Essays in Network Macroeconomics

by

Owen Levi Jetton

A dissertation accepted and approved in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

in Economics

Dissertation Committee:

Bruce McGough, Chair

David Evans, Core Member

Jeremy Piger, Core Member

Youchang Wu, Institutional Representative

University of Oregon

Spring 2026

©2026 Owen Levi Jetton

All rights reserved.

DISSERTATION ABSTRACT

Owen Levi Jetton

Doctor of Philosophy in Economics

Title: Essays in Network Macroeconomics

Chapter I lays the groundwork for a class of economic model that utilizes static networks to simulate the sharing of information. By amending a simple cobweb model to account for an information network, it is shown using computational simulation methods how variations of networks—and information at large—relate to economic conditions in an adaptive learning setting. Overall, as networks become more interconnected, economic volatility decreases and the speed of convergence to equilibrium increases. These results provide further evidence for the importance of information availability in economic dynamics.

Chapter II considers the role expectations play in the spatial diffusion of monetary shocks through a network economy. Using an overlapping generations network model, it is shown that changing the assumption of expectations from rational to adaptive, flips the economic response of a monetary expansion. We consider the Pareto efficiency of this economy, and how expectations change the welfare effects of monetary policy. Different network structures are explored, and we discuss how monetary conditions and convergence to steady-state relate to these network structures. The heterogeneous agent case is considered, and the response is shown to depend crucially on the spatial organization of agents, leading to varying short and medium-run effects.

Chapter III tests the theoretical results of Chapter II in the context of the European Union. Using the European Commission Survey of Expectations, we categorize countries as forward or backward-looking using a Bayesian Model Averaging method. A Network Informed Structural VAR (NISVAR) model demonstrates the EU's varying responses to country-specific monetary expansions. Supportive evidence for the theoretical model's results are found: the economy's response to monetary expansions depends on the forward-lookingness of the expectations those receiving the expansion. Monetary expansions have a greater effect on output, and a lesser effect on inflation, when directed at areas that employ anchored, backward-looking expectations.

ACKNOWLEDGMENTS

I would like to thank the members of my dissertation committee for the invaluable advice and counsel they have provided throughout the development of these papers: David Evans, Jeremy Piger, Youchang Wu, and especially my advisor, Bruce McGough.

I would also like to thank my family—Martin, Julie, Cole, and Reid—and all of the pets who have been there for me across my academic career: Tangy and Cocomelon (sisters who still don't get along), Socrates (the wise), Kepler and Galileo (brothers who do get along), and Clyde (just the best).

TABLE OF CONTENTS

Chapter Page

1 Shared Private Information Adaptive Learning in Static Networks 11

1.1 Introduction and Literature Review 11

1.2 Cobweb Model 18

1.2.1 Set up 18

1.2.2 The Restricted Perceptions Equilibrium 19

1.2.3 Stability 21

1.2.4 Adaptive Learning 21

1.2.5 Solving for the Equilibrium Beliefs Matrix 22

1.2.6 Full and Private Information Cases 23

1.2.7 Model Summary 24

1.3 Simulating the Cobweb Model 24

1.3.1 Full Information Case 25

1.3.2 Shared Private Information Case 26

1.3.3 Private Information Case 27

1.3.4 Comparing Parameter Effects of Weather, β_j 28

1.3.5 Half-Life Estimation and Convergence Speed 29

1.3.6 Price Variance and Economic Volatility 30

1.4 The Simulation 31

1.4.1 Simulation Results 34

1.4.2 Discussion 39

1.5 Conclusion 41

2 Spatial Interactions of Expectations with Applications to Monetary Policy 43

2.1 Introduction 43

2.2 An Overlapping Generations Network Model 48

2.2.1 OLG Model Set Up . . . 48
2.2.2 Finite-Memory Forecasting Learning . . . 51
2.2.3 Comparing the Rational Expectations and Learning Models . . . 53
2.3 Monetary Expansion in Circular Graph . . . 54
2.3.1 Set up . . . 54
2.3.2 Results . . . 55
2.3.3 Discussion . . . 57
2.3.4 Pareto Efficiency . . . 58
2.4 Network Structure . . . 61
2.4.1 General Dynamics . . . 62
2.4.2 Network Simulation . . . 64
2.4.3 Results . . . 65
2.5 Expectation Heterogeneity . . . 70
2.5.1 Fixed-Portion Heterogeneity . . . 70
2.5.2 Results . . . 72
2.5.3 Discussion . . . 74
2.6 Conclusion . . . 75

3 Country Specific Monetary Shocks in Europe: Network Informed Structural Vector Autoregression Analysis . . . 76
3.1 Introduction . . . 76
3.2 Spatial Expectations Data . . . 77
3.3 Quantizing Inflation Expectations . . . 78
3.4 Categorizing Countries . . . 79
3.5 Balance Tests . . . 83
3.6 Country Level Model . . . 84
3.6.1 Structural Vector Autoregression . . . 85
3.6.2 Impulse Response Results . . . 86

3.7 Aggregated Model . . . 88
3.7.1 NISVAR . . . 88
3.7.2 Impulse Response Results . . . 90
3.8 Discussion . . . 91
3.9 Conclusion . . . 92

A Chapter I Appendix . . . 94
A.1 Proof of Proposition 1.1. . . . 94
A.2 Proof of Proposition 1.2. . . . 95
A.3 Proof of Proposition 1.3 . . . 98
A.4 Proof of Proposition 1.4. . . . 99
A.5 Proof of Proposition 1.5. . . . 100

B Chapter II Appendix . . . 101
B.1 Derivation of Equation (2.15) . . . 101
B.2 Infinite Horizon Problem . . . 102
B.3 Proof of Proposition 2.1. . . . 103
B.4 Proof of Proposition 2.2. . . . 104

C Chapter III Appendix . . . 105
C.1 Data Sources and Description . . . 105
C.2 Country Level Impulse Response Functions . . . 108
C.3 Aggregate Impulse Response Functions . . . 111

REFERENCES . . . 116

LIST OF FIGURES

Figure Page

1.1 Beliefs held by agents on each island under full information. 25

1.2 Beliefs held by agents on each island under shared information. 27

1.3 Beliefs held by agents on each island under private information. 28

1.4 Market price given identical weather. 31

1.5 Example network and adjacency matrix with $J = 5$. 33

1.6 Statistics regarding prices in economies with 5 islands 36

1.7 Statistics regarding prices in economies with 10 islands 37

1.8 Statistics regarding the speed of convergence in economies with 5 islands 38

1.9 Statistics regarding the speed of convergence in economies with 10 islands 39

2.1 Plots of $p_t(j)$ for $j \in V$ in periods following monetary expansion. 55

2.2 Plots of $q_t(j)$ for $j \in V$ in periods following monetary expansion. 56

2.3 Real GDP and Price Level over time. 56

2.4 Levels of variables relative to steady-state values. 58

2.5 Lifetime Utility for agents born in time $t = 0, ..., 5$ 60

2.6 Lifetime Utility for generations born in time $t = -10, ..., 40$ 61

2.7 A demonstrative tree graph (A.) that is of the type excluded from this exercise, and an amended version (B.) that includes cycles. 63

2.8 Time paths of the price level and aggregate real output across 100 simulations. . . . 66

2.9 Time paths of aggregate real output and the price level across 100 simulations, normalized to the initial steady-state levels. 68

2.10 Estimated correlation statistics between nodes' network centrality and the nodes' variables' convergence speed to the new equilibrium after the monetary expansion. . . . 69

2.11 Monetary expansion with heterogeneous agents organized into clusters. Rational Expectations agents are centered around the monetary shock. 72

2.12 Monetary expansion with heterogeneous agents organized into clusters. Adaptive expectations agents are centered around the monetary shock. 73
2.13 Monetary expansion with heterogeneous agents dispersed throughout the economy. 74
3.1 Comparing forward/backward-looking classifications of BMA to the method in Lyziak & Mackiewicz-Lyziak (2014) (May 2001 to July 2012) 81
3.2 BMA estimated τ^j (January 2010 to December 2019) 82
3.3 SVAR IRFs of Industrial Production Growth and Inflation, respectively, in percent. 87
3.4 SVAR IRFs of Aggregate Industrial Production Growth and Inflation, respectively, in percent. 90
C.1 Compared sample-averaged industrial production and price level with Euro Area time series of the same variables. 107
C.2 Country level impulse response functions for Industrial Production for backward looking countries. 108
C.3 Country level impulse response functions for Industrial Production for forward looking countries. 109
C.4 Country level impulse response functions for Inflation for backward looking countries. 110
C.5 Country level impulse response functions for Inflation for forward looking countries. 111
C.6 Aggregate impulse response functions for Industrial Production for backward looking countries. 112
C.7 Aggregate impulse response functions for Industrial Production for forward looking countries. 113
C.8 Aggregate impulse response functions for Inflation for backward looking countries. 114
C.9 Aggregate impulse response functions for Inflation for forward looking countries. . 115

LIST OF TABLES

Table Page

1.1 RPE Beliefs where b_k^j represents the RPE belief agents on island j have about island k's weather's effect on the price. 26

1.2 RPE Beliefs in private information case 27

1.3 β_j represents parameters effect of w_{t-1}^j on p_t 28

1.4 Estimated Half-Lives of b_k^j in the full information case as beliefs approach the REE. 29

1.5 Estimated Half-Lives of b_k^j in shared private information case as beliefs approach the RPE. 30

1.6 Estimated Half-Lives of b_k^j in private information case as beliefs approach the RPE. 30

1.7 Centrality measures for the network in Figure 1.5. 33

2.1 Steady-state prices and output by graph type with $\sum_j \bar{m}(j) = J = 40$ agents. 67

3.1 Countries and their estimated τ^j for the time period 2010-2019. 83

3.2 Balance test between τ^j and relevant variables. 84

Chapter I: Shared Private Information Adaptive Learning in Static Networks

1.1 Introduction and Literature Review

How does limited access to information affect the dynamics, convergence, and stability of a production economy where information is shared through a static network? The rational expectations hypothesis, as originally formalized, "asserts that the economy generally does not waste information, and that expectations depend specifically on the structure of the entire system" (Muth, 1961). In an economy with boundedly rational agents whose information is restricted as a result of the structure of the entire system, it has been shown that rational expectations can be learned through econometric methods. However, in other structures of economies with boundedly rational agents and incomplete information, agents naturally shift the economy towards a non-rational expectations equilibrium. This Chapter develops a model where agents and information are structured in a static and often incomplete network and are locked in a perpetually and symmetrically uninformed state. We explore the heterogeneous expectations that naturally arise and, as a result of the restricted perceptions of different types of agents, the non-rational expectations equilibrium of this model.

This Chapter stands on the shoulders of multiple vibrant fields of literature. Most importantly, it introduces a reasonable information structure into Adaptive Learning—that of shared private information between agents in a fixed and static network. Sharing private information in a network provides a more realistic view than strictly private information models: while people have private knowledge and information about the economy, this knowledge is often shared—whether purposefully or incidentally—with other agents. Using static networks as a foundation for this model allows us to capture a wide range of information availability structures in between private and complete information.

The rational expectations equilibrium (REE) was first explored in a cobweb model of production in Muth (1961), where producers' expectations of the price of the produced good next period are equal to the realized price—this is known as rational expectations (RE). The ques-

tion of whether merely boundedly rational agents could learn how to form RE—rather than be gifted them by a benevolent modeler—in a cobweb model was answered by Bray & Savin (1986). They show that when firms form expectations using ordinary least squares or Bayesian methods, the economy converges to the REE with probability 1 for "economically plausible conditions." The authors laid the groundwork for Adaptive Learning (AL), a recursive econometric method by which agents form and update beliefs and expectations. The quintessential textbook on that subject, Evans & Honkapohja (2001), begins its first example with a cobweb model. To further demonstrate the importance of the cobweb model to AL, another introductory text to the topic, Evans & McGough (2020), utilizes the cobweb model pragmatically to demonstrate its versatility across subjects in AL.

AL has always been formulated with agents operating on incomplete information. Bray & Savin note that since agents fail to account for the feedback of their beliefs, these agents are estimating a misspecified model. AL with incomplete information was extended further to include asymmetric information where economic agents have different information sets and make their decisions based on respective information. Granato, Guse, & Wong (2008) find that under AL, an information diffusion process alters their asymmetric cobweb model significantly, even introducing multiple learnable equilibria. The information structure introduced by Nakagawa (2015) is one of private information where a finite number of "types" of agents exist and each make decisions based on their private information. The author finds that the conditions for learnability of the equilibrium are less stringent under the private information structure.

AL with asymmetric information is comparable to AL with heterogeneous agents where at least one "type" of agent has incomplete information. The variability and asymmetry of information yield heterogeneously misspecified forecasting models. There is a rich literature on AL models with heterogeneous expectations—asymmetrical information is but a subset of the AL literature.

Heterogeneous initial conditions lead to what Honkapohja & Mitra (2006) coin "transient heterogeneity," as the results are identical to the representative agent model concerning learnabil-

ity, equilibrium, and stability. Evans & Honkapohja (1996) and Giannitsarou (2003) demonstrate that the conditions for convergence between heterogeneous and representative agents are identical.

A "fixed fraction" heterogeneity method is used in Honkapohja & Mitra (2006) where structural heterogeneity is constructed by assigning different groups of agents "masses" (a fixed portion) that sum to 1. The "types" of agents in Nakagawa (2015) are given identical masses, meaning if there are n agents, each group has a mass of $1/n$. The Real Business Cycle (RBC) model in Giannitsarou (2003) is also developed with heterogeneity in perceived laws of motion stemming from exogenously laid fixed portions. Branch & McGough (2004, 2009, 2011) develop similar models and show that the introduction of heterogeneous expectations changes the dynamics of the model significantly.

Additionally, there is "heuristic heterogeneity," introduced by Brock & Hommes (1997), where the portion of agents utilizing each expectation formation method is endogenous to the method's performance within the model, often measured by mean square errors and a multinomial logit function. The authors demonstrate the chaotic nature of their cobweb model with heuristic heterogeneity under different parameters. Anufriev, Assenza, Hommes, & Massaro (2013) utilize heuristic heterogeneity in a New Keynesian (NK) model and make the case for more aggressive monetary policy under heterogeneous expectations than in the baseline homogeneous expectations case. Hommes & Lustenhouwer (2015) explore a similar model of endogenous central bank credibility, and find that when the zero lower bound is accounted for, expectation-induced liquidity traps can arise, during which the central bank loses credibility. These papers show how the inclusion of heterogeneous expectations can result in important policy prescriptions.

One heterogeneous agent AL equilibrium concept that has flourished—and is the focus of this Chapter—is the *Restricted Perceptions Equilibrium* (RPE). In general, an RPE is "An equilibrium between optimally misspecified beliefs and the stochastic process for the economy" (Branch (2006)). Most elementary examples of RPE demonstrate the equilibrium in a homogenous expec-

tations model, where the representative agent's perceived law of motion is misspecified. Though first explicitly named and articulated in Sargent (1993), many earlier economic and AL models were RPE models, as noted by Branch (2006). The equilibrium concept exists independent of the AL literature as any solutions to an expectations model with misspecified beliefs can be considered an RPE. Additionally, the equilibrium concepts in the heterogeneous AL models above can be considered under the purview of RPE.

Branch (2004, 2022) provides thorough examples and discussions of the RPE. It is a reasonable alternative to REE that takes bounded rationality into account by considering the reasonable cognitive abilities of economic agents and is therefore consistent with Muth's original Rational Expectations Hypothesis. This is the Cognitive Consistency Principle: "the way people forecast should be on par with a good econometrician" (Branch (2022)). This principle is a fundamental key to AL, and the RPE concept articulates this principle in various reasonable economic circumstances.

The equilibrium concept of RPE is robust outside of theoretical frameworks. Branch & McGough (2011) show using quarterly U.S. data, that including heterogeneity yields endogenous propagation of business cycles that yields a closer fit than the representative agent model. Slobodyan & Wouters (2012) find that an optimal Dynamic Stochastic General Equilibrium Vector Autoregression (DSGE-VAR) model in an RPE setting is the best-performing learning model for U.S. economic data. The repeated successful calibration of RPE models with data further cements RPE as a powerful economic concept. Kurz, Jin, & Motolese (2003) show that a model where agents can be heterogeneously optimistic or pessimistic—meaning their forecast is above or below the empirical probability, respectively—replicates the U.S. business cycle data well. There is also a strong empirical ground for the assumption of heterogeneous expectation formation. Examining the University of Michigan inflation expectations survey data, Branch (2007) finds empirical evidence for mixed uncertainty, and heuristic heterogeneity, across household expectations.

Beyond AL, the topic of incomplete information—the lack of society-wide common knowl-

edge—in macroeconomics has been thoroughly explored. Lucas (1972) develops a model with perpetually and symmetrically uninformed agents that make decisions based on different information sets. In this context, familiar relationships between macroeconomic variables, such as the Phillips Curve and the equation of exchange, are derived. Incomplete information has also been articulated to include "rational inattention," as developed by Sims (2003). Sims demonstrates that a utility-maximizing agent in an economy where information's accuracy decreases as it moves through a "channel" away from the source, engages in a higher level of consumption smoothing in response to an income shock relative to an agent with full information. This implies agents employ a greater level of caution when information is imperfect. Demertzis & Hallett (2008) develop a macroeconomic model of asymmetric information that results in heterogeneous macroeconomic expectations. In their model, agents cannot know the central bank's beliefs without paying a price, and the authors find that the gains from rational expectations are minimal, meaning macroeconomic losses from asymmetric information are negligible.

One natural way to consider asymmetric information is through an incomplete network where agents are limited in their access to information and their ability to act upon other agents. *Network Games: Theory, Models, and Dynamics* (Özdağlar & Menache, (2011)) provides a thorough introduction to networks in game theory, discussing multiple problems that involve strategic games where players are connected via networks. Dijkstra & van Assen (2013) develop a network game where agents can cooperate for the benefit of public goods. The authors derive multiple propositions regarding the relationship between characteristics of the model and resulting equilibria, generally favoring more connected networks, though results are marginal. Zeng, et. al. (2019) develop an asymmetric "active deception game" where "attackers" attempt to dismantle an infrastructure network—here, information and actions are both asymmetric. Their model also demonstrates the importance of a highly connected network.

When applied to a laboratory setting, researchers have found contradictory results regarding the converged-upon equilibrium when information travels through a network. In Keser, Ehrhart, & Berninghaus (1998), the authors perform a laboratory experiment where players coordinate on

strategies in large groups or by communicating through neighbors in a network to land on mutually beneficial coordination equilibria that vary in risk. The authors find that players quickly coordinate on the *risk*-dominant equilibrium when subjected to the limiting network. This result is similar to the over-smoothing of consumption under rational inattention as modeled in Sims (2003)—agents are more cautious under incomplete information. Contradictory results are found by Boun My, Willinger & Ziegelmeyer (2001) in a similar lab setting; their experiment finds that agents coordinate on the *payoff*-dominant equilibrium under both local (incomplete network) and global (complete network) interaction. Additionally, the authors of the latter paper find no evidence that the convergence speed to equilibria varies between local and global interaction, again contradicting Keser et. al. (1998).

Though empirical economic work concerning networks is rare, the results provide insights into how economies organized into networks impact the real world. For example, De Masi (2009) analyzes the Italian banking system as a network and concludes from the data that the heterogeneity of the banking system aids interbank market efficiency—the topology of the Italian banking system network mitigates the probability of bankruptcy. The author's work is included in Naimzada, Stefani, & Torriero (2009) which provides a broad overview of graph theory, networks, and its application to economics. Additionally, an analysis of Chinese immigrant community networks by Rauch & Trindade (2002) concludes that the networks significantly increase international trade volumes for commodities whose prices are not publicly available—this provides strong empirical evidence that information becomes more public through the sharing of private information in a network. Jensen (2007) similarly demonstrates how the introduction of phones into the South Indian fishing industry allowed for sharing information that dramatically decreased price dispersion.

The use of networks has been applied to cobweb models in the context of genetic algorithms and programming. Arifovic (1994) constructs a genetic algorithm to model the learning of economic agents in a cobweb model. Genetic algorithm learning differs from AL in that agents learn the viability of beliefs within a decision tree network rather than through econometric methods.

The author finds that the algorithms replicate the results of laboratory experiments. This work is expanded upon by Chen & Yeh (1996) with an evolutionary network. The authors find an inverse relationship between the stability and the adaptability of an equilibrium. It is important to note that the networks in these papers refer to the creation of possible predictors by the genetic programming agents rather than a topological structure of information transmission, as this Chapter explores.

This Chapter contributes to the fields of incomplete and asymmetric information within the broader scope of AL. Both AL and graph theory operate in the realm of linear algebra, and this Chapter exploits that overlap to its fullest extent; most of the vocabulary used concerning graph theory is borrowed from *Networks, Topology and Dynamics* (Naimzada, Stefani, & Torriero (2009)), however, the vocabulary's application to AL is our own. Beyond AL, this Chapter adds to the literature of perpetually and symmetrically uninformed macroeconomic models and provides a baseline introduction to the methodology behind applying information networks to these models.

The model in this Chapter is novel in its encompassing and broadening of past models. A clear example is the model of AL under private information by Nakagawa (2015). This new model accounts for a wide variety of information structures within an economy, from private to full information, and everything in between where agents share their private information relative to the network. Additionally, the model in the Chapter allows for groups observing different exogenous variables to be unequal in size.

This Chapter also helps answer questions regarding convergence speed under local and global interaction. As discussed, laboratory experiments find conflicting results regarding convergence speeds when the interaction is local or global. Furthermore, we explore the effects that determinants of the topology of a network—such as size, configuration, and centrality—have on the dynamics of the resulting equilibrium. Investigating these questions helps us better understand how different geometric formations of networks affect the dynamics of economies with shared private information. Given the cobweb model's status in the literature of heterogeneous expectations and AL, we find it an appropriate model for analyzing the outcomes of an economy with shared pri-

vate information in a static network.

This Chapter continues as follows. Section **1.2** constructs the shared private information cobweb model, derives the RPE and stability conditions, discusses the incorporation of adaptive learning, lays out a method for solving for equilibrium beliefs under incomplete information, as well as discusses the full information REE case, and private information RPE case.

Section **1.3** explores a demonstrative example of the model. Section **1.4** lays out the methodology employed for simulating the cobweb model and deriving results regarding economic volatility and convergence speed. The results are presented and discussed along with their consequences. Finally, section **1.5** concludes.

1.2 Cobweb Model

1.2.1 Set up

Consider a cobweb model of production where firms choose the quantity supplied in the market in time t at time $t-1$.

Consumers in this market observe the market price and have linear demand for the good:

$$D(p_t) = A - Bp_t + \varepsilon_t \tag{1.1}$$

where $A, B > 0$, and ε_t is a mean zero *i.i.d.* shock.

A continuum of firms $i \in [0,1]$ exist in this market and each face the cost function:

$$c(q_t^i, w_{t-1}^i) = (1/C)\left(q_t^{i2}/2 + B\gamma\ w_{t-1}^i q_t^i\right)$$

where q_t^i is the quantity produced for the market at time t, w_{t-1}^i is the weather shock experienced by firm i in the previous period, and $C,\ \gamma > 0.$. Since the market price p_t is not observable at time $t-1$, the firm maximizes profit relative to their expectation of the future price, p_t^e. The profit-maximizing firm supplies:

$$s(p_t^{i,e}, w_{t-1}^i) = C\ p_t^{i,e} - B\gamma w_{t-1}^i \tag{1.2}$$

This model diverges from other cobweb production models in the distribution of information available to the firms. Firms are organized into $J \geq 1$ "islands" such that the proportion θ_j of firms $\left(\theta_j \in (0,1] \; s.t \; \sum_j^J \theta_j = 1\right)$ is located on island j. Since all firms on any given island are identical in information and production, we need only to consider the supply of individual islands, denoted $s(p_t^{j,e}, w_{t-1}^j)$, which is identical in form to (1.2).

The islands are in a *network* formulated by the $J \times J$ adjacency matrix M where:

$$M_{j,k} = \begin{cases} 1 & \text{if island } j \text{ observes island } k\text{'s weather shocks} \\ 0 & \text{if not} \end{cases}$$

Assume every island observes its own weather shock, $M_{j,j} = 1 \; \forall j$.

The vector of weather shocks at time $t-1$, $w'_{t-1} = (w_{t-1}^1, ..., w_{t-1}^J)$ is drawn from the random distribution $\mathcal{D}(0_{J\times 1}, \Sigma)$, where Σ is a $J \times J$ variance-covariance matrix with elements $\sigma_{j,k}$ such that $\sigma_{j,j} > 0$ and $\sigma_{j,k} \neq 0 \; \forall j,k$. The covariance between the weather on any two islands may be negative or positive, and the weather shocks on any two islands may be covariant whether or not agents on these islands can observe each other. Additionally, note that since Σ is a variance-covariance matrix, covariances are symmetric: $\sigma_{j,k} = \sigma_{k,j} \; \forall$ pairs, (j,k).

The aggregate supply in this economy is:

$$S(p_t^e, w_{t-1}) = \sum_j^J \theta_j s(p_t^{j,e}, w_{t-1}^j) \tag{1.3}$$

Setting up the market clearing condition, (1.1) = (1.3), and solving for p_t, yields:

$$p_t = \frac{A}{B} - \frac{C}{B} \sum_j \theta_j p_t^{j,e} + \gamma \sum_j \theta_j w_{t-1}^j + \frac{1}{B} \varepsilon_t \tag{1.4}$$

1.2.2 The Restricted Perceptions Equilibrium

Assume firms have a *perceived law of motion* (PLM) for the market price that they use to formulate their expectations. This PLM is a linear function of the observable weather shocks:

$$p_t^{j,e} = a^j + b^{j\prime}\hat{w}_{t-1}^j \tag{1.5}$$

where $b^{j\prime} = (b_1^j, ..., b_J^j)$ and b_k^j is the belief firms on island j have about island k's weather's effect on the market price, and a^j is the constant coefficient. Additionally, $\hat{w}_{t-1}^j$ represents the vector of weather shocks *observed by* firms on island j at time $t-1$, i.e. the k^{th} element of $\hat{w}_{t-1}^j$ is w_{t-1}^k if $M_{j,k} = 1$, and 0 otherwise. Generally, $\hat{w}_{t-1}^j = M_j \circ w_{t-1}$, where M_j is the j^{th} column of M, and $\circ$ is the Hadamard (element-wise) product operator.

Let $\mu \equiv \frac{A}{B}$, $\alpha \equiv -\frac{C}{B}$, and $\tilde{\varepsilon}_t \equiv \frac{1}{B}\varepsilon_t$. Without loss of generality, we can set the intersection of supply and demand as the new origin, $\mu = 0$. Let $\mathcal{A} = [a^1\ a^2\ ...\ a^j]'$, $\theta' = [\theta_1, ..., \theta_J]$, and $\mathcal{B} = (b^1 b^2 ... b^J)'$.

Proposition 1.1.[1] *In a cobweb economy where information is restricted to a network described by the adjacency matrix M, if firms possess a perceived law of motion (PLM) of prices as linear function of observable shocks, then the actual law of motion (ALM) of prices is determined by:*

$$p_t = \alpha\theta'\mathcal{A} + \theta'\left[\alpha\left(\mathcal{B} \circ M\right) + \gamma I_J\right] w_{t-1} + \tilde{\varepsilon}_t \tag{1.6}$$

Note that in cases of incomplete information, if $M_{j,k} = 0$ for at least one pair j, k, according to equation (1.5), it does not matter what the beliefs firms on the island j have about firms on the island k, since the k^{th} value of $\hat{w}^j$ is 0, meaning the $b_k^j \hat{w}^j = 0$, regardless of the value of b_k^j. To simplify many of our future equations, we impose the following restriction: $b_k^j = 0\ \forall$ pairs of islands j, k such that $M_{j,k} = 0$. Additionally, this means $(\mathcal{B} \circ M) = \mathcal{B}$

Proposition 1.2.[2] *With ALM (1.6), denoting the* Moore-Penrose Pseudo-Inverse *of a matrix with a "+" superscript, and defining* Θ *as:*

[1] Proof provided in **A.1** of the Appendix
[2] Proof provided in **A.2** of the Appendix

$$\Theta \equiv \begin{bmatrix} \left[\Sigma \circ (1_{J\times 1} \cdot M_1')\right] \left[\Sigma \circ (M_1 \cdot M_1')\right]^+ \\ \left[\Sigma \circ (1_{J\times 1} \cdot M_2')\right] \left[\Sigma \circ (M_2 \cdot M_2')\right]^+ \\ \vdots \\ \left[\Sigma \circ (1_{J\times 1} \cdot M_J')\right] \left[\Sigma \circ (M_J \cdot M_J')\right]^+ \end{bmatrix}$$

The Restricted Perceptions Equilibrium (RPE) beliefs, $\mathcal{A}^E$ and $\mathcal{B}^E$, are unique and satisfy:

$$\mathcal{A}^E = 0_{J\times 1}, \quad \mathcal{B}^E = \Big(\alpha(I_J \otimes \theta^{\mathcal{B}}) + \gamma(I_J \otimes \theta')\Big)\Theta, \quad \theta^{\mathcal{B}} \equiv \gamma(\theta' \otimes \theta')\Theta[I_J - \alpha(\theta' \otimes I_J)\Theta]^{-1} \tag{1.7}$$

1.2.3 Stability

Proposition 1.3.[3] *The RPE is stable as long as the eigenvalues of the following $J^2 \times J^2$ matrix have negative real parts:*

$$\frac{d\text{vec}(T)}{d\text{vec}(\mathcal{B})} = \alpha(\Theta' \otimes I_J)\Phi\text{vec}(I_J)(I_J \otimes \theta') - I_{J^2}$$

Where Φ is the commutation matrix that solves: $vec(I \otimes \theta'\mathcal{B}) = \Phi(vec(I) \otimes \theta'\mathcal{B})$.

1.2.4 Adaptive Learning

Assume firms on their respective islands use *recursive least squares* (RLS) as described in Evans and Honkapohja (2001) to compute their beliefs: the constant a^j and the $J \times 1$ vector b^j, as described in (1.5). Every island believes the effect of weather it cannot see has an exogenous effect on the price: *i.i.d.* normal and mean 0. For ease of notation, let:

$$y_t^j = \begin{pmatrix} a_t^j \\ b_t^j \end{pmatrix} \quad \text{and} \quad x_t^j = \begin{pmatrix} 1 \\ \hat{w}_t^j \end{pmatrix}$$

Agents' beliefs (as summarized by y_t^j) can be found with the following RLS equations:

[3] Proof provided in **A.3** of the Appendix

$$y_t^j = y_{t-1}^j + g_t(R_t^j)^+ x_{t-1}^j \ (p_t - y^{j\prime}{}_{t-1} \cdot x_{t-1}^j)$$
$$R_t^j = R_{t-1}^j + g_t(x_{t-1}^j \ x^{j\prime}{}_{t-1} - R_{t-1}^j)$$

Given initial values of $\hat{w}_0^j$.

R^j is an estimate of the second moment matrix for x^j, island j's available weather data. We have to use the pseudo-inverse to account for cases of incomplete information where we have rows/columns of 0's in R^j. The value g_t is the *gain coefficient*, it determines the degree to which new data is valued compared to old data.

1.2.5 Solving for the Equilibrium Beliefs Matrix

In cases of incomplete information, these regressions suffer from multicollinearity and omitted variable bias (OVB). The former will not affect the bias of the RLS estimators from parameter values, and since the islands are merely concerned with prediction and not causal inference, multicollinearity is not an issue here. RLS with a specified constant or decreasing gain coefficient ensures y_t^j converges to its OLS expected value asymptotically. This expected value is an unbiased estimate of the projection but is biased from the parameter effect of w^j on p due to the omitted variables. We will refer to this parameter at β_j.

Proposition 1.4.[4] *The RPE beliefs satisfy the following $J + J^2$ equations:*

$$\beta_j = \alpha\left(\theta_1 b_j^1 M_{1,j} + \theta_2 b_j^2 M_{2,j} + \cdots + \theta_J b_j^J M_{J,j}\right) + \theta_j \gamma \qquad \forall j \in [1, J]$$

$$b_k^j = M_{j,k}\left[\frac{\sigma_{1k}}{\sigma_k}(\beta_1 - M_{j,1} b_1^j) + \frac{\sigma_{2k}}{\sigma_k}(\beta_2 - M_{j,2} b_2^j) + \right.$$
$$\left. \cdots + \beta_k + \cdots + \frac{\sigma_{Jk}}{\sigma_k}(\beta_J - M_{j,J} b_J^j)\right] \qquad \forall j, k \in [1, J]$$

From Proposition 1.4, it is clear how beliefs can be affected by weather on unseen islands. The intuition of OVB comes in handy here as we can see that $\partial b_i^j / \partial \beta_k \neq 0 \ \forall i \in [1, ..., J]$ whether

[4] Proof provided in **A.4** of the Appendix

or not firms on island j can view the weather on island k, in cases of incomplete information. The RLS methods result in firms compensating for the omitted variables with the beliefs that are present. The direction of this compensation depends on the parameters of the model (namely, Σ, α, and γ).

