Judging Unlikely Conjunctions

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Abstract

Several procedures were used to elicit direct numerical estimates of the probabilities associated with various events created by the conjunction of three independent subevents. However the question was asked, many respondents showed a misunderstanding of the conjunction rule. Less than one half met the minimal criterion of consistently assigning a probability to the conjunction that was no larger than that associated with the least likely constituent event. As a result, subjects as a whole greatly overestimated the conjunctive probability. When attention was restricted to individuals who had followed the conjunction rule, a tendency remained to overestimate the smallest probabilities, relative to the calculated values. Subsidiary results concerned the effects on judgment of wishful thinking, the similarity of the constituent events, and the source of the constituent events. The implications of these results for eliciting and presenting the probabilities of unlikely events are also discussed.

Judging Unlikely Conjunctions

A recurrent task in modern life is dealing with unlikely but consequential sequences of events. These may be positive events, such as picking the winner of three successive horse races (on an Exacta ticket) or escaping all the radar traps while speeding down a 100-mile stretch of freeway. Or, they may be negative events, such as (a) being one of the unfortunate individuals who drink water contaminated by a toxic landfill, suffer an adverse reaction, _and_ succumb to its effects, or (b) having a malfunctioning valve go undetected in a nuclear power plant, requiring it in an emergency situation, _and_ having the backup valve be unavailable because of routine servicing.

In order to make effective decisions regarding such contingencies, it is essential to assess and then appreciate the low probabilities associated with them. Even if one has a very good idea of what the payoff is for the Exacta or the suffering associated with a toxic reaction, what one does about them may be very different if the chances of everything going right (or wrong) is 0.01, 0.0001, or 0.000001. Unfortunately, people seem to have considerable difficulty judging such probabilities. Given the rarity of such events, the existence of difficulties should not be entirely surprising. By definition, such events occur so infrequently that individuals have little direct, hands-on experience with them. For understanding, they must rely on observing the experience of others or listening to the reports of experts. The former is likely to be unsystematic, the latter to be motivated by concerns other than informing laypeople. Moreover, the probabilities of many unlikely events are hard to assess accurately even for experts who devote their lives to the task (Fischhoff, in press, and references therein).

One judgmental difficulty that can be traced to the biased nature of lay experience is a tendency to exaggerate the likelihood of unlikely events (e.g., causes of death) that are disproportionately reported in the news media (Combs & Slovic, 1979) or are unusually common in their personal lives (Lichtenstein, Slovic, Fischhoff, Layman & Combs, 1978). People seem not to realize (or at least they are unable to compensate for) the extent to which reporting and observation are driven by processes different than random sampling (Tversky & Kahneman, 1973).

A second problem is the tendency to overestimate the probability of unlikely events (Attneave, 1953; Hogarth, 1975; Lichtenstein et al., 1978; Petrusic, Cousins & Corbin, 1984; Starr & Whipple, 1981). This bias could also be caused by undue salience of the rare occurrences of such events. Or, it could reflect difficulties with using any very small numbers (Poulton, 1968, 1977; Slovic & Lichtenstein, 1971), perhaps expressed in the assignment of some too-high epsilon to any event deemed at all possible.

A third threat to assessing the probabilities of unlikely conjunctions is the possibility that assessments will be biased by the desires of the assessor, reflecting wishful thinking or its unnamed complement (Irwin, 1953; Langer, 1982; Weinstein, 1980; Zakay, 1983).[1] Although any assessment could, in principle, be influenced by such biasing processes, the ambiguity and confusion surrounding unlikely conjunctions may render them particularly vulnerable to the intrusion of extraneous considerations. The magnitude of the consequences associated with those unlikely events that do attract attention may create unusual needs to see things in a particular way.

A final set of possible judgmental difficulties is unique to unlikely events that are seen as arising from the conjunction of a series of events. The foremost of these is that people frequently overestimate the probability of such compound events (Bar Hillel, 1973; Cohen, Chesnick & Haran, 1971; Slovic, 1969; Slovic, Fischhoff & Lichtenstein, 1976; Tversky & Kahneman, 1983). Such overestimation seems to reflect psychological processes such as the persuasiveness of coherent scenarios, the non-salience of the unlikely weak links in otherwise likely scenarios, insensitivity to the extent to which individually likely links can be cumulatively improbable, and anchoring of perceptions in the likelihood of initial probable events. For example, Slovic (1969) found that people were much less willing to play a gamble that paid them off if a single event with probability $\underline{p}$ occurred than to play a gamble requiring the joint occurrence of four events with probability $\underline{p}$(exp 1/4).

An obvious strategy for dealing with the last of these worries is to avoid decomposition. Whenever possible, a single event should be considered, so that conjunctions never arise. Such a strategy could be pursued by individuals attempting to organize their own thoughts or by communicators attempting to tell people about low probability events.

Thus, one talks about the Exacta or the loss-of-coolant accident, rather than the sequence of subevents leading to them. A drawback of this strategy is that it conflicts with an obvious strategy for dealing with the overall problem of the non-intuitive nature of low-probability events: avoid such events by replacing them with the conjunction of more likely events (for which people have more accurate perceptions).

Determining the relative magnitudes of these competing threats to the understanding of low probability events is the primary goal of the present studies. The events to be judged (in all but one case) are variants on that depicted in Figure 1, showing three "spinners," each of which must end up in the black sector for the overall event to occur. In many respects, such spinners would seem to constitute very simple and understandable events. Until a consequential outcome (e.g., the payoff of a lottery) is associated with them, spinners are quite neutral, in the sense of not evoking any emotional responses or associations that might complicate the (cognitive) probability assessment process. Used individually, spinners are such a common and transparent data generator that they are used routinely in children's games and TV quiz shows. Decision analysts have sufficient confidence in people's ability to judge the size of spinner sectors that they use them to elicit their clients' probability assessments (Spetzler & Staël von Holstein, 1975; Staël von Holstein & Matheson, 1979; Wallsten & Budescu, 1983). Indeed, they seem so straightforward that a recent survey sponsored by the Environmental Protection Agency (Smith, Desvousges & Freeman, 1985) used the conjunction of two spinners, showing "risk of exposure" and "risk of death if exposed," to represent the overall "personal risk" arising from hazardous wastes. This particular procedure was derived with the aid of intensive "focus group" interviews in which laypeople expressed satisfaction with this mode of presentation.

Insert Figure 1 about here

Thus, there are strong *a priori* reasons and substantial anecdotal evidence suggesting that people understand the probabilities associated with single, and perhaps even multiple, spinners. If this is the case, then displays such as Figure 1 might prove to be generally useful representational devices, allowing the concretization of probabilities that are hard to understand in the abstract and too small to be shown on a single spinner. If this is not the case, then one might expect even more trouble with less schematic and well-defined events. Spinners seem immune to most of the threats to the comprehension of very small probabilities listed above. For example, there should be no problem of exposure to unrepresentative samples of information. However, the abstractness of the conjunction operation in this context may be difficult for subjects for whom that is not a well-developed concept. Empirical grounds for concern may be found in Bar-Hillel (1973), Cohen et al. (1971), and Slovic (1969), all of whom observed substantial overestimation of the probabilities associated with the conjunctions of similarly schematic events.[2] The present studies attempt to replicate and extend those results.

