

THREE ESSAYS ON ADAPTIVE LEARNING,
INSTITUTIONS AND MULTIPLE
EQUILIBRIA

by

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This dissertation examines the role that institutions play in the existence of multiple equilibria in models of economic development. In addition, it examines the dynamics of transition between such equilibria. In the first chapter of this dissertation, I build a dynamic model of institutional choice, wherein the government invests in the legal infrastructure in response to the need for the protection of output from appropriation. A unique equilibrium exists only under commitment, not under discretion. This would suggest that a measure of institutional quality must not only consider the extent to which current policies protect property rights but also include the ability of the government to commit to reform in the long run.

The second chapter of this dissertation examines the effect of adaptive learning on stability and transitional dynamics between multiple equilibria in a

growth model with human capital externalities. I find that there are two equilibria, one a poverty trap with no education. Only the poverty trap is locally stable under learning. However, productivity shocks are not sufficient to generate transitions between the equilibria. Indeed, productivity shocks must lie below a threshold in order for the economy to escape the poverty trap. These escape paths do not allow the economy to transition to the upper steady state. I propose instead the use of shocks to expectations to permit such a transition.

The third chapter of this dissertation presents an empirical test for the role that human capital and institutions may play in transitions between equilibria by estimating a Markov-switching regression. This methodology allows me to characterize both distinct growth regimes and transitions between them. I explore the effects of time-varying institutional measures and human capital on transition probabilities. I find that political and economic institutions are similar in their effects on transitions and that the time variation in the institutional measure increases the probability of identifying both miracle growth and stagnation regimes. Furthermore, human capital has a significant effect on switches between miracle growth, stable growth and stagnation.

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CHAPTER I

INTRODUCTION

Since the end of World War II and the subsequent wave of independence in many African and Asian countries, one of the most frequently discussed stylized facts of economic growth has been the large income gap between developed and developing countries. Despite a few exceptions, the ratio of GDP per capita in Western Europe and Latin America has increased from 1.8 in 1820 to 2.42 in 1960 and 3.38 in 2006. The more striking divergence is between Western Europe and Africa, where the ratio increased from 2.96 in 1820 to 13.1 in 2006. The origin of this "Great Divergence" in income per capita has been the subject of an intense debate, wherein both economic and political institutions, along with human capital formation have played a fundamental role. Institutions in this context have been very broadly defined as the rules and norms which govern both economic and political behavior. In the empirical literature, those institutions which govern economic behavior are typically thought of as determining the extent to which property rights are protected. The dominant theoretical explanation for these stylized facts has been the existence of poverty traps, due to threshold effects in various state variables, including economic institutions and human capital.

My purpose in this dissertation is twofold. First, I seek to further explore the role that economic institutions and human capital externalities play in the formation of poverty traps and in transitions out of such traps. Second, I examine

the extent to which variation in the quality of institutions and human capital accumulations across countries can account for differences in growth patterns, rather than simply differences in average economic growth over the long run.

In the first essay, which is the second chapter of this dissertation, I consider the role of the quality of institutions in the formation of poverty traps. The previous theoretical literature finds that the poor quality of institutions would necessarily generate poverty traps, and contribute to persistent differences in both institutions and incomes between countries. While these results are most appealing, given persistent income gaps between countries, yet these models either completely suppress the dynamic aspects of the government's optimization problem, or institutions are treated as a binary rather than a continuous variable. Yet, current empirical work and discussions of the impact of institutions on growth assume institutional persistence, as well as a continuum of institutional quality. I incorporate these assumptions into a model where the government invests in legal infrastructure, which serves the purpose of protecting output from appropriation. In the case where the government is able to commit to a sequence of such investment over the infinite horizon, I find a unique equilibrium. Discretionary policy however does permit the formation of a poverty trap. I thus propose expanding the definition of institutions relevant to economic growth to include the ability of the government to commit to reform in the long run.

In the second essay, which is the third chapter of this dissertation, I turn my attention to the role of human capital externalities in the formation of poverty traps. Azariadis and Drazen (1990) find that in a model of economic growth that human capital externalities lead to the formation of a poverty trap. The lower steady state corresponding to this poverty trap is locally stable, while that with positive levels of investment in human capital is saddle-path stable. In this chapter,

I first characterize the global dynamics of this model under perfect foresight. Then, I allow for stochastic shocks to aggregate productivity, and study the role of expectations in transition dynamics. I find that some forms of bounded rationality, specifically econometric learning, will permit escape from the poverty trap, but will not necessarily lead to a transition to the upper steady state. Instead, the economy transitions to an equilibrium of self-sustaining oscillations in human capital investment. The primary goal of this second essay is to illustrate the role that expectational shocks may play in determining whether countries escape from poverty traps.

The third essay, contained in the fourth chapter of this dissertation, discusses an empirical test of the role that institutions and human capital may play in explaining differences in patterns of economic growth across countries. I estimate a Markov regime switching regression for panel data on economic growth. I compare the results of employing a static measure of economic institutions as the sole explanatory variable for the transition probability matrix, with employing time-varying measures of institutions and human capital accumulation as explanatory variables. I find that time-varying measures of institutions yield an improvement in fit over static measures. Furthermore, in allowing economic and political institutions as well as human capital accumulation to affect transitions between growth regimes, I find that all three significantly affect such transitions. Thus, while political institutions and human capital frequently have no significant effect on average economic growth across countries, they do play an important role in the dynamics of growth.

CHAPTER II

COMMITMENT, ENDOGENOUS INSTITUTIONS AND MULTIPLE EQUILIBRIA

II.1 Introduction

The causes of the 'Great Divergence' of incomes between advanced and developing economies has been the source of much debate within the empirical growth literature. Most of the existing studies attempt simply to explain cross-sectional differences in income per capita, and attribute the income gap largely to differences in total factor productivity. Hall and Jones (1997) provide empirical evidence that the levels of economic performance vary considerably across countries, with differences persistent over time. Despite controlling for accumulation of human and physical capital, the Solow residual still varies significantly across countries.

Beginning with the work of Alchian, Demsetz and North in the early 1970s, economic historians have pointed to economic conditions as causes of changing institutions and to the evolution of institutions as efficiency enhancing and thus improving economic outcomes. What are institutions in this context? North defines institutions as "the rules of the game in a society or, more formally, the humanly devised constraints that shape human interaction." Typically, institutions are divided into two categories - economic and political. Political institutions regulate the limits of political power, and determine how political power changes hands. Economic institutions determine the degree of property rights enforcement and

govern contractual arrangements. These institutions then influence the structure of economic incentives and thus the efficiency of resource allocation, particularly to productive capital and labor. A lack of sufficient institutions will drive a wedge between private and social returns to activities in an economy, resulting in suboptimal allocations of resources.

Hall and Jones form a proxy for this wedge between private and social returns by combining two indices – an index of government antidiversion policies, and an index measuring openness to trade. Their measure of institutions is thus the average of these two indices. There is certainly a positive correlation between this measure and output per worker. They then propose that the quality of "social infrastructure" or of institutions acts as a "deep variable" in determining the level of output per worker, and thus regress logged output per worker on the measure of social infrastructure. They find that a difference of .01 in their measure of social infrastructure is associated with a 5% difference in output per worker. This result would seem to be robust to instrumenting for the measure of institutions.

Institutions and their role in economic development have been the focus of a flurry of empirical activity since Hall and Jones (1999). Acemoglu has played a significant role in continuing to emphasize the importance of institutions, and AJR (2001) points to the persistence of institutions as the primal cause of long-run economic growth, a conclusion which is supported by a number of other cross-country studies. Jeffrey Sachs has written, along with co-authors, a series of empirical papers, pointing to the direct effects of geographical factors, including the role of disease burden in diminishing economic growth, on divergence of income between countries. Glaeser et al (2004), following the argument of Lipset (1960), provides evidence to cast doubt on the conclusions of Acemoglu and others, and proposes instead that both political and economic institutions may be the

consequence of development, and especially of human capital accumulation.

However, Glaeser does not presume to say that his study provides well-founded evidence in favor of Lipset's perspective, and one interpretation of his work may simply be that empirical specifications which rely exclusively on the identification of deep variables (such as institutional and geographical measures) to the exclusion of physical and human capital may be inadequate to accurately describe the political and economic processes of development.

The existing theoretical literature is similarly lacking in a unified approach to the subject. The first definitive research efforts in this area, including Grossman and Kim (1996) and Tornell(1997), considered security of property rights without exclusively modeling institutions. Economies are considered in a kind of Hobbesian state of nature, where there is no explicit role of government, and where security is determined through competition between interest groups. Zak (2002) introduced expropriation, and security expenditures by the government, into an overlapping-generations model, where the rate of expropriation was dependent on both criminal effort and on security expenditures. Institutions in the other papers presented here were primarily modeled in this fashion, where government expenditures in each period directly influence the likelihood of theft. Gradstein (2004) as well as Hoff and Stiglitz (2002) allow for discrete states of such institutions, requiring either sufficient political constituency or tax revenues to move from one to the other.

Current empirical work, including the studies referred to above, frequently assume that institutions persist. Institutional transitions are sometimes implemented as the deliberate outcome of bargaining among a small number of elite groups, or because of changes in the balance of power between the elite and ordinary citizens. Following this line of thought, more recent work by Acemoglu and

Robinson (2007) considers a model in which economic institutions are determined by the combination of de facto political power and a stochastic process.

Alternatively, institutional change may either take place either as a decentralized process by which changes in practice occur informally among large groups of agents, or as a formal process occurring through the government. This essay will focus on the latter channel of institutional change.

The approach taken in this chapter is most similar to Zak (2002), in that institutions are introduced to secure property, in a model where agents choose between productive honest labor and theft of output. The government must optimally choose the level of investment into institutions governing property rights in order to maximize the utility of the representative household. However, I build the model in an infinite-horizon setting, in which the level of property rights is a continuous stock variable, thus differing from both Gradstein and Zak. This introduces additional persistence in institutions, or property protection, and makes a key distinction between current government policies and institutions. Furthermore, in considering the dynamic optimization problem of the government in choosing investment in the legal infrastructure which protects property rights, I analyze both equilibria that are attainable when the government can commit to a sequence of policies, and those that occur when policy is set by discretion in each period.

When the government is in fact capable of committing to policy, in order to calculate the optimal level of investment in property rights each period, I must solve an optimization problem that is quite similar in nature to that presented in the optimal taxation literature. In solving the model therefore, I will rely extensively on the methods developed in this literature, and in particular that proposed by Chari et al (1991, 1994). Furthermore, I compare the results in terms of household welfare in the steady state when policy is set under commitment and under discretion, when

the policymaker cannot commit to a sequence of policies. In contrast to the existing literature, which largely considers property rights to be either a discrete variable, or a flow variable, not a continuous stock variable, I find that under commitment there is a single feasible steady state. Multiple steady states only exist when discretionary policy is employed.

The chapter is organized as follows. Section (II.2) reviews the optimal fiscal policy literature. Section (II.3) is devoted to the key assumptions of the model. Section (II.4) sets up the government's optimal policy problem under commitment, using the approach discussed in Chari et al (1994). Section (II.5) proposes an alternative approach in which a steady state may more easily defined, following Benigno and Woodford (2006). Section (II.6) characterizes the steady state under commitment. Section (II.7) sets up the policy problem and analyzes the steady states under discretionary policy. Section (II.8) compares the results under discretion and commitment. Finally, Section (II.9) concludes.

II.2 Literature Review: Optimal Fiscal Policy

The model presented in this chapter is, in some sense, a variant of the traditional dynamic optimal taxation problem called a Ramsey problem, with a corresponding solution called a Ramsey plan. In the Ramsey problem, a benevolent government attempts to maximize households' welfare subject to raising revenues via distortionary taxation. Government spending is typically taken as exogenous. In designing optimal tax policies, the government must consider the equilibrium reactions of households and firms to policies. As a benchmark, let us consider this problem when there exists a technology permitting the government to commit to a policy at some initial period for all time. Lucas and Stokey (1983) analyze this form

of the problem in a stochastic framework without physical capital. Optimal taxes are determined by decentralizing the following problem: the government maximizes lifetime utility of the representative household across time and across states, subject to a feasibility or resource constraint, and subject to a constraint which I will refer to henceforth as an implementation constraint, following the nomenclature of Chari et al (1991). This implementation constraint is obtained by combining the household budget constraint with its own first order conditions, as well as the market price for labor. Imbedded in the latter two constraints are all necessary conditions for the resulting allocation to maximize household welfare. They therefore provide a complete description of the set of competitive equilibrium allocations attainable through feasible policies.

The above process for determining the Ramsey equilibrium expresses allocations as functions of λ , the Lagrangian multiplier on the implementation constraint. In addition, as shown by Chari et al (1991 and 1994), the first order conditions for the optimal allocations are different in the initial period (when policies are committed to for all future periods) than in subsequent periods. To make this issue clearer, let us consider a non-stochastic version of the optimal policy problem from Chari et al (1994).¹ The government aims to set policy, to satisfy their own budget constraint, given an exogenous sequence of government spending g_t , and to maximize the discounted lifetime utility of the representative household, where period utility is increasing in consumption c_t and decreasing in labor supply l_t .

¹My dating conventions differ slightly from Chari et al. That is, I refer to capital purchased in period t for use in period $t + 1$ as k_{t+1} . This is to make the notation here consistent with that in later sections of this chapter. I also note that lower-case variables indicate per capita variables

The policy optimization problem is given by

$$\max_{c_t, k_{t+1}, l_t} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \quad (\text{II.1})$$

subject to

$$c_t + g_t + k_{t+1} = F(k_t, l_t) + (1 - \delta)k_t \quad (\text{II.2})$$

and

$$\sum_t \beta^t [U_{c,t} c_t + U_{l,t} l_t] = U_{c,0} [R_{k,0} k_0 + R_{b,0} b_0] \quad (\text{II.3})$$

where $R_{k,t} = 1 + (1 - \phi_{k,t})(r_t - \delta_k)$ is the gross return on capital in period t and $R_{b,t}$ is the gross return to the one-period risk-free bonds issued by the government in the previous period. $\phi_{l,t}$ is the tax rate on wages and $\phi_{k,t}$ is the tax rate on income from capital.