1.2.6 Full and Private Information Cases

Consider the special cases of full information (that all firms can view all firms' weather shocks) and private information (firms can only view their own weather shock). In the former, from (1.4) we can find the REE price:

$$p_t^{REE} = \frac{1}{1-\alpha}\left[\mu + \gamma\theta' w_{t-1}\right] \tag{1.8}$$

This is comparable to the REE in Evans and McGough (2020, p.9), where $\theta' w_{t-1}$ can be thought of as the aggregate weather.

Proposition 1.5.[5] *In the full information case, beliefs converge to*

$$a^{j,\,REE} = 0, \;\; b_i^{j,\,REE} = \frac{\gamma}{1-\alpha}\theta_i, \; \forall j, i$$

We can find the RPE in the private information case, which is comparable to the solution in Proposition 1.2 with $M = I_J$. Given the linear PLM, we get the following ALM:

$$p_t = \alpha\theta'\mathcal{A} + \theta'\left[\alpha\left[b_1^1 \;\; b_2^2 \;\; \dots \;\; b_J^J\right]' + \gamma 1_{J\times 1}\right] w_{t-1} + \tilde{\varepsilon}_t \tag{1.9}$$

Proposition 1.6.[6] *In the private information case, beliefs converge to:*

$$a^{j,\,RPE} = 0 \;\forall\; j, \quad b_i^{j,\,RPE} = 0 \text{ if } j \neq i$$

$$b_j^{j,\,RPE} \text{ solve: } \; b_j^j = \sum_i \frac{\sigma_{ij}}{\sigma_j}\beta_i, \;\; \beta_j = \alpha\theta_j b_j^j + \theta_j\gamma \;\; \forall j$$

As noted previously, the case of private information adaptive learning is explored in Nakagawa (2015). The parallel between the model discussed in that paper and this model is evident if

[5] Proof provided in **A.5** of the Appendix

[6] Is a result of Proposition 1.4. with $M = I_J$

$M = I_J$ and $\theta_j = 1/J \; \forall j \in [1, ..., J]$. The benefit of this model in the private information case is that it applies to *any* proportion of islands $\theta_j \in (0, 1] \; s.t. \; \sum_j^J \theta_j = 1$, and not just equally weighted proportions.

1.2.7 Model Summary

Given the setup of firms separated into islands in a network, where islands use linear PLMs with available data dependent upon the network, we derived the RPE and stability conditions. Additionally, we provided a method for solving for RPE beliefs and presented two propositions yielding equilibrium beliefs in full and private information cases. While this teaches us much about the model, it is unclear from the equations how information is related to equilibrium convergence or economic volatility.

1.3 Simulating the Cobweb Model

Consider a shared private information network cobweb model with the following matrix and weight vector:

$$\Sigma = \begin{bmatrix} 0.1 & 0.05 & 0.02 & -0.01 \\ 0.05 & 0.12 & 0.03 & -0.04 \\ 0.02 & 0.03 & 0.15 & 0.02 \\ -0.01 & -0.04 & 0.02 & 0.08 \end{bmatrix}, \quad \theta = \begin{bmatrix} 1/10 \\ 1/4 \\ 1/3 \\ 19/60 \end{bmatrix}$$

For this simulation, islands possess heterogeneous initial beliefs determined by a random matrix where element i, j is independently distributed according to $N(0, B^{-2})$ if $M_{i,j} = 1$ and 0 otherwise—notice that this is also the distribution according to which price shocks are determined: $\tilde{\varepsilon}_t = B^{-1}\varepsilon_t$ where $\varepsilon_t \sim N(0, 1)$. Initial beliefs about the regression intercept, a^j, are randomly distributed in the same manner. As discussed, initial heterogeneity has no long-term effects on the system as it only yields transient effects. Beliefs regarding the second-moment matrix R^j as discussed in section **1.2.4** are initially set to a matrix of 0's for all firms. Firms utilize con-

stant gain learning, meaning $g_t = g \ \forall t$.

While in the rest of the Chapter, the terms "incomplete information" and "shared private information" are used interchangeably, the former term is not used here since both the private and shared private information cases are cases of "incomplete information." However, the analysis after this section regarding "incomplete information" shelters the private and shared private cases under its umbrella.

1.3.1 Full Information Case

To begin, consider $M = 1_{4\times4}$, the full information case. Equilibrium beliefs on all islands are identical since their information sets and econometric methods are identical. Using *Proposition 1.5*, we know that these beliefs will converge to:

$$a^{j,\,REE} = 0, \quad b_1^{j,\,REE} = \frac{1}{15}, \quad b_2^{j,\,REE} = \frac{1}{6}, \quad b_3^{j,\,REE} = \frac{2}{9}, \quad b_4^{j,\,REE} = \frac{19}{90} \quad \forall j \in \{1,2,3,4\}$$

Additionally, we can confirm that beliefs converge to the equilibrium values with the graph below.

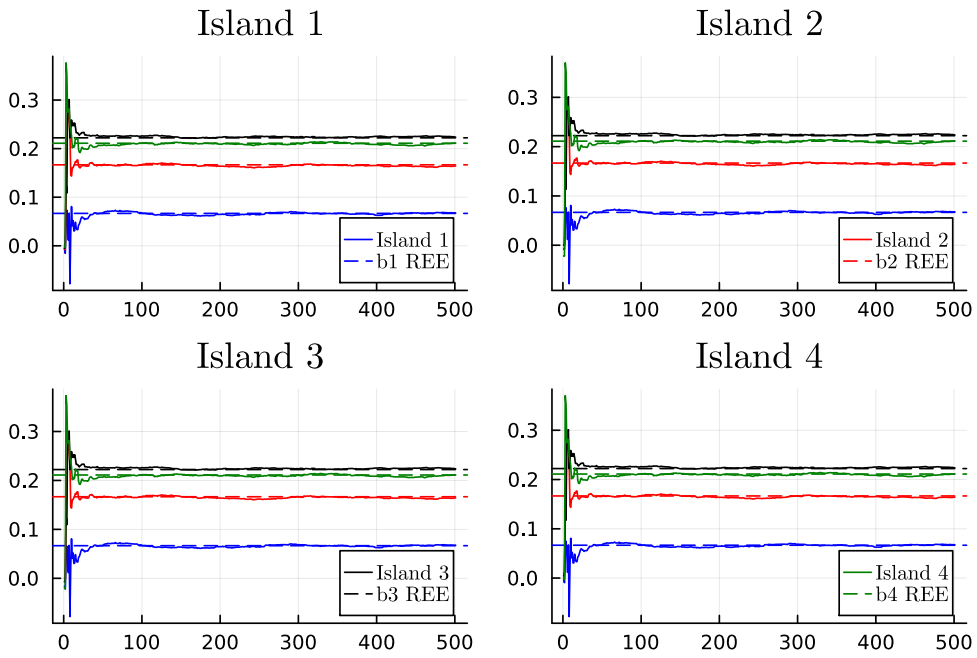


Figure 1.1. Beliefs held by agents on each island under full information.

This simulation is run over $T = 500$ periods, with $\alpha = -0.5,\ \gamma = 1,\ \tilde{\epsilon} \sim N(0, 0.01),\ g = 0.01$. These parameters are the same for the rest of this section.

The symmetry of beliefs across islands should be noted, as was expected. Islands begin with heterogeneous beliefs, though this is difficult to parse from this figure since initial beliefs are tightly generated and converge to similar values quickly.

1.3.2 Shared Private Information Case

Now suppose we restrict a few pathways of observation, resulting in the following adjacency matrix:

$$M = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Using *Proposition 1.2.*, we present the values of the RPE beliefs in Table 1.1.

	b_1	b_2	b_3	b_4
b^1	0.07783	0.08903	0.28128	0
b^2	0.07719	0.26759	0	0.32392
b^3	0.13669	0	0.27403	0.12906
b^4	0	0.20708	0.23812	0.22449

Table 1.1. RPE Beliefs where b_k^j represents the RPE belief agents on island j have about island k's weather's effect on the price.

We show the adaptive learning beliefs of agents on all 4 islands in Figure 1.2 after a simulation with $T = 500$ periods. There is initially a wide variance of beliefs but they quickly converge to the respective RPEs listed in Table 1.1. This simulation uses the same weather shocks that islands experience in Figure 1.1.

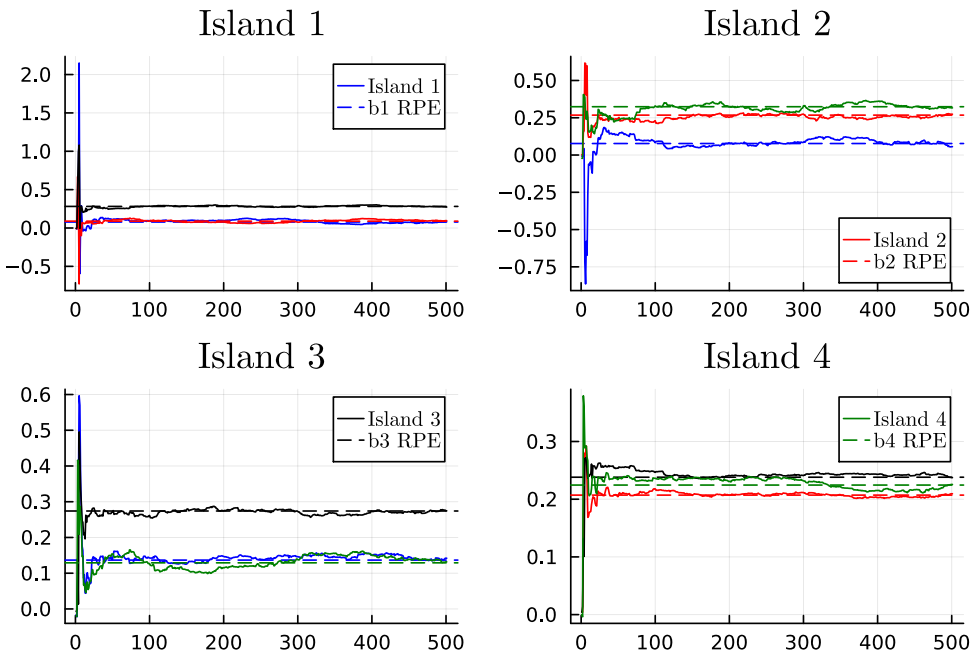


Figure 1.2. Beliefs held by agents on each island under shared information.

1.3.3 Private Information Case

Now suppose firms can only view weather on their island: $M = I_4$. Using the parameter values given, we use *Proposition 1.6.* to solve for RPE beliefs:

	b_1	b_2	b_3	b_4
b^1	0.22559	0	0	0
b^2	0	0.23244	0	0
b^3	0	0	0.36581	0
b^4	0	0	0	0.22722

Table 1.2. RPE Beliefs in private information case

We simulate the cobweb model given the same weather shocks as before.

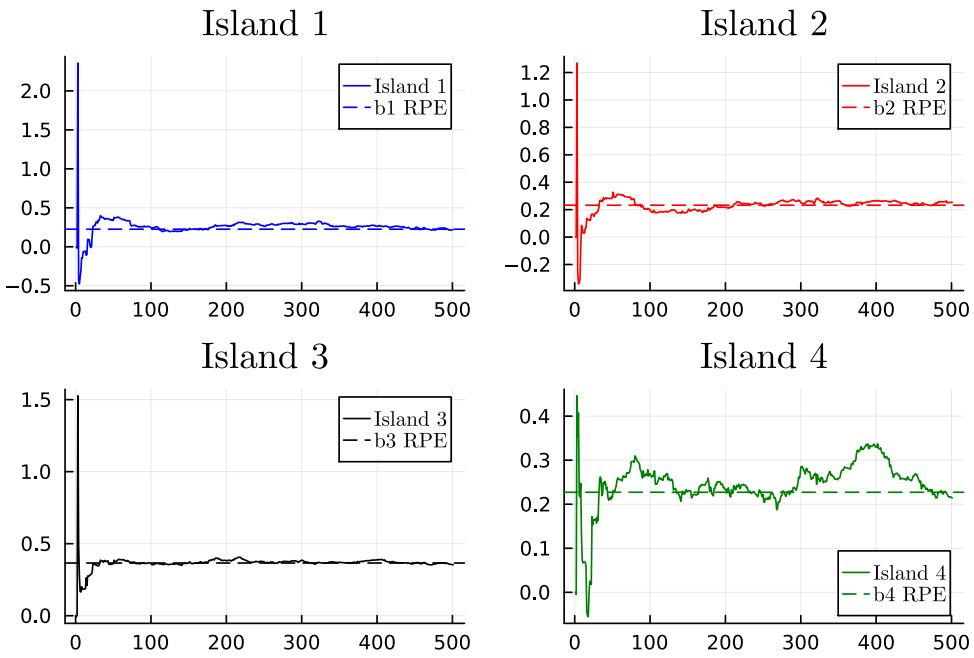


Figure 1.3. Beliefs held by agents on each island under private information.

We see significant detours from the equilibrium across all islands, notably by firms on Island 4, though all other islands appear to safely converge to their RPE values quickly.

1.3.4 Comparing Parameter Effects of Weather, β_j

By defining β_j as in *Proposition 1.4*, we can see how the parameter effects of weather on price change by information set-up. These parameter effects are shown in Table 1.3.

Info:	β_1	β_2	β_3	β_4
Full	0.06667	0.16667	0.22222	0.21111
Shared	0.06368	0.17931	0.23589	0.21912
Private	0.08872	0.22094	0.27237	0.28069

Table 1.3. β_j represents parameters effect of w^j_{t-1} on p_t

Except for β_1 in the Shared Private Information case, all β_j's in the Shared and Private information cases are greater than the respective β_j in the Full information case. This implies that $\partial p_t / \partial w^j_{t-1}$ is generally greater in the cases of incomplete information, meaning these economies will experience greater price volatility. As we demonstrate later, this is indeed the case.

1.3.5 Half-Life Estimation and Convergence Speed

We can further compare these information cases by considering the estimated half-lives of the beliefs as they converge to the equilibrium values, using the following method:

$$\text{Estimate AR(1) Model: } b^j_{k,\,t} = \lambda_0 + \lambda_1 b^j_{k,\,t-1} + \text{error}_t$$
$$\text{Estimated Half Life of } b^j_k = -\frac{ln(2)}{ln(|\lambda_1|)}$$

We use this as a measure of convergence speed. There is an inverse relationship between convergence speed and half-life, as a lower half-life implies beliefs a shorter time lapse between beliefs far from equilibrium to those near equilibrium. In this and the following tables, each row represents the half-life of beliefs held by agents on island j, and each column represents the half-life of beliefs regarding the weather on island k.

	b_1	b_2	b_3	b_4
b^1	0.914775	1.7833	1.39325	0.716026
b^2	0.876984	1.74235	1.4186	0.717714
b^3	0.971739	1.75518	1.42806	0.713732
b^4	0.885502	1.79308	1.38088	0.725675

Table 1.4. Estimated Half-Lives of b^j_k in the full information case as beliefs approach the REE.

In Table 1.4, we see a clear pattern: beliefs regarding any given island converge at similar rates across all islands. This can be seen by the low apparent variance in each column. The discrepancies arise because of heterogeneous initial conditions.

The half-lives of beliefs in the shared private information case are reported in Table 1.5.

Here, there is less of a discernible pattern across or between islands' beliefs. There is wide variance across rows and columns, suggesting little agreement between islands regarding the weather on any given island. We should also note that certain beliefs converge faster under shared private information than under full information, such as all of the beliefs held by firms on Island 1. Therefore, we cannot conclude that convergence speed monotonically increases with the avail-

	b_1	b_2	b_3	b_4
b^1	0.34672	0.17440	0.79059	0
b^2	4.35986	1.43702	0	3.46856
b^3	2.96341	0	0.91159	1.90139
b^4	0	1.73434	1.77267	0.94701

Table 1.5. Estimated Half-Lives of b_k^j in shared private information case as beliefs approach the RPE.

ability of information.

We also estimate the half-lives of beliefs in the private information case.

	b_1	b_2	b_3	b_4
b^1	0.41723	0	0	0
b^2	0	0.70002	0	0
b^3	0	0	0.33395	0
b^4	0	0	0	4.77650

Table 1.6. Estimated Half-Lives of b_k^j in private information case as beliefs approach the RPE.

Here, we see a relatively quick convergence of beliefs by islands 1, 2, and 3, with island 4 once again as a notable outlier.

Simply by looking at the tables in this example, there appears to be no clear advantage to full information when it comes to convergence speed. A further and broader inquiry into these models is needed to uncover possible advantages of full information.

1.3.6 Price Variance and Economic Volatility

Another variable of interest is price. The relationship between information availability and economic volatility can be measured by observing the market price in each simulated economy.

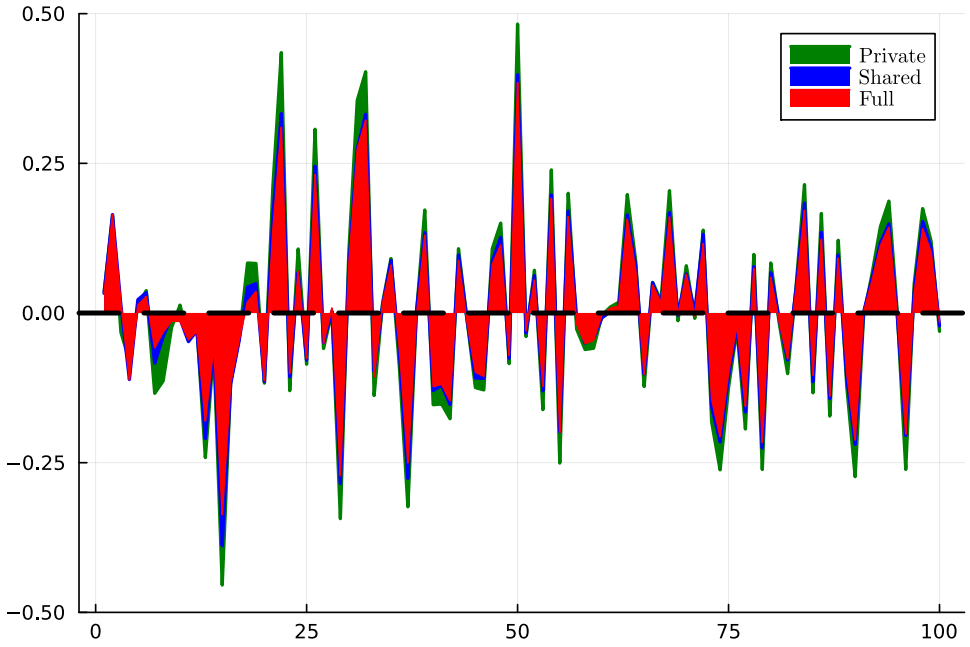


Figure 1.4. Market price given identical weather.

Price Variances: Full Info = 0.01836, Shared Info = 0.02039, Private Info = 0.02961

It is clear that economies follow similar trends throughout their existence, as is expected given identical weather patterns. However, it is also clear that the price under private information is more sensitive to weather shocks than the price under full information, as can be seen by the magnitude of spikes. The same is true for the price under shared private information appearing situated between the two extremes: more volatile than the price under full information, and less than the price under private information. This implies a negative relationship between economic volatility and information availability.

While this example is demonstrative of the differences between full, shared, and private information, it is not a comprehensive look into this model. A more intensive simulation is needed to probe the relationships between convergence speed and economic volatility in this model.

1.4 The Simulation

To simulate many static information network cobweb models of size J, each iteration begins with a random vector that determines θ. Each value of θ is drawn from a uniform $(0, 1)$ distribution

and normalized by the sum of those values such that all elements of $\theta \in (0,1)$ and $\sum_j \theta_j = 1$. Additionally, each iteration has a variance-covariance matrix, Σ, that determines the weather on the islands. $\Sigma = \mathcal{S} \times \mathcal{S}'$ generates Σ where $\mathcal{S}$ is a $J \times J$ random matrix with *i.i.d.* elements drawn from a normal distribution $N(0,1)$. Therefore, each period t, the $J \times 1$ vector of weather is drawn from the described distribution: $w_t \sim N(0_{J\times 1}, \Sigma)\ \forall t$.

Within each iteration, N *sub*iterations are simulated with different network formations to ensure that the differences between networks are driven by their varied formations and not by θ or Σ. Each network's adjacency matrix is simulated randomly according to an Erdős–Rényi (ER) random graph, where each connection between islands has a fixed and *i.i.d.* probability p of occurring or not. The number of neighbors any island has is therefore distributed according to a binomial distribution with $J-1$ trials and probability p. However, since an ER random graph M does not ensure $M_{j,j} = 1\ \forall j$—as is required by this model—the network is adjusted such that $M_{j,j} = 1\ \forall j$. While the ER model can have low clustering, the benefit of generating networks with this model is the wide variance in the number of nodes and resulting degrees generated, and it provides a useful baseline case.

Within each subiteration, a cobweb model is run in the same manner as in section **1.3**, once with the ER random graph, and again with a full information economy given identical weather. Using the generated θ, Σ, and M, the adaptive learning cobweb model is simulated for T periods. Data is collected on the centrality of nodes within each network regarding the eigenvector, betweenness, closeness, and degree centrality. Additionally, a weighted variant of degree centrality concerning θ is measured. These allow us to calculate correlations between individual islands' centrality within the network and variables of interest.

Eigenvector Centrality is a measure of a node's influence on the broader network as a result of the eigenvalue corresponding to each node. (Bonacich & Roy, 1986)

Betweenness Centrality, is a measure of how often each specific node occurs in the shortest path between any two other nodes chosen at random. (Brandes, 2001)

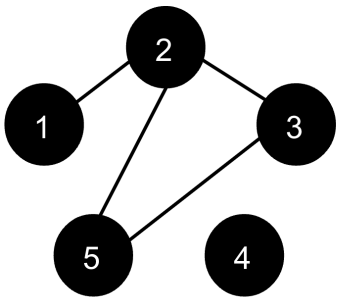
Closeness Centrality computes the average length of the shortest path between the node of

interest and all other nodes in the network. (Bavelas, 1950)

Degree Centrality measures the number of links a node has relative to other nodes (Metcalf & Casey 2016).

Weighted Degree Centrality (W.D.C.) is the normalized product of Degree Centrality and node-individual weights, which in this model are determined by θ. This provides an idea of the importance of islands within the context of the model devoid of any bias that θ may introduce.

For example, consider the following network, assuming self-connectedness:



$$M = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Figure 1.5. Example network and adjacency matrix with $J = 5$.

The centrality measures for the nodes in this network are as follows:

Node	Eigenvector	Betweenness	Closeness	Degree	W.D.C.
1	0.281845	0.0	0.45	0.5	0.5 (θ_1/δ)
2	0.611628	0.333333	0.75	1.0	1.0 (θ_2/δ)
3	0.522721	0.0	0.5625	0.75	0.75 (θ_3/δ)
4	0.0	0.0	0.0	0.25	0.25 (θ_4/δ)
5	0.522721	0.0	0.5625	0.75	0.75 (θ_5/δ)

Table 1.7. Centrality measures for the network in Figure 1.5.
$\delta = 0.5\theta_1 + \theta_2 + 0.75\theta_3 + 0.25\theta_4 + 0.75\theta_5$

In this example, it is clear from the network and adjacency matrix that **Node 4** is the least connected, as its only connection is to itself. All of the centrality measures rank **4** in last, or tied for last. Additionally, **Node 2** appears the most connected and central—this is reflected by all of the centrality measures ranking **2** highest.

Additionally, a numerical measurement of the aggregate network is considered. This is a normalized Linear Connectiveness Measure (LCM) that sets the least connected network (an identity adjacency matrix) to 0, and the fully connected network ($M = 1_{J\times J}$) to 1. The following equation achieves this:

$$LCM(M_{J\times J}) = \frac{\left(\sum_j \sum_k M_{j,k}\right) - J}{J^2 - J}$$

This can be interpreted to represent how similar any given network is to the fully connected (full information) case. One may notice that this measurement is proportional to the average degree of the network—i.e. the average number of neighbors each island has within a network. This measurement is used instead due to its interpretability and normalization.

In each cobweb simulation, data is collected regarding the volatility of the economy, the price prediction errors of each island, and the speed of convergence of beliefs. The volatility is measured as the variance in the price in the last 100 periods. The price prediction error is the difference between the realized price and the islands' expected price. The speed of convergence of beliefs is a measure of the half-life of beliefs. Finally, we look at how the centrality measures of individual nodes and how the LCM of the full network correlate with the variables of interest.

1.4.1 Simulation Results

For each network size J, a total of $\mathcal{N}$ iterations are run, each with N subiterations, and each subiteration for T periods.

In the results below, $\mathcal{N} = 1000$, $N = 100$, $T = 500$, $p = 0.5$ for the following sizes of networks: J = 5 and 10. Additionally, we use the following parameters for all cobweb simulations: $\alpha = -0.5$, $\gamma = 1$, $\tilde{\epsilon} \sim N(0, 0.01)$, $g = 0.01$.

Economic Volatility

1.6 (a) and 1.7 (a) are density plots of the price variance separated by whether or not that economy has complete or incomplete information. The difference between the densities shows that complete information economies experience less volatile prices, though the difference ap-

pears minor. 1.6 (b) and 1.7 (b) show the density of the differences *between* price variances of incomplete and full information economies *given* the same weather. Here, the advantage of full information economies becomes far clearer: when facing identical exogenous shocks, the price variance in incomplete information economies is *always* higher than in full information economies.

1.6 (c) and 1.7 (c) give the density for the correlation statistic between the LCM and the price variance of an incomplete information economy. These density plots support the narrative that the more connected a network economy becomes, the less economic volatility is experienced.

1.6 (d) and 1.7 (d) concern the correlation statistics between individual islands' centrality measure and the variance of that islands' price prediction error. These plots are all centered near 0. This suggests that there is no advantage regarding price prediction for islands that are more central to the network. However, this assumes erroneous price prediction errors would stray from 0 in negative and positive directions, meaning it does not rule out systemic under or overestimation.

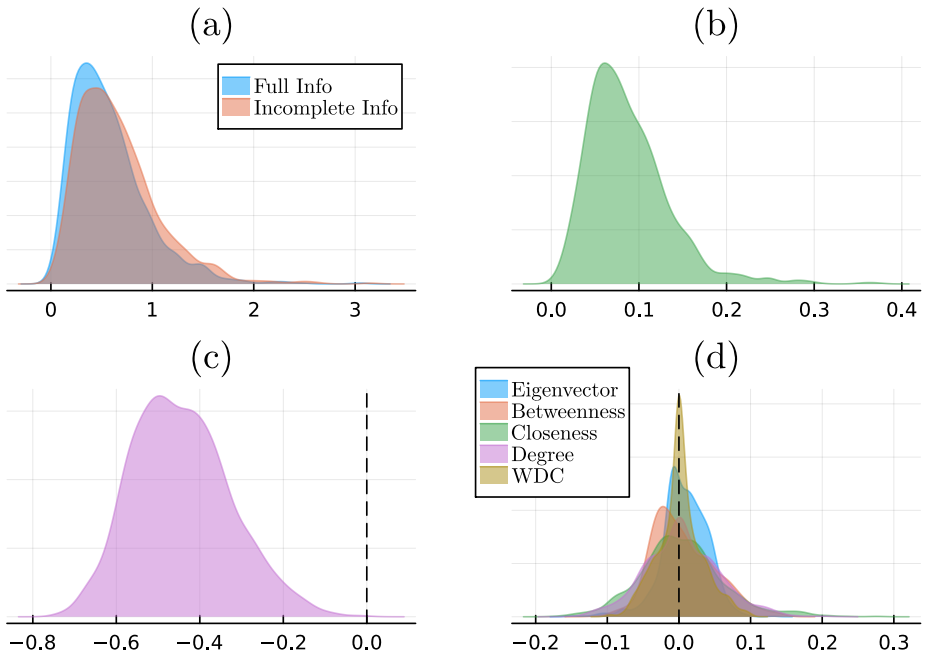


Figure 1.6. Statistics regarding prices in economies with 5 islands

(a) Price variance in last 100 periods

(b) Difference between incomplete and full info price variance given same weather

(c) Correlation statistic between LCM and price variance

(d) Correlation statistic between islands' centrality measure and price prediction error variance

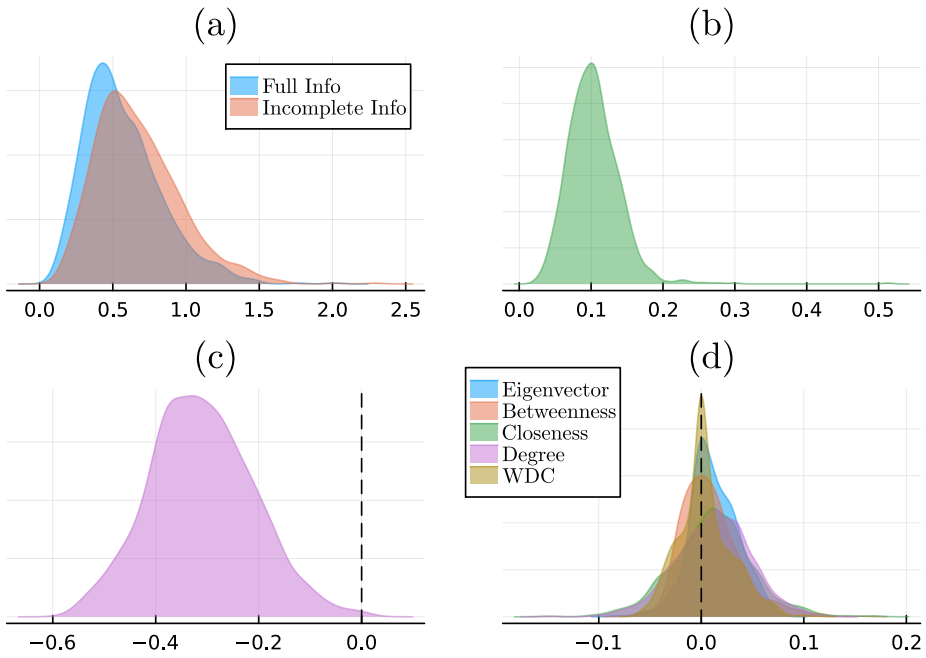


Figure 1.7. Statistics regarding prices in economies with 10 islands

(a), (b), (c), (d), see Figure 1.6.

Speed of Convergence

1.8 (a) and 1.9 (a) show the density of the average half-life of beliefs in economies with full and incomplete information. It is clear that beliefs converge faster in full information economies, and the variance of belief half-lives is smaller as well. This narrative is more pronounced in 1.8 (b) and 1.9 (b) which show the difference between the average half-life of an incomplete economy and a full information economy given the same weather. Given identical exogenous shocks, the average half-life of beliefs is around 1.6 periods *shorter* in full information economies than in incomplete information network economies with 5 islands, and around 1.9 periods shorter in 10 island economies.

1.8 (c) and 1.9 (c) report the distribution of correlation statistics between the average half-life of beliefs in an economy and the LCM of that economy's network. The distribution's negative bias further shows that the more connected a network economy is, the faster beliefs converge. 1.8 (d) and 1.9 (d) demonstrate this as well for individual islands, showing the correlation between islands' centrality measures and islands' average half-life of beliefs. This indicates a clear advan-

tage for islands' centrality regarding the speed of convergence to equilibrium.

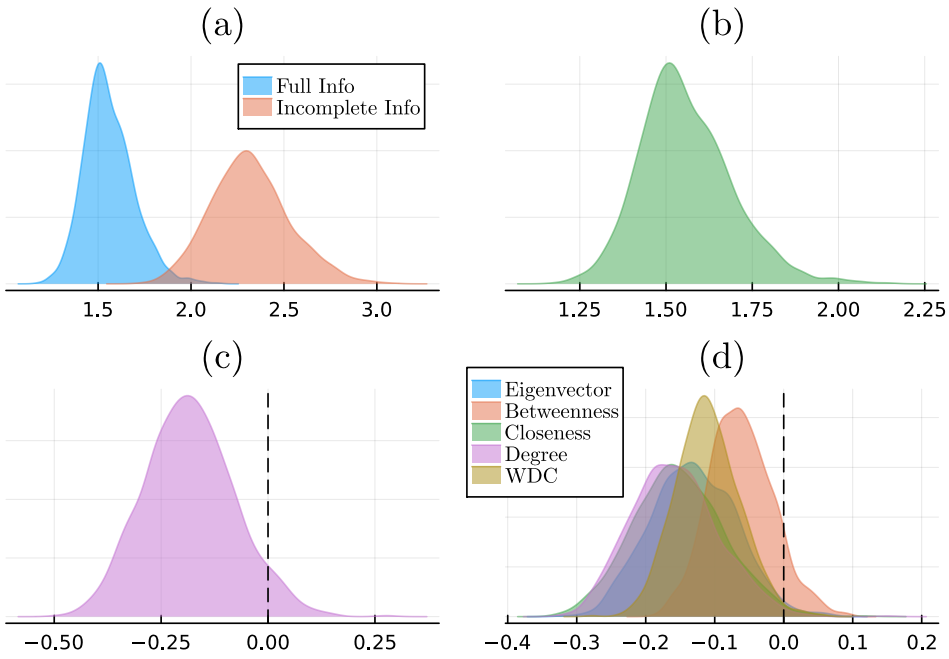


Figure 1.8. Statistics regarding the speed of convergence in economies with 5 islands

(a) Average belief half-life

(b) Difference between incomplete and full info average half-life given same weather

(c) Correlation statistic between LCM and average half-life

(d) Correlation statistic between islands' centrality measure and islands' average half-life

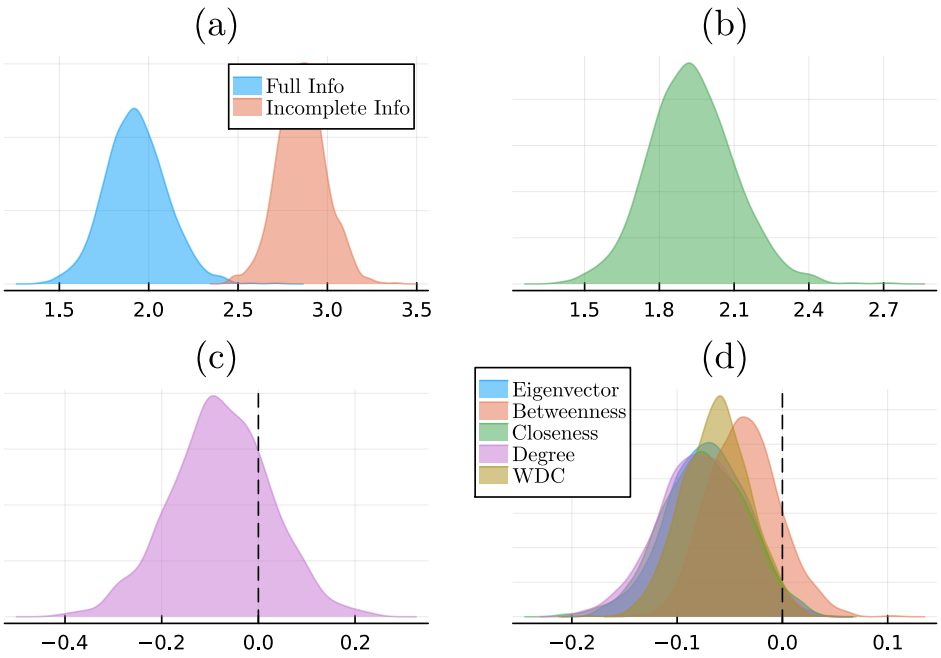


Figure 1.9. Statistics regarding the speed of convergence in economies with 10 islands (a), (b), (c), (d), see Figure 1.8.