Overview of Studies

The *basic* study in the present series asks subjects to judge the probabilities associated with the joint occurrence of all three spinners, such as those in Figure 1, landing in the black. Those assessments can then be compared with the probabilities computed from measurements of the sectors on the spinners. Although straightforward, this procedure might be questioned on a number of grounds. One is that subjects may understand the event intuitively, yet be unable to express their understanding in terms of numerical probabilities, despite our efforts to clarify how to use that response mode for small probabilities. A second concern is that the events are so abstract as to evoke unreal or unrepresentative behavior. More consequential events might improve performance by making the task more meaningful--or degrade it by allowing the intrusion of extraneous, non-cognitive factors. A third, and related concern is that the test stimulus (Figure 1) may be a little too tidy. It would be an unusual real-life conjunction that involved three events of precisely equal magnitude. A fourth concern is that people may misperceive the size of the black sector on the individual spinners, leading to misestimation of their conjunction even among individuals who do understand conjunctions.

The first of these concerns, regarding people's ability to employ the response mode, is addressed within the basic study with supplementary response modes that offer subjects alternative ways to express their understanding. One of these asks for the probability that at least one of 12 three-spinner events will occur, which should evoke a much larger number than that required by the individual three-spinner events. A second asks for the number of three-spinner events that respondents would expect to occur were all 12 carried out, a response mode that requires an integer response between 0 and 12. These responses can be compared with the probabilities computed from the spinners themselves to provide additional perspectives on subjects' understanding. They can also be compared with values for these complex events that are computed from subjects' probability assessments for the individual events in order to examine the internal consistency of subjects' judgments.

Concern over the response mode is also addressed in a separate study which provides subjects with sets of three blank spinners (i.e, circles with dots in the middle) and asks them to color black sectors on them so as to create events with a particular target probabilities. In this format, subjects do not have to use numerical probabilities, only to comprehend them. In order to evoke some of these subjects' intuitive notions of how probability judgments may be biased, they were also asked to create events having the same target probability, but appearing to have particularly large or small conjoint probabilities.

The second concern, the abstractness of the events, is addressed by the addition of a lottery group, which performs the same task as the basic group but is asked to imagine winning $100 if all three spinners in a set ended up in the black. If this hypothetical concretization has any effect, it should be to make the event more meaningful. Should that happen, then the danger would arise of it becoming too meaningful, evoking an involvement in the event capable of distorting perceptions of its likelihood (e.g., wishful thinking). Thus, this manipulation might affect the overall level of probability assessments, as well as their accuracy. Subjects were also asked to estimate how much they would pay to play each lottery. In addition to reinforcing the notion that the spinners represent a lottery, this question provides an additional response mode for eliciting perceptions of probabilities without requiring the use of very small numbers.

In the lottery condition, the event was described as involving simultaneous play of the three spinners. In the sequential lottery condition, the spinners were described as being played one at a time, with the play continuing only if each ends up in the black. Although both descriptions lead to the same probability for the conjunction of three "blacks," each might lead to a different mental representation of the events. For example, the sequential version may focus attention on the initial spinners, anchoring the overall probability assessments on them. The result may be higher probability assessments if the initial spinner is the most likely in the set, and lower assessments if it is the least likely (Ronen, 1973). The two representations may also register somewhat differently on the cognitive processes dealing with conjunctions. Perhaps the sequential version will highlight the need for each of the events to occur (or else one doesn't even get to try the next one); perhaps the trail through its successful events will create a coherent story which tends to obscure the alternative (Peterson, 1973; Slovic, Fischhoff & Lichtenstein, 1976; Tversky & Kahneman, 1973).

The multiple events condition elicited probability assessments for the conjunction of three events described solely by their numerical probability of occurrence. Given the possible advantages of the spinner representations and people's apparent difficulty with numerical probabilities, this context might be expected to produce the worst performance of all. However, it is worth including as a baseline against which the hoped-for benefits of the spinners can be compared, and as an alternative method should estimating the size of the black sector on the spinners prove problematic (the fourth concern in the list given earlier).

Within each condition, several different representations of each compound probability were given. They were created by systematically varying the variance of the black sectors on the spinners, from a tidy equal-probability version (like that in Figure 1) to a version with striking differences. Increasing the variance produces a set of spinners having more black area and a most-likely event with a larger probability, both changes that might increase the apparent likelihood of the conjunction. It also produces a smaller least-likely event, that might decrease the perceived likelihood of the conjunction. There appear to be no studies on the decomposition of probabilities which could provide an empirical basis for choosing between these predictions. In each case, the least likely event in each set was always presented first. If the tendency for overall assessments to anchor on initial events (Ronen, 1973) is repeated here, then greater variance will mean reduced perceived probability.

As a supplement to these studies using artificial events, a pair of conditions was created describing the spinners as representing the risks associated with leakage from a hazardous waste landfill (following Smith et al., 1985). The first spinner describes the risk of leakage, the second the risk of exposure to leaked chemicals, and the third the risk of illness from exposure. Thus, the context concretized the compound event as a series of sequential events. In the <u>general risk</u> condition, subjects were asked to consider the risks posed by the landfill to some other individual; in the personal risk condition, they themselves were to be exposed. The contrast between these conditions offered another opportunity to look at the possible effects of wishful thinking on probability assessments.

A final condition required subjects first to produce constituent events entirely from their own experience, such as "an international event that has a 75% chance of occurring." Their conjunctions then involved the co-occurrence of three such events, drawn from different areas of life (e.g., an international events, a sports event, a meteorological event). Although the previous contexts have all concerned events that do occur in life (hazardous wastes, gambles), conceivably subjects might have a better "feel" for events that they themselves produce. Moreover their juxtaposition in a compound event should seem truly coincidental, thereby overcoming any possible tendency for subjects' mental representation of the other events to violate the independence assumption.

Method

Stimuli

Four decompositions were used to achieve each of three target probabilities. The two probabilities were: 0.002, 0.016, and 0.125. The decompositions were derived by taking the following factors of the target probability, *p*: (A) p(exp 1/3), p(exp 1/3), p(exp 1/3); (B) p(exp 1/2), p(exp 1/4), p(exp 1/4); (C) p(exp 1/2), p(exp 1/3), p(exp 1/6); (D) p(exp 2/3), p(exp 1/4), p(exp 1/12). Table 1 shows the resulting probabilities, along with their sum (which reflects the total area of the black sectors and which can be seen to increase with the variance in the decomposition). A single randomly chosen order was used for presenting the sets in all conditions.

Insert Table 1 about here

Instructions

Spinners

Multiple spinners. To introduce them to the task, subjects in the basic condition were shown three spinners, presenting probabilities of 0.595, 0.354, and 0.707 (and a joint probability of 0.149). They were told:

> Here is a set of three spinners. Imagine that all three were spun simultaneously. What is the probability that the arrows on all three would end up in the dark area? Please answer as carefully as you can. Do not hesitate to give fractional estimates (e.g., 0.785, 0.006, 0.076). For example, 0.785 would mean that there is a 78.5% chance of all three spinners ending up in the black. An estimate of 0.006 would correspond to a 0.6% chance, or 6 chances in 1,000, meaning that if the three spinners were spun 1,000 times, all three would end up in the black on 6 occasions. Answering 0.076 means that you think that all three will end up in the black 76 times out of 1,000, or 7.6% of the time. ________________

Each of the 12 three-spinner sets of test stimuli were introduced by, "Now consider this set of spinners:" and followed by, "What is the probability that the arrows on all three would end up in the dark area?"

The lottery condition varied these instructions by changing the two initial sentences to read, "Imagine a lottery using these three spinners. If the arrow ends up in the dark portions of all three spinners, you win $100." Immediately before the answer blank, they were asked "What is the probability that you would win?" After assessing the probability, they were asked, "How much would you be willing to pay to play such a lottery?" Their test stimuli were introduced with "Now consider the following lottery" and followed by, "If the arrow ends up in the dark portions of all three spinners, you win $100. What is the probability that you would win? __________ How much would you be willing to pay to play such a lottery? __________"

For subjects in the sequential lottery condition, the second sentence of the lottery instructions was replaced by the following:

> First, Spinner A is spun. If the arrow ends up in the dark area, then Spinner B is spun. If not, then the lottery is over. If the arrow on B also ends up in the dark area, then Spinner C is spun. If not, then the lottery is over. If the arrow on Spinner C as well ends up in the dark area, then you will win $100.