Equation (II.2) is simply a resource constraint, and may also be obtained by combining the household and government budget constraints with the rental rates on capital and labor. Equation (II.3), referred to by Chari et al (1994) as an implementation constraint, is a necessary constraint to assure that the allocation that solves the above optimization problem is also a solution to the household's optimization problem, given the tax rates on capital and labor income. To see this, I observe that the household's budget constraint is given by

$$c_t + b_{t+1} + k_{t+1} \leq (1 - \phi_{lt})w_t l_t + R_{b,t} b_t + R_{k,t} k_t \quad (\text{II.4})$$

Now I premultiply the household budget constraint by ψ_t , the Lagrangian multiplier on the budget constraint in the household optimization problem, and sum over all

periods. The Euler equations for capital and bonds imply that

$$\psi_t k_{t+1} = \beta \psi_{t+1} R_{k,t+1} k_{t+1} \quad (\text{II.5})$$

$$\psi_t b_{t+1} = \beta \psi_{t+1} \quad (\text{II.6})$$

These are accompanied by the corresponding transversality conditions

$$\lim_{t \rightarrow \infty} \psi_t b_t = 0 \quad (\text{II.7})$$

$$\lim_{t \rightarrow \infty} \psi_t k_t = 0 \quad (\text{II.8})$$

Equations (II.5-II.8) can then be used to eliminate capital and bonds for every period except for $t = 0$, and I obtain

$$\sum_{t=0}^{\infty} \beta^t \psi_t (c_t - (1 - \phi_{it}) l_t) = \psi_0 (R_{k,0} k_0 + R_{b,0} b_0) \quad (\text{II.9})$$

The household first order conditions with respect to consumption and labor supply imply

$$u_{c,t} = \psi_t \quad (\text{II.10})$$

$$-(1 - \phi_{it}) w_t = \frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} \quad (\text{II.11})$$

Combining equations (II.9-II.11), I can obtain the implementation constraint, equation (II.3).

We note that initial marginal utility of consumption ($U_{c,0}$) is included explicitly in the right-hand side of the implementation constraint, in the expression for the value of the initial capital and bond holdings. The initial consumption allocation thus plays a slightly different role in the government's problem, leading to

the two sets of first-order conditions. A series of papers by Pierpaolo Benigno and Michael Woodford, in particular Benigno and Woodford (2006) show that the policy problem defined by Chari et al is equivalent to a two-stage problem. In the two-stage problem, I define

$$W_0 = \sum_T \beta^T [u_{c,T} c_T + u_{l,T} l_T] \quad (\text{II.12})$$

where W_0 is the value (in units of marginal utility) of the initial asset holdings of the representative household. In the first stage of the problem, the allocations (c_0, l_0, k_1) as well as a commitment \bar{W}_1 are chosen, given exogenously determined capital income tax rate and rate of return on bonds for the initial period. In the second stage, allocations would be chosen to maximize lifetime utility $U_t = \sum_T \beta^{T-t} u(c_T, l_T)$, subject to a resource constraint and the commitment chosen in the previous period. The second stage may be rewritten as a recursive problem, and thus allows us to more easily define a steady state.

II.3 The Model

The economy consists of a large number of individuals and firms. Individuals inelastically supply labor and capital to firms, and both purchase and appropriate output from them. The proportion of working hours spent in appropriation is $(1 - \sigma_t)$. In choosing the portion of their time spent in appropriation of output, individuals must consider the extent of protection of property rights via the legal system. We capture the extent of property rights protection by a variable P_t that I call legal infrastructure. Legal infrastructure refers to the system of laws, as well as the physical and human capital invested in the judicial and legal enforcement systems. P_t is a continuous stock variable, to capture small improvements in laws

and the quality of courts and of legal enforcement over time, as well as persistence of practices within the legal system.

Effective labor is equal to the product of the size of the labor force and the proportion of time each worker spends in productive activities. A constant-returns-to-scale technology is available to transform effective labor $\sigma_t L_t$ and capital K_t into output via the production function $F(K_t, \sigma_t L_t)$. The output can be used either for private consumption C_t , new capital K_{t+1} , or for new legal infrastructure, P_{t+1} , which serves to improve protection of output produced by firms. Feasibility requires that

$$C_t + K_{t+1} + P_{t+1} = F(K_t, \sigma_t L_t) + (1 - \delta_k)K_t + (1 - \delta_p)P_t \quad (\text{II.13})$$

where δ_k is the depreciation rate on capital, and δ_p is the depreciation rate on legal infrastructure. Investment in legal infrastructure is financed by proportional taxes on wages and capital income and by bonds. Let ϕ_{lt} denote the tax rate on labor income, and ϕ_{kt} the tax rate on capital income. Let B_{t+1} denote the debt issued in period t and $R_{b,t+1}B_{t+1}$ denote the debt service payment in the subsequent period.

Constant returns to scale implies that I may consider the following transformation of the aggregate production function

$$f\left(\frac{k_t}{\sigma_t}\right) \equiv \frac{1}{\sigma_t L_t} F(K_t, \sigma_t L_t) \quad (\text{II.14})$$

where $k_t = \frac{K_t}{L_t}$. If I assume a constant, exogenous population growth rate of n , then the feasibility constraint may be written in per capita terms

$$c_t + (1 + n)k_{t+1} + (1 + n)p_{t+1} = \sigma_t f\left(\frac{k_t}{\sigma_t}\right) + (1 - \delta_k)k_t + (1 - \delta_p)p_t \quad (\text{II.15})$$

where, as previously stated, the lower case refers to per capita variables.

Now I consider the behavior of households and firms, and then proceed to specify the optimization problem of policymakers in the Ramsey problem.

II.3.1 Households

My preference structure is standard. I assume that there are many identical consumers, each of whom maximizes the discounted expression for utility,

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \quad (\text{II.16})$$

In each period, while consumers supply labor and capital inelastically, they may choose both their level of consumption, c_t , and the proportion of their time they wish to devote to honest, productive activities. The remainder of their labor time they will spend in appropriating output from the firm. Their labor income consists of wage income, w_t , and appropriation income $\tau(1 - \sigma_t)$, where σ_t is the proportion of their time allocated to productive activities, τ is the rate of return to appropriation. In each period, households face some probability that they will be caught by the government in the act of appropriation. If they are caught, all their labor income, including any appropriated output, will be forfeited to the firm. With probability $h(p_t, \sigma_t)$ they will retain all their income. Only "legal" income is taxed. The budget constraint for period t is thus given by

$$\begin{aligned} h(p_t, \sigma_t) \cdot ((1 - \phi_{lt})w_t + \tau(1 - \sigma_t)) + (1 + (1 - \phi_{kt})(r_t - \delta_k))k_t + R_{b,t}b_t \\ \geq c_t + (1 + n)k_{t+1} + (1 + n)b_{t+1} \end{aligned} \quad (\text{II.17})$$

Here, I treat each household as a continuum of agents, so that each household receives the expected value of labor income from each agent. This avoids issues of risk which are tangential to the focus of this chapter. I make the following simplifying assumptions. First, the utility function is strictly concave and satisfies the Inada conditions. Second, the probability function $h(p_t, \sigma_t)$ behaves as follows:

$$h(p_t, \sigma_t) \in [0, 1] \quad \forall \sigma_t, p_t \quad (\text{II.18a})$$

$$\lim_{p_t \rightarrow 0} h(p_t, \sigma_t) = 1 \quad \forall \sigma_t, p_t \quad (\text{II.18b})$$

$$\lim_{p_t \rightarrow \infty} h(p_t, \sigma_t) = 0 \quad \text{for } \sigma_t < 1 \quad (\text{II.18c})$$

$$\lim_{\sigma_t \rightarrow 1} h(p_t, \sigma_t) = 1 \quad \forall p_t \quad (\text{II.18d})$$

$$\lim_{\sigma_t \rightarrow 0} h(p_t, \sigma_t) = 0 \quad \forall p_t \quad (\text{II.18e})$$

$$h_{\sigma,t} \geq 0 \quad \forall \sigma_t, p_t \quad (\text{II.18f})$$

$$h_{p,t} \leq 0 \quad \forall \sigma_t, p_t \quad (\text{II.18g})$$

$$\lim_{\sigma_t \rightarrow 1} h_{\sigma,t} = 0 \quad (\text{II.18h})$$

These conditions are sufficient to impose interior solutions on c_t and σ_t . I note for the reassurance of the reader that the set of functions satisfying equations (II.18a-II.18h) is non-null. Later in the chapter, I will employ the specific functional form:

$$h(p, \sigma) = \frac{1}{1 + \frac{\theta}{\sigma} p^\gamma (1 - \sigma)^\gamma} \quad (\text{II.19})$$

For now, however, I shall continue with the general form $h(p, \sigma)$. The household first-order conditions are then given by a static condition,

$$h_{\sigma,t}((1 - \phi_{lt})w_t + \tau(1 - \sigma_t)) = h(p_t, \sigma_t)\tau \quad (\text{II.20})$$

the Euler equation for physical capital,

$$((1+n)u_{c,t} - \beta u_{c,t+1}(1 + (1 - \phi_{k,t+1})(r_{t+1} - \delta_k)))k_{t+1} = 0 \quad (\text{II.21})$$

and the Euler equation for bonds

$$((1+n)u_{c,t} - \beta u_{c,t+1}R_{b,t+1})b_{t+1} = 0 \quad (\text{II.22})$$

Equations (II.21) and (II.22) are conventional Euler equations with respect to capital and bonds respectively. In equation (II.20) the household is simply equating the marginal cost and benefit from an increase in the time spent productively, σ_t . $h_{\sigma,t}((1 - \phi_t)w_t + \tau(1 - \sigma_t))$ gives the marginal benefit of σ in the form of increased retained income. $h(p_t, \sigma_t)\tau$ gives the marginal cost, in the form of reduced income from appropriation.

II.3.2 Firms

Perfectly competitive markets imply that firms take rental rates of labor and capital, as well as the return on appropriation, as given. Since firms are not able to observe the appropriation of output by workers, the representative firm maximizes in each period the expected value of its profit

$$\max_{L_t} F(K_t, \bar{\sigma}_t L_t) - h(p_t, \sigma_t)(w_t L_t + \tau(1 - \bar{\sigma}_t)L_t) - r_t K_t \quad (\text{II.23})$$

where $\bar{\sigma}_t$ is the firm's expectation of the proportion of time workers will devote to productive activities. In equilibrium this will be equal to σ_t .

The return to capital and labor thus equal their marginal products, namely

$$r_t = F_K(K_t, \sigma_t L_t) = f'\left(\frac{k_t}{\sigma_t}\right) \quad (\text{II.24})$$

$$h(p_t, \sigma_t)(w_t + \tau(1 - \sigma_t)) = F_L(K_t, \sigma_t L_t) = \sigma_t f\left(\frac{k_t}{\sigma_t}\right) - k_t f'\left(\frac{k_t}{\sigma_t}\right) \quad (\text{II.25})$$

II.3.3 Competitive Equilibrium

In order to fully specify the competitive equilibrium, I must describe the income and expenditure of the government. I assume that the government can borrow by issuing a one-period risk-free real bond. Per capita government debt evolves according to the law of motion

$$(1 + n)b_{t+1} = R_{b,t}b_t + (1 + n)p_{t+1} - (1 - \delta_p)p_t - \phi_{lt}h(p_t, \sigma_t)w_t - \phi_{kt}(r_t - \delta_k)k_t \quad (\text{II.26})$$

where p_0, b_0, k_0 are given, as are $\phi_{k,0}$ and $R_{b,0}$. Our first objective is to understand the behavior of the economy under exogenously given sequences of policies, $\{p_t, \phi_{l,t}, \phi_{k,t}\}_{t=0}^{\infty}$. The economy will be fully characterized by this sequence of policies, the household budget constraint (III.22), the household first-order conditions (II.20-II.22), the rental rates (II.24), and the law of motion of government debt (II.26). The household budget constraint, the government budget constraint and the rental rates may be combined to obtain a resource constraint, which, following Chari et al, I will refer to as a feasibility constraint. Consider now the steady state equilibrium, given by c, σ, k, b and the policy variables p, ϕ_l, ϕ_k, R_b . This steady state is characterized by the following set of first-order conditions and

the feasibility constraint:

$$\sigma f\left(\frac{k}{\sigma}\right) = (n + \delta_k)k + (n + \delta_p)p + c \quad (\text{II.27})$$

$$h_\sigma \left((1 - \phi_l) \frac{\sigma f\left(\frac{k}{\sigma}\right) - kf'\left(\frac{k}{\sigma}\right)}{h(p, \sigma)} + \phi_l \tau (1 - \sigma) \right) = h(p, \sigma) \tau \quad (\text{II.28})$$

$$(1 + n) = \beta \left(1 + (1 - \phi_k) \left(f'\left(\frac{k}{\sigma}\right) - \delta_k \right) \right) \quad (\text{II.29})$$

The Euler equation for bonds implies that $b = 0$ or $R_b = (1 + n)/\beta$. So that this steady state is feasible, I require that the policy variables are such that the government budget constraint is satisfied. The Euler equation for bonds implies that the following constraint must hold:

$$(1 + n) \left(\frac{\beta - 1}{\beta} \right) b = (n + \delta_p)p - \phi_l \sigma f\left(\frac{k}{\sigma}\right) + (\phi_l - \phi_k) k f'\left(\frac{k}{\sigma}\right) + \phi_l h(p, \sigma) \tau (1 - \sigma) + \phi_k \delta_k k \quad (\text{II.30})$$

From the Euler equation for capital, equation (II.29), I find that k and σ are linearly related, or that $k = B\sigma$, where I define

$$B \equiv f'^{-1} \left(\frac{1 + n - \beta(1 - (1 - \phi_k)\delta_k)}{\beta(1 - \phi_k)} \right) \quad (\text{II.31})$$

As a result, I may eliminate c and k from the steady state equations (II.27-II.29).