1.4.2 Discussion

Through a brute force computationally intensive method, we uncover the consequences of this model. Common themes across the results are advantages full information economies have over those with incomplete information. This result is consistent with much of the previous literature on the subject, as discussed in Section **1.1**. This Chapter makes contributions to this hypothesis regarding the topics of economic volatility and speed of convergence to equilibrium, which we show are, respectively, lower and faster under full information. These results also confirm that as networks become more connected, economic volatility decreases, and the rate of convergence increases.

There appears to be no systematic correlation between the variance of the expectational errors individual islands make and that islands' "centrality," as measured by multiple centrality measures. This implies a *shared disadvantage* across all agents under incomplete information. These results also provide a counter-intuitive result that islands that are more "central" to the net-

work do not experience a systematic advantage regarding predicting future prices under incomplete information. This phenomenon may be explained by the fact that as islands become more central, other islands become less relatively central—meaning these centrally located islands exist in an economy with less informed agents and the central agents are not accounting for the uninformed decision-making of the "outsider" agents. Further inquiries into these models should investigate this phenomenon and discover if the relationships found hold for larger networks.

A clear advantage exists for more central islands when it comes to the speed at which their beliefs converge to the RPE. However, since this advantage does not appear to induce more accurate price predictions, it can be questioned how powerful of an advantage convergence speed provides. Additionally, the WDC measure appears to give results closer to 0 than the unweighted degree centrality in all cases, meaning apparent benefits afforded to islands may be driven by θ. Though, the WDC is still safely biased from 0 concerning the speed of convergence in 1.8 (d) and 1.9 (d).

Considering the differences between the plots with 5 or 10 islands, we see that the mean correlation between the LCM and economic volatility or speed of convergence is closer to 0 in the 10 islands simulations. This can be interpreted as smaller networks experiencing greater gain when connections are added as opposed to larger networks. This intuitively follows from the fact that 1 additional connection in a 5 island economy increases the LCM by $1/10^{\text{th}}$, but only by $1/45^{\text{th}}$ in a 10 island economy, meaning the discrete nature of these networks is likely responsible for this difference.

Additionally, regarding convergence, there is a larger distinction between the half-lives of beliefs in 10 island economies than in 5 island economies. This is also reflected in 1.8 (b) and 1.9 (b) as the latter appears centered around a greater mean than the former. This means that the difference between convergence speed in incomplete and full information economies *increases* in larger economies. This result may be motivated by the generation process for networks chosen, as a 5 node ER random graph with $p = 0.5$ is more likely to yield all possible graphs after 1000 generations than a 10 node ER random graph. This would mean that high LCM's are less likely

in a 10 node ER random graph, therefore the sample of networks will be farther from the full information case.

Further research should apply shared private information in static networks to other model settings, such as the various DSGE models discussed in Section **1.1**. Additionally, researchers can relax many of the initial assumptions in this Chapter to investigate the ways that information networks affect economies. Such as relaxing the assumption of a static network and allow for the endogenous evolution of the networks, the assumption of undirected graphs networks (allowing for asymmetric M's), or the assumption of perfectly observed information—i.e. the modeler allows for information interpretation errors, as in the real world, one's view of others' information is far noisier than that of their own. Whether these errors are white noise or systematically biased is up to the modeler, though we believe both are worthwhile inquiries.

One possibly overlooked part of this Chapter, section **1.2.5**, has broader econometric applications regarding omitted variable bias. OVB, as typically taught and discussed, simply concerns the case of two correlated variables and the omission of one variable from a regression. However, as expected, OVB occurs when variables are omitted from a regression where more than two correlated variables construct the endogenous variable. The bias of the regression parameters is more difficult to calculate (or sign) than the elementary two-variable case. The technique discussed in **1.2.5** and *Proposition 1.4.* can be applied to the general OVB case to solve for the expected value and bias of regression estimates.

1.5 Conclusion

This Chapter calls attention to a void in economic theory regarding implementing information networks into dynamics, mathematical economic models. We explore the concept of "shared private information" in a cobweb model of production that yields useful insights into how the sharing and spread of information affects economies. This Chapter successfully marries graph theory and adaptive learning, two camps of mathematics for which we have reason to believe overlap exists in the real world. The mathematical model in this Chapter provides a useful baseline for

the incorporation of information network graphs into economic models. Another advantage of this model is its ability to easily investigate private and full information cases simply by changing the adjacency matrix. The shared use of linear algebra in economics and graph theory allows for smoother notation and simpler equations than an uninformed researcher beginning this inquiry might expect.

Economic volatility, as measured by the market price variance, is lower in the full information case than in the incomplete information case, demonstrating the importance of information availability in economic dynamics. Additionally, we demonstrate a shared disadvantage across all agents under incomplete information regarding price prediction errors, as these errors appear uncorrelated with firms' relative availability of information (centrality). The speed of convergence of beliefs, as measured by the estimated half-life of beliefs over time, increases the more connected an island becomes. It is also true that the more central an island is, the faster that island's beliefs converge to the RPE. Therefore, full-information Rational Expectations economies converge faster and are more stable than shared private information network economies under adaptive learning.

Chapter II: Spatial Interactions of Expectations with Applications to Monetary Policy

2.1 Introduction

General economic regional heterogeneity leads us to expect asymmetric responses to monetary policy across regions, and the empirical literature supports this view (Dominguez-Torres & Hierro, 2019). Empirical and theoretical economics have well-established the existence of heterogeneous expectations and monetary policy responses across regions. This Chapter proposes expectations as a plausible channel for regional variation of monetary policy responses, asking the following question: What role do expectations play in the effects of monetary shocks on a network economy?

The process by which a monetary shock ripples through the economy remains elusive. In a series of papers, Anthonisen (2010, 2013, 2018) explores an overlapping generations (OLG) model within the context of a network where agents buy from, and sell to, agents in nodes adjacent to themselves. The model tracks how the effects of monetary policy ripple through markets when one area of the economy experiences a positive money supply shock. The model finds real effects of monetary policy on output, though one that runs counter to most classical economic models. An expansion of the money supply to the economy in steady-state *decreases* short-run output; expansionary monetary policy has a contractionary effect.

The Anthonisen OLG model has yet to be deeply explored, and as this Chapter shows, the model has rich potential. This Chapter contributes to the monetary policy and OLG literature by digging into this model, subverting many of the assumptions made in the model's inaugural paper. Changing the assumption of rational expectations allows us to explore the role that expectations play in this OLG model, and how expectations affect the economy's response to monetary policy. We consider how these responses relate to welfare, as well as many different variations of the network structure, and how heterogeneous expectations affect the economic response to monetary policy within this model.

Anthonisen (2010) considers a "helicopter money" monetary expansion, which here refers

to a one-time permanent increase in the money supply—as if currency were dropped from a helicopter. The term, originally coined by Milton Friedman (Friedman, 1969) as a thought experiment for how the money supply affects the economy, has long been compared to actual economic policies. Reis & Tenreyro (2022) discusses cases where helicopter money may be an effective economic policy, such as for meeting an inflation target in a near-zero lower bound economic state.

Reis & Tenreyro (2022) considers the conditions under which the effects of helicopter money are neutral, i.e. having no effect on real output, in a stylized model: if (1) demand is satiated, (2) conventional monetary policy keeps inflation steady, and (3) there is no redistribution of permanent income across agents. While there is no long-run redistribution in the Anthonisen model, there is a short-to-medium run redistribution effect that is partly responsible for the real effect of the money supply. The second condition is also not met, and as is later argued, the dramatic rise in prices relative to money holdings is the progenitor of the output contraction. Though, as this Chapter discusses, the change in prices is a result of agents' expectations.

All previous uses of the network economy in Anthonisen (2010) have employed perfect foresight rational expectations. Mandel, Taghawi-Nejad, & Veetil (2019) recognizes the unrealistic and remarkable feats that the rational expectations assumption requires in a network economy. The authors consider monetary shocks to a subset of the population in a production-network economy, with firms that do not anticipate future changes to prices or demand, and short-sighted expectations based on local information. These shocks result in uneven price effects throughout the economy, and there is a similar "wrong" directional response, this time in prices.

The terms "expansionary" and "contractionary" are used to describe certain fiscal and monetary policies because of their commonly understood effects on the economy. Monetary policies such as buying bonds, or fiscal policies such as lowering tax rates, are labeled as expansionary because these policies are understood to stimulate spending in the economy; increasing output and thereby *expanding* the economy. Contractionary policies do the opposite. However, this narrative has been challenged by theoretical and empirical economists who propose that under cer-

tain circumstances, what is commonly called "expansionary policy" may decrease output, yielding *contractionary* effects.

Widely cited literature reviews on this topic reveal the historical consensus: expansionary policies are expansionary, and contractionary policies are contractionary. One of these, Ramey (2019), reviews the recent literature regarding fiscal multipliers and refines previous estimates. While the newer estimates are smaller in absolute value, the analysis still safely rejects any negative effects of expansionary fiscal policy. Alessandri, Venditti, & Jordà (2023) estimates the monetary multiplier, the effect of monetary policy on employment and credit, and find estimates in the same direction—though noting asymmetric effects for positive vs. negative shocks.

However, the Anthonisen Model does not stand alone in producing theoretical results that run counter to the common narrative around expansionary and contractionary macroeconomic policy. Ahtiala (1998) finds that depending on assumptions regarding countries' trade balances, the direct effect of expansionary monetary policy may lower output in the short and long run. On the flip side, Giavazzi & Pagano (1990) showcases two empirical examples of when contractionary macroeconomic policies were followed by GDP increases during the late 20th century Europe. Makin (1998) provides theoretical examples of when contractionary policy can yield expansionary results. The empirical evidence in Ilzetzki, Mendoza, & Végh (2013) suggests that in countries with a high debt-to-GDP ratio, expansionary government spending has negative effects on output. Feasibly counterintuitive—in the sense of running counter to prevailing economic models—effects of macroeconomic policy have been considered for decades.

The Anthonisen Model and the open-economy model in Ahtiala (1998) employ rational expectations and never violate that assumption. Challenges to the rational expectations hypothesis have been proposed, namely considerations of bounded rationality and adaptive learning. At least since the late 1990s, expectations of economic agents have been considered in macroeconomic policy. Brayton, Mauskopf, Reifschneider, & Tinsley (1997) discusses the vital role that expectations play in the Federal Reserve System United States economic model that the U.S. Central Bank began using around that time. Expectations continue to be frequently cited during Federal

Reserve speeches (Federal Reserve, 2024) and Open Market Committee minutes (Board of Governors of the Federal Reserve System, 2024). Given the important role that expectations have in macroeconomic policy, convenient assumptions of rationality may lead policymakers astray. The model in Hommes, Massaro, Assenza, & Anufriev (2013) makes the case for more aggressive monetary policy under heterogeneous expectations than under baseline homogeneous expectations, meaning heterogeneous expectations also have important implications for monetary policy.

Heterogeneity in economic responses to monetary policy across space is a well-documented empirical phenomenon. Carlino & DeFina (1998) finds different effects at the regional level in the U.S., and Di Giacinto (2003) expands this analysis to look at each state with similar results. Furceri, Mazzola, & Pizzuto (2019) finds evidence that these asymmetric responses across the U.S. are magnified by spatial correlations. Beraja, Fuster, Hurst, & Vavra, J. (2017) proposes one possible mechanism for these varying responses: heterogeneity in housing equity. Their empirical evidence finds that this heterogeneity explains some of the spatial heterogeneous effects of monetary policy in response to the financial crisis. This phenomenon is not exclusive to the United States. The analysis in Fraser, Macdonald, & Mullineux (2014) looks at Australian states, and finds heterogeneous responses. Chapter III of this dissertation discusses evidence of heterogeneous responses to monetary policy in Europe.

Expectation heterogeneity across countries has been repeatedly confirmed in Europe using the expectation survey data in the European Commission Business and Consumer Survey. Chapter III discusses this survey in more detail as well. In short, economic expectations vary widely across countries.

Optimal monetary policy rules have been found to vary depending on the expectation formation scheme employed by agents in an economy. Gaspar, Smets, & Vestin (2010) finds differences in optimal monetary policy under rational expectations and expectations formed by adaptive learning—using least squares equations to form expectations. A corollary of this result is that economic responses to monetary policy depend on how agents form their expectations. Evans & Honkapohja (2009) argues that monetary policy needs to react more aggressively to infla-

tion when there are persistent deviations from rational expectations; i.e. economic responses are weaker when the central bank's policies are optimal under rational expectations, yet the expectations held by agents in the economy are persistently not rational. These and similar papers show that expectations add a layer of complexity for policymakers.

A line of logic emerges from this literature review that launches into this Chapter's inquiry:

1. Heterogeneous responses to monetary policy occur across geographic regions.
2. People in these countries have different expectations.
3. Expectations play an important role in the economic response to monetary policy.

Therefore, expectations may be a motivating factor behind heterogeneous responses to monetary policy. Knowing how expectations affect responses to monetary policy is important for policymakers seeking to effectively implement monetary policy towards given ends.

We find that economic responses to monetary policy, on the output and inflation sides, depend on whether expectations are "*forward*" or "*backward*-looking." Having backward-looking expectations means one forms their expectations for the future using past information, whereas forward-looking expectations are less myopic, and are less anticipatory of variations in trends. The theoretical model shows this difference explicitly in the homogeneous expectations case. We consider other aspects within this model, as well, such as various spatial organizations and heterogeneous expectations; across these variations, a common theme holds: the larger the portion of economic agents who are backward-looking, the *larger* the effect of monetary policy on output, and the *smaller* the effect on inflation. When considering how these responses relate to welfare, we find that the welfare loss is greater under backward-looking expectations, despite the increase in output.

The contributions of this paper are beyond those considered in other inquiries regarding monetary policy in network economies—such as Anthonisen (2010, 2013, 2018), Mandel, Taghawi-Nejad, & Veetil (2019), or Mandel & Veetil (2021)—due to our focus on the role of expectations. Consideration of heterogeneous expectations and the organization of such agents is original, along with consideration of Pareto efficiency, convergence as it relates to network structure,

and expectational stability in a monetary network economy. These inquiries greatly expand upon monetary policy in network economies, and provide valuable insights for the field.

In Anthonisen (2010), rational expectations are assumed in the network economy, meaning that while the actions of agents are reasonably restricted to a network, perfect foresight is still granted. In these economies, rational expectations are a far stronger assumption than in standard economic models. As a result of the utility function used in the model, the transfer of money is not dependent on prices. Therefore, any changes to how agents form expectations will not affect the flow of money. Adaptive expectations will, however, affect prices and output.

In the following section, we consider changes to agents' expectations in the Anthonisen OLG Model and provide further motivation for variation in responses to monetary policy based on expectations. Section **2.2** also briefly discusses the welfare implications of monetary policy in this model. Section **2.3** explores monetary policy through simulating the model, and also considers Pareto inefficiencies caused by the monetary expansion. Section **2.4** investigates how network structures affect the economy's steady state and convergence to equilibrium after a monetary shock. The heterogeneous expectations case is considered in Section **2.5**, joining rational and adaptive expectations agents in the spatial OLG model.

2.2 An Overlapping Generations Network Model

2.2.1 OLG Model Set Up

In the Anthonisen OLG Model, agents produce, sell, and buy goods within a network.

Let $\Gamma(V,E)$ be a fixed graph with the set of V vertices and E edges. Denote the vertices/nodes of the graph as $j \in V$ where $V = \{1,...,J\}$ where $J = |V|$ is the cardinality of the set V. Let $N(j)$ denote the set of *neighboring* agents to vertex j. A vertex i is in the set $N(j)$ if there exists an edge $\{j,i\} \in E$ connecting j and i. Let $\eta^j = |N(j)|$ denote the number of vertices neighboring j. $\Gamma(V,E)$ has a corresponding adjacency matrix A_Γ. Elements $a_{i,j}$ of A_Γ equal 1 if there exists an edge $\{j,i\} \in E$ connecting i and j, and $a_{i,j} = 0$ otherwise. All networks considered in this Chapter are *undirected*, meaning $A_\Gamma = A'_\Gamma$.

Agents live for two periods; they are considered "young" in the first and "old" in the second. When young, agents produce and sell their produced goods to neighboring old agents in exchange for fiat currency. The received currency becomes income the young agents carry into the next period. When old, agents buy (and instantly consume) goods from neighboring young agents. The set of neighbors agents sell to, and the set of neighbors agents buy from, are identical, fixed, and exogenously imposed by the graph Γ. Agents born in period t at vertex j have the following expected lifetime utility function, as originally in Anthonisen (2010):

$$E_t^j \ \eta^j \left(\prod_{i \in N(j)} c_{t+1}(j,i) \right)^{1/\eta^j} - L_t(j)^\alpha \tag{2.1}$$

Where $L(j)$ is the quantity of labor supplied when young, with $\alpha > 1$. $c_{t+1}(j,i)$ is the quantity of the consumption good purchased and consumed from neighbor i in $t+1$ by old agents at j.

A young agent produces according to:

$$q_t(j) = L_t(j) \tag{2.2}$$

Let $m_t(j)$ represent the number of units of currency held by the old agent at vertex j at the outset of period t, and $m_t = \{m_t(j) \mid j \in V\} \in \mathbb{R}^J$. The aggregate money supply is constant unless otherwise stated. In all periods except the first, the cash balances held by the old agent at $j \in V$ are obtained as the proceeds from the sale of their output in the previous period:

$$m_{t+1}(j) = p_t(j) \ q_t(j) \tag{2.3}$$

Where $p_t(j)$ is the price of the good produced at j in t. The budget constraint faced by the old agent at j in period $t+1$ is then:

$$\sum_{i \in N(j)} p_{t+1}(i) \ c_{t+1}(j,i) = m_{t+1}(j) \tag{2.4}$$

Maximizing (2.1) subject to (2.2 - 2.4) yields the decisions:

$$q_t(j) = \left(\frac{p_t(j)}{\alpha E_t^j \prod_{i \in N(j)} p_{t+1}(i)^{1/\eta^j}} \right)^{1/(\alpha-1)} \tag{2.5a}$$

$$c_{t+1}(j,i) = \frac{m_{t+1}(j)}{\eta^j p_{t+1}(i)} \quad \forall i \in N(j) \tag{2.5b}$$

As we can see in (2.5b), the Cobb-Douglas preferences cause agents to split their money holdings into η^j equal parts, and spending each part on each neighbor, exhausting their income.

The market-clearing condition for goods at vertex j is

$$q_t(j) = \sum_{i \in N(j)} \frac{m_t(i)}{\eta^i \, p_t(j)} \tag{2.6}$$

(2.6) and (2.5a) yield the market-clearing price:

$$p_t(j) = \left(\alpha E_t^j \prod_{i \in N(j)} p_{t+1}(i)^{1/\eta^j} \right)^{1/\alpha} \left(\sum_{i \in N(j)} \frac{m_t(i)}{\eta^i} \right)^{(\alpha-1)/\alpha} \tag{2.7}$$

$$p_t(j) \equiv \mathcal{P}^j \left(\{ E_t^j p_{t+1}(i), m_t(i) | i \in N(j) \} \right)$$

The market clearing quantity is then:

$$q_t(j) = \left(\alpha E_t^j \prod_{i \in N(j)} p_{t+1}(i)^{1/\eta^j} \right)^{-1/\alpha} \left(\sum_{i \in N(j)} \frac{m_t(i)}{\eta^i} \right)^{1/\alpha} \tag{2.8}$$

$$q_t(j) \equiv \mathcal{Q}^j \left(\{ E_t^j p_{t+1}(i), m_t(i) | i \in N(j) \} \right)$$

The distribution of money holdings over time is independent of prices. Given a graph $\Gamma(V, E)$ with adjacency matrix A_Γ with a common number of neighbors ($\eta^* = \eta^j = \eta^i \; \forall \; i, j$), the flow of money is then determined by:

$$m_{t+1} = \frac{1}{\eta^*} A_\Gamma m_t \tag{2.9}$$

With the exception of Section **2.4**, the remainder of this Chapter considers *k-regular* graphs, where all vertices have the same number of neighbors: $\eta^* > 1$. The steady-state prices and quantities at each vertex (when $\bar{m} = M/J$ units of currency is held at each vertex, with M being the aggregate money supply) are:

$$\bar{p}(j) = \bar{m} \cdot \alpha^{\frac{1}{\alpha-1}}, \;\; \bar{q}(j) = \alpha^{\frac{1}{1-\alpha}}, \;\; \forall j \in V$$

Under rational expectations, prices in this model can be written in vector form by iterating a logged version of (2.7) forward. The anticipated prices disappear from the equation in the limit, and we are left only with constants and information regarding the future path of money holdings:

$$\ln(p_t^{RE}) = \left(\frac{1}{\alpha-1}\right)\ln(\alpha)1_{J\times 1} + \left(\frac{\alpha-1}{\alpha}\right)\sum_{s=0}^{\infty}\frac{1}{\alpha^s}(A')^s \ln\left(A^s m_{t+1}\right) \tag{2.10}$$

As in **Appendix A** of Anthonisen (2010)[7], where $A = (1/\eta^*)A_\Gamma$, and p_t and m_{t+1} represent the $J \times 1$ vectors of agents' prices and money holdings, respectively. This yields the unique bounded equilibrium prices under rational expectations.

The long-run dynamics of (2.9) tell us that—since the eigenvalues of A fall within the unit circle—absent exogenous monetary changes, money holdings eventually equate across the economy. Therefore, absent such changes, the economy returns to its steady-state quantities, which do not depend on the money supply.

2.2.2 Finite-Memory Forecasting Learning

When considering different ways economic agents may form their expectations within a model, the concept of "learning" typically appears. When economic agents "learn" it means they form expectations using data from the economy, and this learning is "adaptive" if agents consistently update their beliefs. Economic agents possess a belief about how variables are related within the economy, what is called their "perceived law of motion" (PLM). Under Adaptive Learning, these

[7]One adjustment made is A' instead of A in the summation term. In a k-regular graph, $A' = A$, but when we later consider other types of graphs, this distinction will become important.

PLMs are estimated statistically and consistently updated as new data appears. Since these expectations factor back into how economic variables are actually determined, the resulting *actual* law of motion (ALM) of the economy depends on the PLM.

Many forms of PLMs exist and have been widely considered. The common recursive-least-squares method in *Learning and Expectations* of regressing data on a slope and intercept will not work within the Anthonisen model since it is nonstochastic—there are no random variables within the model. The implication of this is that in the steady-state, asymptotically, regressors exhibit perfect multicollinearity, violating the assumptions of least-squares analysis.

Since the Anthonisen model is nonstochastic, consider the following general finite-memory forecasting rule:

$$E_t^j p_{t+1}(i) \equiv \psi_{i,t}^j \left(p_{t-1}(i), ..., p_{t-\tau}(i)\right) \tag{2.11}$$

That is, there exists some function $\psi_{i,t}^j$ that young agents at vertex j use in period t to forecast the price at neighboring vertex i in period $t+1$, and this function depends on the past $\tau \geq 1$ prices at the vertex i. Whatever form this function takes, it represents the PLM for agents at j. The parameter τ is the "finite-memory" part of this PLM, assuming expectations only depend on some fixed number of past prices.

Combining (2.7) with (2.11) yield the actual law of motion (ALM) of prices, $\mathcal{P}^j$, and (2.8) with (2.11) yield the ALM of quantities, $\mathcal{Q}^j$:

$$p_t(j) = \mathcal{P}^j \left(\{p_{t-s}(i),\ m_t(i) \mid i \in N(j),\ s = 1, ..., \tau\}\right) \tag{2.12a}$$

$$q_t(j) = \mathcal{Q}^j \left(\{p_{t-s}(i),\ m_t(i) \mid i \in N(j),\ s = 1, ..., \tau\}\right) \tag{2.12b}$$

Therefore, given an initial distribution of prices, p_0, and money holdings, m_0, (2.9), (2.12a), and (2.12b) describe the dynamics of the economy under finite-memory forecasting rules.

A classic example that will be considered here is that an agent's expectation for a given future price is the average of that price over the previous τ periods.

$$\psi^{j}_{i,t}\left(p_{t-1}(i),...,p_{t-\tau}(i)\right) = \tau^{-1}\sum_{s=t-\tau}^{t-1} p_s(i) \tag{2.13}$$

Note that this expectation is *formed* in period t, as in (2.11). In the homogeneous expectation case, $\psi^{j}_{i,t} = \psi^{k}_{i,t}$ if j and k are both neighbors of i; we will define this expectation as $\psi_{i,t}$. This is comparable to a recursive-least-squares Adaptive Learning method of forming expectations under finite-memory, but where agents regress on a constant, which is the same as taking an average—no assumptions of multicollinearity are violated in this case, and note that the average is unweighted here.

2.2.3 Comparing the Rational Expectations and Learning Models

In **Appendix A** of Anthonisen (2010), the following inequality is proven under rational expectations:

$$GDP^{RE}_{t} = \sum_{j=1}^{J} q^{RE}_{t}(j) \leq \alpha^{\frac{1}{1-\alpha}} J \tag{2.14}$$

Meaning aggregate output is maximized in the steady state. However, under adaptive expectations, this inequality is no longer guaranteed, as the upper bound on aggregate output depends on expectations and money holdings. This upper bound becomes:[8]

$$GDP^{AE}_{t} = \sum_{j=1}^{J} q^{AE}_{t}(j) \leq \sum_{j=1}^{J}\left(\frac{1}{\alpha}\right)^{\frac{1+\alpha}{\alpha}} m_{t+1}(j)\left[\sum_{i=1}^{J}\frac{a_{j,i}}{\psi_{i,t}}\right] \tag{2.15}$$

Where $a_{j,i}$ denotes element in row j, column i of A. Now, the upperbound on the economy's aggregate output depends on temporal forces—money holdings and expectation formation schemes—in striking opposition to the upperbound in the rational expectations case. Under the finite-memory forecasting discussed above, where expectations depend on past prices, current output is integrally tied to past prices and quantities.

A Note on the Time Scale

In the theoretical monetary policy literature, OLG models are rarely used, as the effects of mon-

[8] Proof is provided in the B.1. of the Appendix.

etary policy would appear to take generations to actualize, which is in opposition to empirical data. In **B.2.** of the appendix, we adapt this model for infinitely lived agents and find analogous results, where prices are slightly higher and quantities slightly lower than in the OLG case, scaled by a constant transformation of the discount rate: $(1/\beta)^{1/\alpha}$ and $(\beta)^{1/\alpha}$ respectively, with the discount rate $\beta \in (0,1)$. Given this model's existing presence in the network literature, and to not overcomplicate a model that does not need such complications, we stick to the OLG model for the remainder of the Chapter.

2.3 Monetary Expansion in Circular Graph

To investigate the effects of monetary policy within this model, we replicate **Example 4.1** from Anthonisen (2010)—which considers a one-time permanent increase to the money supply in a circular graph—and compare the rational expectations results with those from an economy using finite-memory forecasting.

2.3.1 Set up

Consider the OLG network economy defined by $\Gamma(V,E)$, with $V = \{1,...,40\}$ and a circular graph ($\eta^j = 2\ \forall j$). In the initial steady state bounded equilibrium, we normalize money such that $\bar{m}(j) = 1$, which gives us $\bar{q}(j) = 1/2$ and $\bar{p}(j) = 2\ \ \forall j \in V$ with $\alpha = 2$. An additional unit of money is given to old agents at nodes $j = 8,9,...,17$ (10 total), and the economy's response is tracked.

We graph the distribution of prices and quantities by showing the price/quantity for each vertex in multiple periods (analogous to Figures 5 and 7 in Anthonisen (2010)), and the aggregate responses of price level and real GDP (analogous to Figures 4 and 6 in Anthonisen (2010)).[9]

Anthonisen finds (and the results are replicated here) that under rational expectations, "[A] monetary shock always reduces the economy's real GDP... That is, in equilibrium, the economy's real GDP is at its global maximum when it is in its steady state. Therefore, any shock that pro-

[9]The new steady-state price is: $\bar{p}(j) = 2.5\ \forall j \in V$. This comes from the total money supply being 50 after the shock, which eventually equalizes among everyone ($\bar{m}(j) = 50/40$) and $\bar{p}(j) = 2\,\bar{m}(j)$, as 2 was the old steady state price.

duces a non-steady-state distribution of cash will lower the economy's real GDP" (Anthonisen (2010)).

For the Finite-Memory Forecast Learning agents, we consider 3 values for $\tau: 1, 5, 10$. This allows us to examine the effects that expectations have on the price and output responses to the positive monetary shock, ranging from naive ($\tau = 1$) to heavily anchored ($\tau = 10$). Curiously, when agents employ finite-memory forecasts, the economy responds to a positive monetary shock in the neoclassical tradition with short-run increases in Real GDP and long-run increases in the price level. The more heavily anchored the expectations of economic agents when struck with a monetary expansion, the longer output effects the monetary expansions last.

2.3.2 Results

Figures 2.1 - 2.3 report the results of this exercise. The different colored lines display the expectation method utilized by agents in each scenario. The integers on the horizontal axes of Figues 2.1 and 2.2 denote the vertices j, with each graph of those figures representing a different time period.

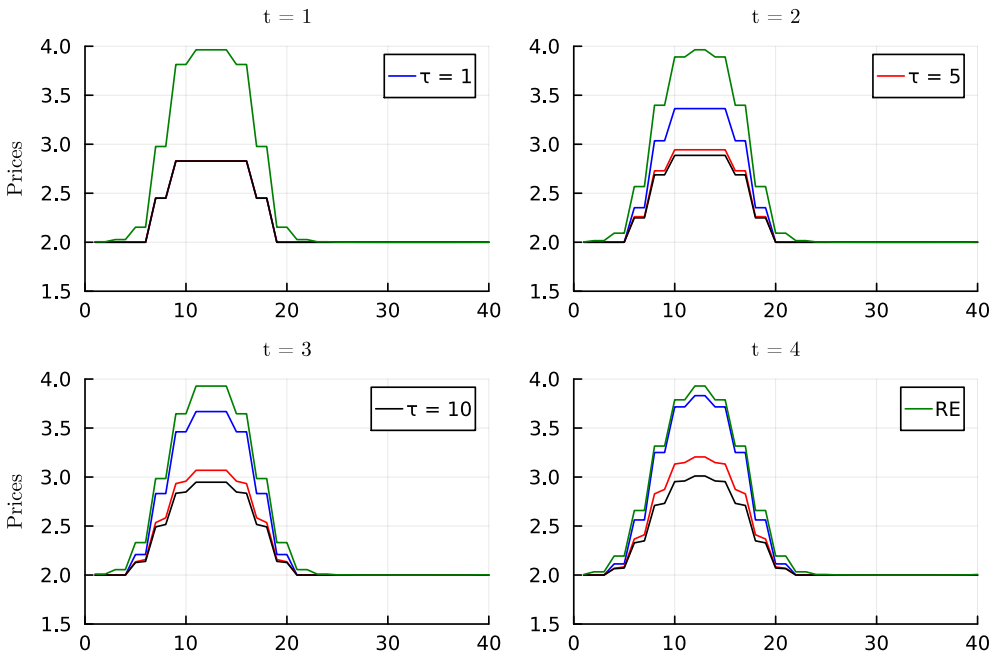


Figure 2.1. Plots of $p_t(j)$ for $j \in V$ in periods following monetary expansion.

