt indicates a lower volume of $\sim 1 \text{ km}^3$ per km arc distance.

Oxide	Weight % for basalts	Weight % for dacites
SiO ₂	50.40	65.37
TiO ₂	1.98	0.54
Al ₂ O ₃	16.28	17.9
FeO	10.55	3.84
MnO	0.15	0.08
MgO	6.84	1.52
CaO	8.55	4.63
Na ₂ O	3.91	4.78
K ₂ O	1.34	1.29

Table A.1. **Composition of typical basalts and dacites at Mount St. Helens used to calculate seismic velocity.** The basaltic composition is from Blatter et al., 2017, and dacitic composition is from Leeman et al., (2005).

A.0.6 Heat flow. Quantifying magmatic origins of heat flow anomalies in the central Cascades is complicated by strong near-surface meteoric circulation (Saar and Manga (2004)). This is reflected by non-conductive geothermal gradients at the arc axis reflecting lateral advection of heat (Ingebritsen et al. (1989)). The regional compilation of (Ingebritsen and Mariner (2010)) shows localized high heat flow coincident with Quaternary volcanic vents throughout the

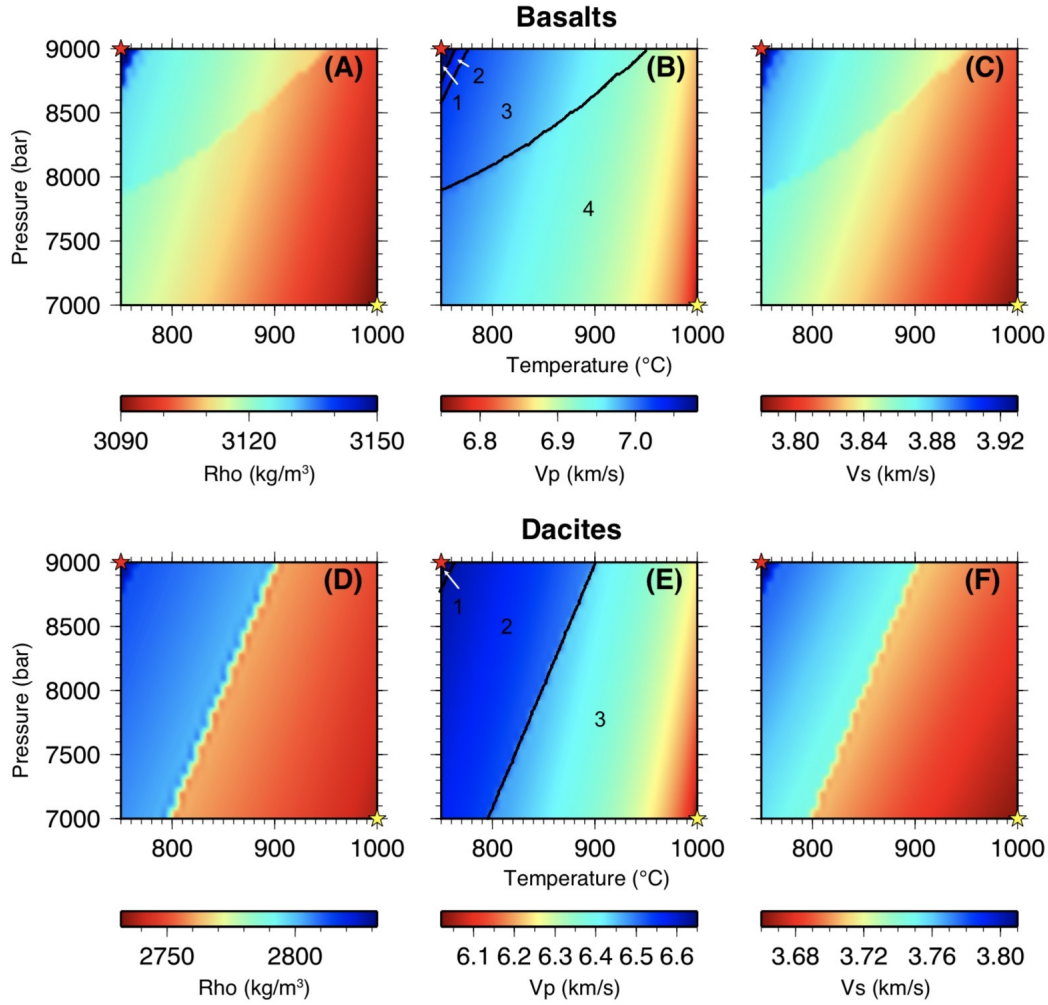