We are left with one key equation in σ and the policy variables:

$$h_\sigma \left((1 - \phi_l) \frac{\sigma f(B) - B\sigma f'(B)}{h(p, \sigma)} + \phi_l \tau (1 - \sigma) \right) = (h(p, \sigma))^2 \tau \quad (\text{II.32})$$

One can show under optimal policy that $\phi_k = 0$, and that $R_b = (1 + n)/\beta$. In this

case, using equation (II.30), equation (II.32) may be rewritten as

$$h_\sigma \left(\sigma f(B) - B\sigma f'(B) - (n + \delta_p)p - \frac{(1+n)(1-\beta)}{\beta}b \right) = h(p, \sigma)\tau \quad (\text{II.33})$$

This equation completely determines the steady state of the economy in the case of exogenous b, p where ϕ_l is chosen to satisfy the government budget constraint. We can now employ the assumptions made about the probability function $h(p, \sigma)$ to more fully characterize the steady state(s). These assumptions imply the following limiting behavior of the left-hand side of equation (II.33)

$$\lim_{\sigma \rightarrow 0} h_\sigma(\sigma f(B) - B\sigma f'(B) - (n + \delta_p)p - (R_b - (1+n))b) \leq 0 \quad (\text{II.34})$$

$$\lim_{\sigma \rightarrow 1} h_\sigma(\sigma f(B) - B\sigma f'(B) - (n + \delta_p)p - (R_b - (1+n))b) = 0 \quad (\text{II.35})$$

However, $w = \sigma f(B) - B\sigma f'(B)$, and under optimal policy $\phi_k = 0$ so that investment in p is wholly funded through taxes on wages. Since I impose that $\phi_l \leq 1$ then $\sigma f(B) - B\sigma f'(B) - h(p, \sigma)\tau(1 - \sigma) \geq (n + \delta_p)p + \frac{(1+n)(1-\beta)}{\beta}$. It may be shown that while equation (II.33) generally admits two solutions, the lower solution implies that $\phi_l > 1$.

As $h(p, \sigma)$ is increasing in σ , the right-hand side of the equation will be increasing in σ for all values of σ . This would imply the possibility of two steady states of σ for a given value of p . In the case where $\lim_{\sigma \rightarrow 0} h_\sigma = 0$ the lower steady state would occur at $\sigma = 0$. Otherwise, there will generally be two steady states such that $\sigma > 0$ for a given value of p . The plot in figure (II.1) is generated for the

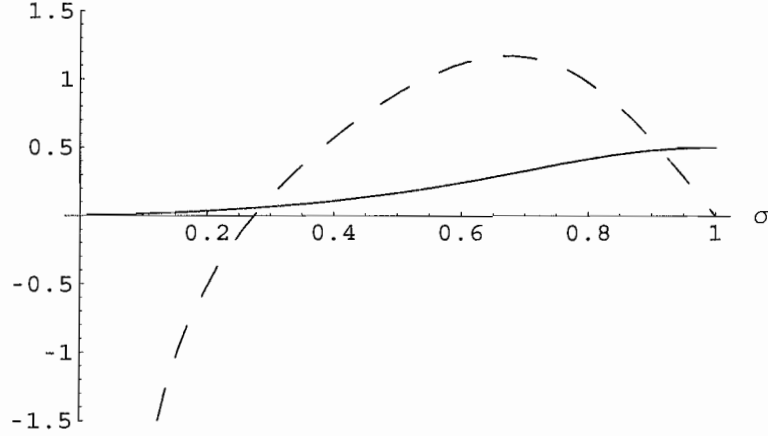


Fig. II.1: Plot of values of left-hand/right-hand sides of equation(II.33)for $p = 2$. The dashed line indicates the value of the left-hand side as σ is varied, while the solid line indicates the value of the right-hand side.

case $b = 0$, assuming the functional forms

$$h(p, \sigma) = \frac{1}{1 + \frac{\theta}{\sigma} p^\gamma (1 - \sigma)^\gamma}$$

$$f\left(\frac{k}{\sigma}\right) = \left(\frac{k}{\sigma}\right)^\alpha$$

and the parameter values presented in table(II.1). We can observe in figure (II.1) the existence of two solutions to equation (II.33), but as argued above, only the upper steady state will satisfy the restriction that $\phi_l \leq 1$.

α	1/3
n	0.03
β	0.98
δ_k	0.1
δ_p	0.1
θ	1
γ	2
τ	0.5

Tab. II.1: Baseline values of key parameters.

In order to satisfy our assumptions regarding its behavior, $h(p, \sigma)$ must have the property that h_σ is initially increasing in σ for low values of σ , but that for high values of σ , h_σ is decreasing. In particular, there will exist a σ^* for which h_σ is maximized. The value σ^* will be increasing in p . The marginal effect of σ on retained income will thus attain a peak and then start to decline as σ approaches unity. As property rights improve, or p increases, the peak will be attained for a higher value of σ . (Figure II.2) The behavior of the expected return to appropriation, $h(p, \sigma)\tau$ is much simpler, as it is simply monotonically increasing in σ and decreasing in p . From the previous arguments with regard to the behavior of the marginal effect of σ on retained labor income, and the return to appropriation, I may conclude that the solution for σ will be increasing in p . From the household's perspective, taking wages as given, an increase in property rights will tend to reduce the relative return in appropriation, resulting in a lower fraction of time spent in appropriation. An increase in τ will increase the upper solution for σ , the only feasible solution. This can be seen by implicitly differentiating σ with respect to τ in equation (II.33) to obtain

$$\frac{d\sigma}{d\tau} = \frac{h(p, \sigma)}{h_{\sigma\sigma}(\sigma f(B) - Bf'(B)\sigma - (n + \delta_p)p - (R_b - (1 + n))b) + h_\sigma(f(B) - Bf'(B) - \tau)}$$

For high values of σ , $h_{\sigma\sigma} < 0$, which can result in $d\sigma/d\tau < 0$.

Consumption in the steady state can be directly determined from the resource constraint, given p and the level of time spent in appropriation σ

$$c = \sigma f(B) - (n + \delta_k)B\sigma - (n + \delta_p)p$$

If social welfare is measured by the lifetime utility of the representative household in

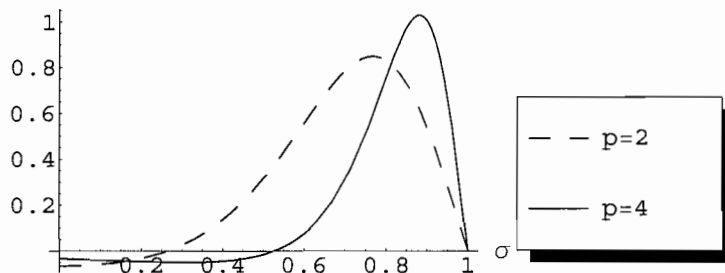


Fig. II.2: Marginal effect of increase in σ on retained labor income

the steady-state, it will be given by

$$U = \frac{1}{1 - \beta} u(c)$$

If the utility function is monotonically increasing in consumption, then an analysis of the variation of household welfare in the steady state will be equivalent to the variation of steady-state consumption. The effect of an improvement of legal infrastructure on steady-state consumption or

$$\frac{dc}{dp} = (f(B) - (n + \delta_k)B) \frac{d\sigma}{dp} - (n + \delta_p) \quad (\text{II.36})$$

will depend on how σ responds to an increase in p . It will be highly responsive to increases in legal infrastructure when p is low, but not for relatively high values of p . This is due to the fact σ will rapidly approach the upper limit of $\sigma = 1$, where $h_\sigma = 0$, regardless of the value of p . As a result, in the steady state, consumption will be maximized for a relatively low level of p .

As the return to appropriation τ increases, households will choose to allocate more time to appropriation, reducing σ for a given level of p . Now address the

responsiveness of p to σ . Differentiate (II.33) with respect to p to obtain:

$$\frac{d\sigma}{dp} = \frac{h_p\tau + (n + \delta_p)h_\sigma - h_{\sigma p} \left(\sigma f(B) - \sigma Bf'(B) - (n + \delta_p)p - \frac{(1+n)(1-\beta)b}{\beta} \right)}{h_\sigma(f(B) - Bf'(B)) + h_{\sigma\sigma} \left(\sigma f(B) - Bf'(B)\sigma - (n + \delta_p)p - \frac{(1+n)(1-\beta)b}{\beta} \right)}$$

The denominator of the expression will be positive for all values of σ except when σ is sufficiently close to 0 or 1, so that

$$h_{\sigma\sigma} \left(\sigma f(B) - Bf'(B)\sigma - (n + \delta_p)p - \frac{(1+n)(1-\beta)b}{\beta} \right) < 0$$

As a result, except for the case where σ is either sufficiently small or large, $d\sigma/dp$ declines as τ increases. In the latter cases, an increase in the return to appropriation will increase $d\sigma/dp$. The increased sensitivity of σ to p at lower levels of legal infrastructure in the case of high return to appropriation will imply a higher level of p for which steady-state consumption is maximized, which one can observe in figure (II.3). Let us define p^* as the value of p for which c will be maximized for a given value of τ . Similarly, define $c^* = \max_p c(p, \tau)$.

$$\frac{dc^*}{d\tau} = c_p \frac{dp^*}{d\tau} + c_\tau$$

At c^* , $c_p = 0$, so that $dc^*/d\tau = dc(p, \tau)/d\tau$. The function $c(p, \tau)$ will be declining in the second argument, since σ declines as τ increases, causing income will be reduced. As a result, c^* will be decreasing in τ .

While the preceding analysis is certainly intuitive, and allows us to characterize the effect of policy on the steady state, it does not allow us to determine optimal policy, which is set in a dynamic framework. To find optimal steady state levels of p , I must solve the Ramsey problem.

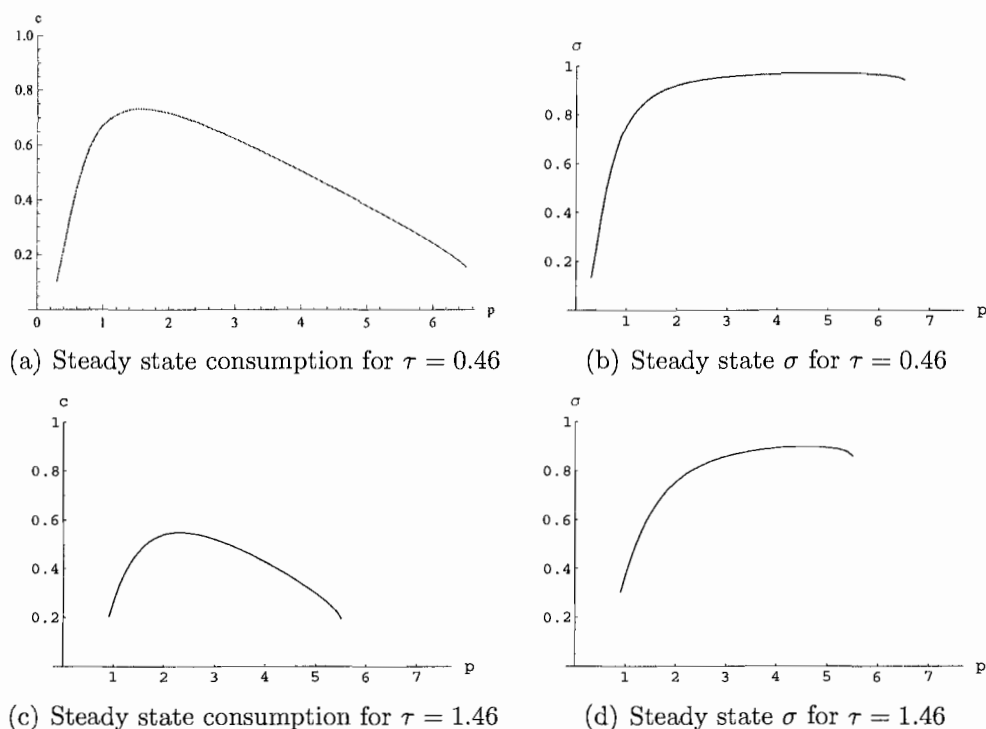


Fig. II.3: Effect of τ on steady state values of c, σ for $b = 0$

II.4 The Ramsey Problem

Consider now the problem faced by the government in setting optimal policy. For simplicity, I assume that there is an institution through which the government can bind itself to a particular sequence of policies. Since policies need to account for consumer and firm responses to policies, allocations and rental rates on labor and capital will be given by sequences of functions that associate allocations and prices with policies.

Definition II.1. A Ramsey equilibrium is a policy $\{p_{t+1}, b_{t+1}, \phi_{l,t}, \phi_{k,t}, R_{b,t}\}_{t=t_0}^{\infty}$, an allocation rule $\{c(\cdot), k(\cdot), \sigma(\cdot)\}$ and price rules $w(\cdot)$, and $r(\cdot)$ such that

- i. The policy maximizes the discounted lifetime utility of the representative household (II.16), subject to the government budget constraint (II.26), where allocations and rental rates are given by the rules defined above.

- ii. For every policy, the corresponding allocation and rental rates will maximize (II.16), subject to the household budget constraint (III.22).
- iii. For every policy, the rental rates satisfy the profit-maximizing conditions given by (II.24) and (II.25).

Proposition II.1. *The allocations of consumption, physical capital and time spent in appropriation in a Ramsey equilibrium solve the Ramsey allocation problem*

$$\max_{c_t, \sigma_t, k_{t+1}, p_{t+1}} \sum_t \beta^t u(c_t) \quad (\text{II.37})$$

subject to the following constraints:

i. Resource constraint

$$c_t + (1+n)k_{t+1} + (1+n)p_{t+1} = \sigma_t f\left(\frac{k_t}{\sigma_t}\right) + (1-\delta_k)k_t + (1-\delta_p)p_t \quad (\text{II.38})$$

ii. Implementation constraint

$$\sum_t \beta^t u_{c,t} \left(c_t - \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \tau \right) = u_{c,0} (R_{b,0} b_0 + (1 + (1 - \phi_{k0})(r_0 - \delta_k)) k_0) \quad (\text{II.39})$$

where $R_{b,0}$ and ϕ_{k0} are taken as given. ²

Proof. In section (II.3.3), I showed that by combining the household budget constraint, the government budget constraint, and the competitive rental rates of labor and capital, I obtained the feasibility condition (II.38). Multiplying the

²This assumption is to avoid the typical result of the Ramsey problem that the government will have the incentive to set the initial tax rate on capital as large as possible.

household budget constraint (III.22) by ψ_t , summing over t

$$\sum_t \beta^t \psi_t [h(p_t, \sigma_t)((1 - \phi_{lt})w_t + \tau(1 - \sigma_t)) + (1 + (1 - \phi_{kt})(r_t - \delta_k))k_t + R_{b,t}b_t - c_t - (1 + n)k_{t+1} - (1 + n)b_{t+1}] = 0 \quad (\text{II.40})$$

The Euler equations for capital and bonds

$$(1 + n)\psi_t k_{t+1} = \beta \psi_{t+1} (1 + (1 - \phi_{k,t+1})(r_{t+1} - \delta_k))k_{t+1} \quad (\text{II.41})$$

$$(1 + n)\psi_t b_{t+1} = \beta \psi_{t+1} R_{b,t+1} b_{t+1} \quad (\text{II.42})$$

and the corresponding transversality conditions may be used to eliminate to eliminate capital and bonds, with the exception of $t = 0$. Combining equations (II.40-II.42), and slightly rearranging, I may obtain

$$\begin{aligned} \sum_t \beta^t \psi_t (c_t - h(p_t, \sigma_t)((1 - \phi_{lt})w_t + \tau(1 - \sigma_t))) \\ = \psi_0 (R_{b,0}b_0 + (1 + (1 - \phi_{k0})(r_0 - \delta_k))k_0) \end{aligned} \quad (\text{II.43})$$

The first-order condition for consumption implies that $u_{c,t} = \psi_t$. The first order condition with respect to σ_t may then be used to obtain w_t in terms of σ_t and p_t . Equation (II.43) may thus be transformed into the implementation constraint (II.39). Thus, equations (II.38) and (II.39) are necessary conditions that any Ramsey equilibrium must satisfy.