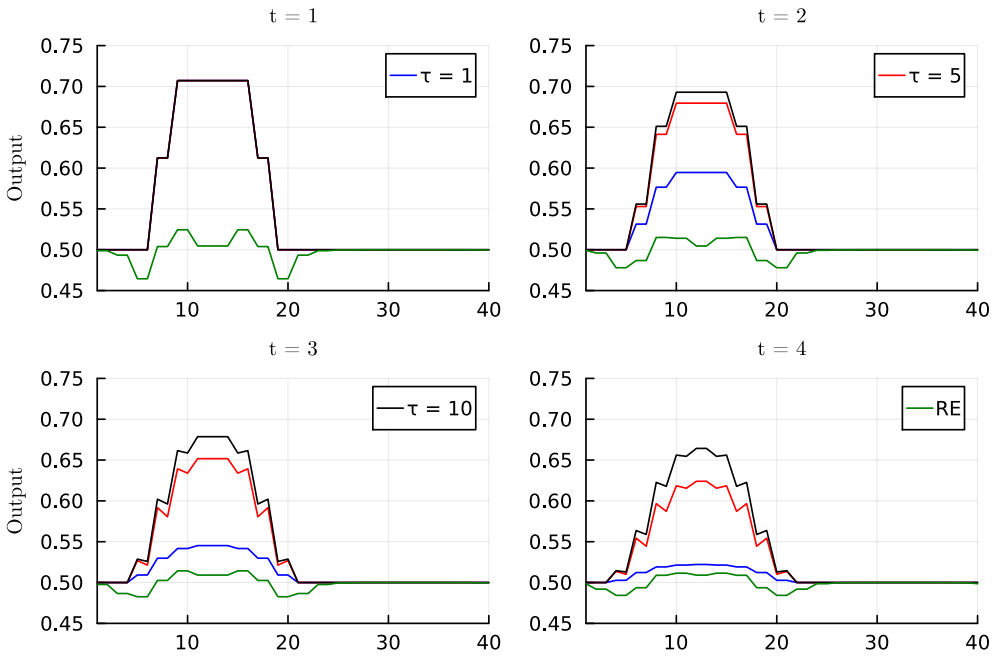


Figure 2.2. Plots of $q_t(j)$ for $j \in V$ in periods following monetary expansion.

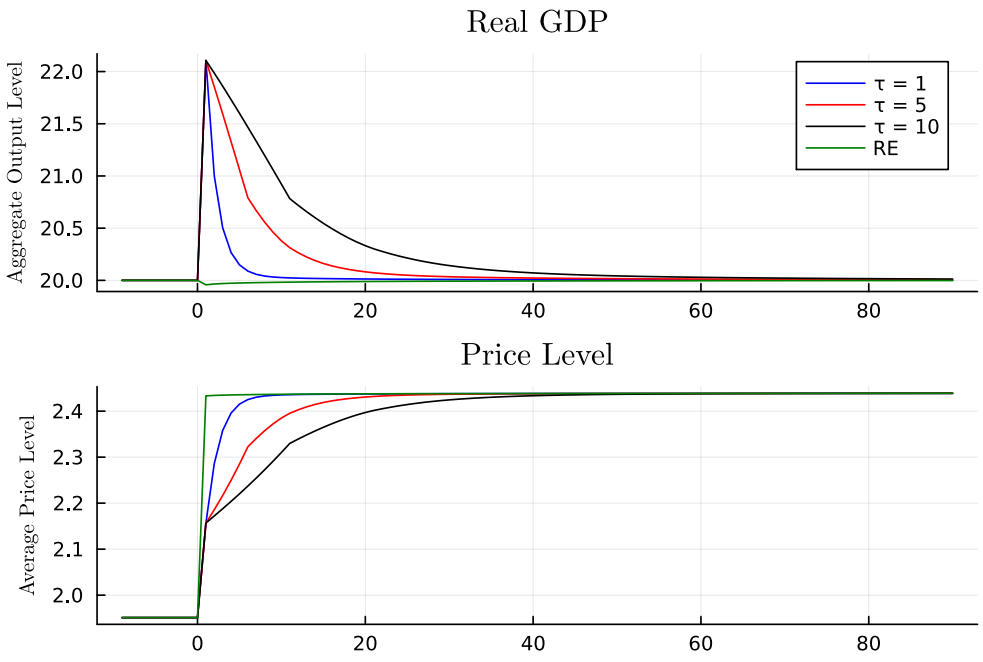


Figure 2.3. Real GDP and Price Level over time.

$$GDP_t = \sum_j q_t(j) \text{ and } P_t = \frac{1}{J} \sum_j p_t(j)$$

It is clear from these figures that agents' forecasting methods have strong implications for the

effects of a monetary expansion. With regard to the price level, prices rise the most dramatically in the RE economy. Under finite-memory forecasting, prices rise faster the less anchored expectations are (i.e. the smaller τ is). As shown in the graphs of Figure 2.1, eventually, the prices across the economies become indistinguishable from each other. There are even more stark differences between outcomes with regard to output, as in Figure 2.2., as the economy's response varies in both direction and magnitude. The initial, first-period, reactions in prices and quantities in all learning cases are identical because in the period the shock hits ($t = 1$) the expectations are identical across all agents regardless of τ, as the economy was in a steady state prior to the shock.

Across all economies where finite-memory forecasting is utilized, Real GDP increases in response to the monetary expansion. Additionally, there are clear differences between the effects the monetary expansion has on output in economies with more anchored adaptive expectations. Economies with higher τ's experience a Real GDP level above the steady state value for longer, as evidenced by Figure 2.3.

2.3.3 Discussion

Consider the following graphical explanation for why the RE economy experiences an immediate contraction and the learning economy an expansion, as given by Figure 2.4.

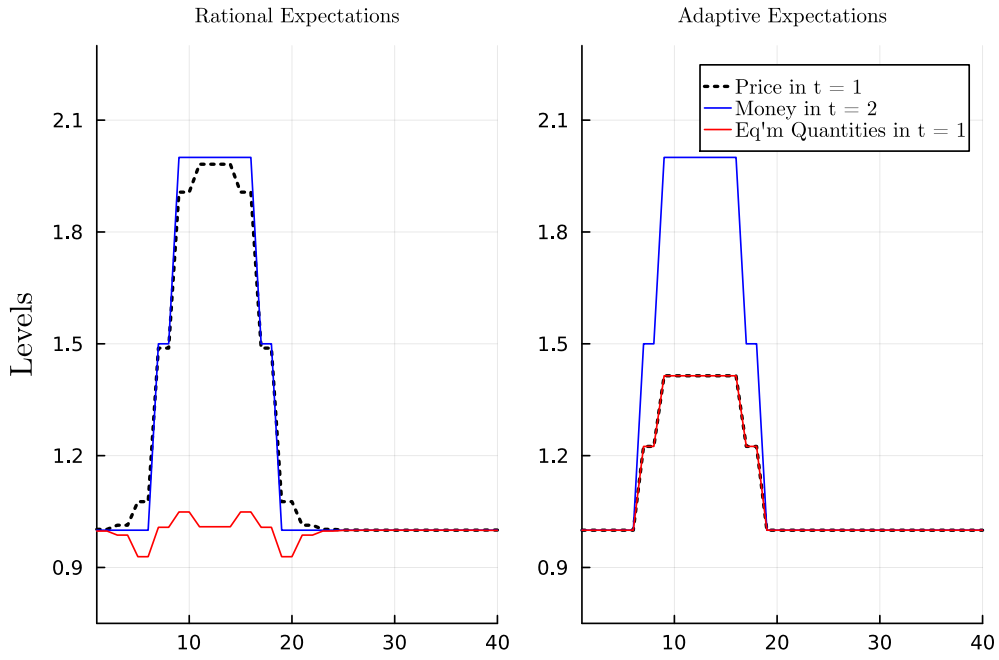


Figure 2.4. Levels of variables relative to steady-state values.

The "Adaptive Expectations" graph is identical across values of τ.

In the RE graph, (denoting these relative values with the hat/carrot) where the increase in $\hat{p}(j)$ is less than the increase in $\hat{m}(j)$, $\hat{q}(j)$ increases (as is the case for all values in the adaptive expectations case). Where the increase in $\hat{p}(j)$ is greater than the increase in $\hat{m}(j)$, $\hat{q}(j)$ decreases. Recall from (2.3) that $q_t(j) = m_{t+1}(j)/p_t(j)$. This decrease happens at the border between where the monetary expansion occurred and where it didn't. The bordering agents charge the same market clearing price to their neighbors with and without the extra money, leading to relative prices changes greater than money changes. Since those decreases near the edge of the expansion outweigh the small increases near the center of the expansion, aggregate Real GDP falls.

2.3.4 Pareto Efficiency

OLG models are famous for often having Pareto-inefficient competitive equilibria, as Balasko & Shell (1980) discusses. Additionally, the welfare effects of monetary policy in OLG models have been considered. Mino & Shibata (2000) find, using an endogenous growth OLG model, that

an increase in the rate of money growth is "generally" not Pareto-improving, but may be under certain conditions and policies. However, the model in this Chapter differs from other common OLG models in a few key ways: restricted trade through a network, no population growth, capital, or saving decisions. Therefore, it is unclear how conditions of Pareto-optimality in OLG models will translate into this model.

Here, we compare the competitive equilibrium allocations under rational and backward-looking expectations as they relate to Pareto-efficient allocations, along with the welfare effects of monetary policy.

Definition. A Pareto Efficient Allocation*, $\{\mathbf{C}_t, \mathbf{Q}_t\}$, in any given period of this Overlapping Generations Network economy is a set of consumption allocations $\mathbf{C}_t = \{\{c_t(j,i)\}_{(j,i)\in V\times V}\}$ and labor allocations $\mathbf{Q}_t = \{q_t(j)\}_{j\in V}$ that is feasible, meaning $q_t(j) = \sum_{i\in N(j)} c_t(i,j)$ is satisfied for all $j \in V$, such that there does not exist another feasible allocation where the generated utility is weakly greater for all agents, and strictly greater for at least one agent.*

The feasibility condition restricts a Social Planner to transferring goods to neighbors within the network: goods produced by the young agent at node j can only be transferred to the old agents at nodes in the set $N(j)$ within the same period. This makes the Planner's problem quite different from that of the household in a competitive market. There is no saving or money in such a problem.

Proposition 2.1.[10] *In the OLG network economy, the steady state competitive equilibrium allocation is the same as Social Planner's optimal solution when the Planner has no preferences across agents or time periods.*

Comparing Pareto Efficiencies of Allocations with a Monetary Expansion

We can compare the Lifetime Utilities of agents in the rational expectations and finite-memory forecasting cases for agents born around the time of the monetary expansion. The monetary expansion occurs at the end of period 0, meaning we expect differences in utility to be the most prominent around that expansion. This is demonstrated in Figure 2.5.

[10]Proof in **B.3. of the Appendix**

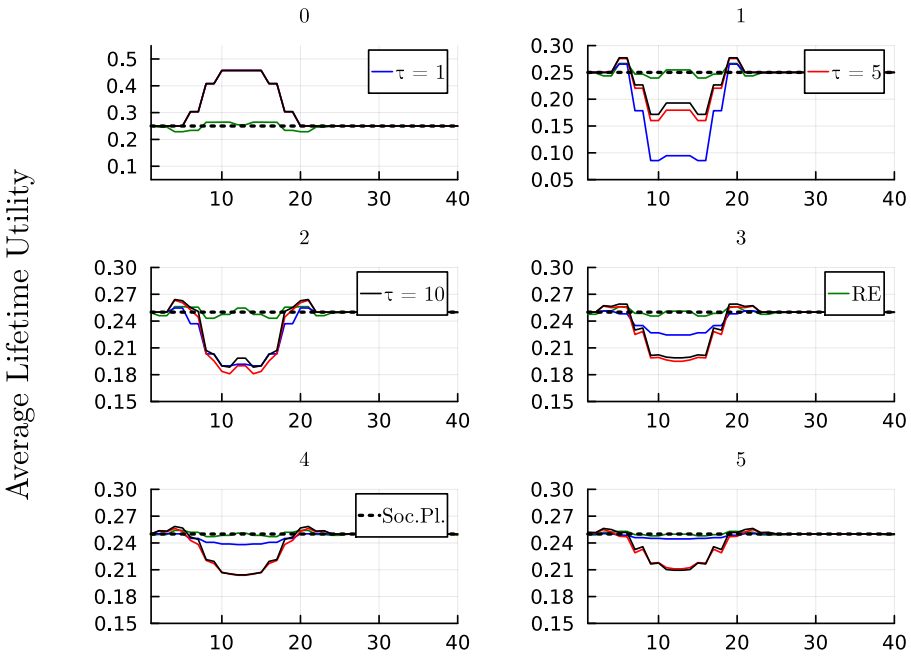


Figure 2.5. Lifetime Utility for agents born in time $t = 0, \ldots, 5$

From Figure 2.5, there is a clear Pareto dominance of the allocations reached under adaptive expectations over the allocation under rational expectations for agents born in period $t = 0$. The Learning allocations Pareto dominate the Social Planner's allocation, but only for one generation of agents. After this first generation, there is no clear pattern of Pareto dominance of one allocation over another as differentiated by expectation-formation methods.

The main beneficiaries of the monetary expansion under Learning are obviously the generation born in period 0 who received the extra money. This generation produced the lower steady-state quantities when young, and were then gifted the monetary expansion to spend when old, meaning no variable decreased their utility.

The Pareto dominance of the allocation for agents born in period 0 over the Social Planner allocation is not inconsistent with *Proposition 2.1*, as that Proposition excludes preference for certain generations or age groups. While changes to the assumption of the Social Planner's generation and agent non-preference could certainly lead to the Social Planner divvying out any of the allocations in Figure 2.5, such a problem is beyond the scope of this dissertation.

To further understand why the allocations resulting from the monetary expansion are not a

decision the Social Planner would make, it is helpful to look at the Average Lifetime Utility of generations across time.

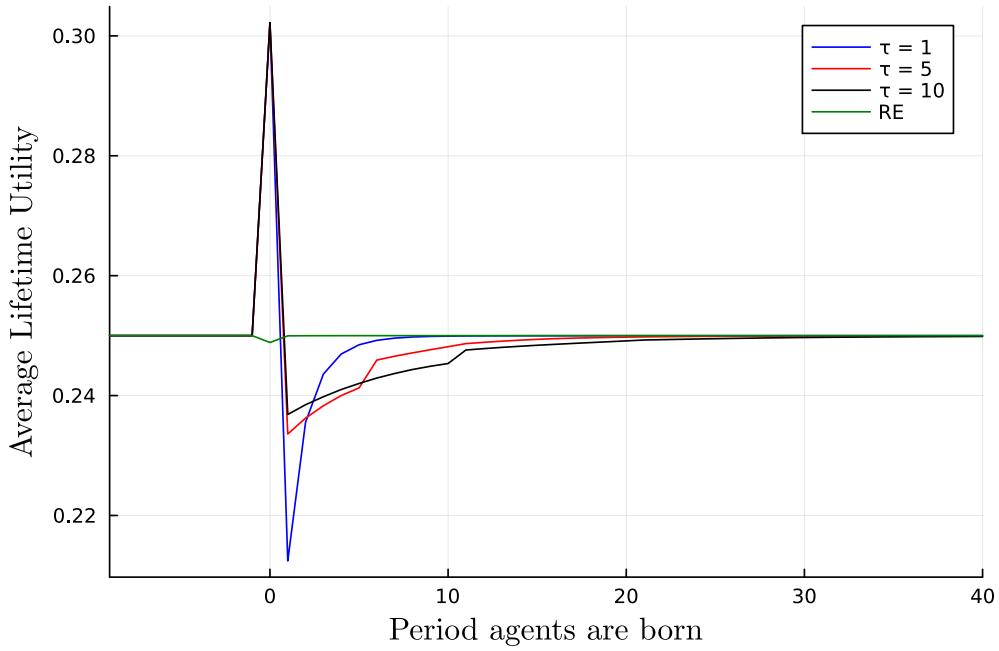


Figure 2.6. Lifetime Utility for generations born in time $t = -10, \ldots, 40$

As Figure 2.6 shows, in the adaptive expectations cases, the welfare benefits of higher output are outweighed by the disutility of labor required to produce that output, yielding a welfare *loss* in the long run. Therefore, the output allocation and distribution that results from the monetary expansion is suboptimal. Additionally, though output decreases under rational expectations and increases under backward-looking forecasts, the aggregate welfare loss resulting from a monetary expansion is greater under backward-looking expectations. Herein lies a benefit of rational expectations in this model: the effects of a monetary expansion are less welfare-reducing under rational expectations than under learning.

2.4 Network Structure

So far, our analysis has been restricted to *k-regular*, circular graphs. This section aims to answer a different question: how does the network structure change the economy's reaction to the monetary expansion? Intuition provides little insight into how these structures will affect the econ-

omy's reaction, and how different types of expectations within the economy impact the overall dynamics. Additionally, the knowledge of the effects of these structures' forms on the macroeconomy aids inquiries into the importance of network structures that the circular graph does not provide in isolation.

There is another question of interest that cannot be answered with the circular graph: how does the centrality of a network's node affect the convergence of the price and quantity variables at that node? In other words, does being more centrally located within the economy affect how quickly that area's economy converges to the new equilibrium, post-monetary expansion? How does this vary by expectations of agents within the economy?

This section discusses how this model differs from the model above, what those changes imply mathematically, how the simulation is performed, which variables are collected during the simulation, and the results.

2.4.1 General Dynamics

Considering the network OLG model in a more general case network case—rather than the *k-regular* graph restriction in Anthonisen (2010)—requires a only few changes to the model in Section **2.2**. In generalizing that model, equations (2.1) through (2.8) still hold. Recall the matrix A that determines how money holdings evolve ($m_{t+1} = Am_t$) where each agent splits their income equally according to the number of neighbors. This consumption decision results from the initial maximization problem and is independent of network structure, as can be seen by (2.5b). Therefore, in this general case, money dynamics operate similarly: for adjacency matrix A_Γ, the transition matrix A is now defined as the *column normalized* version of A_Γ:

$$A = A_\Gamma \cdot \text{diag}(1/\eta^1,\ 1/\eta^2, \ldots,\ 1/\eta^J)$$

This reweighs each column j of A_Γ by the inverse of the number of neighbors j has—making each entry of column j of A equal to 0 or $1/\eta^j$—which replicates the equal splitting of incomes by all agents within the model. This yields a steady-state money supply vector ($\bar{m}$) that is the *pos-*

itive eigenvector associated with the eigenvalue of 1 of the matrix A.

An additional requirement needs to be placed on the graphs we consider to ensure convergence to $\bar{m}$: the existence of at least one *cycle* in the graph, meaning there exists at least one node where there exists a path between that node and itself that does not repeat other nodes. If a graph has no cycles (sometimes called a "tree"), then it is necessarily periodic[11] and, therefore, will not converge to a unique steady-state from any non-steady-state distribution. Figure 2.7 presents a demonstrative example of an acyclical "tree" graph in **(A.)**, however when an additional edge is added (between 3 and 8), we get graph **(B.)** in Figure 2.7, which contains cycles.

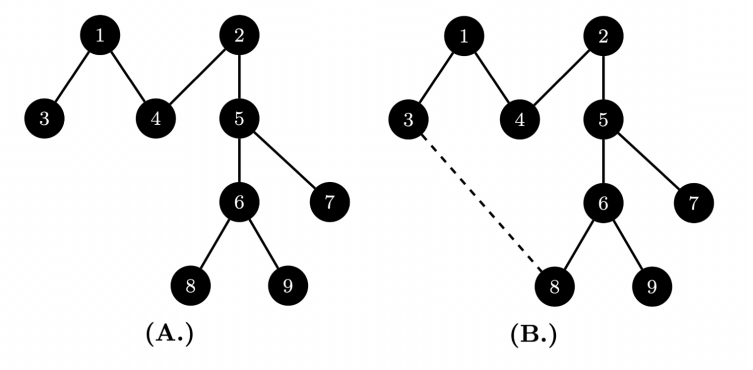


Figure 2.7. A demonstrative tree graph (A.) that is of the type excluded from this exercise, and an amended version (B.) that includes cycles.

Future research should consider the consequences of acyclic network economies.

The following exercise will compare rational expectations (RE), as formed by (2.10), to the finite-memory forecasting case, using (2.13) with $\tau = 1$. To keep this exercise consistent with Section **2.3**, we rescale the vector $\bar{m}$ by a constant so that $\sum_j \bar{m}(j) = J$ *before* the monetary expansion. This ensures the aggregate money supply is the same in both exercises.

Prices in any given period can be generalized in the vectorized and logged version of (2.7):

$$\ln(p_t) = \left(\frac{1}{\alpha}\right)\ln(\alpha)1_{J\times 1} + \left(\frac{\alpha-1}{\alpha}\right)\ln(Am_t) + \left(\frac{1}{\alpha}\right)A'\ \ln\left(p^e_{t+1}\right) \tag{2.16}$$

[11]Tree graphs are *bipartite*, which makes them periodic. See Aldous & Fill (2001) pg. 63. Also note that Tree graphs are the only connected and undirected graphs that lead to periodic Markov Chains, which makes them a special case here.

$\ln(p^e_{t+1})$ is the logged-vector of price expectations. Note the weak response that the dynamic system in (2.16) has to expectations. With $\alpha > 1$, all values of $(1/\alpha)A'$ are strictly between 0 and 1. All simulations in this Chapter yield convergence and stability, and while we do not provide the case for global stability, *Proposition 2.2* shows local stability under a common adaptive expectation case.

Proposition 2.2.[12] *Let* $\Gamma(V, E)$ *be a fully connected graph, with at least one cycle, and* $m_0 > 0$ *be an initial distribution of money holdings. Under homogeneous, naive expectations* ($E^j_t\, p_{t+1}(i) = p_{t-1}(i)\ \forall j, i \in V$)*, the economy is locally stable.*

Regardless of expectation formation method, the log-vector of steady-state prices can be written as:

$$\ln(\bar{p}) = \left(I_J - \frac{1}{\alpha}A'\right)^{-1}\left[\left(\frac{1}{\alpha}\right)\ln(\alpha)1_{J\times 1} + \left(\frac{\alpha-1}{\alpha}\right)\ln(\bar{m})\right] \tag{2.17}$$

2.4.2 Network Simulation

To keep the results similar to the analysis above, we make sure that the graphs generated are *fully connected* (meaning there exists a path between every pair of vertices), *undirected* (meaning edges have no specified direction, i.e. money flows both ways), and the expected value of neighbors for each vertex is 2. This is done by generating an Erdős–Rényi random graph—where each connection between islands has a fixed and *i.i.d.* probability **p** of occurring or not—excluding graphs that are not connected, and setting $\mathbf{p} = 2/J$.

Within each iteration, the economy is simulated with $J = 40$ for 100 periods (10 pre-monetary expansion, and 90-post). The monetary expansion again consists of giving an additional unit of money to 10 agents ($j = 11, ..., 20$), however, note that these agents are no longer guaranteed to be closely connected. Data is collected on 4 different (but similar) measures of centrality, as well as the "half-life" of the price and output at each node for the last 90 periods. The measures of centrality used are *Eigenvector*, *Betweenness*, *Closeness*, and *Degree* Centrality, as described in Section **1.4**. The half-life estimate is a measure of the speed of convergence from the old steady-state

[12] Proof provided in **B.4.** of the appendix.

to the new steady-state—the smaller the half-life estimate, the faster the "decay" / convergence.

Half-Life

We consider the estimated half-lives of the prices and quantities for each node j as these variables converge to the new steady-state values, from the period directly following the monetary expansion, to the end of the sample, using the following method:

$$\text{Estimate AR(1) Model:} \quad y_t(j) = \lambda_0 + \lambda_1 y_{t-1}(j) + \text{error}_t$$
$$\text{Estimated Half Life of} \quad y_t(j) = -\frac{ln(2)}{ln(|\lambda_1|)}$$

Where $y_t(j)$ represents the price or quantity at node j in period t.

2.4.3 Results

We generate 100 random, fully-connected networks, simulate the economy in the RE and $\tau = 1$ cases within that network, and report the results here.

The top graph in Figure 2.8 shows aggregate real output (as $\sum_j q_t(j)$), and the bottom graph shows the price level, estimated as the average price across all 40 agents in the RE and $\tau = 1$ economy simulations.

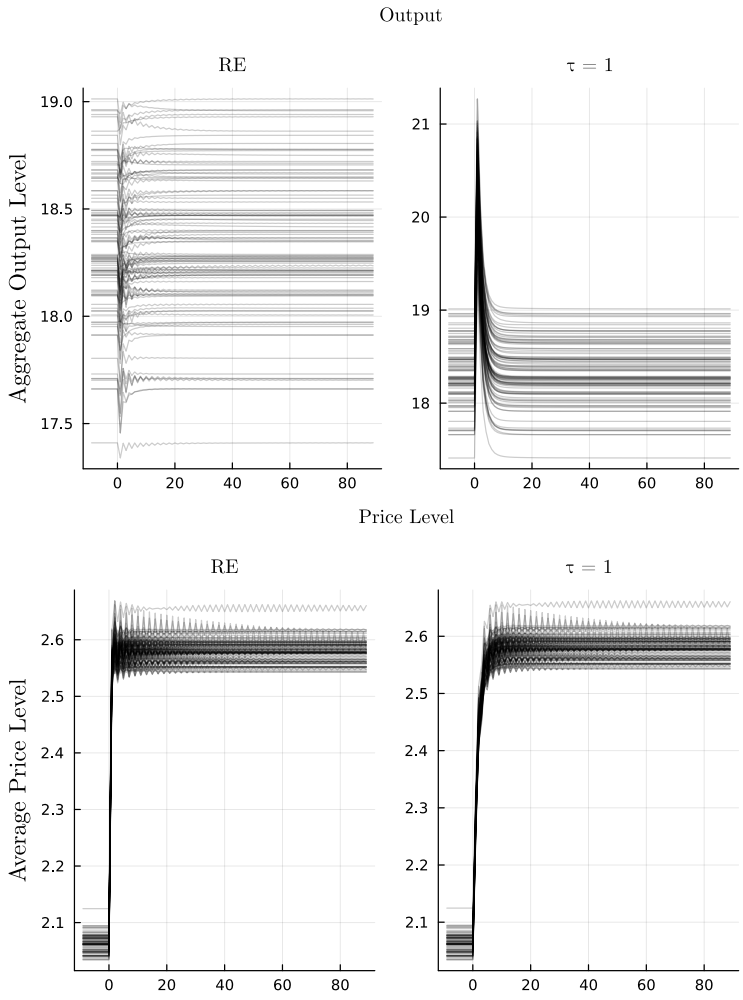


Figure 2.8. Time paths of the price level and aggregate real output across 100 simulations.

The first obvious difference is that despite having the same number of agents and aggregate money supply, different economies have different steady-state output and price levels. We know the initial $\bar{m}$ will vary by networks, which will translate to steady-state price vectors, as determined by (2.17), that will vary greatly, even causing the sum of $\bar{p}$ to vary. These simulated graphs appear to have steady-state output levels *below* and price levels *above* those from the circular graph.

Preliminary simulations suggest that any connected *k-regular* graph (such as the circular graph of before or a fully connected graph) maximizes the steady-state output while minimizing the steady-state price level. Star graphs (where 1 node sits in the center and all others are only

connected to that central node) with one additional connection (to eliminate periodicity) appear to minimize steady-state output and maximize steady-state price level. When all outer nodes in a star graph are connected to two other outer nodes, we get a wheel graph, which is also considered. All other graphs tested (including those in the simulation) yield steady state values between the extremes of the *k-regular* and *Star + 1*.

Graph Type	mean $\bar{p}(j)$	$\sum_j \bar{q}(j)$
Circle	2	20
Fully Connected	2	20
Simulation Average	2.063	18.334
Wheel	2.33	15.018
Star + 1	3.6	7.7

Table 2.1. Steady-state prices and output by graph type with $\sum_j \bar{m}(j) = J = 40$ agents.

While there appears to be a pattern of a higher steady-state price level corresponding with a lower output, this increase and decrease is not guaranteed to be uniform across networks. In fact, some of the networks appearing in the simulation have higher price levels and output than others. For example, the graph with the highest steady-state output in the simulation is not the same graph that yields the lowest price level.

Since (2.17) does not depend on expectations, the varying price levels will be the same across the RE and τ cases within each network. Unfortunately, it is difficult to glean clear conclusions regarding the effects of monetary policy across these economies from Figure 2.8. To more clearly analyze these models, consider the same variables, now normalized by their pre-monetary expansion steady-state levels. These are shown in Figure 2.9.

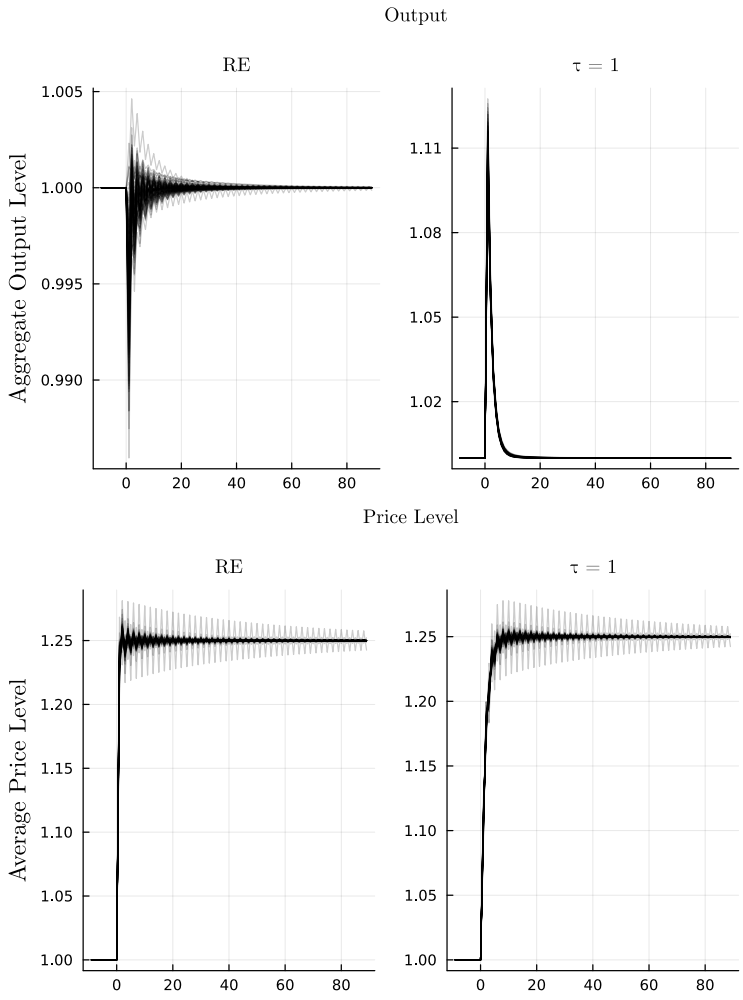


Figure 2.9. Time paths of aggregate real output and the price level across 100 simulations, normalized to the initial steady-state levels.

The striking difference between the rational and adaptive expectations cases is still present here: the RE economy often experiences an output contraction post-monetary expansion, while the adaptive economy sees a short-run increase in output. Monetary neutrality is still present in the long-run, though there appears to be much more variation in the RE economy. One key difference here is that the steady-state output is no longer an upper bound in the RE economy, as is the case in a *k-regular* graph. This means that Proposition 3 in Anthonisen (2010) (equation (2.14) in this paper) does not generalize to other types of networks, even in the RE case.[13]

[13] Anthonisen (2010) made use of the fact that the *columns and rows* of A sum to 1 in the *k-regular* graph case. In the general case, however, only the columns of A sum to 1, and the sum of the rows can be less or greater than 1.

For the price level, much like in the circular graph example, the RE economy convergences to the new steady-state price level more quickly than the $\tau = 1$ economy. However, now the convergence is no longer a monotonic approach to the new steady-state price level, as oscillations are present. All of these economies converge from a pre-expansion steady-state price level ($\bar{p}_{pre}$) to a post-expansion price level equal to $(50/40) \cdot \bar{p}_{pre}$.

Regarding convergence, Figure 2.10 reports the results of the correlation statistics between the above centrality measures and the estimated half-lives of prices and quantities at each node, from each of the 100 simulations. Recall that the smaller the half-life, the faster the convergence.