Figure A.50. Predicted V_p , V_s , and ρ with respect to the variations of pressure and temperature condition for the solid phase using PerpleX, both for basalts (A-C) and dacites (D-E). The red and yellow stars define the maximum and minimum value of each parameter to define a range used in the calculation. The black lines in B and E separate the 2D space into several domains where slightly different mineral assemblages are stable. See Table S2 for the detailed mineral assemblages for each domain.

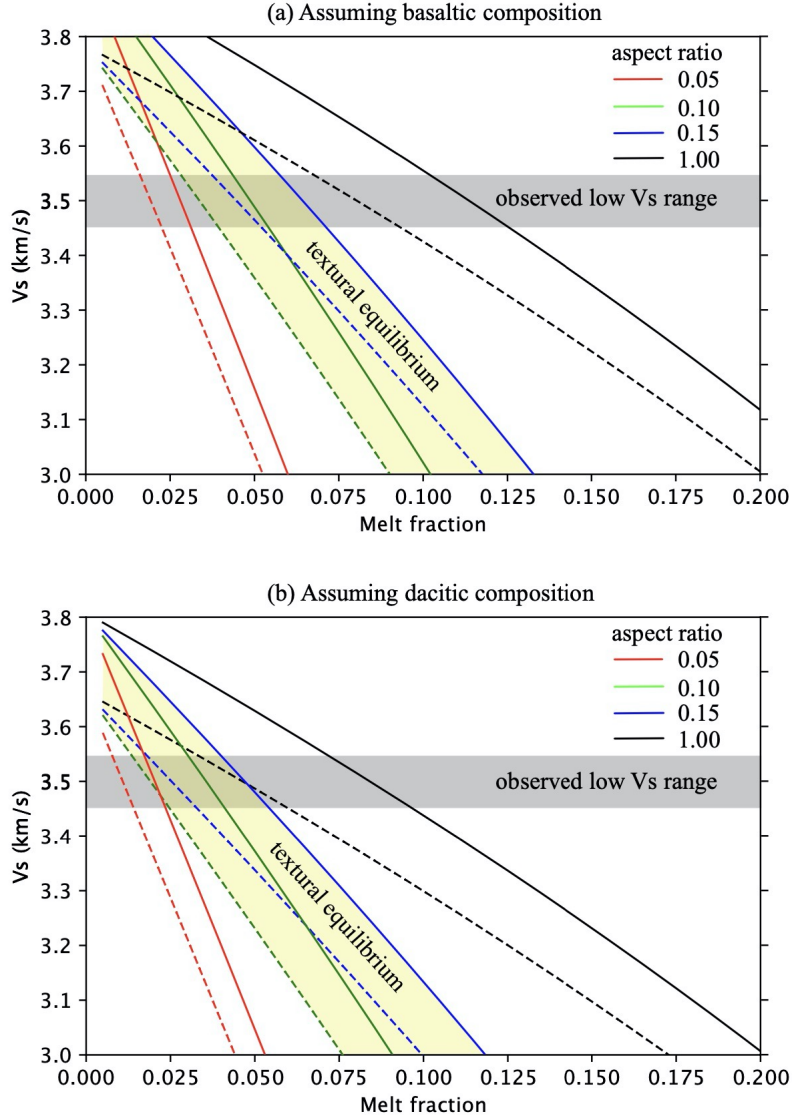


Figure A.51. Modelled V_s and melt fraction relationship for basalts (A) and dacites (B). The color of the curves represents the different aspect ratios assumed during the calculation, with the textural equilibrium regime of aspects ratio between 0.1-0.15 highlighted in yellow. The solid lines are from calculations using the maximum values of elastic parameters (red stars in Figure A.50) and the dashed lines are from the minimum values. The grey-shaded region indicates V_s range observed in this study.