Now, given any allocation that satisfies (II.38) and (II.39), I may construct sequences of bond holdings and policies such that these allocations satisfy the first-order conditions of the household's optimization problem. An allocation $\{c_t, \sigma_t, k_{t+1}, p_{t+1}\}_{t=0}^{\infty}$ will determine the rental rates of labor and capital, according

to:

$$r_t = f'\left(\frac{k_t}{\sigma_t}\right) \quad (\text{II.44})$$

$$h(p_t, \sigma_t)(w_t + \tau(1 - \sigma_t)) = \left(\sigma_t f\left(\frac{k_t}{\sigma_t}\right) - k_t f'\left(\frac{k_t}{\sigma_t}\right)\right) \quad (\text{II.45})$$

To construct the bond allocations, multiply the household budget constraint (III.22) by ψ_t and sum overall periods following r . Slightly rearranging, I obtain

$$\begin{aligned} \sum_{t=r+1}^{\infty} \beta^t \psi_t [(1 + (1 - \phi_{kt})(r_t - \delta_k))k_t - (1 + n)k_{t+1} + R_{b,t}b_t - (1 + n)b_{t+1}] \\ = \sum_{t=r+1}^{\infty} \beta^t \psi_t [c_t - h(p_t, \sigma_t)((1 - \phi_{lt})w_t + \tau(1 - \sigma_t))] \end{aligned} \quad (\text{II.46})$$

Following the same steps as I used in deriving the implementation constraint, in particular, employing the Euler equations for capital and bonds, equation (II.46) may be transformed to

$$\beta^{r+1} \psi_{r+1} [(1 + (1 - \phi_{k,r+1})(r_{r+1} - \delta_k))k_{r+1} + R_{b,r+1}b_{r+1}] = \sum_{t=r+1}^{\infty} \beta^t \psi_t \left[c_t - \tau \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \right] \quad (\text{II.47})$$

But the household first order condition with respect to capital and bonds implies that

$$(1 + n)\beta^r \psi_r (b_{r+1} + k_{r+1}) = \beta^{r+1} \psi_{r+1} [R_{b,r+1}b_{r+1} + (1 + (1 - \phi_{k,r+1})(r_{r+1} - \delta_k))k_{r+1}] \quad (\text{II.48})$$

As a result, I obtain

$$(1 + n)\beta^r \psi_r (b_{r+1} + k_{r+1}) = \sum_{t=r+1}^{\infty} \beta^t \psi_t \left[c_t - \tau \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \right] \quad (\text{II.49})$$

Or bonds issued in period r are given by

$$b_{r+1} = \sum_{t=r+1}^{\infty} \beta^{t-r} \frac{u_{c,t} \left(c_t - \frac{(h(p_t, \sigma_t))^2 \tau}{h_{\sigma,t}} \right)}{(1+n)u_{c,r}} - k_{r+1} \quad (\text{II.50})$$

The method of construction of the bond allocations implies that the household first-order conditions and budget constraint will be satisfied, provided that the return to bonds is given by

$$R_{b,t} = 1 + (1 - \phi_{kt})(r_t - \delta_k) \quad (\text{II.51})$$

for periods in which $b_t > 0$. □

The Lagrangian for the Ramsey problem can thus be written as

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t & \left(U(c_t) + \tilde{\psi}_t (\sigma_t f \left(\frac{k_t}{\sigma_t} \right) + (1 - \delta_k)k_t + (1 - \delta_p)p_t - (1+n)(k_{t+1} + p_{t+1}) - c_t) \right. \\ & \left. - \lambda u_{c,t} \left(c_t - \frac{(h(p_t, \sigma_t))^2 \tau}{h_{\sigma,t}} \right) \right) \\ & + \lambda u_{c,0} \left(R_{b,0} b_0 + \left(1 + (1 - \phi_{k0}) \left(f' \left(\frac{k_0}{\sigma_0} \right) - \delta_k \right) \right) k_0 \right) \quad (\text{II.52}) \end{aligned}$$

$\tilde{\psi}_t$ is the time-varying Lagrange multiplier on the feasibility constraint, as opposed to ψ_t , the Lagrange multiplier on the budget constraint in the household's optimization problem. λ is the multiplier on the implementation constraint (II.39), and is thus constant over time. The first order conditions for this problem imply that for $t \geq 1$ the Lagrange multiplier on the feasibility constraint may be given by

$$\tilde{\psi}_t = u_{c,t} - \lambda \left(u_{c,t} + u_{cc,t} \left(c_t - \tau \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \right) \right) \quad (\text{II.53})$$

Were the implementation constraint not binding, then $\lambda = 0$, and the Lagrange

multiplier on the feasibility constraint would simply equal the marginal utility of consumption at the optimum, a familiar result from the household optimization problem. When the implementation constraint binds, then the expression must include a correction term, equal to the marginal effect of an increase in c_t on the value of the implementation constraint. This captures the necessity for policy to be set so that the household attains its optimal allocation. In particular, the allocations in $c_t, k_{t+1}, \sigma_{t+1}$ obtained as solutions to the Ramsey problem, will be solutions to the household's optimization problem, under the policies $p_t, \phi_{k,t}, \phi_{l,t}$ corresponding to the Ramsey equilibrium. Having obtained $\tilde{\psi}_t$, the following static condition must hold

$$\tilde{\psi}_t \left(f \left(\frac{k_t}{\sigma_t} \right) - \frac{k_t}{\sigma_t} f' \left(\frac{k_t}{\sigma_t} \right) \right) + \lambda u_{c,t} \tau \left(2h(p_t, \sigma_t) - \left(\frac{h(p_t, \sigma_t)}{h_{\sigma,t}} \right)^2 h_{\sigma\sigma,t} \right) = 0 \quad (\text{II.54})$$

along with two dynamic equations, the first order conditions with respect to capital and property rights respectively

$$-(1+n)\tilde{\psi}_t + \beta\tilde{\psi}_{t+1} \left(1 - \delta_k + f' \left(\frac{k_{t+1}}{\sigma_{t+1}} \right) \right) = 0 \quad (\text{II.55})$$

$$\begin{aligned} \beta\lambda u_{c,t+1} \tau \left(2h(p_{t+1}, \sigma_{t+1}) \frac{h_{p,t+1}}{h_{\sigma,t+1}} - \left(\frac{h(p_{t+1}, \sigma_{t+1})}{h_{\sigma,t+1}} \right)^2 h_{\sigma p,t+1} \right) \\ + \beta\tilde{\psi}_{t+1} (1 - \delta_p) - (1+n)\tilde{\psi}_t = 0 \end{aligned} \quad (\text{II.56})$$

For $t = 0$, the Lagrangian multiplier on the feasibility constraint will be given by

$$\begin{aligned} \tilde{\psi}_0 = u_{c,0} - \lambda \left(u_{c,0} + u_{cc,0} \left(c_0 - \tau \frac{(h(p_0, \sigma_0))^2}{h_{\sigma,0}} \right) \right) \\ + \lambda u_{cc,0} \left(R_{b,0} b_0 + \left(1 + (1 - \phi_{k,0}) \left(f' \left(\frac{k_0}{\sigma_0} \right) - \delta_k \right) \right) k_0 \right) \end{aligned} \quad (\text{II.57})$$

The following first order condition with respect to σ_0 must also hold:

$$\begin{aligned} \tilde{\psi}_0 \left(f \left(\frac{k_0}{\sigma_0} \right) - \frac{k_0}{\sigma_0} f' \left(\frac{k_0}{\sigma_0} \right) \right) + \lambda u_{c,0} \tau \left(2h(p_0, \sigma_0) - \left(\frac{h(p_0, \sigma_0)}{h_{\sigma,0}} \right)^2 h_{\sigma\sigma,0} \right) \\ - \lambda u_{c,0} (1 - \phi_{k,0}) \left(\frac{k_0}{\sigma_0} \right)^2 f'' \left(\frac{k_0}{\sigma_0} \right) = 0 \quad (\text{II.58}) \end{aligned}$$

The first order conditions with respect to capital and property rights for $t=0$ will be identical in form to those for $t \geq 1$.

II.5 An Alternative Approach to the Ramsey Equilibrium

The approach taken by Chari et al has the property that the policy problem is not recursive, and in particular, that the first-order conditions in the initial period differ from those in subsequent periods. Let us now consider an alternative approach, advocated by Benigno and Woodford (2006), which has the technical advantage of making the optimal policy stationary. To proceed I slightly alter the set of initial conditions. That is, rather than taking the initial tax rate on capital as given, I instead assume that there is a pre-existing commitment with regard to the value of initial household assets.

Definition II.2. The value of consumption net of wage income over periods $T \geq t$ is given by

$$W_t \equiv \sum_{T=t}^{\infty} \beta^{T-t} u_{c,T} \left(c_T - \frac{(h(p_T, \sigma_T))^2}{h_{\sigma,T}} \tau \right) \quad (\text{II.59})$$

\bar{W}_t is defined to be the pre-existing commitment regarding the value of household assets in period t .

Proposition II.2. *Given $\{k_{t_0}, p_{t_0}, b_{t_0}, \bar{W}_{t_0}\}$, consider the sequential optimization problem in which $\{x_t\} = \{c_t, \sigma_t, k_{t+1}, p_{t+1}, W_{t+1}\}$ are chosen for each period $t \geq t_0$ to maximize the function $J(x_t, \bar{W}_{t+1})$, subject to the feasibility constraint (II.38) and*

$$\bar{W}_t = u_{c,t} \left(c_t - \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \tau \right) + \beta \bar{W}_{t+1} \quad (\text{II.60})$$

given the values for $\{k_t, p_t, \bar{W}_t\}$ determined in the previous period. The function $J(\cdot)$ is defined by:

$$J(x_t, \bar{W}_{t+1}) \equiv u(c_t) + \beta V(k_{t+1}, p_{t+1}, \bar{W}_{t+1}) \quad (\text{II.61})$$

where $V(k_{t+1}, p_{t+1}, \bar{W}_{t+1})$ denotes the maximum attainable value of

$$U_t = \sum_{T=t}^{\infty} \beta^{T-t} u(c_T) \quad (\text{II.62})$$

subject to the feasibility and implementation constraints mentioned above. The allocation chosen in this manner will maximize the lifetime utility of households subject to the feasibility constraint (II.38) and the implementation constraint $W_{t_0} = \bar{W}_{t_0}$.

Proof. Consider the Ramsey problem:

$$\max \sum_{t=t_0}^{\infty} \beta^{t-t_0} u(c_t) \quad (\text{II.63})$$

subject to (II.38) and $W_{t_0} = \bar{W}_{t_0}$, given $\{k_{t_0}, p_{t_0}\}$. We now proceed by comparing the allocations chosen as the solution to the above problem to the outcome of a 2-stage problem. In the first stage of this problem, choose $\{x_{t_0}, \bar{W}_{t_0+1}\}$ to maximize

$J(x_{t_0}, \bar{W}_{t_0+1})$ such that the feasibility constraint holds and

$$\bar{W}_{t_0} = u_{c,t_0} \left(c_{t_0} - \frac{(h(p_{t_0}, \sigma_{t_0}))^2}{h_{\sigma,t_0}} \tau \right) + \beta \bar{W}_{t_0+1} \quad (\text{II.64})$$

In the second stage, $\{x_t\}$ is chosen to maximize U_{t_0+1} given the feasibility constraint and $W_{t_0+1} = \bar{W}_{t_0+1}$. We note that the solution of this two-stage problem is feasible as a Ramsey equilibrium, in that it satisfies the feasibility and implementation constraints. Clearly, both the first and second stage satisfy feasibility. Then, I can substitute $W_{t_0+1} = \bar{W}_{t_0+1}$ into equation (II.64), to obtain $W_{t_0} = \bar{W}_{t_0}$. It then remains only to show that there cannot be any other sequence of allocations $\{\tilde{x}_t\}$ that satisfies the constraints of the Ramsey problem and attains a higher level of utility U_{t_0} . Suppose that there does exist such a $\{\tilde{x}_t\}$, and let \tilde{W}_{t_0+1} be the implied value for W_{t_0+1} , \tilde{U}_{t_0+1} the implied forward-looking utility in period $t_0 + 1$ from the sequence of allocations, and \tilde{U}_{t_0} the implied value of U_{t_0} . By hypothesis $\tilde{U}_{t_0} > U_{t_0}$. We note that $\{x_t\}$ satisfies the constraints of the first-stage problem. Because the allocation is feasible and is consistent by construction with the precommitment \tilde{W}_{t_0+1} , I must have

$$V(\tilde{k}_{t_0+1}, \tilde{p}_{t_0+1}, \tilde{W}_{t_0+1}) \geq \tilde{U}_{t_0+1} \quad (\text{II.65})$$

Since the allocation also satisfies the constraints of the first-stage problem, then

$$J(\tilde{x}_{t_0}, \tilde{W}_{t_0+1}) \geq \tilde{U}_{t_0} \quad (\text{II.66})$$

We must then conclude that

$$J(\tilde{x}_{t_0}, \tilde{W}_{t_0+1}) > U_{t_0} \quad (\text{II.67})$$

which contradicts the assumption that $\{x_t\}$ solved the first-stage optimization. This

contradiction implies that $\{x_t\}$ represents a Ramsey equilibrium. Furthermore, the second-stage problem as described here is of the same form as the Ramsey problem. The same proof can be used to show that it is equivalent to a similar two-stage problem. By induction, I may establish that (x_t, \bar{W}_{t+1}) solve a similar "first-stage" problem to that described in this proof. The proposition follows. \square

A similar approach may be used to show that the policy problem as originally defined (with a constraint on the capital tax rate, rather than on the value of initial assets) is also equivalent to a two-stage problem. In the first stage, $(c_{t_0}, \sigma_{t_0}, k_{t_0+1}, p_{t_0+1})$ and \bar{W}_{t_0+1} are chosen, given the initial capital tax rate and the initial levels of capital and bonds. For $t \geq t_0$, policy is chosen to maximize U_{t_0+1} subject to the feasibility constraint and $W_{t_0+1} = \bar{W}_{t_0+1}$. Proposition (II.2) implies that the second stage of this problem is characterized by stationary optimal policies, for which I can define a steady state equilibrium.