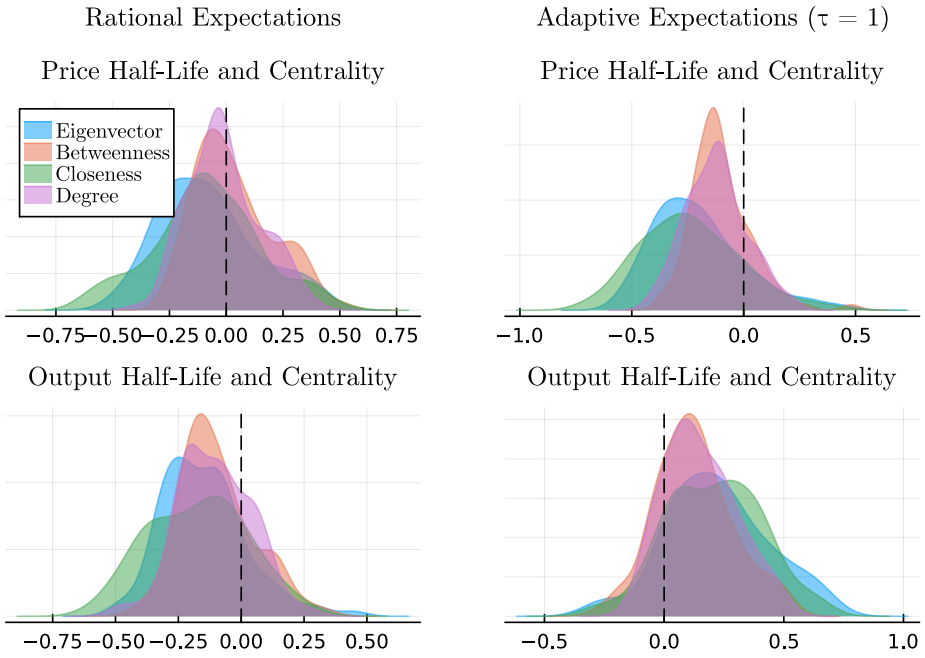


Figure 2.10. Estimated correlation statistics between nodes' network centrality and the nodes' variables' convergence speed to the new equilibrium after the monetary expansion.

In the RE case, price convergence appears to *not* depend on centrality. This is caused by the "far-in-the-future," forward-looking forecasts of these agents, as even "distant" agents anticipate the higher future price level. This is no longer the case in the adaptive economy, where more central agents converge to the new price level faster—this is because these agents are working with more data, and adjust their pricing behavior in response to these changes much quicker than those with little data.

On the output side, the expectations play an even greater role. In the RE economy, output production converges to the steady-state faster for more central agents, while in the adaptive economy, the convergence of output is slower for more centrally located agents. This simulation helps us further understand how the economy's network structure interacts with expectations after a monetary expansion.

2.5 Expectation Heterogeneity

A natural question that arises after witnessing the striking differences above is what happens when some agents are rational and others employ adaptive expectations? This section tackles this question in the context of the circular *k-regular* graph.

One way expectation heterogeneity has been incorporated into economic models is "fixed portion" heterogeneity, where groups of agents use various expectation formation methods, never changing that method. Here, we consider the case that a portion $\gamma \in (0,1)$ of agents are *rational* (employ rational expectations), while the remaining $(1-\gamma)$ agents use a finite-memory forecasting method. Both the level of γ and the *distribution* of these groups across the economy within the network have important implications for the economy's output response to a monetary expansion.

2.5.1 Fixed-Portion Heterogeneity

Assuming the rational agents believe everyone was rational, all agents' expectation for $p_{t+1}(i)$ would be exponential of the i^{th} entry of the vector $ln(p^{RE}_{t+1})$, as determined by equation (2.10). However, in the case that some nodes have agents with adaptive expectations and others with rational expectations, this equation no longer holds.

Under heterogeneous expectations, the j^{th} element of the vector $[A' \ln\left(p^e_{t+1}\right)]$ depends on the expectations of agents at node j. The j^{th} entry of $[A' \ln\left(p^e_{t+1}\right)]$ is $1/2\left[\ln(p_{t+1}(i)) + \ln(p_{t+1}(k))\right]$ if j is rational and has 2 neighbors: i and k. The j^{th} entry of $[A' \ln\left(p^e_{t+1}\right)]$ is $1/2\left[\ln(\psi_{i,t}) + \ln(\psi_{k,t})\right]$ if agent j uses adaptive expectations. Let $\bar{\gamma}$ be a $J \times 1$ vector such that the j^{th} entry of $\bar{\gamma}$ is 1 if j

is rational and 0 otherwise. Also, let ψ_t be the vector of finite memory forecasts regarding each agent's price at time t. We can then write (2.16) as:

$$\begin{aligned}\ln(p_t) = &\left(\frac{1}{\alpha}\right)\ln(\alpha)1_{J\times 1} + \left(\frac{\alpha-1}{\alpha}\right)\ln(Am_t) + \left(\frac{1}{\alpha}\right)[(1_{J\times 1} - \bar{\gamma}) \circ A' \ln(\psi_t)] \\ &+ \left(\frac{1}{\alpha}\right)[\bar{\gamma} \circ A' \ln(p_{t+1})]\end{aligned} \tag{2.18}$$

Again, $\circ$ is the Hadamard element-wise produdct. In the simulation, the rational expectations are computed by iterating (2.18) forward. Together with the assumption that the rational agents ignore adaptive expectations in future periods, we get the following equation[14]:

$$\begin{aligned}\ln(p_t) = &\left(\frac{1}{\alpha}\right)\ln(\alpha)1_{J\times 1} + \left(\frac{\alpha-1}{\alpha}\right)\ln(Am_t) + \left(\frac{1}{\alpha}\right)[(1_{J\times 1} - \bar{\gamma}) \circ A' \ln(\psi_t)] \\ &+ \left(\frac{1}{\alpha}\right)\bar{\gamma} \circ \left[\sum_{s=1}^{\infty}\left(\frac{1}{\alpha^{s+1}}\right)\ln(\alpha)1_{J\times 1} + \left(\frac{\alpha-1}{\alpha^{s+1}}\right)(A')^{s+1}\ln\left(A^{s+1}m_t\right)\right]\end{aligned} \tag{2.19}$$

This tells us how RE agents form their expectations.

Again we use a circular *k-regular* graph with $J = 40$ agents. As stated, the level of γ and the physical distribution of agents will both be important factors in the effect of a monetary expansion on the economy. The levels of γ we consider are $1/4$, $1/2$, and $3/4$. The first distribution we consider is "clusters" of agents, where each group is packed together, and only 2 members of each group are neighbors with members of the other group. The second distribution we label "dispersed," where heterogeneous agents are spread throughout the economy. The exact manner of this dispersion is discussed below.

The monetary expansion will consist of an additional unit of money given to the old agents as islands $j = 8, 9, ..., 17$ (10 total). The adaptive agents will use $\tau = 1$.

[14] Also note that $A' \cdot 1_{J\times 1} = 1_{J\times 1}$.

2.5.2 Results

Clustered Agents

First, consider clustering RE agents centered around the agents given the monetary shock, with the remaining agents being adaptive. The results of this economy are shown in Figure 2.11.

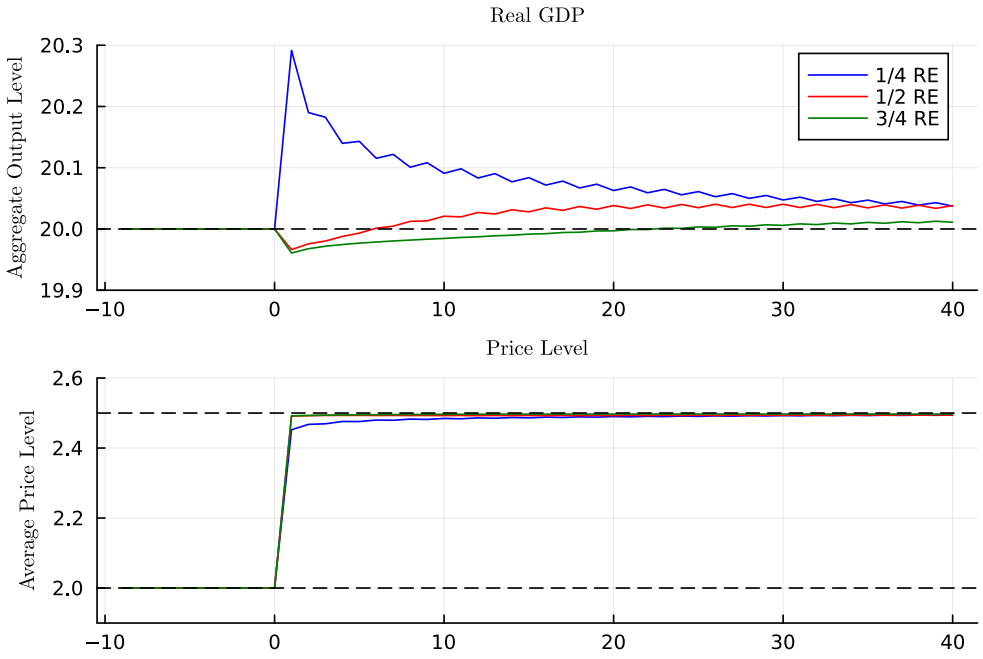


Figure 2.11. Monetary expansion with heterogeneous agents organized into clusters. Rational Expectations agents are centered around the monetary shock.

The initial response varies by the portion of rational agents. In both the 1/2 and 3/4 rational cases, the economy experiences an initial contraction with aggregate real GDP below the steady-state level, followed by a longer period of expansion with aggregate real GDP above the steady-state level. This can be interpreted as considering the rational agents who first receive the monetary expansion as contractionary and the learning agents as expansionary. As the money flows from contractionary agents to expansionary ones, the learning agents increase production many periods after the initial shock. This subsequent expansion is noticeably smaller in the 3/4 rational case.

Regarding the price level, the $1/4$ RE case again stands out, showing a much slower adjustment to the new steady-state price level.

Keeping with agents being organized into clusters, consider the case that the adaptive agents

are centered around the monetary shock. Figure 2.12 shows the resulting responses in these economies.

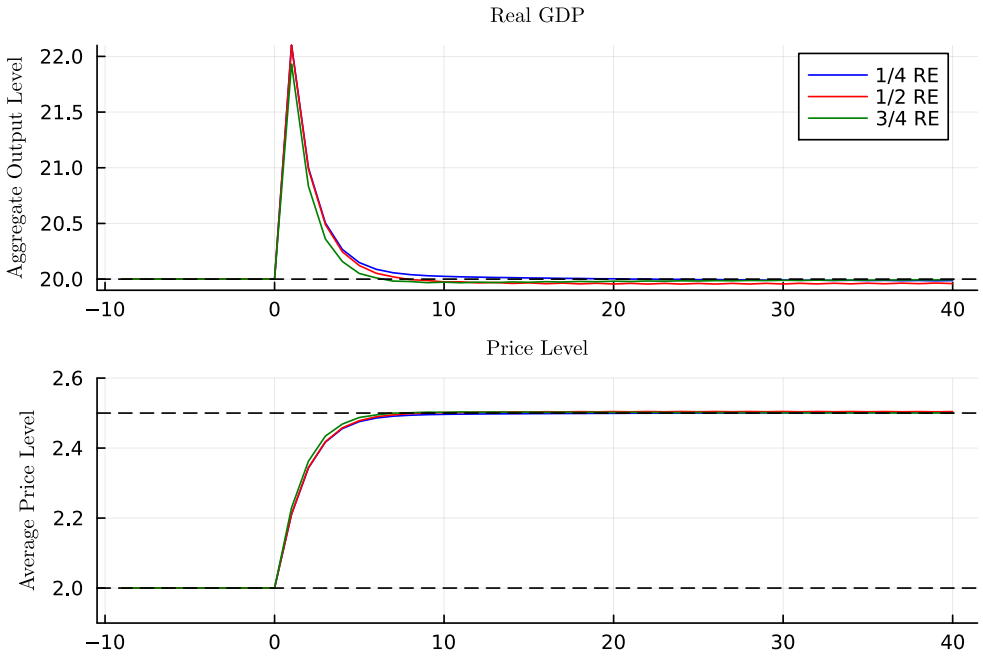


Figure 2.12. Monetary expansion with heterogeneous agents organized into clusters. Adaptive expectations agents are centered around the monetary shock.

Here, regardless of the proportion of rational agents, the economy responds with an initial expansion. However, after output returns to its steady-state value, it drops below that steady-state value and slowly increases back to it. This, much like the previous case, shows that while the initial response may be in one direction, in the medium-run (the time between the immediate reaction and the return to the steady-state,) output may rebound in the other direction.

Regarding the price level, there is little noticeably different between the cases here.

Dispersed Agents

Consider the case that heterogeneous agents are dispersed throughout the economy. Here, starting from agent 1, agents will be ordered as $(A,\ A,\ A,\ B)$, when $\gamma = 1/4$ or $3/4$ (where A is a member of the majority group, and B the minority), and $(A,\ B,\ A,\ B)$ when $\gamma = 1/2$. The results of this economy are seen in Figure 2.13.

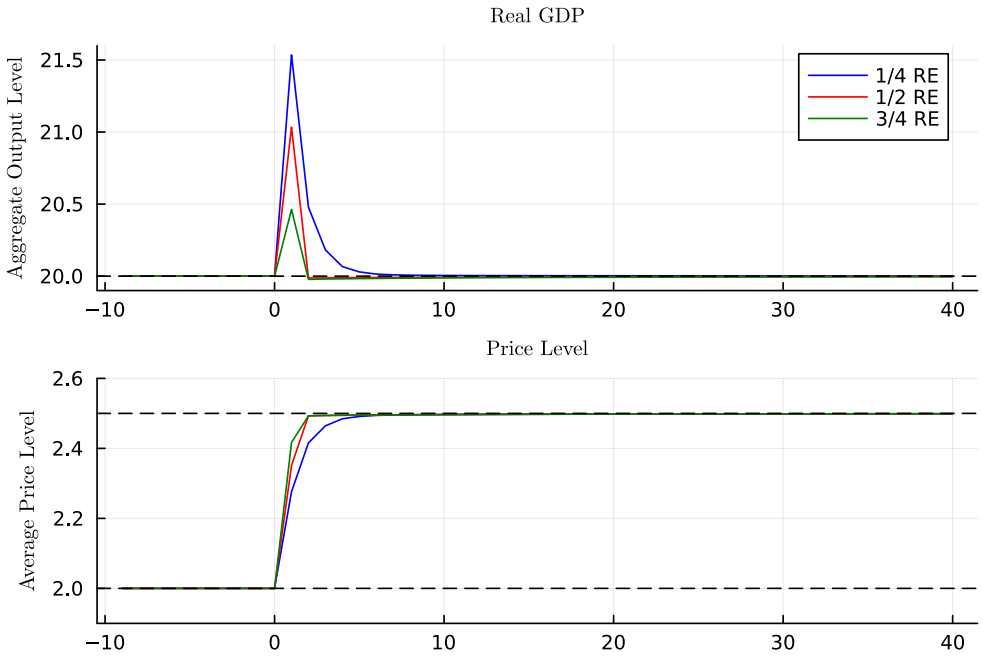


Figure 2.13. Monetary expansion with heterogeneous agents dispersed throughout the economy.

Here, all economies respond with initial expansions of output above the steady-state level. However, in the $1/2$ and $3/4$ RE cases, this expansion is followed by a dip slightly below the steady-state level. Regarding the price level, the level in the $1/4$ RE case adjusts to the new steady-state slower than in the other two cases.

2.5.3 Discussion

A common theme across these simulations is that agents employing rational expectations respond to a monetary expansion by decreasing their output, and adaptive agents respond by increasing their output. The reasons for this are discussed in Section **2.3.3**, but the implications become more clear in the heterogeneous case. Initial reactions in output and price level appear to depend on which group *receives* the monetary expansion. On the output side, the larger the portion of RE agents. Startling jumps to the new steady-state price level are witnessed when the RE agents are gifted the shock, and more gradual transitions are seen when the adaptive agents are the target of the shock.

We also find in multiple cases that the economy's short-term reaction may differ from the

medium-run reaction to the monetary shock. This depends on the spatial organization of agents. In the homogeneous agent case, the initial shock is followed by a uniform increase or decrease to the steady-state.

2.6 Conclusion

Generalizing the OLG network model in Anthonisen (2010) reveals rich layers, ripe for analysis. This Chapter shows that many of the previous results gleaned from this model are a consequence of rational expectations. For example, the cap on aggregate output that rational expectations places on this economy is flexible, and depends on expectations. The result that aggregate output decreases in response to a monetary expansion is found to be caused by the far-in-the-future looking rational expectations. Under adaptive expectations, the model shows the classical result: monetary expansions increase output in the short-run and the price level in the long run.

This Chapter considers welfare within this economy, and it is shown that the steady-state competitive equilibrium is Pareto efficient, and that all monetary expansions yield aggregate decreases in welfare. The network simulation shows us that the steady-state level of prices and output depend on the network. We also find that centrality interacts with expectations in regard to convergence to steady-state after a monetary expansion. The heterogeneous expectation case teaches us that the general conclusions of the homogeneous case hold under heterogeneity, but both the proportion of groups and the organizations of these groups matters for the economic response.

This model demonstrates that the assumption of rational expectations does not always align with classical beliefs about monetary policy. However, the validity of this conclusion in the real world cannot be reached absent empirical data. To test the legitimacy of the results in this Chapter, we utilize empirical VAR models in the next Chapter.

Chapter III: Country Specific Monetary Shocks in Europe: Network Informed Structural Vector Autoregression Analysis

3.1 Introduction

The empirical exercise in this Chapter is not an attempt to calibrate the model from Chapter II. Rather, the goal of this Chapter is to establish the conclusion the theoretical model gives us: that monetary shocks given to areas with agents who employ forward-looking, rational expectations are (a) *different from* and (b) *lesser in magnitude* than monetary shocks given to areas with agents who use backward-looking expectations.

Since the European Union (EU) is the largest monetary union in the world, it is a great candidate for estimating heterogeneous responses to monetary policy. Peersman (2004) and Cavallo & Ribba (2015) find that the member countries of the EU have well-integrated economies with near-synchronized business cycles. However, that does not mean monetary policy responses are homogenous across countries. Anzuini & Levy (2007) finds that the three largest economies added to the EU in 2004—the Czech Republic, Hungary, and Poland—respond less to monetary policy shocks than older members of the EU, while Jarociński (2010) finds similar responses to monetary policy across older and newer adoptees of the Euro.

As for mechanisms driving this heterogeneity, multiple dimensions have been considered. Coricelli, Égert, & MacDonald (2006) finds that the composition of manufacturing sectors within EU countries plays a large role in determining these heterogeneous responses. Additionally, the distribution of small firms across sectors can affect a country's response to monetary policy via credit offered to these firms. Other channels of heterogeneity can be ruled out. Elbourne & de Haan (2004, 2006) find a little-to-non-existent link between indicators of countries' financial structures and their response to monetary policy for a host of current and future EU members. One difference that has not been explored as a driver of heterogeneous responses to monetary policy is the difference in the formation of expectations.

The European Commission Business and Consumer Survey polls from up to 32 countries in

Europe to estimate consumer sentiments, including inflation expectations. This survey has been used to help answer a wide variety of economic questions. The assumption of rational expectations has been safely debunked in most of the countries surveyed (Szyszko, M., Rutkowska, A., & Kliber, A., 2020). Overall, it appears that European households are "sluggish" in updating their information; Döpke, Dovern, Fritsche, & Slacalek (2008) estimates this information update occurs about once every 18 months. Comparing the EU before and after the 2008 financial crisis, Lyziak & Mackiewicz-Lyziak (2014) finds that expectation errors increased during the financial crisis, but improved in the period after. A common result across these papers is that of heterogeneous expectations across countries, a result found in this Chapter as well.

The empirical exercise in this Chapter supports the theoretical conclusion that economic responses to monetary policy depend on the expectations of the economy. Using the European Commission Survey of Expectations, we categorize countries' expectations as forward or backward-looking with a Bayesian Model Averaging method. We consider the impulse response functions within two models: (1) responses of a country's output and inflation to a monetary policy expansion within said country, and (2) responses of the aggregate economy's output and inflation to country-specific monetary policy expansions.

In Section 6, we categorize countries as forward or backward-looking, and test the hypothesis empirically in the EU using multiple Structural VAR models.

The effects of a one-time increase in the money supply for economies near the zero-lower bound (ZLB) will be considered. As "helicopter money" has been considered as possible expansionary monetary policy for ZLB economies, we find the decade from January 2010 to December 2019 in Europe an appropriate and relevant for analysis.

3.2 Spatial Expectations Data

One difficulty in estimating spatial expectations' effect on the economy is the dearth of data detailing expectations by location. While popular inflation expectation surveys such as the New York Fed Survey of Consumer Expectations and the University of Michigan's Survey of Con-

sumers divide the United States into 4 large regions, many others do not report any level of location data. The most detailed relevant survey publicly available is the European Commission Business and Consumer Survey, a monthly survey of perceived and expected inflation, reporting responses from up to 32 countries in Europe, starting in 1985.

To categorize regions as more forward or more backward-looking in their expectations, data describing inflation expectations along with respondents' regions is required. The monetary and trading union intertwining these countries' economies makes Europe a great candidate for this research. Respondents to the Commission's survey are asked to report their perception of inflation over the previous 12 months, and their expectation of inflation over the next 12 months *in comparison* to their perceived inflation. Respondents are given the following choices: "*Prices will...*" (1) *Increase more rapidly*, (2) *Increase at the same rate*, (3) *Increase at a slower rate*, (4) *Stay about the same*, (5) *Fall*, or finally (6) *Don't know*.

3.3 Quantizing Inflation Expectations

Macroeconomists have considered methods of quantizing these qualitative answers for decades. The two most commonly employed methods are the "Probability" and "Regression" methods. The Probability Method uses the portion of respondents for each answer as a maximum likelihood estimate of the area under the distribution of expectations as bounded between relevant thresholds—Batchelor & Orr (1988) were the first to apply this method to the Commission's survey. The Regression Method assumes respondents' opinions concerning present inflation map to future inflation, and is discussed further in Pesaran (1987).

Both methods assume unbiased inflation perceptions, but do not imply unbiased (*rational*) expectations. Given that whatever method chosen will be applied to heterogeneous countries over many years, we use the method with the fewest restricting assumptions, which here is the Regression Method. Unlike the Probability Method, the Regression Method makes no assumptions regarding the distribution of inflation expectations. Additionally, issues arise in the Probability Method when no respondents select answers on the extreme ends, effectively ignoring this data,

while the Regression Method makes use of this response as data.

First, we assign the portion of respondents reporting the different answers (reweighted with respect to those who chose "Don't know") respectively to the following variables: A_i^p for inflation perceptions, and A_i^e for inflation expectations, for responses $i = \{1, \ldots, 5\}$. We estimate the following regression for each country j, where π_t^j is the Eurostat HICP monthly inflation estimate in that country.

$$\pi_t^j = \alpha_{up} \sum_{i=1}^{3} A_{i,t}^p + \alpha_{down} A_{5,t}^p \tag{3.1}$$

The estimated coefficients, $\hat{\alpha}_{up}$ and $\hat{\alpha}_{down}$ are used to compute inflation expectations as:

$$\pi_t^{j,e} \equiv \hat{\alpha}_{up} \sum_{i=1}^{3} A_{i,t}^e + \hat{\alpha}_{down} A_{5,t}^e \tag{3.2}$$

Not every country included in the Commission's survey will be included due to missing data.[15]

Equations (3.1) and (3.2) are estimated on a 5 year rolling basis, *including* inflation and expectation data before 2010. For example, inflation expectation for country j in January 2012, is estimated with α_{up} and α_{down} estimated using HICP and inflation perceptions data between July 2009 and June 2014 (60 months). The estimated $\hat{\alpha}_{up}$ and $\hat{\alpha}_{down}$ are then used to calculate $\pi_{Jan-2012}^{j,e}$ using (3.2) with the reported expectations portions: $A_{i,Jan-2012}^e$.

3.4 Categorizing Countries

Gerberding (2009) relates rational expectations to being forward-looking. We exploit this dichotomy of "forward" vs. "backward"-looking expectations to draw parallels to the theoretical OLG model in Chapter II. Lyziak & Mackiewicz-Lyziak (2014), estimates the "*degrees of forward-lookingness*" for many European countries. However, we believe the method in that paper lacks the flexibility needed to estimate different degrees of inflation averaged expectations. A fitting alternative that we develop here is a Bayesian Model Averaging method.

To categorize how forward or backward-looking a country is, we estimate τ for each country.

[15] Countries with insufficient expectation data: Montenegro, North Macedonia, Albania, Serbia, and Ireland.

Recall from Chapter II that τ determines how backward-looking a country is. However, here, a country's τ can be negative, implying forward-looking expectations. Let $\hat{\tau}^j$ be the estimated τ for country j. This estimation assumes inflation expectations are some average of past or future year-over-year inflation rates. The following regressions are run for integer values of τ^j from -12 to 12.[16]

$$\pi^e_{j,t} = \beta^{\tau,j} \frac{1}{|\tau^j|} \sum_{k=\tau^j}^{-1} \pi_{j,t-k} \quad \text{if } \tau^j \leq -1$$

$$\pi^e_{j,t} = \beta^{0,j} \pi_{j,t} \quad \text{if } \tau^j = 0$$

$$\pi^e_{j,t} = \beta^{\tau,j} \frac{1}{\tau^j} \sum_{k=1}^{\tau^j} \pi_{j,t-k} \quad \text{if } \tau^j \geq 1$$

Denote each of these regressions as model $\mathcal{M}_i$ (where i denotes the value of $\tau^j + 13$, to begin at $i = 1$ and continue up to $i = 25 \equiv N$ total models estimated). We estimate τ^j with:

$$\hat{\tau}^j = \sum_{i=1}^{N} \tau^j \cdot \Pr(\mathcal{M}_i | \pi^j) \tag{3.3}$$

This makes $\hat{\tau}^j$ a probability-weighted sum of the model-specific measures of τ^j. $\Pr(\mathcal{M}_i|\pi^j)$ is the posterior probability that model $\mathcal{M}_i$ is true, conditional on the inflation data, π^j. As in Morley & Piger (2012), this posterior distribution is computed as:

$$\Pr(\mathcal{M}_i|\pi^j) = \frac{\exp(SIC_i) \cdot \Pr(\mathcal{M}_i)}{\sum_{i=1}^{N} \exp(SIC_i) \cdot \Pr(\mathcal{M}_i)}$$

Where SIC_i is the Schwarz Information Criterion from model $\mathcal{M}_i$.[17] Additionally, we assign equal weight to the prior model probability, $\mathcal{M}_i$.

We test the flexibility of this method by comparing it with the method in Lyziak & Mackiewicz-

[16] Time period restrictions were applied after calculating the average on the RHS of these equations. This ensures identical sample sizes for all models.

[17] The SIC is computed as $SIC_i = -\frac{1}{2}\left[N \cdot log(SSR_i) + K_i \cdot log(N)\right]$ as in Davidson & MacKinnon (2004), where SSR_i is the Sum-of-Squared-Residuals, and K_i is (typically) the number of estimated coefficients, in model $\mathcal{M}_i$. Since K_i is meant to be a penalty—and all models here have 1 coefficient by Footnote 16—we use $K_i = 1$ for $\tau^j = 0$ and $K_i = 1/|\tau^i|$ otherwise; this way, K_i is a measure of flexibility, which is lower for τ's of greater magnitude.

Lyziak (2014). The estimated parameter α_2 measures the "*degree of forward-lookingness*," and ranges from 0 to 1, with 1 being completely forward-looking, and 0 completely backward-looking. Using the same sample period, (May 2001 to July 2012) we use the BMA method across values of τ ranging from -12 to 12.

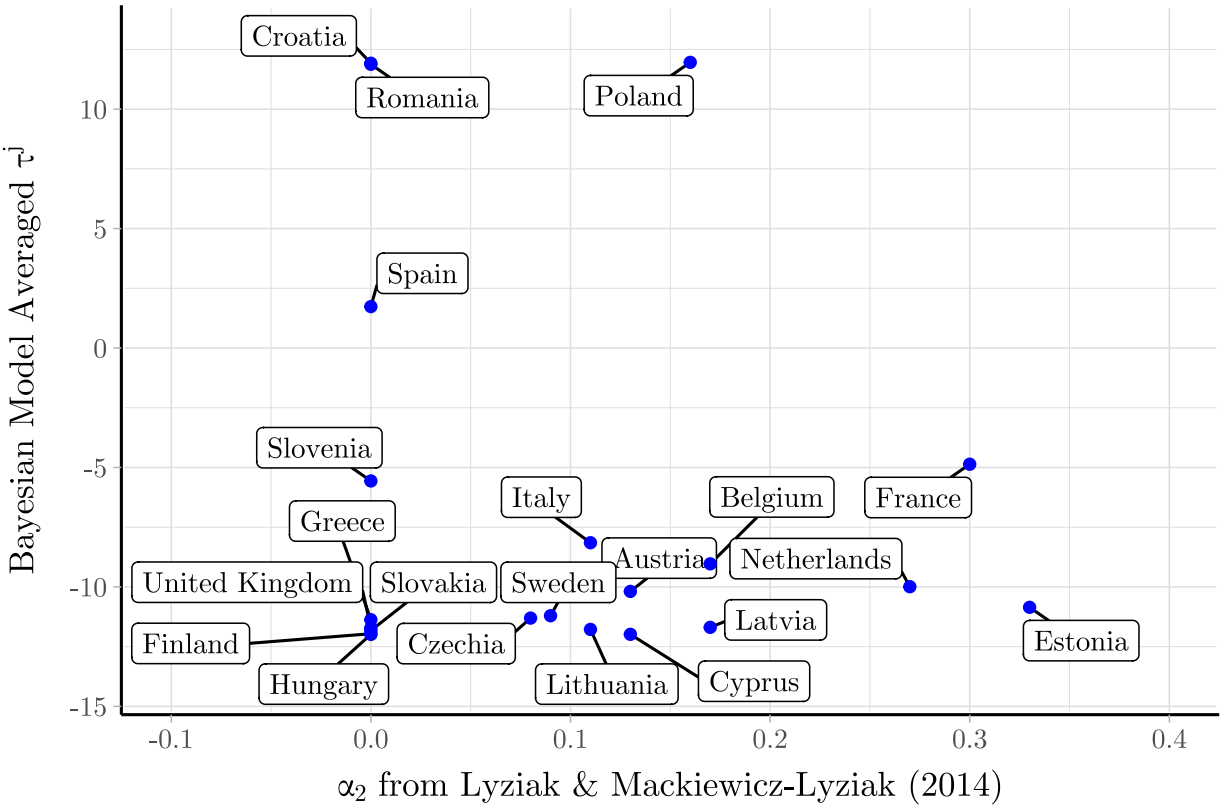


Figure 3.1. Comparing forward/backward-looking classifications of BMA to the method in Lyziak & Mackiewicz-Lyziak (2014) (May 2001 to July 2012)

There is a weak negative relation between the estimated parameters methods, which is expected, and shows agreement between the models—negative τ's and high α countries are considered forward-looking by both methods. The Lyziak method of categorization seems to favor backward-looking classification—about $1/3^{\text{rd}}$ of countries yield the corner solution of $\alpha_2 = 0$—while the BMA method appears to favor forward-looking classification—only 4 countries are categorized as backward-looking in this time frame. However, when each method categorizes a country away from its apparent *tilt*, the other method agrees with that classification, with Poland

as an obvious outlier.

Differences may also stem from different methods of quantifying expectations, where Lyziak & Mackiewicz-Lyziak (2014) uses probability method, along with many assumptions regarding the distribution of expectations and what comprises "*moderate inflation.*" It should be noted that the BMA result agrees with Gerberding (2009) and Lyziak (2010) that European countries' expectations are generally forward-looking.

Turning to the time sample of January 2010 to December 2019, the BMA method is applied. The results are reported in Figure 3.2 and Table 3.1

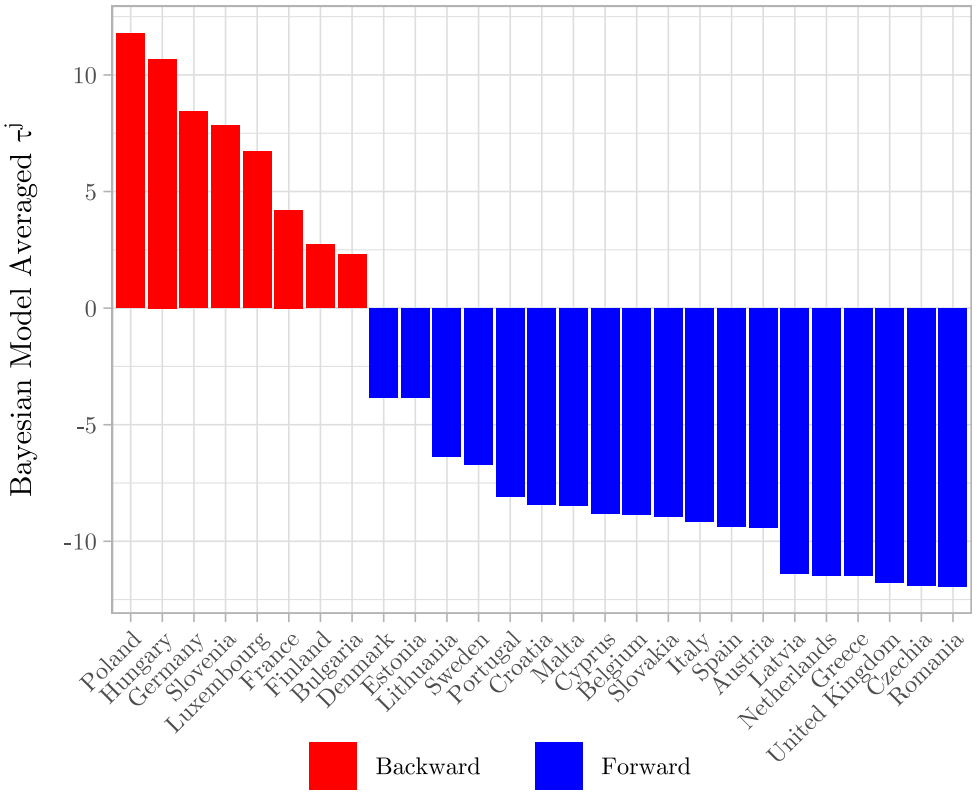


Figure 3.2. BMA estimated τ^j (January 2010 to December 2019)

Country (j)	Estimated τ^j
Poland	11.79
Hungary	10.68
Germany	8.42
Slovenia	7.82
Luxembourg	6.72
France	4.21
Finland	2.71
Bulgaria	2.28
Denmark	-3.81
Estonia	-3.85
Lithuania	-6.37
Sweden	-6.71
Portugal	-8.09
Croatia	-8.41
Malta	-8.46
Cyprus	-8.82
Belgium	-8.86
Slovakia	-8.93
Italy	-9.17
Spain	-9.37
Austria	-9.41
Latvia	-11.38
Netherlands	-11.48
Greece	-11.49
United Kingdom	-11.76
Czechia	-11.91
Romania	-11.92

Table 3.1. Countries and their estimated τ^j for the time period 2010-2019.