The remainder of their questionnaire was the same except that each test stimulus was introduced by "Now consider the following lottery, which would be played in the same way, beginning with A and then going on with B and then C (if possible)."

For the multiple events group, the initial two sentences and example stimulus were replaced by:

Imagine three events.

Event A has a 60% chance of occurring.
Event B has a 35% chance of occurring.
Event C has a 71% chance of occurring.

What is the probability that all three of these events will occur? Their probability question was phrased as, "The probability of A and B and C occurring is __________."

Compound predictions. After assessing the probability for the 12 sets, subjects in the basic form were told, "Look back now over all 12 sets of spinners. If all 12 were spun, how many would expect to end up with all three arrows in the dark area?" The numbers from 0 to 12 were listed in a row with the instruction to "Please circle your answer." They were also asked, "If all 12 sets were spun, what is the probability that at least one would end up with all three arrows in the dark area?" Appropriate adaptations were made for the other three groups.

Single Spinners. Subjects in the three spinners groups were then shown 12 single spinners, representing all those used at least once in the 12 three-spinner sets, as well as the three conjoint probabilities for those sets (i.e., 0.125, 0.016, 0.002). The basic group was told:

> Here are 12 additional spinners, not in sets. For each, estimate the probability that the arrow would end up in the dark area if it were spun separately. Again, please answer as carefully as possible and do not hesitate to use fractional estimates.

Their question was "The probability of the arrow ending up in the dark area is __________." The two lottery groups received similar instructions, except that the second sentence was replaced by "Imagine that each represents a lottery, whereby you win $100 if the arrow for that spinner ends up in the dark area. For each, estimate the probability that you would win." Their question was, "What is the probability that you would win?" They were also asked, "How much would you be willing to pay to play such a lottery?"

After the 12 spinners, subjects in the basic condition were asked, "If all 12 of these individual spinners were spun simultaneously, how many would you expect to end up in the dark areas?", followed by the numbers from 0 to 12, and "What is the probability that <u>at least one</u> will end up in the dark area?" Comparable questions with appropriate wording changes were given to the two lottery groups.

Although the notion of a lottery was not mentioned to either the basic group or the multiple events group at the time they were making probability assessments, the final questions in their forms told them to go back to the three-event sets and "Imagine that each is now a lottery and you will win $100 if <u>all three</u> spinners end up in the black" (or "if all three events occur"--for the multiple events group). They were then asked, "How much would you be willing to pay to play Set X? $__________"

<u>Creating Lotteries</u>

Subjects here were told,

> Imagine that you are designing a lottery. The way that participants play is by simultaneously spinning three spinners like these [the introductory examples used elsewhere]. If the arrows on all three spinners end up in the black area, then the participants win. You can make the size of the dark area in each spinner as large or as small as you wish. All three could have the same shape, or all three could have different shapes.

The first exercise provided three 3.8 cm (1.5 in) circles with a black dot in the middle. They were told,

> Please color in these three spinners so that participants would

> have a 25% chance of winning a lottery. That is, there should be a 25% chance of all three arrows landing in the black area. You may find yourself changing your mind while creating the spinners. If so, please make it clear which shapes you have settled upon.

Subsequent questions asked for probabilities of 12.5%, 4.2%, 1.5%, and 0.2% (which was also stated as "2 chances in 1,000").

Finally, they were told,

> While creating these five sets of spinners, you may have noticed that there were several different ways of creating a lottery with each chance of winning. Perhaps it occurred to you that some versions would seem more attractive to potential participants than others. That is, although different versions might have the same objective probability of producing a win, some might seem more likely to produce a win. Could you now repeat the preceding task, producing what you believe to be the most attractive lottery for producing each probability of winning.

Five tasks analogous to the proceeding ones followed.

Hazardous wastes

The substantive context of these questionnaires was patterned after that used by Smith et al., (1985) in their attempts to determine the benefits that people attribute to controlling the risks of hazardous waste sites. They described those risks in terms of the conjunction of two spinners, one giving "risk of exposure" (also labeled "possible pathways") and one giving "risk of death if exposed" (also labeled "heredity and health"). A third spinner gave the conjunction of the two, called "combined risk: exposure and death" (also labeled "personal risk"). Beneath each spinner, the associated probability appeared in both ratio and percentage form. In order to be able to test subjects' judgments, these probabilities were deleted. In order to preserve comparability with the preceding experiments, a three-event partition was used, the first two of which represented the decomposition of Smith et al.'s first event into two independent constituent events.

The personal risks form of this questionnaire read:

> Recently, the news media have given a good deal of coverage to the health risks posed by the leakage of hazardous chemicals from landfills. For the threat that one of these facilities posed to you to be realized, a number of things would have to happen: First, the chemicals would have to leak. Then, you would have to be exposed to them (e.g., by drinking groundwater that they have contaminated). Finally, you would have to become ill as a result of the exposure. That is, each of three uncertain events would have to happen.
>
> One way of thinking about such uncertain events would be in terms of a set of spinners, like the following. The chances of the landfill leaking toxic chemicals would be like spinning the first spinner and having its arrow end up in the dark area. The chances of your being exposed to those chemicals would be like spinning the second spinner and having its arrow end up in the dark area. The chances of your becoming ill as the result of that exposure would be like spinning the third spinner and having its arrow end up in the dark area.

They were then shown the test three-event stimulus and asked, "If these risks described a situation near you, what would you estimate to be the probability that <u>all three</u> of these events would happen." The use of the probability scale was then described and followed by "The probability of all three events occurring (and your becoming ill) is:____" The remaining questions were adapted appropriately. The <u>general risks</u> form described these risks as applying to "an individual."

<u>Mixed Events</u>

On the first page of this questionnaire, subjects were told,

> For each of the following questions, please describe an event that fits the accompanying description. By "event," we mean things such as, "I will have no cavities during my next dental exam." "It will rain in Eugene at least once during this coming April." "Margaret Thatcher will be Prime Minister after the next Parlimentary elections in Britain." or "The Ducks will be next year's PAC-10 football champions."

The five events were specified as: (A) "an international event that has a 75% chance of occurring;" (B) "a personal event that has a 10% chance of occurring;" (C) "a meteorological event that has a 5% chance of occurring;" (D) "a political event on the national level that has a 25% chance of occurring;" (E) "an event from the world of entertainment that has a 35% chance of occurring." After events B-E, they were told to "make this an event that is unrelated to the previous one(s)."

The numerical values of these probabilities were adapted from the single-event probabilities used with the middle-variance stimuli, after rounding them off to more customary values. Their order of presentation followed that used in evaluation of individual spinners.

For each event in the conjunction-estimation portion of the task, subjects were asked to describe briefly three of the simple events (e.g., B, D and E) and then to assess the probability that all three would occur. The first conjunction involved events B, E, and D; its probability was 0.013, compared with 0.016 prior to the rounding. The second conjunction in volved B, C, and F, with probability 0.0018 (compared with 0.0020). The third involved A, D, and F, equal to 0.131 (compared with 0.125). The fourth involved A, C, and F, equal to 0.013 (compared with 0.016).

Subjects and Administration

Subjects were recruited through advertisements in the University of Oregon student paper requesting paid volunteers for "experiments in judgment and decision making." They received $5 for participating in 1 1/4 hours of unrelated tasks of which this was one. They were divided roughly equally between men and women, with the mean age of the former 24 and of the latter 21. On previous occasions in which more detailed demographic information was obtained, subjects recruited in this way have been found to be roughly two-thirds students and one-third otherwise associated with the university community (e.g., spouses, staff, hangers-on). Almost all have some university education, with the median being 15 years of schooling.

