

The Sound of Ions: Using Trapped Atomic Ion Motion for Quantum Computation
and Sensing

by

JEREMY M. METZNER

A dissertation accepted and approved in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

in Physics

Dissertation Committee:

Daniel Steck

Chair

David Allcock

Advisor & Core member

Tien-Tien Yu

Core Member

George Nazin

Institutional Representative

University of Oregon
Spring 2024

© 2024 Jeremy M. Metzner
All rights reserved.

DISSERTATION ABSTRACT

Jeremy M. Metzner

Doctor of Philosophy in Physics

Title: The Sound of Ions: Using Trapped Atomic Ion Motion for Quantum Computation and Sensing

Encoding qubits in the electronic states of atoms has enabled the ability to perform computations, sense the environment, and gain a deeper understanding of other physical systems through quantum simulation. In the trapped ion platform, each of these applications is made possible, or enhanced by coupling the qubit to the motion of the trapped ions. The motion can be used to do more than just mediate interactions with qubits and is itself a resource for computation, sensing and simulation. The work presented here focuses on methods for manipulation and entanglement of trapped ion motional states in a spin-independent way while retaining the spin to enable measurement of the quantum state of the motion. We have shown the ability to use a set of spin-independent operations including displacement, beam splitter, squeezing and two-mode squeezing, to sense the phase of the oscillator states with precision exceeding the standard quantum limit (SQL) by up to 5.9 dB and approaching the ultimate Cramér-Rao bound. With qubits still generally required for utilizing the motional states for quantum information experiments, qubit measurement makes it difficult to preserve any motional state during state detection. We have developed the technique of encoding both states in a metastable manifold which could enable methods to preserve particular motional states of longer-chains during state detection. We present preliminary results demonstrating this preservation of motional states using a mixed atomic-species trap at Lincoln Lab. The metastable encoding also provides the unique ability to

engineer and explore non-Hermitian qubit Hamiltonians, where we have shown the ability to break the quantum speed limit. This dissertation includes co-authored material that is pending publication.

ACKNOWLEDGEMENTS

I would like to start by thanking my advisor David Allcock for allowing me to be a part of growing a new lab, for sharing his organizational skills and his great depth of knowledge of the trapped ion field. I really appreciate all the time and effort spent reviewing my work and making sure it was presentable. I also thank Dave Wineland for being a co-advisor, always providing feedback on work and always taking an interest what was going on in the lab. A special thanks to my starting lab mates Alex Quinn and Daniel Moore for sharing the load of all of those packages and always providing stimulating conversations, especially in the early days trapped in B61 together. I appreciate all of the members of the lab that I have worked with, Sean Brudney, Gabe Gregory, Evan Ritchie, Oliver Miller and Vikram Sandhu for being a part of the process and always being an ear for my complaints. Thanks also to outside collaborators Yogesh Joglekar, Shaun Burd and Colin Bruzewicz for sharing their knowledge. Finally I want to thank my family and friends. I want to thank Claire Albrecht and Matt Ball for helping me to survive my first year of classes, being my community in the department and always keeping me from getting ahead of myself. A special shoutout to my wife Sahlea Tubbeh, who wasn't my wife the whole time I was working on my PhD, but was my rock and source of happiness the whole time. Thanks to my parents and in-laws for always taking the time to come visit and share in the magic of Oregon and for always making an effort to understand what I work on.

To my wife Sahlea. . .

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	16
1.1. Trapped ion harmonic motion	16
1.2. Harmonic oscillators and qubits	17
1.3. Effects of qubit measurements	18
1.4. Interferometry with trapped ions	20
1.5. Thesis layout	22
II. QUANTUM HARMONIC OSCILLATORS	24
2.1. Fock basis ladder	24
2.2. Wigner function representation	27
2.3. Oscillator modes with ion chains	28
2.3.1. Motional modes	29
2.3.2. Mixed mass crystals	30
2.4. Driven harmonic oscillators and parametric modulation	32
2.4.1. Multi-mode operators	34
2.5. Gaussian states	36
2.5.1. Displacement and coherent states	37
2.5.2. Squeezing and single mode squeezed states	38
2.5.3. Phase operator	40
2.5.4. Beam splitter	42
2.5.5. Two-mode squeezing and two mode squeezed vacuum	44
2.5.6. Thermal states	46
2.6. Non-Gaussian states	47

Chapter	Page
2.6.1. Fock states	48
2.6.2. Cross-Kerr interaction	48
2.6.3. Tri-squeezing	53
III. ATOMIC SPIN QUBITS IN OSCILLATORS	55
3.1. Calcium-40	55
3.2. Qubit encodings	56
3.2.1. Ground-state qubits	56
3.2.2. Optical qubits	57
3.2.3. Metastable qubits	58
3.3. Zeeman qubits in the $D_{5/2}$ Manifold	58
3.4. State preparation	59
3.5. Single qubit rotations	61
3.5.1. RF magnetic fields	61
3.5.2. Raman transitions	63
3.6. Spin-motion coupling	65
3.6.1. Lamb-Dicke regime	65
3.6.2. Carrier	66
3.6.3. Red sideband (RSB)	66
3.6.4. Blue sideband (BSB)	67
3.7. State detection	67
3.7.1. Mode thermalization	69
IV. EXPERIMENTAL SETUP	71
4.1. Rod trap	71
4.2. RF electronics	74
4.2.1. Helical resonator	74

Chapter	Page
4.2.2. Squareatron amplitude stabilization	75
4.2.3. Filtering AM Noise (Squareatron 5000)	76
4.2.4. Tuning board	76
4.2.5. Amplifier	77
4.2.6. Temperature stabilization	78
4.3. Laser systems	79
4.3.1. Laser rack	80
4.3.2. Photoionization	82
4.3.3. Locking lasers to a wavemeter	85
4.3.4. Raman laser system	88
4.4. Motional state cooling	88
4.4.1. Doppler cooling	89
4.4.2. EIT cooling	91
4.4.3. Resolved sideband cooling	94
4.5. Electronic drives	96
4.5.1. Resonant drive	96
4.5.2. Parametric drive resonator circuit	96
4.5.3. Drive filtering	97
4.5.4. Pulse shaping	97
V. NON-HERMITIAN HAMILTONIANS	99
5.1. Unitary dynamics and quantum speed limit	99
5.2. Lindblad model and master equation solver	100
5.3. Post-selection and non-Hermitian Hamiltonians	102
5.4. Finding the exceptional point	106
5.5. Legget-garg inequality	107

Chapter	Page
5.6. Violation of quantum speed limit	109
VI. LASER-FREE CONTROL OF TRAPPED-ION QUANTUM HARMONIC OSCILLATORS	114
6.1. State characterization	114
6.2. Coherent states	115
6.3. Thermal states	116
6.4. Squeezed states	117
6.5. Two-mode squeezed states	118
6.6. Beam splitter	119
6.7. Off-resonant motional coupling	120
VII. PROTECTED MODE INVESTIGATION FOR MID- CIRCUIT MEASUREMENT	122
7.1. Mode structure and protected modes	122
7.2. Surface electrode trap with mixed species ion chain	124
7.3. Heating rates	126
7.3.1. Sideband thermometry	126
7.4. Motional Fock state Ramsey interferometry	130
7.5. Perturbations to protection	131
7.5.1. Radiation pressure	132
7.5.2. Stray electric fields	133
VIII.SU(2) AND SU(1,1) INTERFEROMETRY FOR PRECISION PHASE MEASUREMENT	135
8.1. Verification of two-mode squeezed state with beam splitter	138
8.2. Cramer-Rao bounds	139
8.3. Theoretical bounds	144

Chapter	Page
IX. CONCLUSION AND OUTLOOK	149
9.1. Phase insensitive displacement amplification with two- mode squeezed states	149
9.1.1. Potential experimental implementation	151
9.2. Phase distributions	152
9.3. Correlation measurements below the $\mathcal{CH}$ bound	154
9.4. Continuous variable quantum information processing	155
APPENDIX: SQUAREATRON 5000 AND TUNING BOARD	157
REFERENCES CITED	162

LIST OF FIGURES

Figure	Page
1.1. Qubit and motion visualization	19
1.2. Interferometer geometries	22
2.1. Mixed mass motional modes	32
2.2. single mode Gaussian operations	42
2.3. Action of a beamsplitter	44
2.4. Two mode squeezed state visualization	46
2.5. Fock state Wigner functions	48
2.6. Kerr interaction resonances	51
2.7. Wigner functions of two modes after Kerr interaction	53
2.8. Wigner function of tri-squeezed state	54
3.1. $^{40}\text{Ca}^+$ level structure	56
3.2. Zeeman m qubit	59
3.3. Laser and RF m qubit operations	61
3.4. Raman transition theory	64
3.5. State detection count histogram	69
4.1. Render of our ion trap	72
4.2. Photograph of our trap	73
4.3. Squareatron AM noise measurement	77
4.4. RF chain temperature stabilization	79
4.5. Photograph of laser rack	81
4.6. Laser rack drawer	82
4.7. Rendering of photoionization board	84

Figure	Page
4.8. Image of the photoionization board	84
4.9. OSA board image	87
4.10. Raman beam delivery optics	89
4.12. EIT schematic and results	94
4.13. Parametric modulation setup	98
5.1. non-Hermitian Hamiltonian in $^{40}\text{Ca}^+$	103
5.2. non-Hermitian and Lindblad model comparison	105
5.3. K3 measurement results	108
5.4. Branching ratio effects on maximum K3	110
5.5. Breaking the quantum speed limit	111
6.1. Displaced state qubit probabilities	116
6.2. Squeezed state qubit probabilities	118
6.3. Two mode squeezed state qubit probabilities	119
6.4. Beamsplitter calibration	120
6.5. Off resonant motional effects on qubit probabilities	121
7.1. motional mode spectroscopy	124
7.2. Lincoln standard surface electrode trap	125
7.3. Multi ion sideband probabilities	129
7.4. two ion motional interferometry pulse sequence	131
7.5. Motional interferometry phase fringes	132
8.1. Motional interferometer experimental setup	137
8.2. Two mode squeezed state verification	139
8.3. Phase fringes of different motional state interferometers	141
8.4. Interferometer sensitivities	147
9.1. Single mode displacement amplification	151

Figure	Page
9.2. amplification phase sensitivity	152
A.1. Squareatron 5000 electronic schematic	158
A.2. Squareatron tuning board schematic	159
A.3. Squareatron and tuning board PCB render	160
A.4. Squareatron and tuning board image	161

LIST OF TABLES

Table	Page
1. two ion sideband qubit probabilities	127

CHAPTER I

INTRODUCTION

1.1 Trapped ion harmonic motion

Understanding the nature of harmonic oscillators is an essential part of understanding basic physics given that many physical systems can be modeled as harmonic oscillators (Cao & Voth, 1995) including the electromagnetic field (Bennett, Barlow, & Beige, 2015). Systems near a potential energy minimum can often be approximated by harmonic oscillations with an appropriate Taylor expansion about that minimum. This means many physical systems can be modeled as a harmonic oscillator. In the quantum regime, where we work near minimum energy states the harmonic approximation works very well for many systems and quantum harmonic oscillators are a part of many different platforms (J. Aasi et al. (Ligo Scientific Collaboration), 2013; Oliver et al., 2005; Regal, Teufel, & Lehnert, 2008). Independent of the platform, quantum harmonic oscillators are being used for sensing of weak signals (quantum metrology) (C. Caves, 1981; McCormick et al., 2019), mediating interactions with other systems (J. I. Cirac & Zoller, 1995; Schmidt-Kaler et al., 2003) and even for processing of quantum information (Flühmann et al., 2019; Lloyd & Braunstein, 1999). Further development of the techniques used to control these systems will likely lead to a deeper understanding of quantum mechanics and provide avenues for new applications of these technologies.

In this thesis I present work that extends the current capabilities of the control and readout of trapped ion oscillators. Trapped ions represent some of the most ideal harmonic oscillators given that they can be trapped in deep and highly harmonic potentials. Trapping an ion in vacuum provides exquisite isolation from the environment which enables ion oscillations with long quantum coherence

times. Significant work has been done to demonstrate the ability to cool and coherently control the ion motion (Meekhof, Monroe, King, Itano, & Wineland, 1996; Wineland, Drullinger, & Walls, 1978). The dynamics of these oscillator states can be controlled using laser-based methods, as well as with static and oscillating magnetic field gradients, where these techniques also effect the atomic internal electronic states (Leibfried et al., 2002). We develop techniques that involve modulation of trapping electric field gradient, which enables manipulation of these oscillator states (Heinzen & Wineland, 1990) in a way that doesn't effect the qubit. These techniques allow for the generation of different types of classical and quantum states that are useful for metrology and computation. Parametric modulation of the trapping potential is one of these techniques which allows for the ability to couple two distinct oscillator states of a single trapped-ion and generate entangled states of motion (Leibfried et al., 2002). Generation of entangled motional states and or qubits can provide a resource for metrology at the Heisenberg limit and is a necessary component in quantum computation. The techniques and interactions presented here are not uniquely possible with trapped-ions but leveraging some of the strengths of this platform can enable a more direct path to utilizing quantum harmonic oscillators.

1.2 Harmonic oscillators and qubits

Trapping ions in three dimensions means three harmonic oscillators per ion. With each ion comes a single qubit which can be coupled by utilizing the collective motion of any of the oscillator modes. The collective motion gives rise to discrete normal modes (James, 1998), which like single ion oscillators can be manipulated and used for metrology and information processing. The state of these oscillators are difficult to detect directly. The common method of detecting the motional states is to couple them to the ion's internal electronic states using

laser fields (J. I. Cirac & Zoller, 1995; Vogel & Filho, 1995). This technique is possible when a interaction field is applied with an effective wavelength that is not significantly greater than the size of the motional wavepacket (Meekhof et al., 1996). This can be done with laser fields or with strong magnetic field gradients (Mintert & Wunderlich, 2001; Ospelkaus et al., 2011). The simplest method for this coupling is to use a field that couples two levels of the ion’s electronic structure and is detuned by the motional frequency (J. Cirac, Parkins, Blatt, & Zoller, 1996; Meekhof et al., 1996). This requires a suitable choice of a two-level system chosen from a vast range of different energy levels. The initial (uncoupled) full state of the qubit and the oscillator can be written as

$$\psi = \left(\cos(\theta/2) |\downarrow\rangle + \sin(\theta/2) e^{i\phi} |\uparrow\rangle \right) \otimes \sum_{n=0}^{\infty} c_n |n\rangle, \quad (1.1)$$

where θ and ϕ are arbitrary real parameters, $|\downarrow\rangle, |\uparrow\rangle$ represent the two different energy levels of the qubit, and c_n are the amplitude of each oscillator Fock state $|n\rangle$. The qubits can be visualized with the Bloch sphere representation shown in Fig. 1.1, with the equally spaced levels of the Harmonic oscillator shown. There are several options when it comes to choosing two levels to encode a qubit in trapped-ions with frequency scales ranging from MHz to THz and different advantages to each encoding. The ability to take advantage of different encodings could help to scale quantum systems without incorporating additional physical systems (Allcock et al., 2021).

1.3 Effects of qubit measurements

Building a quantum computer or a quantum sensor requires incorporating quantum systems into larger devices which follow the rules of quantum mechanics but provide the ability to measure a quantum state and give a classical outcome (Nelsen & Chuang, 2001). This scheme requires the ability to change

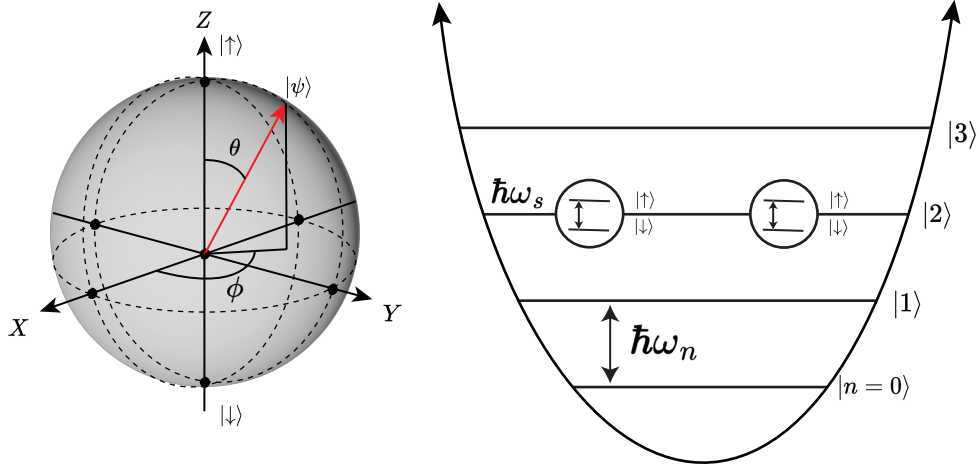


Figure 1.1. a) Bloch sphere representation of a qubit, where qubit populations are measured in the Z basis and angles θ and ϕ give the phase relationship between qubit states. b) Harmonic potential that confines two trapped ions represented with the circles and internal states. Energy eigenstates of the harmonic potential are evenly spaced levels.

from quantum coherences to classical measurement results by collapsing the wavefunction into a discrete output. This interface between quantum and classical, to our knowledge, means updating the description of the quantum state in a non-unitary way (Bassi, Lochan, Satin, Singh, & Ulbricht, 2013). The choice of when and where this update happens is critical for preserving a certain desired quantum state. In practice this is done by combining coherent control of qubit states and the ability to detect a subset of the full quantum system. The non-unitary measurement often leads to disturbances of quantum states which should be preserved and updated depending on the outcome of the measurement. There are techniques that enable the ability to preserve the quantum state stored in the electronic states of ions, by decoupling those states from the state detection process. The preservation of the quantum state of certain oscillators is more difficult. When using lasers to detect the state of a qubit, in the event of absorption and emission of photons, the position and momentum of the ion gets kicked

which drives the quantum state of the oscillators towards a classical mixed state (Wineland et al., 1978). Consequences of the geometrical structure of the normal modes of ion chains means detection of certain ions internal states can be done without perturbing every oscillator state of the ions. Understanding the limits of this geometrical protection will help to enable classical feedback schemes that could assist in the error correction schemes involving bosonic codes.

1.4 Interferometry with trapped ions

Some early physics experiments involved interfering two classical sources of light to make measurements about the universe, such as the existence of the ether or the speed of light (Kennard, 1922; Michelson & Morley, 1887). The field of light being described as a harmonic oscillator implies that harmonic oscillators, like trapped ions, can be used for interferometry. In modern times, interferometers are used to measure many physical properties like length, in order to detect gravitational wave detection (J. Aasi et al. (Ligo Scientific Collaboration), 2013), but can also be used to measure properties of the laser used, like wavelength and linewidth (Monchalín et al., 1981). We will use these same concepts to measure the phase of trapped ion harmonic oscillators. Using narrow band lasers enables sub-wavelength resolution of the parameter that is being measured. Using quantum resources can lead to improved sensitivities that can get below the shot-noise limit of the light source called the standard quantum limit (SQL) (Demkowicz-Dobrzański, Jarzyna, & Kołodyński, 2015). The basic strategy is to allow to one or more modes of electromagnetic fields, or oscillator wave packets, to interfere and to measure the interference pattern seen by some detector. The phase of the interacting fields can be varied experimentally, or by some physical phenomenon, which can be detected as a shift in the interference pattern. The interference pattern and the sensitivity to phase shifts depends on the physical implementation

of the interferometer. The earliest interferometer is the Michelson interferometer, used to verify the lack of an ether (Michelson & Morley, 1887). More recently, the Mach-Zehnder interferometer was developed which has two outputs which can be measured (Jakeman, Oliver, & Pike, 1975); the Michelson and Mach-Zehnder interferometer geometries are shown in Fig. 1.2 . The signal detected from these interferometers gets stronger when using stronger fields, however the scaling of the sensitivity, which depends on the signal to noise ratio, is set by the measurement technique being used. Quantum resources enable the ability to measure phases at the fundamental Heisenberg limit by reducing noise below the SQL. Entanglement of quantum systems can help in reaching the fundamental limits in sensitivity, which involves using multiple modes that are highly correlated, but reducing the quantum fluctuations in a single mode can also improve sensitivity. Having the ability to implement a variety of interaction elements and input states enables the ability to employ different interferometers for detection of different phases e.g. single mode, common mode, difference mode. These techniques, in combination with displacement detection are important for progress towards fully characterizing the state of the modes of interest, or characterizing operations that transform the state.

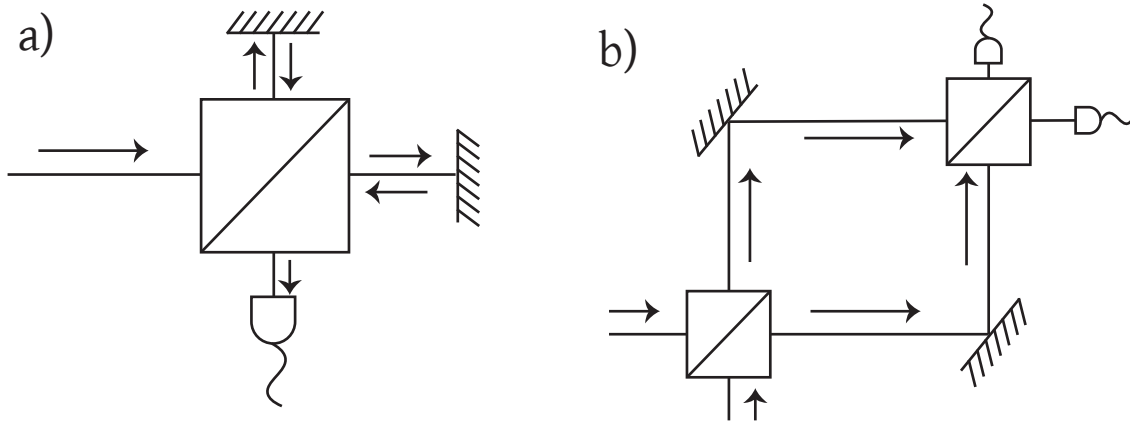


Figure 1.2. a) Michelson interferometer with a single beam splitter element and a single accessible output. b) the Mach-Zehnder interferometer which enables two inputs and the ability to measure two outputs with only one additional beam splitter element.

1.5 Thesis layout

The topics in this thesis cover development of techniques for control of trapped-ion mechanical oscillator states using laser-free methods as well as methods for preservation of oscillator states during readout. Metastable qubits provide the toolbox for measurement in this work and will be discussed in detail along with the experimental studies that are possible using these states.

Chapter 2 reviews the theory of quantum harmonic oscillators with a focus of parametric modulation to couple two independent modes of motion.

Chapter 3 describes the possible encoding schemes for qubits for atoms with a similar structure to Ca^+ . This includes techniques for coupling these two-level systems and performing state detection of the spin states or spin-motion coupling for detection of motional states.

Chapter 4 covers the experimental systems that are used throughout this work. This chapter covers a range of systems starting from how to produce ions from neutral atoms to how the state of the ion is detected.

Chapter 5 is a presentation of work to demonstrate non-Hermitian dynamics and the quantum limits that they can exceed. This work is made possible with the use of metastable qubits which enables a dissipative channel to engineer these interactions. This chapter includes work pending publication co-authored by A. Quinn, S. Brudney, I.D. Moore, J.E. Muldoon, S. Das, D.T.C Allcock and J.N. Joglekar.

Chapter 6 focuses on the mechanisms for generating and calibrating experiments using the motional state of the trapped ion. This chapter connects heavily to the theories in Chapter 2 to bring those motional operations and characterization to the experiment.

Chapter 7 discusses a method for preserving a particular motional state during detection of the internal qubit state. This work relies on the mode geometry of longer ion chains and is an exploration of the techniques to determine the protection that this particular mode has.

Chapter 8 is a presentation of $SU(2)$ and $SU(1,1)$ interferometers using the motional states of trapped ions. The limit of the sensitivity is discussed and presented alongside the experimental results. This chapter includes work pending publication that is co-authored by A. Quinn, S. Brudney, I.D. Moore, S.C. Burd, D.J Wineland and D.T.C Allcock.

Chapter 9 is the outlook for future work with the motional states of trapped ions. This includes discussion of potential experiments using two-mode squeezed states for quantum amplification and how to measure their correlations. An outlook for potential implementations of continuous variable quantum computing is included.

CHAPTER II

QUANTUM HARMONIC OSCILLATORS

Following from the realization that many physical systems can be modeled as harmonic oscillators, it is necessary to discuss the mathematical details of this model. Mechanical harmonic oscillators are defined beginning with a system that is subject to a restoring force that is proportional to its displacement x from equilibrium. This force is given by Hooke's law $F = -kx$ with spring constant k which corresponds to a potential $V(x) = \frac{1}{2}kx^2$. When considering the kinetic and potential energy of this system we have the full Hamiltonian for a one-dimensional harmonic oscillator

$$H = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2x^2, \quad (2.1)$$

where we introduce the oscillator frequency defined as $\omega = \sqrt{\frac{k}{m}}$ with oscillator mass m and momentum p . This Hamiltonian describes the total energy of the system and leads to oscillations of the position and momentum given by

$$x(t) = A \sin(\omega t + \phi), \quad (2.2)$$

$$p(t) = Am\omega \cos(\omega t + \phi), \quad (2.3)$$

where A, ϕ are the amplitude and phase of the oscillations and depend on the initial conditions. This classical description is very useful for analysis of classical systems, however a quantum treatment is necessary to get a deeper understanding these systems.

2.1 Fock basis ladder

Before we can describe the ladder of energy states available to the quantum harmonic oscillator we must first make the transformation to working with operators $\hat{x}_r$ and $\hat{p}_r$ (Gerry & Knight, 2005) instead of canonical variables of

position and momentum x, p . The full Hamiltonian is now written as

$$\hat{H} = \frac{\hat{p}_r^2}{2m} + \frac{1}{2}m\omega^2\hat{x}_r^2. \quad (2.4)$$

These canonical operators must satisfy the commutation relation $[\hat{x}_r, \hat{p}_r] = i\hbar$, where $\hbar$ is Planck's constant. We can now rewrite the position and momentum operators in terms of dimensionless constants and use quanta creation and annihilation operators $\hat{a}^\dagger, \hat{a}$ giving

$$\hat{x} \equiv \sqrt{\frac{m\omega}{2\hbar}}\hat{x}_r = \frac{1}{2}(\hat{a}^\dagger + \hat{a}) \quad (2.5)$$

and

$$\hat{p} \equiv \sqrt{\frac{1}{2m\omega\hbar}}\hat{p}_r = \frac{i}{2}(\hat{a}^\dagger - \hat{a}). \quad (2.6)$$

We will define $x_0 = \sqrt{\frac{\hbar}{2m}}$ which is the extent of the ground state wavefunction. Using these transformations we can now re-write the Hamiltonian in terms of its second-quantization form

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + 1/2). \quad (2.7)$$

At this point, it is useful to define the commutation relation for creation and annihilation operators as $[\hat{a}^\dagger, \hat{a}] = 1$. From this commutation relation we can define the number operator $\hat{N} = \hat{a}^\dagger\hat{a}$. This operator is connected to the Fock states where the eigenstates of $\hat{N}$ are defined to be Fock states $|n\rangle$ where $n \in \{0, 1, 2, 3, \dots\}$. The associated eigenvalues are given by $(n + 1/2)\hbar\omega$ and we make the observation of the ground state energy is $\frac{\hbar\omega}{2}$, which is an energy offset and has no effect on the dynamics. Gaining or losing n quanta of energy is accomplished with successive application of the creation (raises n) or annihilation (lowers n) operators onto the state $|n\rangle$. For a ground state to exist we terminate the ladder at $|0\rangle$ with any further application of the lowering operator resulting in zero. These relations are written out completely as

$$\hat{N} |n\rangle = n |n\rangle \quad (2.8)$$

$$\hat{a} |0\rangle = 0 \quad (2.9)$$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle \quad (2.10)$$

$$\hat{a} |n\rangle = \sqrt{n} |n-1\rangle. \quad (2.11)$$

Now that the Fock state has been defined we can also talk about the density matrix representation as this is useful to discuss certain states of the harmonic oscillator.

We will stick to pure states where we can define the density matrix as

$$\rho = |\psi\rangle \langle\psi|. \quad (2.12)$$

The Fock state ladder forms an orthonormal basis with $\langle n|m\rangle = \delta_{n,m}$ and $\sum_n |n\rangle \langle n| = \mathbb{1}$. The density operator representation of the oscillator is then written as

$$\rho = \sum_{n,m=0}^{\infty} b_{mn} |m\rangle \langle n| \quad \text{and} \quad b_{mn} = \langle m|\rho|n\rangle. \quad (2.13)$$

Before moving on to other representations of quantum harmonic oscillators it is worth mentioning consequences of multi-mode systems given that this is a focus of the work presented here. These relations are summed up by the fact that operators can only operate on a Hilbert space that contain the states that the operators belong to,

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij} \quad (2.14)$$

$$[\hat{a}_i, \hat{a}_j] = [\hat{a}_i^\dagger, \hat{a}_j^\dagger] = 0. \quad (2.15)$$

Finally, the creation and annihilation operators in a multi-dimensional space change the eigenvalues of the number operator,

$$N = \sum_i n_i = \sum_i \hat{a}_i^\dagger \hat{a}_i. \quad (2.16)$$

2.2 Wigner function representation

Moving away from the number basis we can begin to speak in more classical analogs of position and momentum. The position $|x\rangle$ and momentum $|p\rangle$ basis plays a key role in discussion of phase space and the oscillator state $|\psi\rangle$ can be written in this basis as

$$|\psi\rangle = \int_{-\infty}^{\infty} dx \psi(x) |x\rangle, \quad (2.17)$$

where $\psi(x)$ is a complex wavefunction that gives weight to each position eigenstate and can be written as $\psi(x) = \langle x|\psi\rangle$. The position and momentum representation are connected by a Fourier transform given by

$$\phi(p) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx e^{-2ixp} \psi(x). \quad (2.18)$$

From this relation we can begin to see naturally how the Heisenberg uncertainty principal arises. If we were to consider a particle that has a position that is perfectly well defined given by a delta function $\delta(x)$ then when we take the Fourier transform we get

$$\mathcal{F}(\delta(x)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dx \delta(x) e^{i\omega x} = \frac{1}{2\pi}, \quad (2.19)$$

which is a constant function spread across all momentum space evenly. With this in mind, we define the Heisenberg uncertainty relation for position and momentum as

$$\Delta x \Delta p \geq \frac{1}{4} |\langle [\hat{x}, \hat{p}] \rangle|^2 = \frac{\hbar^2}{4}, \quad (2.20)$$

where $\Delta x = \langle \hat{x} - \langle \hat{x} \rangle^2 \rangle$ is the variance of the position. We will now define the Wigner function (Wigner, 1932) for an arbitrary density matrix $\hat{\rho}$

$$W(x, p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \left\langle x + \frac{1}{2}q \left| \hat{\rho} \right| x - \frac{1}{2}q \right\rangle e^{ipq/\hbar} dq, \quad (2.21)$$

where $|x \pm \frac{1}{2}q\rangle$ are eigenstates of the position operator. It is important to point out that this is a quasi-probability distribution (Gerry & Knight, 2005) function and not the only one that can be used to represent the state, but is the one used in this thesis. If we assume a pure state defined above then we get

$$W(x, p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \psi^*(x - \frac{1}{2}q) \psi(x + \frac{1}{2}q) e^{ipq/\hbar} dq. \quad (2.22)$$

Without writing out all the details it is important to note that by integrating over position or momentum we get

$$\int_{-\infty}^{\infty} W(x, p) dp = |\psi(x)|^2 \quad \text{and} \quad \int_{-\infty}^{\infty} W(x, p) dx = |\phi(p)|^2 \quad (2.23)$$

which are the probability density for position x and momentum p . Working backwards we can note that like a probability density the Wigner function is real valued, but in contrast the Winger function can take on negative values which is a signature of quantum states and is why it is referred to as a quasi-probability distribution.

2.3 Oscillator modes with ion chains

Generally ion traps are operated with more than one ion in order to perform 2-qubit gates, as well as allow for sympathetic cooling (Kielpinski et al., 2000). For a system of N ions, the potential can be written as a sum of the quadratic potentials generated by the trap and the Coulomb potential due to the charge of the ions.

$$V = \frac{M}{2} \sum_{n=1}^N \sum_{i=1}^3 \omega_i^2 x_{ni}^2 + \frac{Q^2}{8\pi\epsilon_0} \sum_{\substack{n,m=1 \\ m \neq n}} \left[\sum_{i=1}^3 (x_{ni} - x_{mi})^2 \right]^{-\frac{1}{2}} \quad (2.24)$$

The secular frequencies of the harmonic motion are set by the strength of the confinement. With weak axial confinement, the ions will create a linear crystal. When this confinement strength is increased the ion crystal undergoes a phase transition to other structural phases (Yan et al., 2016). We will remain in the linear crystal regime for the work in our lab and for the purposes of this analysis, under the assumption that

$$\omega_1 = \omega_2 = \omega_3/\sqrt{\alpha}, \quad (2.25)$$

where $\alpha \ll 1$ for linear crystals, and 1,2,3 are x, y, z respectively.

2.3.1 Motional modes. For N ions there are $3N$ motional modes of the ion crystal. The motion is analyzed by expanding the potential to second order about the equilibrium positions denoted by $\bar{x}_{ni}$ (Marquet, Schmidt-Kaler, & James, 2003), where n indexes the ion and i the dimension. These positions are determined by solving the equations

$$\left. \frac{\partial V}{\partial x_{ni}} \right|_0 = 0. \quad (2.26)$$

It is more convenient to work with the dimensionless equilibrium position $u = \bar{x}_{ni}/l$ that is normalized to a natural length scale in this problem.

$$l = \left(\frac{Q^2}{4\pi\epsilon_0 M \omega_3^2} \right) \quad (2.27)$$

This distance is approximately equivalent to the cubed distance between ions in a two ion crystal. The displacements of the ions from their equilibrium position are written as

$$q_{ni}(t) = x_{ni}(t) - \bar{x}_{ni}. \quad (2.28)$$

Expanding about these equilibrium positions the Lagrangian is approximated as

$$\begin{aligned}
L &= \frac{M}{2} \sum_{n=1}^N \sum_{i=1}^3 \dot{q}_{ni}^2 - V_0 - \frac{1}{2} \sum_{m,n=1}^N \sum_{i,j=1}^3 \left. \frac{\partial^2 V}{\partial x_{mi} \partial x_{nj}} \right|_0 q_{mi} q_{nj} \\
&\approx \frac{M}{2} \left[\sum_{n=1}^N \dot{q}_{n3}^2 - \omega_3^2 \sum_{m,n=1}^N A_{mn} q_{m3} q_{n3} \right] + \frac{M}{2} \sum_{i=1}^2 \left[\sum_{n=1}^N \dot{q}_{ni}^2 - \omega_3^2 \sum_{m,n=1}^N B_{mn} q_{mi} q_{ni} \right],
\end{aligned} \tag{2.29}$$

where V_0 has no impact on the motion and is ignored and the tensors A_{mn} and B_{mn} are given by

$$A_{mn} = \begin{cases} 1 + 2 \sum_{\substack{p=1 \\ p \neq m}}^N \frac{1}{|u_m - u_p|^3} & \text{if } m = n \\ \frac{-2}{|u_m - u_p|^3} & \text{if } m \neq n \end{cases} \tag{2.30}$$

$$B_{mn} = \left(\frac{1}{\alpha} \delta_{mn} \right) - \frac{1}{2} A_{mn}. \tag{2.31}$$

The eigensystem of A_{mn} and B_{mn} describe the axial and radial motion respectively.

The eigenvectors are the same for both, meaning all 3 dimensions have the same mode structure. However, the oscillation frequencies of the radial modes are different.

2.3.2 Mixed mass crystals. In the section above it was assumed that all ions in the crystal have the same mass and charge. Having different species of ions allows each species to have a different role given its different electronic structure. The normal modes are still eigenvectors of the A_{mn} , but we must now re-normalize to account for the mixed mass (Kielpinski et al., 2000). This analysis makes the assumption of an odd number of ions and that the center ion has a different mass, which will be labeled by the index n_c with mass m , all others having mass M . This treatment is useful for the work presented in Chapter VII. The

Lagrangian is now written as

$$L = \frac{m}{2} \dot{q}_{n_c 3}^2 + \frac{M}{2} \left[\sum_{\substack{n=1 \\ n \neq n_c}}^N \dot{q}_{n3}^2 - \omega_3^2 \sum_{m,n=1}^N A_{mn} q_{m3} q_{n3} \right] + \frac{M}{2} \sum_{i=1}^2 \left[\frac{m}{2} \dot{q}_{n_c i}^2 + \sum_{\substack{n=1 \\ n \neq n_c}}^N \dot{q}_{ni}^2 - \omega_3^2 \sum_{m,n=1}^N B_{mn} q_{mi} q_{ni} \right]. \quad (2.32)$$

The mass dependence is removed by using the transformation $Q_{ni} = q_{ni} \sqrt{m_n}$ where $\mu = m/M$. This produces the re-normalized Lagrangian

$$L = \frac{1}{2} \sum_{n=1}^N \dot{Q}_{n3}^2 - \frac{\omega_3^2}{2} \sum_{m,n=1}^N A'_{mn} Q_{m3} Q_{n3} + \frac{1}{2} \sum_{i=1}^2 \left[\sum_{n=1}^N \dot{Q}_{ni}^2 - \omega_3^2 \sum_{m,n=1}^N B'_{mn} Q_{mi} Q_{ni} \right] \quad (2.33)$$

where

$$A'_{mn} = \begin{cases} A_{mn} & m, n \neq n_c \\ A_{mn}/\sqrt{\mu} & m \text{ or } n = n_c, m \neq n \\ A_{mn}/\mu & m = n = n_c \end{cases} \quad (2.34)$$

and the same modification is true for B'_{mn} . One notable consequence of having as mixed mass ion chain is that there is no longer a rigid body mode where each ion has the same amplitude oscillation; this is shown in Fig.2.1. This enables the possibility of these modes, normally called center of mass, exchanging energy with other modes, which is not possible with equal mass ions.

		Same Mass		Ca-Sr-Ca		Sr-Ca-Sr	
		Axial modes	Frequencies	Vectors			
Z1		$\omega_{zz} = 2.41\omega_z$	$(0.41, -0.82, 0.41)$	$1.95\omega_z$	$(0.55, -0.62, 0.55)$	$3.26\omega_z$	$(0.27, -0.92, 0.27)$
Z2		$\omega_{zr} = 1.73\omega_z$	$(-0.71, 0, 0.71)$	$1.73\omega_z$	$(-0.71, 0, 0.71)$	$1.73\omega_z$	$(-0.71, 0, 0.71)$
Z3		ω_z	$(0.58, 0.58, 0.58)$	$0.83\omega_z$	$(0.44, 0.78, 0.44)$	$1.09\omega_z$	$(0.65, 0.38, 0.65)$
		Radial modes					
X1		$\omega_{xz} = \sqrt{\omega_x^2 + \frac{\omega_z^2}{2} - \frac{\omega_{zx}^2}{2}}$	$(0.41, -0.82, 0.41)$	$\sqrt{\omega_x^2 + \frac{\omega_z^2}{2} - \frac{\omega_{zx}^2}{2}}$	$(0.55, -0.62, 0.55)$	$\sqrt{\omega_x^2 + \frac{\omega_z^2}{2} - \frac{\omega_{zx}^2}{2}}$	$(0.27, -0.92, 0.27)$
X2		$\omega_{xr} = \sqrt{\omega_x^2 + \frac{\omega_z^2}{2} - \frac{\omega_{xr}^2}{2}}$	$(-0.71, 0, 0.71)$	$\sqrt{\omega_x^2 + \frac{\omega_z^2}{2} - \frac{\omega_{xr}^2}{2}}$	$(-0.71, 0, 0.71)$	$\sqrt{\omega_x^2 + \frac{\omega_z^2}{2} - \frac{\omega_{xr}^2}{2}}$	$(-0.71, 0, 0.71)$
X3		ω_x	$(0.58, 0.58, 0.58)$	$\sqrt{\omega_x^2 + \frac{\omega_z^2}{2} - \frac{\omega_{x3}^2}{2}}$	$(0.44, 0.78, 0.44)$	$\sqrt{\omega_x^2 + \frac{\omega_z^2}{2} - \frac{\omega_{x3}^2}{2}}$	$(0.65, 0.38, 0.65)$

Figure 2.1. Normal modes and their oscillation frequencies for 3 ion chains. The normal mode vectors give the amplitude of oscillation described by q_{ni} . Results for $^{40}\text{Ca}^+ / ^{88}\text{Sr}^+ / ^{40}\text{Ca}^+$ chains and $^{88}\text{Sr}^+ / ^{40}\text{Ca}^+ / ^{88}\text{Sr}^+$ chains are also shown, which are of interest for experiments presented here.

2.4 Driven harmonic oscillators and parametric modulation

Now that the oscillator states can be described mathematically we can work out how different oscillator states are generated. Specifically we can consider what types of operations we can generate using an ion trap. The geometry of macroscopic ion traps generally enable the ability to easily generate linear and quadratic potentials. Starting with linear potentials we can see what types of quantum operators we generate. Assuming a potential of the form $\hat{V}(t) = F \frac{\hat{x}}{x_0} \sin(\omega_d t + \phi)$ the full Hamiltonian governing these dynamics is written as

$$\hat{H} = \hbar\omega(\hat{a}^\dagger \hat{a} + 1/2) - F(\hat{a} + \hat{a}^\dagger) \sin(\omega_d t + \phi), \quad (2.35)$$

where we have transformed to working with the creation and annihilation operators. If we move into an interaction picture where we are rotating in phase

with the oscillator we get

$$\hat{H}_I = -Fx_0(\hat{a}e^{i\omega t} + \hat{a}^\dagger e^{-i\omega t}) \sin(\omega_d t + \phi). \quad (2.36)$$

We can see from this transformation, we get terms that rotate at different rates, $\omega_d + \omega$ and $\omega_d - \omega$. In the case that the coupling rate, defined by $g_d = Fx_0/\hbar$ is much slower than the mode frequency $g_d \ll \omega$ then the fast rotating terms $\omega + \omega_d$ can be ignored, called the rotating wave approximation (RWA). The interaction Hamiltonian is then reduced to

$$\hat{H}_I = -\frac{\hbar g_d}{2i}(\hat{a}e^{-i\delta t + i\phi} - \hat{a}^\dagger e^{i\delta t - i\phi}). \quad (2.37)$$

The interaction detuning is defined as $\delta = \omega_d - \omega$ and we can see the effect on the state when we tune into resonance with the oscillator $\omega_d = \omega$. In this case we get a time independent Hamiltonian and the unitary evolution operator can be written as

$$\hat{U}_I = \exp(i\hat{H}_I t/\hbar) = \exp\left(-\frac{g_d t}{2i}(\hat{a}e^{i\phi} - \hat{a}^\dagger e^{i\phi})\right), \quad (2.38)$$

which is the definition of the displacement operator (Carruthers & Nieto, 1965) $\hat{D}(\alpha = \frac{g_d t}{2}e^{i\phi})$ where the amplitude depends linearly on the duration the drive is applied for. This operator allows for linear translations anywhere in phase space while maintaining minimum uncertainty and generates states with a common classical analog (Alonso et al., 2016). This technique is used widely in the trapped ion field in order to do things like calibrate motional frequencies. Next we will look at higher order operations that can generate states with more of a quantum nature.