3.5 Balance Tests

An understandable concern with this identification method is endogeneity between economic conditions and the forward-lookingness of expectations. For example, if countries with lower or more stable inflation were categorized as more backward-looking, it would be fallacious to conclude from impulse response function analysis that backward-looking expectations are responsible for the low and stable inflation response to a monetary expansion.

Here, we test the relationships between the estimated τ^j's and the relevant variables in the SVAR estimation via simple regression estimation. The explanatory variables are the means and standard deviations of the relevant variables to test the relationship between levels and variability

of the variables with the forward-lookingness of expectations. The mean and standard deviation transformations are calculated *after* the variable transformations discussed in Section **3.6.1**.

Table 3.2 reports the regression coefficient estimates and respective standard deviations. This is similar to a "balance test" to show that treatment and control groups are comparable in observable variables. Statistically insignificant coefficient estimates imply that different economic conditions do not affect the forward-lookingness of expectations.

Variable	Interest	Output	Price	Energy	Money
Mean	2.257	-32.03	-40.37	-26.058	0.7979
	(1.713)	(43.38)	(231.43)	(18.880)	(0.7549)
Standard Deviation	4.389	10.663	-45.024	-26.007	31.540
	(3.330)	(34.420)	(135.546)	(80.828)	(24.826)

Table 3.2. Balance test between τ^j and relevant variables.

All estimated relationships are statistically insignificant. Therefore, levels of interest rates, and log-levels of output, price, energy-price, and money supply, as well as standard deviations in all these variables, are not associated with countries' expectations' forward-lookingness.

3.6 Country Level Model

Historically, literature examining regional heterogeneous effects of monetary policy use Structural VAR (SVAR) models, as is the case in Carlino & DeFina (1998) and Giacinto (2003). Now that we have categorized countries as being forward or backward-looking in their expectations, we inquire into whether these groups of countries react differently to monetary expansions, and what those differences look like. Anzuini & Levy (2007) similarly compares how recent additions to the EU react differently to monetary policy shocks than older member countries. This section employs a comparable method, using an SVAR model of each country included in this sample, and recording the impulse response of output and price level to a positive shock to the country's money supply.

3.6.1 Structural Vector Autoregression

Consider an economy (j) with policy interest rate $i_{j,t}$, industrial production $Y_{j,t}$, price level $P_{j,t}$, energy price $X_{j,t}$, and a money supply in the local currency of $M_{j,t}$. All variables (with the exception of the interest rate) are translated to log-levels.[18]

Let $Z_t = [i_{j,t}\ y_{j,t}\ p_{j,t}\ x_{j,t}\ m_{j,t}]'$. In total, we have $K = 5$ variables. The SVAR model is:

$$AZ_t = \left(\tilde{C}_1 Z_{t-1} + ... + \tilde{C}_q Z_{t-q}\right) + B\varepsilon_t$$

where $\varepsilon \sim N(0, I_K)$. The matrices A and B describe the contemporaneous relationships between variables and disturbances, respectively.

$$Z_t = C_1 Z_{t-1} + ... + C_q Z_{t-q} + v_t \tag{3.4}$$

Where $C_i = A^{-1}\tilde{C}_i$ and $v_t = A^{-1}B\varepsilon_t$. Using the lag operator $Z_t L^p = Z_{t-p}$, (3.4) can be simplified to:

$$Z_t = \left[I_K - C_1 L - ... - C_q L^q\right]^{-1} v_t = \left(\sum_{s=0}^{\infty} \Phi_s\right) v_t$$

and finally:

$$Z_t = \sum_{s=0}^{\infty} \Psi_s \varepsilon_{t-s} \tag{3.5}$$

Equation (3.5) yields the impulse response function for this model, where $\Psi_s = \Phi_s A^{-1} B$. The optimal number of lags between 1 and 10 is chosen according to the SIC as previously defined. We use the same structural restrictions as in Anzuini & Levy (2007). Recall that $v_t = C_B \varepsilon_t$, where $C_B = A^{-1}B$. These restrictions are of the form:

[18] Specifically, $y_{j,t} = \log(Y_{j,t})$, $p_{j,t} = \log(P_{j,t})$, $x_{j,t} = \log(X_{j,t})$, and $m_{j,t} = \log(M_{j,t})$

$$\begin{bmatrix} v^i_{j,t} \\ v^y_{j,t} \\ v^p_{j,t} \\ v^x_{j,t} \\ v^m_{j,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & c_{1,3} & 0 & c_{1,5} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & c_{3,2} & 1 & 0 & 0 \\ c_{4,1} & c_{4,2} & c_{4,3} & 1 & c_{4,5} \\ c_{5,1} & c_{5,2} & c_{5,3} & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon^i_{j,t} \\ \varepsilon^y_{j,t} \\ \varepsilon^p_{j,t} \\ \varepsilon^x_{j,t} \\ \varepsilon^m_{j,t} \end{bmatrix}$$

The first line is a money supply equation within each country. The second line represents adjustment costs in industrial production. The third is a NK Phillips Curve. The fourth uses these variables' relation to energy price as an arbitrage equation, as in Anzuini & Levy (2007). The final line represents money supply within country j.

There are many noticeable differences between this open-economy New Keynesian SVAR and the OLG network model considered in Chapter II of this dissertation. However, they are fundamentally answering the same question: How do expectation formation methods determine regional effects of monetary expansions? Therefore, we believe the conclusions from the OLG and the SVAR models are both relevant for this inquiry.

More detail on the data sources is provided in Appendix **C.1.**.

3.6.2 Impulse Response Results

The impulse response functions (IRFs) are categorized according to whether the country of interest has an estimated $\tau > 0$ (backward-looking) or $\tau < 0$ (forward-looking). The estimated IRF for group G in period $t \geq 0$ for all countries j in group G is computed as:

$$IRF^G_t(z) = |G|^{-1} \sum_{j \in G} IRF_t(z_j) \tag{3.6}$$

Where z_j is the variable of interest (here, either industrial production or inflation: $z_j \in \{y_j, p_j\}$) in country j. The IRFs below are in response to country-specific expansions of the money supply of 10%. The group-averaged IRFs are shown in Figure 3.3.

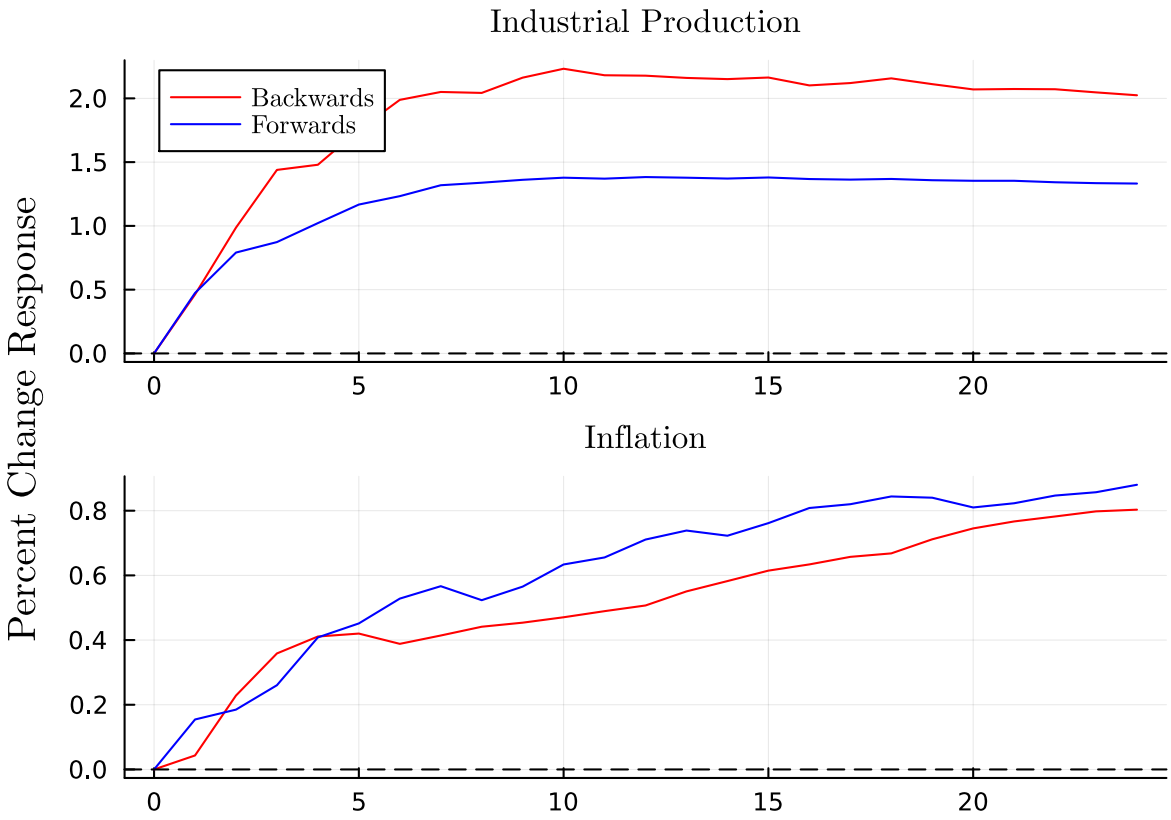


Figure 3.3. SVAR IRFs of Industrial Production Growth and Inflation, respectively, in percent.

This figure displays supportive evidence for the theoretical results: monetary expansions in backward-looking regions yields greater effects on output, while monetary expansions in forward-looking regions yield lesser effects on output. Unlike in the OLG model, forward-looking countries see positive effects on output, which, as discussed in Section **2.1**, is the conventional response. The responses on the inflation side appear comparable, with slightly higher values among forward-looking countries, as the OLG model predicts.

The responses within the individual countries to their respective money supply shocks (for all 27 countries) can be found in **C.2.** of the Appendix. As is clear from those figures, all backward-looking countries experience positive responses to industrial production, while forward-looking countries experience mixed responses. On the inflation side, both groups experience mixed responses.

3.7 Aggregated Model

Here we consider an empirical model that is more comparable to the OLG model. In this model, all countries in the sample as belonging to one aggregate economy. We shock the money supplies of each country, and observe how the *aggregate* economy's output and price level reacts to these shocks. However, we have no reason to expect that these reactions will differ from those in the theoretical model. The results of Section **2.5** show that the OLG model with heterogeneous agents will still see higher inflation and lower output in reaction to a money supply expansion, the larger the portion of rational agents in the economy.

Consider an aggregate economy made up of the J regions considered in Section **3.6**. Aggregate industrial production and price level are computed as averages (prior to log-transformations) across the j regions. The interest rate is the policy rate set by the European Central Bank, and the energy variable is an aggregated variable, as explained in **C.1.** of the Appendix. The VAR of interest is determined by equation (3.5) with logged variables $Z_t = [i_t \; y_t \; p_t \; x_t \; m_{1,t} \; ... \; m_{J,t}]'$. Therefore, we're including aggregate estimates of the interest rate, industrial production, the price level, and energy price, but with individual money supply estimates from the sample countries. In total, we have $K = J + 4$ variables.

3.7.1 NISVAR

Structural VAR models impose theoretically informed restrictions on the model. Since this model is motivated by the OLG network economy in Chapter II, where money flows through the connected network, a network restriction on money transit between countries is apt. Here, we add a network-informed layer to the SVAR restriction matrix in the previous section. Comparably, the Network Informed Structural VAR (NISVAR) restrictions are described as the vector v_t relates to the vector ε_t:

$$\begin{bmatrix} v_t^i \\ v_t^y \\ v_t^\pi \\ v_t^x \\ v_{1,t}^m \\ \vdots \\ v_{J,t}^m \end{bmatrix} = \begin{bmatrix} 1 & 0 & c_{1,3} & c_{1,4} & & a_i^M & \\ 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & c_{3,2} & 1 & 0 & 0 & \dots & 0 \\ c_{4,1} & c_{4,2} & c_{4,3} & 1 & & a_x^M & \\ & & & 0 & & & \\ a_M^i & a_M^y & a_M^\pi & \vdots & & T_M & \\ & & & 0 & & & \end{bmatrix} \begin{bmatrix} \varepsilon_t^i \\ \varepsilon_t^y \\ \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_{1,t}^m \\ \vdots \\ \varepsilon_{J,t}^m \end{bmatrix}$$

The final J lines are money demand equations for each country, showing how each region reacts to changes from the monetary authority, output, inflation, and as money is traded across regions. The column vectors a_M^i, a_M^y, and a_M^π, where $a_M^i = [a_{M,1}^i \ \dots \ a_{M,J}^i]'$ (and likewise for a_M^y and a_M^π), represent reactions of the money supply to the policy rate, aggregate output, and aggregate inflation by each country, respectively.

The $J \times J$ matrix T_M captures the spatial passage of money across borders between countries. It has values of 1 on the diagonal, and coefficient $c_{j+4,k}$ as the element in the j^{th} row and k^{th} column of the matrix *if* regions j and k are *sufficient* trade partners, and the element is 0 otherwise. By "sufficient" is meant the number of trade partners included is such that the model is *just identified*. With 27 countries, $K = 31$, the model will be just identified with $K(K+1)/2 = 496$ restrictions. After the above restrictions are applied, including the 12 closest trade partners for each country in the model (including itself makes 13 restrictions per country,) the sufficient number of restrictions is met. These trading partners are determined by column of T_M as the top 12 importers in 2015, according to the OEC, by import value. As a top importer to country k, $c_{j+4,k}$ determines how much of the money shock in country j is due to imports to country k (thereby, money is exported to country j). This allows the model to capture the flow of money between the regions, similar to the transition matrix A in the OLG model from Chapter II.

3.7.2 Impulse Response Results

The optimal number of lags for the VAR is 3 as determined by the SIC. More detail on the data used is available in **C.2.** of the Appendix. The IRFs below are in response to country-specific expansions of the money supply of 10%, grouped again according to equation (3.6). The group-summed IRFs are displayed in Figure 3.4, for shocks to forward and backward-looking countries.

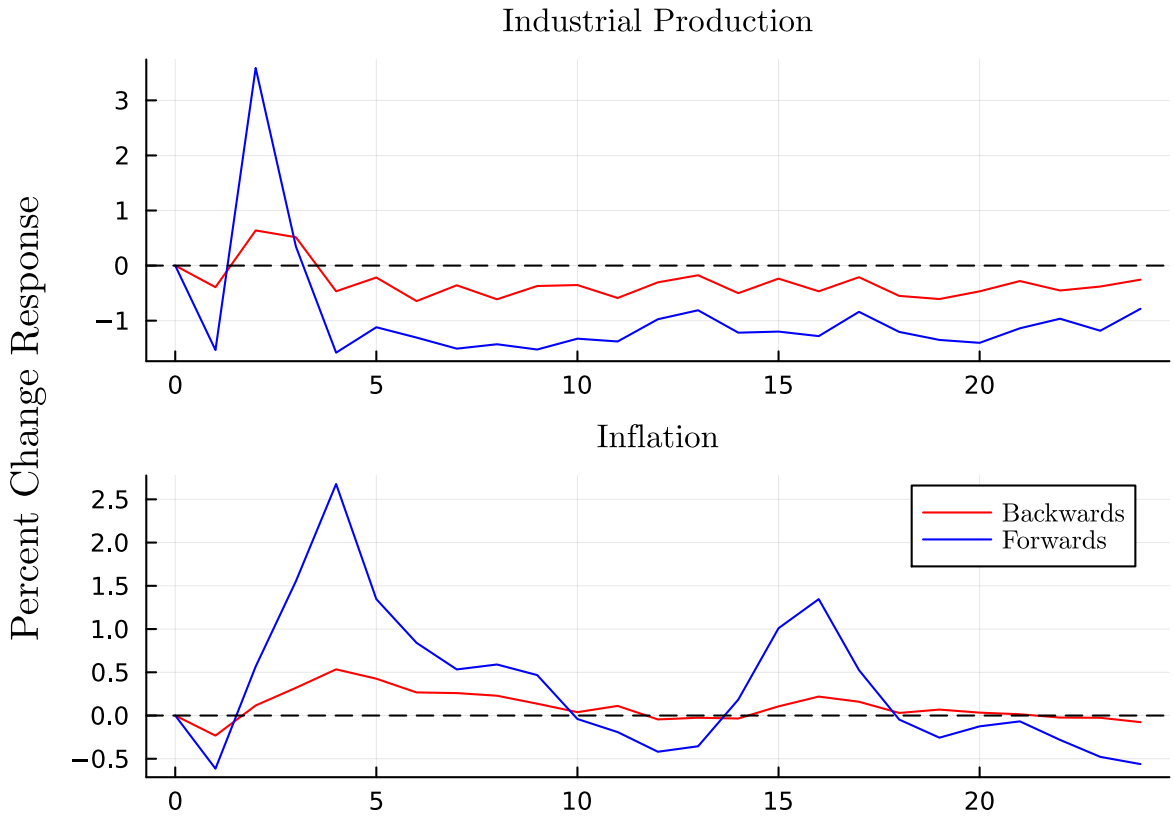


Figure 3.4. SVAR IRFs of Aggregate Industrial Production Growth and Inflation, respectively, in percent.

Responses to individual country money supply shocks (for all 27 countries) can be found in **C.3.** of the Appendix. The industrial production IRFs appear less clearly monotonic or trend inducing as in the country-level cases. The inflation IRFs tell a story that appears to agree with the theoretical narrative: shocks to forward-looking countries yield greater positive responses to the price level, whereas the shocks to backward-looking countries are far less determinate.

3.8 Discussion

The empirical exercises above display supportive evidence for the theoretical results, namely that country-specific monetary expansions will yield different results that depend on the expectations of those receiving the money. In the Country and Aggregate-level models, the inflation impulse responses match the theory closely. As the OLG model predicted, when forward-looking agents receive a monetary expansion, the aggregate economy experiences higher (on average) and more volatile inflation than when that expansion is given to backward-looking economies.

On the industrial production side, the models are in disagreement with each other regarding the sign of the response. The Country-Level model agrees with the theoretical model in showing backward-looking countries having *greater* positive expansions, though unlike the model, it also shows forward-looking countries responding positively to the shock. The Aggregated Model shows that monetary expansions to individual countries yield negative responses in Industrial Production. This matches the OLG model for the forward-looking countries, and agrees with the theoretical model in the sense of the response being higher among backward-looking countries.

The incongruity of the results may be due to the cointegration of the money supply variables. For example, 22 of the sample countries have Germany ranked as their number 1 importer, (4 as their 2nd, and 1 as its 4th,) and Germany is categorized a backward-looking country, meaning regardless of the country experiencing the expansion, much of this money will soon flow to Germany, where backward-looking expectations will theoretically react by increasing output. Heterogeneity of expectations within countries could be another factor for this positive response.

The conclusion that forward-looking countries will experience higher inflation may appear to disagree with Lyziak & Mackiewicz-Lyziak (2014), which finds that forward-looking economies will not experience high inflation. Even though the authors report that this correlation is weak and note two exceptions—Estonia and Latvia—our results are not necessarily in opposition to such a conclusion. Their paper refers to measured trend inflation, whereas we are considering positive monetary shocks to these forward-looking countries' money supply. Existing theoretical and empirical results present no claims about the relationship between forward-lookingness and

inflation outside of reactions to monetary shocks. Regardless of the venn diagram of conclusions reached by these models, the responses appear different between the groups.

Coricelli (2006) considers an "*expectations channel*" in monetary transmission in the first stages of a monetary policy change, as expectations are responsible for how the yield curve reacts to policy. The analysis provided here suggests that the expectation channel expands beyond the bond market. Anyone interested in the effects of a monetary policy change ought to consider the expectations of those affected, as these play a critical role in the economy's response.

Future research should consider how expectations change after a monetary shock of the form considered in this Chapter hits an economy. In a laboratory setting, Petersen (2015) finds statistically insignificant responses of expectations to monetary policy shocks. The assumption that expectation formation methods remain consistent before and after a monetary shock may have large implications for how an economy would respond. Empirical analyses of expectation data have shown that agents' forecasting functions change over time (Branch 2004, 2007), and as Lyziak & Mackiewicz-Lyziak (2014) shows, European countries became more forward-looking after the financial crisis, implying economic events have real effects on how expectations form. An endogenous forecast switching model à la Brock & Hommes (1997) in a spatial setting may be a helpful theoretical starting point for future researchers.

Arguably, the most important stepping stone needed to advance this research is more spatially-informed expectation data. Including more detail regarding surveyees' location will be of great benefit to the macroeconomic understanding of how expectations are formed and how responses to economic policies depend on the expectations of affected agents.

3.9 Conclusion

Monetary expansions are less effective at increasing output in regions that employ rational expectations than in regions utilizing backward-looking expectation formation schemes. This Chapter presents this hypothesis in a spatial overlapping generations model and with empirical analysis. The theoretical model compares the economic responses to a helicopter money supply expansion

when agents employ rational or adaptive expectations. We find strong theoretical evidence for this hypothesis.

We use EU expectation data to categorize countries into forward/backward-looking, and estimate impulse response functions using two SVAR models to document economic output and inflation responses to country-specific money supply expansions. Looking at expansions within countries, we find support for the theoretical conclusion that monetary policy shocks to regions with forward-looking expectations increase output *less* and increase prices *more* than monetary policy shocks to regions with backward-looking expectations. The Network-Informed SVAR imposes a trade network restriction on the model, and finds that money supply expansions in forward-looking regions yield greater effects on prices and greater *negative* effects on output, further supporting the theoretical results.

Duran & Karahasan (2022) considers spatial heterogeneity to be "a severe issue that reduces the effectiveness of monetary policy." While there will always be winners and losers of monetary policy, the analysis in this Chapter implies that a strategic Central Bank could potentially target locations where agents are backward-looking for monetary policy to yield optimal economic output and inflation responses—though, issues of equity and distribution ought to be considered before such policy is realized to ensure desired increases in welfare.

Appendix A

A.1 Proof of Proposition 1.1.

Let $\mu = \frac{A}{B}$, $\alpha = -\frac{C}{B}$, and $\tilde{\varepsilon}_t = \frac{1}{B}\varepsilon_t$. Without loss of generality, we can set the intersection of supply and demand as the new origin, $\mu = 0$. Plugging (1.5) into (1.4) gives us the *actual law of motion* (ALM):

$$p_t = \alpha \sum_j \theta_j \left[a^j + b^{j\prime} \hat{w}^j_{t-1} \right] + \gamma \sum_j \theta_j w^j_{t-1} + \tilde{\varepsilon}_t \tag{A.1}$$

Where $\hat{w}^j_{t-1} = M_j \circ w_{t-1}$, where M_j is the j^{th} column of M, and $\circ$ is the Hadamard (element-wise) product operator. To be clear, $\hat{w}^j_{t-1}$ is the vector of weathers observed by island j at time $t-1$, and w^j_{t-1} is the numerical value representing the weather on island j at time $t-1$.
Dropping the $t-1$ subscripts for ease of notation, we can simplify the function for the market price to:

$$\begin{aligned}
p_t = {} & \alpha\theta_1 \left[a^1 + b^1_1 M_{1,1} w^1 + b^1_2 M_{1,2} w^2 + \ldots + b^1_J M_{1,J} w^J \right] + \\
& \alpha\theta_2 \left[a^2 + b^2_1 M_{2,1} w^1 + b^2_2 M_{2,2} w^2 + \ldots + b^2_J M_{2,J} w^J \right] + \\
& \ldots \\
& \alpha\theta_J \left[a^j + b^J_1 M_{J,1} w^1 + b^J_2 M_{J,2} w^2 + \ldots + b^J_J M_{J,J} w^J \right] + \\
& \gamma \left[\theta_1 w^1 + \theta_2 w^2 + \ldots + \theta_J w^J \right] + \tilde{\varepsilon}_t \\
= {} & \alpha \left[\theta_1 a^1 + \theta_2 a^2 + \ldots + \theta_J a^J \right] + \\
& w^1 \left[\alpha(\theta_1 b^1_1 M_{1,1} + \theta_2 b^2_1 M_{2,1} + \ldots + \theta_J b^J_1 M_{J,1}) + \theta_1 \gamma \right] + \\
& w^2 \left[\alpha(\theta_1 b^1_2 M_{1,2} + \theta_2 b^2_2 M_{2,2} + \ldots + \theta_J b^J_2 M_{J,2}) + \theta_2 \gamma \right] + \\
& \ldots \\
& w^J \left[\alpha(\theta_1 b^1_J M_{1,J} + \theta_2 b^2_J M_{2,J} + \ldots + \theta_J b^J_J M_{J,J}) + \theta_J \gamma \right] + \tilde{\varepsilon}_t
\end{aligned}$$

Let $\mathcal{B} = (b^1 b^2 \ldots b^J)'$, meaning $\mathcal{B}$ is a $J \times J$ matrix where row j is the vector b^j and column k is

the vector of beliefs all firms have regarding island k's effect on the market price.

$$\mathcal{B} = \begin{bmatrix} b_1^1 & b_2^1 & \dots & b_J^1 \\ b_1^2 & b_2^2 & \dots & b_J^2 \\ \vdots & & \ddots & \\ b_1^J & b_2^J & \dots & b_J^J \end{bmatrix}$$

Setting $\mathcal{A} = [a^1\ a^2\ ...\ a^j]'$, and $\theta' = [\theta_1, ..., \theta_J]$ the ALM can therefore be simplified to:

$$p_t = \alpha\theta'\mathcal{A} + \theta'\left[\alpha\left(\mathcal{B} \circ M\right) + \gamma I_J\right] w_{t-1} + \tilde{\varepsilon}_t \tag{A.2}$$

Q.E.D.

A.2 Proof of Proposition 1.2.

Begin by considering the PLM firms possess, as determined by (1.5).

In the case of incomplete information (i.e. $\hat{w}^j \neq w$ for at least one island $j \in \{1, J\}$), the PLM is missing data present in the ALM, meaning the forecasting method is *misspecified*. To calculate the *T-mapping* of the beliefs, we need to project the ALM onto the available information. For firms on the island j, using the orthogonality condition that the regression error, $(p_t - T(a^j) - T(b^j)' \cdot \hat{w}^j_{t-1})$ where $T(a^j,\ b^j)$ is the mapping of beliefs for firms on Island j, must be uncorrelated with the exogenous variables in a least-squares learning context, we get:

$$\begin{aligned} &E\left[p_t - T(a^j) - T(b^j)' \cdot \hat{w}^j_{t-1}\right] = 0 \\ &E\left[\left(p_t - T(a^j) - T(b^j)' \cdot \hat{w}^j_{t-1}\right)\hat{w}^{j\prime}_{t-1}\right] = 0_{1\times J} \\ &E\left[\left(\alpha\theta'\mathcal{A} + \theta'\left[\alpha\mathcal{B} + \gamma I_J\right] w_{t-1} - T(a^j) - T(b^j)' \cdot \hat{w}^j_{t-1}\right)\hat{w}^{j\prime}_{t-1}\right] = 0_{1\times J} \end{aligned}$$

With E being the mathematical expectations operator. Since $\tilde{\varepsilon}_t$ is mean 0, we can eliminate it from this step. We replace 0 with the $1 \times J$ zero-vector in the second line because we multiply by the transpose of the $J \times 1$ vector $\hat{w}^j_{t-1}$.

First, we will deal with the constant term. This projection simplifies to:

$$T(a^j) = \alpha\theta'\mathcal{A}$$

A fixed point in this T-map $\forall\ j \in J$ will yield the vector a in equilibrium. Since this constant does not depend on information, its equilibrium value will be the same for all islands. This is a fixed point $\bar{a}$ such that:

$$T(\bar{a}) = \alpha\left[\theta_1\bar{a} + \theta_2\bar{a} + \ldots\theta_J\bar{a}\right]$$

Since $0 < \theta_j < 1$ and $\sum_j \theta_j = 1$, there are *no* non-zero solutions for $\bar{a}$ as long as $\alpha \neq 1$. Therefore, in the RPE, $\bar{a} = 0$ and $\mathcal{A} = 0_{J\times 1}$.

Now we can consider the T-map for beliefs b^j. Consider the difference between $(w \cdot \hat{w}^{j\prime})$ and $(\hat{w}^j \cdot \hat{w}^{j\prime})$. Recall that $\hat{w}^j = (w \circ M_j)$, where M_j is the j^{th} row of M, and that $E\ (w \cdot w') = \Sigma$.

$$\begin{aligned} E\ (\hat{w}^j \cdot \hat{w}^{j\prime}) =& E\ (w \circ M_j) \cdot (w \circ M_j)' \\ & E\ (w \cdot w') \circ (M_j \cdot M_j') \\ & \Sigma \circ (M_j \cdot M_j') \end{aligned}$$

$$\begin{aligned} E\ (w \cdot \hat{w}^{j\prime}) =& E\ (w \circ 1_{J\times 1}) \cdot (w \circ M_j)' \\ & E\ (w \cdot w') \circ (1_{J\times 1} \cdot M_j') \\ & \Sigma \circ (1_{J\times 1} \cdot M_j') \end{aligned}$$

Note that since the values of elements of M_j are either 1 or 0, then the element $(M_j \cdot M_j')_{l,k} = 1$ *IFF* $M_{j,l} = M_{j,k} = 1$. The T-map further simplifies to:

$$\theta'\left[\alpha\mathcal{B} + \gamma I_J\right]\left[\Sigma \circ (1_{J\times 1} \cdot M_j')\right] - T(b^j)'\left[\Sigma \circ (M_j \cdot M_j')\right] = 0_{1\times J}$$

$$\Rightarrow T(b^j)'\left[\Sigma \circ (M_j \cdot M_j')\right] = \theta'\left[\alpha\mathcal{B} + \gamma I_J\right]\left[\Sigma \circ (1_{J\times 1} \cdot M_j')\right] \tag{A.3}$$

In cases of incomplete information, we cannot invert $\left[\Sigma \circ (M_j \cdot M_j')\right]$, since incomplete in-

formation requires $M_{j,k} = 0$ for at least one pair j, k. In such a case, the k^{th} row *and* column of $(M_j \cdot M_j')$ must be 0's, so $\det(M_j \cdot M_j') = 0$ and the matrix not invertable.

To get the T-map by itself, take the *Moore–Penrose Pseudo-inverse* of $\left[\Sigma \circ (M_j \cdot M_j')\right]$, call this $\left[\Sigma \circ (M_j \cdot M_j')\right]^+$. Multiply this on both sides of (A.3). The LHS of (A.3) equation becomes:

$$T(b^j)' \left[\Sigma \circ (M_j \cdot M_j')\right] \left[\Sigma \circ (M_j \cdot M_j')\right]^+ = T(b^j)'$$

This is because $\left[\Sigma \circ (M_j \cdot M_j')\right] \cdot \left[\Sigma \circ (M_j \cdot M_j')\right]^+$ is a $J \times J$ amended identity matrix where the diagonal element on the k^{th} row/column = $M_{j,k}$. This is a general result of pseudo-inverses of square matrices with symmetric columns/rows of 0's. Multiplying the LHS by this amended identity matrix yields $T(b^j)'$ in equilibrium because if $M_{j,k} = 0$, then $T(b_k^j) = 0$.