Questionnaires were completed in self-paced groups of approximately 50. The different spinner multiple events conditions were alternated in packets given to subjects in early to mid-1984. Because of a collating error, somewhat fewer copies of the multiple-events questionnaire were distributed. The two hazardous-events forms were alternated and distributed in the same manner in 1985. The mixed-event forms were given to a subset of these subjects later in the same session to allow assessment of within-subject consistency in response patterns.

Results and Discussion

Spinners

Coherence

Results. The probability of the conjunction of a series of events can be no larger than the probability of the least likely of those events, considered singly. Any case in which subjects' responses failed to maintain this relationship was interpreted as a violation of coherence.[3] Table 2 shows the distributions of such violations across the four groups. Perhaps the most striking feature of these results is the small percentage of subjects who consistently passed this rather lenient coherence criterion. Over all groups, only 22.3% of subjects gave conjoint probabilities no greater than the smallest single-event probability for all 12 stimulus sets. About half that percentage of subjects always violated the rule. Indeed, the table reveals a curvilinear relationship, with most subjects consistently either obeying or violating the rule for most of the stimuli. Categorizing subjects as coherent if they had two or fewer violations (out of 12 possible) and as noncoherent if they had ten or more yields 45.2% coherent and 26.7% noncoherent subjects. Only 28.1% of subjects showed one of the seven (of 12 possible) intermediate degrees of coherence.

Insert Table 2 about here

Although it is never very large, the degree of coherence seems to vary across questionnaires. The percentage of coherent subjects varies from the lottery group's 35.8% to the basic group's 54.8%, with the sequential lottery (44.4%) and multiple events (44.1%) groups in between. The percentage of noncoherent subjects ranges from the basic group's 12.9% to the multiple event's group's 44.4%, with the sequential lottery (20.3%) and lottery (37.7%) groups in between. The differences in the distributions of coherent subjects across groups is statistically significant (chi-squared = 19.63; df = 6; $p < .005$).

Discussion. By and large, these subjects have demonstrated a distressingly low level of understanding of one of the basic laws of probability theory. Even by our rather lenient categorization scheme, less than half of the subjects produced generally coherent responses. Although this result is consistent with the lack of understanding observed in previous studies, it is reasonable to ask whether it was somehow induced by the details of the present methodology. It is my reading of the questionnaires that they ask the probability assessment question in four different ways, each of which seems fairly reasonable and straightforward, not unlike questions that might arise in everyday life. For what it is worth, they were not designed with the intent of demonstrating noncoherence and this high level came as quite a surprise (and substantially complicated the data analysis).

The best performance was found in the basic group, which might be seen as having a relatively abstract setting compared to the lottery groups. With the lottery format, somewhat better performance was observed when the event was described as happening sequentially, although differing by a factor of five (0.068 vs 0.342).

Inserts Tables 3a, 3b, and 3c about here

One pattern emerging in all three tables is that the different conditions had no consistent effects on estimates. Three-way ANOVAs (Form X Probability X Decomposition) yielded non-significant ($p > .20$) F-ratios for the Form factor in each case. Thus, there was no evidence of a wishful thinking effect increasing (or its complement decreasing) the probabilities associated with the lottery versions; nor was there any significant overall difference between the probabilities evoked by numerical and pictorial stimuli. This was true both with the (seemingly more sophisticated) coherent subjects and the noncoherent subjects, as well as the conditions as a whole.

Not surprisingly, there was a highly significant ($p < .0001$) Probability effect in all three tables. Nonetheless, the differences between the assessments evoked by stimuli associated with the different target probabilities were much smaller than they should have been. The best aggregate performance is found with the coherent subjects. However, even there, the 0.125 stimuli evoked mean responses only ten times (instead of 60 times) larger than the 0.002 stimuli. Moreover, the assessed probabilities were quite different from the target probabilities, especially for the smaller values which were the focal topic of this investigation. The mean probability assigned to the 0.002 conjunction was at least five times that value, and was much higher for the noncoherent subjects and the pooled results.[4]

There was a strong Decomposition effect within the coherent subjects group ($F = 15.35$; $df = 3,261$; $p < .0001$),[5] reflecting the tendency for assessments to increase as the variance of the decomposition decreased. Overall, the no-variance decomposition (A) evoked probabilities that were twice as large as the high-variance decomposition. Apparently, subjects were more attuned to the fact that the high variance decomposition included one very small probability than that the no variance decomposition offered no very large probability and the smallest sum of probabilities (and total area, with the spinners).

This pattern was tempered by a modest interaction with Form ($F = 2.68$; $df = 9,261$; $p < .01$) for the coherent subjects. The most discrepant form was the multiple events, in which the four decompositions produced similar mean probabilities. Thus, whatever aspects of spinners led to the (normatively inappropriate) decomposition effect observed in the other groups, was not evoked by the numerical representation of this group.[6]

Discussion. The pooled results reveal a rather dismal picture. All probability assessments are too high and too much alike, given the different target probabilities. Much of this poor performance is, of course, attributable to the noncoherent group, where matters are on the average much worse. Although the coherent group's performance is decidedly better, it, too, has trouble with the smaller probabilities. Moreover, these subjects alone were sensitive to the (normatively irrelevant) way in which the target probabilities were decomposed.

Thus, the probabilities of unlikely conjunctions were consistently misassessed with four different, reasonably straightforward presentations, designed, in part, to facilitate understanding. It did not help to use a graphic representation instead of a numerical one, to make the conjunctive event a lottery instead of a consequence-less event, or to describe the lottery as happening sequentially instead of simultaneously. Even though performance was much better among subjects who consistently followed the conjunction rule, the fact remains that they were but a minority of all subjects. The pooled responses might be interpreted as the perceptions that would be evoked by presenting such stimuli to individuals from this population, without any additional explanation. The responses of the coherent subjects might represent the level of understanding that could be achieved with tutoring in the basic rules of conjunction.

Compound Multiple Events

Results. After judging the 12 three-event sets individually, subjects considered two aspects of what would happen were all the events to be played: (a) the number that they would win (in terms of the lottery instructions) and (b) the probability that they would win at least one. Formally, *a* should be equal to the sum of the probabilities of the individual events and *b* should be equal to one minus the product of one minus each of the event probabilities. Based on the objective probabilities, the former is equal to 0.572 and the latter is equal to 0.455. Of course, to the extent that subjects do understand how the probabilities of the individual stimuli compound, their personal computations would presumably be based on their own estimates of the individual stimulus probabilities, rather the calculated values. Because they overestimated those values, subjects should expect to win more than one half a lottery and to have more than a 0.45[5] chance of winning at least one.

Table 4 presents subjects' estimates of these quantities, along with the values computed from their responses to the individual stimuli. The number that subjects expect to win is in all cases much higher than the expectation computed from either the objective probabilities of the individual stimuli or from their subjective probabilities. The higher individual probabilities assigned by the noncoherent subjects are reflected in their higher expected number of wins, although the difference is much less than one would expect. In fact, the noncoherent subjects' win estimate is not inconsistent with that calculated from their individual event estimates. By contrast, the coherent subjects' estimates are too high by a factor of three, meaning that the small probabilities of the individual events mount up too fast.[7]

Insert Table 4 about here

The exaggerated probabilities of "success" on the left-hand side of Table 4 are replaced by underestimated probabilities on the right-hand side. All groups thought that winning at least one event was less likely than they should have, given their individual event probabilities. Thus, they did not see quite how likely it was that not all of a series of unlikely events would fail to occur. The discrepancies were particularly great for the noncoherent subjects, for whom at least one success should have been a virtual certainty.