In order to get higher order operations we need to implement higher order potentials at the location of the ion. Harmonic potentials, as described above, are how the ions are confined in space. We can modulate this potential to implement a Hamiltonian with $\hat{x}^2$ given by

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + 1/2) - \hbar g \sin(\omega_s t - \theta)(\hat{a}^{\dagger 2} + \hat{a}^2 + 2\hat{a}^\dagger\hat{a} + 1). \quad (2.39)$$

In the same manner as for linear potentials, we move into a interaction frame rotating at ω and drop fast rotating terms at $(2\omega + \omega_s)$ and ω_s which reduces $\hat{H}_I$ to

$$\hat{H}_I = -\frac{\hbar g}{2i}(\hat{a}^2 e^{-2i\omega t + \omega_s t - i\theta} - \hat{a}^{\dagger 2} e^{2i\omega t - i\omega_s t + i\theta}). \quad (2.40)$$

Time dependence of this Hamiltonian is removed by the choice of modulation frequency to be $\omega = 2\omega_s$,

$$\hat{H}_I = \frac{i\hbar g}{2}(\hat{a}^2 e^{-i\theta} - \hat{a}^{\dagger 2} e^{i\theta}). \quad (2.41)$$

In this rotating frame we can write the unitary evolution operator when applied for a duration t as

$$\hat{U}_I = \exp(i\hat{H}_I t/\hbar) = \exp\left(\frac{gt}{2}(\hat{a}^2 e^{i\theta} - \hat{a}^{\dagger 2} e^{i\theta})\right). \quad (2.42)$$

This unitary is defined as the squeezing operator $\hat{S}(\xi)$ and the squeezing parameter ξ is given by $\xi = gte^{i\theta}$ (Heinzen & Wineland, 1990). The states generated by these operators will be described in further detail in the next section, but first we will take a look at the types of operators than can be generated involving more than one mode of oscillation.

2.4.1 Multi-mode operators. When considering multiple orthogonal modes of oscillation of a single trapped ion we can generalize the oscillator Hamiltonian to be

$$\hat{H}_0 = \sum_{i \in \{x, y, z\}} \hbar\omega_i(\hat{a}_i^\dagger \hat{a}_i + 1/2). \quad (2.43)$$

In the same manner as above we can apply an oscillating potential that will now be generalized to be $U(\mathbf{r}, t) = U(\mathbf{r}) \cos(\omega t + \phi)$. The multi-mode interactions can be seen by expanding this potential to second order

$$U(\mathbf{r} + \delta\mathbf{r}) \simeq U(\mathbf{r}_0) + \sum_{i \in \{x, y, z\}} \frac{\partial U}{\partial i} \Big|_{\mathbf{r}=\mathbf{r}_0} \delta r_i + \frac{1}{2} \sum_{i \in \{x, y, z\}} \frac{\partial^2 U}{\partial i \partial j} \Big|_{\mathbf{r}=\mathbf{r}_0} \delta r_i \delta r_j, \quad (2.44)$$

where we can drop terms besides second order terms using the same rotating wave approximation, reducing the potential to

$$U(\mathbf{r} + \delta\mathbf{r}) \simeq 2^{-\delta(i_a, i_b)} \sum_{i \in \{x, y, z\}} \frac{\partial^2 U}{\partial i \partial j} \Big|_{\mathbf{r}=\mathbf{r}_0} \delta r_i \delta r_j. \quad (2.45)$$

This curvature expression has several terms where the non-cross (x^2, y^2, z^2) terms give rise to the single mode interactions. In order to couple two different modes of oscillation we need a terms that looks like $\hat{x}\hat{y}$ where $\hat{x}$ and $\hat{y}$ are the position operators for the two orthogonal modes of oscillation. The interaction Hamiltonian is then written as

$$\hat{H}_I = -\hbar g \sin(\omega_{tm}t - \phi)(\hat{x}\hat{y}). \quad (2.46)$$

If we define the x mode of oscillation to have frequency ω_x with annihilation operator $\hat{a}$ and the y mode to have frequency ω_y with annihilation operator $\hat{b}$, we can transform into the rotating frame. Making a transformation into a frame at ω_x and ω_y is possible given that $\hat{a}$ and $\hat{b}$ commute, given by

$$\begin{aligned} e^{i/\hbar \hat{H}_0 t} \hat{a} e^{-i/\hbar \hat{H}_0 t} &= \hat{a} e^{-i\omega_x t} \\ e^{i/\hbar \hat{H}_0 t} \hat{b} e^{-i/\hbar \hat{H}_0 t} &= \hat{b} e^{-i\omega_y t}. \end{aligned} \quad (2.47)$$

The full interaction Hamiltonian is then expanded to be

$$\hat{H}_I = -\hbar g \sin(\omega_{tm}t - \phi) [\hat{a}^\dagger \hat{b}^\dagger e^{it(\omega_x + \omega_y)} + \hat{a} \hat{b} e^{-it(\omega_x + \omega_y)} + \hat{a}^\dagger \hat{b} e^{it(\omega_x - \omega_y)} + \hat{a} \hat{b}^\dagger e^{-it(\omega_x - \omega_y)}] \quad (2.48)$$

and we now have access to different resonant interactions. We will first let $\omega_{tm} = \omega_x - \omega_y$ and can drop terms that oscillate at $2\omega_{x,y}$ using the RWA. This interaction is typically called the beam splitter interaction which enables coherent exchange of energy between modes (Gorman, Schindler, Selvarajan, Daniilidis, & Häffner, 2014) and is written as

$$\hat{H}_{BS} = \hbar g (\hat{a} \hat{b}^\dagger e^{i\phi} + \hat{a}^\dagger \hat{b} e^{-i\phi}). \quad (2.49)$$

The other choice of modulation frequency we can make is to set $\omega_{tm} = \omega_x + \omega_y$ and ignore the same fast oscillating terms. This interaction is generally called two-mode squeezing and generates highly correlated oscillator states and is expressed as

$$\hat{H}_{TMS} = \hbar g(\hat{a}^\dagger \hat{b}^\dagger e^{i\phi} + \hat{a} \hat{b} e^{-i\phi}). \quad (2.50)$$

All of the interactions described in this section can be used for implementations of quantum computation and quantum metrology in a variety of platforms, but require different methods to produce. In the following section these states that are generated from these interactions will be described in more detail along with potential use cases.

2.5 Gaussian states

In order to talk about the states generated with the Hamiltonians given above, it is necessary to work again in the language of phase space and the Wigner function. In order to do this we must define the most relevant quantities of the Wigner function, which are the statistical moments of the quantum state. The first moment is sometimes called the displacement vector, but more simply described as the mean value

$$\bar{x} := \langle \hat{x} \rangle = \text{Tr}(\hat{x}\rho), \quad (2.51)$$

where again the quantum state is described by the density operator ρ . The second moment is called the covariance matrix V with the elements defined by

$$V_{ij} := \frac{1}{2} \langle \{\Delta \hat{x}_i, \Delta \hat{x}_j\} \rangle, \quad (2.52)$$

with $\Delta \hat{x}_i := \hat{x}_i - \langle \hat{x}_i \rangle$ and $\{, \}$ is the anti-commutator. The variance in the quadrature operators $\hat{x}$ is then just the diagonal elements of this matrix

$$\begin{aligned} V_{ii} &= V(\hat{x}_i) \\ V(\hat{x}_i) &= \langle \hat{x}_i^2 \rangle - \langle \hat{x}_i \rangle^2. \end{aligned} \quad (2.53)$$

It is also useful to note that through this formalism the uncertainty relation appears as $V(\hat{x})V(\hat{p}) \geq \frac{\hbar}{2}$. For Gaussian states alone the quantum state can be completely characterized by these two moments $\hat{\rho} = \hat{\rho}(\bar{x}, V)$ and the Wigner function can take the general form

$$W(x) = \frac{\exp\{[-(1/2)(x - \bar{x})^T V^{-1}(x - \bar{x})]\}}{(2\pi)^N \sqrt{\det V}}, \quad (2.54)$$

which is that of a Gaussian function. This is another place where the Wigner negativity is an important aspect of the quantum state as a state is only Gaussian if and only if its Wigner function is non-negative. With Gaussian states roughly mathematically defined we can again take a look at the operators that we can apply to these states. Each of the above operators is unitary and transforms the state according to $\hat{\rho} \rightarrow U\hat{\rho}U^\dagger$ or just $|\psi\rangle \rightarrow U|\psi\rangle$ for the pure states we are considering. All of these operators are considered Gaussian operators if they transform a Gaussian state into another Gaussian state and each one of these operators and the final state is described below.

2.5.1 Displacement and coherent states. The displacement operator has already been defined above but we will discuss in more detail the effect of this operator on the vacuum state of an oscillator in terms of the displacement parameter α . The magnitude and phase of α is set by the driving field coupling, duration and phase. More generally we can define $\alpha = (x + ip)/2$ which can help to clarify the action in phase space. Application of this operator makes the transformation $|\alpha\rangle = \hat{D}(\alpha)|0\rangle$ which maintains the same variance as the vacuum state ($V = I$) but shifts the mean value ($\bar{x} = d_\alpha$) which is clear when looking at the Wigner function of the coherent state,

$$W(\alpha, \beta) = \frac{2}{\pi} e^{-2|\alpha - \beta|^2}. \quad (2.55)$$

Coherent states are also the eigenstates of the annihilation operator $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ and when expanded in the number basis can be written as

$$|\alpha\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (2.56)$$

When making a measurement of the phonon number of a coherent state α , like photon counting a laser field, the probabilities are given by a Poisson distribution

$$P_n = |\langle n|\alpha\rangle|^2 = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}, \quad (2.57)$$

where the mean value $\langle \hat{n} \rangle = \sum_n n P_n = |\alpha|^2$ which is equal to the variance.

The number distribution and Wigner function can both be visualized in 2.2a).

Coherent states are used widely in trapped ions (Gorman et al., 2014; Mihalcea, 2023), appearing in the entanglement of spin states (Mølmer & Sørensen, 1999), but they will also be important for performing classical interferometry experiments that use classical laser fields.

2.5.2 Squeezing and single mode squeezed states. We can now discuss in more detail the effect of the squeezing operator that has been defined above. In optics, a single photon in a pump field at frequency 2ω is spit into 2 photons at frequency ω via a non-linear medium. The same effect happens with trapped ions where the driving field at 2ω generates phonon pairs of the motion, but without the inefficiencies of using a non-linear medium. Since this operator is second order in the annihilation and creation operators the state that is generated is a superposition of the even Fock states $|2n\rangle$ and that state can be written in the number basis as

$$|\xi\rangle = \frac{1}{\sqrt{\cosh(r)}} \sum_{n=0}^{\infty} -e^{in\theta} \frac{\sqrt{(2n)!}}{2^n n!} \tanh(r)^n |2n\rangle. \quad (2.58)$$

The even state populations are expressed as

$$P_{2n} = \frac{(\tanh r)^{2n}}{\cosh r} \frac{(2n)!}{(2^n n!)^2}. \quad (2.59)$$

The effects of squeezing are more obvious in phase space where the Wigner function is given as

$$W(r, \theta, \alpha) = \frac{2}{\pi} \exp \left(-2e^{2r} \operatorname{Re}[e^{-i\theta/2}\alpha]^2 - 2e^{-2r} \operatorname{Im}[e^{-i\theta/2}\alpha]^2 \right) \quad (2.60)$$

which is a two-dimensional Gaussian with different variances which can be seen in 2.2b). In order to get the mean phonon number (expectation of the number operator) it is easiest to first see how the squeezing operator transforms the annihilation and creation operator, given by the Bogoliubov transformations (Bogoljubov, 1958):

$$\hat{S}^\dagger(\xi) \hat{a} \hat{S}(\xi) = \hat{a} \cosh r - \hat{a}^\dagger e^{i\theta} \sinh r \quad (2.61)$$

$$\hat{S}^\dagger(\xi) \hat{a}^\dagger \hat{S}(\xi) = \hat{a}^\dagger \cosh r - \hat{a} e^{-i\theta} \sinh r \quad (2.62)$$

Using these transformations the expectation value of the number operator evaluates simply to $\langle \hat{n} \rangle = \langle 0 | \hat{S}^\dagger(\xi) \hat{a}^\dagger \hat{a} \hat{S}(\xi) | 0 \rangle = \sinh^2 r$. Unlike the displacement operator, squeezed states are fixed at the origin with expectation values of position and momentum remaining zero ($\langle \xi | \hat{x} | \xi \rangle = \langle \xi | \hat{p} | \xi \rangle = 0$). Since the mean value of these Gaussian distributions remain unchanged it is more interesting to look at the quadrature variances. At a phase $\theta = 0$ the quadrature operators $\hat{\mathbf{q}} = (\hat{x}, \hat{p})^T$ are transformed under the mapping $\hat{\mathbf{q}} \rightarrow \mathbf{S}(r) \hat{\mathbf{q}}$ where

$$\mathbf{S}(r) := \begin{pmatrix} e^{-r} & 0 \\ 0 & e^r \end{pmatrix}, \quad (2.63)$$

where the covariance matrix is then given by $V = \mathbf{S}(r) \mathbf{S}(r)^T = \mathbf{S}(2r)$. The effect is to shrink one quadrature variance below the quantum shot noise limit and stretch the other all while maintaining the Heisenberg uncertainty relation. Having a mechanism to shrink one quadrature variance below the shot noise limit is a key ingredient in metrology experiments that achieve sensitivity below the standard

quantum limit that is achievable with coherent states. (C. Caves, 1981) The phase of the the squeezing operator sets the quadrature that is squeezed and can be used for amplification protocols at certain phases.

2.5.3 Phase operator. Before moving onto the effects of two-mode operations it is worth discussing the phase operator and its effect on oscillator states. The phase of a quantum state is what metrology experiments may aim to sense and given the need understand this phase it is worth discussing. The phase of a quantum state is generally tracked with a local oscillator in the experimental control system, which can be provided by a laser or some other oscillator. Phase can naturally be added in optical experiments by passing a field through a medium or increasing optical path length but looks different with trapped ions. Since the ions are fixed in space, instead of increasing an optical path length, additional time delays can be added, as well as changing the trapping potential to shift the oscillator frequency, or moving the local oscillator off resonant with a frame rotating with the oscillator for some fixed time. With a digitally programmable oscillator, this can be done by advancing the phase programmed to the device. No matter the method, the effect is to implement a free propagation Hamiltonian of $\hat{H} = 2\theta\hat{a}^\dagger\hat{a}$. This looks just like the bare oscillator Hamiltonian and is made up of the number operator that preserves the energy of the state. The unitary phase operator is given as

$$\hat{R}(\theta) = \exp(-i\theta\hat{a}^\dagger\hat{a}) \quad (2.64)$$

and transforms the annihilation operator by another Bogoliubov transformation by $\hat{a} \rightarrow e^{-i\theta}\hat{a}$. The mapping for the quadratures is given by $\hat{\mathbf{q}} \rightarrow \mathbf{R}(\theta)\hat{\mathbf{q}}$ where

$$\mathbf{R}(\theta) := \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (2.65)$$

which is a simple rotation matrix with angle θ . In phase space, for a coherent state, this looks like a rotation about the origin with angle θ as shown in 2.2a). Later we will implement experiments to detect these sort of phase rotations with sensitivities beyond the SQL.

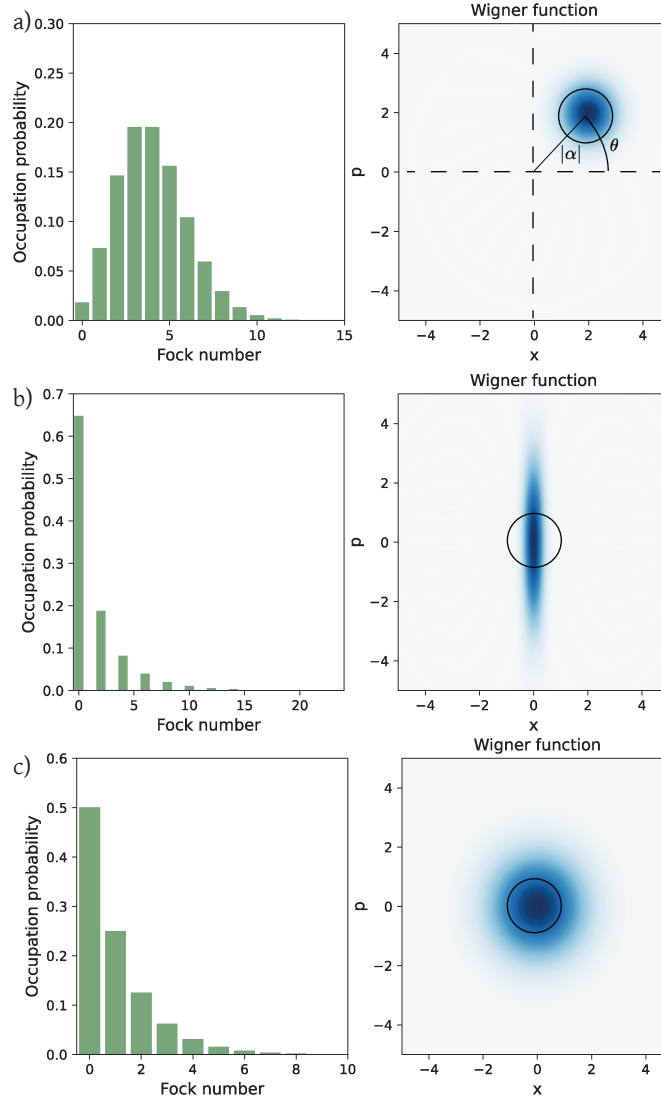


Figure 2.2. a) number distribution and Wigner function of a coherent state with $\alpha=2$. The displacement operator translates the Gaussian centered at 0 by magnitude $|\alpha|$ with phase rotations shown by the state subtending the angle θ . Variance of the state is shown by the black circle b) number distribution and Wigner function of a squeezed state with $r=1$. Population of only the even Fock states can be seen clear and variance of the squeezed state being below that of a coherent state only in one quadrature. c) number distribution and Wigner function of a thermal state with $\bar{n}=1$, where the increased variance of the the SQL (coherent state) can be seen clearly

2.5.4 Beam splitter.

The two-mode unitary is a key part of quantum optics and is the basis for most interferometers. The beam splitter unitary operator

is given as

$$\hat{B}(\theta) = \exp\left[\theta(\hat{a}^\dagger \hat{b} - \hat{a} \hat{b}^\dagger)\right], \quad (2.66)$$

where θ sets the transmissivity of the beam splitter $\tau = \cos^2 \theta$ which is fixed to between 0 and 1. In the trapped ion platform we have the ability to tune this value to whatever we want by tuning the coupling rate or the interaction time, where $\tau = 1/2$ is usually the default in optics, shown in 2.3. The annihilation operators are again transformed under a Bogoliubov transformation by

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} \rightarrow \begin{pmatrix} \sqrt{\tau} & \sqrt{1-\tau} \\ -\sqrt{1-\tau} & \sqrt{\tau} \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}. \quad (2.67)$$

It can be seen from the form of the transformation matrix there is a similarity to rotation matrices. In the most simplified description this view is a consequence of the beam splitter belonging to the SU(2) group which will be discussed more in detail later. The mapping for the quadrature operators $\hat{\mathbf{q}} := (\hat{x}_a, \hat{p}_a, \hat{x}_b, \hat{p}_b)^T$ is given by $\hat{\mathbf{q}} \rightarrow \mathbf{B}(\tau)\hat{\mathbf{q}}$ where

$$\mathbf{B}(\tau) := \begin{pmatrix} \sqrt{\tau}\mathbf{I} & \sqrt{1-\tau}\mathbf{I} \\ -\sqrt{1-\tau}\mathbf{I} & \sqrt{\tau}\mathbf{I} \end{pmatrix}, \quad (2.68)$$

where $\mathbf{I}$ is the identity matrix.

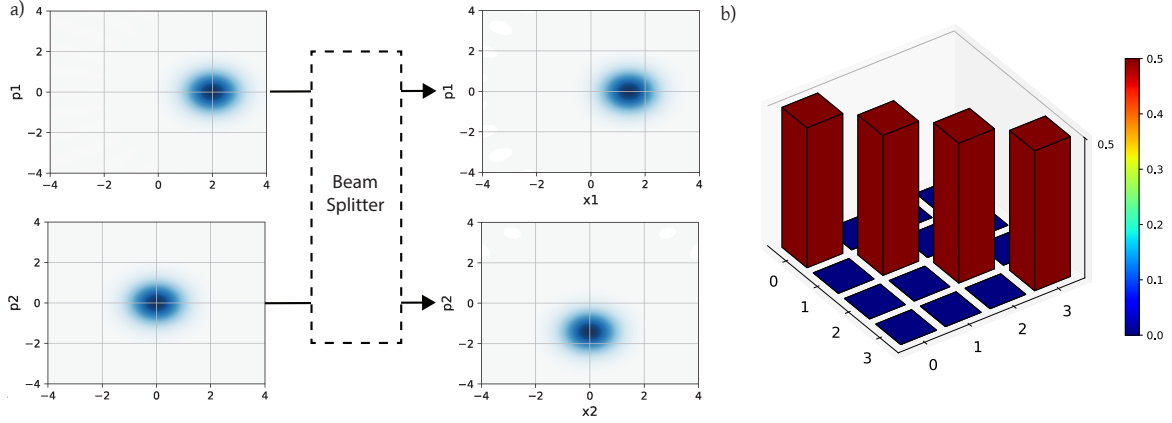


Figure 2.3. a) action of the beam splitter when $\tau=1/2$ on the input state $|\alpha=2,0\rangle$. Total energy (phonon) number is conserved in this process. b) covariance matrix for the multi-mode state shown that the two modes are left unentangled after the beam splitter operation.

2.5.5 Two-mode squeezing and two mode squeezed vacuum. In a similar fashion to the single mode squeezing where a pump photon at 2ω is split into two photons, now those photons are generated into two different modes with the pump field at $\omega_a + \omega_b$. As shown above, this can be done with a driving field that has some cross curvature terms that implement a bilinear terms $\hat{a}^\dagger \hat{b}^\dagger$ in the Hamiltonian. The corresponding Gaussian unitary is given by

$$\hat{S}_2 = \exp\left[r(\hat{a}\hat{b} - \hat{a}^\dagger\hat{b}^\dagger)/2\right], \quad (2.69)$$

where r described the amount of two-mode squeezing and is defined as $r = 2gt$.

For the quadrature operators $\hat{\mathbf{q}} := (\hat{x}_a, \hat{p}_a, \hat{x}_b, \hat{p}_b)^T$ the mapping for the two-mode squeezer is given by $\hat{\mathbf{q}} \rightarrow \mathbf{S}_2(r)\hat{\mathbf{q}}$ where

$$\mathbf{S}_2(r) := \begin{pmatrix} \cosh r \mathbf{I} & \sinh r \mathbf{Z} \\ \sinh r \mathbf{Z} & \cosh r \mathbf{I} \end{pmatrix}, \quad (2.70)$$

where $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The output state when applied to vacuum is sometimes called an Einstein-Podolsky-Rosen (EPR) state where

$$|TMSV\rangle = \sqrt{1 - \lambda^2} \sum_{n=0}^{\infty} (-\lambda)^n |n_a, n_b\rangle \quad (2.71)$$

and $|n_a, n_b\rangle$ now describes the number state of two modes with $\lambda = \tanh r$. The question of quadrature variance becomes more interesting with this state with the covariance matrix given by

$$\mathbf{V}_{TMSV} = \begin{pmatrix} \nu \mathbf{I} & \sqrt{\nu^2 - 1} \mathbf{Z} \\ \sqrt{\nu^2 - 1} \mathbf{Z} & \nu \mathbf{I} \end{pmatrix}, \quad (2.72)$$

where $\nu = \cosh r$ describes the quadrature variance. The reduced variance is not seen in the quadratures of a single mode but it can be checked that

$$V(\hat{x}_-) = V(\hat{p}_+) = e^{-2r} \quad (2.73)$$

where $\hat{x}_- := (\hat{x}_a - \hat{x}_b)/\sqrt{2}$ and $\hat{p}_+ := (\hat{p}_a + \hat{p}_b)/\sqrt{2}$ are new EPR quadrature variables. Working in this new basis enables squeezing to be more clearly seen and is visualized in 2.4 The consequences of these new quadrature variances means that for $r > 0$ the correlations between these two modes beat the quantum shot noise limit. This entangled Gaussian state is another common state used in quantum optics and the elements used to generate this state also play a role in interferometry.

Before moving on, it is worth discussing the phonon number probabilities and what it means to look at a single mode of this entangled state. The joint phonon probability distribution for the two-mode squeezed vacuum only has diagonal elements

$$P(n1, n2) = P(n, n) \delta_{n_1, n} \delta_{n, n_2} \quad (2.74)$$

and those elements are given by

$$P(n, n) = \frac{(\tanh r)^{2n}}{\cosh^2 r}. \quad (2.75)$$

If a measurement of only a single mode is made, this is equivalent to tracing over a single mode, the result is $\text{Tr}_b[\hat{\rho}] = \hat{\rho}_a^{th}(\bar{n})$, which is a thermal state with average phonon number of $\bar{n} = \sinh^2 r$ (C. M. Caves, Zhu, Milburn, & Schleich, 1991). This means that the mean phonon number of single mode is related to the amount of two-mode squeezing, which will be important later.

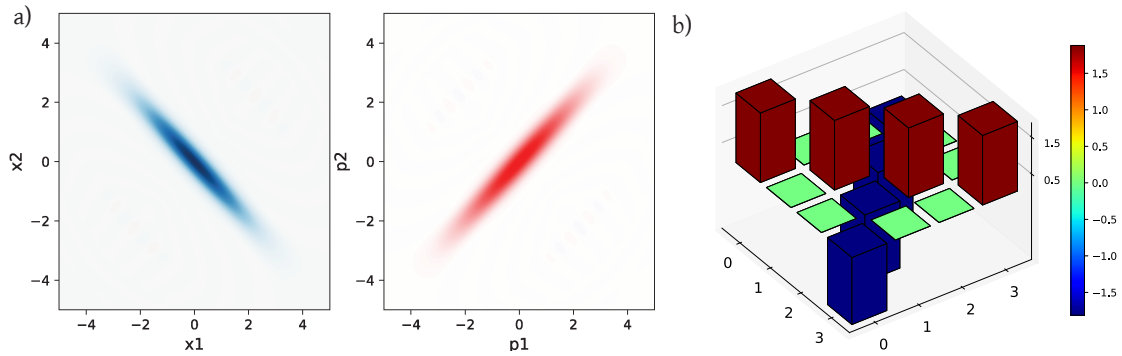


Figure 2.4. a) Wigner function representation of a two-mode squeezed state in the EPR-variable basis. The squeezing is seen in the mode correlations and not single mode quadratures. b) correlation matrix for a two-mode squeezed state with off-diagonal elements giving proof of mode correlations.

2.5.6 Thermal states. Given the presence of thermal states when making measurements of two-mode squeezed states, it is relevant to include some discussion of these states. Thermal states are not only present when working with two-mode squeezed states but are common in trapped ions and exist as a consequence of the ions coupling to the environment or after some dissipative operation. These Gaussian states are not pure states and the mixed state density matrix is given by

$$\hat{\rho}^{th} = (1 - e^{-\hbar\omega\beta}) \sum_{n=0}^{\infty} e^{-\beta\hbar\omega n} |n\rangle \langle n|, \quad (2.76)$$

where $\beta = 1/k_B T$ with k_B being the Boltzmann constant and temperature T . Measurements of the average phonon number ($\bar{n}$) can be made by using the motional sidebands (Bergquist, Itano, & Wineland, 1987). The temperature relates to $\bar{n}$ by,

$$T = \frac{\hbar\omega}{k_B} \frac{1}{\ln\left[\frac{\bar{n}}{1+\bar{n}}\right]}, \quad (2.77)$$

which allows the density matrix to be re-written in terms of $\bar{n}$ to be

$$\hat{\rho}^{th} = \frac{1}{1+\bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{1+\bar{n}} \right)^n |n\rangle \langle n|. \quad (2.78)$$

The phonon number distribution is then given by

$$P_n = \frac{1}{1+\bar{n}} \left(\frac{\bar{n}}{1+\bar{n}} \right)^n. \quad (2.79)$$

The Gaussian nature of these states can again be seen by looking at the Wigner distribution given by

$$W(\bar{n}, \alpha) = \frac{2}{\pi} \frac{1}{1+2\bar{n}} \exp[-2|\alpha|^2/(1+2\bar{n})]. \quad (2.80)$$

Both quadrature variances are equal, giving a circular state like a coherent state, however the variances grow above those of a coherent state with increasing $\bar{n}$ which can be seen in 2.2c).

2.6 Non-Gaussian states

Gaussian states are exceedingly common in quantum optics and are a key ingredient in continuous variable quantum computing (CVQC) and are a resource for quantum metrology. Despite this, non-Gaussian states are necessary to complete a universal set of operations for CVQC (Lloyd & Braunstein, 1999) and can be advantageous in some quantum metrology experiments (Wolf et al., 2019). All of these states require higher order interactions, being at least third order in the annihilation and creation operators, or a coupling to a discrete variable system like a qubit. The potentials necessary to implement these higher order interaction are

generally weak without a more electrodes (Hou et al., 2022). A few non-Gaussian states will be mentioned here for their potential relevance.

2.6.1 Fock states. Fock states, sometimes called number states, have already been mentioned heavily since it is natural to work in a basis where states are described as a superposition of Fock states (Gerry & Knight, 2005). Shown in the number probability distributions in 2.2, Gaussian states are given by a superposition of Fock states, but single Fock states can be generated and are non-Gaussian. The Wigner function for these single Fock states is given by

$$W(x, p) = \frac{(-1)^m}{\pi} e^{-(x^2+p^2)L_m(2(p^2+x^2))}, \quad (2.81)$$

where $L_m(x)$ denotes the m -th degree Laguerre polynomial. Figure 2.5 highlights the non-Gaussian nature of these state through the Wigner negativity.

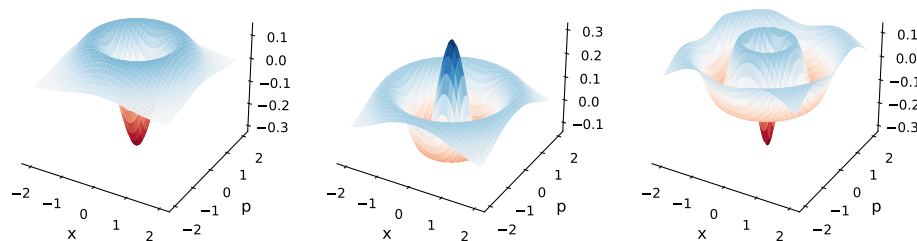


Figure 2.5. Wigner functions for the $n = 1, 2, 3$ states which highlight the Wigner negativity and the spherical symmetry of the states

2.6.2 Cross-Kerr interaction. With a single ion there are approximately no anharmonic terms in the potential experienced by an ion. When considering the problem of scaling the capabilities of quantum systems using the motional states of trapped ions, it is necessary to start working with more ions. There is more physics to consider when working with more than one ion. In Chapter VII there is discussion of how the geometries of the modes can be used

advantageously when detecting the internal state of the ion. In this section we will explore the interactions that arise from considering the additional Coulomb interaction present with multiple ions.

The Kerr interaction is a non-linear interaction that appears when considering multiple ions. This Hamiltonian is of the form $\hat{H}_{Kerr} \sim \xi \hat{n}_a \hat{n}_b$ where ξ is the coupling strength between mode a and mode b with number operators $\hat{n}_a$ and $\hat{n}_b$ (the number operator being given by $\hat{n}_a = a^\dagger a$). This interaction can be found by expanding the potential, from eq. 2.24, to fourth order, producing an additional term

$$-\frac{M\omega_3^2}{2l} \sum_{m,n,p} C_{m,n,p} q_{p3} (2q_{m3}q_{n3} - 3q_{m1}q_{n1} - 3q_{m2}q_{n2}). \quad (2.82)$$

This term describes coupling between different ions. The coupling strength is given by the tensor C_{mnp} which is written as

$$C_{mnp} = \sum_{\substack{q=1 \\ q \neq m}}^N \frac{(u_q - u_m)(\delta_{mn} - \delta_{qn})(\delta_{mp} - \delta_{qp})}{(u_m - u_q)^4}. \quad (2.83)$$

This complete Hamiltonian can be re-written in terms of the normal modes using the transformation

$$X_n = \sum_{i=1}^{3N} A_{ni} q_i, \quad (2.84)$$

where X_n is the normal mode vector, q_m is the position vector, and A_{mn} is the matrix of eigenvectors. Using this transformation the Hamiltonian can be written as $H = H_0 + H_I$, where H_0 describes the uncoupled collective oscillations, and H_I describes the perturbation that can couple modes (Marquet et al., 2003). This term is written as

$$H_I = \frac{M\omega_3^2}{2l} \sum_{i,j,k=1}^N D_{ijk} Z_k (2Z_i Z_j - 3X_i X_j - 3Y_i Y_j), \quad (2.85)$$

where i, j, k now index the mode. The mode-mode coupling coefficients $D_{\alpha\beta\gamma}$ are defined as

$$D_{\alpha\beta\gamma} = \sum_{\nu,\sigma,\lambda=1}^N C_{\nu\sigma\lambda} b_{\nu}^{(\alpha)} b_{\sigma}^{(\beta)} b_{\lambda}^{(\gamma)}, \quad (2.86)$$

where the $b_n^{(p)}$ are the eigenvectors of A_{mn} . Quantization of this Hamiltonian involves replacing the position operators with their ladder operator representation. Ignoring terms that have zero matrix elements for population transfer between some initial and final state, the Hamiltonian can be written in the interaction representation as

$$\hat{H}_i \approx -3\epsilon\hbar\omega_3 \sum_{i,j,k=1}^N \frac{D_{ijk}}{\sqrt[4]{\gamma_i\gamma_j\mu_k}} \left[2\hat{a}_k(\hat{b}_i^\dagger\hat{b}_j + \hat{c}_i^\dagger\hat{c}_j)e^{i\delta^{(-)}\omega_z t} + \hat{a}_k(\hat{b}_i^\dagger\hat{b}_j^\dagger + \hat{c}_i^\dagger\hat{c}_j^\dagger)e^{i\delta^{(+)}\omega_z t} \right] + h.a., \quad (2.87)$$

where ϵ characterizes the strength of the non-linear interaction, and γ_i, μ_k are the radial and axial eigenvalues and is given by

$$\epsilon = \frac{1}{4\sqrt{2}} \left[\frac{\hbar\omega_3}{\alpha_{fsc}^2 M c^2} \right]. \quad (2.88)$$

The mode coupling resonance condition is given by $\delta^{(\pm)} = \sqrt{\gamma_i} \pm \sqrt{\gamma_j} - \sqrt{\mu_k}$.

This condition puts a constraint on what α can be for resonant mode coupling.

Resonances can only occur for

$$\alpha = \frac{16\mu_k}{4\mu_k^2 + \mu_i^2 + \mu_j^2 - 8\mu_k + 4\mu_k\mu_i + 4\mu_k\mu_j - 2\mu_i\mu_j}. \quad (2.89)$$

There is also a bound placed on what α can be. When α is too large, the matrix B_{mn} is no longer positive definite, which is the result of unstable transverse oscillation modes. This is the boundary for when a linear crystal transforms into a "zig-zag" type structure. There is also a minimum value of α for which resonant

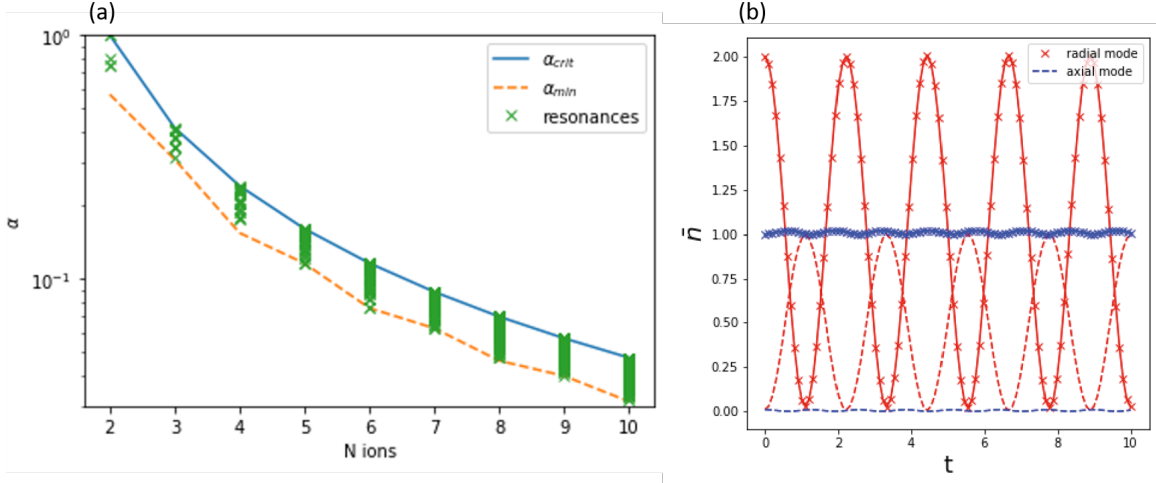


Figure 2.6. (a) shows the phase boundary for trapped ions. Above the critical value of α , the crystal is no longer linear. Each of the green x's represent a possible resonant mode coupling from the Kerr interaction. There are no resonances below the minimum value of α . (b) shows the non-dispersive regime where coherent exchange of phonons occur between coupled modes. This simulation is of a 3-ion crystal with the radial mode prepared in the $n = 1$ or $n = 2$ Fock state and the axial mode prepared in a $\bar{n} = 0.01$ thermal state.

mode coupling is no longer possible. This minimum value is given by

$$\alpha_{min} = \begin{cases} \frac{16\mu_N}{\mu_N(9\mu_N + 2\mu_{N-1} - 8) + \mu_{N-1}^2} & \text{if } N \text{ is odd} \\ \frac{4}{3\mu_N - 2} & \text{if } N \text{ is even} \end{cases} \quad (2.90)$$

The constraint on α limits the modes that can resonantly couple; Fig.2.6. shows the boundaries for this coupling. There are also two different types of resonances given in this problem. $\delta^{(-)}$ is associated with the creation of a radial phonon and simultaneous annihilation of both a axial and a radial phonon and is referred to as a resonance of the first kind. $\delta^{(+)}$ is the detuning from resonances of the second kind which creates two radial phonons in different modes by annihilating an axial phonon, with the resonance condition conserving energy. The coupling for this type of resonance tends to be exceedingly weak (Marquet et al., 2003).

There are several applications for the Kerr interaction. It could be used as a non-Gaussian operation, but other possible uses could include non-destructively reading out the motional modes of the ions S. Ding, Maslennikov, Hablützel, Loh, and Matsukevich (2017b). There are also different regimes of this interaction, near resonance with $\delta \sim 0$ and $\delta \gg \xi$. In the first regime there is coherent exchange of phonons between coupled modes. The second is the dispersive regime S. Ding, Maslennikov, Hablützel, Loh, and Matsukevich (2017a); S. Ding et al. (2017b); Home, Hanneke, Jost, Leibfried, and Wineland (2011); Nie, Roos, and James (2009) where there is no coherent exchange of phonons, and instead there is a frequency shift of the coupled modes relative to the occupation number of that coupled mode. I built out a set of simulations using the QuTiP python package Johansson, Nation, and Nori (2013) to fully characterize this interaction and understand the quantum mechanical evolution of states and extended the theory to include mixed-species ion chains. Evolution Under this Hamiltonian is shown in Fig. 2.7, which highlights the Wigner negativity associated with non-Gaussian states.

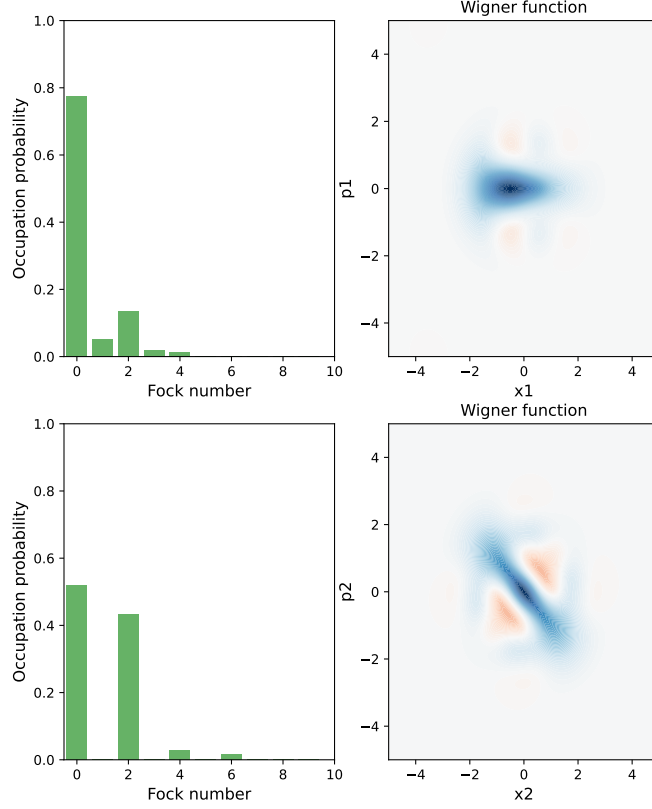


Figure 2.7. a) Number probability distribution and Wigner function for the axial mode highlighting the non-Gaussian nature of the state of some characteristic interaction time with the radial mode b) radial mode number probability distribution and Wigner function shown that only even Fock states are populated and more clearly showing Wigner negativity.

2.6.3 Tri-squeezing. The final non-Gaussian state worth mentioning is the tri-squeezed state. Generation of this state could be pursued with methods similar to parametric modulation. In this case, a cubic potential would need to be applied where additional trap electrodes have been used to implement this type of potential for mode coupling (Hou et al., 2022). Modulation of this potential at 3ω leads to only population of the $|3n\rangle$ states and has a 60° rotational symmetry

in phase space. Only a little exploration of this state has been done previously, and some using laser field interactions (Sutherland & Srinivas, 2021) have been shown Băzăvan et al. (2024), but knowledge of applications is underdeveloped.