Therefore, the projection can be simplified to:

$$T(b^j)' = \theta' \left[\alpha \mathcal{B} + \gamma I_J\right] \left[\Sigma \circ (1_{J\times 1} \cdot M_j')\right] \left[\Sigma \circ (M_j \cdot M_j')\right]^+ \tag{A.4}$$

Stack the T-maps of the individual islands to get the aggregate T-map: $T(\mathcal{B})' = (T(b^1)\ T(b^2)\ \ldots\ T(b^J))$ and define the matrices Θ and $\Theta_{\mathcal{B}}$ as follows:

$$\Theta \equiv \begin{bmatrix} \left[\Sigma \circ (1_{J\times 1} \cdot M_1')\right] \left[\Sigma \circ (M_1 \cdot M_1')\right]^+ \\ \left[\Sigma \circ (1_{J\times 1} \cdot M_2')\right] \left[\Sigma \circ (M_2 \cdot M_2')\right]^+ \\ \vdots \\ \left[\Sigma \circ (1_{J\times 1} \cdot M_J')\right] \left[\Sigma \circ (M_J \cdot M_J')\right]^+ \end{bmatrix}$$

$$\Theta_{\mathcal{B}} \equiv (I \otimes \theta' \mathcal{B})\Theta$$

Therefore:

$$T(\mathcal{B}) = \alpha \Theta_{\mathcal{B}} + \gamma (I \otimes \theta')\Theta \tag{A.5}$$

We know that $T(b_k^j) = 0\ \forall\ j, k,$ such that $M_{j,k} = 0$. Meaning $T(\mathcal{B})$ is of the form $(\mathcal{B} \circ M)$. The RPE of this economy is pinned down by the equilibrium beliefs matrix $\mathcal{B}^E = \left(\mathcal{B}^E \circ M\right)$ that

solves:

$$\mathcal{B}^E = \alpha(I \otimes \theta' \mathcal{B}^E)\Theta + \gamma(I \otimes \theta')\Theta \tag{A.6}$$

This can be simplified further. First, solve for $\theta' \mathcal{B}^E$:

$$\begin{aligned}
\theta' \mathcal{B}^E &= \alpha(\theta' \otimes \theta' \mathcal{B}^E)\Theta + \gamma(\theta' \otimes \theta')\Theta \\
\theta' \mathcal{B}^E &= \alpha(\theta' \mathcal{B}^E)(\theta' \otimes I)\Theta + \gamma(\theta' \otimes \theta')\Theta \\
\theta' \mathcal{B}^E [I - \alpha(\theta' \otimes I)\Theta] &= \gamma(\theta' \otimes \theta')\Theta \\
\theta' \mathcal{B}^E &= \gamma(\theta' \otimes \theta')\Theta[I - \alpha(\theta' \otimes I)\Theta]^{-1} \equiv \theta^{\mathcal{B}}
\end{aligned}$$

Plug this back into (A.6), with simplification, gives us (1.7):

$$\mathcal{B}^E = \Big(\alpha(I \otimes \theta^{\mathcal{B}}) + \gamma(I \otimes \theta')\Big)\Theta, \quad \theta^{\mathcal{B}} \equiv \gamma(\theta' \otimes \theta')\Theta[I - \alpha(\theta' \otimes I)\Theta]^{-1}$$

Since all of the matrices and vectors in (1.7) are unique (including the pseudo-inverse matrices in Θ), the equilibrium beliefs matrix $\mathcal{B}^E$ is unique.

Q.E.D.

A.3 Proof of Proposition 1.3

The E-stability ODE of this system is:

$$\dot{\Big(\mathcal{A}, \mathcal{B}\Big)} = T\Big(\mathcal{A}, \mathcal{B}\Big) - \Big(\mathcal{A}, \mathcal{B}\Big)$$

$$\dot{\Big(\mathcal{A}, \mathcal{B}\Big)} = \Big(\alpha\theta' \mathcal{A},\ \alpha(I \otimes \theta' \mathcal{B})\Theta + \gamma(I \otimes \theta')\Theta\Big) - \Big(\mathcal{A}, \mathcal{B}\Big)$$

Vectorize $T(\mathcal{B})$ to get:

$$\begin{aligned}\text{vec}(T(\mathcal{B})) &= \alpha\text{vec}\Big((I\otimes\theta'\mathcal{B})\Theta\Big)+\gamma\ \text{vec}\Big((I\otimes\theta')\Theta\Big)\\ &= \alpha(\Theta'\otimes I)\text{vec}\Big(I\otimes\theta'\mathcal{B}\Big)+\gamma\ \text{vec}\Big((I\otimes\theta')\Theta\Big)\\ &= \alpha(\Theta'\otimes I)\Phi\text{vec}(I)(I\otimes\theta')\text{vec}(\mathcal{B})+\gamma\ \text{vec}\Big((I\otimes\theta')\Theta\Big)\end{aligned}$$

Where Φ is the commutation matrix that solves: $\text{vec}(I\otimes\theta'\mathcal{B}) = \Phi(\text{vec}(I)\otimes\theta'\mathcal{B})$. Note that $\theta'\mathcal{B} = (I\otimes\theta')\text{vec}(\mathcal{B})$.

Differentiating tells us that the RPE is stable as long as the eigenvalues of the following $J^2\times J^2$ matrix have negative real parts:

$$\frac{d\text{vec}(T)}{d\text{vec}(\mathcal{B})} = \alpha(\Theta'\otimes I)\Phi\text{vec}(I)(I\otimes\theta') - I_{J^2}$$

Q.E.D.

A.4 Proof of Proposition 1.4.

We can compute the mathematical expectation of any belief coefficient vector y^j as estimated by RLS, using the following steps:

First, let β^j be the parameter coefficient of w^j on price p as in (1.6):

$$\beta_j = \alpha(\theta_1 b_j^1 M_{1,j} + \theta_2 b_j^2 M_{2,j} + \ldots + \theta_J b_j^J M_{J,j}) + \theta_j\gamma \tag{A.7}$$

Then, the beliefs vector b^j and constant a^j held by agents on the island j will solve the following system of equations, as is required by OLS:

$$E\ [\alpha\left[\theta_1 a^1+\theta_2 a^2+...+\theta_J a^J\right]+\beta_1 w_{1,t}+\beta_2 w_{2,t}+...+\beta_J w_{J,t}+\tilde{\epsilon}$$
$$-a^j-M_{j,1}b_1^j w_{1,t}-M_{j,2}b_2^j w_{2,t}-...-M_{j,J}b_J^j w_{J,t}]=0,$$

$$E\ [M_{j,1}w_{1,t}(\alpha\left[\theta_1 a^1+\theta_2 a^2+...+\theta_J a^J\right]+\beta_1 w_{1,t}+\beta_2 w_{2,t}+...+\beta_J w_{J,t}+\tilde{\epsilon}$$
$$-a^j-M_{j,1}b_1^j w_{1,t}-M_{j,2}b_2^j w_{2,t}-...-M_{j,J}b_J^j w_{J,t})]=0,$$
$$\vdots \qquad\qquad \vdots$$
$$E\ [M_{j,J}w_{J,t}(\alpha\left[\theta_1 a^1+\theta_2 a^2+...+\theta_J a^J\right]+\beta_1 w_{1,t}+\beta_2 w_{2,t}+...+\beta_J w_{J,t}+\tilde{\epsilon}$$
$$-a^j-M_{j,1}b_1^j w_{1,t}-M_{j,2}b_2^j w_{2,t}-...-M_{j,J}b_J^j w_{J,t})]=0$$

As showed previously, $a^j \to 0$. Recall that $E[w_j w_k]=\sigma_{j,k}$, $M_{j,k}\times M_{j,k}=M_{j,k}$, and $M_{j,k}b_k^j=b_k^j$. Keeping these in mind, the above equations simplify to:

$$b_1^j=M_{j,1}\left[\beta_1+\frac{\sigma_{12}}{\sigma_1}(\beta_2-M_{j,2}b_2^j)+...+\frac{\sigma_{1J}}{\sigma_1}(\beta_J-M_J^j b_{j,J})\right]$$
$$b_2^j=M_{j,2}\left[\frac{\sigma_{12}}{\sigma_2}(\beta_1-M_{j,1}b_1^j)+\beta_2+...+\frac{\sigma_{2J}}{\sigma_2}(\beta_J-M_{j,J}b_J^j)\right]$$
$$\vdots \qquad\qquad \vdots$$
$$b_J^j=M_{j,J}\left[\frac{\sigma_{1J}}{\sigma_J}(\beta_1-M_{j,1}b_1^j)+\frac{\sigma_{2J}}{\sigma_J}(\beta_2-M_{j,2}b_{j,2})+...+\beta_J\right]$$

These, along with $\beta_1,\beta_2,...,\beta_J$ as defined by (A.7) provide a system of equations that yield RPE beliefs. Note that this solution is unique and varies greatly by J and M.

Q.E.D.

A.5 Proof of Proposition 1.5.

Firstly, note that beliefs held by firms on island j regarding island i's weather's effect on the price will be symmetric across all $j \in 1,\ldots,J$: $b_i^1=b_i^2=...=b_i^J$ in equilibrium. This is because, in the full information case, all islands have identical information sets and are utilizing identical

econometric methods. Call this value b_i and the vector of beliefs $b = [b_1\ b_2\ ...\ b_J]'$. Consider mapping the linear PLM to the REE ALM (A.1):

$$T(a^j) = \alpha\theta'\mathcal{A}$$

$$T(b)' = \theta'\left[\alpha\ b\ 1_{1\times J} + \gamma I_J\right]$$

For any b_i, we get:

$$T(b_i) = \alpha\left(\theta_1 b_i + \theta_2 b_i + ... + \theta_J b_i\right) + \theta_i\gamma$$

The fixed points of these equations yield equilibrium beliefs:

$$a^j = 0, \quad b_i = \frac{\gamma}{1-\alpha}\theta_i$$

Plugging these beliefs into the ALM yields the REE price.

Q.E.D.

Appendix B

B.1 Derivation of Equation (2.15)

Proof of the following inequality under Learning (equation 2.15):

$$GDP_t^{AE} = \sum_{j=1}^{J} q_t^{AE}(j) \leq \sum_{j=1}^{J}\left(\frac{1}{\alpha}\right)^{\frac{1+\alpha}{\alpha}}\left(m_{t+1}(j)\right)\left[\sum_{i=1}^{J}\frac{a_{j,i}}{\phi_{i,t}}\right]$$

Begin by rewriting equation (2.8) as:

$$q_t(j) = \left(\frac{m_{t+1}(j)}{\alpha E_t^j\prod_{i\in N(j)} p_{t+1}(i)^{1/\eta^j}}\right)^{1/\alpha} = \left(\frac{m_{t+1}(j)}{\alpha\prod_{i\in N(j)}\psi_{i,t}^{1/\eta^j}}\right)^{1/\alpha}$$
$$= \left(\frac{m_{t+1}(j)}{\alpha\psi_{1,t}^{a_{j,1}}\psi_{2,t}^{a_{j,2}}\ldots\psi_{J,t}^{a_{j,J}}}\right)^{1/\alpha} = \left(\frac{1}{\alpha}\right)^{1/\alpha}\left(\frac{m_{t+1}(j)}{\psi_{1,t}^{a_{j,1}}\psi_{2,t}^{a_{j,2}}\ldots\psi_{J,t}^{a_{j,J}}}\right)^{1/\alpha}$$

Where $a_{j,i}$ denotes element in row j, column i of $A = \eta^{*-1}A_\Gamma$.

The following inequality between the arithmetic mean and geometric mean (the AM-GM inequality) holds for set of non-negative $\{y_n\}_{n=1}^N$ and positive number k

$$(y_1 \cdot y_2 \cdot \ldots \cdot y_N)^{1/k} \leq \frac{1}{k}(y_1 + y_2 + \ldots + y_N)$$

And the inequality between the harmonic mean and geometric mean (the HM-GM inequality) holds for set of non-negative $\{y_n\}_{n=1}^N$ and set of positive numbers $\{k_n\}_{n=1}^N$ such that $\sum_n k_n = 1$:

$$\frac{1}{y_1^{k_1} \cdot y_2^{k_2} \cdot \ldots \cdot y_N^{k_N}} \leq \frac{k_1}{y_1} + \frac{k_2}{y_2} + \ldots + \frac{k_N}{y_N}$$

Apply these inequalities to our new equations for $q_t(j)$

$$\begin{aligned} q_t(j) &= \left(\frac{1}{\alpha}\right)^{1/\alpha} \left(\frac{m_{t+1}(j)}{\psi_{1,t}^{a_{j,1}} \psi_{2,t}^{a_{j,2}} \ldots \psi_{J,t}^{a_{j,J}}}\right)^{1/\alpha} \\ &\leq \left(\frac{1}{\alpha}\right)^{1/\alpha} \left(\frac{1}{\alpha}\right) m_{t+1}(j) \left[\prod_{i=1}^{J} \frac{1}{\psi_{i,t}^{a_{j,i}}}\right] \quad \text{(AM-GM)} \\ &\leq \left(\frac{1}{\alpha}\right)^{\frac{1+\alpha}{\alpha}} m_{t+1}(j) \left[\sum_{i=1}^{J} \frac{a_{j,i}}{\psi_{i,t}}\right] \quad \text{(HM-GM)} \end{aligned}$$

Summing this final inequality across all nodes yields (2.15).

Q.E.D.

B.2 Infinite Horizon Problem

Consider an infinitely lived agent who exists in the same network model as described in **Section 2.2**. Every period, the agent produces and sells to adjacent agents *and* buys and consumes goods produced by their neighbors at the perfect competition market clearing prices. The difference between this and the conventional Ramsey model with labor decisions (beyond the network) is that there is a lag between when agents gain their income and when they consume using that income. This agent's problem is represented by the following equations and conditions:

$$\max_{c_t(j),\, L_t(j)} \; E_0^j \sum_{t=0}^{\infty} \beta^t \left[\eta^j \left(\prod_{i \in N(j)} c_t(j,i) \right)^{1/\eta^j} - L_t(j)^{\alpha} \right]$$

$$s.t.\; p_t(j) q_t(j) = m_{t+1}(j)$$

$$\sum_{i \in N(j)} p_{t+1}(i)\; c_{t+1}(j,i) = m_{t+1}(j)$$

$$L_t(j) = q_t(j)$$

$$\alpha > 1$$

Solving this problem yields the following decisions, analogous to (2.5a,b):

$$q_t(j) = \left(\frac{\beta p_t(j)}{\alpha E_t^j \prod_{i \in N(j)} p_{t+1}(i)^{1/\eta^j}} \right)^{1/(\alpha-1)}$$

$$c_{t+1}(j,i) = E_t^j \; \frac{m_{t+1}(j)}{\eta^j p_{t+1}(i)} \; \forall i \in N(j)$$

The market clearing prices and quantities are determined by the following equations:

$$p_t(j) = \left(\frac{\alpha}{\beta} E_t^j \prod_{i \in N(j)} p_{t+1}(i)^{1/\eta^j} \right)^{1/\alpha} \left(\sum_{i \in N(j)} \frac{m_t(i)}{\eta^i} \right)^{(\alpha-1)/\alpha}$$

$$q_t(j) = \left(\frac{\alpha}{\beta} E_t^j \prod_{i \in N(j)} p_{t+1}(i)^{1/\eta^j} \right)^{-1/\alpha} \left(\sum_{i \in N(j)} \frac{m_t(i)}{\eta^i} \right)^{1/\alpha}$$

Which are the same as (2.7) and (2.8), scaled by $(\beta)^{-1/\alpha}$ and $(\beta)^{1/\alpha}$ respectively. If $0 < \beta < 1$ (as convention ascribes) then this economy sees slightly higher prices and lower quantities. Therefore, the dynamics will be analogous to the dynamics of the OLG model, simply scaled.

B.3 Proof of Proposition 2.1.

Social Planner's Problem: If we assume that the planner has no preferences time periods, Pareto weights λ_j^{old} for the old agents at node j and λ_j^{young} for the young agent at node j, the problem of

the Social Planner is as follows:

$$\max_{\{\mathbf{C}_t,\, \mathbf{Q}_t\}} \sum_{j \in V} \lambda_j^{old}\, \eta^j \left(\prod_{i \in N(j)} c_t(j,i) \right)^{1/\eta^j} - \lambda_j^{young} q_t(j)^{\alpha}$$

$$s.t. \quad q_t(j) = \sum_{i \in N(j)} c_t(i,j) \;\; , \;\; \forall j \in V$$

$$\alpha > 1$$

This results in a consumption allocation satisfying:

$$\lambda_j^{old} \frac{\left(\prod_{h \in N(j)} c_t(j,h) \right)^{1/\eta^j}}{c_t(j,i)} = \lambda_k^{old} \frac{\left(\prod_{h \in N(k)} c_t(k,h) \right)^{1/\eta^k}}{c_t(k,i)} = \lambda_i^{young}\, \alpha\, q_t(i)^{\alpha-1}$$

Where $j, k \in N(i)$.

$$c_t(j,i) = \left(\frac{\lambda_k^{old}}{\lambda_j^{old}} \frac{\left(\prod_{h \in \{N(k)/i\}} c_t(k,h) \right)^{1/\eta^k}}{\left(\prod_{h \in \{N(j)/i\}} c_t(j,h) \right)^{1/\eta^j}} \right)^{\frac{\eta^j}{1-\eta^j}} c_t(k,i)^{\frac{1-\eta^k}{\eta^k} \frac{\eta^j}{1-\eta^j}}$$

If we constrain our analysis to *k-regular* graphs ($\eta^j = \eta^k = \eta^*$) and assume that the planner has no preferences across agents ($\lambda_j^{old} = \lambda_k^{old} = \lambda_i^{young}\ \forall\ j, k \in N(i)\ \forall\ i \in V$), then

$$c_t(j,i) = c_t(k,i) = \left(\frac{1}{\eta^*} \right) \alpha^{\frac{1}{\alpha-1}}$$

Q.E.D.

B.4 Proof of Proposition 2.2.

Beginning with (2.16), take the derivative with respect to the expectation vector. Note that (2.16) is only true under homogeneous expectations.

$$\frac{\partial}{\partial\ ln(p_{t-1})}\ln(p_t) = \frac{\partial}{\partial\ ln(p_{t-1})}\left[\left(\frac{1}{\alpha}\right)\ln(\alpha)1_{J\times 1} + \left(\frac{\alpha-1}{\alpha}\right)\ln\left(Am_t\right) + \left(\frac{1}{\alpha}\right)\left[A'\ ln(p_{t-1})\right]\right]$$
$$= \left(\frac{1}{\alpha}\right)A'$$

Since all values of A' are between 0 and 1, and row-wise sum to 1, all eigenvalues of A', $\lambda_{A'}$ are $0 \leq |\lambda_{A'}| \leq 1$. Scaled by $1/\alpha$ with $\alpha > 1$ ensures the eigenvalues are within the unit circle. Therefore, the dynamic system is locally stable.

Q.E.D.

Appendix C

C.1 Data Sources and Description

The Data

- Country level consumer price index and money supply: *www.theglobaleconomy.com*
  - Note: *theglobaleconomy* has incomplete data for this time period for Austria, Belgium, Croatia, Cyprus, Estonia, and Romania. However, this data is available from *www.eulerpool.com*. When these websites overlap with present data, the money supply data matches perfectly.
- Monthly energy price data
  - For country-level: *countryeconomy.com*'s "Consumer prices of petroleum products - Gas oil" series from each country. This weekly data is averaged to get monthly measures.
  - Note that Croatia's 'Gas oil' series begins in 2013, therefore all data prior to 07/01/2013 is *predicted* by matching FRED's "Harmonized Index of Consumer Prices: Gas for Croatia" series with a simple linear time series model. This is done prior to the log-transformation.
  - For aggregate model: *www.tradingeconomics.com*'s Euro Area Energy Prices series

- Interest rate data
  - For countries in the Euro Zone: The European Central Bank's target interest rate is taken from the *ecb*'s website
  - For countries outside the Euro Zone: *countryeconomy.com*'s "key rates"
- Country-level industrial production is seasonal and calendar-adjusted monthly manufacturing data from *eurostat*.

As Figure C.1. shows, the time series for Y_t and P_t closely match their counterpart aggregate series for the Euro Area, generally. This makes the analysis in this Chapter highly relevant for the Euro Area, not just the countries included.

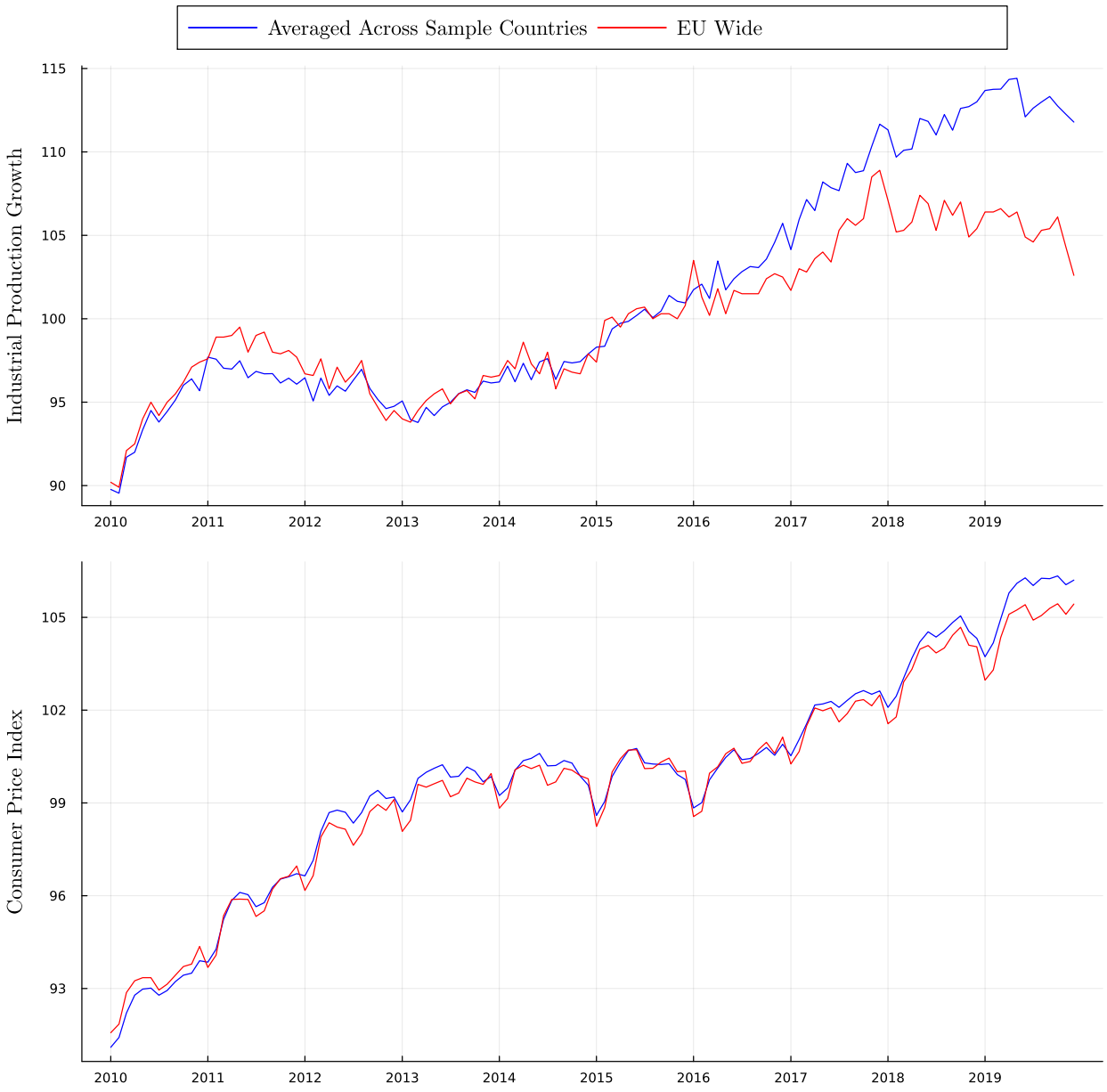


Figure C.1. Compared sample-averaged industrial production and price level with Euro Area time series of the same variables.

These time series have the respective correlation statistics 0.959 and 0.997, respectively. While not every country in Europe nor the European Union is represented, the analysis is still highly relevant to the continent.

C.2 Country Level Impulse Response Functions

Country level impulse response functions for Industrial Production (y_j) for the backward-looking countries are shown in Figure C.2. and for forward-looking countries in C.3.

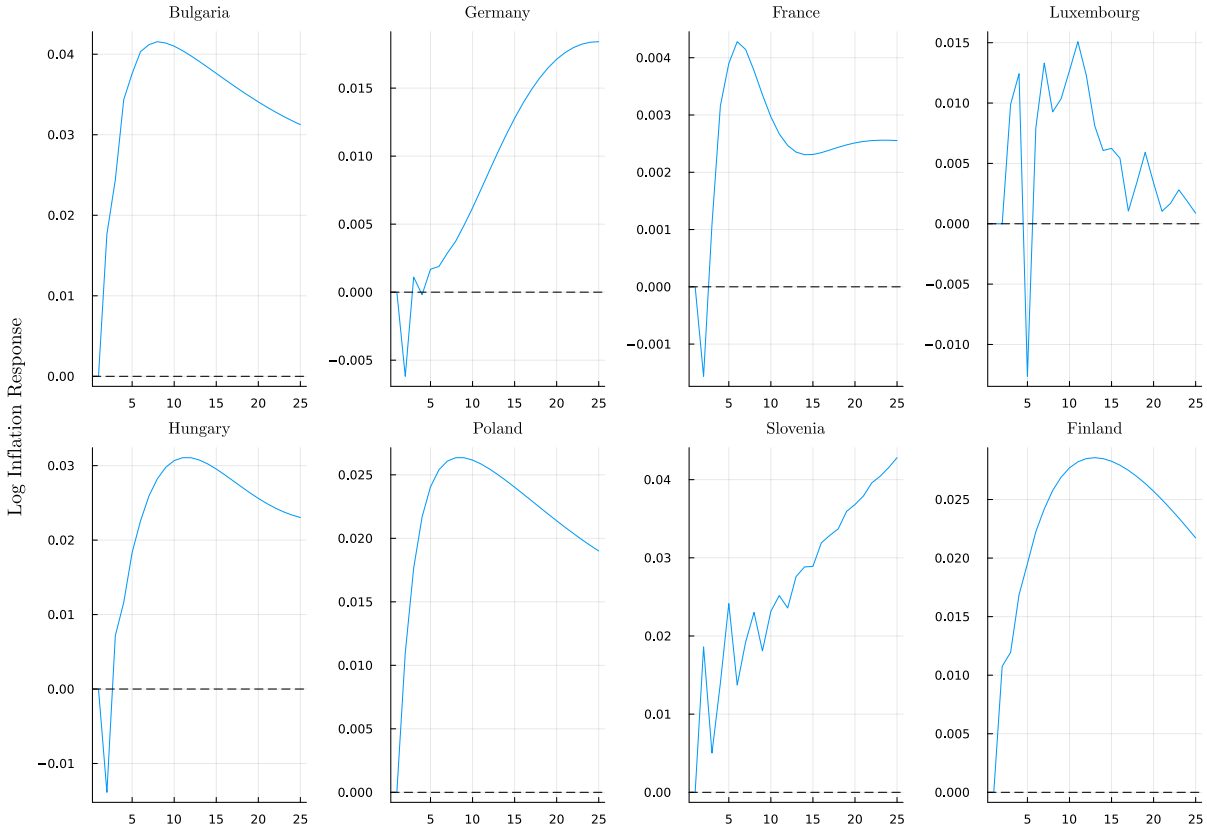


Figure C.2. Country level impulse response functions for Industrial Production for backward looking countries.

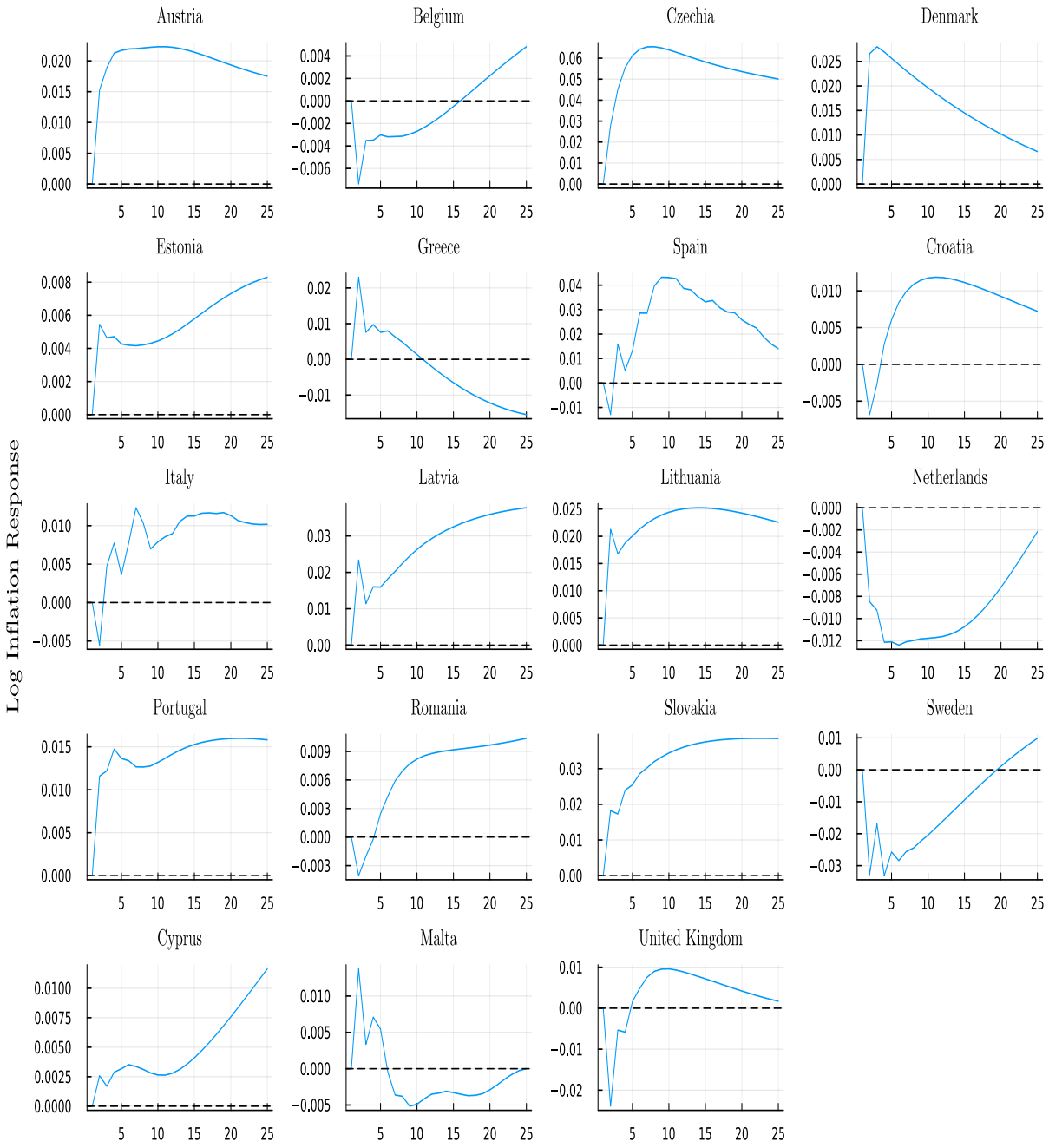


Figure C.3. Country level impulse response functions for Industrial Production for forward looking countries.

Country level impulse response functions for Inflation (p_j) for the backward looking countries are shown in Figure C.4. and for forward looking countries in C.5.

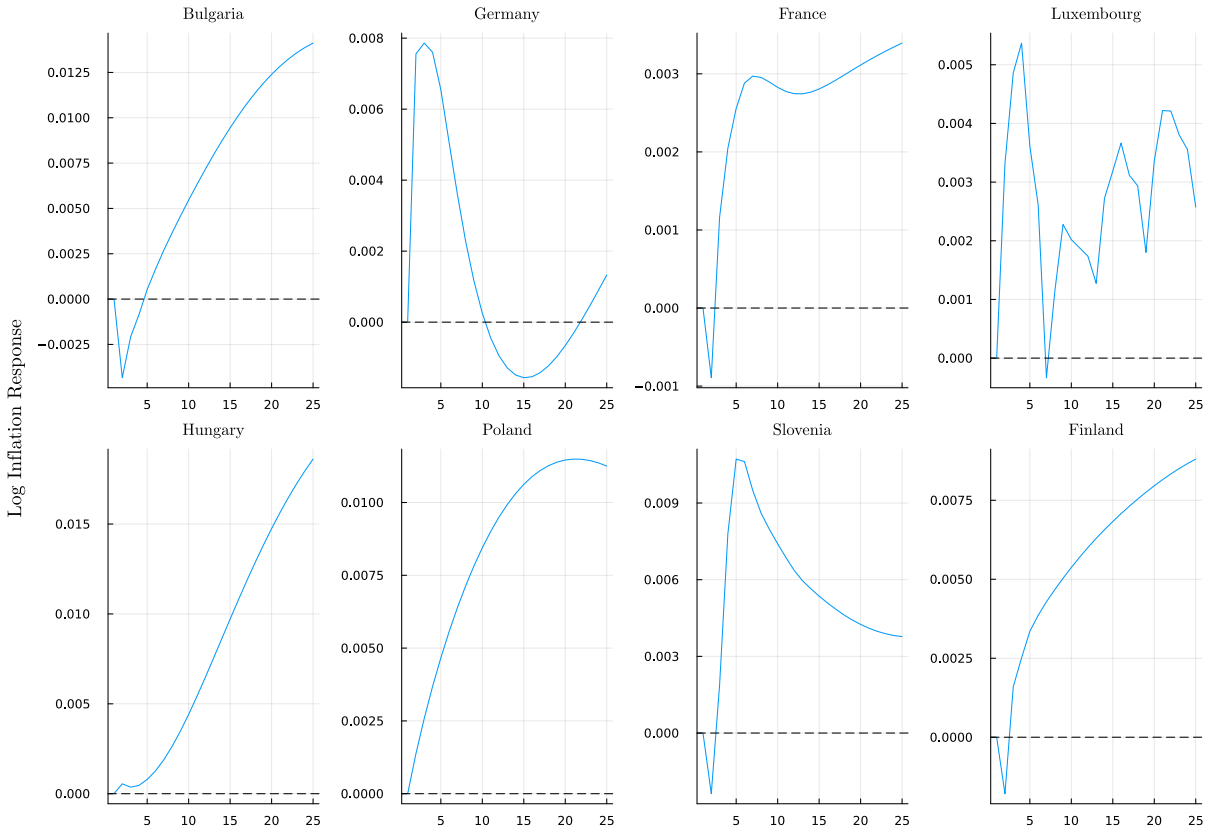


Figure C.4. Country level impulse response functions for Inflation for backward looking countries.