Within each group, subjects who generally assigned higher probabilities to individual events tended to expect a higher number of wins. For all subjects, these correlations were 0.534, 0.579, 0.372, and 0.402, for basic, lottery, sequential lottery, and multiple events, ners (comprising the spinners that had appeared in the multiple-event stimuli as well as ones representing the three target probabilities). For only four of these 36 comparisons were there significant differences (alpha = .05) between the estimates of coherent and noncoherent subjects. As a result, the higher multiple-event estimates of the noncoherent subjects cannot be attributed simply to their having exaggerated the single-event probabilities. Subjects in the lottery and sequential lottery groups also estimated the amount that they would pay to play each of these single-event lotteries. Here, too, there were few significant differences (three in 24 cases) between coherent and noncoherent subjects, providing evidence that they saw the single events similarly.

Table 6 presents the mean responses of subjects in these groups. The probability assessments are strikingly similar to one another. There was, therefore, no tendency for the lottery context to induce wishful thinking or its opposite. Given the relative lack of ambiguity in the probabilities of these events, they should have provided much less opportunity than the multiple-stimulus events for subjects' perceptions to be influenced by such motivational factors.

Insert Table 6 about here

The probability responses in Table 6 are also fairly similar to the measured probabilities for the spinners, with the only systematic deviation being a tendency to overestimate the smallest probabilities. Compared with the responses in Table 3, these assessments are somewhat more accurate than those evoked by the multiple-stimulus events from the coherent subjects and much better than those evoked from the noncoherent subjects.

respectively ($p < .05$, in each case). These correlations were considerably weaker within the coherent and noncoherent subgroups, suggesting that the overall correlation reflected primarily the tendency for noncoherent subjects to provide higher estimates of both types.[8] There was also a tendency for subjects who gave high individual probabilities to assign a high probability to having at least one success, with correlations of 0.321, 0.350, 0.385, and 0.407, for the groups in the order listed above. This, too, seemed largely attributable to the overall differences between the coherent and noncoherent subjects.[9]

Discussion. Given the difficulty of the mental arithmetic required by these tasks, the responses that they evoked should only be evaluated qualitatively for the understanding that they reveal. In this light, they show some rough kinds of consistency, in that subjects assigning higher probabilities to the individual stimuli tended to expect more successful events and a greater chance of at least one success. In other respects, however, there are substantial discrepancies between subjects' estimates for these compound events and those computed either from the objective probabilities or their subjective probabilities for the individual events.

Willingness to Pay

Results. Table 5 shows the mean amount that subjects were willing to pay to play each of the 12 three-spinner events.[10] On the average, noncoherent subjects were willing to pay about twice as much as coherent subjects, although few of the differences for individual estimates were statistically significant (alpha = .05). Although the direction of this difference is consistent with noncoherent subjects' higher probability the result was only marginally significant statistically (chi-squared = 4.18; df = 2; $p < .20$). The multiple events condition might be thought of as the most difficult, because it was the most abstract, or as the easiest, because it gave the probabilities numerically and with the least extraneous detail. Subjects knowing the conjunction rule (and its application in this case) would have a clear upper limit on their responses. It produced the most clearly differentiated performance, with 16 coherent subjects, 16 noncoherent subjects, and only 4 in the middle. Perhaps those who understood the rule at all understood it best here.

It follows from our definition of coherence, that the less coherent subjects will tend to give higher probabilities. As a result, variations in the responses across the different conditions can be predicted, in part, by the proportions of coherent subjects in each. Therefore, the results that follow are reported not only for the conditions as a whole, but separately for the coherent and noncoherent subjects within them. Comparisons within those subpopulations will render cross-condition analyses more meaningful, as well as create a profile of these two, possibly distinct subjective approaches to probability.

Multiple Event Probabilities

Results. Table 3a shows the mean probabilities assigned by all subjects in the four conditions to the 12 three-event stimuli, organized by target probability and decomposition. Tables 3b and 3c show the subsets of those responses produced by consistently coherent and noncoherent subjects. By definition, the noncoherent subjects produced much higher assessments than did the coherent ones, with the overall mean assessments, its magnitude is not commensurate with how much higher their probabilities were.

Insert Table 5 about here

In every case, the mean response of Multiple Events subjects was higher than those for the two lottery groups (which were quite similar). Given the similar probability assessments of these groups, it appears as though the experience of thinking about the events initially as lotteries somehow makes (lottery) subjects more risk averse. Perhaps it makes them more sensitive to budgetary constraints or less sanguine about the chances of winning (even if they do assign similar probabilities).

In general, subjects' estimates were less than the (subjective) expected value of the lotteries based on their subjective probabilities (indicating risk aversion) and greater than the (objective) expected value based on the computed probabilities (indicating risk seeking). These estimates were roughly proportionate to the subjective probabilities. They were essentially unrelated to the decomposition.

Discussion. Subjects' willingness to pay to play these lotteries was closely related to their estimates of success, suggesting that their probability estimates were relatively meaningful to them. On the average, they were quite risk averse, especially for the noncoherent subjects (whose probabilities were higher than the overall averages in the table).

Single-Event Probabilities

Results. Subjects in the three groups that considered multiple spinners also judged the probabilities associated with 12 single-spin-

The amounts subjects were willing to pay to play these single-event lotteries increased monotonically with their assessments of the likelihood of winning them. Interpreting their absolute assessments literally shows a high degree of risk aversion. For all but the smallest probabilities, subjects were willing to pay about one-quarter of the lotteries' expected value.

Were all 12 single spinners to be spun, subjects in all three groups expected approximately four successes. This expectation is quite similar to the 3.6 successes obtained by adding the probabilities assigned to the individual events. Apparently, subjects had a better feeling for how these medium-range probabilities "added up" than they had for the smaller probabilities associated with the multiple-spinner events (where they overestimated the number of successes). On the other hand, subjects failed to realize the extent to which these individually uncertain events provided a cumulative certainty of having at least one success. Their mean probability was 0.67, compared with the 0.996 computed from their single-spinner assessments.

Discussion. The orderliness of subjects' responses to the single-stimuli events showed that they had little difficulty with the concepts of spinners, probabilities, or lotteries. The precision of their probability assessments suggests that single spinners may be an effective way of communicating probabilities (although not necessarily any more effective than simply giving numerical values). Most significantly, both coherent and noncoherent subjects were better able to assess the small probabilities of unlikely events when those were presented as single events than as multiple events, even though this meant judging very small sectors of spinners.

Creating Lotteries

Results

The top rows of Table 7 show the mean probabilities appearing on each of the spinners created by subjects instructed to produce the target probabilities at the top. The "fair" probabilities are the ones that they initially provided; the "attractive" ones were produced subsequently in response to the request to make the lottery seem more attractive, while holding its probability constant.

With the fair lotteries, the probabilities assigned to the three spinners were quite similar, whereas with the attractive lottery, the initial spinner had a higher probability associated with it. Thus, in the aggregate, these subjects believed that other individuals' probability assessments were anchored on the first spinner that they saw (so that making it more likely made the conjunction seem more likely). Consistent with this serial differentiation of the spinners, there was greater variance among the attractive sets. One measure of the variance is the range of the standard (z) scores associated with the three spinners in a set. With 0.250 as a target probability, the means of these z-scores were 1.22 and 1.55 for fair and attractive sets, respectively. For the other target probabilities, the respective means were: 0.125 (1.50, 1.78), 0.042 (1.74, 1.69), 0.015 (1.44, 1.69), 0.002 (0.91, 1.13). For the three larger target probabilities, there was no tendency for attractiveness to be achieved by increasing the actual probability of the event. With the two lower targets, the attractive version of each spinner had a higher probability than its fair counterpart.[11]

These differences between the fair and attractive probabilities were far outshadowed by the differences between each and the target probabilities that subjects were asked to create. The middle section of the table shows three measures of the conjoint probabilities associated with the subject-created spinners: (a) the arithmetic mean of the products of the three spinners produced by individual subjects, (b) the geometric mean of the products of individual subjects' spinners (which reduces the influence of subjects with extreme responses, (c) the product of the mean probabilities associated with the spinners (as appear in the table above). By any measure, subjects created probabilities that were much too small. The bottom portion of the table shows the ratio of their created conjoint probabilities to the intended ones. These ratios are always greater than one and range up to enormous deviations with the smallest target probabilities.