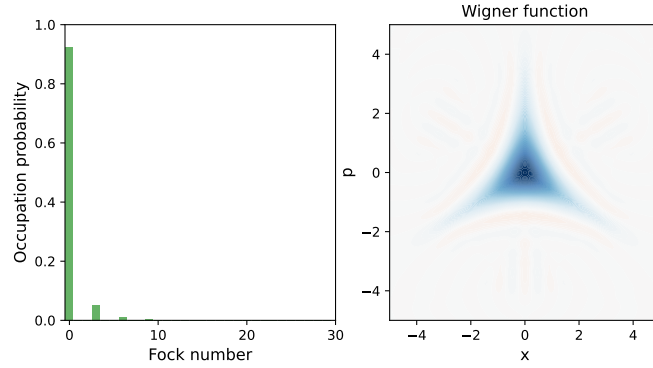


Figure 2.8. Number probability distribution and Wigner function for the tri-squeezed state showing the 3-fold rotation symmetry

CHAPTER III

ATOMIC SPIN QUBITS IN OSCILLATORS

Any charged particle can generally be trapped in a linear Paul trap with the proper choice of trapping parameters. For small excitations, a particle trapped in this potential can generally be treated as a quantum harmonic oscillator. Alkaline earth elements are most commonly trapped in ion traps, due to their simple electronic structure, with a single valence electron for singly ionized species. This structure makes state discrimination straightforward. Choice of atomic isotope is usually taken from the stable isotopes (longer lived radioactive isotopes can be used) that exist with reasonable abundances. The choice of isotope is also important for encoding qubits as many isotopes have nuclear spin that gives rise to hyperfine structure. All of these considerations dictate the experimental setups that are necessary to prepare, control and readout the chosen ion.

3.1 Calcium-40

The ion work described in this thesis has been done entirely with $^{40}\text{Ca}^+$. This isotope has 20 protons and 20 neutrons which give it zero nuclear spin meaning no hyperfine structure has to be considered. This species has transitions between different orbitals that are at wavelengths where semiconductor diode lasers are available. Calcium ions have a ground state $S_{1/2}$ manifold, short-lived $P_{1/2}$ and $P_{3/2}$ manifolds and metastable $D_{3/2}$ and $D_{5/2}$ manifolds. Transition wavelengths and Zeeman sub-levels are shown in Fig. 3.1

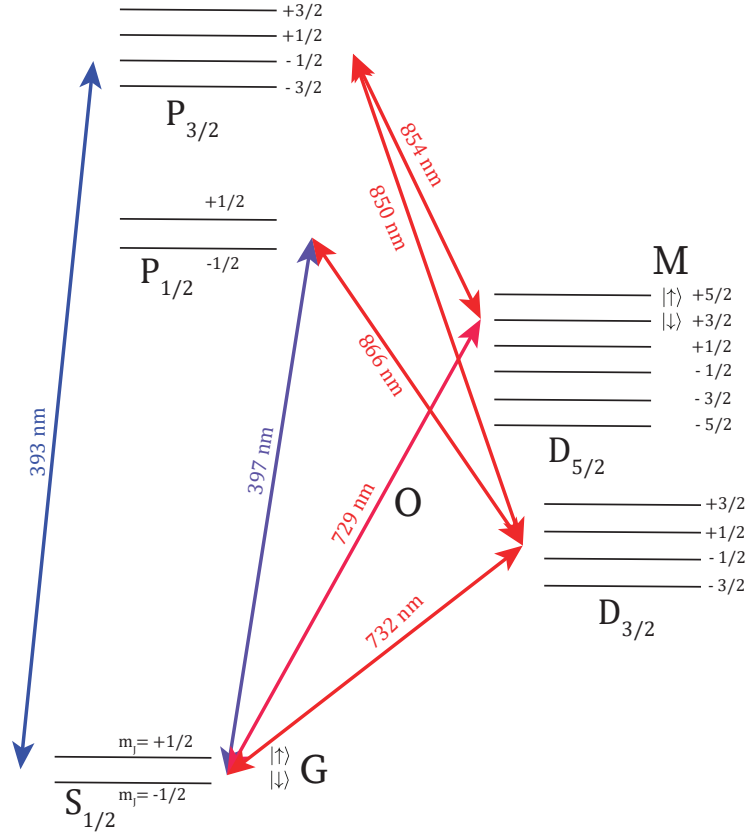


Figure 3.1. Atomic structure of $^{40}\text{Ca}^+$ showing different orbitals with their Zeeman sub-levels, which are non-degenerate when a magnetic field is applied. Also shown are the wavelengths of the transitions. OMG labels the different qubits that can be used that make use of different states.

3.2 Qubit encodings

Given the large number of electronic states that exist within atoms, the choice of qubit encoding is an important decision. The very short lifetimes of certain states makes them unsuitable for qubit levels. The common encodings used in atoms, with a similar structure to a calcium ion, fall into what is called the ‘OMG’ architecture (Allcock et al., 2021). How the different encodings work and what the letters in this acronym stand for are explained in the following sections.

3.2.1 Ground-state qubits. The G in OMG stands for ground state qubit. Ground state qubits are generally split by MHz-GHz depending on

if these are Zeeman qubits or hyperfine qubits. These qubit states can be coupled with microwaves or with two-photon stimulated-Raman transitions. In order for the state of the qubit to be determined, these qubits require “shelving” of one of the states to another level that does not participate in the cycling transition. A cycling transition uses a closed number of electronic levels and enables continuous absorption and stimulated emission in order to collect photons. This enables the ability to discriminate which qubit state is “bright” or “dark”. This type of shelving naturally requires the use of another metastable state that does not decay for the duration of the state readout. The ability to drive this transition coherently leads to another natural qubit encoding.

3.2.2 Optical qubits. Optical qubits, or O qubits, generally consist of one qubit state being in the ground state manifold with the other being encoded in a metastable manifold, separated by the energy of an optical photon. The transition for this qubit in $^{40}\text{Ca}^+$ is an electric quadrupole transition with a narrow linewidth, given the relatively long life of the metastable state, and can be directly coupled with a single optical wavelength photon. The laser used for coupling these qubit states generally needs to be linewidth narrowed by locking to a high finesse cavity. Readout of optical qubits does not require any extra shelving steps as the “bright” and “dark” states are naturally distinguishable. Having one state, with optical qubits, or two states, with ground state qubits, that are naturally coupled to the cycling transition means that operations that involve fluorescence couple directly to qubit states. This is a technical difficulty when using multiple ions of a single species as all qubits are projected when performing state detection. Many quantum computing implementations use two atomic species with spectrally resolved transitions (Home, 2013) to address this problem. In order to get around this

limitation, it is possible to encode both qubit states into a manifold where both qubit states are not affected by beams that drive fluorescence.

3.2.3 Metastable qubits. Qubit states encoded into metastable states, M qubits, require either Raman or microwave couplings, with qubit splittings also in the MHz-GHz range, similar to G qubits. This encoding scheme enables the ability to preserve quantum states when attempting to drive the cycling transition, but has some other technical difficulties. Zeeman qubit energy levels are made non-degenerate with the application of a magnetic field, but qubit splittings are naturally degenerate. This is not an issue in the ground state with only two Zeeman sublevels, but with more Zeeman sublevels in the metastable manifold this would be a problem. With degenerate energy splittings it would be impossible to only drive a two level transitions in the metastable manifold. The highest or lowest two energy states can be isolated with the application of an AC Stark shift that couples only to the other four levels (Sherman et al., 2013). State discrimination also requires moving population from one qubit state out of the metastable state, called deshelling. We have done significant work to implement qubits in metastable states with the ability to perform single qubit operations and perform state detection. The specific scheme for how this encoding works is shown in Fig. 3.2

3.3 Zeeman qubits in the $D_{5/2}$ Manifold

For the work presented here, our qubit is encoded as follows:
 $|1\rangle = |D_{5/2}, m = +5/2\rangle$ and $|0\rangle = |D_{5/2}, m = +3/2\rangle$. All the energy levels in isotopes without nuclear spin are linearly dependent on the magnetic field, for the magnetic fields used in this work. This dependence of the qubit states on the magnetic field makes any qubit encoding first order sensitive to fluctuations in the magnetic field. This makes working with Zeeman qubits more difficult given the

shorter coherence times without effective magnetic shielding in our setup. However, these qubits are straightforward for us to prepare and control, as compared to $^{43}\text{Ca}^+$ which has 48 $D_{5/2}$ states, and are sufficient for our experiments in this work, which makes this qubit an ideal candidate for initial exploration of using metastable qubits.

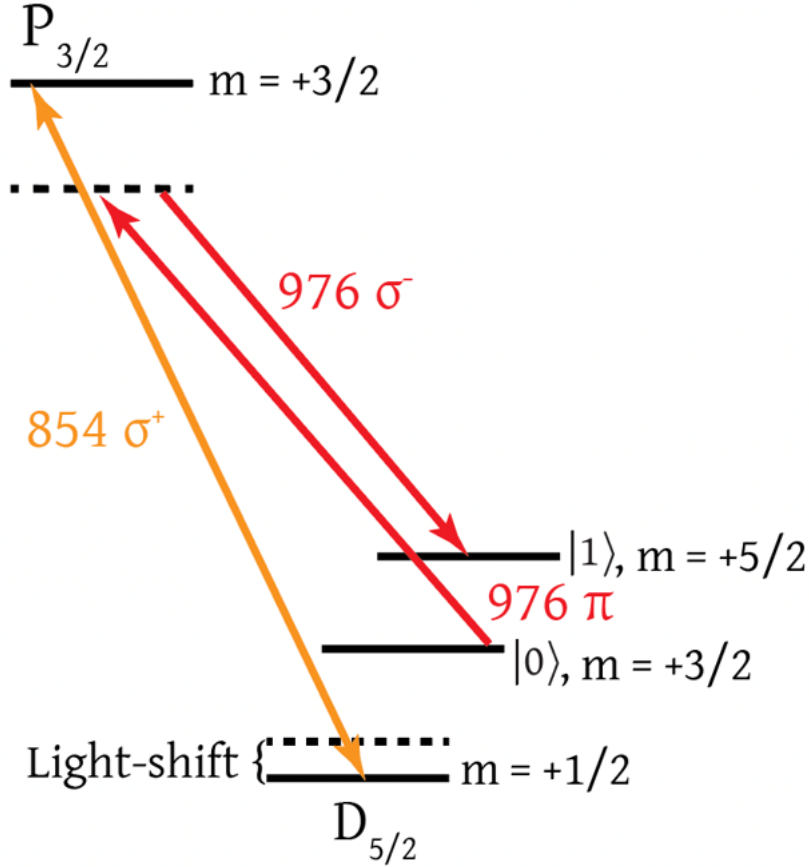


Figure 3.2. The $D_{5/2}$ Zeeman qubit. The qubit is isolated with 854 nm light shift beam and the gates are driven with the 976 nm Raman laser beams, or an RF magnetic field (not shown).

3.4 State preparation

Coherent qubit operations are the basis for performing quantum information experiments. In atomic systems we do not have a simple two-level system, requiring that we first must address the problem of population outside the qubit manifold.

Optical pumping is the method used independent of encoding in O,M, or G. O and G have the same preparation scheme as they both involve the same ground state level for one of the qubit states. This process can happen with a single round of optical pumping.

Optical pumping requires using a transition where the population ends in a fixed final state and cannot leave it on the timescale of the experiments. The cycling transition in $^{40}\text{Ca}^+$ involves applying 397 nm laser radiation on the $S_{1/2} \rightarrow P_{1/2}$ transition. Unfortunately, without another laser the population would not be entirely in a single state because 6% of the population will decay from the $P_{1/2}$ into the $D_{3/2}$ manifold. This can be addressed with 866 nm laser radiation to re-pump population from $D_{3/2} \rightarrow P_{1/2}$. Choosing the 397 nm light to be σ^+ polarized and the 866 nm light to have $\sigma^{+/-}$ components, leaves all population in $S_{1/2} m_j = +1/2$. This is the final step needed to get the O and G qubits prepared.

There are a few more rounds of optical pumping necessary to prepare the metastable qubit without using a 729 nm laser to drive the quadrupole transition (Sherman et al., 2013). After preparing $m_j = +1/2$ in $S_{1/2}$, 393 nm radiation polarized with σ^+ will drive population to highest energy Zeeman level in $P_{3/2}$, $m_j = +3/2$. Population will only decay into the $D_{5/2}$ manifold $\simeq 5\%$ of the time, spreading into $m_j = +5/2, +3/2, +1/2$ depending on the polarizations chosen. All population can be pumped to the $m_j = +5/2$ by using 854 nm π polarized radiation to re-pump the other states. Given the probabilistic nature of this process, repeating the whole sequence multiple times results in high fidelity state preparation, this sequence is shown in Fig. 3.3 for the qubit of choice in this work.

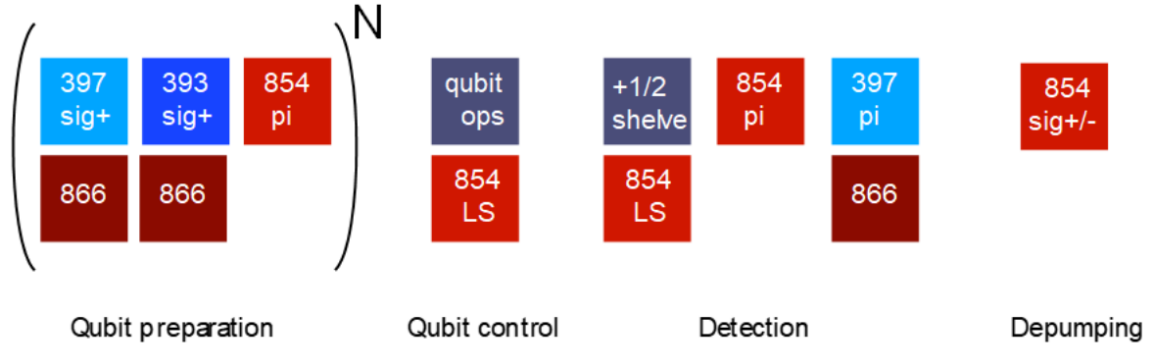


Figure 3.3. [Figure made by Alex Quinn] Laser and RF operations on the $^{40}\text{Ca}^+$ Zeeman qubit in the $D_{5/2}$ manifold. The preparation pulse sequence is performed N times, until preparation fidelity has saturated and further rounds of pumping don't significantly change the fidelity. The qubit operations (using the 976 nm Raman beams) may then be performed while the 854 nm light shift beam is on. To read out the qubit, we first shelve the $m = +3/2$ population in the $m = +1/2$ state and then deshelve this population with an 854 nm π -polarized laser beam. We then check for population in the $S_{1/2}$ and $D_{3/2}$ manifolds with the 397 nm and 866 nm laser beams. Finally, we can pump into the $S_{1/2}$ manifold with the 854 nm σ^- -polarized laser beam.

3.5 Single qubit rotations

The first step in realizing quantum gates is performing single qubit rotations. This primitive is the one of the requirements for being able to perform universal computations and is a stepping stone to performing two qubit gates. These qubit operations require the ability to couple two energy states of a quantum system in order to transfer population from one state to the other. The method used to couple these states has a large impact of the performance of the quantum computer. The three common methods have yielded near perfect single qubit operations (Ballance, Harty, Linke, Sepiol, & Lucas, 2016; Srinivas et al., 2021), but can have different potential benefits or drawbacks. What follows is a discussion of two methods used in the experiments described here.

3.5.1 RF magnetic fields. Using RF radiation to couple qubits states is a very attractive approach given that this method is straight forward

to implement in the trapped ion platform. Additionally, this method does not suffer from issues arising in steering the optical beams towards trapped ions and controlling the laser frequency, phase and amplitude noise. Addressing individual ions with this technique is non-trivial, but some techniques has been demonstrated (Srinivas et al., 2023). In order to use RF radiation we use a trap electrode that runs along the trap axis. A current is passed through this electrode which generates a dynamic magnetic field $B(x, t) = \cos(\omega_b t)B(x)$ with a gradient of the amplitude perpendicular to the trap axis. The ion and the magnetic field are coupled via the matrix element μ_x of the magnetic dipole moment. Writing the magnetic field with position x about the equilibrium position of the ion x_j plus a small displacement Δx we have $B(x) = B_j + \Delta x B'$. We can write the Hamiltonian corresponding to the qubit's harmonic motion and coupling to the magnetic field as

$$H = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega_n a^\dagger a + \sigma_x \cos(\omega_b t)\mu_x B_j + \sigma_x \cos(\omega_b t)\mu_x B'[q(a^\dagger + a)], \quad (3.1)$$

with Pauli operators σ_z and σ_x , mode angular frequency ω_n and $q = \sqrt{\hbar/(2m\omega_n)}$ being the ion's spatial extent in the ground state. Using the RWA this Hamiltonian is re-written as

$$H = \hbar\sigma_+(\Omega e^{-i\delta t} + \Omega_n a e^{-i(\delta+\omega_n)t}) + h.c. \quad (3.2)$$

with the detuning $\delta = \omega_b - \omega_0$ and the Rabi frequencies $\Omega = B_j\mu_x/(2\hbar)$ and $\Omega_n = B'\mu_x q/(2\hbar)$ (Ospelkaus et al., 2011). When the RF frequency ω_b is brought on resonance with the qubit frequency the first term in the above equation dominates the dynamics. This enables arbitrary single qubit rotations by simply tuning the field strength and/or interaction time. The second term corresponds the motional sidebands, which gives a method for a coupling between the internal states and the oscillator states given by $\sigma_+ a$. This coupling is only possible with a magnetic field gradient B' and requires a large magnetic field gradient to achieve a strong coupling

which is not accessible in our setup. The effects of spin-motion coupling will be discussed in later sections.

3.5.2 Raman transitions. Despite the advantages of using RF radiation for coupling two levels systems, the long wavelength of this radiation means a strong magnetic field gradient is required to enable spin-motion coupling. This coupling is the method for generating spin-spin entanglement and requires more complicated trap engineering as compared to laser methods. Laser beams that drive stimulated-Raman transitions can be used instead, which have shown some of the highest fidelity one and two qubit gates (Ballance et al., 2016). Besides the technical challenges of using laser beams, it is also important to understand the off-resonant spontaneous Raman scattering from using these beams (Ozeri et al., 2007). This effect is suppressed at large detunings and we will explore below how this coupling works to better understand the limitations. This type of two level coupling requires an third excited state to exist that can be adiabatically eliminated in the description by proper choice of laser parameters. The existence of the excited state is what gives rise to the possibility of stimulated-Raman transitions. The most common arrangements of these levels is the Λ scheme, where the two lower energy states (ground states) have a much smaller splitting than the detuning from the excited state. The Hamiltonian for this coupling can be written as

$$H = \Delta_1 |2\rangle \langle 2| + \Delta_2 |3\rangle \langle 3| + \Omega_{12}(\sigma_1 e^{i\delta t} + \sigma_1^\dagger e^{-i\delta t}) + \Omega_{32}(\sigma_2 + \sigma_2^\dagger), \quad (3.3)$$

where we can choose $|1\rangle$ to be at zero energy. $|2\rangle$ is the excited state and $|3\rangle$ is the other “ground” state. $\Delta_{2,3}$ are detunings of the lasers with respect to the dipole transition frequency to the excited state. This energy structure and laser tuning

can be seen in Fig. 3.4. There is a two-photon resonance when $\delta = 0$ or

$$\omega_{L_1} - \omega_{L_2} = \frac{E_1 - E_2}{\hbar} \quad (3.4)$$

where $\omega_{L_{1,2}}$ is the laser frequency of the field coupling $|1\rangle, |3\rangle$ to $|2\rangle$. The single photon Rabi frequencies are $\Omega_{12,32}$ which drive transitions with operator $\sigma_{1,2} = |1, 3\rangle \langle 2|$.

Population is prevented from getting into $|2\rangle$ with successful adiabatically elimination of this state. When the detuning from the intermediate excited level is sufficiently large this state can be ignored and the problem is reduced to an effective two-level system. This is qualitatively argued by thinking about the time-energy uncertainty principle such that when the atom transitions to this “virtual” state at energy Δ , it remains in this state for a time $\sim 1/\Delta$. In the event that $\Delta \gg \Omega_{12}, \Omega_{32}, \delta$ this time is much shorter than the evolution time for $|1\rangle \rightarrow |3\rangle$. This makes the dynamics dominated by the slow dynamics such that the probability amplitude in $|2\rangle$ adiabatically follows that of states $|1\rangle$ and $|3\rangle$.

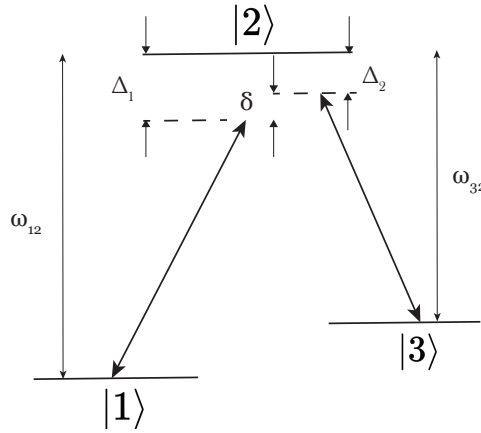


Figure 3.4. A three level Λ type structure with states $|1\rangle, |2\rangle, |3\rangle$. Transition frequencies between $|1\rangle, |3\rangle$ and $|2\rangle$ are ω_{12}, ω_{32} with coupling lasers detuned by Δ_1, Δ_2 and relative detuning δ

3.6 Spin-motion coupling

So far the motion of the ion hasn't been thoroughly addressed when considering coupling of a two-level system. Ignoring the motion is reasonable to do when treating the ion like a point particle with a fixed location given by its equilibrium position. In reality there ion position is described by a wavefunction with a non-zero spatial extent such that the ion feels a position dependent force. Even with this consideration it is possible to ignore this force if we assume the the wavelength of the applied field is much larger than the wavefunction extent of the motional state, such that the ion sees no effective gradient. This consideration of the ion motional wavefunction and the wavelength of the field factor into what is called the Lamb-Dicke regime (Meekhof et al., 1996).

3.6.1 Lamb-Dicke regime. In one dimension, if we consider the position of the ion x , we can write the coupling of a field ion about the equilibrium position as

$$kx = kx_{eq} + \eta(a + a^\dagger) \quad (3.5)$$

where

$$\eta \equiv kx_0 \quad (3.6)$$

with k being the wave vector of the field and $x_0 = \sqrt{\hbar/2m\omega_x}$ being the spread of the ground state wavefunction of the ion with mass m . The energy transferred to the ion from a photon of the field absorbed by the atom is given by $\hbar\omega_x\eta^2 = \hbar^2k^2/2m$. The Lamb-Dicke parameter η therefore describes how strongly the field couples to a mode of the ion. Operation within the Lamb-Dicke regime is satisfied as long as $\langle\psi|k(x - x_{eq})|\psi\rangle \ll 1$ where $|\psi\rangle$ is the motional state wavefunction. This criteria is to just re-state that the amplitude of the ion's motion is small compared

to the wavelength. This criteria is essential for the validity of the Hamiltonians we use to describe the spin-motion coupling.

When the internal levels are coupled by an electric field the Hamiltonian is written as

$$H = -\boldsymbol{\mu}_d \cdot \mathbf{E}(x, t) = \hbar\Omega(\sigma_+ + \sigma_-)(e^{i(kx - \omega t + \phi)} + e^{-i(kx - \omega t + \phi)}) \quad (3.7)$$

with electric dipole moment $\boldsymbol{\mu}_d$. When moving into the interaction representation we get

$$H_I = -\hbar\Omega\sigma_+ \exp(i[\eta(ae^{-i\omega_x t} + a^\dagger e^{i\omega_x t}) - \delta t + \phi]) + h.c. \quad (3.8)$$

with $\delta \equiv \omega - \omega_0$. Now, considering the Lamb-Dicke regime we can make the approximation that

$$\exp(i\eta(ae^{-i\omega_x t} + a^\dagger e^{i\omega_x t})) \approx 1 + \eta(ae^{-i\omega_x t} + a^\dagger e^{i\omega_x t}). \quad (3.9)$$

With this approximation we get effectively three different terms we can identify, each with their own unique effects.

3.6.2 Carrier. When $\delta = 0$, the first order term dominates and the other terms can be ignored making the RWA. This term simplifies to

$$H_{carrier} = \hbar\Omega(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi}) \quad (3.10)$$

which drives population between qubit states without affecting the motional state..

3.6.3 Red sideband (RSB). When $\delta = -\omega_x$ the carrier term retains a time dependence which will now be ignored with the same RWA when $\omega_x \gg \Omega$.

There are second order terms that no longer have time dependence that are given by

$$H_{RSB} = \eta\hbar\Omega(\sigma_+ a e^{i\phi} + \sigma_- a^\dagger e^{-i\phi}). \quad (3.11)$$

When starting in a lower internal energy state, this Hamiltonian leads to a flip of the qubit state to the higher energy state while subtracting a phonon from the

motion. This transition occurs with Rabi frequency

$$\Omega_{n \rightarrow n-1} = \Omega \sqrt{n} \eta \quad (3.12)$$

which is lower than the carrier Rabi frequency when $\eta \ll 1$.

3.6.4 Blue sideband (BSB). The last resonant condition for second order terms is when $\delta = \omega_x$. The resulting Hamiltonian is now

$$H_{BSB} = \eta \hbar \Omega (\sigma_+ a^\dagger e^{i\phi} + \sigma_- a e^{-i\phi}), \quad (3.13)$$

which now when starting in the lower energy state, drives a state flip and adds a phonon to the motion. The Rabi frequency for this transition is given by

$$\Omega_{n \rightarrow n+1} = \Omega \sqrt{n+1} \eta. \quad (3.14)$$

These Hamiltonians are known more commonly as the Jaynes-Cummings and anti-Jaynes-Cummings Hamiltonians and are very important for engineering many interactions in trapped-ions and other platforms (Shore & Knight, 1993). In our work, this interactions will be the foundation of how to determine the motional state, but they can be used to create spin-spin entanglement and be used for a host of other interesting physics with longer ion chains.

3.7 State detection

State readout essentially entails turning all qubit encodings into optical qubits. This is required because it enables a state-dependent fluorescence. In order to perform state detection with metastable qubits one of the qubit states must first be deshelled. The $|\downarrow\rangle$ qubit state is deshelled using a pulse of 854 nm laser which is π polarized meaning the laser does not couple to the $|\uparrow\rangle$ state. Given $\sim 5\%$ of the population will return to the $D_{5/2}$ $m_j = +5/2, +3/2, +1/2$ states after decaying from the $P_{3/2}$ manifold, shelving the $|\downarrow\rangle$ population in the $m_j = +1/2$ is required to prevent a readout error. This sequence is shown in Fig. 3.3. After the $|\downarrow\rangle$ state is deshelled the cycling transition with 397 and 866 nm lasers, will make population

in this state fluoresce, making this the “bright” state and leaving any population in the metastable manifold “dark”. The ground state is “bright” because we can collect and count photons at 397 nm.

In order to assign a state based on the number of photons that are counted, a threshold is assigned (Myerson et al., 2008). The bright state will scatter photons that are collected with some efficiency η at a rate of R_B , but detectors also have a dark rate detection rate of R_D . These fixed rates provide Poissonian statistics given by

$$P(n) = \frac{\lambda^n e^{-\lambda}}{n!}, \quad (3.15)$$

where n is the number of photons and $\lambda = R t_b$, the product of the detection rate R and the detection time bin t_b .

There are inherently errors in this method of state detection. The average readout error is $\epsilon = (\epsilon_B + \epsilon_D)/2$, where ϵ_B is the fraction of events where an ion prepared in the bright state was measured to be in the dark state (likewise for ϵ_D). A statistical treatment is used to determine the threshold for how many photons there need to be for the ion to be considered bright. This threshold is chosen based on the intersection of the bright and dark Poissonian distributions.

$$n_{threshold} = \frac{t_b(R_b - R_d)}{\ln\left(\frac{R_b}{R_d}\right)} \quad (3.16)$$

This intersection point shows how longer detection times can help to reduce readout errors, but will ultimately be limited by the lifetime of the qubit. Larger bright count rates and longer detection time bins helps separate the means of the Poissonian distributions.

This method can be extended to more than 1 ion but does not provide the ability to distinguish which ion is bright, only the number of bright ions. The thresholds are decided again with the intersection of Poissonian distributions given

by the count rate for the number of bright ions, and can be seen in Fig. 3.5 with a 2 ms readout time. The overlap is not equivalent for each distribution and can become significant with a larger number of ions as the distributions spread more and more. For long ion chains this method will lead to significant readout errors and other techniques are required. For all the work in this thesis this method provides readout fidelity $>99\%$ for 1 ion.

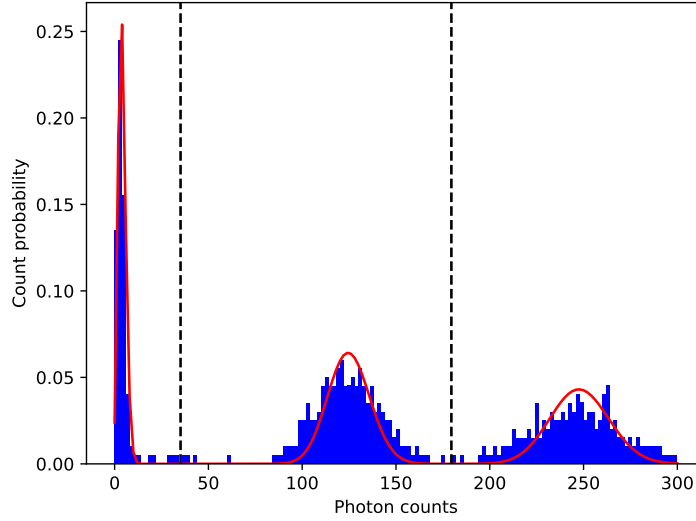


Figure 3.5. Histogram of counts recorded in 100 shots across 81 iterations of an arbitrary experiment. The red curves show the theoretical Poissonian distributions for a given count rate and detection period of 1 ms. Black vertical dash lines show the intersection points between the 0/1 bright state and the 1/2 bright state.

3.7.1 Mode thermalization. The cycling transition, which used to discriminate between bright and dark states, means that millions of photons are scattered from the ion every second. This is a very powerful technique for determining the internal state of the ion, but there are consequences for the motional states of the ion. Motional states, that have been prepared into some non-thermal state, are driven towards a thermal state by scattering of photons when a laser is tuned to the red sideband. We have already seen that the recoil energy is

the energy given to the oscillator from the photon and depends on the mass and frequency of the ion and illuminating field.

This effect of photon scattering makes it impossible to preserve motional states and perform readout of qubit states. We will consider in Ch. VII how this problem can be overcome with longer ion chains for particular geometries. In Ch. IV we will explore how this same effect of photon scattering can be used to cool motional states that start in a relatively large thermal state.

CHAPTER IV

EXPERIMENTAL SETUP

Many experimental systems are used in order to trap, cool, manipulate and detect the state of ions. An easy to follow discussion of how an ion trap works can be found in (Foot, 2005) and will not be exhaustively covered here. This chapter will provide descriptions and applications of experimental systems that enable quantum information processing.

4.1 Rod trap

Ions, in this work, are trapped using a room temperature linear Paul trap in our lab with a schematic shown in Figure 4.1. A view of the physical trap is given in Figure 4.2. The ion-electrode separation ' r_0 ' is 0.75 mm, the needle-to-needle spacing ' a ' is ~ 3 mm, and the rods have a diameter ' $2r_e$ ' of 0.5 mm. In order to minimize the rate of background gas collisions, we keep the trap under ultra-high vacuum, reaching pressures of order 10^{-11} Torr.

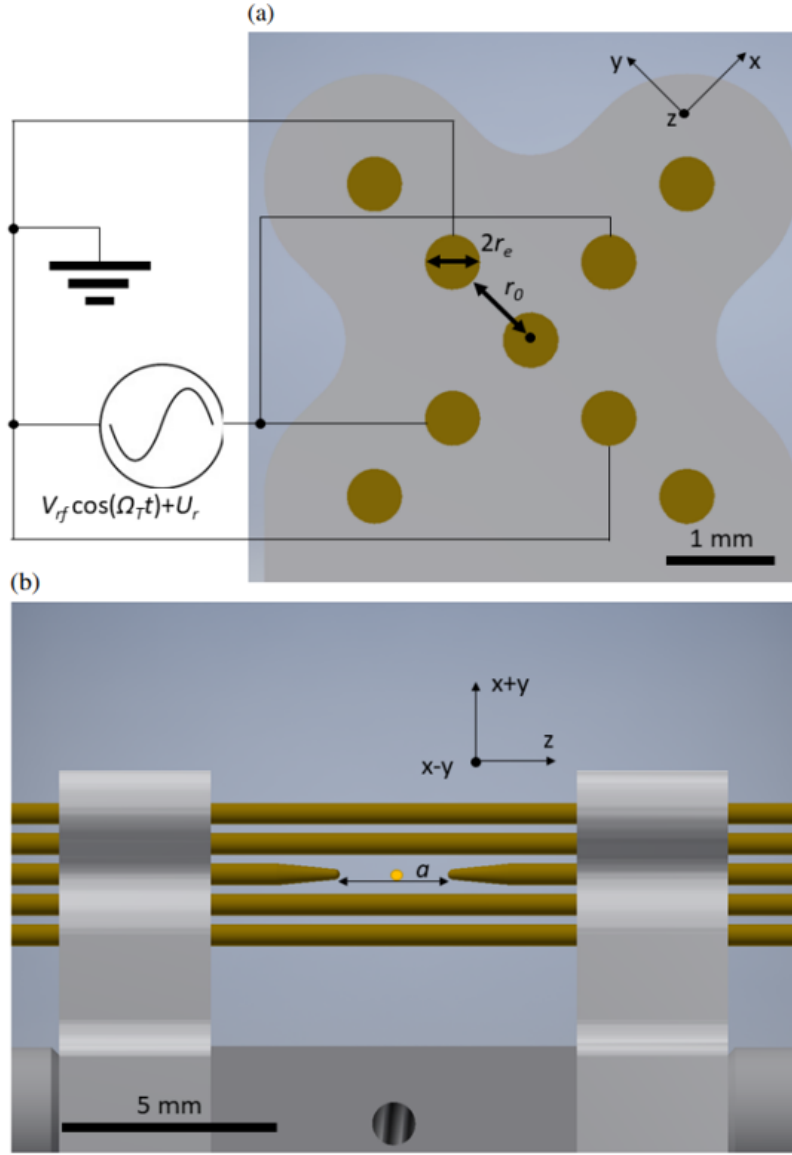


Figure 4.1. (a) Schematic view of our linear rod trap along the trap axis. The gold circles are cross-sections of the different rods in the trap. In the figure, r_e is the electrode radius and r_0 is the trap axis to electrode surface distance or trap radius. Two opposing inner rods are connected to the oscillating voltage V_{rf} at frequency Ω_T with offset U_r and the other two inner rods are held at ground. (b) Schematic side view of the linear rod trap where a is the tip-to-tip distance between the needles and the yellow dot is an oversized ion giving its approximate location in the trap. Following the axes in figure (a), the upward direction in (b) is the $(x + y)$ axis and the out-of-the-page direction is the $(x - y)$ axis.

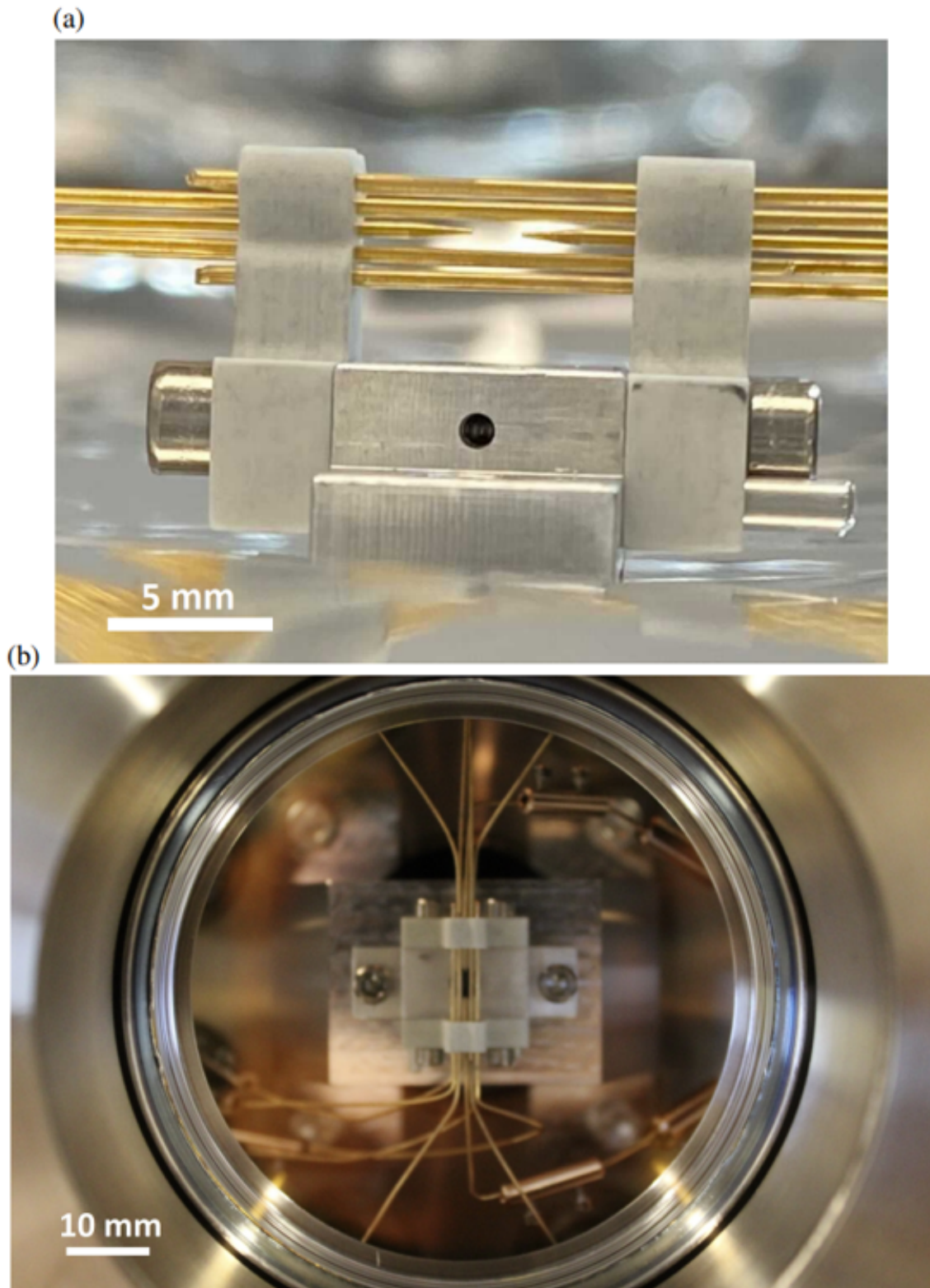


Figure 4.2. (a) The assembled rod trap. (b) The rod trap mounted in the vacuum chamber, as seen through a viewport.

4.2 RF electronics

This section discusses how the trapping potential is generated and the requirements for stable trapping, without getting into the full theory of how an ion trap works. The potential to trap an ion of mass m and charge e can be written as

$$\psi = \frac{e^2 V_0^2 \eta^2}{4mr^4 \Omega_{rf}^2} (x^2 + y^2). \quad (4.1)$$

Ω_{rf} is the RF angular frequency of the trapping potential, r is the ion electrode separation, V_0 is the amplitude of the RF voltage and η is an efficiency factor that depends on the geometry of the trap. This true potential provides a ponderomotive pseudopotential with an oscillation secular frequency of

$$\omega_s = \frac{eV_0\eta}{\sqrt{2}mr^2\Omega_{rf}} \quad (4.2)$$

The equations of motion of the ion is given by the Mathieu equations and have stability criteria that depend on several of these parameters. For common masses of atoms used in an ion trap, the trapping potential must be an AC potential, in radio frequency band, for stable trapping. Depending on the mass of the ion, the geometry of trap, the trap frequency and desired secular frequency, necessary trapping voltages for stable trapping can vary from V to kV. Ion traps can trap room temperature atoms with trap depths of order 10 eV which enable days long ion lifetimes. Increasing the trapping voltage also increases the mode frequency which can enable more efficient cooling of motional modes. Working with excessively high voltages should be avoided to prevent electrical breakdown and experimentally intrusive temperatures due to power dissipated in the trap.

4.2.1 Helical resonator. The helical resonator used in these experiments is a quarter wave resonator, meaning one end of the coil is ground and the other end is at the voltage maximum. This resonator has an antenna

coil on the input that is inductively coupled to the main coil, which allows for impedance matching with a high ratio of output to input voltage. A large voltage step up and narrow bandwidth of the resonator are desired to lower the input power requirements and reduce noise coupling to the trap.

The length and diameter of the resonator and main coil length are all parameters that can effect the resonant frequency and the quality factor of the resonator. After testing, a 6" diameter resonator provided a suitable quality factor and resonant frequency. The design follows a lumped circuit element model (Siverns, Simkins, Weidt, & Hensinger, 2012), but manual adjustment of antenna and resonator coils is necessary to get the best possible quality resonator. The resonator was characterized using a tracking generator coupled to the resonator and looking for the resonant drop in reflected power on the spectrum analyzer. The quality factor is given by the full width at half max (-3 dB) of the resonance. The voltage step up is measured as a ratio of the input to output voltage levels. Capacitive loads, like a scope probe, changes the impedance, so a calibrated capacitive divider was used to reduce the capacitive load to a negligible level. We have achieved a quality factor as high as 150 and a step up of nearly 80 with a resonant frequency of 14.13 MHz. With a 1 W amplifier you have ~ 10 V amplitude in a $50\ \Omega$ system. With a voltage step up of 80 this gives approx 0.8 kV, and a ~ 1.8 MHz radial secular frequency.

4.2.2 Squareatron amplitude stabilization. Running an RF ion trap for coherent control of ions, with long motional coherence times, is not as simple as connecting a waveform generator to the helical resonator when using radial modes. While this method works when using axial modes that are not coupled to this potential, waveform generators alone can introduce significant noise, which leads to motional decoherence. The goal is to achieve ultra-low noise RF

signals, which provide the trapping strength and enable the motional coherence time we need. Amplitude modulation (AM) noise leads to fluctuations of the trapping frequencies, and causes motional decoherence. The motional coherence time τ_c , assuming white noise, is related to the AM noise power spectral density N_0^{AM} (Harty, 2013) by

$$\tau_c^{-1} = \frac{1}{2} \omega_r^2 \frac{N_0^{AM}}{P_c}. \quad (4.3)$$

P_c is the power in the RF carrier and ω_r is the motional frequency of the mode of interest.

4.2.3 Filtering AM Noise (Squareatron 5000). The Squareatron 5000 is a circuit that filters the AM noise of a clock source and produces an ultra-low noise RF tone (10s of MHz) and was originally designed at Oxford University (Harty, 2019). Modifications were made to the original design to improve performance and enable greater flexibility. The circuit diagrams are included in Appendix A. The board is designed to use a highly saturated amplifier to “clip” the input signal, producing a constant amplitude square-wave. This element is combined with a pass band filter to strip the harmonics and output the original frequency sine wave with the AM noise greatly reduced.

The basic design of the circuit involves using a low phase noise logic converter (LTC6957) (“LTC6957 Low Phase Noise, Dual Output Buffer/Driver/Logic Converter”, 2017) on the input and a low drift precision reference to have a stable power supply. This combination of devices generates a low noise output with no way to tune the output amplitude.