II.6 Steady State under Optimal Policy with Commitment

Finding the steady state will correspond to finding, for an initial level of debt b , the initial commitment \bar{W} and initial levels of physical capital and legal infrastructure (k, p) such that the optimal sequence of allocations is a constant allocation and set of policies $\{c, \sigma, k, p, b, \phi_k, \phi_l, \bar{W}\}$, where b, k, \bar{W} are identical to the initial conditions.

Rewriting the optimization problem as a Lagrangian, I find

$$\mathcal{L}_{t_0} = \sum_{T=t_0}^{\infty} \left(u(c_T) + \psi_T \left(\sigma_T f \left(\frac{k_T}{\sigma_T} \right) + (1-\delta_k)k_T + (1-\delta_p)p_T - (1+n)k_{T+1} - (1+n)p_{T+1} - c_T \right) - \lambda u_{c,T} \left(c_T - \frac{(h(p_T, \sigma_T))^2}{h_{\sigma,T}} \tau \right) \right) + \lambda \bar{W}_{t_0} \quad (\text{II.68})$$

The first-order conditions with respect to c_t, σ_t are then given by

$$(1 - \lambda)u_{c,t} - \tilde{\psi}_t - \lambda u_{cc,t} \left(c_t - \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \tau \right) = 0 \quad (\text{II.69})$$

$$\tilde{\psi}_t \left(f \left(\frac{k_t}{\sigma_t} \right) - \frac{k_t}{\sigma_t} f' \left(\frac{k_t}{\sigma_t} \right) \right) + \tau \lambda u_{c,t} \left(2h(p_t, \sigma_t) - \left(\frac{h(p_t, \sigma_t)}{h_{\sigma,t}} \right)^2 h_{\sigma\sigma,t} \right) = 0 \quad (\text{II.70})$$

These are accompanied by two Euler-like equations, for k_{t+1} and p_{t+1} respectively:

$$-(1 + n)\tilde{\psi}_t + \beta\tilde{\psi}_{t+1} \left(1 - \delta_k + f' \left(\frac{k_{t+1}}{\sigma_{t+1}} \right) \right) = 0 \quad (\text{II.71})$$

$$\begin{aligned} \beta\tau\lambda u_{c,t+1} \left(2h(p_{t+1}, \sigma_{t+1}) \frac{h_{p,t+1}}{h_{\sigma,t+1}} - \left(\frac{h(p_{t+1}, \sigma_{t+1})}{h_{\sigma,t+1}} \right)^2 h_{\sigma p,t+1} \right) \\ - (1 + n)\tilde{\psi}_t + \beta\tilde{\psi}_{t+1}(1 - \delta_p) = 0 \end{aligned} \quad (\text{II.72})$$

In a steady-state solution, these conditions reduce to the following system of equations:

$$(1 - \lambda)u_c - \psi - \lambda u_{cc} \left(c - \frac{(h(p, \sigma))^2}{h_\sigma} \tau \right) = 0 \quad (\text{II.73})$$

$$\tilde{\psi} \left(f \left(\frac{k}{\sigma} \right) - \frac{k}{\sigma} f' \left(\frac{k}{\sigma} \right) \right) + \lambda\tau u_c \left(2h(p, \sigma) - \left(\frac{h(p, \sigma)}{h_\sigma} \right)^2 h_{\sigma\sigma} \right) = 0 \quad (\text{II.74})$$

and

$$1 + n = \beta \left(1 - \delta_k + f' \left(\frac{k}{\sigma} \right) \right) \quad (\text{II.75})$$

$$\tilde{\psi}(1 + n - \beta(1 - \delta_p)) = \beta\tau\lambda u_c \left(2h(p, \sigma) \frac{h_p}{h_\sigma} - \left(\frac{h(p, \sigma)}{h_\sigma} \right)^2 h_{\sigma p} \right) \quad (\text{II.76})$$

Equation (II.75) and the household first order condition for capital (II.29)

imply together that the tax rate on capital will be equal to zero in the steady state. Equations (II.73)-(II.76) together with the steady versions of the feasibility and implementation constraint and a constraint on \bar{W} to make the commitment feasible

$$\sigma f\left(\frac{k}{\sigma}\right) = c + (n + \delta_k)k + (n + \delta_p)p \quad (\text{II.77})$$

$$(1 - \beta)\bar{W} = u_c\left(c - \frac{(h(p, \sigma))^2}{h_\sigma}\tau\right) \quad (\text{II.78})$$

$$\bar{W} = u_c\left(R_b b + \frac{1+n}{\beta}k\right) \quad (\text{II.79})$$

form a system of equations which determine the steady state. The steady state tax rate on wages is determined by combining the household first order condition for σ with the market wage rate to obtain

$$(1 - \phi_l)\left(\sigma f\left(\frac{k}{\sigma}\right) - kf'\left(\frac{k}{\sigma}\right)\right) + \phi_l h(p, \sigma)\tau(1 - \sigma) = \frac{(h(p, \sigma))^2}{h_\sigma}\tau \quad (\text{II.80})$$

The following budget constraint gives the relationship between ϕ_l and b

$$(1 + n - R_b)b = (n + \delta_p)p - \phi_l\left(\sigma f\left(\frac{k}{\sigma}\right) - kf'\left(\frac{k}{\sigma}\right)\right) + \phi_l h(p, \sigma)\tau(1 - \sigma) \quad (\text{II.81})$$

The law of motion for government debt, equation (II.26), implies that b will be determined by the history of the economy prior to convergence to the steady state. The steady state allocation, c, σ, k, p will depend on b , and thus is history-dependent. In order to obtain numerical results for the steady state under full commitment, I consider the case of log utility. In my initial analysis, I assign to the parameters $\alpha, n, \beta, \delta_k, \delta_p, \theta, \gamma$ the values listed in Table II.1.

In section III.4, I considered the effect of p , taken at that point to be exogenous, on steady-state consumption and time spent in appropriation, σ . To

more clearly compare the results under exogenous policy to steady state consumption in the Ramsey equilibrium, let us define $c(p)$ as the steady state consumption as a function of legal infrastructure p , where $\phi_k = 0$ and $b = 0$. Furthermore, let $c^* = \max_p c(p)$, and let c_r denote the steady state value of consumption in the Ramsey equilibrium. Suppose that $u(c_r) > u(c^*)$. By the definition, the steady state allocation of the Ramsey equilibrium satisfies the household FOCs given policies p, ϕ_l, ϕ_k . Therefore, c^* violates its own definition. We can therefore state that $u(c^*) \geq u(c_r)$. Furthermore, c^* , and corresponding σ^*, p^*, k^* must satisfy the steady-state versions of the implementation and feasibility constraint, since these are derived from the household first order conditions. As a result, I can use the solution to the following maximization problem to find c^*

$$\max_{c, \sigma, p} u(c) + \psi \left(\sigma f \left(\frac{k}{\sigma} \right) - (n + \delta_k)k - (n + \delta_p)p - c \right) - \lambda u_c \left(c - \tau \frac{(h(p, \sigma))^2}{h_\sigma} \right) + \lambda \bar{W} \quad (\text{II.82})$$

given the steady state Euler equation, which implies $k = B\sigma$. The first order conditions of this problem with respect to c and σ are identical to those obtained for the steady state of the Ramsey equilibrium. The FOC with respect to p is given by

$$\psi(n + \delta_p) = \lambda u_c \tau \left(2h(p, \sigma) \frac{h_p}{h_\sigma} - \left(\frac{h(p, \sigma)}{h_\sigma} \right)^2 h_{\sigma p} \right) \quad (\text{II.83})$$

which is identical in form to equation (II.76), and is identical to equation (II.76) for $\beta = 1$. Thus, as β approaches 1, c_r approaches c^* .

We saw in our analysis of the steady state with exogenous policy, that while there were in general two potential solutions to the first order condition for σ , the lower value of σ implied a value of ϕ_l which was not feasible, i.e. $\phi_l \geq 1$. In my

numerical analysis, I found that for a given value of τ there was a single value of p that maximized steady-state consumption. One could use these results to argue for the existence of a unique steady state under policy with commitment. Alternatively, I may reduce the equations (II.73 - II.76) to a two-dimensional system in σ and p , given by

$$\begin{aligned} (f(B) - Bf'(B)) \left((1 - \lambda(p, \sigma))u_c(p, \sigma) - \lambda(p, \sigma)u_{cc}(p, \sigma) \left(c(p, \sigma) - \tau \frac{(h(p, \sigma))^2}{h_\sigma} \right) \right) \\ + \lambda(p, \sigma)\tau u_c(p, \sigma) \left(2h(p, \sigma) - \left(\frac{h(p, \sigma)}{h_\sigma} \right)^2 h_{\sigma\sigma} \right) = 0 \end{aligned} \quad (\text{II.84a})$$

and

$$(1 - \beta) \frac{1 + n}{\beta} (b + B\sigma) = c(p, \sigma) - \tau \frac{(h(p, \sigma))^2}{h_\sigma} \quad (\text{II.84b})$$

First, consider the behavior of equation (II.84b), which is simply the implementation constraint in the steady state. From the Euler equation, $k = B\sigma$, implying that the left-hand side is linear and increasing in σ , and independent of p . On the left-hand side, consumption will be linear in both σ and p . Income from labor, given by $\tau(h(p, \sigma))^2/h_\sigma$, is increasing in σ , decreasing in p and has the following limiting behavior

$$\lim_{\sigma \rightarrow 0} \frac{(h(p, \sigma))^2}{h_\sigma} = 0 \quad (\text{II.85})$$

$$\lim_{\sigma \rightarrow 1} \frac{(h(p, \sigma))^2}{h_\sigma} = \infty \quad (\text{II.86})$$

As a result, there is a unique value of σ for which $c - \tau(h(p, \sigma))^2/h_\sigma$ is maximized. Since for σ close to both 0 and 1, consumption net of labor income will be negative. There will thus be at most 2 solutions to equation (II.84b) with the case of no solutions occurring for p sufficiently low or high. For the moment, let us refer to

these two solutions as $\sigma_l(p, b)$ and $\sigma_h(p, b)$. An increase in p , i.e. in the capital invested in legal infrastructure, will lower steady steady-state consumption for every value of σ , and will similarly lower labor income. The value of the right-hand side of equation (II.84b) will decline for every value of σ . Consequently, both σ_l and σ_h will be increasing in p .

The behavior of equation (II.84a) is more complicated, and the reasoning is considerably less straightforward. However, numerical analysis shows that for a range of parameter values, and in particular, for a range of values of τ , there are at most 2 solutions in σ to the equation, where the upper solution is decreasing in p and the lower solution is increasing in p . In particular, when the capital invested in the legal infrastructure approaches 0, the lower solution will approach 0 and the upper solution will approach 1. This implies that there will be at most two solutions to the set of equations, which one can see in figure II.4. We refer to these two solutions as $\{\sigma_{lr}, p_{lr}\}$ and $\{\sigma_{hr}, p_{hr}\}$. In general, however, the solution $\{\sigma_{lr}, p_{lr}\}$ will in general be infeasible, requiring that $\phi_l > 1$.

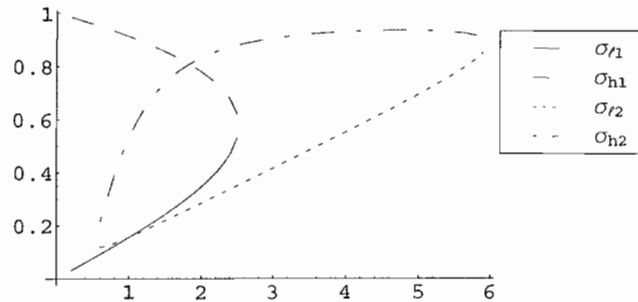


Fig. II.4: Possibility of multiple steady states under commitment: Solutions to the equations (II.84a-II.84b) for $b = 0$ and $\tau = 1$. σ_{l1} and σ_{h1} are the solutions to (II.84a), while σ_{l2} and σ_{h2} indicate the solutions to (II.84b).

In section (III.4), I showed that c^* was decreasing in τ . The dependence of the steady state on τ is entirely driven by its influence on the household's choice of σ_t . In particular, an increase in τ increases the return on appropriation relative to

wage income. The reduction in σ would imply a decline in steady state capital stock and consumption. The government could in theory increase the tax rate and improve the legal infrastructure in order to restore σ to its original level. However, an increase in the tax rate, would also increase the relative return from appropriation, reducing the net effect of the increase in the tax rate and in p on σ . In addition, maintaining a higher level of p would reduce steady state consumption. An increase in legal infrastructure will thus be optimal but σ will still decline.