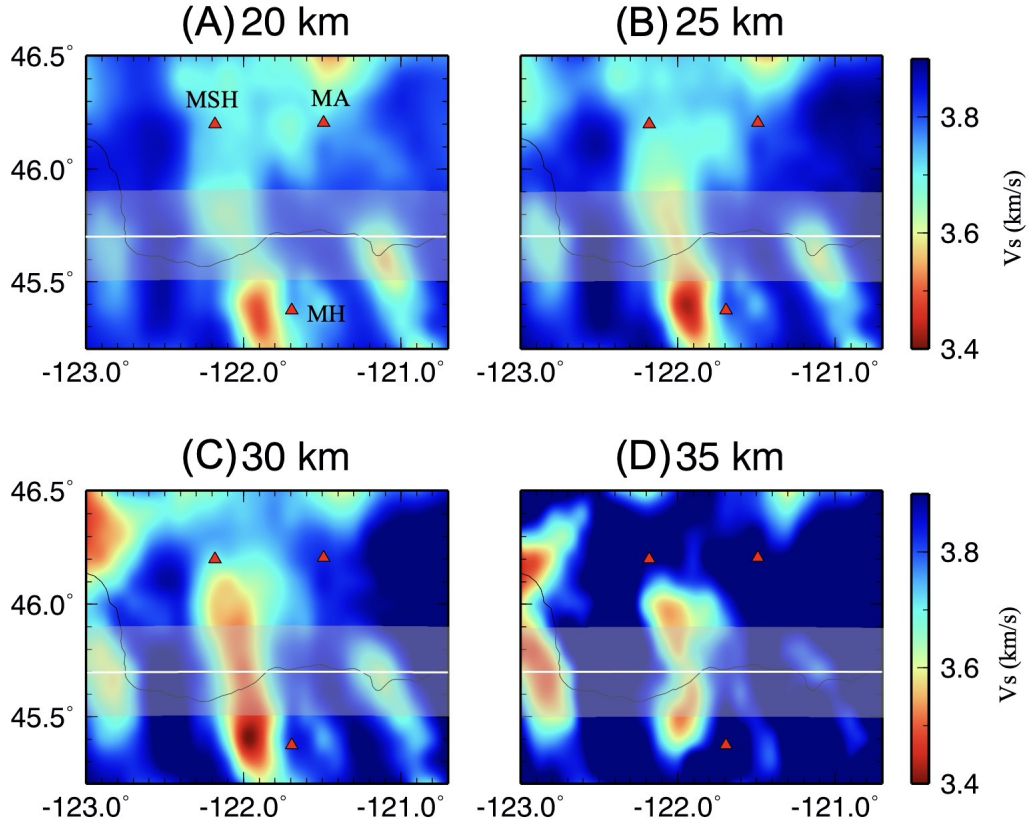


Figure A.52. Regional maps of V_s at 20 km (A), 25 km (B), 30 km (C), and 35 km (D) depths, respectively. The red-filled stars denote the volcanoes of Mount St. Helens (MSH), Mount Adams (MA), and Mount Hood (MH). The grey-shaded region highlights the corridor where V_s values are averaged to produce the transect shown in Figure 39C. The color of the curves represents the different aspect ratios assumed during the calculation, with the textural equilibrium regime of aspect ratio between 0.1-0.15 highlighted in yellow. The solid lines are from calculations using the maximum values of elastic parameters (red stars in Figure A.50) and the dashed lines are from the minimum values. The grey-shaded region indicates V_s range observed in this study.

Domain	Basalts	Dacites
1	almandine, spessartine, enstatite, diopside, jadeite, albite, sanidine, ilmenite, hercynite	almandine, spessartine, kyanite, sphen, diopside, albite, sanidine, anorthite, quartz
2	forsterite, almandine, spessartine, diopside, jadeite, albite, sanidine, ilmenite, hercynite	almandine, spessartine, kyanite, diopside, albite, sanidine, anorthite, quartz, ilmenite
3	forsterite, fayalite, almandine, spessartine, diopside, albite, sanidine, ilmenite, hercynite	almandine, spessartine, enstatite, diopside, albite, sanidine, anorthite, quartz, ilmenite
4	fayalite, almandine, spessartine, enstatite, diopside, albite, sanidine, ilmenite, hercynite	

Table A.2. **The mineral assemblages stabilized at different P-T conditions.** The number and corresponding domains are specified in Figure A.50 panels **B** and **E**.

arc (with increasing correlation for Holocene age vents, (O’Hara et al. (2020))).