4.2.4 Tuning board. Having the ability to tune the amplitude of the output directly allows for tuning of the ion trap secular frequencies. This will be critical for CVQC in as mode frequency tuning is required for operations

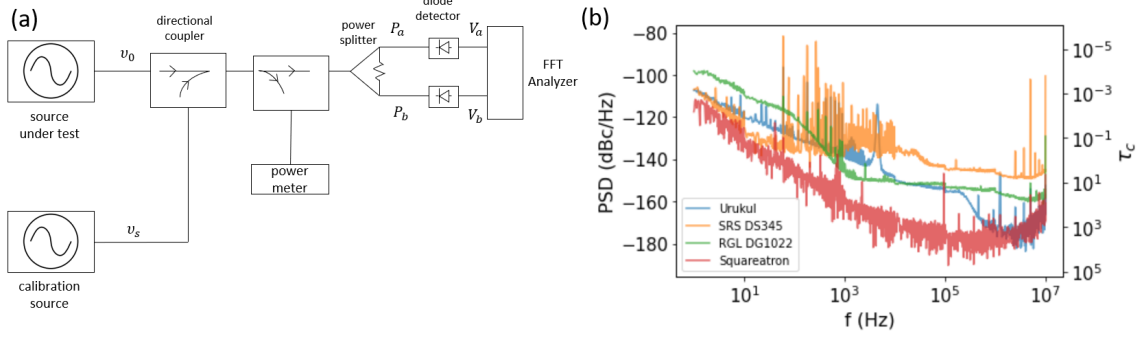


Figure 4.3. (a) The setup for measuring AM noise is shown (Enrico, 2006). A calibration source at frequency ν_s is used to calibrate the power spectral density (PSD) to be in terms of the RF carrier power at frequency ν_0 . A power splitter splits the signal into P_a and P_b and two diode detectors are used to pick out the AM noise from the carrier. The two channel setup allows for a correlated measurement and gives a noise level below the noise floor of each detector. (b) shows the measured AM noise power densities typical lab devices. The right hand ordinate gives the motional coherence time for a 3 MHz mode that would be obtained with white amplitude noise at the level given by the left hand ordinate.

like coupling modes with the Kerr interaction, amongst other operations. To achieve this goal, I have designed and tested a circuit board that attaches to the Squareatron (Metzner, 2020). A 16-bit digital to analog converter (DAC) has a reference supplied by the stable power reference from the Squareatron and can be programmed to adjust the power rail of the logic converter. The circuit diagram is shown in Appendix A. This additional circuit was tested and has shown that it does not inject noise into the Squareatron. We would like to have coherence times that are longer than the heating rate for the mode of interest. This would require coherence times of at least 100 ms for our heating rates, which are shown in section 4.4.3. Fig. 4.3 shows the motional coherence times above this threshold and orders of magnitude better than typical lab devices.

4.2.5 Amplifier. The necessary power to trap an ion is well above what the Squareatron can output. The helical resonator, described in section 4.2.1,

provides a significant voltage step up, however, an amplifier is still necessary after the Squareatron to get to the power levels we need. The Minicircuits ZX60-100VH+, 36 dB gain amplifier is used to get to the desired powers. The amp is connected to the Squareatron with attenuators to avoid compression and output over 30 dBm of power. To increase the secular frequencies, which might be desirable for faster operations involving the motion, even more power is necessary, which likely also requires an increase in the trap RF frequency for stability. To achieve this, we have designed a 4 Watt amplifier that has two four-way splitters and four amplifiers. The splitters divide and combine 6 dB of power coherently before and after the amplifiers; this allows us to run at a higher output power without compression. This parallel amplifier has 1 dB compression at 34 dBm out power and saturates at 38 dBm. The AM noise test with both of these amplifiers show the noise levels are not noticeably above the levels without an amplifier. Note that the 4 amp configuration divides the additive AM noise from the amp by 4 (Scott, 2015).

4.2.6 Temperature stabilization. After assembly of the Squareatron and amplifier was completed, the combined temperature coefficient was measured to be -1300 ppm/k, which was determined by measuring the change in the output power as a function of the temperature (controlled with a hotplate). Power fluctuations are very important to stabilize due to the direction conversion from RF power to trap frequency. Fast fluctuations can lead to motional decoherence and slow drifts need to be calibrated constantly. We have measured a secular frequency response of -3 Hz/mK, which matches closely to the measure temperature coefficient. This means we require few mK temperature stability to the frequencies from drifting significantly on the time scale of the experiment. Fig. 4.12a) shows a housing that was constructed that hosts a Peltier TEC connected

to a thermostat. Both the Squareatron and amplifier are in thermal contact a bulk plate of aluminum and the whole box is filled with foam insulation. The Thermostat (*Thermostat*, 2022) runs a PID loop that controls the TEC in order to reach a target temperature. PID loop parameters are automatically tuned to minimize temperature oscillations. Fig. 4.12b) shows the temperature stability that was achieved after the thermostat is used.

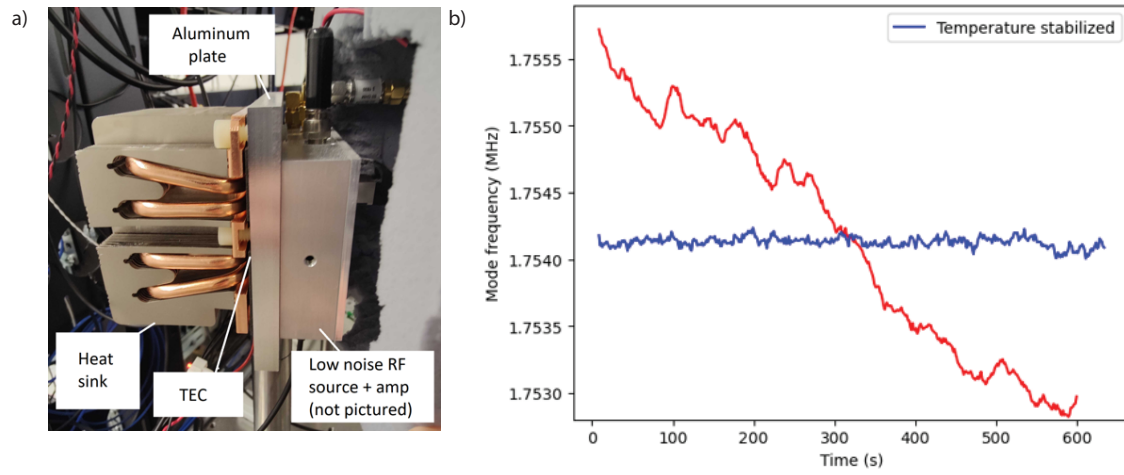


Figure 4.4. a) combined mounting of Squareatron and amplifier in thermal contact with an aluminium plate. The aluminum plate has a heat sync mounted on top of a TEC to help stabilize the temperature. The whole mounting is placed inside and insulated box to prevent airflow from causing temperature fluctuations. b) time series data of the trap secular frequencies with and without the active temperature stabilization showing temperature stabilization reducing fluctuations to <100 Hz over hours.

4.3 Laser systems

There are a broad range of lasers required to go from neutral Calcium to ground state cooled and coherently manipulated ions. A large fraction of the physical space in the lab can be dominated by laser heads and beam delivery optics. To minimize the space required for this, we have built rack mounted optical setups that contain all the lasers and optics to deliver laser beams to several different traps. This setup also enables active locking each laser to a fixed

wavelength and monitoring of the lasing mode. Each of the laser systems used in the experiment are detailed in the following sections.

4.3.1 Laser rack. All the lab’s laser systems are stored in one room with three racks: a rack containing drawers which house our Toptica Littrow ECDL lasers (Model ‘DL Pro’), a rack containing Toptica laser controllers (Model ‘DLC Pro’) and photoionization laser board, and a rack housing the wavemeter and laser diagnostic equipment. These rack mounted drawers can be seen in Figure 4.5, with the standard laser setup inside the rack drawer shown in Figure 4.6. In each drawer, a Toptica laser is mounted to an optical breadboard. The Toptica laser parameters are set by the controllers mounted in the adjacent rack. The beam is redirected, from the laser head, by a periscope to be level with half-inch optics mounts. In order to prevent amplified stimulated emission a few nanometers away from the lasing mode from reaching the optical fiber, the beam is then reflected from a diffraction grating before being sent down a path of polarizing beam splitters, where half-waveplates allow us to adjust how much power goes down each arm. These separated beams are then coupled into fibers to be delivered to the AOM boards in the trap rooms. One of the beams on each board is sent to the wavemeter and optical spectrum analyzer(OSA) on the OSA board, where the wavemeter is used to stabilize the laser frequencies.



Figure 4.5. The laser racks. Contains our breadboard laser systems used for ionization, cooling, preparation, and readout.

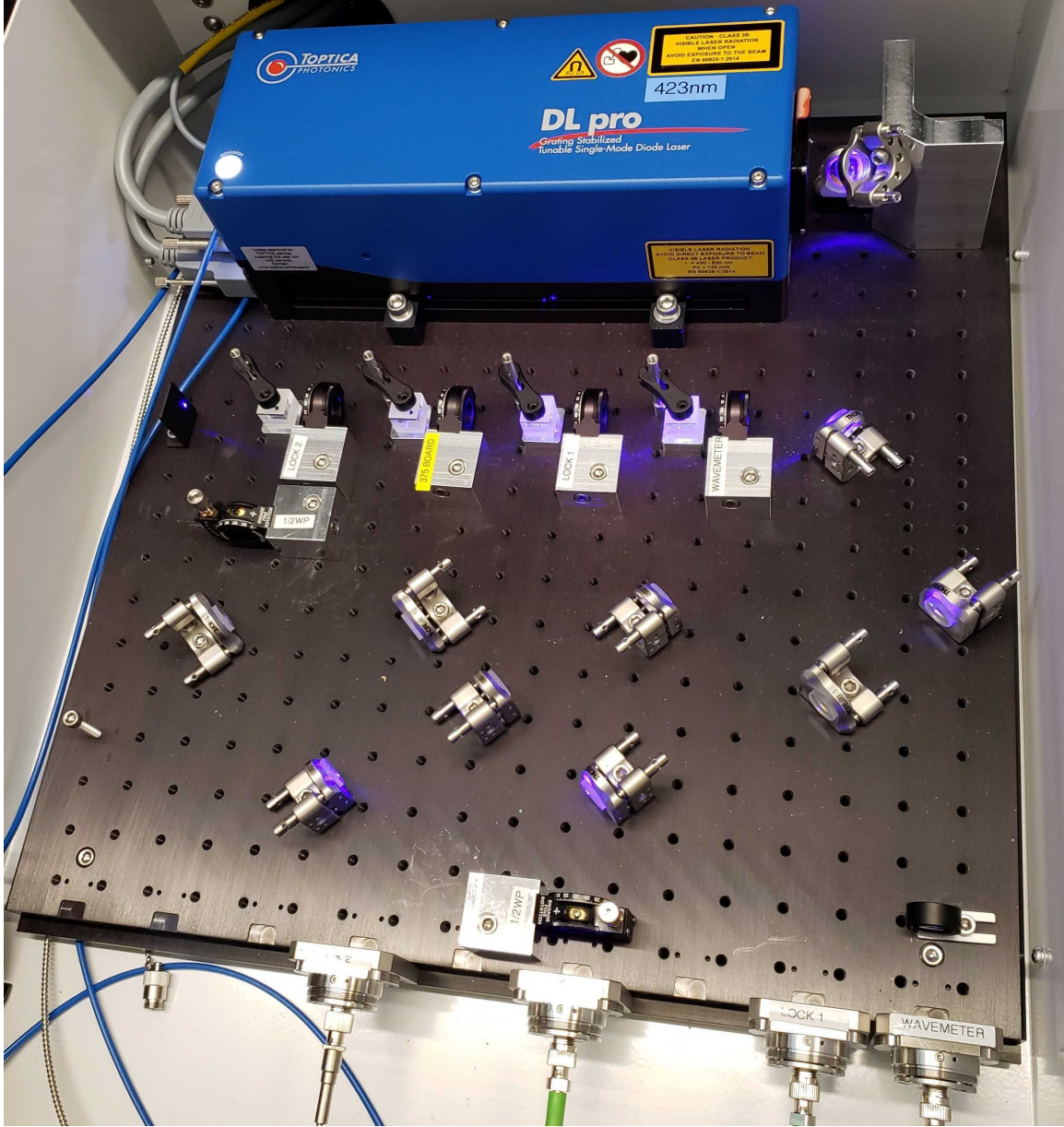


Figure 4.6. The inside of a laser rack drawer. The beam from a Toptica laser is picked off by multiple polarizing beam splitters and is coupled into multiple fibers to be sent to the trap and the OSA board.

4.3.2 Photoionization. Photoionization is used in our lab instead of electron bombardment in part because photo-ionization allows for isotope selectivity. Photo-ionization also reduces the charging of insulating parts of the trap which causes drifting electric fields. Photo-ionization has been shown to be $\sim 10^4$

times more efficient (Lucas et al., 2004) than electron bombardment, leaving fewer neutral atoms that are deposited onto the trap. Neutral atoms are loaded into an oven, made from a steel tube, as granules of calcium. About 4 amps is passed through the oven heating, vaporizing, and ejecting the neutral atoms from a hole in the side. The atoms are ejected into the interaction region where the lasers are used for ionization. Two lasers at 423 nm and 375 nm are necessary. The 423 nm laser will excite the neutral to a high lying $4s4p^1P_1$ level at which point the 375 nm laser will take it to the continuum.

An optical bread board that has the 375 nm laser mounted on it and the 423 nm laser brought on from a optical fiber is used to combine these two laser beams for delivery to the trapping setup. Four different traps can be run by using beam splitters to pick off a quarter of the 375 nm laser power, shown in Fig. 4.7 and Fig. 4.8. The 423 and 375 nm lasers are brought together using a dichroic mirror and coupled into a fiber that is delivered to the trap. Shutters are also mounted in front of each fiber coupler so the lasers can be shut off once the required number of ions are loaded into the trap.

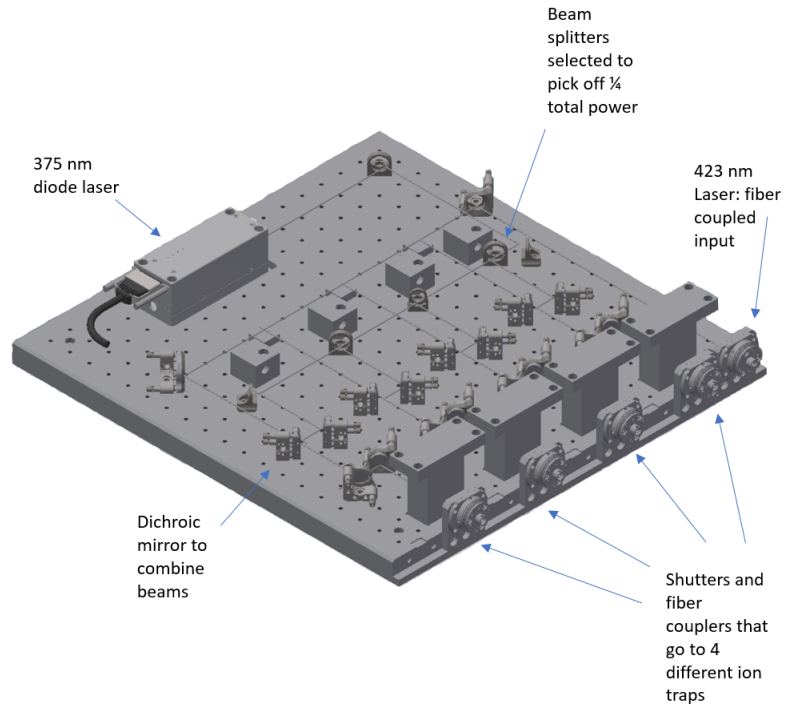


Figure 4.7. A 3D rendering of the photoionization board showing the 4 possible trap connections and the optics to combine the 375 nm and 423 nm light.

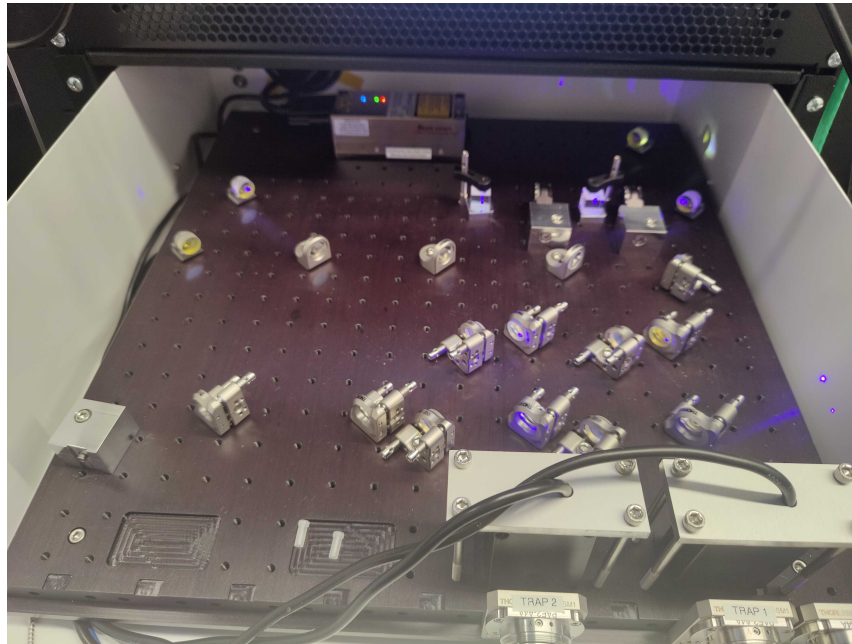


Figure 4.8. Image of the photionization board mounted in a drawer with two output collimators and shutters installed.

4.3.3 Locking lasers to a wavemeter. Analysis of the lasing mode and wavelength of each laser is made possibly by fiber coupling the individual lasers to two different fiber switches. The fiber switches are designed to operate in the range of the UV and near-infrared wavelengths that the lasers lie within. The output of the fiber switches are coupled onto the OSA board. This board has two different scanning Fabry-Pérot interferometers (ThorLabs SA200) for each of the primary wavelength ranges used in our lab.

An additional fiber coupler couples the light to a wavemeter, via a photonic crystal fiber (PCF) that is single mode for all wavelengths, to monitor and correct drift in frequency of the lasers using the Oxford wavemeter analysis and display software (WAnD) (Oxford, 2019). An error signal is generated, for a proportional-integral-derivative (PID) loop to stabilize, by recording the actual wavelength and taking the difference with a given reference wavelength. The laser grating has a piezo attached which can be fed back on to stabilize the wavelength. The optimal gain for each laser was found by ramping the gain and monitoring the laser frequency variance over a 1 minute window. The wavemeter is temperature stabilized and has proven to drift by less than 100 MHz per year allowing for the wavemeter to operate without regular calibration with a frequency reference like the ion.

ECDLs can commonly start lasing on multiple modes, which can be prevented by monitoring the lasing mode and tuning the laser when necessary. The laser mode is monitored with the scanning Fabry-Pérot cavities, which is driven by a piezo electric crystal with a photodiode to collect the transmitted light. The piezo is used to sweep the length of the cavity which causes light to be transmitted only when the cavity length is resonant with the laser mode. To operate the OSA board electronics including a piezo driver and a photodiode amplifier are necessary.

These electronics and cavity have been characterized to compare the Finesse to the cavity specifications, as well as the signal to noise ratio (SNR) of the photodiode to compare it to the amplifier design spec. The cavity has a calculated Finesse between 210-270 for both cavities which matches well to the specification. The photodiode amplifier output is connected to a digital multi-meter (DMM) (Keithley DMM6500). The DMM is triggered off of the AWG that generates the ramping voltage which scans the cavity piezo. The measured voltages as displayed along with the reference detuning on the WAnD software Oxford (2019) The completed configuration can be seen in Fig. 4.9.

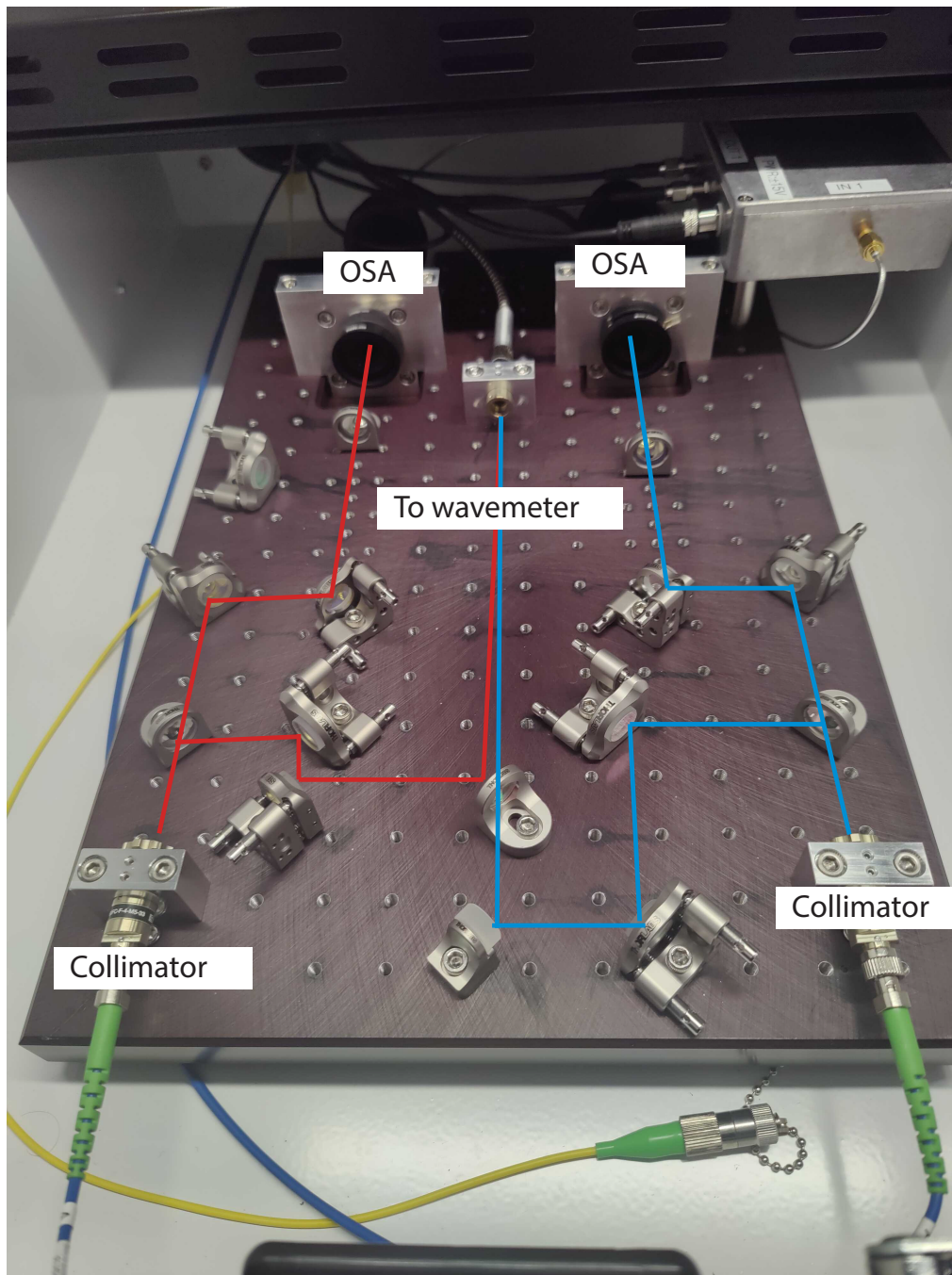


Figure 4.9. A picture of the OSA board mounted in a drawer showing the input collimators, the Fabry-Perot cavities and the output collimator to the wavemeter.

4.3.4 Raman laser system. The laser system that enables metastable coherent operations is different from the other laser systems presented here. The wavelength of this laser is 976 nm which means it is 122 nm detuned from $D_{5/2} \leftrightarrow P_{3/2}$ resonance in calcium. This large detuning reduces the spontaneous Raman scattering rate, but means a significant amount of power is required to get two qubit gate operations on a timescale faster than decoherence. While two photons from a single beam is sufficient to drive Raman transitions, as long as the polarization has π/σ^- components, we built an orthogonal geometry, see Fig. ??, to have a non-zero Δk that is in the radial plane. This geometry allows for coupling to both of the radial modes, but not the axial mode.

The 976 nm laser setup for this work was using a free space (IPS I0976SB0750B) direct-diode laser. This laser is a volume-holographic (VHG) hybrid external cavity laser (HECL) mounted in a butterfly package. This package is mounted on a Koheron industrial laser controller which manages temperature and laser current. The free space laser is directed into a Newport ISO-04-980-MP optical isolator to prevent optical feedback. The beam is then split into two paths with a waveplate and a beamsplitter. Each separate path is focused through an AOM to have separate switching and frequency control of the individual beams. There is an optical pickoff on one path which sends light to the OSA board for laser monitoring. Both beams are then collimated into fibers which are delivered onto raised platforms on either side of the trap. Piezo mirrors and waveplates provide the spatial and polarization degrees of freedom to get the desired polarizations aligned onto the ion.

4.4 Motional state cooling

In order to be able to perform operations with ions with high fidelity, it is in general necessary to first cool the ions to their motional ground state. This process,

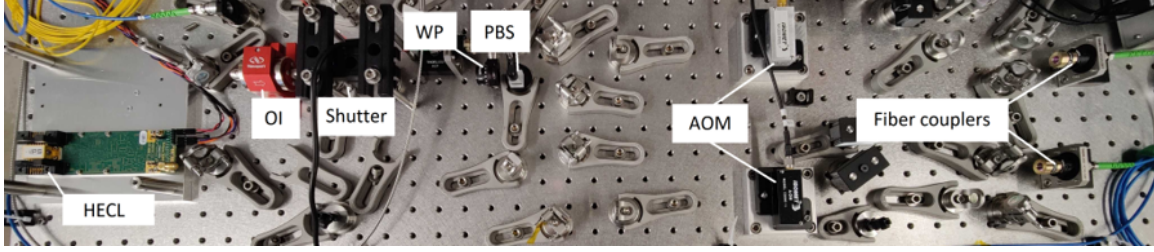


Figure 4.10. Optical setup for Raman beam delivery showing the laser head (HECL) the optical isolator (OI) a shutter, to prevent beam leakage onto the ion (not used in work presented here), waveplate (WP) and polarizing beam splitter (PBS) for relative power control in the two arms, acousto-optic modulators (AOM) for frequency control, and fiber couplers to get the light to the ions.

in trapped ions, generally makes up the majority of the experimental duty cycle and consists of multiple stages. This section is devoted to discussing the theory and implementation behind the stages of cooling that are used in these experiments.

4.4.1 Doppler cooling. Doppler cooling is a critical part of trapping ions, even when not specifically worried about operations on the motional states. This type of cooling is very fast and enables the very long lifetimes of trapped ions by preventing heating out of the trap.

The Hamiltonian considered here is the same as Eq. 3.7, which is a simple Dipole coupling Hamiltonian where the motion has to be taken into account due to the non-zero wavepacket extent. The excited state in this case has a spontaneous decay rate Γ , which can be considered in the full Lindblad master equation discussed in section 5.2. The spontaneous emission has a mechanical effect that does not effect the ion's momentum on average due to the spatial symmetry of the emission (Eschner, Morigi, Schmidt-Kaler, & Blatt, 2003). We have already seen that this emission does come with a recoil energy $E_{spont} = \hbar\omega_R\alpha$, where ω_R is the emitted photon angular frequency and α describes the average component of the recoil energy on the motional axis (Wineland & Itano, 1979). Spontaneous emission has a dipole pattern which does not effect all motional degrees of freedom equally.

The spontaneous emission spectrum can be seen by sweeping the laser detuning across the atomic resonance. In theory this couples a given state $|g, n\rangle$ to several excited states $|e, m\rangle$ with each of their own Rabi frequencies (Eschner et al., 2003). The additional resonances, which are the motional sidebands, are separated by the trap frequency which means that the spectral resolution of the resonances depend on the relation between the line width Γ and the trap frequency ω_x . For Doppler cooling, the 397 nm line is ~ 21.6 MHz wide making the sideband transitions impossible to resolve (the weak confinement limit). We will consider only transitions on the first red and blue sideband as the transition rates of higher order transitions are greatly suppressed in the Lamb-Dicke regime.

In order to consider cooling in this scattering process, we must consider the rates at which transitions are driven on the red and blue sideband. The rate to go from $|g, n\rangle \rightarrow |g, n+1\rangle$ is given by $R_+ = (n+1)A_+$ and the rate for $|g, n\rangle \rightarrow |g, n-1\rangle$ is $R_- = nA_-$, where

$$A_{\pm} = \frac{\Omega^2}{\Gamma} \eta^2 [\cos^2(\theta) W(\Delta \mp \omega_x) + \alpha W(\Delta)] \quad (4.4)$$

with $W(\Delta) = 1/(4\Delta^2/\Gamma^2 + 1)$, which is found in Eschner et al. (2003). $R_{\pm}$ have two terms that describe either absorption on the sideband and emission on the carrier or vice versa with their own rates and relative probability. In order to see the dynamics of the vibrational state we can look at the rate of change of the average vibrational number $\langle n \rangle = \sum_{n=0}^{\infty} n \langle n | \rho | n \rangle$

$$\frac{d}{dt} \langle n \rangle = -(A_- - A_+) \langle n \rangle + A_+, \quad (4.5)$$

found in Eschner et al. (2003). This equation has a steady state solution as long as $A_- > A_+$, which is to say as long as the scattering rate on the red sideband is greater than that on the blue sideband. This relation is always true when $\Delta < 0$ (when the laser is tuned to a lower frequency than the atomic resonance). The

solution to this equation is given as

$$\langle n \rangle(t) = \langle n \rangle(0) \exp[-(A_- - A_+)t] + \bar{n} \{1 - \exp[-(A_- - A_+)t]\}, \quad (4.6)$$

where $\langle n \rangle(t)$ is the vibrational quantum number at time t and $\bar{n} = A_+/(A_- - A_+)$ is the steady-state value. In the weak confinement limit, we get Doppler cooling and find the maximum ratio of A_-/A_+ is obtained at $\Delta = -\Gamma/2$ which gives the smallest steady-state temperature ($\bar{n}$) (Wineland et al., 1978). With the ideal two-level system, this steady state temperature is given as $\bar{n} \simeq \Gamma/2\omega_x$. This mathematical description of motional heating and cooling will be important in Chapter VII, where heating rates are measured. Eq. 4.6 can be fit to heating rate data to extract the Lamb-Dicke parameter.

In the experiment, Doppler cooling is a little bit more complicated. As mentioned in section 3.6, there is a lower lying D manifold that must be re-pumped with 866 nm light. There are also multiple Zeeman sub-levels that population is spread amongst. The natural linewidth is also not observed due to the Doppler broadening from finite temperatures of the ion (Wineland & Itano, 1979). When the ion is first ionized, it is very hot and has a linewidth significantly broader than the natural linewidth. In order to still cool hot ions efficiently a far red detuned 397 nm beam is used. The 397, 866 nm beams are brought in 45° to the trap axis which enables cooling of all modes, seen in ???. Radial mode temperatures, have a Doppler limit of $\bar{n} \simeq 10$ at 1.8 MHz. In order to get below the Doppler limit other cooling techniques are required.

4.4.2 EIT cooling. Electromagnetically Induced Transparency (EIT) cooling is another method of atomic cooling that can reach near ground state temperatures (Eschner et al., 2002). This method of cooling is unique from the method above, in that some slightly different physics is used to alter the

spontaneous emission spectrum to suppress scattering on the blue sideband, which leads to heating.

EIT cooling is similar to a Raman transition and involves a three-level system and involves cancellation of absorption on one transition induced by simultaneous coherent driving of another transition. This type of dark resonance is straight forward to realize in trapped ion systems given the large number of available states (Lechner et al., 2016). The effect arises from an interference of the two pathways to the excited level. This method suppresses the carrier transition on $|g, n\rangle \rightarrow |e, n\rangle$ and enhances absorption on the cooling transition $|g, n\rangle \rightarrow |e, n-1\rangle$ (Eschner et al., 2002). The method for EIT cooling consists of a ground state $|g\rangle$ that is coupled to a metastable state $|r\rangle$ with an intense “pump” laser with Rabi frequency Ω_r and detuning $\Delta_r = \omega_r - \omega_{re}$, where ω_{re} is the bare atomic transition wavelength. The third state is an excited state $|e\rangle$ which is coupled to $|r\rangle$ with a “probe”(cooling) laser with Rabi frequency Ω_g and detuning $\Delta_g = \omega_g - \omega_{ge}$. The spectrum for this excited state transition is a Fano-like profile (Marangos, 1998) with a zero at $\Delta_g = \Delta_r$ and is an asymmetric profile when $\delta_r \neq 0$. This profile encapsulates the cooling nature of this setup. When considering the motion this zero of the Fano-like profile corresponds to the carrier transition $|g, n\rangle \rightarrow |e, n\rangle$. When $\Delta_r > 0$ with a suitable Rabi frequency the spectrum can be altered to have $|g, n\rangle \rightarrow |e, n-1\rangle$ transition on the maximum of the profile and the $|g, n\rangle \rightarrow |e, n+1\rangle$ transition lying near zero excitation probability (Eschner et al., 2002). The strong coupling laser also induces an AC stark shift that shifts the metastable state by

$$\delta = (\sqrt{\Delta_r^2 + \Omega_r^2} - |\Delta_r|)/2. \quad (4.7)$$

The carrier transition is eliminated and red sideband absorption is maximized when

$$\Delta_g = \Delta_r \quad \text{and} \quad \delta \simeq \omega_x. \quad (4.8)$$

Making use of these dipole transition means that the achievable scattering rates can be significant and lead to rapid cooling. This types of cooling also couples to all modes of motion that have overlap with the k vectors of the lasers used. This means this method is very effective at cooling all modes quickly when compared to resolved sideband cooling, described below.

In the experiment, $|g\rangle |e\rangle$ are the two Zeeman sub-levels of the $S_{1/2}$ manifold and $|r\rangle$ being a Zeeman sublevel in the $P_{1/2}$ manifold. The pump is a σ^+ beam with the probe beam being π polarized. The π polarized beam couples both ground state manifolds to both metastable Zeeman states, a large Rabi asymmetry $\Omega_r \gg \Omega_g$ helps suppress other transitions being driven. Finding optimal beam tunings can be difficult to determine experimentally. In order to tune up the experiment a basic type of thermometry can be performed as a function of the pump and probe detuning. At a set detuning, the metastable qubit can be prepared and a red sideband driven. In the event that that ion is in the ground state, the red sideband does nothing. This enables the detuning to be scanned to find which detuning minimizes the bright state probability as shown in Fig. 4.12. The pump and probe beam are detuned 150 MHz from the atomic resonance, with ideal cooling when the beams are detuned approximately 4 MHz from each other. The radial modes are cooled from the start mean occupations number of $\bar{n} \simeq 20$ to $\bar{n} \simeq 1$ in 1 ms with 2.7 uW of power in the pump beam and 0.15 uW in the probe.

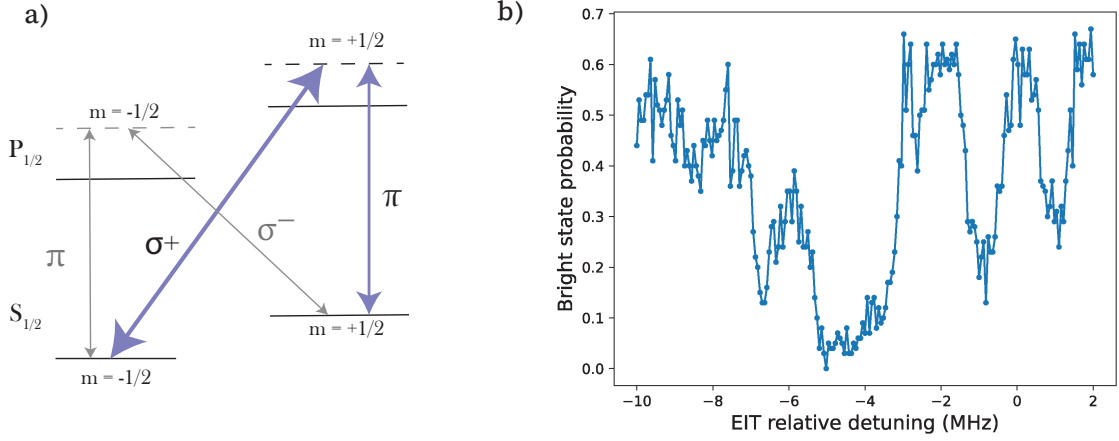


Figure 4.12. a) Level structure of the $^{40}\text{Ca}^+$ transitions used in the experiment for EIT cooling. Cooling is implemented using the Zeeman sublevels of the $S_{1/2} \leftrightarrow P_{1/2}$ dipole transition. All polarizations used in the experiment are shown. Grey transitions do not contribute to EIT cooling. b) Qubit bright state probability after applying a red sideband as a function of the relative EIT beam detuning. Probabilities near zero indicate where cooling to ground state is maximized for one of the radial modes.

4.4.3 Resolved sideband cooling.

Resolved sideband cooling works in a manner similar to the Doppler cooling. When For $\Gamma < \omega_x$ (strong confinement limit) the motional sidebands are well resolved. A laser can be tuned to the red motional sideband at $\Delta = -\omega_x$. We perform sideband cooling after the metastable qubit has been prepared. The linewidth of the qubit transition can then be arbitrarily controlled depending on the pulse times and the Rabi frequency. A π pulse on this transition will subtract a single vibration quanta and flip the qubit state. In order to dissipate energy from the system a dissipative reset of the original qubit state is required. With this procedure, very small values of $\bar{n}$ can be achieved. For $\Gamma \ll \omega_x$, $\bar{n} \simeq (\Gamma/\omega_x)^2 \ll 1$ (Eschner et al., 2003), which shows with increased mode frequencies it is possible to get even cooler. In our experiment, the Rabi frequency and pulse times used limit the linewidth Γ of the transition to be

~ 1 kHz, which means for radial mode frequencies of 1.8 MHz, final mean occupation number of $\bar{n} \simeq 3 \times 10^{-4}$ is in principle possible.

The use of metastable qubits makes this procedure slightly more complicated. The state preparation scheme laid out in section 3.4 means that on average there is ~ 20 photons scattered to prepare the qubit. This means that preparing the qubit from the ground state of motion will add some vibrational quanta. The Lamb-Dicke parameter for this scatter is $\sim \eta = 0.01$ and the energy added will be $\sim \eta^2 \beta$, where β is the number of photons scattered. This will limit the temperature to be $\bar{n} \sim 0.1$.

The full procedure of resolved-sideband cooling involves preparing the qubit, a π pulse on the red sideband and then using the 854 nm laser to deshelve population in one qubit state and reset the qubit. This is repeated several times to prepare a motional state near the ground state. It is possible to get slightly closer to the ground state by doing a post-selection procedure to project closer into the ground state. In this case, after the 854 deshelling pulse is used, most of the population left in the initial qubit state is associated with the ground state. This means that only running experiments where the qubit is measured to be in the initial state, after cooling, will give a slightly lower initial average vibrational quanta. This same process can be repeated several times to get closer to the ground state. It is important to note that this procedure is much slower than Doppler cooling. It takes $30 \mu\text{s}$ to prepare the qubit, around $50 \mu\text{s}$ to perform the red sideband pulse and another $30 \mu\text{s}$ to deshelve the qubit. This whole procedure is repeated 20 before diminishing returns. This full sequence can take several milliseconds. In order to speed up the cooling rate EIT cooling is used to bridge the gap between the Doppler limit and the ground state.

A combination of these cooling techniques has enabled us to achieve mode temperatures of $\bar{n} < 0.15$ in 7-8 ms, including Doppler, EIT and resolved sideband cooling, which corresponds to over 90% in the ground state. Cooling times are longer than when only cooling a single mode. We have also measured the heating rate using the sideband ratio technique shown in Bergquist et al. (1987). Heating rates for the radial modes are $\dot{\bar{n}} = 3\text{quanta/sec}$.

4.5 Electronic drives

In order to apply the oscillating forces described in Chapter 2, the ion trap needs to be connected electronically to a source that provides the necessary potentials. The electronics drives in this section only couple to the radial modes. RF filters are also implemented to reduce noise that can lead to decoherence of these motional modes.

4.5.1 Resonant drive. The resonant drive involves applying an oscillating force at the frequency of the harmonic oscillator. This force (known as the “tickle”) is implemented by applying an oscillating potential to one of the the DC electrodes of the ion trap, shown in Fig. 8.1. The potential is provided using a Urukul direct digital synthesizer (DDS) Kasproicz et al. (2022) controlled by an FPGA and ARTIQ software and gateway Kasproicz et al. (2020). This DDS enables precision control of the amplitude, frequency, and phase of the applied potential. This generates a force $F(t) = qE_0 \sin(\omega t + \phi)$ where E_0 is the component of the electric field along the motional mode of interest. This also means that the tickle drive does not couple to all radial modes equally. This force is a generator of the displacement operator and with a arbitrary amplitude and duration can generate displacements of arbitrary size.

4.5.2 Parametric drive resonator circuit. In order to generate second order motional operations, we modulate the trapping potential using the

RF electrodes of the trap. Applying this potential to the RF electrodes ensures it is aligned with the RF null. In order to apply this potential to the RF electrodes, it is necessary to connect a DDS to the ground of the helical resonator. In order to prevent opening the trapping RF to the DDS, parallel resonant tank circuits are used, shown in Fig. 4.13. These tank circuits are resonant with twice the mode frequency at 3.6 MHz and the difference of the single ion mode frequencies at 30 kHz. The circuits are off resonant at the trap RF frequency which ensures preventing high impedance to the DDS. Tuning the DDS frequency means that any of two-mode squeezing, single mode squeezing or beam splitter Hamiltonians can be realized. Coupling rates of these Hamiltonians are not expected to be the same due to different coupling coefficients and the mode geometry.

4.5.3 Drive filtering. The parametric drive circuit relies on strong modulation of the potential curvature with any noise in the driving electronics causing excitation of the ion secular motion. In order to prevent this, this signal source needs to be filtered. In order to filter out frequencies near the secular frequency without filtering the beam splitter frequency, the squeezing and beam splitter signal are provided from different sources and combined after the filters. A custom high-pass elliptical filter on a custom PCB was built to filter out low frequency noise from the DDS with a cutoff around 3 MHz.