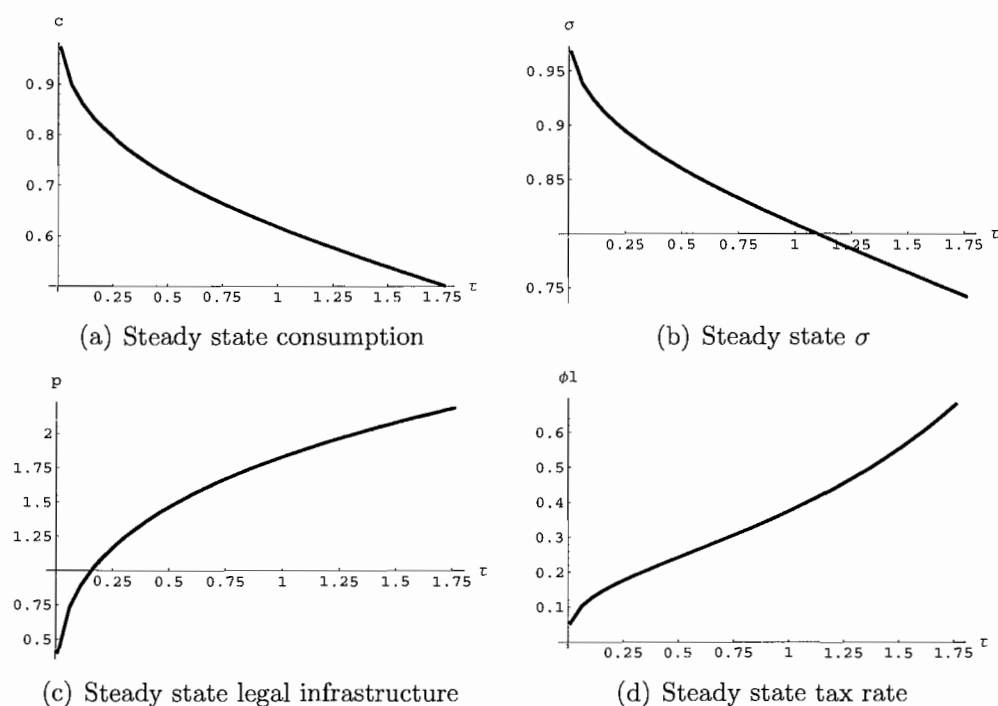


Fig. II.5: Effect of τ on steady state of Ramsey equilibrium

II.7 Discretionary Policy

II.7.1 Discretion as the first period of Ramsey problem

Now consider the case where the government is unable to commit to a sequence of policies for all time. Instead, they are only able to commit one period in

advance. Specifically, they choose the capital tax rate and the level of investment in the legal infrastructure for the subsequent period, and then set current labor tax rates in order to satisfy the government budget constraint. In this initial approach to the optimal policy problem under discretion, they assume that they will be able to commit to optimal policy in subsequent periods.

Rather than using an implementation constraint such as equation (II.39) based on initial endowments in capital and bonds, the government uses the following constraint in each period, to assure that the allocation found as a solution to the discretionary policy problem is in fact a solution to the household's optimization problem:

$$\sum_{T=t}^{\infty} \beta^{T-t} u_{c,T} \left(c_T - \frac{(h(p_T, \sigma_T))^2}{h_{\sigma,T}} \tau \right) = u_{c,t} (R_{b,t} b_t + (1 + (1 - \phi_{k,t})(r_t - \delta_k)) k_t) \quad (\text{II.87})$$

The derivation of this constraint is isomorphic to that for equation (II.39), with the exception that rather than summing over all time, I am simply considering the household's first-order conditions and budget constraint for $T \geq t$. The constraint simply represents the fact that present and future consumption in excess of period labor income must be funded by income from assets with which the representative household begins period t . The Lagrangian for the discretionary policy problem may thus be written

$$\begin{aligned} \mathcal{L}_t = & \sum_{T=t}^{\infty} \beta^{T-t} \left(u(c_T) + \tilde{\psi}_T \left(\sigma_T f \left(\frac{k_T}{\sigma_T} \right) + (1 - \delta_k) k_T + (1 - \delta_p) p_T \right. \right. \\ & \left. \left. - (1 + n)(k_{T+1} + p_{T+1}) - c_T \right) - \lambda u_{c,T} \left(c_T - \frac{(h(p_T, \sigma_T))^2}{h_{\sigma,T}} \tau \right) \right) \\ & + \lambda u_{c,t} \left(R_{b,t} b_t + \left(1 + (1 - \phi_{k,t}) \left(f' \left(\frac{k_t}{\sigma_t} \right) - \delta_k \right) \right) k_t \right) \quad (\text{II.88}) \end{aligned}$$

The first order condition with respect to c_t takes the form

$$u_{c,t} - \tilde{\psi}_t - \lambda \left(u_{c,t} + u_{cc,t} \left(c_t - \frac{(h(p_t, \sigma_t))^2}{h_{\sigma,t}} \tau \right) \right) + \lambda u_{cc,t} \left(R_{b,t} b_t + \left(1 + (1 - \phi_{k,t}) \left(f' \left(\frac{k_t}{\sigma_t} \right) - \delta_k \right) \right) k_t \right) = 0 \quad (\text{II.89})$$

The first-order condition with respect to σ_t can similarly be written as

$$\tilde{\psi}_t \left(f \left(\frac{k_t}{\sigma_t} \right) - \frac{k_t}{\sigma_t} f' \left(\frac{k_t}{\sigma_t} \right) \right) + \lambda u_{c,t} \tau \left(2h(p_t, \sigma_t) - \left(\frac{h(p_t, \sigma_t)}{h_{\sigma,t}} \right)^2 h_{\sigma\sigma,t} \right) - \lambda u_{c,t} (1 - \phi_{k,t}) \left(\frac{k_t}{\sigma_t} \right)^2 f'' \left(\frac{k_t}{\sigma_t} \right) = 0 \quad (\text{II.90})$$

Finally, I obtain two Euler-like equations, the first order conditions with respect to k_{t+1} and p_{t+1}

$$-(1+n)\tilde{\psi}_t + \beta\tilde{\psi}_{t+1} \left(1 - \delta_k + f' \left(\frac{k_{t+1}}{\sigma_{t+1}} \right) \right) = 0 \quad (\text{II.91})$$

$$\beta\lambda\tau u_{c,t+1} \left(2h(p_{t+1}, \sigma_{t+1}) \frac{h_{p,t+1}}{h_{\sigma,t+1}} - \left(\frac{h(p_{t+1}, \sigma_{t+1})}{h_{\sigma,t+1}} \right)^2 h_{\sigma p,t+1} \right) - (1+n)\tilde{\psi}_t + \beta\tilde{\psi}_{t+1}(1 - \delta_p) = 0 \quad (\text{II.92})$$

By comparing the above equation with the household Euler condition, one may find once again that the steady capital tax rate will be equal to zero. The latter conditions may be combined with the resource constraint and equation (II.87) to obtain the following system of equations characterizing the steady state under

discretionary policy

$$\begin{aligned} \psi &= (1 - \lambda)u_c - \lambda u_{cc} \left(c - \frac{(h(p, \sigma))^2}{h_\sigma} \tau \right) \\ &\quad + \lambda u_{cc} \left(R_b b + \left(1 + f' \left(\frac{k}{\sigma} \right) - \delta_k \right) k \right) \end{aligned} \quad (\text{II.93a})$$

$$\begin{aligned} \psi \left(f \left(\frac{k}{\sigma} \right) - \frac{k}{\sigma} f' \left(\frac{k}{\sigma} \right) \right) + \lambda \tau u_c \left(2h(p, \sigma) - \left(\frac{h(p, \sigma)}{h_\sigma} \right)^2 h_{\sigma\sigma} \right) \\ - \lambda u_c \left(\frac{k}{\sigma} \right)^2 f'' \left(\frac{k}{\sigma} \right) = 0 \end{aligned} \quad (\text{II.93b})$$

$$\psi(1 + n - \beta(1 - \delta_p)) = \beta \lambda \tau u_c \left(2h(p, \sigma) \frac{h_p}{h_\sigma} - \left(\frac{h(p, \sigma)}{h_\sigma} \right)^2 h_{\sigma p} \right) \quad (\text{II.93c})$$

$$(1 + n) = \beta \left(1 - \delta_k + f' \left(\frac{k}{\sigma} \right) \right) \quad (\text{II.93d})$$

$$c + (n + \delta)k + (n + \delta_p)p = \sigma f \left(\frac{k}{\sigma} \right) \quad (\text{II.93e})$$

$$\frac{1}{1 - \beta} \left(c - \frac{(h(p, \sigma))^2}{h_\sigma} \tau \right) = R_b b + \left(1 - \delta_k + f' \left(\frac{k}{\sigma} \right) \right) k \quad (\text{II.93f})$$

As in the case of commitment, I may reduce the equations characterizing the steady state to a two-dimensional system:

$$\begin{aligned} (f(B) - Bf'(B)) \left((1 - \lambda(b, p, \sigma))u_c(p, \sigma) - \lambda(b, p, \sigma)u_{cc}(p, \sigma) \left(c(p, \sigma) - \tau \frac{(h(p, \sigma))^2}{h_\sigma} \right. \right. \\ \left. \left. - \frac{1 + n}{\beta} (b + B\sigma) \right) \right) + \lambda(p, \sigma) \tau u_c(p, \sigma) \left(2h(p, \sigma) - \left(\frac{h(p, \sigma)}{h_\sigma} \right)^2 h_{\sigma\sigma} \right) \\ - \lambda u_c(p, \sigma) B^2 f''(B) = 0 \end{aligned} \quad (\text{II.94a})$$

and

$$(1 - \beta) \frac{1 + n}{\beta} (b + B\sigma) = c(p, \sigma) - \tau \frac{(h(p, \sigma))^2}{h_\sigma} \quad (\text{II.94b})$$

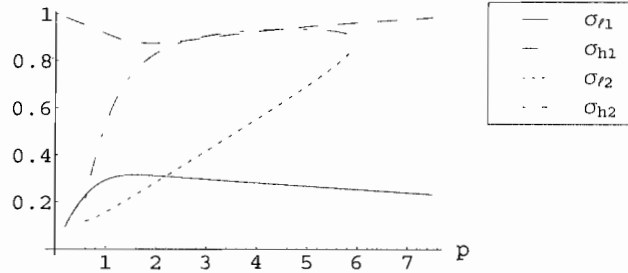


Fig. II.6: Possibility of multiple steady states under discretion: Solutions to the equations (II.94a-II.94b) for $b = 0$ and $\tau = 1$. σ_{l1} and σ_{h1} are the solutions to (II.94a), while σ_{l2} and σ_{h2} indicate the solutions to (II.94b).

The behavior of the two solutions to (II.94b) is identical to that of (II.84b).

The key difference in the Ramsey problem first-order conditions under discretion and commitment is that in the case of discretion policymakers consider the effect of policy on the value of assets in every period. In determining the sequence of allocations $\{c_T, \sigma_T, k_{T+1}, p_{T+1}\}_{T=t}^{\infty}$, the policymakers must balance dual objectives of maximizing lifetime utility and maximizing the current value of assets in order to relax the implementation constraint. The second objective causes the introduction of additional terms in the FOCs for c_t and σ_t . To see the intuition behind this, I observe that an increase in c_t , while increasing current-period utility, will also lower the marginal utility of consumption. If the current value of assets is given by

$$u_{c,t} (R_{b,t} b_t + \left(1 + (1 - \phi_{k,t}) \left(f' \left(\frac{k_t}{\sigma_t} \right) - \delta_k \right) k_t \right))$$

then the increase in c_t will reduce the value of these assets. Similarly, an increase in σ_t both increases wage income, and by increasing the interest rate, increases the current value of household assets. Under discretion, policy-makers will have an

incentive to decrease current consumption and increase σ_t , relative to their values under commitment. In the steady state, for relatively low values of p , numerical analysis shows that σ_{h1} will initially decrease as p increases, and in fact, will not differ significantly from its value under commitment. However, σ_{h1} will be increasing in p once property rights exceed a certain threshold (Figure (II.6)). This behavior may be justified by the fact that when property rights are high, wage income is relatively more important than income from appropriation, and the tax base will thus be higher. It becomes less costly to set investment in legal infrastructure and tax rates so that households spend less time in appropriation. Under discretion, the government will have an added incentive to do so.

The above characterization of the loci of solutions to equations (II.94a-II.94b) implies the possibility of four steady states, two with relatively low values of σ and two with relatively high values. The lower two steady states will not be feasible, except for very low values of τ , as they would require $\phi_l > 1$. However, the upper two steady states are indeed feasible for a broad range of values of τ , the return to appropriation.

II.7.2 Markov Perfect Equilibria

The approach to discretionary policy outlined in the previous section has the disadvantage that it assumes that despite having perfect foresight, governments and households will continue to assume throughout time that the government is always in the first period of a new commitment to policy. The framework of Markov Perfect Equilibria, as used by Klein et al (2003) and Ortigueira and Pereira (2007), assumes that governments are conscious of their inability to commit. Instead, fiscal policy will be chosen to maximize the lifetime utility of the representative household, given the policy functions of the sequence of future governments.

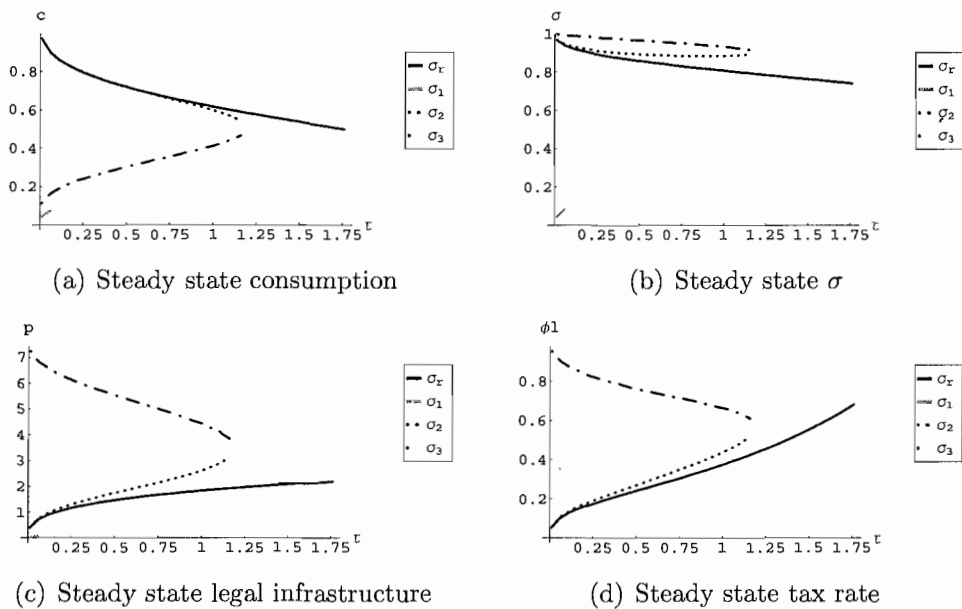


Fig. II.7: Effect of τ on steady state of equilibrium under discretionary policy. σ_r corresponds to the equilibria under commitment, while σ_1 , σ_2 and σ_3 correspond to the 3 possible steady states under discretionary policy

Equilibria will correspond to the identical policy function being used by current and future governments.