<u>Discussion</u>

Despite the difference in procedures, having subjects create lotteries revealed the same misunderstanding of conjoint probabilities as did having them just judge probabilities. In both conditions, they failed to realize just how unlikely the conjunction of a set of uncertain events would be. Where comparisons are possible, even the magnitude of the bias is similar. Subjects in the other groups saw three spinners of 0.125 (target .002, Decomp A) and thought that they represented a conjoint probability of 0.08;[12] subjects here created three spinners with roughly those probabilities in order to produce (what they thought was) a probability of 0.042. Subjects seeing three spinners of 0.25 thought that their conjoint probability was 0.13; subjects created three spinners of roughly 0.2 in order to achieve a conjoint probability of 0.125. Subjects seeing three 0.50 spinners thought that they amounted to 0.34; subjects created three of approximately 0.40 to achieve 0.25. Moreover, these values are also similar to those observed in other studies. Cohen et al. (1971) found that subjects estimated the conjoint probability of three 0.5 events to be 0.30 (rather than 0.125) and the conjoint probability of three 0.25 events to be 0.11 (rather than 0.016). Bar-Hillel (1973) found a probability of 0.15 associated with three events of 0.25.

Hazardous Wastes

In most respects, responses to the two hazardous waste conditions were strikingly similar to one another. The absolute differences between the mean values assigned by all subjects to the conjunctions were 0.005, .004, and .019 for stimulus probabilities of .002, .016, and .125, respectively. There was no significant form effect for the conditions as a whole, or for either the coherent or noncoherent subsets. Nor was Condition involved in any significant interactions. Thus, there was no evidence of wishful thinking, or any related processes, evoked by considering the risks as personal rather than as the lot of an anonymous individual. Table 8 presents mean values for the pooled conditions. The corresponding ANOVAs show a significant effect for Probability. There was, however, no significant effect for Decomposition anywhere. Thus the tendency observed earlier for the no variance decomposition to elicit higher probabilities was absent here.

Insert Table 8 about here

The pooled results here are fairly similar to those found in the corresponding bottom sections of Tables 3a–3c, pooling the results of all subjects viewing these stimuli in more benign contexts. The percent-age of coherent and noncoherent subjects here are also within the ranges observed earlier, being 49.6% and 23.5%, respectively. Although these values are toward the high performance ends of those ranges, established by the basic group, they do not seem very impressive given the realism of the events involved and the weakness of the definition of coherence.

In other respects as well, these subjects resembled their predecessors, with the greatest similarities being to the basic group. The expected number of successes was higher for coherent subjects (3.49) than noncoherent ones (5.77), reflecting consistency with the general ordering of their probabilities although much above the actual expectation in both cases. Their estimates of the probability of at least one win also showed this differentiation and produced values akin to the higher performance end of the range observed above; the means were .446 and .668 for the coherent and noncoherent groups, respectively. Thus, this concretization evoked no better performance than did the minimal context of the basic condition.

Mixed Events

The 43 subjects who completed the mixed events forms provided a wide variety of events in the prescribed categories and with the prescribed probabilities. Although the accuracy of these predictions might be of interest for other purposes (e.g., Fischhoff & MacGregor, 1982), for present purposes all that matters is that they seem right to the individuals producing them. The procedure is being tested as a way to communicate very low probabilities per se, not the very low probabilities of particular events.

All in all, it was not very effective in that role. In 42.4% of their assessments, subjects assigned higher probabilities to the conjunction than to the least likely of the constituent events. Only 32.6% provided coherent responses to all four questions, whereas 37.2% were always noncoherent. These percentages are within the ranges observed with the previous conditions, and toward their low performance end. Violation levels were quite high for each of the four questions, with no clear relationship to the size of the conjoint probability. Table 9 presents mean responses are presented for all these subjects along with means for all spinners subjects receiving either the same decomposition (C) or any decomposition. In this respect, too, the numbers are quite comparable. As before, however, these overall means represent the responses of few individual subjects. Rather, they are the weighted means of subjects showing varying degrees of coherence who respond similarly and are present in similar proportions in all of these studies.

Insert Table 9 about here

All subjects completing the present forms had, earlier in the same experimental session, completed one of the hazardous waste forms. Across subjects, there was a correlation of .58 between the proportions of noncoherent responses in the two conditions, indicating a consistent individual tendency to follow or violate the conjunction rule.

General Discussion

The predominant conclusion of these experiments is that many people have difficulty with the notion of conjunction, which emerges in group statistics as consistent exaggeration of the probability of conjunctions. This occurs whether the events are described numerically or with graphic spinners, whether the events have no attendant consequences or are described as having significant consequences (either positive, a lottery, or negative, a health risk), whether the events occur simultaneously or sequentially, whether subjectively or objectively measured values are used for the constituent events, and whether the events are created by the experimenter or by the subject. In overall means, the magnitude of the overestimation varied from a factor of two for intermediate probabilities to a factor of 10 to 100 for very small probabilities. Even the magnitude of the bias was similar across conditions here and with results observed elsewhere. A complementary process is found in subjects' underestimation of the probability that at least one of the set of 12 uncertain events will occur, both for the single-stimulus and the multiple-stimulus events. This can be seen as overestimation of the probability of the conjunction of a series of 12 non-occurrences.

At the individual subject level, fewer than a quarter of subjects passed the (seemingly minimal) coherence test of never giving a conjoint probability greater than the smallest single-event probability in a set. Although the level of coherence varied some across conditions, there was no clear trend. With the 12-event conditions, the highest levels were observed with the basic and hazardous waste stimuli, which might be seen as most and least abstract, respectively. The correlation between the proportions of coherent responses among subjects who participated in both the hazardous wastes and the mixed events forms suggest that it may be more productive to see individual, rather than situational, correlates of coherence. With the present seemingly straightforward questions and presentations, the rates were reasonably similar in all conditions. Within each condition, secondary analysis revealed a consistent difference with regard to the one demographic variable available: the percentage of female subjects was always higher in the noncoherent subpopulation than in the coherent subpopulation.[13] As there is no reason to assume that the life experiences or thought processes of the sexes differ in this regard, a plausible attribution is that this difference reflects the relative amount of education in analytical subjects such as mathematics, physics, and engineering. Even if the conjunction rule itself is not taught, such training may facilitate discerning the logical structure of tasks, such as the nominal implication of the conjunction rule used here as a criterion of coherence.

Among the coherent subjects, there was a modest tendency with the spinners tasks to attribute higher probabilities to the no variance decompositions. Apparently, a concomitant of their general sophistication was a particular sensitivity to the presence of very small probabilities in a set (as was found with the higher variance stimuli).[14] Where there was variance in a set, the smallest probability stimulus was always presented first, an arrangement that the creating-lotteries subjects seemed to believe would reduce its attractiveness.

The stakes associated with an event had no obvious effect on the probability assigned to it. The lack of a wishful thinking effect here might be contrasted with the common belief in its existence and its observation in correlational studies. The latter often show higher probabilities associated with more favorable events. In correlational designs, however, it is difficult to disentangle biases in the recruitment of information from biases in its interpretation. The present ambiguous stimuli provided identical information to all subjects, varying only the consequences. It would be instructive to repeat this manipulation with real, rather than hypothetical stakes.