4.5.4 Pulse shaping. Another significant concern performing these operations is off-resonant driving from features being narrowly split. The radial modes are commonly only split of 10s of kHz, which means resonant interactions are only split by 10s of kHz. Filtering is not effective for frequencies so close to the carrier. When using a DDS, by default the pulse have a $\sim$ ns ramp time, giving almost ideal square pulses. The spectral decomposition of a square pulse is a Sinc^2 function with significant power in side lobes. This can result in off resonant

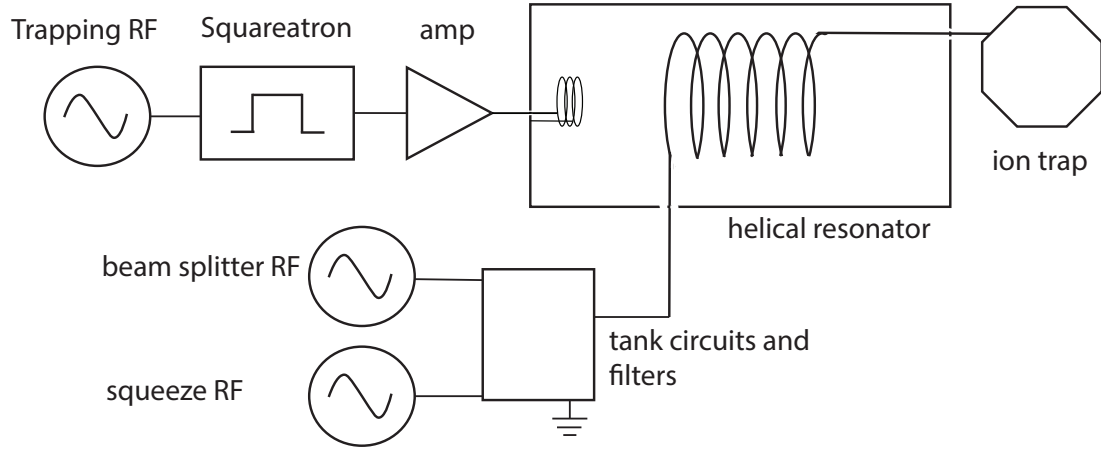


Figure 4.13. Diagram of the trapping RF connected to the Squareatron, amplifier and helical resonator. Beam splitter and squeezing RF are connected to an elliptical filter and tank circuits. The helical resonator is connected to the ion trap.

driving of single mode and two mode squeezing simultaneously. These effects can be mitigated by shaping the pulse. In order to shape the pulses, an RF multiplier board (ADL5932-EVALZ) is used. This board takes the RF input from the DDS and multiplies it with a DC ramp to remove the spurious spectral components. The DC ramp is provided by an AWG (Rigol DG1022Z) that is triggered by a TTL pulse at the beginning of every experimental sequence. A trapezoidal window is used in the experiment with a rise time of $15\ \mu\text{s}$. This rise time is sufficient to reduce power, in the frequency components that are $>100\ \text{kHz}$ from the carrier, to levels where off-resonant driving was not observed.

CHAPTER V

NON-HERMITIAN HAMILTONIANS

Quantum information experiments generally involve two separate but equally important processes, the coherent unitary dynamics used for transformation of the quantum state and the dissipative dynamics that enable measurements of this state. Combining unitary dynamics with dissipative operations enables exploring an entirely different type of Hamiltonian. These non-Hermitian Hamiltonians are relatively unexplored and could provide benefits for quantum information processing. The work presented in this chapter explores the dynamics of a non-Hermitian Hamiltonian and was done together with Alex Quinn who ran the experiments and is co-first author on work pending publication (Quinn et al., 2023).

5.1 Unitary dynamics and quantum speed limit

Before discussing how to implement non-Hermitian Hamiltonians and what that means, it is important to discuss, in more detail, unitary evolution and the corresponding limits. We have already seen that given any state $|\eta\rangle$ in a Hilbert space $\mathcal{H}$, there is some other state $|\psi\rangle$ in $\mathcal{H}$ such that under unitary evolution, $|\eta\rangle = U|\psi\rangle$. This mapping of a Hilbert space onto itself is true if

$$UU^\dagger = I, \tag{5.1}$$

and in addition the inner product is preserved under unitary evolution,

$$(U|w\rangle)^\dagger U|\phi\rangle = \langle w|U^\dagger U|\phi\rangle. \tag{5.2}$$

This preservation of the inner product will also only be true when

$$U^\dagger U = I. \tag{5.3}$$

These two equalities, eq. 5.1 and eq. 5.3, show that $U^\dagger$ is equivalent to the inverse U^{-1} of U . In order to tie this back to Hamiltonians and time evolution we use the

density operator representation of Schrödinger's equation

$$\dot{\rho}(t) = -\frac{i}{\hbar}[H, \rho(t)], \quad (5.4)$$

known as the Von Neumann equation. In this representation, H is the Hamiltonian governing the system and can be time dependent. The solution to this equation is in general given by the same unitary operators described above. This unitary operator $U(t; t_0)$ propagates the state of the system from some initial state at t_0 to some final state $\rho(t)$ at some time t ,

$$\rho(t) = U(t; t_0)\rho(t_0)U^\dagger(t; t_0). \quad (5.5)$$

In many of the cases we have seen in Chapter II, if H is time-independent the solution is given by $U(\tau) = \exp[-iH(\tau)/\hbar]$, for all t, t_0 with $\tau = t - t_0$.

After looking at the operators that evolve some given state to another, it is reasonable to ask, what is the fastest rate achievable to go from the initial state to the final state? When considering the minimum time needed for any state of a system to evolve into an orthogonal state $\tau_\perp$, it has been shown that with a fixed average energy E it is always true that

$$\tau_\perp \geq \frac{\hbar}{4E}. \quad (5.6)$$

This can be put in terms of a Rabi frequency Ω as $\tau_{SQL} = \frac{\pi}{2\Omega}$. This is the minimum transit time for a given state to reach an antipodal state. This is equivalent to doing a full rotation on the Bloch sphere.

5.2 Lindblad model and master equation solver

Real physical system can only be treated with the description above in the limits of perfectly isolated systems. In reality we have to deal with open quantum systems, where the system is interacting with the environment. Being able to perfectly describe the environment and the systems coupling to the environment can be very difficult and makes this problem significantly more complicated and

often requires numerical techniques. The full Hamiltonian is instead written as

$$H = H_S + H_E + H_{int}, \quad (5.7)$$

where H_S is the Hamiltonian of just the system of interest, H_E is the environment Hamiltonian, and H_{int} describes the interaction between the two. Under these conditions, if the solution to $U(\tau)$ is known, the dynamics of the system of interest can still be drawn from a larger state ρ by tracing over the environment's degrees of freedom

$$\rho_S(t) = \text{Tr}_E[\rho(t)]. \quad (5.8)$$

In order to describe the evolution of the system, the Von Neumann equation can be extended to become the Lindblad master equation,

$$\dot{\rho}(t) = -\frac{i}{\hbar}[H, \rho(t)] + \sum_k \gamma_k \left(L_k \rho(t) L_k^\dagger - \frac{1}{2} \left\{ L_k^\dagger L_k, \rho(t) \right\} \right), \quad (5.9)$$

where ρ and H is the system density operator and Hamiltonian respectively. L_k are Lindblad operators which represent some non-unitary process like decoherence at some rate γ_k . The operator $\{.,.\}$, in eq. 5.9 is the anti-commutator of the operands. The Lindblad operators are generators of incoherent transitions as opposed to coherent transitions given by the Hamiltonian. When working with discrete energy states it is relevant to look at incoherent couplings, sometimes given by natural decay from some excited state $|e\rangle$ to some ground state $|g\rangle$. This process is mediated by the Lindblad operator

$$L_\downarrow = |g\rangle\langle e|, \quad (5.10)$$

with $|g\rangle = L_\downarrow |e\rangle$, but $L_\downarrow^\dagger = |e\rangle\langle g| \neq L_\downarrow$. The Lindblad master equation is used to approximate the evolution of the density operator with a weak coupling to a memory-less environment. This Lindblad model will be the starting point for

realizing non-Hermitian Hamiltonians. A incoherent (dissipative) channel will be a necessary but a non-sufficient condition for realizing this type of Hamiltonian.

5.3 Post-selection and non-Hermitian Hamiltonians

Experimental realization of a incoherent channel, which can be combined with coherent control, requires considered selection of qubit encodings. The use of metastable qubits enables an almost ideal realization of a non-Hermitian Hamiltonian. With most qubit encodings it is possible to use coupling to the short lived P manifolds for dissipation, however population generally returns to the qubit states, which scrambles the coherent qubit dynamics. Population can be coherently coupled to the $S_{1/2}$ ground state which enables dissipation, but this limits the achievable effective dissipation rate given that the coherent transfer rate must be sufficiently slow to prevent population from coming back to the qubit. This effectively limits the possible parameter regime to $\Omega > \gamma$.

With these considerations in mind, using a single $^{40}\text{Ca}^+$ ion with states $|\uparrow\rangle \equiv |m = +5/2\rangle$ and $|\downarrow\rangle \equiv |m = +3/2\rangle$ within the metastable $D_{5/2}$ manifold as the two-level system is ideal (Fig. 5.1a) (Sherman et al., 2013). Through post-selection, we realize the non-Hermitian Hamiltonian ($\hbar = 1$)

$$H(\gamma) = J\sigma_x + i\gamma\sigma_z \quad (5.11)$$

as follows. The Hermitian Rabi drive $J\sigma_x \equiv \Omega(|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|)$ is implemented by using resonant radio frequency pulses at the qubit frequency. Additionally, the $|\downarrow\rangle$ state, called the lossy-state, is coupled to the auxiliary, short-lived $P_{3/2}$ state $|A\rangle$ using π -polarized light with pulse-strength Ω_A , where population then primarily (93.5%) decays to the $S_{1/2}$ ground-state $|g\rangle$ with decay rate γ_g . The dissipative dynamics of the four levels $\{|\uparrow\rangle, |\downarrow\rangle, |A\rangle, |g\rangle\}$ are described by a Lindblad equation with two Hermitian drives $\Omega\sigma_x$ and $\Omega_A(|\uparrow\rangle\langle A| + |A\rangle\langle\uparrow|)$, and three spontaneous-

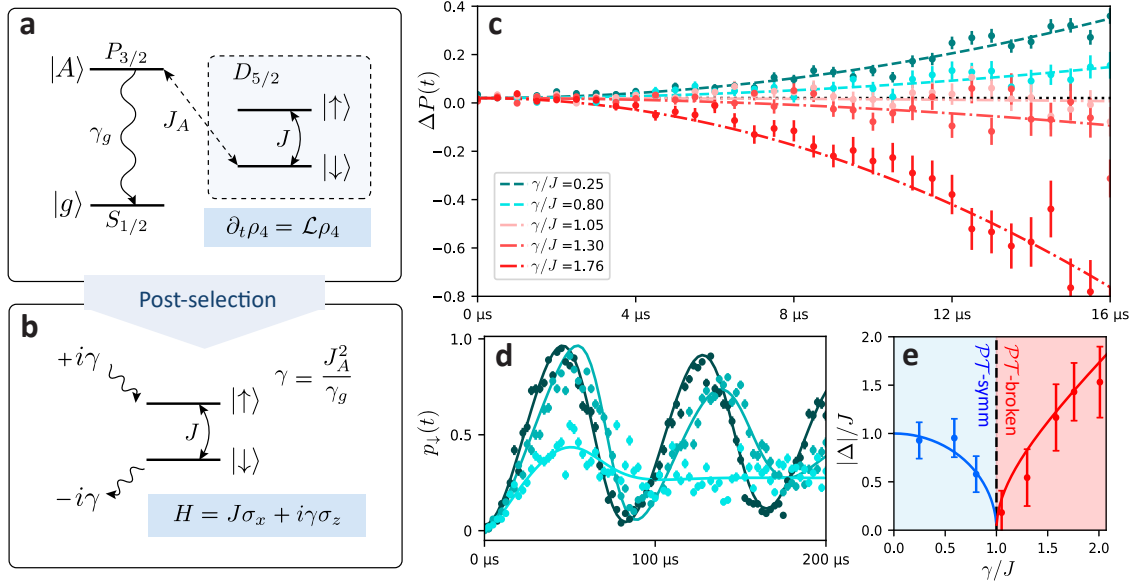


Figure 5.1. (figure from Quinn et al. (2023)) **Two-level non-Hermitian trapped ion.** **a.** Four levels of a $^{40}\text{Ca}^+$ ion $\{|\uparrow\rangle, |\downarrow\rangle, |A\rangle, |g\rangle\}$ have a density matrix $\rho_4(t)$ that is governed by Lindblad equation with two coherent drives and three spontaneous-emission dissipators, most dominant of which, to the $|g\rangle$ level, is shown. **b.** Post-selection, i.e. eliminating the short-lived $|A\rangle$ level and conditioning the ion on not being in the $|g\rangle$ level, leads to a non-Hermitian qubit within the $\{|\uparrow\rangle, |\downarrow\rangle\}$ manifold; both Rabi drive J and the anti-Hermitian term γ in Eq.(5.11) can be independently varied. **c.** Population-transfer difference $\Delta P(t)$ measured at short times $t \lesssim 15 \mu s$ shows the transition from the $\mathcal{PT}$ -symmetric phase ($\Delta P > 0$) to a $\mathcal{PT}$ -broken phase ($\Delta P < 0$), with the EP at $\gamma_{\text{EP}} = J$. **d.** The $|\downarrow\rangle$ -level probability $p_\downarrow(t)$ measured over 200 μs shows three skewed oscillations that are increasingly affected by the backflow to the qubit manifold as γ increases ($\gamma/J = 0.18$ dark green, $\gamma/J = 0.37$ medium, $\gamma/J = 0.73$ light blue-green) **e.** $|\Delta|$ extracted from parabolic fits to $\Delta P(t)$ shows the expected transition at the EP. (Data: symbols, theory: solid lines, best-fit: broken lines; error bars in **c-d** are s.d. from 400 shots.)

emission dissipators, the most dominant being $\sqrt{\gamma_g} |g\rangle\langle A|$. When $\gamma_g \gg J_A$, adiabatically eliminating the auxiliary level and conditioning the ion on not being in ground state generates the anti-Hermitian potential $i\gamma\sigma_z = i\gamma(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|)$ with $\gamma = J_A^2/\gamma_g \ll \gamma_g$ (Fig. 5.1b).

In order to model this system, it is simpler and has proven sufficient to work with a three level system with $|\downarrow\rangle, |\uparrow\rangle, |e\rangle$, with a qubit coupling rate Ω and Lindblad operator $L_{\downarrow,e} = |e\rangle\langle\downarrow|$ at rate γ . In order to capture some of the experimental imperfections, it is necessary to take into account the $\sim 5\%$ of the population that decays back into the $D_{5/2}$ manifold from the $P_{3/2}$. There are two different Lindblad operators that can be used to capture this effect, $L_{e,\downarrow} = |\downarrow\rangle\langle e| = L_{\downarrow,e}^\dagger$ and $L_{e,\uparrow} = |\uparrow\rangle\langle e|$. These processes will have rates that are proportional to γ and constrained by the specific Zeeman branching ratio, $\gamma_\downarrow = 0.039\gamma$ and $\gamma_\uparrow = 0.015\gamma$. There is some population that can go to lower Zeeman sublevels, but including a fourth state was found to have a negligible effect.

Using Qutip (Johansson et al., 2013), the Lindblad master equation can be used to include a bare qubit coupling with Lindblad operators. This can be compared directly to using the non-Hermitian Hamiltonian given in eq. ?? seen in Fig. 5.2 In order to simulate post selection, the norm needs to be re-defined. The final state after some time evolution is given by

$$|\psi\rangle = a|\downarrow\rangle + b|\uparrow\rangle + c|e\rangle, \quad (5.12)$$

and the norm will be redefined from $N = |a|^2 + |b|^2 + |c|^2$ to $N = |a|^2 + |b|^2$. This re-normalization is used to just capture the probabilities of the population remaining within the qubit manifold.

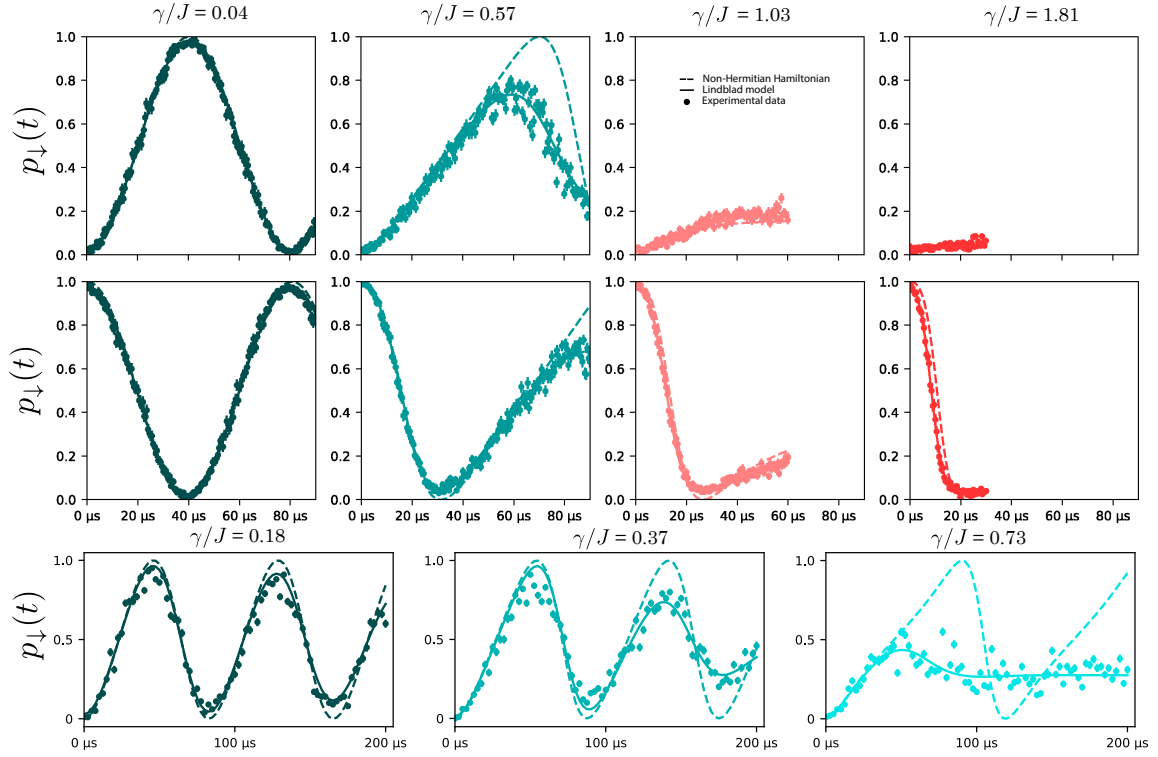


Figure 5.2. (figure from Quinn et al. (2023)) A comparison of non-Hermitian Hamiltonian versus a Lindblad model across a range of different values of γ/J . The first two rows show results starting in different initial states. The third row gives longer time dynamics, highlighting the strong deviation from the non-Hermitian model at long times, especially at larger values of γ/J

5.4 Finding the exceptional point

One of the significant features of this type of non-Hermitian Hamiltonian $H(\gamma)$ is the parity-time ($\mathcal{PT}$) symmetry with $\mathcal{P} = \sigma_x$ and complex conjugation as the $\mathcal{T}$ -operator (Naghiloo, Abbasi, Joglekar, & Murch, 2019). Its eigenvalues $\pm\Delta = \pm\sqrt{J^2 - \gamma^2}$ change from real ($\mathcal{PT}$ -symmetric phase) to imaginary ($\mathcal{PT}$ -broken phase) at the exceptional-point (EP) degeneracy $\gamma = \gamma_{\text{EP}} = J$; both J, γ are experimentally determined and can be independently controlled. Post-selection preserves the state-norm in the qubit manifold, and leads to a nonlinear Schrödinger-like equation for the qubit (Brody & Graefe, 2012; Varma, Muldoon, Paul, Joglekar, & Das, 2023) with solution given by

$$|\psi(t)\rangle = \frac{G(t) |\psi(0)\rangle}{\sqrt{\langle\psi(0)| G^\dagger(t) G(t) |\psi(0)\rangle}}, \quad (5.13)$$

where $G(t) = \cos(\Delta t)\mathbb{1}_2 - iH \sin(\Delta t)/\Delta$ is the non-unitary time-evolution operator. Traditionally, the qubit level probabilities $p_\downarrow(t)$ and $p_\uparrow(t) = 1 - p_\downarrow(t)$ are measured at times $0 \leq t \lesssim T_\Delta \equiv 2\pi/|\Delta|$ to characterize the transition across the EP. In our setup, the small fraction (5.87%) of the population from the auxiliary state decays back into the $D_{5/2}$ manifold means at times $t \sim T_\Delta$ this backflow creates deviations from Eq.(5.11) as the effective description. This approach thus becomes ill-suited to observe the transition (Fig. 5.1d).

Instead, we use the sign of the population-transfer difference $\Delta P(t) \equiv P_\gamma(t) - P_J(t)$ to differentiate the $\mathcal{PT}$ -symmetric phase ($\Delta P > 0$) from the $\mathcal{PT}$ -broken phase ($\Delta P < 0$) without determining the eigenvalue gap $|2\Delta|$ (L. Ding et al., 2021). Here $P_\gamma(t) \equiv |\langle\downarrow| G(t) |\uparrow\rangle|^2 = (J^2/\Delta^2) \sin^2(\Delta t)$ and $P_J(t) \equiv |\langle-| G(t) |+\rangle|^2 = (\gamma^2/\Delta^2) \sin^2(\Delta t)$ denote population transfers to respective antipodal states in the two bases, and $|\pm\rangle \equiv (|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}$. Thus $\Delta P(t) = \sin^2(\Delta t)$, obtained at even short times $|\Delta|t \ll 1$, changes sign

at the EP and allows determination of $\mathcal{PT}$ -symmetry breaking transition. The population transfers $P_\gamma(t), P_J(t)$ are related to the level-transition probabilities $p_\downarrow, p_-$ and the exponentially decaying successful-post-selection fractions $F_\uparrow, F_+$ — four experimentally measured quantities — as follows:

$$P_\gamma(t) = e^{2\gamma t} F_\uparrow(t) p_\downarrow(t); P_J(t) = e^{2\gamma t} F_+(t) p_-(t). \quad (5.14)$$

Figure 5.1c shows that the measured $\Delta P(t)$ changes from positive to negative as γ traverses across the EP at $\gamma_{\text{EP}} = J$. Detecting this transition through a single time-instance data in a non-Hermitian qubit is an embodiment of quantum advantage over classical $\mathcal{PT}$ -symmetric dimers that require data over time $t \sim T_\Delta$.

5.5 Legget-garg inequality

The general protocol for measuring the LG parameter K_3 is schematically shown in Fig. 5.3a. We use $Q = \sigma_z$ as the dichotomous observable with eigenvalues ± 1 and corresponding one-dimensional projectors $|\uparrow\rangle\langle\uparrow|$ and $|\downarrow\rangle\langle\downarrow|$. With the judicious choice of lossy-state as the initial state and equally-spaced times $t_1 = 0$, $t_2 = t$, $t_3 = 2t$, the LG parameter becomes

$$K_3(t, \gamma) = C(t) + F(t) - C(2t), \quad (5.15)$$

where, unlike the unitary case, $C_{21} = C(t)$ and $C_{32} = F(t)$ are distinct functions. Starting from one, $K_3(t)$ reaches the Lüder bound of 1.5 at $Jt = \pi/6 \approx 0.523$ for a Hermitian qubit. At small γ , expansion around this point gives $K_3 \approx K_3^L + 7\sqrt{3}\gamma/(8J)$ thereby exceeding the Lüder bound. Near the EP, similar analysis predicts that $\max K_3$ occurs at $Jt \approx 0.35$ and approaches its algebraic maximum in a vanishingly small window at short times $t \propto \gamma^{-1} \ln(2\gamma/J)$ deep in the $\mathcal{PT}$ -broken region $\gamma/J \gg 1$. Optimizing K_3 over the space of $\{|\psi(0)\rangle, Q\}$ significantly broadens the window where K_3 supersedes the Lüder bound. However, in that case,

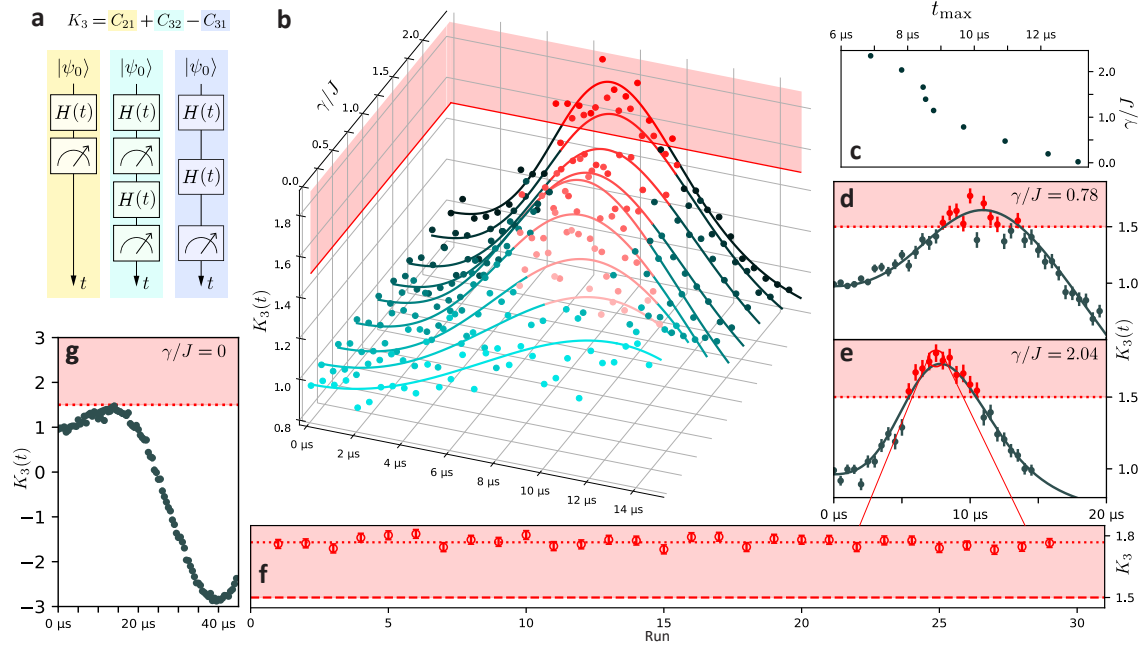


Figure 5.3. (figure from Quinn et al. (2023)) **Super-quantum correlations in $K_3(t)$.** **a.** General protocol for determining $K_3(t)$ comprises three, pairwise two-time projective measurements of a dichotomous observable. **b.** Measured $K_3(t, \gamma)$ exceeds the Lüder bound of 1.5 over a wide range of γ/J . Different shades of green ($K_3 \leq 1.5$) and red ($K_3 > 1.5$) represent different γ/J values. **c.** With judicious choice of the lossy-state as the initial state, the max K_3 occurs at time $t_{\max}(\gamma)$ shorter than the unitary-limit value $Jt = \pi/6$. Thus, coherent, non-unitary dynamics generate stronger correlations faster. **d., e.** Time-series for $K_3(t)$ below the EP ($\gamma = 0.78J$) and above the EP ($\gamma = 2.04J$) show that the location of max K_3 shifts to lower time-values as γ is increased. **f.** For a given $(t, \gamma/J)$, the measured K_3 remains stable over time across experiments carried out over five hours, thereby showing the robustness of super-quantum correlations in coherent, non-unitary dynamics. A red dotted line shows the average $K_3 = 1.768(8)$ across these runs. In each run, the Lüder bound, shown by the red dashed line, is exceeded by at least 10.5 standard deviations. **g.** $K_3(t)$ in the unitary case spans the range from 1.5 to -3 (Emary, Lambert, & Nori, 2013). (data: symbols; theory: lines; each point denotes the average of 400 shots.)

these stronger correlations take longer time than $Jt = \pi/6$ to develop (Varma et al., 2023).

Figure 5.3b shows the experimentally measured $K_3(t, \gamma)$ time-series with γ/J values ranging across the EP with $K_3 > 1.5$ marked in varying shades of red that indicate increasing γ . Correspondingly, varying shades of green indicate K_3 values that remain below the Lüder bound. As γ increases, the time at which K_3 is maximum shortens, to less than half its unitary-limit value at $\gamma/J \approx 2$ (Fig. 5.3c). A close-up time-series across the EP shows this trend clearly (Fig. 5.3d,e). Our observed values reach a maximum $K_3 = 1.768(8)$, clearly superseding the Lüder bound of 1.5. The coherent, non-unitary dynamics of a $\mathcal{PT}$ -symmetric qubit, thus, spawn temporal correlations that are stronger than the Lüder bound, in a time that is shorter than one dictated by the unified quantum speed limit (Levitin & Toffoli, 2009; Margolus & Levitin, 1998).

Accessing parameter region where K_3 can approach its algebraic maximum is challenging for two reasons. First, increasing γ by ramping up the J_A drive suppresses the successful post-selection fraction; second, at a fixed value of γ , reducing J slows down the dynamics, amplifies deviations from the non-Hermitian, $\mathcal{PT}$ -symmetric qubit model, and suppresses $\max K_3$ relative to predictions from such a model. The effects of the branching ratio in the backflow can be seen in Fig. 5.4 where the maximum achievable value of K_3 is shown as a function of the branching ratio.

5.6 Violation of quantum speed limit

The excesses above 1.5 in $K_3(t)$ arise primarily from the temporal variation of the norm of $G(t)$ in Eq.(5.13). For a unitary qubit, the Lüder bound emerges from a constant speed-of-evolution on the Bloch sphere (Campaoli, Pollock, & Modi, 2019; Levitin & Toffoli, 2009; Margolus & Levitin, 1998). Being equal for

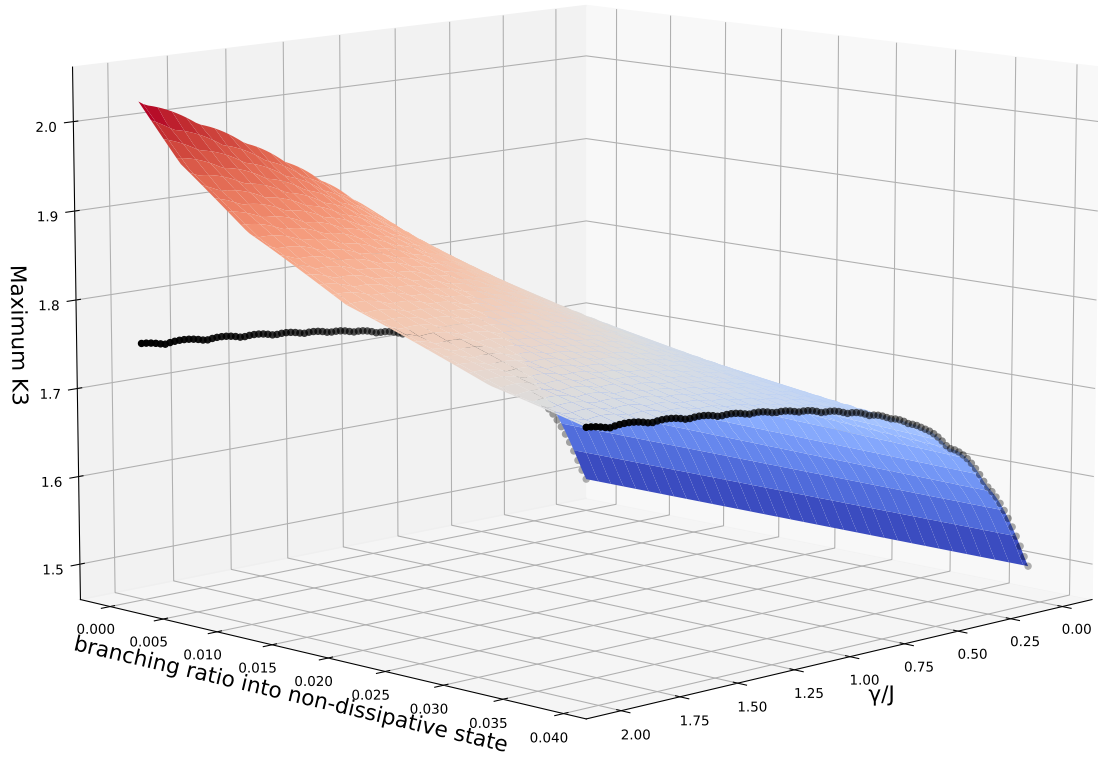


Figure 5.4. Surface contour of the Maximum value of $K3$ possible across different values of γ/J taking into account the branching ratio into the non-dissipative state. The black line highlights the true experimental value, which prevents reaching higher values of $K3$.

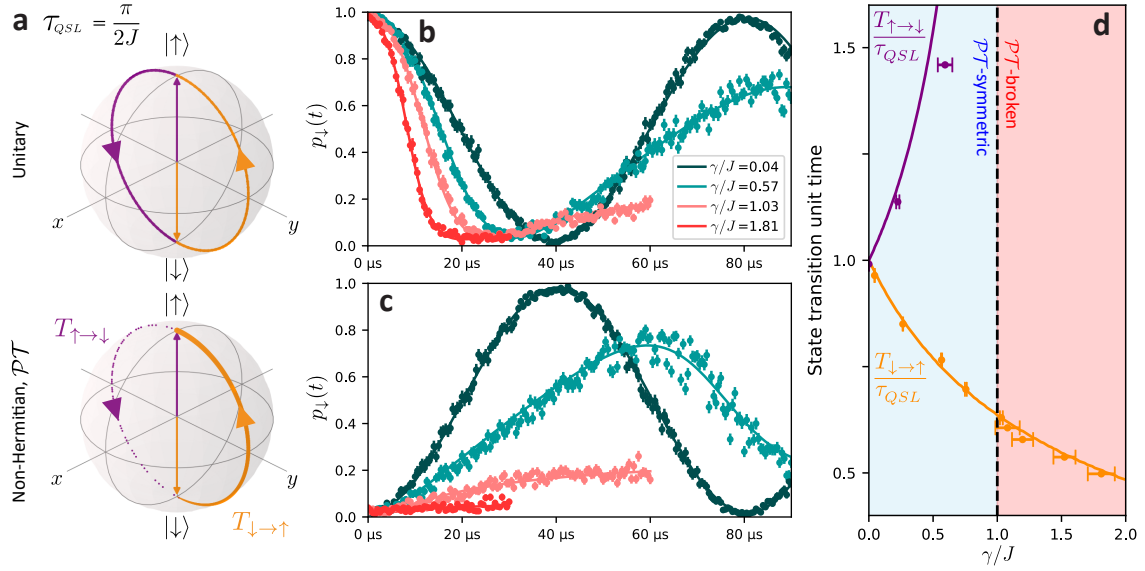


Figure 5.5. (figure from Quinn et al. (2023)) **Moving faster than quantum-speed-limit on the Bloch sphere.** **a.** In unitary case, the transit times to and from an antipodal state are equal, bounded below by unified quantum speed limit $\tau_{QSL} = \pi/(2J)$; coherent dynamics of $\mathcal{PT}$ -symmetric Hamiltonian break this reciprocity. **b.** Starting from the lossy state, measured $p_{\downarrow}(t)$ shows a faster transit to the non-lossy state $|\uparrow\rangle$ as γ is increased; theory lines are obscured by the data symbols. **c.** In contrast, starting in the non-lossy state, measured $p_{\downarrow}(t)$ shows a slower return from the $|\uparrow\rangle$ state. The incomplete transfer to the lossy-state $|\downarrow\rangle$ at $\gamma = 0.57J$ also reflects the backflow to the qubit manifold. **d.** Measured transit time $T_{\downarrow\rightarrow\uparrow}$ shows a 50% reduction relative to its minimum mandated value τ_{QSL} ; the error in measured $T_{\uparrow\rightarrow\downarrow}(\gamma)$ is due to the backflow and the incomplete Rabi flop (Supplementary Information). Data: symbols; theory: lines.

antipodal states, for a two-level system, it results in the unified lower bound $\tau_{\text{QSL}} = \pi/2J$ for the minimum transit-time required to reach an antipodal state (Fig. 5.5a). The coherent, non-unitary dynamics generated by $G(t)$ break this reciprocity, and the new, non-reciprocal transit times satisfy $T_{\downarrow \rightarrow \uparrow}(\gamma) \leq \tau_{\text{QSL}} \leq T_{\uparrow \rightarrow \downarrow}(\gamma)$. As γ is increased, $T_{\downarrow \rightarrow \uparrow}$ is suppressed from τ_{QSL} to $1/J$ at the EP, to $\gamma^{-1} \ln(4\gamma/J)$ for $\gamma \gg J$. The return time satisfies

$$T_{\uparrow \rightarrow \downarrow}(\gamma) + T_{\downarrow \rightarrow \uparrow}(\gamma) = \frac{\pi}{\Delta(\gamma)} \quad (5.16)$$

when $\gamma < J$, and diverges at the EP. Past the EP, a complete transit from the non-lossy-state $|\uparrow\rangle$ to the lossy-state $|\downarrow\rangle$ cannot occur.

Measured occupations $p_{\downarrow}(t)$ show that the antipodal transit time is shortened when starting in the $|\downarrow\rangle$ state (Fig. 5.5b) and increased when starting in the $|\uparrow\rangle$ state (Fig. 5.5c) as γ/J increases. The extracted transit-time $T_{\downarrow \rightarrow \uparrow}(\gamma)$ monotonically decreases showing a 50% reduction across the EP, while $T_{\uparrow \rightarrow \downarrow}$ diverges as the EP is approached (Fig. 5.5d). This non-reciprocal transit is a signature that a non-Hermitian Hamiltonian commingles salient features of unitary and dissipative dynamics.

By implementing a two-level non-Hermitian Hamiltonian in a single, dissipative ion, we have demonstrated stronger temporal correlations that supersede all known models comprising unitary or dissipative, linear quantum maps Budroni and Emary (2014); Budroni, Moroder, Kleinmann, and Gühne (2013). These correlations are generated faster than those permitted by standard quantum speed limits Levitin and Toffoli (2009); Margolus and Levitin (1998). While making clear that non-Hermiticity is necessary for superseding the Lüder bound, we have laid to rest implicit assumptions regarding the special role played by the EP in maximizing the LG parameter Karthik, Akshata Shenoy, and Devi (2021); Lu et al. (2024);

Naikoo, Kumari, Banerjee, and Pan (2021); Wu et al. (2023) (Supplementary Information).

These dual features of the coherent, non-unitary dynamics generated by $\mathcal{PT}$ -symmetric, non-Hermitian Hamiltonians – stronger correlations, faster – may pave the way for new models of quantum correlations and entanglement generation.

CHAPTER VI

LASER-FREE CONTROL OF TRAPPED-ION QUANTUM HARMONIC OSCILLATORS

Chapter II has laid out the theoretical and experimental toolbox for operators that generate a range of classical and non-classical states of trapped ion motion without using laser fields. With that foundation, we can discuss the method of analyzing the states that these operators generate in order to calibrate experiments and perform useful measurements. The results of these calibration are critical for a full understanding of the experiments laid out in chapter VIII.

6.1 State characterization

The primary method for extracting information from an ion is to use qubits. This allows access to a single bit of information that when averaged over many experimental runs, gives insight into processes that affect the state of the qubit. This means that in order to gain information about the motional states that have been prepared, the sideband interaction are critical for mapping information about the motional state onto the qubit state. We have already seen that using the BSB interaction leads to transitions between $|n, \downarrow\rangle$ and $|n + 1, \uparrow\rangle$ when starting from a Fock state. This means that probability of measuring the spin state $|\downarrow\rangle$ after applying the BSB Hamiltonian for some time t is given by

$$P_{\downarrow}(t) = |\langle \downarrow | \psi(t) \rangle|^2 = \cos^2 \left(\frac{\sqrt{n+1}\Omega t}{2} \right). \quad (6.1)$$

When the ion is in some superposition of Fock states the probability to measure $|\downarrow\rangle$ must include the probabilities of all Fock states and is given as

$$P_{\downarrow}(t) = \sum_{n=0}^{\infty} P_n \cos^2 \left(\frac{\sqrt{n+1}\Omega t}{2} \right), \quad (6.2)$$

where P_n is the probability of finding the Harmonic oscillator is some Fock state $|n\rangle$ before the application of the BSB Hamiltonian. In order to take into account

spin and motional decoherence, full numerical simulations of the master equation can be done with phenomenological decay factors taken into account. Under consideration of motional decoherence alone, caused by white frequency noise, the qubit probabilities can be re-written as

$$P_{\downarrow}(t) = \frac{1}{2} \left(1 + \sum_{n=0}^{\infty} P_n e^{-\gamma_n t} \cos(\sqrt{n+1} \Omega t) \right), \quad (6.3)$$

where the decay constant $\gamma_n = \gamma_0 \sqrt{n+1}$. This means motional states that occupy larger Fock states will decay more rapidly. This generally puts a constraint on the size of the states that can be generated with phase coherence in the experiment.

The RSB Hamiltonian is more useful when trying to make measurements of the ground state population. When applying this Hamiltonian with Rabi frequency Ω for time t the qubit probabilities are given as

$$P_{\downarrow}(t) = \frac{1}{2} \left(1 + \sum_{n=0}^{\infty} P_n e^{-\gamma_n t} \cos(\sqrt{n} \Omega t) \right). \quad (6.4)$$

From this, it is clear that the Rabi frequency $\Omega \sqrt{n}$ is zero when in the ground state. This makes sense as you cannot subtract a phonon from the vacuum state. This type of measurement will be best when we consider time reversal protocols that should return the motional state to the ground state as this Hamiltonian can be applied to determine if return to the ground state was successful.

6.2 Coherent states

Generation of coherent state without the use of lasers has already been described in section 2.5.1 using an oscillating electric field at the mode frequency for a duration t . The BSB analysis can be seen in Fig. 6.1 which shows a collapse and revival of the contrast which is expected for this type of Jaynes-Cummings interaction (Berman & Ooi, 2014) with coherent states. The coupling rate of the motional state and the qubit state is set by the Rabi frequency and Lamb-Dicke parameter is a fit parameter in Fig. 6.1a). Fitting the theory to the data also

enables the coherent state amplitude α to be extracted. In order to determine the coherent state amplitude α as a function of drive duration at a fixed amplitude, a linear fit is applied. The slope of this line is used to determine the coupling rate of the displacement Hamiltonian. The linear relationship between drive duration and coherent state amplitude is shown in Fig. 6.1b). This calibration is important to determine coherent state amplitude that sets the sensitivity for some interferometers in Ch. VIII.