Interpolated heat flow variations around the Gorge (C. F. Williams and DeAngelo (2008)) are displayed in Figure A.45B and Figure 39B.

Cascades arc-centered heat flow anomalies likely arise from magmatic intrusions at depth (Blackwell, Steele, Kelley, and Korosec (1990)). Here we show that the magnitude of peak heat flow is consistent with long-lived magma flux into the lower/mid crust at depths spanning observed low seismic V_s velocities. While a joint inversion of geophysical data that accounts for fluid advection of heat is needed to capture the details of regional heat flow, a simple 1D model similar to that used in previous work (Till et al. (2019)) is sufficient to argue that the presence of anomalous heat at the arc axis is likely due to intrusions at depth, and consistent with the seismically inferred melt at present day.

We estimate Cascades geothermal gradient via solution to

$$k_T \frac{d^2 T(z)}{dz^2} = P(z), \quad (\text{A.12})$$

with T temperature as a function of depth z , subject to $T(0) = 13$ C (yearly average for Portland, OR) and $k_s dT/dz|_{z=0} = q_{surf}$, k_T is the effective conductivity in the near-surface. We vary a_s through the range of heatflow observed in the gorge, $q_{surf} \in [45, 85]$ mW/m². $P(z)$ is a radiogenic heat production term, of the generic form

$$P(z) = \rho H_0 \exp(-z/\lambda), \quad (\text{A.13})$$

with H_0 the radiogenic heat production and λ a scale length for concentration in the shallow crust (Turcotte and Schubert (2014)), with ρ an average crustal density. We experiment with and without this term to demonstrate its effects qualitatively.

Figure SA.53 shows several representative conductive geotherms that demonstrate the likelihood of melt as inferred from seismic tomography.

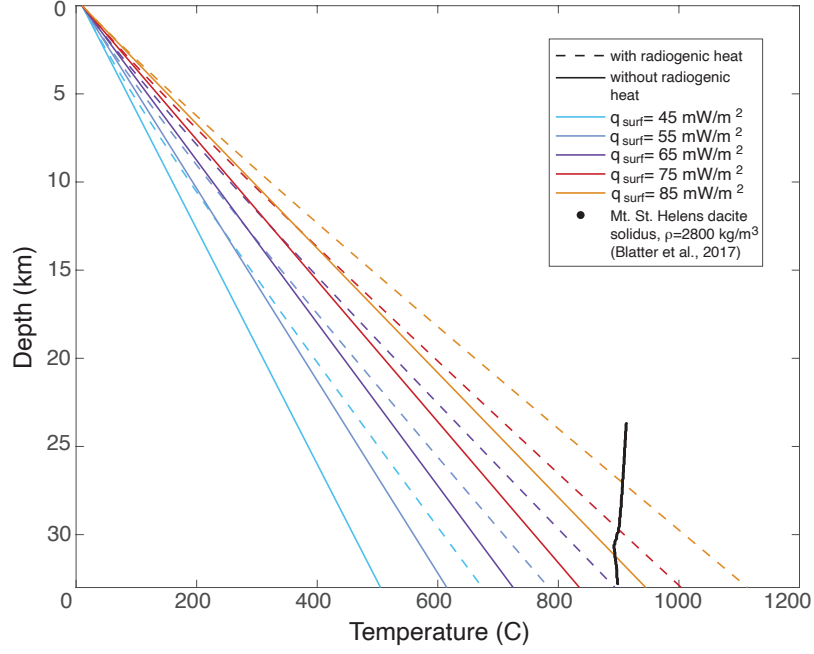


Figure A.53. Illustrative conductive geotherms with and without radiogenic heat production ($H_0 = 9 \times 10^{-10}$ W/kg). Curve colors correspond to assumed surface heat flow, spanning the range observed in the Gorge. The black curve is Dacite solidus with 5 weight percent water (Blatter et al. (2017)), computed in PerpleX. The curves are lower bounds for geothermal gradient as they neglect heat associated with intrusions, and hydrothermal circulation in the shallow crust (Ingebritsen et al. (1989)), supporting our seismic inference of lower-crustal melt at present.

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