Within this overall picture of problematic responses, there were also some signs of appropriate sensitivity. Subjects' probability assessments were strongly correlated with the presented values even if they different greatly from their absolute values. At least with the single-stimulus events, their estimates of the number of "successes" were consistent with their subjective probabilities. Similarly consistent were the amounts that subjects were willing to pay to play these gambles (showing a general tendency toward risk aversion when evaluated in terms of subjective probabilities). Their probability assessments for the single-stimulus events were quite accurate, indicating that subjects had little difficulty with the concepts of simple probability, lotteries, and spinners. Whether such performance is reassuring or the least that one would expect from people able to survive in the modern world is a topic for discussion.

In addition to the theoretical insight that they provide into judgmental processes, these results suggest one strong practical suggestion:

presenting low probability events as the conjunction of more likely events is not a good way to communicate information about them. It is preferable to find some way to represent the low probability itself. Where both conjunctions and summary probabilities are provided (e.g., Smith et al., 1985), many subjects are likely to be surprised by the contrast between what they would expect the probabilities to be (given the constituent events) and what the presenter claims that they are. It is unclear whether recipients would respond to this contrast by rejecting one of the two values, by getting confused, or by achieving some deeper understanding of conjunction that allows them to reconcile the expected and presented values. Where the constituent events are drawn from subjects' lives (as was the case directly with the mixed events condition and as might be the case with the hazardous wastes stimuli), any misperception of those events probabilities would only complicate matters. In individual cases, the result might be countervailing biases. However, it would take a large act of faith to build on that hope.

A more realistic hope is being able to add the conjunction rule to people's repetoire of statistical intuitions (e.g., Nisbett, Krantz, Jepson & Kunda, 1983). What would seem to be needed here is education that helps one to discern the logical (or statistical) structure of diverse situations (Beyth-Marom, Dekel, Gombo & Shaked, 1985). Attempts to achieve better performance without conferring some deeper understanding have not proven very successful (Fischhoff, 1982; Fischhoff & Bar-Hillel, 1984). Attempts to teach conjunction quickly in the course of presenting information about specific low probability events risk confusing people about the task at hand without effectively enabling them to cope with future tasks.

Footnotes

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1. Candidate terms might be "disconsolate thinking," "depressive thinking," "wishless thinking," "hopeless thinking," and "dreadful thinking."

2. The artificiality of the events in these studies suggests that this bias arises without the sort of rich, causal interpretation underlying the "conjunction fallacy" described by Tversky & Kahneman (1983).

3. For the spinners groups, our standard of comparison for the probability of the least likely event was the measured area of the black sector on the spinners. In order to accommodate any problems arising from subjects' inability to estimate the area of sectors, estimates as much as 0.02 more than the smallest spinner's probability were not considered as violations. This is quite a lenient criterion, insofar as the other two spinners in each set always had probabilities that were obviously less than 1.0.

4. Within all three tables, there was a modest Form X Probability interaction, which reached significance in the coherent subject and

pooled results. It seems, in part, attributable to a restriction of the range of responses with the multiple-events form. Although this might suggest that the numerical presentation reduced the discriminability of the different probabilities, the effect and evidence are too weak to do more than raise that as a suggestion. For the pooled results, $F = 2.84$ ($df = 6,400$; $p < .01$); for the coherent subjects, $F = 2.99$ ($df = 6,174$; $p < .01$); for the noncoherent subjects, $F = 1.65$ ($df = 6,104$; $p < .15$).

5. After being diluted by the absence of an effect among the noncoherent subjects ($F = 2.37$; $df = 3,156$; $p > .05$), this emerged as a weakly significant effect in the pooled results ($F = 3.65$; $df = 3,600$; $p < .02$). Given the noisiness of these responses, no additional interpretation seems appropriate.

6. One possible interpretation (suggested by Lita Furby) of the decomposition effect with this group is that coherent subjects understand the limiting role of the smallest probability in the set and use it as an anchor, adjusting their overall estimate downward to accommodate the two other events. As elsewhere (e.g., Tversky & Kahneman, 1974), however, the adjustment process proves inadequate, leaving their final estimates too close to the anchor, whose value increases as the variance of the decomposition decreases.

7. This result provides some small reason for pessimism regarding another possible approach to making small probabilities more comprehensible, namely, representing them by the union of less likely events. Here, the whole is seen as more likely than the sum of its parts.

8. The mean correlation, using Fisher's z transformation, was 0.116 across the four coherent subgroups and 0.273 across the noncoher-

ent subgroups.

9. Mean correlation for the four coherent subjects was 0.132; for the noncoherent groups it was 0.546, a value that was bouyed by inexplicably large correlations in two small groups (n = 8,12).

10. Subjects in the basic group were also asked about their willingness to pay to play the three-spinner events. However, these questions came after they had made probability judgments for the single spinners and many subjects seemed to have been confused regarding which set of 12 events they should be judging. Rather than attempt to discern which subjects were referring to which stimuli, their responses will be ignored.

11. Thus, although ANOVA yielded no overall Task (fair vs. attractive) effect, there was a significant (alpha = 0.005) Target Probability X Task interaction for the sum of the probabilities in Table 7, as well as for each measure of conjoint probability.

12. This is the mean value for all subjects. It is bracketed by the 0.02 for coherent subjects and 0.17 for noncoherent ones. Analogous bracketing values may be found for the other values cited in the text.

13. Over all groups, women constituted 72.1% of the non-coherent subjects, compared with only 48.8% of the coherent ones (z = 3.77).

14. See also Footnote 6.

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Table 1

Stimuli

Decomposition	.002 Spinner 1	2	3	.016 Spinner 1	2	3	.125 Spinner 1	2	3
A (no variance)	.125	.125	.125	.250	.250	.250	.500	.500	.500
		(.375, 11)			(.750, 1)			(1.50, 2)	
B (low variance)	.044	.210	.210	.125	.354	.354	.354	.595	.595
		(.464, 12)			(.837, 4)			(1.54, 9)	
C (mid. var.)	.044	.125	.354	.125	.250	.500	.354	.500	.707
		(.523, 8)			(.875, 7)			(1.56, 10)	
D (high variance)	.016	.210	.595	.063	.354	.707	.250	.595	.841
		(.820, 5)			(1.13, 3)			(1.69, 6)	

Note: Numbers in parentheses refer to the sum of the probabilities in each set and the order in which the set appeared in all questionnaires.

Table 2

Subjects Violating Conjunction Rule

Different Numbers of Times

Group	Number of Violations													
	0	1	2	3	4	5	6	7	8	9	10	11	12	n
basic	18	11	5	7	0	4	4	2	1	2	2	2	4	62
lottery	8	8	3	3	2	1	1	1	3	3	5	8	7	53
sequential lottery	15	6	5	7	2	4	4	2	1	1	3	5	4	59
multiple events	6	5	5	1	1	1	1	0	0	0	2	3	11	36
total	47	30	18	18	5	10	10	5	5	6	12	18	26	210

Table 3a
Mean Responses
All subjects providing complete responses
(probability x 1000)

Group	target probability	Decomposition A	B	C	D	All
basic (N = 60)	002	080	050	048	063	060
	016	113	123	136	087	115
	125	313	306	303	252	293
	All	168	159	163	134	156
Lottery (N = 52)	002	088	075	094	088	086
	016	159	138	185	180	165
	125	397	352	401	336	371
	All	215	188	226	201	208
sequential lottery (N = 56)	002	069	064	065	058	064
	016	112	128	125	089	163
	125	322	308	442	250	334
	All	168	167	210	133	169
multiple events (N = 36)	002	103	102	105	147	114
	016	169	172	151	151	161
	125	335	275	266	256	283
	All	202	183	174	185	186
All (N = 204)	002	083	069	075	083	077
	016	134	137	148	123	135
	125	341	313	360	274	322
	All	186	173	194	160	178