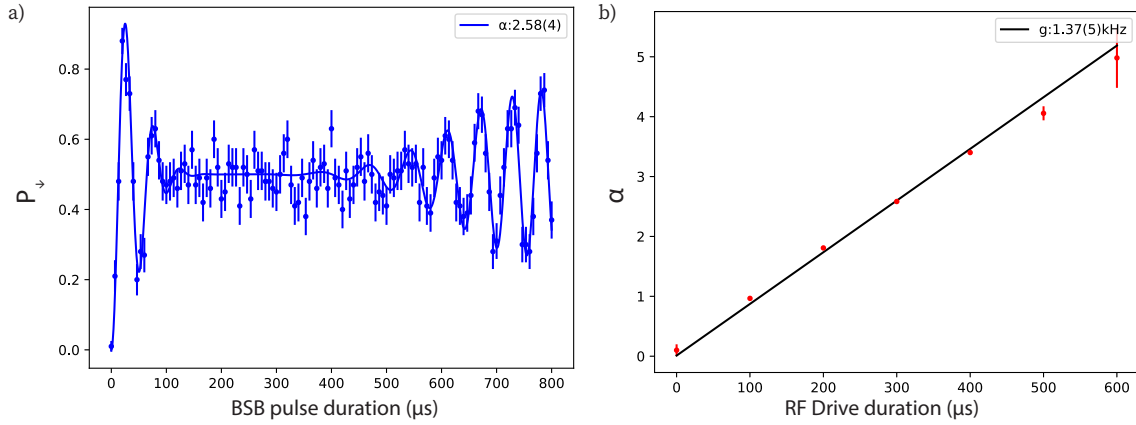


Figure 6.1. (a) Fits to time series data produced by BSB Rabi flopping with the motion in a coherent state. Data reproduces the characteristic collapse and revival of a coherent state. (b) Calibration data for determining the coupling rate of the displacement operations. Data is taken by fitting time series data like in a) for varying durations of the displacement operation. The slope of the linear fit gives the coupling rate.

6.3 Thermal states

After the photon scattering events from cooling sequences, the ion's motion is theoretically in a thermal state. The density matrix, that describes this state, does not have off-diagonal elements (coherences) which is a fully mixed state. Sideband analysis still enables determination of the Fock state populations. When using this type of analysis for thermometry after cooling, it is generally more useful to use the RSB. As the mean phonon number is reduced, the $|\downarrow\rangle$ state population

tends to 1. This enables scanning and finding cooling parameters that maximize the ground state population.

As has been shown in section 2.5.6, thermal states are also a product of tracing over a single mode of a two-mode squeezed state. Using the BSB Hamiltonian, the qubit populations are fit to using numerical simulations to determine the mean phonon number $\bar{n}$. Thermal state fitting can be seen in Fig. 6.3a). The relationship between $\bar{n}$ and the squeeze amplitude r_{tms} is $\bar{n} = 2 \sinh(r_{tms})$. This relationship then gives the size of the two-mode squeezed state when measuring the thermal state.

6.4 Squeezed states

Section 2.5.2 showed how squeezed states can be generated with a parametric drive at twice the mode frequency, applied for a duration t_r . Squeezed vacuum or squeezed coherent states are generated by applying this interaction on either the ground state, or a coherent state. The analysis in that section also showed how this interaction should only populate the even Fock states. Using the BSB analysis enables measuring the squeezed state amplitude r . The parametric coupling strength can also be measured with a method similar to that used for coherent states. Since $r = gt_r$, the slope of a plot of r vs t_r can be fit to give the value of g . The calibration for the experiment involves keeping t_r fixed and varying g with the maximum coupling rate given by the amount of squeezing at full amplitude for the given squeezing time, which is shown in Fig. 6.2.

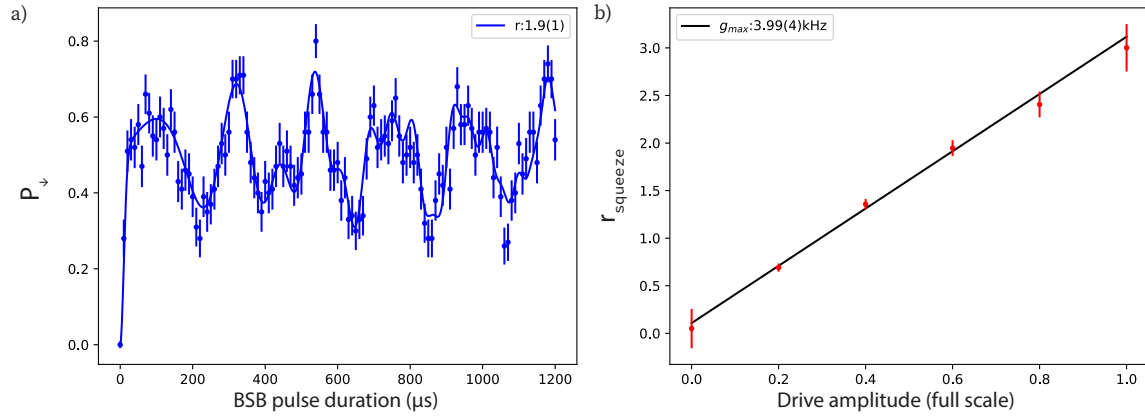


Figure 6.2. (a) Fits to time series data produced by blue sideband Rabi flopping with the motion in a squeezed. (b) Calibration data for determining the coupling rate of the displacement operations. Data is taken by fitting time series data like in a) after varying the amplitude of the driving field with a fixed pulse duration. The maximum coupling rate is taken as the coupling rate when the DDS is at full amplitude.

6.5 Two-mode squeezed states

Given that making a qubit measurement after using a sideband interaction to a single mode is equivalent to a partial trace of a single mode, this state is difficult to measure. Measuring the thermal state can be used to calibrate the experiment so long as there is additional evidence that an actual two-mode squeezed state has actually been generated. Given the commonality of thermal states of motion after cooling, further evidence is necessary. The thermal state measurement was used for calibration once other experiments have been used to verify this operation and is shown in Fig. 6.3. The beam splitter can be used to effectively measure the mode in another basis and can be seen in section 8.1. The other method we employed to verify the coherence of the state, was to implement a time reversal protocol to bring the motion back to the ground state and is shown in section 8.2.

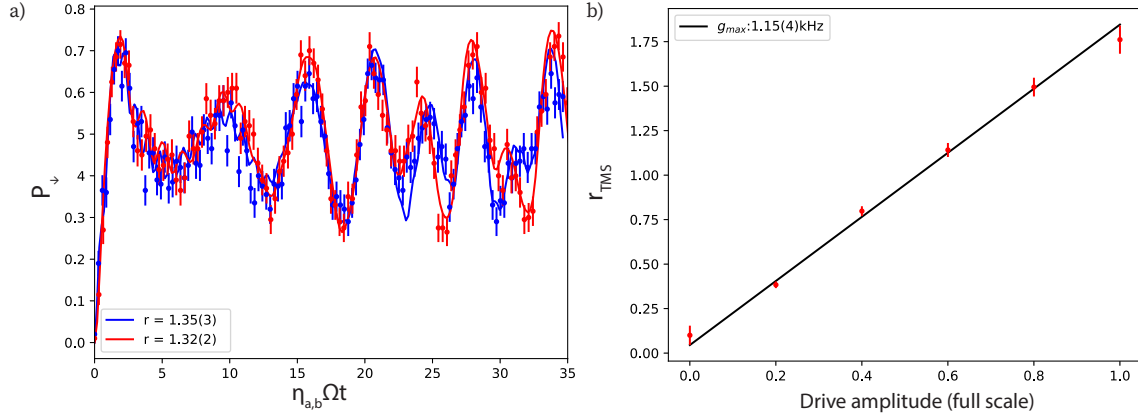


Figure 6.3. (a) Fits to time series data produced by blue sideband Rabi flopping on either radial mode with the motion in a two mode squeezed state. Tracing over a single mode when using sideband flopping produces a thermal state. The x-axis is rescaled with the relative Lamb-Dicke parameter to show overlap in time scales of the oscillations (b) Calibration data for determining the coupling rate of the two mode squeeze operation. Data is taken by fitting time series data like in a) after varying the amplitude of the driving field. The maximum coupling rate is taken as given coupling rate at full amplitude of the DDS.

6.6 Beam splitter

To produce the beam splitter operation described by the Hamiltonian in section 2.5.4, we digitally trigger the drive AWG to output a pulse consisting of an integer number of cycles (30 cycles in the runs in Fig. 6.4) at a fixed amplitude, starting at a phase of zero. We cannot continuously adjust the length of these pulses and therefore must calibrate the pulse amplitude to produce a 50/50 beam splitter pulse. Specifically, we select an amplitude that maximizes the fringe contrast of our SU(2) interferometer (i.e. that allows a pair of 50/50 beam splitter pulses to fully transfer a displacement from one mode to another). The starting phase of each pulse is fixed which means to adjust the relative phase between beam splitter pulses in our SU(2) experiments we adjust the delay time t_{delay} between pulses, giving a phase offset of $(\omega_a - \omega_b)t_{\text{delay}}$, as shown in Fig. 6.4.

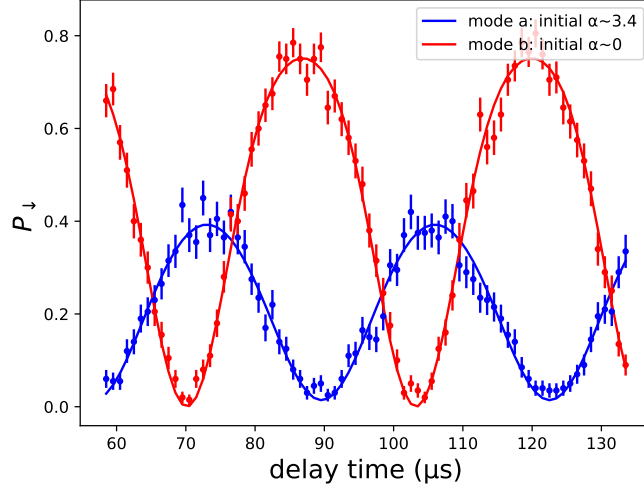


Figure 6.4. Starting with a coherent state in the low-frequency mode, the SU(2) interferometer is performed with a variable delay between beam splitter pulses, allowing coherent transfer of the coherent state between modes. The amplitude of the 50/50 beam splitter pulses used can be calibrated by maximizing contrast on mode *a* fringes. Delay time is time passed between beam splitter pulses.

6.7 Off-resonant motional coupling

Due to the narrow frequency splitting of our motional modes, the sideband interaction results in off-resonant driving of our second mode which is separated in frequency from the first mode by only 33 kHz for the results shown in Ch. VIII. The resulting interaction Hamiltonian is now more accurately written as

$$\hat{H} = \hbar\Omega/2(\hat{\sigma}^+\hat{a}^\dagger e^{i\delta_1 t} + \hat{\sigma}^-\hat{a}e^{-i\delta_1 t} + \hat{\sigma}^+\hat{b}^\dagger e^{i\delta_2 t} + \hat{\sigma}^-\hat{b}e^{-i\delta_2 t}) \quad (6.5)$$

where $\delta_1 - \delta_2 \approx 2\pi \times 33 \text{ kHz}$. There is no known analytical solution for the qubit probabilities under evolution of this Hamiltonian therefore cannot be used. In order to get a model free fit, master equation simulations were performed using QuTiP (Johansson et al., 2013) where each Fock state probability in the initial state is a fit parameter while preserving the trace. Estimates of fit uncertainties with this method proved to be a challenge due to weak dependence on the off-resonant mode. Fitting was performed starting with an ideal state and holding

all Fock states except one constant. The single Fock state was iterated through all Fock states for both mode a and b which may produce an under estimate of the uncertainty. Data where either mode a or b was driven resonantly was fit independently. Figure 6.5 shows the results of the fit Fock state fits with the model state fit to the probabilities for both modes independently and overlaid.

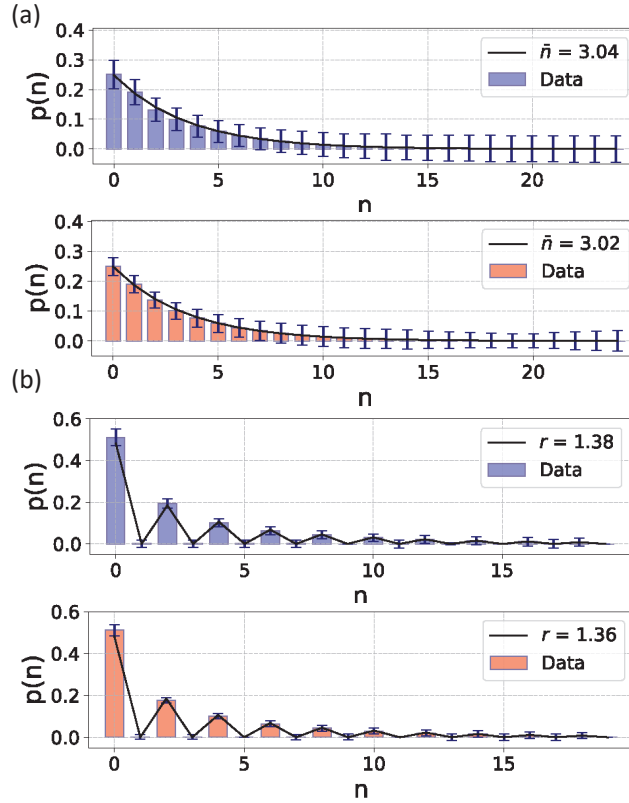


Figure 6.5. (a) Fits to the output of two-mode squeezing under interaction described above. Blue bars indicate mode a and red bars are mode b with associated error bars. Black line is a thermal state model fit to the Fock state fits. (b) Fock state fits to output of the beam splitter after two-mode squeezing. Blue data is mode a and red is mode b with associated error bars. Black line is a squeezed state model fit to the Fock state fits.

CHAPTER VII

PROTECTED MODE INVESTIGATION FOR MID-CIRCUIT MEASUREMENT

In order to run quantum algorithms, there are many instances where it is important to be able to make measurements of a subset of the full systems. This type of measurement can enable things like error correction that can employ a classical feedback mechanism (Ahn, Wiseman, & Milburn, 2003). This problem has been addressed when using spin qubits in different traps and having individual addressing, which is possible by using tightly focused laser beams (Nägerl et al., 1999). This can also be done with multiple ion species (Wright et al., 2016) or the OMG scheme (Allcock et al., 2021), where not all qubits have a state that couples to the detection laser.

This problem of mid-circuit measurement is more complicated when dealing with the motion. With a single ion, the ion is involved in all modes of motion, which means that photon scattering will heat and decohere every mode of motion. Chapter VI showed that with more ions, more interesting mode structure arises. By symmetry, when there is an odd number of ions in a chain, the center ion does not participate in the motion of the symmetric modes. This chapter covers the details of these “protected” modes and the experimental setup that was used to test the limits of the protection offered from photon scattering, when using the ion that doesn’t participate in the motion.

7.1 Mode structure and protected modes

Section 2.3 covers how when considering the Coulomb coupling present with multiple ions, a collective motional structure is present with N linearly independent modes for each principle axis of the trap. The mode structure for 3 ions is also characterized in section 2.3. With a odd number of ions a consequence of the symmetry means that the center ion does not participate in the motion. This

participation is characterized by the vector describing the components of motion of the ion j in the motional mode α . This is important when considering the Lamb-Dicke parameter for each ion in the chain given by

$$\eta_{j,\alpha} = \sqrt{\frac{\hbar}{2m_j\omega_\alpha}} \mathbf{k} \cdot \mathbf{e}'_{j,\alpha}, \quad (7.1)$$

where $\mathbf{e}'_{j,\alpha}$ is this mode-ion vector and $\mathbf{k}$ is the wavevector of the laser. With the center ion in the stretch mode having a zero Lamb-Dicke parameter, it is decoupled from this mode and should not show up in spectroscopy of the motional modes performed on the center ion, which can be seen in Fig. 7.1. For this experimental work, it is important to have the individual addressing of the qubits, in order to be able to investigate the effects of probing only the center ion in the 3 ion chain. This work is made possible by using a mixed species ion chain of $^{40}\text{Ca}^+$ and $^{88}\text{Sr}^+$ trapped ions in a surface electrode trap at Lincoln lab. The mode structure of this mixed species chain can be seen in Fig. 2.1.

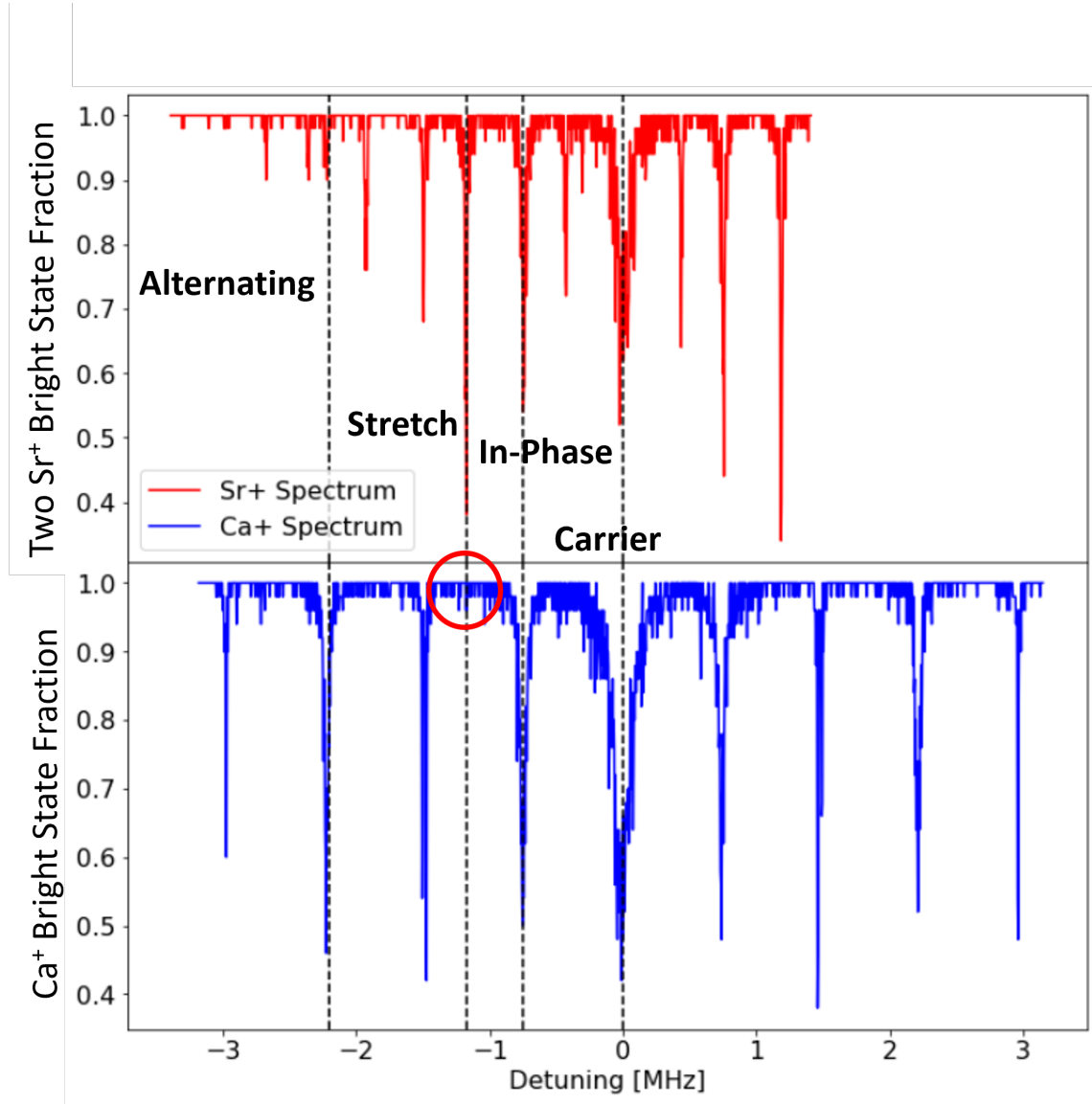


Figure 7.1. Spectroscopy of calcium and strontium ions showing the motional sidebands detuned clearly resolved from the carrier transition. The protected stretch mode is seen in the spectroscopy of the outer strontium ions, but is not present when performing spectroscopy with the center calcium ion.

7.2 Surface electrode trap with mixed species ion chain

The experimental setup at Lincoln Lab was used in order to experimentally verify that this protected mode can be preserved during ion detection, given the mixed species capabilities with this setup. All data was collected by Colin

Bruzewicz and I developed experimental sequences and performed data analysis. More complete details of the experimental setup can be seen in Bruzewicz, McConnell, Stuart, Sage, and Chiaverini (2019). This setup consists of a cryogenic surface-electrode trap with a strong magnetic shield to prevent decoherence of the Zeeman qubits used for this work. This 2-dimensional surface-electrode trap consists of a $2\,\mu\text{m}$ -thick, patterned aluminum layer on a sapphire substrate and can be seen in Fig. 7.2. Using segmented electrodes enables the ability to control the position and twisting of the crystal which affect the mode structure of a chain of ions. The protected mode structure only exists under the assumption that the ion chain lies along the null of the RF potential.

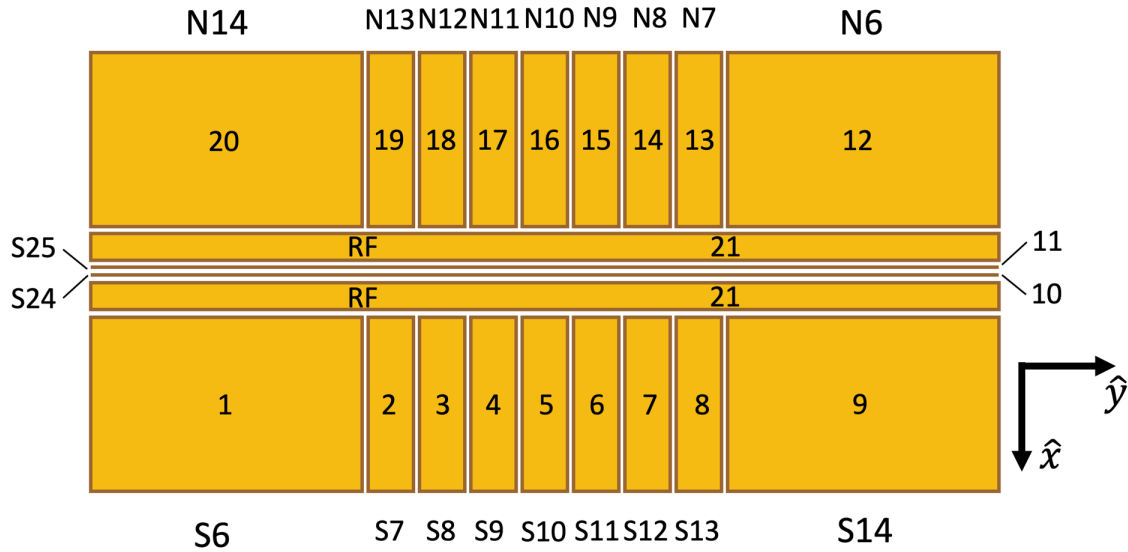


Figure 7.2. [Figure made by Colin Bruzewicz] Schematic of the surface electrode trap used for this experimental work. Trapping RF electrodes are labelled and run along the length of the trap axis. All other electrodes are DC electrodes with individual connections to enable arbitrary control of the ion location.

7.3 Heating rates

A first order check on whether the protected mode is preserved during ion detection is to determine if this mode leaves the ground state after state detection of the center ion. In order to determine this, a heating rate is measured. All modes of motion have stochastic anomalous heating (Deslauriers et al., 2006) that can be modeled with Lindblad operators $\hat{\mathcal{L}}_1 = \sqrt{\dot{n}}\hat{a}$ and $\hat{\mathcal{L}}_2 = \sqrt{\dot{n}}\hat{a}^\dagger$ where $\dot{n}$ is the anomalous heating rate and $\hat{a}(\hat{a}^\dagger)$ are the annihilation (creation) operators for the mode of interest. This type of heating (decoherence) arises from the oscillator coupling to a bath of an infinite number of oscillators at thermal equilibrium (Turchette et al., 2000). This heating rate can be measured by performing thermometry of the ion as a function of time. In addition to the anomalous heating, when considering photon scattering, can experience excitations on the red or blue sideband sideband during fluorescence. Excitations on the red sideband are suppressed when near the ground state, which means carrier and blue sideband excitations dominate. As a consequence, the motion is quickly driven the the Doppler limit where an equilibrium is reached. If the heating rate measured as a function of the duration of the state detection is not significantly different than the anomalous heating rate, it can be assumed that mode heating from state detection is not the primary cause of decoherence.

7.3.1 Sideband thermometry. For a single ion, measuring the mean motional occupation $\bar{n}$ is fairly straight forward using the red and blue sideband. Equation 2.79 showed the populations for a thermal state and eqs. 6.4 and 6.3 gave the qubit probabilities on the red and blue sideband. Taking a ratio of the red and blue sideband qubit probabilities $P_\uparrow^{rsb}(t)$ and $P_\uparrow^{bsb}(t)$ gives (Bergquist et al., 1987)

$$r = P_\uparrow^{rsb}(t)/P_\uparrow^{bsb}(t) = \frac{\bar{n}}{\bar{n} + 1}. \quad (7.2)$$

Populations	red sideband	blue sideband
$ \langle\downarrow\downarrow \Psi\rangle ^2$	$\left(1 - \frac{n}{2n-1}[1 - \cos(g_r\Omega t/2)]\right)^2$	$\left(1 - \frac{n+1}{2n+3}[1 - \cos(g_b\Omega t/2)]\right)^2$
$ \langle\downarrow\uparrow \Psi\rangle ^2, \langle\uparrow\downarrow \Psi\rangle ^2$	$\frac{n}{2(2n-1)} \sin^2(g_r\Omega t/2)$	$\frac{n+1}{2(2n+3)} \sin^2(g_b\Omega t/2)$
$ \langle\uparrow\uparrow \Psi\rangle ^2$	$\frac{n(n-1)}{(2n-1)^2} [1 - \cos(g_r\Omega t/2)]^2$	$\frac{(n+1)(n+2)}{(2n+3)^2} [1 - \cos(g_b\Omega t/2)]^2$

Table 1. Populations of the four different possible qubits states of a two ion systems after resonant red or blue sideband transitions on the state $|\Psi, n\rangle$ with pulse length $t.g_r = \eta_m \sqrt{2(2n-1)}$ and $g_b = \eta_m \sqrt{2(2n+1)}$, where η_m is the Lamb-Dicke parameter of either the center of mass mode η_c or the stretch mode η_s

This ratio of the excitations on the two sidebands is a simple algebraic equation and conveniently does not depend on the sideband pulse duration or Rabi frequency. This ratio tends to 1 for $\bar{n} \gg 1$, which can also be near the Doppler limit, and tends to $\bar{n}$ for $\bar{n} \ll 1$. A simple rearrangement of this expression gives the mean occupation as a function of the sideband ratio

$$\bar{n} = \frac{r}{1-r}. \quad (7.3)$$

For the longer chain of ions considered here, the problem of sideband thermometry is not as straightforward. While there are three ions in the chain, sidebands are only driven on two of them, so this can be considered a two ion problem. For two ions, the sideband probabilities are shown in table 1, taken from (Webster, 2005), for each of the measurement outcomes of the state $|\Psi\rangle$. For two ions, there is not a time and Rabi frequency independent ratio of the sidebands. Despite this limitation, a joint fit of all the qubit probabilities can be performed to extract the mean motional occupation. Fig. 7.3 shows the red and blue sideband probabilities for the three possible measurement outcomes. A fit to the data is used to get the peak height. The peak height for all three outcome probabilities is used to perform a least squares fit and return the mean motional occupation number as well as the pulse duration and Rabi frequency.

The heating rate above the anomalous heating rate depends on the intensity and detuning from atomic resonance of the detection laser. The anomalous heating rate is ~ 1 quanta/sec with heating rates from photon scattering varying from as low as 2 quanta/sec to as high as 90 quanta/sec. The Lamb-Dicke parameter has been estimated to be as low as 10^{-4} which means of order 10^8 photons would need to be scattered to get to a thermal state with $\bar{n} = 1$. As is, this leads to limited heating during a detection cycle and could be reduced further by mitigating effects that break the symmetry of the mode.

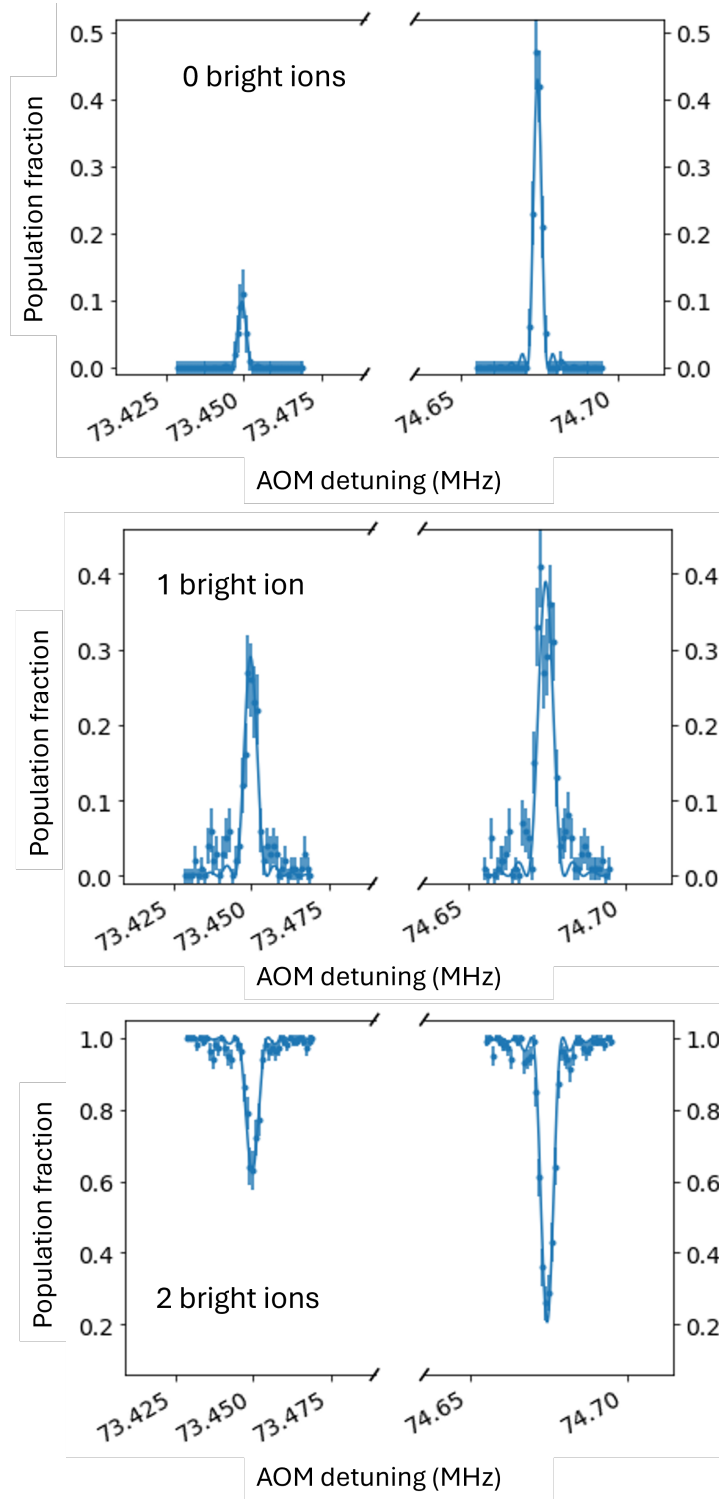


Figure 7.3. Fits to the red and blue sideband transitions on the two Sr^+ ions. The ratio of the sideband peak heights are used in a joint fit to extract the average motional quanta in order to determine a heating rate.

7.4 Motional Fock state Ramsey interferometry

Heating of a motional mode is only one mechanism of decoherence.

Determining the heating rate of the protected mode provides only a lower bound on the possible coherence time. In order to determine the full coherence time, the phase relation between Fock states should be measured. For single ions, a motional Ramsey type experiment is straightforward to implement. Starting with a carrier $\pi/2$ pulse on a pure qubit state and the ground state of motion gives the state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle + |\uparrow\rangle) \otimes |n=0\rangle$. With a π pulse on the blue sideband the spin superposition can be mapped onto the motion, $|\Psi\rangle = |\uparrow\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. A time reversal of this procedure enables to ability measure for how long the phase relationship between the $|0\rangle$ and $|1\rangle$ Fock states can be tracked.

For two ions, it is not possible to perform the π pulse on the blue sideband that gives the above result. A more involved sequence is necessary to remove the spin superposition while giving a motional superposition. Fig. 7.4 shows a pulse sequence involving a blue and red sideband pulse which produces approximately the final state $|\Psi\rangle = |\downarrow\downarrow\rangle \otimes (0.33|0\rangle + 0.93|4\rangle)$. The same time reversal scheme can be used here to measure the motional coherence. It is also important to point out that using a higher Fock state in this experiment means that the expected decoherence rate should be 4x larger than when using a $|0\rangle, |1\rangle$ superposition. The contrast of this signal reduced for this sequence, given the lack of a equal superposition, which means sensitivity is not increased 4x for this interferometer. The phase of the final pulse can be varied to provide fringes which can be used to determine the coherence time. As longer delay times are used there is an expected decay in the contrast of the fringes and this decay can be fit to give a photon scattering-free coherence time. When state detection is performed on the center ion during this Ramsey sequence, any decoherence caused by this detection beam will lead to a more rapid decay

in the contrast. Preliminary results show that there is negligible loss in contrast after $400 \mu s$ of photon scattering, consistent with the heating rate of ~ 4 quanta/sec heating rate. Results can be seen in Fig. 8.3.

Similar results have been shown in Hou et al. (2022), where the beam splitter interaction has been used to coherently swap the state of non-protected modes with the protected modes. This ability has been used to preserve the desired state in the protected mode during state detection.

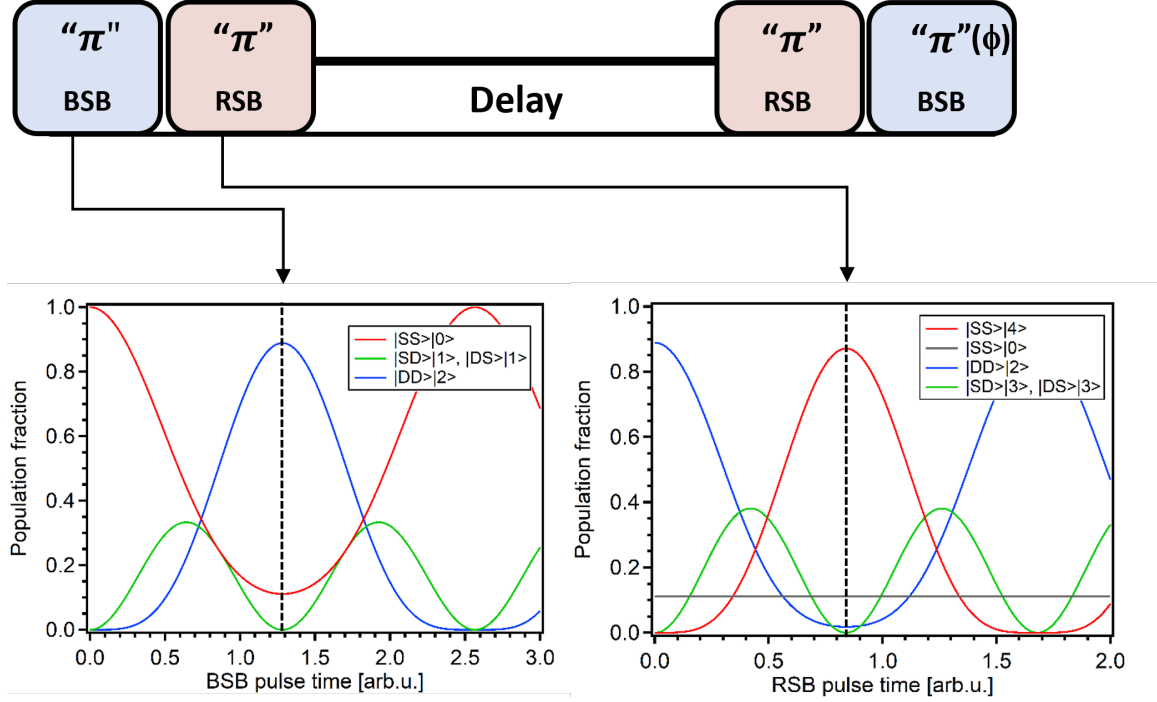


Figure 7.4. [Figure made by Colin Bruzewicz] Pulse sequence used to determine the motional coherence of the stretch mode. Combining red and blue sideband pulses enables the ability to prepare a motional superposition of Fock states while leaving the spin in a single spin state. Quotations are around the π to indicate that these pulses do not achieve exact population swapping.

7.5 Perturbations to protection

In order for the Lamb-Dicke parameter of the center ion to remain zero, the ion chain must be perfectly aligned with the RF null of the trap. There are a number of different ways that the ion chain can become offset from the RF

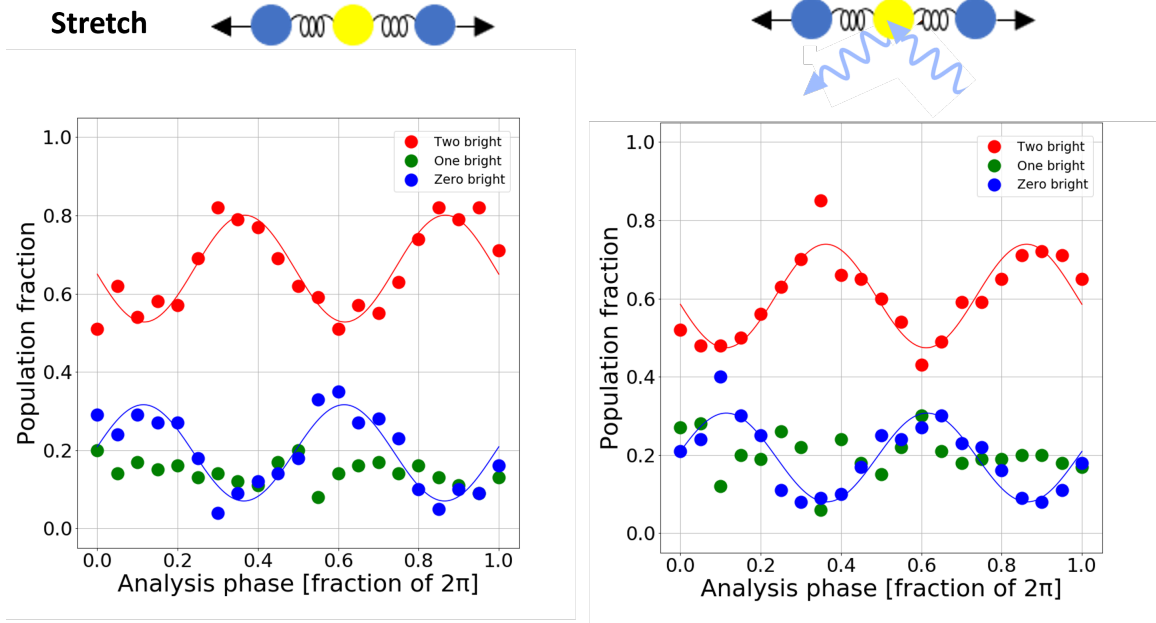


Figure 7.5. Phase scans of the final sideband pulse using the motional interferometer sequence shown in 7.4. The first panel shows fringes after a $400\ \mu\text{s}$ delay without the use of the $397\ \text{nm}$ fluorescing beam. The second panel shows similar fringe contrast after the same amount of delay time but with the fluorescing beam on during the delay.

null. The detection field itself can also exert a force, called radiation pressure, on the ions and cause a breaking in the symmetry. Stray electric fields are the main contributor to an offset and can lead to a significant departure from the ideal mode geometry. These types of offset can generally be detected and compensated for by detecting excess micromotion and leads to additional motional sidebands when performing spectroscopy. Both of these effects have been considered and investigated in this section.

7.5.1 Radiation pressure. State detection is performed using a near resonant laser on a broad dipole allowed transition. This laser exerts a force on the ion from the laser field which time averages to

$$\mathbf{F} = \hbar \mathbf{k} \Gamma \rho_{ee}, \quad (7.4)$$

where $\mathbf{k}$ is the wavevector of the laser, Γ is the natural linewidth of the dipole transition and $\rho_{ee} = \langle e|\hat{\rho}|e\rangle$ is the steady-state population of the excited state. This probability depends on the tuning of the laser and is given by

$$\rho_{ee} = \frac{s/2}{1 + s + (2\Delta/\Gamma)^2} \quad (7.5)$$

where $s = 2|\Omega|^2/\Gamma^2$ is the saturation parameter, Ω is the Rabi frequency and Δ is the laser detuning. For typical laser detunings used for Doppler cooling $\Delta = -\Gamma/2$ and $s = 1$, this force is of order 10^{-20} N and can lead to displacements of the ion from equilibrium of order nm. This shift leads to changes in the mode structure and mode frequency, but from models would only lead to a Lamb-Dicke parameter of the protected mode of less than order 10^{-3} . This small of a Lamb-Dicke parameter would mean that heating induced from this effect would be below the anomalous heating rate and therefore not easy to detect.

7.5.2 Stray electric fields. The presence of unwanted electric fields at the location of the ions is a certainty. It is possible through different techniques to measure the effect of these fields (Berkeland, Miller, Bergquist, Itano, & Wineland, 1998) and use trap electrodes to compensate for them. In order to consider the effects of these fields it is worth briefly discussing the potential that the ion is subject to. The potentials can be separated into static and dynamic potentials:

$$\begin{aligned} \phi_{st}(\mathbf{r}) &= \frac{V}{r_0^2}(\chi_x x^2 + \chi_y y^2 + \chi_z z^2) + \mathbf{E} \cdot \mathbf{r} \\ \phi_{RF}(\mathbf{r}) &= \frac{U_{RF}}{r_0^2}(\kappa_x x^2 + \kappa_y y^2 + \kappa_z z^2) \cos(\Omega_{RF} t), \end{aligned} \quad (7.6)$$

where $\mathbf{r} \equiv (x, y, z)$, r_0 is the distance from the ion to the nearest electrode and $\chi_{x,y,z}, \kappa_{x,y,z}$ are dimensionless shape parameters dependent on trap geometry. $\mathbf{E}$ is a static electric field that is added to account the effects of stray fields. There are constraints on the shape parameters to satisfy Laplace's equation (Griffiths,

1981) and considering the oscillating part of the potential in equation 7.6, the ponderomotive part of the potential is given as

$$\Phi_{pond}(\mathbf{r}, m) = \frac{qU_{RF}^2}{m\Omega_{RF}^2 r_0^4} (\kappa_x x^2 + \kappa_y y^2 + \kappa_z z^2) \quad (7.7)$$

which from the nature of this potential is mass dependent and gives rise to secular oscillations at a frequency

$$\omega_{x,y,z} = \sqrt{\frac{2q^2 U_{RF}^2 \kappa_{x,y,z}}{m^2 \Omega_{RF}^2 r_0^4} + \frac{2qV \chi_{x,y,z}}{mr_0^4}}. \quad (7.8)$$

The mass dependence of this potential is very important for the mixed species ion chain considered here. Any uncompensated electric fields will cause displacement of the ion chain from the RF null, where the displacement of the ions will depend on the depth of the trapping potential and the mass of the ions. In a chain consisting of strontium and calcium ions, the strontium ions have more than double the mass and will be effected more significantly by uncompensated electric fields. Unequal displacements of the ions cause a sort of zig-zag structure in the ion chain and break the symmetry that decouples the center ion from participating in any mode. Besides careful calibration to ensure stray fields have been calibrated, this is another potential benefit to using the OMG architecture where all ions are the same mass and uncompensated stray fields won't break the chain symmetry.