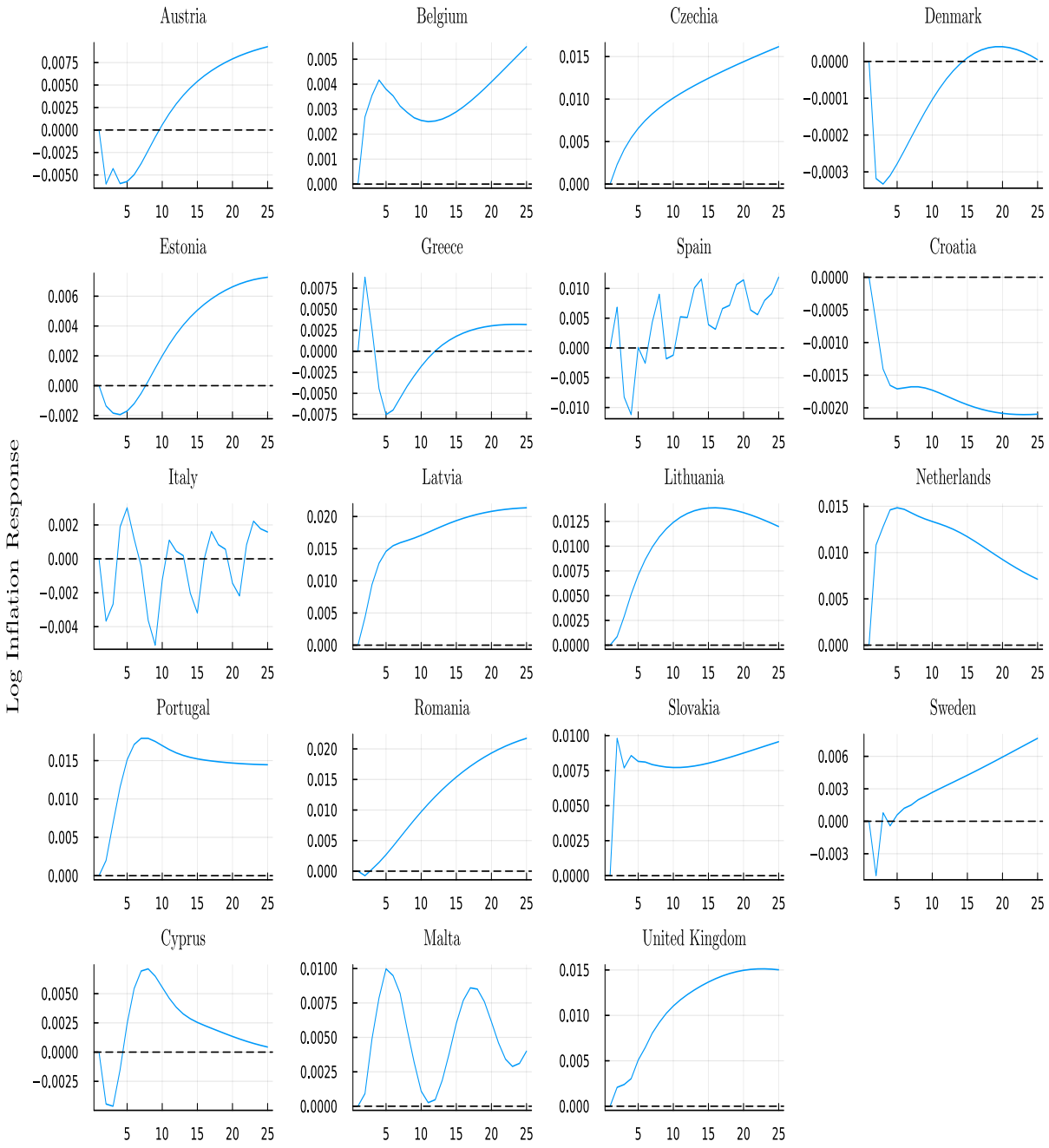


Figure C.5. Country level impulse response functions for Inflation for forward looking countries.

C.3 Aggregate Impulse Response Functions

Aggregate impulse response functions for Industrial Production (y_t) for the shocks to backward looking countries are shown in Figure C.6. and for forward looking countries in C.7.

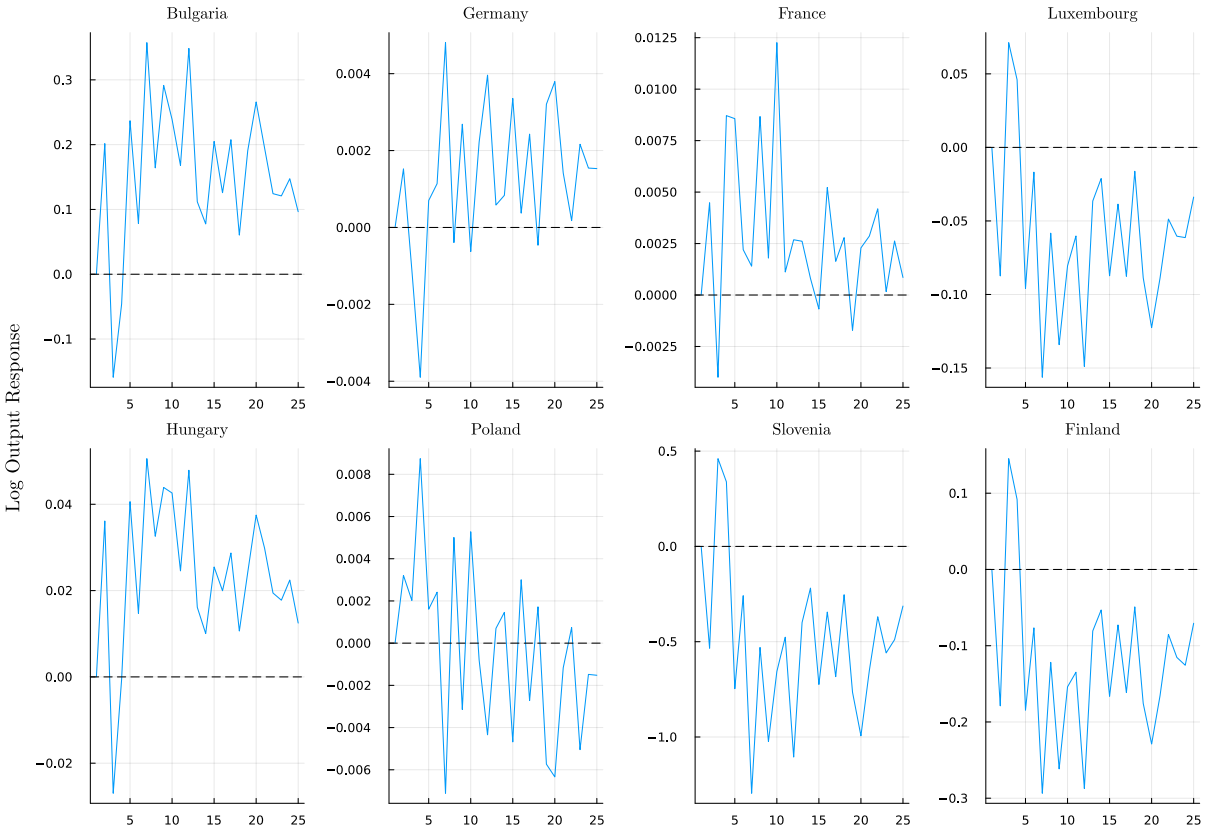


Figure C.6. Aggregate impulse response functions for Industrial Production for backward looking countries.

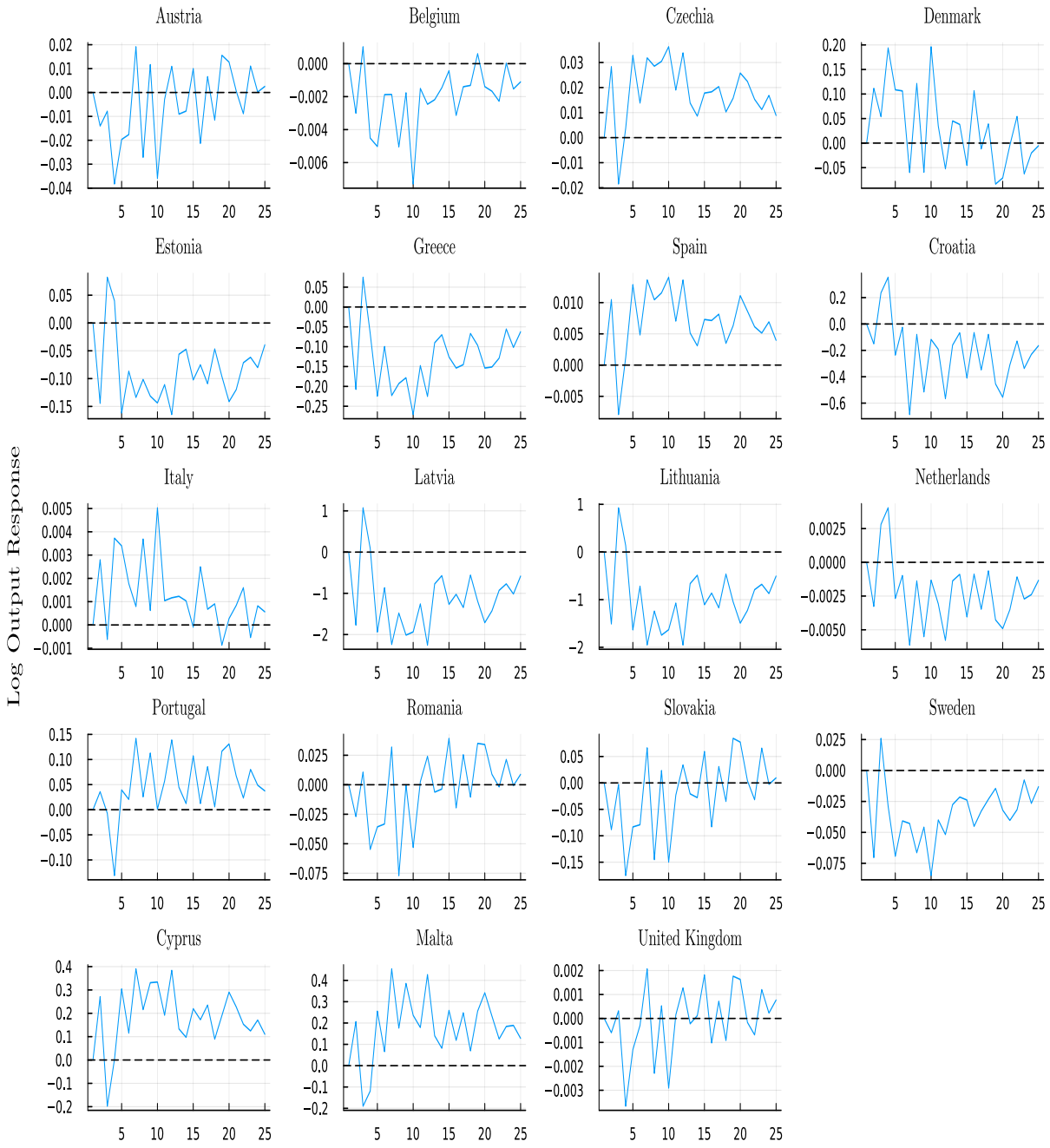


Figure C.7. Aggregate impulse response functions for Industrial Production for forward looking countries.

Aggregate impulse response functions for Inflation (p_t) for the shocks to backward looking countries are shown in Figure C.8. and for forward looking countries in C.9.

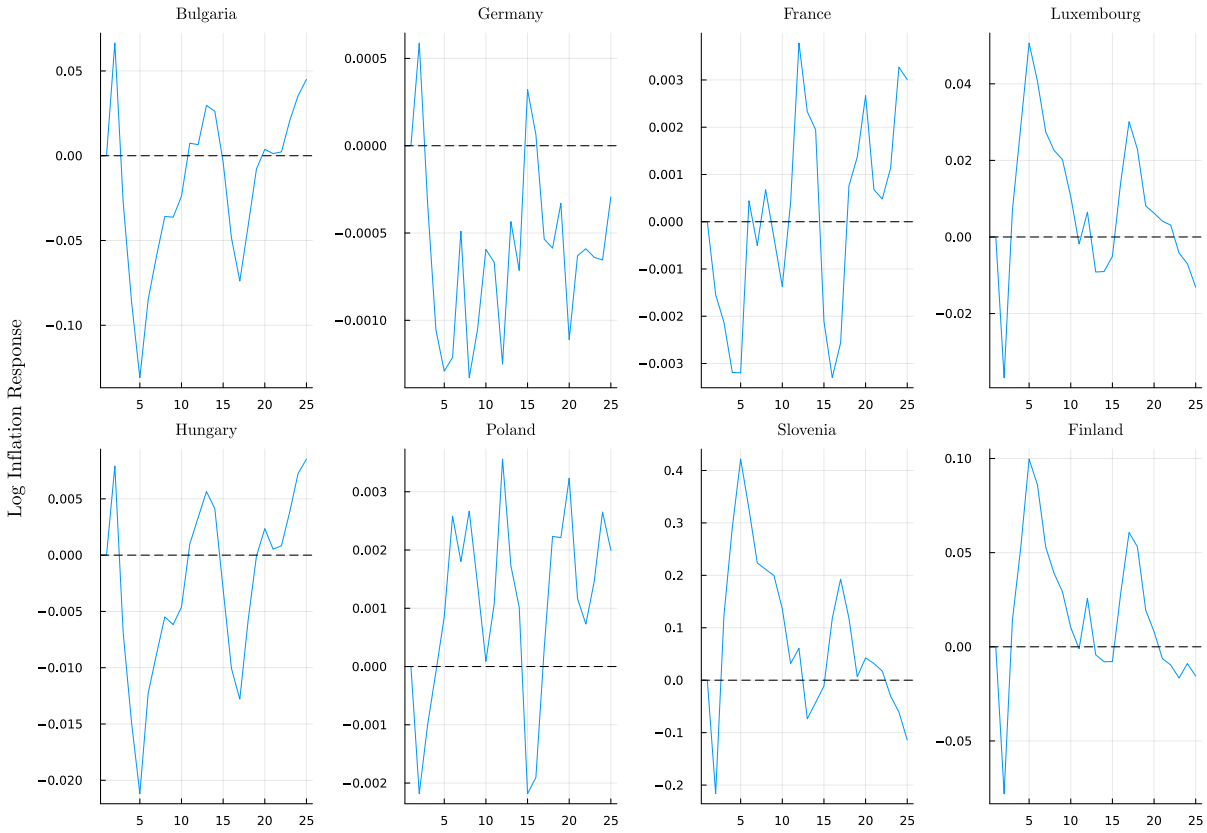


Figure C.8. Aggregate impulse response functions for Inflation for backward looking countries.

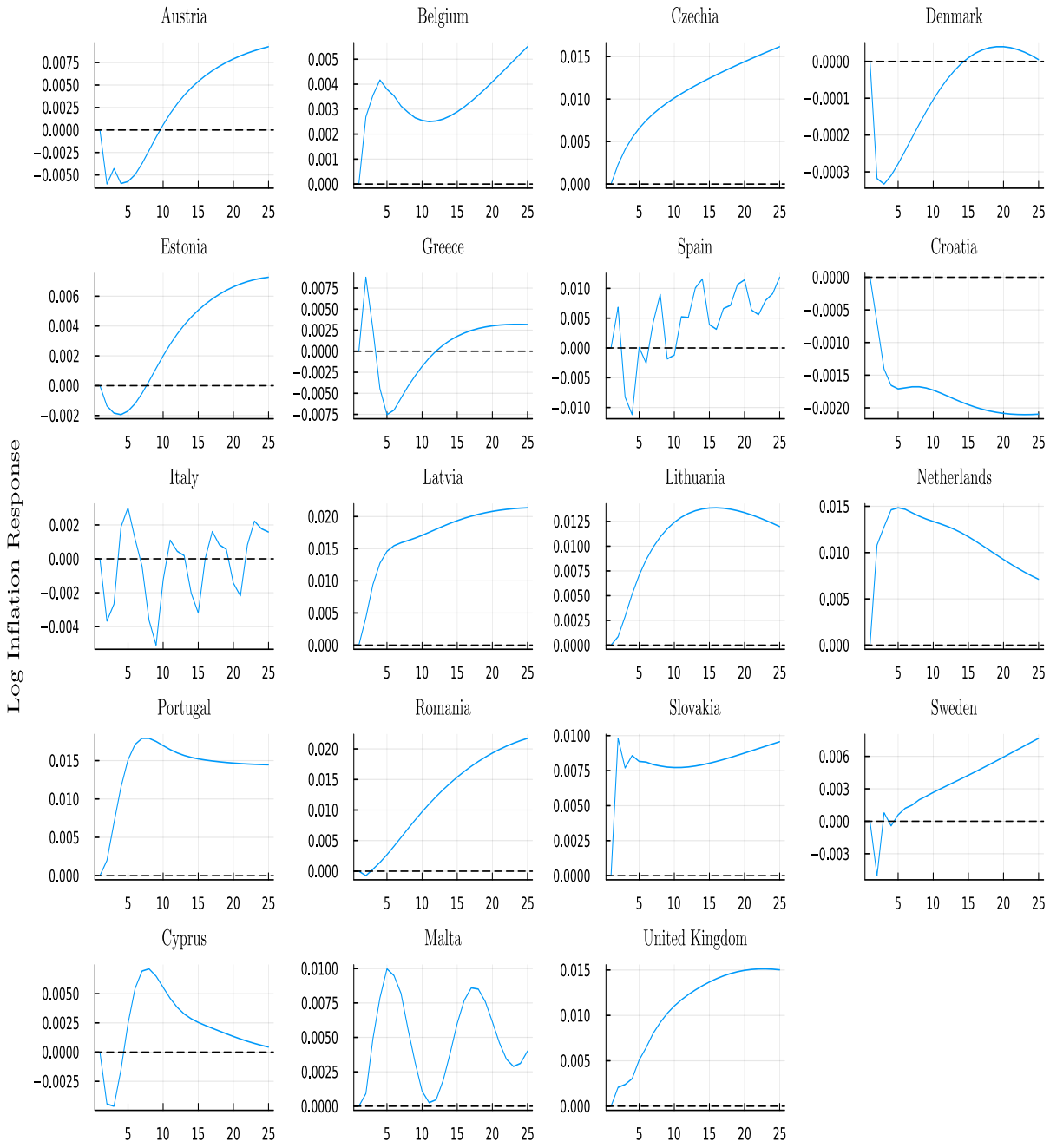


Figure C.9. Aggregate impulse response functions for Inflation for forward looking countries.

References

Ahtiala, P. (1998). Monetary policy under flexible exchange rates: When is expansion contractionary? *International Review of Economics & Finance*, 7(4), 361-378.

Aldous, D., & Fill, J. (2001). Reversible Markov Chains.

Alessandri, P., Jordà, Ò., & Venditti, F. (2025). Decomposing the monetary policy multiplier. *Journal of Monetary Economics*, 152, 103783.

Anthonisen, N. (2010). Monetary shocks in a spatial overlapping generations model. *Journal of Economic Dynamics and Control*, 34(12), 2461-2484.

Anthonisen, N. (2013). On the long-run relationship between inflation and output in a spatial overlapping generations model. *Journal of Economic Dynamics and Control*, 37(12), 2500-2524.

Anthonisen, N. (2018). Sticky prices in a dynamic network economy: A family of counterexamples. *Journal of Mathematical Economics*, 74, 1-20.

Anufriev, M. T. Assenza, C. Hommes, and D. Massaro (2013) Interest Rate Rules and Macroeconomic Stability Under Heterogeneous Expectations, *Macroeconomic Dynamics*, 17, 2013, 1574-1604

Anzuini, A., & Levy, A. (2007). Monetary policy shocks in the new EU members: A VAR approach. *Applied Economics*, 39(9), 1147-1161.

Arifovic, J (1994) Genetic Algorithm Learning and the Cobweb Model, *Journal of Economic Dynamics and Control*, 18, 3-28.

Balasko, Y., & Shell, K. (1980). The overlapping generations model, I: The case of pure exchange without money. *Journal of Economic Theory*, 23(3), 281-306.

Batchelor, R. A., & Orr, A. B. (1988). Inflation expectations revisited. *Economica*, 317-331.

Beraja, M., Fuster, A., Hurst, E., & Vavra, J. (2017). Regional heterogeneity and monetary policy (No. w23270). *National Bureau of Economic Research.*

Board of Governors of the Federal Reserve System. (2024). Minutes of the Federal Open Market Committee, November 6–7, 2024. Retrieved from `https://www.federalreserve.gov/monetarypolicy/fomcminutes20241107.htm`

Bonacich, P., & W.G. Roy (1986) "Centrality, dominance, and interorganizational power in a network structure: Interlocking directorates among American railroads, 1886–1905," *The Journal of Mathematical Sociology*, 12:2, 127-135, DOI: 10.1080/0022250X.1986.9990009

Boun My, K., M. Willinger & A. Ziegelmeyer (2001) Global versus local interaction in coordination games: an experimental investigation. *Bureau d'Economie Théorique et Appliquée*, UDS, Strasbourg.

Branch, W. (2004). The theory of rationally heterogeneous expectations: evidence from survey data on inflation expectations. *The Economic Journal*, 114(497), 592-621.

Branch, W. (2006) Restricted Perceptions Equilibria and Learning in Macroeconomics, in *Colander (2006)*, pp.135-160.

Branch, W. (2007) Sticky Information and Model Uncertainty in Survey Data on Inflation Expectations, *Journal of Economic Dynamics and Control*, 31, 245-276.

Branch, W. (2022) *Misspecification and the Restricted Perceptions Approach*, Technical Report.

Branch, W & G. W. Evans (2006) Intrinsic Heterogeneity in Expectation Formation, *Journal of Economic Theory*, 127, 264-295.

Branch, W. & G. W. Evans (2011) Monetary policy and heterogeneous expectations, *Economic Theory*, Vol. 47, (June 2011), pp. 365-393.

Branch, W. & B. McGough (2004) Multiple Equilibria in Heterogeneous Expectations Models, *The B.E. Journal of Macroeconomics*, 4(1).

Branch, W. & B. McGough (2009) A New Keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*. Volume 33, Issue 5.

Branch, W. & B. McGough (2011) Business Cycle Amplification With Heterogeneous Expectations. *Economic Theory*, volume 47, pages 395–421.

Brandes, U. (2001). A faster algorithm for betweenness centrality. *Journal of Mathematical Sociology*, 25(2), 163-177.

Brayton, F., Mauskopf, E., Reifschneider, D., & Tinsley, P. (1997). The role of expectations in the FRB/US macroeconomic model. *Fed. Res. Bull.*, 83, 227.

Bavelas, A. (1950). Communication patterns in task-oriented groups. *The journal of the acoustical society of America*, 22(6), 725-730.

Bray, M & N. Savin (1986) Rational Expectations Equilibria, Learning, and Model Specification, *Econometrica*, 54, 1129-1160.

Brock, W. A. & C. Hommes (1997) A Rational Route to Randomness, *Econometrica*, 65, 1059-1095.

Carlino, G., & DeFina, R. (1998). The differential regional effects of monetary policy. *Review of economics and statistics*, 80(4), 572-587.

Cavallo, A., & Ribba, A. (2015). Common macroeconomic shocks and business cycle fluctuations in Euro area countries. *International Review of Economics & Finance*, 38, 377-392.

Chen, S-H. & C-H. Yeh (1996) "On the Coordination and Adaptability of the Large Economy: An Application of Genetic Programming to the Cobweb Model," *IFAC Proceedings Volumes*, Volume 29, Issue 1, June–July 1996, Pages 5659-5664

Coricelli, F., Égert, B., & MacDonald, R. (2006). Monetary transmission mechanism in Central and Eastern Europe: Gliding on a wind of change.

Davidson, R., & MacKinnon, J. G. (2004). *Econometric theory and methods* (Vol. 5, pp. 189-196). New York: Oxford University Press.

De Masi, G. (2009) Empirical Analysis of the Architecture of the Interbank Market and Credit Market Using Network Theory, In: Naimzada, A.K., Stefani, S., Torriero, A. (eds) *Networks, Topology and Dynamics. Lecture Notes in Economics and Mathematical Systems*, vol 613. Springer, Berlin, Heidelberg.

Demertzis, M. & A. H. Hallett (2008) Asymmetric Information and Rational Expectations: When is it Right to be 'Wrong'? *Journal of International Money and Finance*, Volume 27, Issue 8, December 2008, Pages 1407-1419

Di Giacinto, V. (2003). Differential regional effects of monetary policy: A geographical SVAR approach. *International Regional Science Review*, 26(3), 313-341.

Dijkstra, J. & M. A.L.M. van Assen (2013) Network public goods with asymmetric information about cooperation preferences and network degree, *Social Networks*, Volume 35, Issue 4, October 2013, Pages 573-582.

Dominguez-Torres, H., & Hierro, L. A. (2019). The regional effects of monetary policy: A survey of the empirical literature. *Journal of Economic Surveys*, 33(2), 604-638.

Döpke, J., Dovern, J., Fritsche, U., & Slacalek, J. (2008). The dynamics of european inflation expectations. *The BE Journal of Macroeconomics*, 8(1).

Duran, H. E., & Karahasan, B. C. (2022). Heterogenous responses to monetary policy regimes: A regional analysis for Turkey, 2009-2019. *Regional Statistics*, 12(3).

Elbourne, A., & De Haan, J. (2004). *Asymmetric monetary transmission in EMU: the robustness of VAR conclusions and Cecchettis legal family theory* (No. 1327). CESifo working paper.

Elbourne, A., & de Haan, J. (2006). Financial structure and monetary policy transmission in transition countries. *Journal of Comparative Economics*, 34(1), 1-23.

Evans, G. W. & S. Honkapohja (1996) Least Squares Learning with Heterogeneous Expectations, *Economics Letters*, Volume 53, Issue 2, November 1996, Pages 197-201

Evans, G. W. & S. Honkapohja (2001) *Learning and Expectations in Macroeconomics*. Princeton University Press, Princeton, New Jersey.

Evans, G. W., & Honkapohja, S. (2009). Expectations, learning and monetary policy: an overview of recent research. *Banco Central de Chile.*

Evans, G. W. & B. McGough (2020) *Adaptive Learning and Macroeconomics*. Oxford Research Encyclopedia of Economics and Finance, November 2020.

Federal Reserve. (2024). Review and Outlook: Chair Jerome H. Powell. Speech presented at "Reassessing the Effectiveness and Transmission of Monetary Policy," an economic symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, WY. Retrieved from `https://www.federalreserve.gov/newsevents/speech/powell20240823a.htm`

Fraser, P., Macdonald, G. A., & Mullineux, A. W. (2014). Regional monetary policy: An Australian perspective. *Regional Studies*, 48(8), 1419-1433.

Friedman, M. (1969). *The optimum quantity of money: and other essays.*

Furceri, D., Mazzola, F., & Pizzuto, P. (2019). Asymmetric effects of monetary policy shocks across US states. *Papers in Regional Science*, 98(5), 1861-1892.

Gaspar, V., Smets, F., & Vestin, D. (2010). Inflation expectations, adaptive learning and optimal monetary policy. *Handbook of Monetary Economics* (Vol. 3, pp. 1055-1095). Elsevier.

Gerberding, C. (2009). Household versus expert forecasts of inflation: New evidence from the European survey data. In *Inflation Expectations* (pp. 136-157). Routledge.

Giannitsarou, C. (2003) Heterogeneous Learning, *Review of Economic Dynamics*, Volume 6, Issue 4, October 2003, Pages 885-906

Giavazzi, F., & Pagano, M. (1990). Can severe fiscal contractions be expansionary? Tales of two small European countries. *NBER Macroeconomics Annual*, 5, 75-111.

Granato, J., E. A. Guse, & M. S. Wong (2008) Learning from the Expectations of Others, *Macroeconomic Dynamics*, 12, 345-377.

Hommes, C., & J. Lustenhouwer (2015) Inflation Targeting and Liquidity Traps under Endogenous Credibility, *Journal of Monetary Economics*, Elsevier, vol. 107(C), pages 48-62.

Honkapohja, S. & K. Mitra (2006) Learning stability in economics with heterogeneous agents, *Review of Economic Dynamics*, Volume 9, Issue 2, April 2006, Pages 284-309

Ilzetzki, E., Mendoza, E. G., & Végh, C. A. (2013). How big (small?) are fiscal multipliers?. *Journal of Monetary Economics*, 60(2), 239-254.

Jarociński, M. (2010). Responses to monetary policy shocks in the east and the west of Europe: a comparison. *Journal of Applied Econometrics*, 25(5), 833-868.

Jensen, R. (2007). The digital provide: Information (technology), market performance, and welfare in the South Indian fisheries sector. The quarterly journal of economics, 122(3), 879-924.

Keser, C., Ehrhart, K-M, & Berninghaus, S. K. (1998) Coordination and local interaction: experimental evidence. *Economics Letters*, 1998, vol. 58, issue 3, 269-275

Kurz, M., H. Jin, & M. Motolese (2003) The role of expectations in economic fluctuations and the efficacy of monetary policy, *Journal of Economic Dynamics and Control* 29 (2005) 2017-2065

Lucas, R. (1972) Expectations and the Neutrality of Money, *Journal of Economic Theory*, 4, 103-124 (1972)

Lyziak, T., & Mackiewicz-Lyziak, J. (2014). Do consumers in Europe anticipate future inflation? Has it changed since the beginning of the financial crisis?. *Eastern European Economics*, 52(3), 5-32.

Makin, T. (1998). When contractionary fiscal policy is expansionary. *Agenda: A Journal of Policy Analysis and Reform*, 419-426.

Mandel, A., Taghawi-Nejad, D., & Veetil, V. P. (2019). The price effects of monetary shocks in a network economy. *Journal of Economic Behavior & Organization*, 164, 300-316.

Mandel, A., & Veetil, V. P. (2021). Monetary dynamics in a network economy. *Journal of Economic Dynamics and Control*, 125, 104084.

Metcalf, L., & Casey, W. (2016). *Cybersecurity and applied mathematics*. Syngress.

Mino, K., & Shibata, A. (2000). Growth and Welfare Effects of Monetary Expansion in an Overlapping-generations Economy. *The Japanese Economic Review*, 51(3), 407-430.

Muth, J. F., (1961) Rational Expectations and the Theory of Price Movements, *Econometrica*, 29, 315-335.

Nakagawa, R. (2015) Learnability of an Equilibrium with Private Information, *Journal of Economic Dynamics and Control*, 59, 58-74.

Naimzada, A. K., S. Stefani, & A. Torriero (2009) *Networks, Topology and Dynamics. Lecture Notes in Economics and Mathematical Systems*, vol 613. Springer, Berlin, Heidelberg.

Özdağlar, A. & I. Menache, (2011) *Network Games: Theory, Models, and Dynamics.* Morgan & Claypool Publishers, 2011.

Peersman, G. (2004). The transmission of monetary policy in the Euro Area: are the effects different across countries?. *Oxford Bulletin of Economics and Statistics*, 66(3), 285-308.

Pesaran, H. (1987). The Limits to Rational Expectations, *Basil Blackwell*, Oxford.

Petersen, L. (2015). Do expectations and decisions respond to monetary policy?. *Journal of Economic Studies*, 42(6), 972-1004.

Ramey, V. A. (2019). Ten years after the financial crisis: What have we learned from the renaissance in fiscal research?. *Journal of Economic Perspectives*, 33(2), 89-114.

Rauch, J. E. & V. Trindade (2002) Ethnic Chinese Networks In International Trade. *The Review of Economics and Statistics*, MIT Press, vol. 84(1), pages 116-130, February.

Reis, R., & Tenreyro, S. (2022). Helicopter money: what is it and what does it do?. *Annual Review of Economics*, 14(1), 313-335.

Sargent, T. J. (1993): *Bounded Rationality in Macroeconomics*. Oxford University Press, Oxford.

Sims, C. A., (2003) Implications of Rational Inattention. *Journal of Monetary Economics* 50 (2003) 665-690

Slobodyan, S., & R. Wouters (2012) Learning in an Estimated DSGE Model. *Journal of Economic Dynamics and Control*, 36, 26-46.

Szyszko, M., Rutkowska, A., & Kliber, A. (2020). Inflation expectations after financial crisis: are consumers more forward-looking?. *Ekonomska istraživanja*, 33(1), 1052-1072.

Zeng, C., B. Ren, M. Li, H. Liu, & J. Chen (2019) Stackelberg game under asymmetric information in critical infrastructure system: From a complex network perspective. *Chaos: An Interdisciplinary Journal of Nonlinear Science* Volume 29, Issue 8 August 2019.