The optimization problem for the government may be formalized as follows³. The Euler equation and the static first order condition with respect to σ , give us expressions for the choice variables c, σ in terms of state variables and policy variables, namely $\sigma = \sigma(K, P, \phi_l)$ and $c = c(K, B, P, \phi_l, \phi_k)$. Given these functions, the government's problem may be formulated in two stages. In the second stage, given tax rates and investment in the legal infrastructure, the government will choose the level of public debt for the subsequent period. This will give public debt as a function of current bond holdings, the aggregate stock of physical capital and legal infrastructure, along with tax rates. In the first step of the government's problem, tax rates and legal infrastructure are chosen subject to the capital

³A slight change in notation is employed in this section. B, P, K all will refer to aggregate values of variables b, p, k , while k refers to the choice of physical capital accumulation by the household

accumulation equation, the government's budget constraint, with public debt and household choice variables being included as functions of state variables, as described above.

The value function for the government then becomes:

$$W(K, B, P) = \max_{\phi_l, \phi_k, P'} \{U(C(K, B, P, \phi_l, \phi_k)) + \beta W(K', B'(K, B, P, \phi_l, \phi_k), P'(K, B, P))\} \quad (\text{II.95})$$

subject to

$$(1+n)K' = \sigma(K, P, \phi_l) f\left(\frac{K}{\sigma(K, P, \phi_l)}\right) + (1-\delta_k)K + (1-\delta_p)P - (1+n)P' - C(K, B, P, \phi_l, \phi_k) \quad (\text{II.96})$$

and

$$(1+n)P' = (1-\delta_p)P + \phi_l h(P, \sigma(K, P, \phi_l)) * \left(\frac{K}{\sigma(K, P, \phi_l)}\right) + \phi_k f'\left(\frac{K}{\sigma(K, P, \phi_l)}\right) + (1+n)B'(K, B, P, \phi_l, \phi_k) - R_b B \quad (\text{II.97})$$

The next step is to obtain what I shall refer to as generalized Euler equations, by combining the first order conditions for the government's maximization problem with appropriate envelope conditions.⁴ The first order condition with respect to legal infrastructure implies that $W'_K = W'_{P'}$, while that with respect to bonds implies $W'_K P'_{B'} = W'_{B'}$. The first order condition with respect

⁴Subscripts indicate a derivative with respect to the variable in the subscript. Primes in the subscript indicate that the derivative is being taken with respect to the value in the next period, while primes in a variable indicate next period values for that variable.

to tax rates implies that:

$$U_c c_{\phi_l} + \beta W'_{K'} \left(\sigma_{\phi_l} \left(f \left(\frac{K}{\sigma} \right) - \frac{K}{\sigma} f' \left(\frac{K}{\sigma} \right) \right) - (1+n)P'_{\phi_l} - (1+n)P'_{B'} B'_{\phi_l} \right) \frac{1}{1+n} - \beta W'_{K'} \left(\frac{c_{\phi_l}}{1+n} \right) + \beta W'_{B'} B'_{\phi_l} + \beta W'_{P'} P'_{\phi_l} = 0 \quad (\text{II.98})$$

A similar first order condition is found for capital taxes. Combining the above first conditions I obtain the following condition, with a similar one to be satisfied once again for capital taxes:

$$(1+n)u_c c_{\phi_l} + \beta W'_{K'} \left(\sigma_{\phi_l} \left(f \left(\frac{K}{\sigma} \right) - \frac{K}{\sigma} f' \left(\frac{K}{\sigma} \right) \right) - c_{\phi_l} \right) = 0 \quad (\text{II.99})$$

In the previous section, for the case of commitment, I found that in a steady state that capital taxes would be equal to zero, and that the value of bonds in the steady state would be equal to their value prior to entering the steady state, so that legal infrastructure would be primarily funded by labor taxes. Ortigueira and Pereira (2007) find that when Markov Perfect Equilibria are applied to case of unbalanced budgets, that one of the multiple steady states existing under discretionary policy will correspond to that obtained as a solution to the Ramsey problem. However, in the case of the model examined in this chapter, this result does not hold.

Proposition II.3. *There does not exist a steady state Markov Perfect equilibrium that is identical to the steady state obtained as a solution to the Ramsey problem*

Proof. In a steady state, the envelope theorem implies that the derivative of the

value function with respect to physical capital must satisfy:

$$\begin{aligned} (1+n)W_K - \beta W_K \left(\sigma_K \left(f \left(\frac{K}{\sigma} \right) - \frac{K}{\sigma} f' \left(\frac{K}{\sigma} \right) \right) - (1+n)P'_K - C_K \right) - (1+n)u_c C_K \\ = \beta W_K (1 - \delta_k + f' \left(\frac{K}{\sigma} \right)) \end{aligned}$$

If it were the case that there existed a steady state Markov Perfect equilibrium identical to that obtained from the Ramsey problem, then the above first order condition would reduce to the Euler equation for the household under no capital taxes. This would imply that:

$$\begin{aligned} (1+n)W_K - 1+n)W_K - \beta W_K \left(\sigma_K \left(f \left(\frac{K}{\sigma} \right) - \frac{K}{\sigma} f' \left(\frac{K}{\sigma} \right) \right) - (1+n)P'_K - C_K \right) \\ - (1+n)u_c C_K = (1+n)W_K \end{aligned}$$

This would be satisfied if both $(1+n)U_C = \beta W_K$ and

$$\sigma_K \left(f \left(\frac{K}{\sigma} \right) - \frac{K}{\sigma} f' \left(\frac{K}{\sigma} \right) \right) - (1+n)P'_K = 0 \quad (\text{II.100})$$

were satisfied. However, the first condition will only be consistent with equation (II.99) if $\sigma_{\phi_l}/c_{\phi_l}$, which will in general only be satisfied when $\sigma_{\phi_l} = 0$. This would be inconsistent with the static first order condition of the household which must be satisfied by σ_{ϕ_l} , except in the extreme case where $\tau = 0$. \square

This result would imply a gap between the steady state under the Ramsey problem and that found under discretionary policy, which is consistent with my findings in the previous subsection in the simpler approach to discretionary policy as simply the first period of optimal policy under commitment. A third possible approach to discretionary policy would be to adapt the Lagrangian setup employed

in the section under commitment. That is, one may think of the problem under discretion as simply maximizing the lifetime utility of the household, subject to the implementation constraint and resource constraint, but with an additional sequence constraints which give the household first order conditions for labor and capital in terms of the policy functions of future government. The government's problem under discretion may thus be thought of as simply a more highly constrained version of the problem under commitment. In particular, governments will no longer be able to take advantage of time inconsistencies in optimal policy, which was the source of zero capital taxes in the steady state under commitment. This justifies the gap between the steady states under commitment and under discretion found under both the previous approaches to discretionary policy.

II.8 Discussion

In the literature on the differences between allocations under discretionary policy and policy under commitment, the explanations work via private sector expectations. Three relevant strands of the literature can be identified. First, there is the optimal fiscal policy strand, which argues that for policy under commitment, capital taxes will be set higher in the first period, and lower in subsequent periods. Under discretion, government behavior is identical to that of the government under commitment during the initial period. As a result, capital taxes will typically be higher under discretion than under commitment. Second, Kydland and Prescott argued that commitment by the central bank to a low average rate of inflation may permit lower inflation than would result from discretionary monetary policy, with little loss of output. The third strand consists of a dynamic reformulation of the Kydland Prescott argument. Under traditional optimal control setting of the policy

problem, the policymaker's optimal action will depend only on the economy's state in the current period. Under discretionary policy, its target variables will depend both on its current actions and on private-sector expectations. Commitment to earlier policy promises will imply that the central bank's behavior will depend on both current and past conditions, and thus introduces inertia into optimal policy rules.

All three strands of the literature share the characteristic that if policy is committed to at the initial period, then this will result in private-sector expectations being derived from certain knowledge of future policy actions in each possible state of the world. Policy may thus be chosen to affect both current allocations and private-sector expectations, which allows for a superior outcome to that attainable under discretionary policy.

Support for forward-looking elements in the New Keynesian models employed by Woodford include first the determination of aggregate demand by real rather than nominal interest rates. Second, aggregate demand will depend on expected long-term real interest rates. These results may be derived from micro foundations, in particular a household Euler equation, which jointly with the resource constraint will determine capital accumulation. While the model of economic growth and property rights determination presented here is deterministic, government policy under commitment affects capital accumulation just as it does in the models in the literature. An increase in legal infrastructure for the subsequent period, p_{t+1} , will decrease the time spent in appropriation in that period, and thus increase returns to both capital and labor. The increase in the interest rate will affect the growth rate of consumption, and thus the accumulation of capital. As in prior work on the difference between discretionary policy and policy under commitment, it may be argued that discretionary policy is suboptimal with respect

to policy under commitment.

Consider now the differences in the policymaker's problem under commitment and discretion. Under commitment, the sequence of policies are set so as to satisfy the implementation constraint in the initial period, thereby ensuring that the household's first order condition will be satisfied throughout subsequent periods. Under discretion, since the government cannot commit to a sequence of policies through time, it must assure in each period that policies are set to satisfy the implementation constraint from that period onwards. An increase in consumption will reduce $u_{c,t}$ and thus decrease the value of initial assets. Discretionary policy thus imposes an additional cost to increasing consumption in the current period. Similarly, decreasing time spent in appropriation will increase current interest rates, or increase the value of initial assets. The benefit of increasing σ_t is thus higher. This implies that a lower steady state exists, with a lower level of consumption, and a higher level of σ , induced by higher levels of legal infrastructure and of taxes on wage income. The policy in this lower steady state is effective, in the sense of discouraging appropriation and allowing for a higher steady state level of capital to be maintained. We assumed that $\lim_{\sigma \rightarrow 1} h_p = 0$, so that investments in the legal infrastructure are less effective as σ approaches 1. Thus, setting taxes and investment in legal infrastructure so that $\sigma > \sigma_r$, as occurs in the lower steady state under discretion, is highly costly, and results in lowered consumption.

While the upper steady state level of σ , σ_3 , is close to its steady state level under commitment, σ_r when τ is close to zero, as the return to appropriation increases, σ_r responds more quickly than σ_3 , given the added incentive under discretion to maintain low levels of appropriation. This causes consumption in the upper steady state under discretion to be lower than it is under commitment. In addition, I note that the upper and lower steady states under discretion approach

each other as τ until the return to appropriation approaches a threshold level, above which no steady state under discretion exists.

II.9 Conclusion

In this chapter, I have developed a model in which legal infrastructure, or the level of protection of property rights, affects the probability of being caught if agents engage in appropriation from their employer. This model has the characteristic that under exogenous policy, there is a single steady state. We embed this in a model of optimal fiscal policy, characterize the steady states under commitment and under discretionary policy. Under commitment, I find that there is a unique feasible steady state, and that welfare, as measured by the utility of the representative household, will be decreasing in the level of return to appropriation. An intuitive result is that taxes and legal infrastructure will also be increasing in τ . When policy is discretionary, that is, when the government is unable to commit to a particular sequence of policies, then there will generally exist at least two steady states. The upper steady state will not generally be significantly different from the steady state under commitment. The lower steady state displays in fact a higher level of legal infrastructure, and less time devoted to appropriation, but this is so costly that consumption is lower than in the upper steady state.

This model differs from the existing theoretical literature on the connection between economic growth and institutions in two key respects. First, legal infrastructure or the level of protection of property rights is explicitly modeled as accumulating over time, and then as explicitly entering into the household's optimization problem via a probability function. Secondly, and perhaps most importantly, I look at both discretion and commitment. The result of multiple

steady states does not occur, as it does in other models, because of strategic complementarities in the optimization problems of households and firms, but rather because of the inability of the government to commit to policy.

This model not only argues in favor of growing protection of property rights during the process of economic development, but also demonstrates the need for commitment to policy over the long-run. In the set-up of the model, this would imply that a sequence of policies, specifically investment in legal infrastructure, be committed to in some initial period of the economy. Alternatively, one may use Woodford's "timeless perspective" approach, and argue instead that the government simply needs to set policy according to the pattern of behavior to which it would have wished to commit itself at a date far in the past. This implies a history-dependent approach to the reform and improvement of a country's legal infrastructure.

In its current form, this model serves as a benchmark for the relationship between economic growth and institutions. Future work will include first the characterization of the dynamics of the model, both in the case of commitment and discretionary policy, including transition dynamics between the two steady states found under discretion. As alluded to earlier, our assumption of an exogenous return to appropriation neglects the effect of development on return to appropriation. A possible extension would thus be to make the return to appropriation proportional to the level of per capita output. This would imply that both interest rates and wages would depend on the level of protection of property rights, and legal infrastructure would enter directly into the Euler equation. The level of steady state physical capital would thus depend both on σ and on p , introducing an additional interaction between property rights and time spent in appropriation. This will undoubtedly lead to richer dynamics and possibly more

steady states, but our analysis suggest that the distinction between results under commitment and discretion will be preserved, arguing for the need of long-term perspective on the part of the governments of developing countries alongside any attempt to improve protection of property rights.