CHAPTER VIII

SU(2) AND SU(1,1) INTERFEROMETRY FOR PRECISION PHASE MEASUREMENT

Given that state motional state preparation, control and readout has been covered in chapters IV, VI, VII, we can discuss methods for using trapped ion motion for quantum sensing. In the following, we demonstrate different classes of interferometers using the motional modes of a single trapped atomic ion. With a demonstration of these interferometers, we show the ability to concatenate several operations on the oscillator states of two modes in a phase coherent manner. Performing these operations with digitally programmable frequency sources for relative phase control simplifies significant calibration challenges when developing more complicated circuits.

Yurke *et al.* (Yurke, McCall, & Klauder, 1986) proposed schemes to reach the Heisenberg limit (Pezzé & Smerzi, 2009) using different classes of interferometers. Here, we work within the framework of the Cramér-Rao bound (Agarwal & Davidovich, 2022; Zwiern, Pérez-Delgado, & Kok, 2010) which sets the ultimate limit for the resolution of a sensing scheme. This bound is described by a parameter variance, instead of the Heisenberg limit which is set directly by the number of resources used, i.e. photon or phonon number (Zwiern *et al.*, 2010), making it more suitable to the limits of the experiments reported here. The required interactions for the interferometers (described below) are colloquially called displacing, beam splitting, single-mode squeezing, and two-mode squeezing. We have already shown the ability to perform these interactions and characterize the states that are produced. Performance of these interferometers can, in practice, be limited only by the motional heating of the ion resulting from fluctuating patch potentials on the trap electrodes (Deslauriers *et al.*, 2006; Hite *et al.*, 2013). Here,

we demonstrate an $SU(2)$ interferometer (Mach-Zender), which has previously been demonstrated in (S. Ding et al., 2017b; Gorman et al., 2014; Leibfried et al., 2002), (coherent input states are used here in contrast to those works), as well as single-mode and two-mode $SU(1,1)$ ¹ interferometers. Each of the interferometers described here is sensitive to different phase shifts, which are single-mode phase ϕ_a (single-mode $SU(1,1)$), difference phase $\phi_a - \phi_b$ ($SU(2)$), or sum phase $\phi_a + \phi_b$ (two-mode $SU(1,1)$), where the a, b labels refer to different modes. We show performance close to the Cramér-Rao bounds for these interferometers.

To demonstrate these different interferometers we use the experimental setup shown in Chapter IV. Experiments are performed using the two ‘radial’ modes of the ion (normal to the trap axis) with frequencies $\omega_a \approx 2\pi \times 1.80$ MHz and $\omega_b \approx 2\pi \times 1.83$ MHz, and oscillator energy eigenstates denoted by $|n_a, n_b\rangle$. The mode splitting can be adjusted by changing a DC potential on the RF electrodes which also rotates the mode angle and can be used to vary the achievable coupling rate of the interactions described below. We use qubit states $|\uparrow\rangle \equiv |m_L = +5/2\rangle$ and $|\downarrow\rangle \equiv |m_L = +3/2\rangle$ within the metastable $D_{5/2}$ manifold which has a lifetime of $\simeq 1.1$ seconds (Kreuter et al., 2005), where m_L is the total angular momentum projection along the direction of the quantization magnetic field of 0.498 G. The qubit transition frequency is $\omega_0 \approx 2\pi \times 2.63$ MHz. This is a magnetic field sensitive qubit with a coherence time of $\simeq 1$ ms, while stable trapping potentials provide motional coherence times over 20 ms. In each experiment the qubit is first prepared in the $|\uparrow\rangle |0, 0\rangle$ with the methods shown in Chapter IV. Mode temperatures correspond to $\bar{n}_{a,b} < 0.15$. Motional state analysis is performed using the techniques described in Ch. VI. The necessary interactions are generated by applying the same

¹The Lie-group labelling is used to refer to the groups that contain the operators that generate these states.

pair of Raman beams that drive the qubit, but detuned above (red sideband) or below (blue sideband), the qubit frequency. The red sideband is again used to verify preparation or return to the motional ground state. The blue sideband interaction is used to fully characterize the generated state populations (Meekhof et al., 1996). Generation of the motional states is accomplished by applying

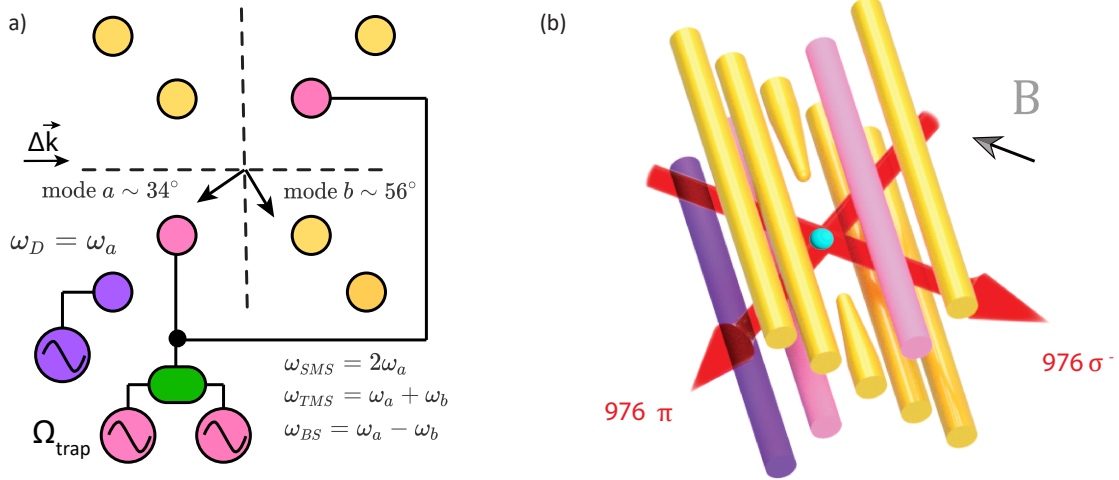


Figure 8.1. (figure from Metzner et al. (2023)) (a) View of trap along the trap axis showing the electrical connections: *pink*—RF quadrupole electrodes, trap and parametric drive, *purple*—displacement drive and DC electrode, *yellow*— other DC electrodes and relevant control electrodes. The green box represents the circuitry necessary to combine the drives. Mode *a* and mode *b* label the two different ‘radial’ mode vectors of a single ion with the angles from the horizontal shown. (b) A 3D rendering of the trap showing direction of the 976 nm laser used for motional state preparation and state characterization, and cyan ball representing the $^{40}\text{Ca}^+$ ion

time-varying potentials to the trap electrodes (see Fig. 8.1) producing a unitary operation described in Ch. II. The displacement operation is produced by applying an oscillating potential, at the mode frequency, to a single trap electrode (purple electrode Fig. 8.1). Single-mode squeezing, two-mode squeezing and the beam splitter are accomplished by applying an oscillating potential at twice the mode frequency ($2\omega_a$), at the sum of the mode frequencies ($\omega_a + \omega_b$) or at the difference frequency ($\omega_a - \omega_b$) respectively (pink electrodes Fig. 8.1). These modulated

potentials yields a generalized squeezing interaction Hamiltonian,

$$\hat{H} = i\hbar\frac{g}{2}[\hat{a}_i\hat{a}_j\exp(i\delta t)\exp(-i\theta) - \hat{a}_i^\dagger\hat{a}_j^\dagger\exp(-i\delta t)\exp(i\theta)] \quad (8.1)$$

where g is the parametric coupling strength, which has a different value for each operation, δ is the detuning from the interaction resonance (either single mode squeezing (mode a or b) or two-mode squeezing), and θ is the phase of the parametric modulation. With the proper choice of detuning this implements the unitary squeezing operator with squeezing parameter magnitude given by $r_{SMS} = gt$ and the two mode squeezing operator with $r_{TMS} = 2gt$. Here, we used a single-mode displacement coupling of $g = 2\pi \times 1.37$ kHz which gives a displacement parameter $\alpha = gt$. The maximum single mode squeezing coupling is $g = 2\pi \times 3.99$ kHz and a maximum two-mode squeezing coupling of $g = 2\pi \times 1.15$ kHz. Maximum coupling refers to a coupling rate calculated from fits to the states with the maximum voltage we used and the relevant pulse time for each operation. The beam splitter coupling is $g = 2\pi \times 0.66$ kHz calculated using calibration data shown in Chapter VI.

8.1 Verification of two-mode squeezed state with beam splitter

In order to verify the proper performance of the beam splitter operation, we use a calibration experiment described in section 6.6. Verification of the two-mode squeezed state is more complicated given we are only able to readout a single mode in a single shot of the experiment. Using the blue sideband readout method, described above, is equivalent to tracing over a single mode and a thermal state is measured (C. M. Caves et al., 1991). This is analogous to measuring a single qubit from a Bell state which produces a random qubit state. Additionally, we can use a beam splitter operation after the two-mode squeezer to disentangle the modes and produce single-mode squeezed states (Fig. 8.2). Performing both of these

measurements verifies we have produced the target two-mode squeezed state and enables the use of these states for interferometric measurements.

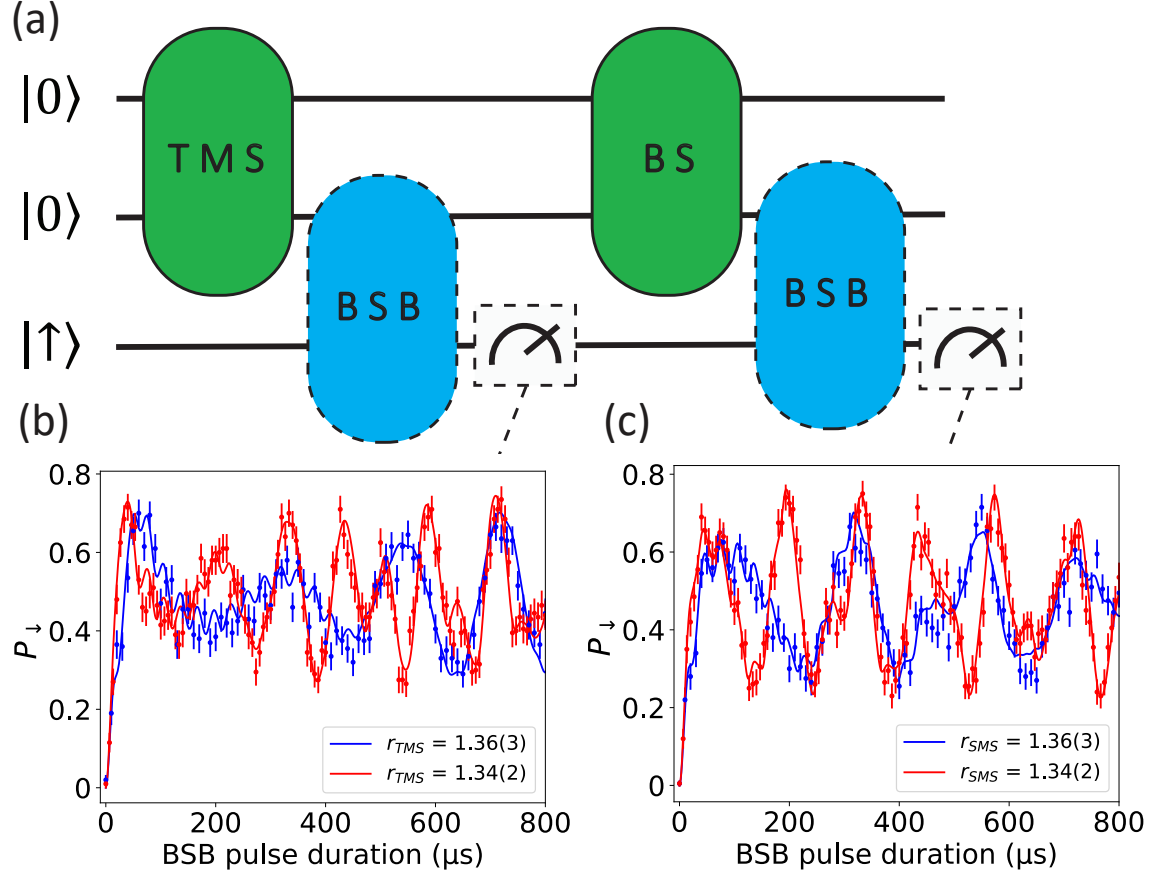


Figure 8.2. (figure from Metzner et al. (2023)) (a) General quantum circuit diagram for characterizing the two-mode squeezed state with blue sideband readout performed at one of two instances, to verify we are producing the expected output (b) Measurement after the two-mode squeezed state and before beam splitter produces a thermal state. (c) applying the beam splitter operation after the two-mode squeezer and then characterizing the state produces two single-mode squeezed states with squeezing parameter r equal to that of the two-mode squeezed state.

8.2 Cramer-Rao bounds

Implementing time reversal protocols is a method for generating these types of interferometers and is a powerful tool for sensing applications (Colombo et al., 2022). These protocols also enable us to verify that we are coherently

generating the desired states by time-reversing back to the initial state. The SU(2) interferometer (Fig. 8.3c) is implemented by first displacing mode a in our experiment, which generates the desired initial coherent state. The beam splitter is then used to generate a superposition of coherent and vacuum states. The phase of the final beam-splitter is controlled by using a variable delay, such that the accumulated phase is $(\omega_a - \omega_b)t_{delay}$. The final beam splitter will completely transfer the phonons to mode b with the correct choice of relative phase, and the red sideband readout will verify we have returned to the ground state (Fig. 8.3c). We use a maximum displacement of $\alpha \simeq |6|$ to maintain a similar maximum $\bar{n}$ in the single and two-mode squeezed states.

The single-mode (Fig. 8.3a) and two-mode (Fig. 8.3b) SU(1,1) interferometers are implemented by using the parametric drive tuned to either twice the frequency of mode a or the sum frequency of mode a and b respectively. To avoid increased decoherence, from long pulse durations, these pulses are run at a fixed duration with the drive voltage being varied to control the size of the state. A second squeezing pulse is then applied (Fig. 8.3b) with a varied phase controlled by the driving electronics. We are limited in the amount of two-mode squeezing we can generate due to our increased sensitivity to common-mode phase fluctuations that this interferometer is sensitive to.

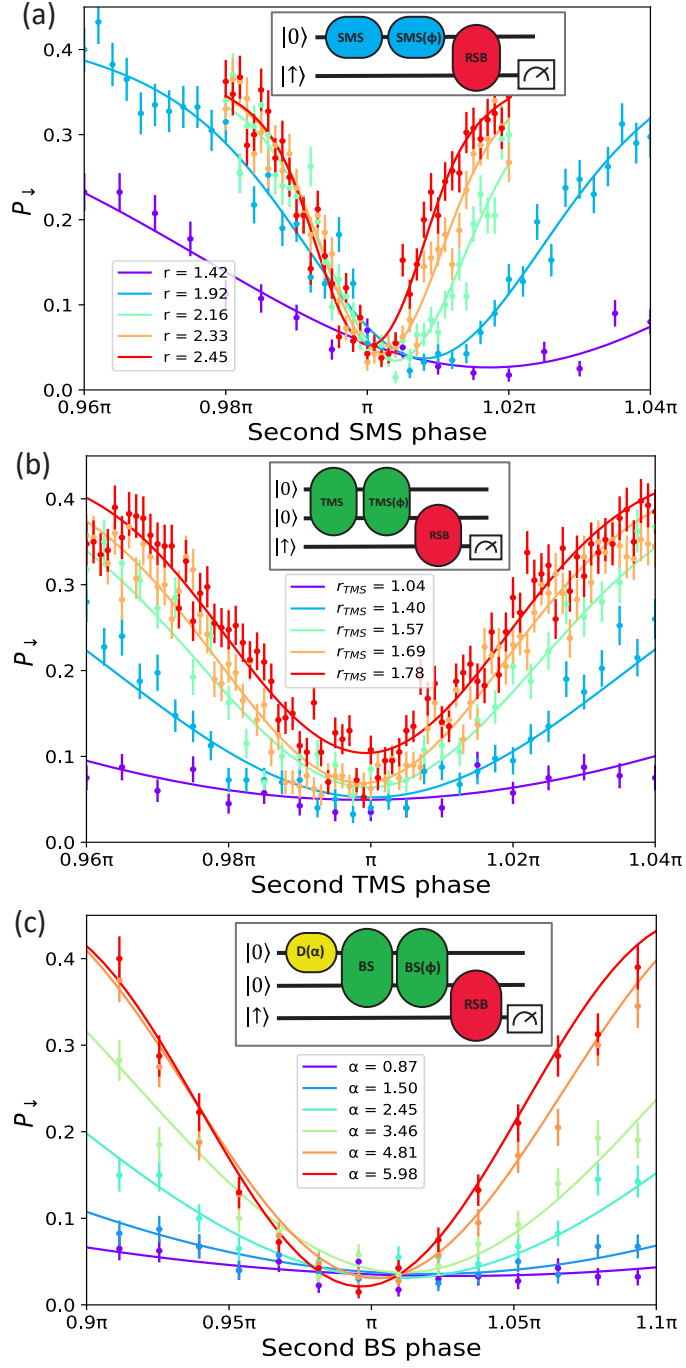


Figure 8.3. (figure from Metzner et al. (2023)) (a) Single-mode SU(1,1) interferometer output for different amounts of squeezing is shown by varying the relative phase of the second squeezing pulse shown in the circuit diagram (inset). (b) repeat of (a) with two-mode squeezing and two-mode SU(1,1) shown in the interferometer circuit inset. (c) SU(2) interferometer output shown for various sizes of initial displacement where relative phase is controlled with a variable delay shown in the SU(2) circuit inset.

In order to determine the sensitivities to small phase shifts that are achievable with these different experiments, we calculate the quantum Fisher information for a pure state (Agarwal & Davidovich, 2022)

$$\mathcal{F}(\phi) = 4(\Delta\hat{\mathcal{G}})^2 \quad (8.2)$$

where $(\Delta\hat{\mathcal{G}})^2$ is the variance in the initial state of the probe (coherent state, squeezed state, two-mode squeezed state) and $\hat{\mathcal{G}} = i[d\hat{U}^\dagger/d\phi]\hat{U}$. For phase sensing $\hat{U} = e^{i\phi a^\dagger a}$ and therefore $\hat{\mathcal{G}} = a^\dagger a$ and the Fisher information is proportional to variance in the phonon number of the probe states. The phase sensitivity is limited by the Cramér-Rao bound (Agarwal & Davidovich, 2022) given by

$$\Delta\phi \geq \frac{1}{\sqrt{\mathcal{F}(\phi)}}. \quad (8.3)$$

This means for the SU(2) interferometer we are limited at

$$\Delta\phi = \frac{1}{\alpha} = \frac{1}{\sqrt{\langle N \rangle_d}} \quad (8.4)$$

where $\langle N \rangle_d$ is the expected mean phonon number for the displacement generated and this limit corresponds to the expected standard quantum limit. The limit for the single-mode SU(1,1) interferometer is given by (Monras, 2006)

$$\Delta\phi = \frac{1}{\sqrt{8\langle N \rangle_{SMS}(\langle N \rangle_{SMS} + 1)}} \quad (8.5)$$

where $\langle N \rangle_{SMS}$ is the expected mean phonon number for the input squeezed state.

The two-mode SU(1,1) limit is given by (Agarwal & Davidovich, 2022)

$$\Delta\phi = \frac{1}{\sqrt{\langle N \rangle_{TMS}(2 + \langle N \rangle_{TMS})}} \quad (8.6)$$

where $\langle N \rangle_{TMS}$ is the expected mean phonon number for the input two-mode squeeze state.

The sensitivities achieved are calculated from the experimental data using the Fisher information (Agarwal & Davidovich, 2022)

$$F(\phi) = \sum_j \frac{1}{P_j(\phi)} \left[\frac{dP_j(\phi)}{d\phi} \right]^2 \quad (8.7)$$

where $P_j(\phi)$ is the probability of getting the experimental results j if the value of the parameter is ϕ . In our experiment with only two possible outcomes, spin-up or spin-down, this simplifies to

$$F(\phi) = \frac{1}{P_\downarrow(1 - P_\downarrow)} \left[\frac{\partial P_\downarrow}{\partial \phi} \right]^2. \quad (8.8)$$

The variance and slope of the signal are expressed analytically and are fit to the phase signals that we generate (Fig. 8.3). These theoretical curves are derived, in section 8.3, without any adverse effects and represent the ideal experimental output (Fig. 8.4). Systematic offsets in the phase fringes, seen in the single-mode squeezing phase scans, from π are due to an additional pseudopotential (Leefer et al., 2017) when the parametric drive is applied. This additional potential accounts for a maximum shift of 10 kHz of the mode frequencies. In the single mode SU(1,1) experiment the interaction frequency is calibrated at the maximum coupling meaning as the coupling is turned down, the mode frequencies decrease and larger phase offsets are produced. This phase offset is calibrated away in the two-mode phase scans by calibrating the mode frequencies at every parametric drive voltage. The maximum phase sensitivity is extracted and plotted against the prediction from the quantum Fisher information (Fig. 8.4). The results we achieve deviate from the ideal limit due to motional and qubit decoherence as well as motional heating. Initial mode temperatures also affects these results and contribute to vertical offsets of the curves in Fig. 8.3. The ideal experimental

output deviates from the CR bound due to the choice of red side-band readout times, where optimal pulse time varies with $\bar{n}$. Red side-band pulse times were not optimized across all $\bar{n}$ due to the constraints on pulse times due to off-resonant driving of other modes and increased pulse length causing further decoherence.

8.3 Theoretical bounds

To obtain an analytical expression for the output of these interferometers we must determine the action of the red sideband on the final state and determine the expectation values for the final qubit projection operator. We calculate this below for the two-mode SU(1,1) case. The single-mode case can be treated in a similar manner. We start by representing the two-mode squeezed state in the Fock basis (Kurochkin, Prasad, & Lvovsky, 2014)

$$|TMS\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r |n, n\rangle = \sqrt{\frac{1}{1+\lambda}} \sum_{n=0}^{\infty} \left(\sqrt{\frac{\lambda}{1+\lambda}} \right)^n |n, n\rangle \quad (8.9)$$

where $\lambda = \sinh r$. We need to determine $|out\rangle = TMS(\phi)TMS|0, 0\rangle$. For phase other than $\phi = \pi$ the output is still a two-mode squeezed state with a different two-mode squeeze parameter r , which we can express as a function of the phase.

$$r(\phi) = \sinh^{-1}(\sinh r_0 \cos \phi/2), \quad (8.10)$$

where r_0 is the input two-mode squeeze state magnitude. The red sideband Hamiltonian is written as (Meekhof et al., 1996)

$$H_{RSB} = \hbar\eta \frac{\Omega}{2} (\sigma^+ a + \sigma^- a^\dagger). \quad (8.11)$$

Ω is the qubit carrier Rabi frequency, η is the Lamb-Dicke parameter (Meekhof et al., 1996) for the given mode, and σ^+ and σ^- are the single qubit raising and lowering operators. The unitary operator for the Hamiltonian can be written as

$$U = e^{\frac{i\beta H_{RSB}t}{\hbar}} \quad (8.12)$$

where $\beta = \eta\Omega t/2$. Expanding this unitary into an infinite series gives

$$|n, n\rangle \rightarrow \sum_{j=0}^{\infty} \frac{(i\beta)^j}{j!} (\sigma^+ a + \sigma^- a^\dagger)^j |\downarrow, n, n\rangle. \quad (8.13)$$

Only terms having equal numbers of raising and lower operators for spin and motion will have a final contribution to the projection, so we can re-write this state

$$U_{RSB} |\downarrow, n, n\rangle = \sum_{j=0}^{\infty} \frac{(i\beta)^j}{j!} (\sigma^+ a \sigma^- a^\dagger)^j |\downarrow, n, n\rangle, \quad (8.14)$$

where $\{j\}$ is only even integers. This leaves the final spin state unaffected and only the number operator $a^\dagger a$ remains, leaving us

$$\begin{aligned} U_{RSB} |\downarrow, n, n\rangle &= \frac{(i\beta)^{2j}}{(2j)!} n^j |\downarrow, n, n\rangle = \\ \frac{(i\beta)^{2j}}{(2j)!} \sqrt{n}^{2j} |\downarrow, n, n\rangle &= \cos(\beta\sqrt{n}) |\downarrow, n, n\rangle. \end{aligned} \quad (8.15)$$

The expectation value of the spin projector is therefore

$$\begin{aligned} \langle \downarrow, n, n | \downarrow \otimes \rangle \langle \downarrow \otimes | \downarrow, n, n \rangle &= \\ \frac{1}{1+\lambda} \sum_{n=0}^{\infty} \cos(\sqrt{n}\eta\Omega \frac{t}{2}) \left(\frac{\lambda}{1+\lambda} \right)^n & \quad (8.16) \end{aligned}$$

For the sake of simplicity we will call this expectation value $\langle \hat{X} \rangle$ and we define the variance

$$\Delta \hat{X}^2 = \langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2. \quad (8.17)$$

For spin projection $\langle \hat{X}^2 \rangle = \langle \hat{X} \rangle$. We also need an expression for the derivative of the expectation value as a function of the phase;

$$\begin{aligned} \frac{d\langle \hat{X} \rangle}{d\phi} &= \frac{d\lambda}{d\phi} \frac{1}{1+\lambda} \\ &\left(\langle \hat{X} \rangle + \frac{1}{1+\lambda} \sum_{n=0}^{\infty} \cos^2(\sqrt{n}\eta \frac{\Omega t}{2}) n \left(\frac{\lambda}{1+\lambda} \right)^{n-1} \right. \\ &\quad \left. \left(1 - \frac{\lambda}{1+\lambda} \right) \right) \end{aligned} \quad (8.18)$$

where $\frac{d\lambda}{d\phi} = -2 \cosh^2 r \sin \phi \sinh^2 r$. The same process is carried out and shown for the coherent state in the SU(2) interferometer as these results differ significantly from the two-mode SU(1,1). A coherent state is expressed in the Fock basis as

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{n!} |n\rangle. \quad (8.19)$$

The red sideband produces a similar output,

$$\begin{aligned} \langle \downarrow, n | \downarrow \otimes \rangle \langle \downarrow \otimes | \downarrow, n \rangle &= \langle \hat{X} \rangle = \\ e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{n!} \cos^2(\sqrt{n}\eta \Omega t/2) \end{aligned} \quad (8.20)$$

with α a function of the phase, $\alpha = \alpha_0 \sin \phi/2$. The variance can again be expressed in terms of this expectation value as in eq. 8.17 and the derivative of $\langle \hat{X} \rangle$ is given as

$$\frac{d\langle\hat{X}\rangle}{d\phi} = e^{|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{n!} \cos^2(\sqrt{n}\eta\Omega t/2)(-\alpha^{2n+1} + n\alpha^{2n-1}) \quad (8.21)$$

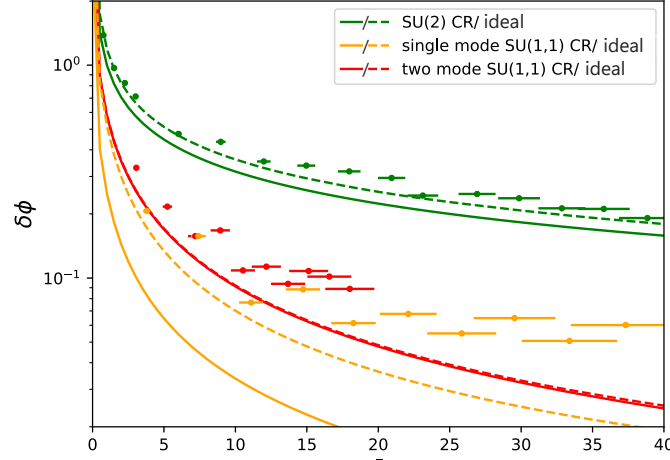


Figure 8.4. (figure from Metzner et al. (2023)) Ideal experimental phase sensitivities ($\delta\phi$) for the SU(2) (dashed green line), single mode SU(1,1) (dashed orange line) and two-mode SU(1,1) (dashed red line) are shown and can be compared to experimental results shown as dots with error bars and Cramér-Rao (CR) bounds in solid lines.

We have demonstrated a wide class of trapped-ion motional state interferometers using a ‘laser-free’ parametric drive achieving single and two-mode SU(1,1) interferometer sensitivities of 5.9(2) dB and 4.5(2) dB below the SQL, respectively and a SU(2) interferometer sensitivity within 0.67(5) dB of the SQL. These results illustrate the measurement of phase shifts near the Cramér-Rao bound for common-mode, relative, and single-mode phase fluctuations. Enhanced sensitivity for measuring these motional phase fluctuations could enable characterization and mitigation of motional frequency noise which can benefit experiments involving motional states. Beside providing a framework for implementing quantum-enhanced sensing protocols, our methods provide the

flexibility to concatenate multiple motional operations into circuits for potential applications in CVQC.

CHAPTER IX

CONCLUSION AND OUTLOOK

Chapter VIII is one example of a method for using two-mode squeezed states for quantum sensing. There are other uses for one and two-mode squeezed states in quantum sensing of displacements (Cardoso, Rossatto, Fernandes, Higgins, & Villas-Boas, 2021) and general cases where reducing quantum fluctuations is necessary to improve signal to noise (Lawrie, Lett, Marino, & Pooser, 2019). These methods, combined with time reversal protocols provide amplification of a signal for detection (Colombo et al., 2022). In addition to making use of two-mode squeezed states, it is important to be able to measure properties of these states, for example, a direct witness of the mode entanglement. This chapter explores some possible experiments that could be performed in our setup to further demonstrate the ability to characterize and make use two-mode squeezed states. It is also important to consider how this work may fit into the larger context of continuous variable quantum computing.

9.1 Phase insensitive displacement amplification with two-mode squeezed states

Previous theoretical and experimental work has shown single mode squeezed states can be used to amplify displacements (Burd et al., 2019; Yurke et al., 1986). By first squeezing the motional ground state, which reduces quantum fluctuations along one quadrature, displacements along this quadrature can be amplified by performing time reversal of the squeezing operation. The state is returned to a minimum uncertainty coherent state with a larger amplitude, $\alpha_f = G\alpha_i$ with gain G . This amplification is given by

$$\hat{D}(\alpha_f) = \hat{S}^\dagger(\xi)\hat{D}(\alpha_i)\hat{S}(\xi), \quad (9.1)$$

where $\hat{D}(\alpha)$ is the displacement operator with amplitude $|\alpha|$ and $\hat{S}(\xi)$ is the single mode squeeze operator with complex squeeze parameter $\xi = re^{i\theta}$. The issues with this scheme is that the final displacement depends on the phase relationship between the squeeze and the displacement, $\alpha_f = \alpha_i \cosh(r) + \alpha_i^* e^{i\theta} \sinh(r)$, which can be seen in 9.1. Only when the displacement is along the squeezed axis do you get maximum amplification, $G = e^r$. If the phase of the displacement is not known this leads to the maximal gain being washed out over the many experimental iterations. Two-mode squeezed states, in theory, can out perform single mode squeezed states when the displacement phase is unknown. This scheme is written as

$$\hat{D}_a(\alpha_f^{(a)})\hat{D}_b(\alpha_f^{(b)}) = \hat{T}^\dagger(\xi)\hat{D}_a(\alpha_i^{(a)})\hat{T}(\xi), \quad (9.2)$$

where now we consider two modes with annihilation (creation) operators $\hat{a}, \hat{b}(\hat{a}^\dagger, \hat{b}^\dagger)$ of the two modes. The displacement operators $\hat{D}_{a,b}(\alpha)$ act independently on either mode and the two-mode squeezing operator is given by $\hat{T}(\xi) = \exp(\xi^* \hat{a} \hat{b} - \xi \hat{a}^\dagger \hat{b}^\dagger)$. Independent of the squeeze and displacement phase relationship the final displacements are $\alpha_f^{(a)} = \cosh(r)\alpha_i^{(a)}$ and $\alpha_f^{(b)} = \sinh(r)\alpha_i^{(a)}$. Given that the displacement and squeezing phase are not a part of this expression, the gain becomes independent of these phases. This means that the displacement phase does not need to be known and a gain of $G = \cosh(r)$ is fixed. This $\cosh(r)$ gain matches what has been shown in Burd et al. (2024) using single mode squeezing, however their protocol requires equal amplitude displacements in each pulse period and would be unsuitable for transient displacements.

Chapter II showed how the beam splitter can be used to effectively rotate the basis and work in the EPR variable basis. This result means that the beam splitter can be used at the end of the above amplification protocol to retrieve full gain. $G = e^r$. The effect of the beam splitter is to transfer the small displacement

in the second mode back to the originally displaced mode. In order to properly make this transformation, another phase is introduced into the protocol. This means that given an unknown displacement phase, the addition of the beam splitter makes this a phase sensitive amplification and suffers the same loss in gain.

The addition of a second mode in this scheme does provide access to a second bit of information. This second bit can be conditioned to potentially increase the gain in the event that the displacement phase is not known. This increase in gain is achieved by post selecting on the secondary mode being in the ground state. The secondary mode is only in the ground state if the action of the beam splitter gives the ideal EPR transformation.

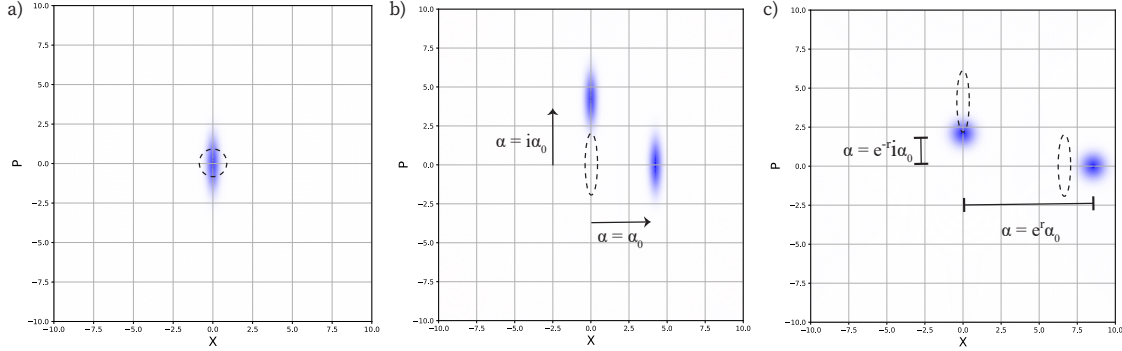


Figure 9.1. a) single mode quadrature squeezing which shows variance reduced below the SQL (black dashed lines) for one quadrature, with variance increased in the other. b) displacement of squeezed state along two different trajectories with a $\pi/2$ phase difference. c) reversal of the squeezing showing amplification of displacement when displaced along the squeezed axis and loss when displaced along anti-squeezed axis.

9.1.1 Potential experimental implementation. Most of the elements for a demonstration of this technique have been performed and shown in Chapter VI and VIII. The last remaining question is how to measure the gain. In Burd et al. (2019), small displacements were used which meant that the final state overlapped significantly with the ground state. This makes a single RSB pulse

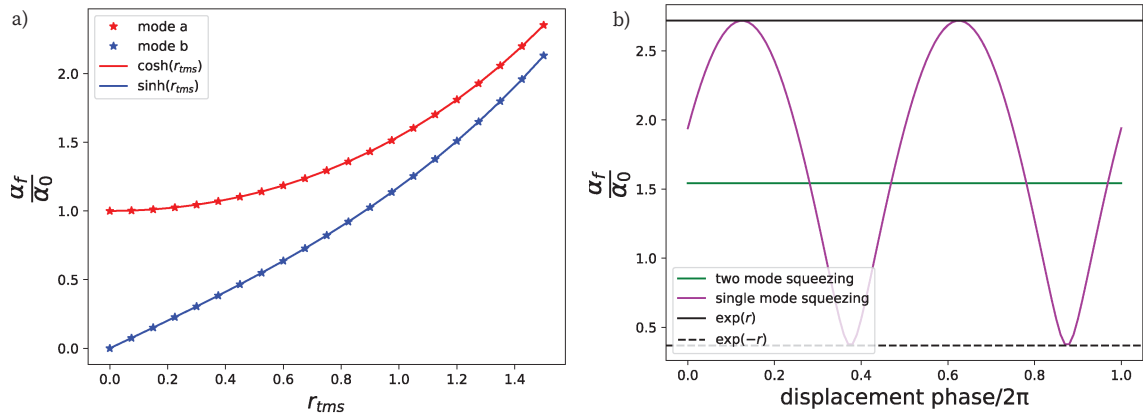


Figure 9.2. a) output amplification of a displacement on mode a, when using two mode squeezing and measuring either mode *a* or mode *b*. b) phase sensitivity comparison of the two amplifications schemes, with a fixed $r_{sms} = r_{tms} = 1$, with varied gain dependent on the displacement phase for single mode squeezing and no sensitivity to the phase when using two mode squeezing, but with gain below the maximum of the single mode squeezing scheme.

ineffective for measuring the gain, which is why a more complicated readout scheme was used in that work. For this demonstration, if larger displacements are used, a single RSB pulse is sufficient to measure the gain. With a fixed RSB pulse duration is used, the qubit probability can be measured as a function of the size of the initial displacement. The slope of this signal can be used to retrieve the gain.

When adding in the beam splitter, there are more optimal beam splitter fractions that can be used. Post-selection is done by performing a RSB pulse on the secondary mode and rejecting shots where the qubit is flipped.

9.2 Phase distributions

Measuring correlations in two-mode squeezed states is not straight forward when it is difficult to directly measure the motional states. Mapping the motion to a qubit also does not provide an obvious solution to how to measure some joint property of two modes. One potential solution has been laid out theoretically in Salvadoray and Sanjay Kumar (1997). Given the types of phase profiles that

are shown in section chapter VIII, there may be an avenue for generating full two-dimensional phase profiles as a method for measuring mode correlations. The phase distribution of a two-mode state $|\psi\rangle$ is

$$P(\theta_1, \theta_2) = \frac{1}{(2\pi)^2} |\langle \theta_1, \theta_2 | \psi \rangle|^2, \quad (9.3)$$

where θ_1 and θ_2 are the phases of the individual modes and $-\pi \leq \theta_1, \theta_2 \leq \pi$.

The two-mode phase state $|\theta_1, \theta_2\rangle$ is defined to be the eigenstate of the two-mode Susskind-Glogower (Susskind & Glogower, 1964) phase operator $\hat{e} = (1 + \hat{a}^\dagger \hat{a})^{-1/2} \hat{a} (1 + \hat{b}^\dagger \hat{b})^{-1/2} \hat{b}$, giving

$$|\theta_1, \theta_2\rangle = \sum_{n_1, n_2=0}^{\infty} \exp[i(n_1 \theta_1 + n_2 \theta_2)] |n_1, n_2\rangle. \quad (9.4)$$

This phase state is non-normalizable and non-orthogonal, but does form a complete set and the phase distribution is normalized:

$$\int_{-\pi}^{\pi} d\theta_1 \int_{-\pi}^{\pi} d\theta_2 P(\theta_1, \theta_2) = 1. \quad (9.5)$$

Measuring the two-mode squeezed state overlap with this phase state is not exactly realizable, however the phase one-dimensional phase distributions that are shown in chapter VIII may be a close enough approximation to be able to measure correlations. The correlations between the phases of the two modes is defined by

$$C_{12} = \langle \theta_1 \theta_2 \rangle - \langle \theta_1 \rangle \langle \theta_2 \rangle. \quad (9.6)$$

We have already seen that the marginal Wigner distribution corresponding to one mode obtained by tracing over the other mode is a thermal state. The phase distribution of this single mode state has the property that $P(\theta_1) = P(-\theta_1)$, meaning that $\langle \theta_1 \rangle = 0$ and by the same argument, $\langle \theta_2 \rangle = 0$. Hence the correlation C_{12} is given entirely by

$$C_{12} = \int_{-\pi}^{\pi} d\theta_1 \int_{-\pi}^{\pi} d\theta_2 \theta_1 \theta_2 P(\theta_1, \theta_2). \quad (9.7)$$

This means that if this phase distribution can be generated, the mode correlations can be measured by integrating over the distribution. Work still needs to be done to determine how well the procedure in chapter VIII is actually approximating a phase distribution. It will also be necessary to individually vary the phase of the individual modes instead of just the sum-mode phase.

9.3 Correlation measurements below the $\mathcal{CH}$ bound

Phase distributions is only one possible avenue to measuring two-mode squeezed state correlations. Another method based on work from Banaszek and Wódkiewicz (1999), is to test nonlocality in phase space. This type of experiment can be performed with a phonon counting type experiment which leads directly to a measurement that is described by the Wigner function. This function is given by joint phonon count correlations. The setup for this experiment will be to take a correlated source of the ion, like the two-mode squeezed state, make conditional displacements on either, or both modes, and then be able to make joint measurements of the modes. The measurement that can be used to do this, which is very adaptable to trapped ions, is to use events when non phonons are registered. This is very similar to a Bell type test with a strict analogy to two-particle coincidence experiments. The adjustable polarizers used in those photon experiments are replaced with coherent displacements of α and β .

The joint probability of no-count events happening simultaneously with both detections is

$$Q_{ab}(\alpha, \beta) = \langle \Psi | \hat{Q}_a(\alpha) \otimes \hat{Q}_b(\beta) | \Psi \rangle, \quad (9.8)$$

where $\hat{Q}_{(ab)}$ are projection operators of a single mode,

$$\hat{Q}(\alpha) = \hat{D}(\alpha) |0\rangle\langle 0| \hat{D}^\dagger(\alpha), \quad (9.9)$$

with displacement operator $\hat{D}(\alpha)$. The full measurement is performed for two setting of the coherent displacement for each mode: zero or α for mode a and zero or β for mode b . This full set of measurements enables calculating the Clauser-Horne combination (Clauser & Horne, 1974):

$$\mathcal{CH} = Q_{ab}(0, 0) + Q_{ab}(\alpha, 0) + Q_{ab}(0, \beta) - Q_{ab}(\alpha, \beta) - Q_a(0) - Q_b(0), \quad (9.10)$$

which for local theories is bounded by the inequality $-1 \leq \mathcal{CH} \leq 0$. With arbitrary control of the size of the displacements and relative phase of the displacements a range of values can be achieved. For equal amplitude displacements $|\alpha| = |\beta| = \mathcal{A}$ and a phase difference $\beta = e^{2i\phi}\alpha$ the values are

$$\mathcal{CH} = -1 + \mathcal{A}e^{-\mathcal{A}} - 2\mathcal{A}e^{-2\mathcal{A}} \sin^2 \phi, \quad (9.11)$$

which violates the inequality when the displacements have opposite phases $\beta = -\alpha$.

Testing this theory with trapped ions would not pose any significant difficulties. Every piece of this scheme has been demonstrated besides the detection scheme. There are several avenues to explore in terms of addressing this detection scheme. One possible avenue is to use two ions and implement different qubit encodings on each ion with the OMG architecture.