Table 3b
Mean Responses
All coherent subjects providing complete responses
(probability x 1000)

Group	target probability	Decomposition A	B	C	D	All
basic (N = 32)	002	016	007	009	005	010
	016	060	030	048	018	039
	125	213	161	167	111	163
	All	097	066	075	045	070
lottery (N = 19)	002	015	005	008	009	009
	016	033	022	030	027	041
	125	263	132	146	097	159
	All	121	053	061	044	070
sequential lottery (N = 24)	002	041	039	027	003	028
	016	029	031	036	019	029
	125	178	144	143	120	146
	All	083	072	068	047	068
multiple events (N = 16)	002	011	011	019	023	016
	016	041	078	030	059	052
	125	144	099	091	108	111
	All	066	063	047	063	060
all (N = 91)	002	022	016	015	008	015
	016	053	037	038	027	039
	125	202	140	143	110	149
	All	093	064	066	049	068

Table 3c
Mean Response
All noncoherent subjects providing complete responses
(probability x 1000)

Group	target probability	Decomposition A	B	C	D	All
basic (N = 8)	002	187	186	247	345	241
	016	249	439	461	333	371
	125	551	672	594	599	604
	All	329	432	434	426	405
lottery (N = 20)	002	181	165	203	214	191
	016	226	274	316	380	289
	125	520	527	571	574	548
	All	309	322	363	389	346
sequential lottery (N = 12)	002	169	112	161	194	157
	016	296	304	278	290	282
	125	500	518	544	393	489
	All	317	311	328	213	312
multiple events (N = 16)	002	168	170	178	290	202
	016	289	279	296	271	284
	125	547	492	489	450	495
	All	335	314	321	337	327
All (N = 56)	002	174	158	193	260	194
	016	262	306	323	323	303
	125	528	536	545	503	528
	All	321	333	354	359	342

Table 4
Mean Judgments for Compound Events

Group		Expected Number of Successes			Probability of at Least One Success		
		Direct	Computed	p<	Direct	Computed	p<
basic	all	3.571	1.889	0.0001	0.535	0.741	0.0001
	coherent	2.767	0.831	0.0001	0.464	0.571	ns
	noncoherent	6.250	4.861	ns	0.616	0.997	0.05
lottery	all	3.551	2.534	0.0001	0.351	0.717	0.0001
	coherent	2.889	0.850	0.0001	0.231	0.456	0.05
	noncoherent	4.850	4.151	ns	0.483	0.972	0.0001
sequential	all	3.093	1.996	0.0001	0.444	0.785	0.0001
lottery	coherent	2.348	0.751	0.0001	0.278	0.527	0.001
	noncoherent	3.727	3.751	ns	0.548	0.986	0.001
multiple	all	3.129	2.234	ns	0.523	0.835	0.0001
events	coherent	2.143	0.621	ns	0.379	0.592	0.01
	noncoherent	4.000	3.900	ns	0.703	0.992	0.0001

Note: Computed values are derived from probabilities assigned to individual stimuli.

Table 5

Willingness to Pay to Play Three-Spinner Events
All Subjects
(in $)

Group	Probability computed	Probability subjective[a]		Decomposition A	B	C	D	All
Lottery	0.002	0.086		0.76	0.63	0.99	1.97	0.74
	0.016	0.165		2.62	2.52	2.40	2.19	1.65
	0.125	0.391		11.62	5.60	7.01	6.55	5.22
			All	5.00	2.92	3.47	3.57	2.54
Sequential Lottery	0.002	0.064		0.85	1.00	1.18	0.81	0.65
	0.016	0.163		2.23	2.09	2.13	1.93	1.43
	0.125	0.334		7.68	6.20	6.13	4.58	4.17
			All	3.59	3.10	3.15	2.44	2.08
Multiple Events	0.002	0.116		4.43	4.38	3.02	4.02	2.66
	0.016	0.161		8.63	5.57	5.44	9.60	4.90
	0.125	0.283		15.92	10.30	11.71	9.39	7.95
			All	9.66	6.75	6.72	7.67	5.17

a These are the mean probabilities from Table 3a.

Table 6

Mean Responses to Single-Event Stimuli

(probabilities multiplied by 1000)

Group	Probability of Spinner[a]											
	<u>002</u>	<u>016</u>	044	063	<u>125</u>	210	250	354	500	595	707	841
Order	12	3	6	8	5	4	9	10	7	2	1	11
<u>Probability of Win</u>												
Basic	007	021	052	092	123	183	243	350	485	595	680	805
Lottery	008	031	067	082	128	188	237	333	492	585	687	800
Sequential Lottery	010	028	061	090	115	171	235	320	499	577	679	805
<u>Willing to Pay ($s)</u>												
Lottery	.12	.34	.67	1.2	2.5	3.6	4.4	6.2	11.	12.	17.	20.
Sequential Lottery	.12	.57	.70	1.1	2.4	2.8	5.1	6.8	14.	15.	18.	27.

[a] Underlining indicates target probabilities in multiple-stimuli events. Of these, only 0.125 was used as a spinner in the multiple-stimuli events.

Table 7

Creating Lotteries
(probabilities multiplied by 1000)

	Target Probability									
	250		125		42		15		2	
	fair	attractive	fair	attractive	fair	attractive	fair	attractive	fair	attractive
Single Probabilities										
Spinner 1	389	394	236	254	103	143	67	107	30	67
Spinner 2	375	284	197	208	147	126	77	90	32	61
Spinner 3	386	311	206	167	108	118	63	92	28	57
Conjoint Probabilities										
mean of products	98.0	51.0	26.0	18.0	5.0	7.0	1.9	2.1	0.3	0.2
geometric mean of products	37.0	23.0	4.0	4.0	0.4	0.4	0.04	0.06	0.001	0.003
product of means	56.0	36.0	9.6	0.8	1.6	2.1	0.3	0.9	0.03	0.02
Ratio of Target to Created Probabilities										
mean of products	2.0	4.9	4.8	6.9	8.4	6.0	7.9	7.1	6.7	11.1
geometric mean of products	6.8	10.9	31.3	31.3	105.0	105.0	375.0	250.0	2000.0	667.0
product of means	4.7	6.9	13.0	14.2	26.3	20.0	50.0	16.7	74.1	87.0

Table 8
Mean Responses
Hazardous Waste Forms--Pooled
(probability x 1000)

Group	target probability	Decomposition A	B	C	D	All
All subject (N = 115)	002	063	071	094	079	077
	016	142	147	155	120	141
	125	285	298	308	295	297
	All	163	172	186	165	172
Coherent (N = 57)	002	012	009	019	017	014
	016	027	034	047	024	033
	125	145	132	137	138	138
	All	061	058	068	060	062
Noncoherent (N = 27)	002	161	179	214	242	199
	016	343	320	359	326	337
	125	518	540	564	552	552
	All	340	346	379	384	362

Table 9

Consistency of Estimates for Subjects and Experimenter-Produced Conjunctions

	MIXED EVENTS						SPINNERS		
Event	Constituent Events			Conjoint Probability	Mean Response	Percent Violations	Comparison Stimulus Prob.	Decomposition C	Decomposition All
1	.10	.25	.50	.013	.177	48.8	.016	.148	.135
2	.05	.10	.35	.002	.128	39.5	.002	.075	.077
3	.35	.50	.75	.131	.320	37.2	.125	.360	.322
4	.05	.35	.75	.013	.184	46.5	.016	.148	.135

Figure Caption

1. Stimuli used for creating compound events.