9.4 Continuous variable quantum information processing

The ability to process information using quantum system is not constrained to only using two discrete energy states (qubits). This paradigm can be scaled to include many discrete states (qudits) and can even be scaled to an infinite number of equally spaced states, or continuous energy spectra (continuous variable). While schemes employing qubits have been widely developed, significantly less effort has been given to realizing other methods of quantum information processing. Many of the elements necessary of a universal set of CV operations have been demonstrated here. A source of non-Gaussian states is necessary for universal computation. This

is already possible with motional sidebands for generating Fock states. Sidebands can also be used to engineer higher order Hamiltonians (Sutherland & Srinivas, 2021). A non-Gaussian operation could be provided by the native Coulomb interaction, but tends to be very slow. Higher order parametric modulation is another avenue for providing this non-linear interaction, but would require at least cubic terms in the trapping potential.

Harmonic oscillators provide the possibility of a CV quantum information processor, but it is also possible to encode qubits into these systems. Bosonic codes exist which can be implemented in trapped ions and provide advantages for error correction over existing schemes. One of these bosonic codes is the Gottesman, Kitaev and Preskill (GKP) code which is a grid state making use of a superposition of squeezed and displaced states. A truncated approximation to this infinite energy state has been generated in the trapped ion platform at ETHZ (Flühmann et al., 2019). The state engineering has been performed using laser based methods, but performance could theoretically be improved with parametric modulation. Current work is focused on generating two GKP state gates. This is especially relevant for the techniques demonstrated here. With the capabilities of current ion traps, CVQC is a paradigm within reach and effort needs to start shifting towards demonstrations of quantum gates for use in quantum algorithms.

APPENDIX

SQUAREATRON 5000 AND TUNING BOARD

This appendix includes the design schematics and PCB layout of the Squareatron 500 and the additional Tuning board. The Squareatron design is modified from the original Oxford design (Harty, 2019) to include pads for the tuning board to connect to; the update design is shown in Metzner (2020). The tuning board is powered by the Squareatron and uses the 3.3 V reference for the DAC. The tuning board has a RJ45 connector mounted to allow for programming from a field-programmable gate array (FPGA) used in the lab.



159

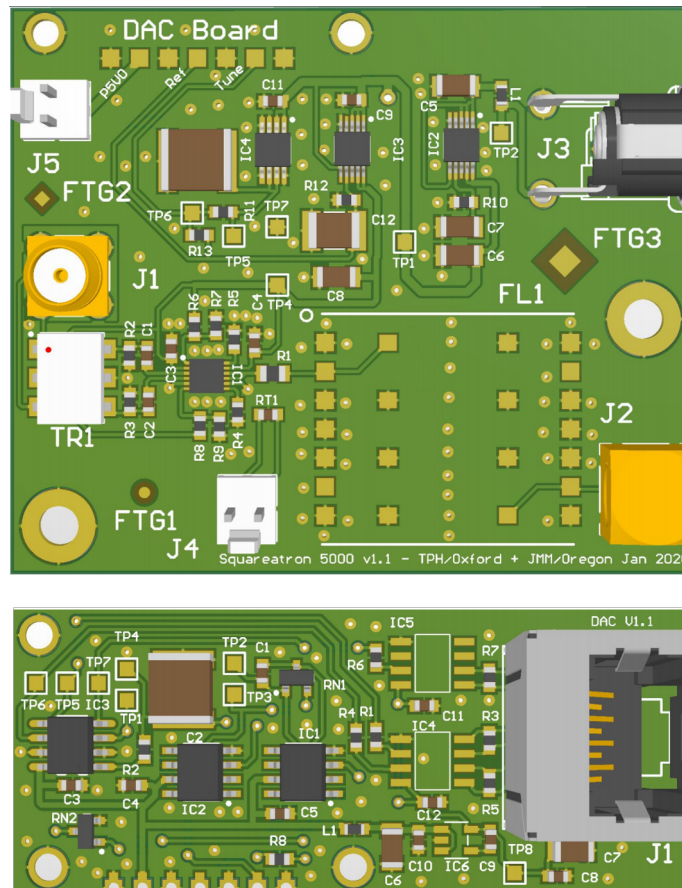


Figure A.3. A rendering the the Squareatron and tuning board PCBs

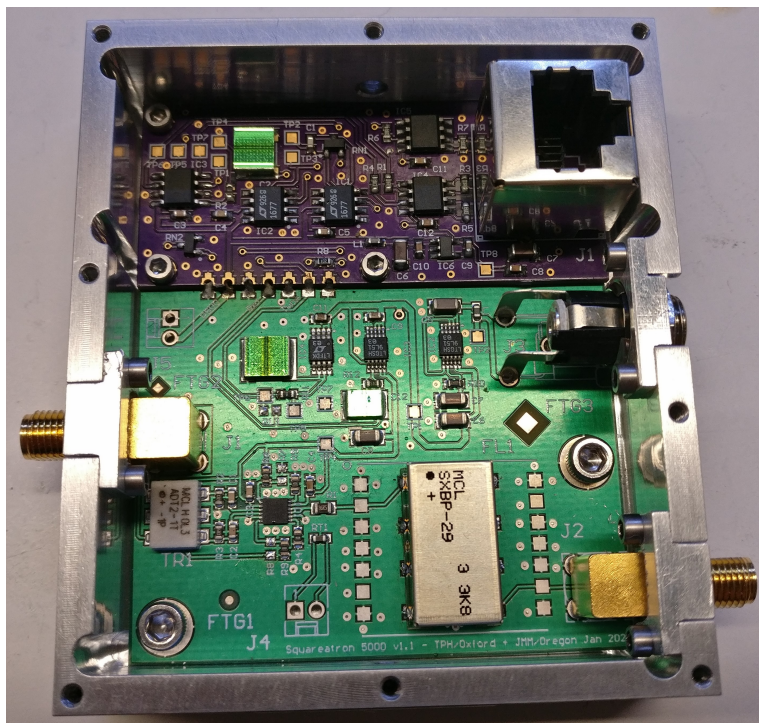


Figure A.4. Image of the Squareatron and tuning board connected and mounted in a box

REFERENCES CITED

- Agarwal, G. S., & Davidovich, L. (2022). Quantifying quantum-amplified metrology via Fisher information. *Phys. Rev. Res.*, *4*(1), L012014. Retrieved from <https://doi.org/10.1103/PhysRevResearch.4.L012014> doi: 10.1103/PhysRevResearch.4.L012014
- Ahn, C., Wiseman, H. M., & Milburn, G. J. (2003). Quantum error correction for continuously detected errors. *Physical Review A*, *67*(5). Retrieved from <https://doi.org/10.1103/PhysRevA.67.052310> doi: 10.1103/physreva.67.052310
- Allcock, D. T., Campbell, W. C., Chiaverini, J., Chuang, I. L., Hudson, E. R., Moore, I. D., ... Wineland, D. J. (2021). omg blueprint for trapped ion quantum computing with metastable states. *Applied Physics Letters*, *119*(21). Retrieved from <https://doi.org/10.1063/5.0069544> doi: 10.1063/5.0069544
- Alonso, J., Leupold, F. M., Solèr, Z. U., Fadel, M., M. Marinelli, Keitch, B. C., ... P., J. H. (2016). Generation of large coherent states by bang-bang control of a trapped-ion oscillator. *Nature Communications*, *7*, 1–8. Retrieved from <https://doi.org/10.1038/ncomms11243> doi: 10.1038/ncomms11243
- Ballance, C., Harty, T., Linke, N., Sepiol, M., & Lucas, D. (2016). High-fidelity quantum logic gates using trapped-ion hyperfine qubits. *Physical Review Letters*, *117*(6). Retrieved from <https://doi.org/10.1103/PhysRevLett.117.060504> doi: 10.1103/physrevlett.117.060504
- Banaszek, K., & Wódkiewicz, K. (1999). Testing quantum nonlocality in phase space. *Physical Review Letters*, *82*(10), 2009–2013. Retrieved from <https://doi.org/10.1103/PhysRevLett.82.2009> doi: 10.1103/physrevlett.82.2009
- Bassi, A., Lochan, K., Satin, S., Singh, T. P., & Ulbricht, H. (2013). Models of wave-function collapse, underlying theories, and experimental tests. *Reviews of Modern Physics*, *85*(2), 471–527. Retrieved from <https://doi.org/10.1103/RevModPhys.85.471> doi: 10.1103/revmodphys.85.471

- Bennett, R., Barlow, T. M., & Beige, A. (2015). A physically motivated quantization of the electromagnetic field. *European Journal of Physics*, *37*(1), 0–11. Retrieved from <https://iopscience.iop.org/article/10.1088/0143-0807/37/1/014001> doi: 10.1088/0143-0807/37/1/014001
- Bergquist, J. C., Itano, W. M., & Wineland, D. J. (1987). Recoilless optical absorption and doppler sidebands of a single trapped ion. *Physical Review A*, *36*(1), 428–430. Retrieved from <https://doi.org/10.1103/PhysRevA.36.428> doi: 10.1103/physreva.36.428
- Berkeland, D. J., Miller, J. D., Bergquist, J. C., Itano, W. M., & Wineland, D. J. (1998). Minimization of ion micromotion in a paul trap. *Journal of Applied Physics*, *83*(10), 5025–5033. Retrieved from <https://doi.org/10.1063/1.367318> doi: 10.1063/1.367318
- Berman, P. R., & Ooi, C. H. (2014). Collapse and revivals in the jaynes-cummings model: An analysis based on the mollow transformation. *Physical Review A*, *89*(3), 033845. Retrieved from <https://doi.org/10.1103/PhysRevA.89.033845> doi: 10.1103/physreva.89.033845
- Bogoljubov, N. N. (1958). On a new method in the theory of superconductivity. *Il Nuovo Cimento*, *7*(6), 794–805. Retrieved from <https://doi.org/10.1007/BF02745585> doi: 10.1007/bf02745585
- Brody, D. C., & Graefe, E.-M. (2012). Mixed-state evolution in the presence of gain and loss. *Phys. Rev. Lett.*, *109*, 230405. Retrieved from <https://link.aps.org/doi/10.1103/PhysRevLett.109.230405> doi: 10.1103/PhysRevLett.109.230405
- Bruzewicz, C. D., McConnell, R., Stuart, J., Sage, J. M., & Chiaverini, J. (2019). Dual-species, multi-qubit logic primitives for Ca⁺/Sr⁺ trapped-ion crystals. *npj Quantum Information*, *5*(1), 1–10. Retrieved from <http://dx.doi.org/10.1038/s41534-019-0218-z> doi: 10.1038/s41534-019-0218-z
- Budroni, C., & Emary, C. (2014). Temporal quantum correlations and leggett-garg inequalities in multilevel systems. *Phys. Rev. Lett.*, *113*, 050401. Retrieved from <https://link.aps.org/doi/10.1103/PhysRevLett.113.050401> doi: 10.1103/PhysRevLett.113.050401

- Budroni, C., Moroder, T., Kleinmann, M., & Gühne, O. (2013). Bounding temporal quantum correlations. *Phys. Rev. Lett.*, *111*, 020403. Retrieved from <https://link.aps.org/doi/10.1103/PhysRevLett.111.020403> doi: 10.1103/PhysRevLett.111.020403
- Burd, S. C., Knaack, H. M., Srinivas, R., Arenz, C., Collopy, A. L., Stephenson, L. J., ... et al. (2024). Experimental speedup of quantum dynamics through squeezing. *PRX Quantum*, *5*(2), 020314. Retrieved from <https://doi.org/10.1103/PRXQuantum.5.020314> doi: 10.1103/prxquantum.5.020314
- Burd, S. C., Srinivas, R., Bollinger, J. J., Wilson, A. C., Wineland, D. J., Leibfried, D., ... Allcock, D. T. (2019). Quantum amplification of mechanical oscillator motion. *Science*, *364*(6446), 1163–1165. Retrieved from <https://www.science.org/doi/10.1126/science.aaw2884> doi: 10.1126/science.aaw2884
- Băzăvan, O., Saner, S., Webb, D., Ainley, E., Drmota, P., Nadlinger, D., ... R., S. (2024). *Squeezing, trisqueezing, and quadsqueezing in a spin-oscillator system*. arXiv: 2403.05471. Retrieved from <https://doi.org/10.48550/arXiv.2403.05471> doi: 10.48550/arXiv.2403.05471
- Campaioli, F., Pollock, F. A., & Modi, K. (2019). Tight, robust, and feasible quantum speed limits for open dynamics. *Quantum*, *3*, 168. Retrieved from <https://doi.org/10.22331/q-2019-08-05-168> doi: 10.22331/q-2019-08-05-168
- Cao, J., & Voth, G. A. (1995). Modeling physical systems by effective harmonic oscillators: The optimized quadratic approximation. *The Journal of Chemical Physics*, *102*(8), 3337–3348. Retrieved from <https://doi.org/10.1063/1.469207> doi: 10.1063/1.469207
- Cardoso, F. R., Rossatto, D. Z., Fernandes, G. P., Higgins, G., & Villas-Boas, C. J. (2021). Superposition of two-mode squeezed states for quantum information processing and quantum sensing. *Physical Review A*, *103*(6), 062405. Retrieved from <https://doi.org/10.1103/PhysRevA.103.062405> doi: 10.1103/physreva.103.062405
- Carruthers, P., & Nieto, M. M. (1965). Coherent states and the forced quantum oscillator. *American Journal of Physics*, *33*(7), 537–544. Retrieved from <https://doi.org/10.1119/1.1971895> doi: 10.1119/1.1971895
- Caves, C. (1981). Quantum-mechanical noise in an interferometer. *Physical Review D*, *23*(8), 1693–1708. doi: 10.1103/PhysRevD.23.1693

- Caves, C. M., Zhu, C., Milburn, G. J., & Schleich, W. (1991). Photon statistics of two-mode squeezed states and interference in four-dimensional phase space. *Phys. Rev. A*, *43*(7), 3854. Retrieved from <https://doi.org/10.1103/PhysRevA.43.3854> doi: 10.1103/PhysRevA.43.3854
- Cirac, J., Parkins, A., Blatt, R., & Zoller, P. (1996). Nonclassical states of motion in ion traps. *Advances In Atomic, Molecular, and Optical Physics*, *37*, 237–296. Retrieved from [https://doi.org/10.1016/S1049-250X\(08\)60102-0](https://doi.org/10.1016/S1049-250X(08)60102-0) doi: 10.1016/s1049-250x(08)60102-0
- Cirac, J. I., & Zoller, P. (1995). Quantum computations with cold trapped ions. *Physical Review Letters*, *74*(20), 4091–4094. Retrieved from <https://doi.org/10.1103/PhysRevLett.74.4091> doi: 10.1103/physrevlett.74.4091
- Clauser, J. F., & Horne, M. A. (1974). Experimental consequences of objective local theories. *Physical Review D*, *10*(2), 526–535. Retrieved from <https://doi.org/10.1103/PhysRevD.10.526> doi: 10.1103/physrevd.10.526
- Colombo, S., Pedrozo-Peñafiel, E., Adiyatullin, A. F., Li, Z., Mendez, E., Shu, C., & Vuletić, V. (2022). Time-reversal-based quantum metrology with many-body entangled states. *Nat. Phys.*, *18*, 925. Retrieved from <https://doi.org/10.1038/s41567-022-01653-5> doi: 10.1038/s41567-022-01653-5
- Demkowicz-Dobrzański, R., Jarzyna, M., & Kołodyński, J. (2015). Quantum limits in optical interferometry. *Progress in Optics*, 345–435. Retrieved from <https://doi.org/10.1016/bs.po.2015.02.003> doi: 10.1016/bs.po.2015.02.003
- Deslauriers, L., Olmschenk, S., Stick, D., Hensinger, W. K., Sterk, J., & Monroe, C. (2006). Scaling and suppression of anomalous heating in ion traps. *Phys. Rev. Lett.*, *97*(10), 103007. Retrieved from <https://doi.org/10.1103/PhysRevLett.97.103007> doi: 10.1103/PhysRevLett.97.103007
- Ding, L., Shi, K., Zhang, Q., Shen, D., Zhang, X., & Zhang, W. (2021, Feb). Experimental determination of $\mathcal{PT}$ -symmetric exceptional points in a single trapped ion. *Phys. Rev. Lett.*, *126*, 083604. Retrieved from <https://link.aps.org/doi/10.1103/PhysRevLett.126.083604> doi: 10.1103/PhysRevLett.126.083604

- Ding, S., Maslennikov, G., Hablützel, R., Loh, H., & Matsukevich, D. (2017a). Cross-Kerr Nonlinearity for Phonon Counting. *Physical Review Letters*, 119(19), 193602. Retrieved from <https://doi.org/10.1103/PhysRevLett.119.193602> doi: 10.1103/PhysRevLett.119.193602
- Ding, S., Maslennikov, G., Hablützel, R., Loh, H., & Matsukevich, D. (2017b). Quantum Parametric Oscillator with Trapped Ions. *Physical Review Letters*, 119(15), 150404. Retrieved from <https://doi.org/10.1103/PhysRevLett.119.150404> doi: 10.1103/PhysRevLett.119.150404
- Emary, C., Lambert, N., & Nori, F. (2013). Leggett–garg inequalities. *Reports on Progress in Physics*, 77(1), 016001. Retrieved from <https://doi.org/10.1088/0034-4885/77/1/016001> doi: 10.1088/0034-4885/77/1/016001
- Enrico, R. (2006). The measurement of AM noise of oscillators. *Proceedings of the IEEE International Frequency Control Symposium and Exposition*, 750–758. Retrieved from <https://doi.org/10.1109/FREQ.2006.275483> doi: 10.1109/FREQ.2006.275483
- Eschner, J., Morigi, G., Keitel, C., Roos, C., Leibfried, D., Mundt, A., ... Blatt, R. (2002). Ground state laser cooling of trapped atoms using electromagnetically induced transparency. *Phys. Rev. Lett.*, 85(21), 4458. Retrieved from <https://doi.org/10.1103/PhysRevLett.85.4458> doi: 10.1142/9789812778307_0041
- Eschner, J., Morigi, G., Schmidt-Kaler, F., & Blatt, R. (2003). Laser cooling of trapped ions. *Journal of the Optical Society of America B*, 20(5), 1003. Retrieved from <https://doi.org/10.1364/JOSAB.20.001003> doi: 10.1364/josab.20.001003
- Flühmann, C., Nguyen, T. L., Marinelli, M., Negnevitsky, V., Mehta, K., & Home, J. P. (2019). Encoding a qubit in a trapped-ion mechanical oscillator. *Nature*, 566, 513–517. Retrieved from <http://dx.doi.org/10.1038/s41586-019-0960-6> doi: 10.1038/s41586-019-0960-6
- Foot, C. (2005). *Atomic Physics*. Oxford University Press.
- Gerry, C. C., & Knight, P. L. (2005). *Introductory quantum optics*. Cambridge University Press.

- Gorman, D. J., Schindler, P., Selvarajan, S., Daniilidis, N., & Häffner, H. (2014). Two-mode coupling in a single-ion oscillator via parametric resonance. *Physical Review A - Atomic, Molecular, and Optical Physics*, 89(6), 062332. Retrieved from <https://doi.org/10.1103/PhysRevA.89.062332> doi: 10.1103/PhysRevA.89.062332
- Griffiths, D. J. (1981). *Introduction to electrodynamics*. Cambridge University Press.
- Harty, T. (2013). *High-fidelity microwave-driven quantum logic in intermediate-field $^{43}\text{Ca}^+$* (Unpublished doctoral dissertation). University of Oxford.
- Harty, T. (2019). *Squareatron 5000*. <https://github.com/OxfordIonTrapGroup/Squareatron-5000>. GitHub.
- Heinzen, D. J., & Wineland, D. J. (1990). Quantum-limited cooling and detection of radio-frequency oscillations by laser-cooled ions. *Phys. Rev. A*, 42(5), 2977–2994. Retrieved from <https://doi.org/10.1103/PhysRevA.42.2977> doi: 10.1103/PhysRevA.42.2977
- Hite, D. A., Colombe, Y., Wilson, A. C., Allcock, D. T., Leibfried, D., Wineland, D. J., & Pappas, D. P. (2013). Surface science for improved ion traps. *MRS Bulletin*, 38(10), 826–833. Retrieved from <https://doi.org/10.1557/mrs.2013.207> doi: 10.1557/mrs.2013.207
- Home, J. P. (2013). Quantum Science and Metrology with Mixed-Species Ion Chains. *Advances in Atomic, Molecular and Optical Physics*, 62, 231–277. Retrieved from <https://doi.org/10.1016/B978-0-12-408090-4.00004-9> doi: 10.1016/B978-0-12-408090-4.00004-9
- Home, J. P., Hanneke, D., Jost, J. D., Leibfried, D., & Wineland, D. J. (2011). Normal modes of trapped ions in the presence of anharmonic trap potentials. *New Journal of Physics*, 13, 073026. Retrieved from <https://doi.org/10.1088/1367-2630/13/7/073026> doi: 10.1088/1367-2630/13/7/073026
- Hou, P., Wu, J., Erickson, S., Cole, D., Zarantonello, G., Brandt, A., ... Leibfried, D. (2022). Coherently coupled mechanical oscillators in the quantum regime. *arXiv: 2205.14841*. Retrieved from <https://doi.org/10.48550/arXiv.2205.14841> doi: 10.48550/arXiv.2205.14841
- J. Aasi et al. (Ligo Scientific Collaboration). (2013). Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light. *Nature Photonics*, 7(8), 613–619. doi: 10.1038/nphoton.2013.177

- Jakeman, E., Oliver, C., & Pike, E. (1975). Optical homodyne detection. *Advances in Physics*, 24(3), 349–405. Retrieved from <https://doi.org/10.1080/00018737500101421> doi: 10.1080/00018737500101421
- James, D. F. (1998). Quantum dynamics of cold trapped ions with application to quantum computation. *Applied Physics B: Lasers and Optics*, 66(2), 181–190. Retrieved from <https://doi.org/10.1007/s003400050373> doi: 10.1007/s003400050373
- Johansson, J., Nation, P., & Nori, F. (2013). QuTiP 2: A Python framework for the dynamics of open quantum systems. *Comp. Phys Comm.*, 184(1234). Retrieved from <http://dx.doi.org/10.1038/s41586-019-0960-6> doi: 10.1016/j.cpc.2012.02.021
- Karthik, H. S., Akshata Shenoy, H., & Devi, A. R. U. (2021, Mar). Leggett-garg inequalities and temporal correlations for a qubit under $\mathcal{PT}$ -symmetric dynamics. *Phys. Rev. A*, 103, 032420. Retrieved from <https://link.aps.org/doi/10.1103/PhysRevA.103.032420> doi: 10.1103/PhysRevA.103.032420
- Kasprowicz, G., Harty, T., Bourdeauducq, S., Jördens, R., Allcock, D., Nadlinger, D., ... Nowicka, D. (2022). Urukul – Open-source Frequency Synthesizer Module for Quantum Physics. *International Journal of Electronics and Telecommunications*, 68(1), 123–128. Retrieved from <https://doi.org/10.24425/ijet.2022.139859> doi: 10.24425/ijet.2022.139859
- Kasprowicz, G., Kulik, P., Gaska, M., Przywozki, T., Pozniak, K., Jarosinski, J., ... Pozniak, K. (2020). Artiq and sinara: Open software and hardware stacks for quantum physics. In *Osa quantum 2.0 conference* (p. QTu8B.14). Optica Publishing Group. Retrieved from <https://opg.optica.org/abstract.cfm?URI=QUANTUM-2020-QTu8B.14> doi: 10.1364/QUANTUM.2020.QTu8B.14
- Kennard, E. H. (1922). The speed of light. *Nature*, 109(2740), 581–582. Retrieved from <https://doi.org/10.1038/109581b0> doi: 10.1038/109581b0
- Kielpinski, D., King, B. E., Myatt, C. J., Sackett, C. A., Turchette, Q. A., Itano, W. M., ... Zurek, W. H. (2000). Sympathetic cooling of trapped ions for quantum logic. *Physical Review A*, 61(3), 8. Retrieved from <https://doi.org/10.1103/PhysRevA.61.032310> doi: 10.1103/PhysRevA.61.032310

- Kreuter, A., Becher, C., Lancaster, G. P., Mundt, A. B., Russo, C., Häffner, H., ... Safronova, M. S. (2005). Experimental and theoretical study of the $3d$ 2D -level lifetimes of $^{40}\text{Ca}^+$. *Phys. Rev. A*, *71*(3), 032504. Retrieved from <https://doi.org/10.1103/PhysRevA.71.032504> doi: 10.1103/PhysRevA.71.032504
- Kurochkin, Y., Prasad, A. S., & Lvovsky, A. I. (2014). Distillation of the two-mode squeezed state. *Phys. Rev. Lett.*, *112*(7), 070402. Retrieved from <https://doi.org/10.1103/PhysRevLett.112.070402> doi: 10.1103/PhysRevLett.112.070402
- Lawrie, B. J., Lett, P. D., Marino, A. M., & Pooser, R. C. (2019). Quantum sensing with squeezed light. *ACS Photonics*, *6*(6), 1307–1318. Retrieved from <https://doi.org/10.1021/acsp Photonics.9b00250> doi: 10.1021/acsp Photonics.9b00250
- Lechner, R., Maier, C., Hempel, C., Jurcevic, P., Lanyon, B. P., Monz, T., ... Roos, C. F. (2016). Electromagnetically-induced-transparency ground-state cooling of long ion strings. *Physical Review A*, *93*(5), 053401. Retrieved from <https://doi.org/10.1103/PhysRevA.93.053401> doi: 10.1103/physreva.93.053401
- Leefer, N., Krimmel, K., Bertsche, W., Budker, D., Schmidt-kaler, F., Fajans, J., & Folman, R. (2017). Investigation of two-frequency Paul traps for antihydrogen production. *Hyperfine Interact.*, *238*, 12. Retrieved from <https://doi.org/10.1007/s10751-016-1388-0> doi: 10.1007/s10751-016-1388-0
- Leibfried, D., DeMarco, B., Meyer, V., Rowe, M., Ben-Kish, A., Britton, J., ... Wineland, D. J. (2002). Trapped-Ion Quantum Simulator: Experimental Application to Nonlinear Interferometers. *Phys. Rev. Lett.*, *89*(24), 247901. Retrieved from <https://doi.org/10.1103/PhysRevLett.89.247901> doi: 10.1103/PhysRevLett.89.247901
- Levitin, L. B., & Toffoli, T. (2009). Fundamental limit on the rate of quantum dynamics: The unified bound is tight. *Phys. Rev. Lett.*, *103*, 160502. Retrieved from <https://link.aps.org/doi/10.1103/PhysRevLett.103.160502> doi: 10.1103/PhysRevLett.103.160502
- Lloyd, S., & Braunstein, S. L. (1999). Quantum Computation over Continuous Variables. *Phys. Rev. Lett.*, *82*, 1784. Retrieved from <https://doi.org/10.1103/PhysRevLett.82.1784> doi: 10.1103/PhysRevLett.82.1784

- Ltc6957 low phase noise, dual output buffer/driver/logic converter (Computer software manual No. TJA1043). (2017). Retrieved from <https://www.analog.com/media/en/technical-documentation/data-sheets/6957fb.pdf> (Rev. B)
- Lu, P., Rao, X., Liu, T., Liu, Y., Bian, J., Zhu, F., & Luo, L. (2024). Experimental demonstration of enhanced violations of leggett-garg inequalities in a $\mathcal{PT}$ -symmetric trapped-ion qubit. *Physical Review A*, *109*(4). Retrieved from <https://doi.org/10.1103/PhysRevA.109.042205> doi: 10.1103/physreva.109.042205
- Lucas, D. M., Ramos, A., Home, J. P., McDonnell, M. J., Nakayama, S., Stacey, J. P., ... Steane, A. M. (2004). Isotope-selective photoionization for calcium ion trapping. *Physical Review A - Atomic, Molecular, and Optical Physics*, *69*(1), 13. Retrieved from <https://doi.org/10.1103/PhysRevA.69.012711> doi: 10.1103/PhysRevA.69.012711
- Marangos, J. P. (1998). Electromagnetically induced transparency. *Journal of Modern Optics*, *45*(3), 471–503. Retrieved from <https://doi.org/10.1080/09500349808231909> doi: 10.1080/09500349808231909
- Margolus, N., & Levitin, L. B. (1998). The maximum speed of dynamical evolution. *Physica D: Nonlinear Phenomena*, *120*(1-2), 188–195. Retrieved from [https://doi.org/10.1016/s0167-2789\(98\)00054-2](https://doi.org/10.1016/s0167-2789(98)00054-2) doi: 10.1016/s0167-2789(98)00054-2
- Marquet, C., Schmidt-Kaler, F., & James, D. F. (2003). Phonon-phonon interactions due to non-linear effects in a linear ion trap. *Applied Physics B: Lasers and Optics*, *76*(3), 199–208. Retrieved from <https://doi.org/10.1007/s00340-003-1097-7> doi: 10.1007/s00340-003-1097-7
- McCormick, K. C., Keller, J., Burd, S. C., Wineland, D. J., Wilson, A. C., & Leibfried, D. (2019). Quantum-enhanced sensing of a single-ion mechanical oscillator. *Nature*, *572*, 86–90. Retrieved from <http://dx.doi.org/10.1038/s41586-019-1421-y> doi: 10.1038/s41586-019-1421-y
- Meekhof, D. M., Monroe, C., King, B. E., Itano, W. M., & Wineland, D. J. (1996). Generation of nonclassical motional states of a trapped atom. *Phys. Rev. Lett.*, *76*(11), 1796 (1996); erratum: *Phys. Rev. Lett.* **77**, 2346. Retrieved from <https://doi.org/10.1103/PhysRevLett.76.1796> doi: 10.1103/PhysRevLett.76.1796

- Metzner, J. (2020). *Squareatron dac board*.
https://github.com/OregonIons/Squareatron_DAC_Board. GitHub.
- Metzner, J., Quinn, A., Brudney, S., Moore, I., Burd, S., Wineland, D., & Allcock, D. (2023). *Two-mode squeezing and $su(1,1)$ interferometry with trapped ions*. Retrieved from <https://doi.org/10.48550/arXiv.2312.10847> doi: 10.48550/arXiv.2312.10847
- Michelson, A. A., & Morley, E. W. (1887). On the relative motion of the earth and the luminiferous æther. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 24(151), 449–463. Retrieved from <https://doi.org/10.1080/14786448708628130> doi: 10.1080/14786448708628130
- Mihalcea, B. (2023). Coherent states for trapped ions: Applications in quantum optics and precision measurements. *CPT and Lorentz Symmetry*. Retrieved from https://doi.org/10.1142/9789811275388_0043 doi: 10.1142/9789811275388_0043
- Mintert, F., & Wunderlich, C. (2001). Ion-trap quantum logic using long-wavelength radiation. *Physical Review Letters*, 87(25), 257904. Retrieved from <https://doi.org/10.1103/PhysRevLett.87.257904> doi: 10.1103/physrevlett.87.257904
- Monchalin, J.-P., Kelly, M. J., Thomas, J. E., Kurnit, N. A., Szöke, A., Zernike, F., ... Javan, A. (1981). Accurate laser wavelength measurement with a precision two-beam scanning michelson interferometer. *Applied Optics*, 20(5), 736. Retrieved from <https://doi.org/10.1364/AO.20.000736> doi: 10.1364/ao.20.000736
- Monras, A. (2006). Optimal phase measurements with pure Gaussian states. *Phys. Rev. A*, 73(3), 033821. Retrieved from <https://doi.org/10.1103/PhysRevA.73.033821> doi: 10.1103/PhysRevA.73.033821
- Moore, I. (2023). *Easy on the ions: Photon scattering errors from far-detuned raman beams in trapped-ion qubits* (Unpublished doctoral dissertation). University of Oregon.
- Myerson, A. H., Szwed, D. J., Webster, S. C., Allcock, D. T., Curtis, M. J., Imreh, G., ... Lucas, D. M. (2008). High-fidelity readout of trapped-ion qubits. *Physical Review Letters*, 100(20), 200502. Retrieved from <https://doi.org/10.1103/PhysRevLett.100.200502> doi: 10.1103/physrevlett.100.200502

- Mølmer, K., & Sørensen, A. (1999). Multiparticle entanglement of hot trapped ions. *Physical Review Letters*, 82(9), 1835–1838. Retrieved from <https://doi.org/10.1103/PhysRevLett.82.1835> doi: 10.1103/physrevlett.82.1835
- Naghiloo, M., Abbasi, M., Joglekar, Y. N., & Murch, K. W. (2019). Quantum state tomography across the exceptional point in a single dissipative qubit. *Nature Physics*, 15(12), 1232–1236. Retrieved from <https://doi.org/10.1038/s41567-019-0652-z> doi: 10.1038/s41567-019-0652-z
- Naikoo, J., Kumari, S., Banerjee, S., & Pan, A. K. (2021). Pt-symmetric evolution, coherence and violation of leggett–garg inequalities. *Journal of Physics A: Mathematical and Theoretical*, 54(27), 275303. Retrieved from <http://dx.doi.org/10.1088/1751-8121/ac0546> doi: 10.1088/1751-8121/ac0546
- Nelsen, M. A., & Chuang, I. L. (2001). *Quantum computation and quantum information*. Cambridge University Press.
- Nie, X. R., Roos, C., & James, D. (2009). Theory of cross phase modulation for the vibrational modes of trapped ions. *Physics Letters, Section A: General, Atomic and Solid State Physics*, 373(4), 422–425. Retrieved from <http://dx.doi.org/10.1016/j.physleta.2008.11.045> doi: 10.1016/j.physleta.2008.11.045
- Nägerl, H. C., Leibfried, D., Rohde, H., Thalhammer, G., Eschner, J., Schmidt-Kaler, F., & Blatt, R. (1999). Laser addressing of individual ions in a linear ion trap. *Physical Review A*, 60(1), 145–148. Retrieved from <https://doi.org/10.1103/PhysRevA.60.145> doi: 10.1103/physreva.60.145
- Oliver, W. D., Yu, Y., Lee, J. C., Berggren, K. K., Lvitov, L. S., & Orlando, T. P. (2005). Mach-Zehnder Interferometry in a Strongly Driven Superconducting Qubit. *Science*, 310, 1653–1658. Retrieved from <https://doi.org/10.1126/science.1119678> doi: 10.1126/science.1119678
- Ospelkaus, C., Warring, U., Colombe, Y., Brown, K. R., Amini, J. M., Leibfried, D., & Wineland, D. J. (2011). Microwave quantum logic gates for trapped ions. *Nature*, 476(7359), 181–184. Retrieved from <https://doi.org/10.1038/nature10290> doi: 10.1038/nature10290
- Oxford. (2019). *Wavemeter analysis and display*. <https://github.com/OxfordIonTrapGroup/wand>. GitHub.

- Ozeri, R., Itano, W., Blakestad, B., Chiaverini, J., Jost, J., Langer, C., ... Wineland, D. (2007). Errors in trapped-ion quantum gates due to spontaneous photon scattering. *Phys. Rev. A*, *75*, 042329. Retrieved from <https://doi.org/10.1103/PhysRevA.75.042329> doi: 10.1103/PhysRevA.75.042329
- Pezzé, L., & Smerzi, A. (2009). Entanglement, nonlinear dynamics, and the Heisenberg limit. *Phys. Rev. Lett.*, *102*(100401), 13–16. Retrieved from <https://doi.org/10.1103/PhysRevLett.102.100401> doi: 10.1103/PhysRevLett.102.100401
- Quinn, A., Metzner, J., Muldoon, J., Moore, I., Brudney, S., Das, S., ... Joglekar, Y. (2023). *Observing super-quantum correlations across the exceptional point in a single, two-level trapped ion*. Retrieved from <https://doi.org/10.48550/arXiv.2304.12413> doi: 10.48550/arXiv.2304.12413
- Regal, C. A., Teufel, J. D., & Lehnert, K. W. (2008). Measuring nanomechanical motion with a microwave cavity interferometer. *Nat. Phys.*, *4*(7), 555–560. Retrieved from <https://doi.org/10.1038/nphys974> doi: 10.1038/nphys974
- Schmidt-Kaler, F., Häffner, H., Riebe, M., Gulde, S., Lancaster, G. P., Deuschle, T., ... Blatt, R. (2003). Realization of the Cirac-Zoller controlled-NOT quantum gate. *Nature*, *422*(6930), 408–411. Retrieved from <https://doi.org/10.1038/nature01494> doi: 10.1038/nature01494
- Scott, K. (2015). *Parallelizing amplifiers improves signal-to-noise performance* (Tech. Rep.). Analog devices.
- Selvadoray, M., & Sanjay Kumar, M. (1997). Phase properties of correlated two-mode squeezed coherent states. *Optics Communications*, *136*(1–2), 125–134. Retrieved from [https://doi.org/10.1016/s0030-4018\(96\)00612-8](https://doi.org/10.1016/s0030-4018(96)00612-8) doi: 10.1016/s0030-4018(96)00612-8
- Sherman, J. A., Curtis, M. J., Szwer, D. J., Allcock, D. T. C., Imreh, G., Lucas, D. M., & Steane, A. M. (2013). Experimental recovery of a qubit from partial collapse. *Phys. Rev. Lett.*, *111*, 180501. Retrieved from <https://link.aps.org/doi/10.1103/PhysRevLett.111.180501> doi: 10.1103/PhysRevLett.111.180501
- Shore, B. W., & Knight, P. L. (1993). The jaynes-cummings model. *Journal of Modern Optics*, *40*(7), 1195–1238. Retrieved from <https://doi.org/10.1080/09500349314551321> doi: 10.1080/09500349314551321

- Siverns, J. D., Simkins, L. R., Weidt, S., & Hensinger, W. K. (2012). On the application of radio frequency voltages to ion traps via helical resonators. *Applied Physics B: Lasers and Optics*, 107(4), 921–934. Retrieved from <https://doi.org/10.1007/s00340-011-4837-0> doi: 10.1007/s00340-011-4837-0
- Srinivas, R., Burd, S. C., Knaack, H. M., Sutherland, R. T., Kwiatkowski, A., Glancy, S., ... et al. (2021). High-fidelity laser-free universal control of trapped ion qubits. *Nature*, 597, 209–213. Retrieved from <https://doi.org/10.1038/s41586-021-03809-4> doi: 10.1038/s41586-021-03809-4
- Srinivas, R., Löschner, C., Malinowski, M., Hughes, A., Nourshargh, R., Negnevitsky, V., ... et al. (2023). Coherent control of trapped-ion qubits with localized electric fields. *Physical Review Letters*, 131(2), 020601. Retrieved from <https://doi.org/10.1103/physrevlett.131.020601> doi: 10.1103/physrevlett.131.020601
- Susskind, L., & Glogower, J. (1964). Quantum mechanical phase and time operator. *Physics Physique*, 1(1), 49–61. Retrieved from <https://doi.org/10.1103/physicsphysiquefizika.1.49> doi: 10.1103/physicsphysiquefizika.1.49
- Sutherland, R. T., & Srinivas, R. (2021). Universal hybrid quantum computing in trapped ions. *Physical Review A*, 104(3), 032609. Retrieved from <https://doi.org/10.1103/physreva.104.032609> doi: 10.1103/physreva.104.032609
- Thermostat*. (2022). <https://github.com/sinara-hw/Thermostat/wiki>. GitHub.
- Turchette, Q. A., Myatt, C. J., King, B. E., Sackett, C. A., Kielpinski, D., Itano, W. M., ... Wineland, D. J. (2000). Decoherence and decay of motional quantum states of a trapped atom coupled to engineered reservoirs. *Physical Review A*, 62(5), 053807. Retrieved from <https://doi.org/10.1103/physreva.62.053807> doi: 10.1103/physreva.62.053807
- Varma, A. V., Muldoon, J. E., Paul, S., Joglekar, Y. N., & Das, S. (2023). Extreme violation of the leggett-garg inequality in nonunitary dynamics with complex energies. *Phys. Rev. A*, 108, 032202. Retrieved from <https://link.aps.org/doi/10.1103/PhysRevA.108.032202> doi: 10.1103/PhysRevA.108.032202

- Vogel, W., & Filho, R. L. (1995). Nonlinear jaynes-cummings dynamics of a trapped ion. *Physical Review A*, 52(5), 4214–4217. Retrieved from <https://doi.org/10.1103/physreva.52.4214> doi: 10.1103/physreva.52.4214
- Webster, S. (2005). *Raman sideband cooling and coherent manipulation of trapped ions* (Unpublished doctoral dissertation). St. John’s College, Oxford.
- Wigner, E. (1932). On the quantum correction for thermodynamic equilibrium. *Physical Review*, 40(5), 749–759. Retrieved from <https://doi.org/10.1103/physrev.40.749> doi: 10.1103/physrev.40.749
- Wineland, D. J., Drullinger, R. E., & Walls, F. L. (1978). Radiation-pressure cooling of bound resonant absorbers. *Physical Review Letters*, 40(25), 1639–1642. Retrieved from <https://doi.org/10.1103/physrevlett.40.1639> doi: 10.1103/physrevlett.40.1639
- Wineland, D. J., & Itano, W. M. (1979). Laser cooling of atoms. *Physical Review A*, 20(4), 1521–1540. Retrieved from <https://doi.org/10.1103/physreva.20.1521> doi: 10.1103/physreva.20.1521
- Wolf, F., Shi, C., Heip, J. C., Gessner, M., Pezzè, L., Smerzi, A., ... Schmidt, P. O. (2019). Motional fock states for quantum-enhanced amplitude and phase measurements with trapped ions. *Nature Communications*, 10, 2929. Retrieved from <https://doi.org/10.1038/s41467-019-10576-4> doi: 10.1038/s41467-019-10576-4
- Wright, J., Auchter, C., Chou, C.-K., Graham, R. D., Noel, T. W., Sakrejda, T., ... Blinov, B. B. (2016). Toward a scalable quantum computing architecture with mixed species ion chains. *Quantum Information Processing*, 15(12), 5339–5349. Retrieved from <https://doi.org/10.1007/s11128-015-1220-9> doi: 10.1007/s11128-015-1220-9
- Wu, C.-W., Zhang, M.-C., Zhou, Y.-L., Chen, T., Huang, R., Xie, Y., ... Chen, P.-X. (2023). *Maximizing temporal quantum correlation by approaching an exceptional point*. arXiv: 2304.06590. Retrieved from <https://doi.org/10.48550/arXiv.2304.06590> doi: 10.48550/arXiv.2304.06590
- Yan, L. L., Wan, W., Chen, L., Zhou, F., Gong, S. J., Tong, X., & Feng, M. (2016). Exploring structural phase transitions of ion crystals. *Scientific Reports*, 6, 1–9. Retrieved from <https://doi.org/10.1038/srep21547> doi: 10.1038/srep21547

- Yurke, B., McCall, S. L., & Klauder, J. R. (1986). SU(2) and SU(1,1) interferometers. *Phys. Rev. A*, *33*(6), 4033–4054. Retrieved from <https://doi.org/10.1103/PhysRevA.33.4033> doi: 10.1103/PhysRevA.33.4033
- Zwierz, M., Pérez-Delgado, C. A., & Kok, P. (2010). General optimality of the Heisenberg limit for quantum metrology. *Phys. Rev. Lett.*, *105*(18), 180402. Retrieved from <https://doi.org/10.1103/PhysRevLett.105.180402> doi: 10.1103/PhysRevLett.105.180402