CHAPTER III

ESCAPING THE POVERTY TRAP: THE EFFECT OF LEAST SQUARES LEARNING

III.1 Introduction

The transition of Western European economies from stagnation to rapid growth and industrialization has long been a puzzle of economic historians. The factors governing this transition from poverty to rapid and sustained growth of incomes are critical not only to our understanding of industrialization but also to the causes of current divergence in cross-country incomes. A number of authors have tried to understand the latter issue by building models that contain multiple steady states for either the level or the growth rate of per capita output. Poverty traps in these models are thought to characterize existing conditions in both preindustrial and less developed economies. The poverty traps are generated in a number of fashions, through institutions, and threshold effects in both physical and human capital. A seminal paper in this area is Azariadis and Drazen (1990) which considers poverty traps generated both by threshold effects in physical capital, and the role played in their formation by the accumulation of human capital. This provides an explanation for the Great Divergence in incomes between countries, but given the stability properties of the poverty trap, cannot explain the transition from stagnation to modern sustained growth.

There has been a more recent surge of activity in the literature on transitions. One strand of this literature is focused on the process of structural change, or a regime switch from an economy dominated by agricultural production, to one in which modern industrial production allows for sustained economic growth. Frequently, as in Hansen and Prescott (2002), and Galor and Weil (2000), this is accompanied by a change in fertility behavior, and increased benefits from education of children. Goodfriend and McDermott (1995) divide history into three epochs: pre-market, pre-industrial market, and industrial growth. The transition to the pre-industrial market occurs primarily through a switch from primitive to specialized technologies as a result of population growth. The expansion in specialization eventually makes it optimal for households to invest in human capital, and thus in technological innovation. In an alternative approach, in which there is a single sector, but human capital accumulation affects technical progress, Galor and Tsiddon (1997) develop an overlapping generations model, in which individuals differ in the level of human capital that they inherit from their parents. Productivity gains are induced by an increase in the average level of human capital. The model has multiple steady states. Structural changes of the dynamic system as a consequence of increases in aggregate human capital result in the elimination of the poverty trap. It replicates the change in growth observed at the onset of the Industrial Revolution and the coincident accumulation of human capital. However, the deterministic character of the model implies that the Industrial Revolution was inevitable, and that the Great Divergence simply resulted from differences in timing of the transition between countries.

Arifovic et al (1997) find similar results that are qualitatively similar, but generated via a different approach. In particular, they use the Azariadis and Drazen (1990) model, but augment it by an adaptive learning process, whereby agents learn

optimal savings rates and education levels. Thus, rather than generating the transition through structural change along the lines of Hansen and Prescott (2002) or Galor and Tsiddon (1997), they no longer assume perfect foresight, and the transition occurs as a result of experimentation by agents. Economies with low initial levels of human capital will spend long periods of time in the poverty trap. But, regardless of initial endowments, all economies will converge with probability one to the high steady state, in which there is constant growth of human capital.

In this chapter, I apply least squares learning, i.e. where agents forecast variables using simple ordinary least squares regressions, to the Azariadis Drazen model. It replicates the results of Arifovic et al (1997) in that it permits an escape path from the no-training poverty trap. However, a transition path to the high-training steady state only exists under perfect foresight. Instead, adaptive learning generates endogenous oscillations in investment in physical and human capital. This chapter is thus related to a rather rich literature on endogenous cycles in either physical or human capital, beginning with Goodwin (1951), in which capacity limits and the requirement of positive investment generates oscillations in physical capital investment. A number of authors have found the potential for cycles and other nonlinear dynamics in overlapping generations models, including Benhabib and Day (1982) and Reichlin (1986). More recently, Matsuyama (1999), Walde (2002), and Kaas and Zink (2007) have found cycles in OLG models with production, either through regime-switching between innovation and capital accumulation, or because of discontinuities in productivity growth as a function of the number of skilled workers. This chapter has a similar switching mechanism, in that individuals switch between investment in physical and human capital. The difference is that the switching is generated through bounded rationality rather than discontinuities in production functions.

The rest of the chapter is organized as follows. Section III.2 briefly discusses the Azariadis-Drazen model. Section III.3 describes genetic algorithms and compares to least squares learning. Section III.4 presents the model and analytical results for perfect foresight dynamics. Section III.5 analyzes stability under learning. Section III.6 describes the escape path and transition dynamics when the economy starts in the no-training poverty trap. Section III.7 concludes.

III.2 Azariadis and Drazen Model

In the overlapping generations model proposed by Azariadis and Drazen, and then used by Arifovic et al (1997), agents chose c_{1t} , consumption in their first period of life, c_{2t+1} , consumption in their second period of life, and τ_t , the fraction of time in the first period spent in training, to maximize lifetime utility. An individual's human capital in their second period of life is given by

$$x_t^i(t+1) = (1 + \gamma(x_t)\tau_t^i)x_t \quad (\text{III.1})$$

where x_t is the average human capital of all agents at time t , and τ_t^i is the fraction of time in training spent by agent i . Azariadis and Drazen show that there are at least two equilibria. In the first, $\tau_t = 0$ for all $t \geq T$, which implies

$$r_{t+1} \geq \gamma(x_t) \frac{w_{t+1}}{w_t} \quad (\text{III.2})$$

$$2k_{t+1} = x_t w_t - c_{1t} \quad (\text{III.3})$$

where r_t and w_t are respectively the rental rate on physical capital and the wage per effective unit of labor, and k_t is physical capital per effective unit of labor. Each worker's effective units of labor in their first period of life are given by $(1 - \tau_t^i)x_t$,

and equal to their human capital accumulation in the second period. Effective physical capital k_t and aggregate effective labor L_t are then given respectively by

$$k_t = \frac{K_t}{N(2 - \tau_t)x_t}$$

$$L_t = N(2 - \tau_t)x_t$$

as all individuals are solving identical optimization problems. The inequality (III.2) implies that the return to investment in physical capital exceeds that to human capital. The second condition (III.3) can be obtained from the requirement that aggregate physical capital K_t be equal to aggregate savings, and to the product of effective labor and physical capital per effective units of labor. Azariadis and Drazen show that there exists a stable, no-training steady state associated with this corner equilibrium.

In the interior equilibrium, $\tau_t > 0$, and the following conditions characterize the equilibrium:

$$\tau_t = \gamma(x_t) \frac{w_{t+1}}{w_t} \quad (\text{III.4})$$

$$(2 - \tau_{t+1})(1 + \gamma(x_t)\tau_t)x_t k_{t+1} = (1 - \tau_t)x_t w_t - c_{1t} \quad (\text{III.5})$$

For this equilibrium, k_t must be such that return to physical and human capital are equal. The second condition has the same interpretation as for the corner equilibrium. There is a second steady state associated with this equilibrium, where k and τ are constant, and $x_t \rightarrow \infty$.

In this model, the dynamics rely inherently on the assumption of perfect foresight accompanied with optimizing behavior by households. Under this assumption, the no-education equilibrium is a steady state, and we will see no

transition from this to the interior equilibrium. But, either stochastic shocks or alternatively small deviations of the agents from optimal behavior under perfect foresight, may result in the possibility of escape from the poverty trap.

Why introduce adaptive learning, or more generally, bounded rationality in expectations, into the Azariadis Drazen model? First of all, much of the work attempting to model transitions out of poverty traps is based on perfect foresight models, and hence ignores the effect of expectations. Yet, if accumulation of human capital is one of the causes of transitions out of poverty trap, then one should not ignore the role of expected earnings and expected returns to education in determining educational attainment. There are at least two possible ways to introduce bounded rationality into the model. The first is to assume that agents choose savings and training rates by trial and error. This is the approach taken by Arifovic et al, who use genetic algorithms to model this process of experimentation. The second approach is to introduce stochastic shocks, and to assume that agents form forecasts of wages and interest rates, upon which they base their choice of savings and training rates. The latter approach is that taken in this chapter.

III.3 Learning and Genetic Algorithms

The learning process in Arifovic et al (1997) is modeled using a genetic algorithm. Genetic algorithms have been primarily used in computing to find solutions to optimization and search problems. They are loosely based on models of genetic change, in which three stages determine the evolution of the population of chromosomes from one period to the next: fitness evaluation, selection and then mutation or genetic operators. In the use of genetic algorithms for adaptive search processes, the relevant population is a set of binary strings, which can experience

similar types of mutation as chromosomes. There is a large body of both theoretical and empirical evidence showing that even for very large and complex search spaces, genetic algorithms can quickly locate strings with high fitness ratings using a population of 50-100 strings.

Arifovic et al (1997) encode decision rules of agents regarding saving and education as binary strings. The fitness of each string is found by solving for the lifetime utility the individual would have attained if it had been in use in the previous period, taking aggregate variables as given. Reproduction then takes place by choosing two strings at random from the population, and choosing the string that has the higher fitness value. This process is repeated to yield a set of decision rules that are on average more fit than those in use in the previous period. Each chosen string is then subjected to two genetic operators: mutation and crossover. This approach to learning yields a permanent transition from the poverty trap in the Azariadis and Drazen model to the upper steady state, where there is sustained growth in human capital.

Riechmann (1999) studies the dynamic properties of genetic algorithms. He argues that selection, mutation and crossover correspond to three learning mechanisms: imitation, communication, and experimentation. Consider the case where the only possible genetic operator is that of selection/imitation. It may be shown that this algorithm is a Markov process, in which uniform populations are absorbing states. That is, all agents will eventually have the same decision rules. This same result holds when the genetic operators include both selection and crossover. Under mutation, there is a positive probability that a single bit of a binary string will be changed. The stochastic nature of the genetic algorithm implies that the Markov process no longer has an absorbing state. However, it may be shown that the system asymptotically converges to a constant probability

distribution of all states.

Arifovic et al (1997) analyze transitions when the economy starts in a poverty trap. Initially, human capital will be very low, and the binary strings which correspond to positive investment in human capital will have low fitness. Suppose that the population was initially uniform, with decision rules corresponding to the lower steady state values of saving and training. Low returns to human capital means that selection will work against any agents investing in human capital through education. Mutation implies that there will sometimes be one or more agents who spent positive amounts of time in school. This will result in increases in aggregate human capital over time. Once human capital is sufficiently high, the fitness of strings associated with positive investment in human capital will increase, until selection works against those who do not invest in human capital. Since human capital does not depreciate, the economy will remain in the neighborhood of the upper steady state.

While the results of Arifovic et al (1997) are quite appealing, in their explanation of the long-run transition of economies out of no-training poverty traps, and in the key role in this transition of social learning, the paper faces several common criticisms of genetic algorithm approaches to learning. In particular, there are no formal convergence results, and convergence of the algorithm may be quite sensitive to choice of parameters. In addition, a literal interpretation of the learning rules is quite difficult, in considering social learning processes rather than real evolutionary processes. In this chapter, I propose to analyze a version of the Azariadis Drazen model, augmented with productivity shocks, where expectations are formed using least squares learning methods. While, this is a quite different approach, in its focus on individual formation of expectations, rather than some process of social learning, it allows for analytic results with respect to stability

under learning, and also for the determination a set of conditions under which transitions will be possible. In addition, the deviation from the assumption of perfect foresight, or rational expectations in the presence of stochastic shocks, is smaller, in the sense that agents will continue to behave optimally, subject to their expectations of factor prices.

III.4 Model

This model closely follows that developed by Azariadis and Drazen (1990), while allowing for stochastic shocks to aggregate productivity. In this economy, individuals live for two periods, working in both periods. Savings in the first period buys physical capital, combined in the subsequent period with the effective labor of both the old and young, under the following production technology

$$Y_t = K_t^\alpha L_t^{1-\alpha} \epsilon_t^\sigma \quad (\text{III.6})$$

where K_t indicates aggregate physical capital in period t and L_t total effective labor in the same period. Production is perturbed in each period by the i.i.d. shocks ϵ_t .

Each worker is endowed with one unit of time in every period. They may increase their effective units of labor by devoting some fraction τ_t of their time endowment in their youth to training. Each individual born at date t inherits the average level of efficiency units, x_t , determined by the decisions of the previous generation. Since all individuals will face an identical maximization problem, we may drop the i superscript, and find that x_t accumulates according to

$$x_{t+1} = x_t[1 + \gamma(x_t)\tau_t] \quad (\text{III.7})$$

In later simulations, I will assume the specific functional form:

$$\gamma(x_t) = \frac{\lambda}{1 + e^{-x_t}} - \frac{\lambda}{2} \quad (\text{III.8})$$

Output per unit of effective labor is given by

$$y_t = k_t^\alpha \epsilon_t^\sigma \quad (\text{III.9})$$

where k_t is once again the ratio of aggregate physical capital to the measure of effective aggregate labor. The rental rates on physical capital and labor are given by, respectively,

$$r_t = \alpha k_t^{\alpha-1} \epsilon_t^\sigma \quad (\text{III.10})$$

$$w_t = (1 - \alpha) k_t^\alpha \epsilon_t^\sigma \quad (\text{III.11})$$

where wages are given per unit of effective labor. Since a period in the overlapping generations model may be thought of as the length of a generation, or 30-40 years, I assume full depreciation of physical capital, or $\delta_k = 1$. Agents maximize their expected lifetime utility, given by

$$\ln(c_{1t}) + \beta E_t \ln(c_{2t+1}) \quad (\text{III.12})$$

subject to budget constraints given by

$$s_t = (1 - \tau_t)x_t w_t - c_{1t} \quad (\text{III.13})$$

$$c_{2t+1} = s_t r_{t+1} + x_{t+1} w_{t+1} \quad (\text{III.14})$$

In addition, I impose the restrictions that $c_{1t} \leq (1 - \tau_t)x_t\omega_t$, and $\tau_t \in [0, 1]$. The Lagrangian for their maximization problem is then given by

$$\begin{aligned} \mathcal{L} = & \ln(c_{1t}) + \beta E_t \ln(c_{2t+1}) \\ & + \lambda_1 (((1 - \tau_t)x_t\omega_t - c_{1t}) E_t r_{t+1} + (1 + \gamma(x_t)\tau_t)x_t E_t \omega_{t+1} - E_t c_{2t+1}) \\ & + \lambda_2 ((1 - \tau_t)x_t\omega_t - c_{1t}) + \lambda_3 \tau_t + \lambda_4 (1 - \tau_t) \end{aligned} \quad (\text{III.15})$$

where equations (III.13-III.14) have been combined into a single budget constraint.

Equation (III.15) implies the following first order condition with respect to c_{1t}

$$c_{1t}^{-1} - \beta E_t c_{2t+1}^{-1} E_t r_{t+1} - \lambda_2 = 0 \quad (\text{III.16})$$

where I have substituted for $\lambda_1 = \beta E_t c_{2t+1}^{-1}$. In the case where $\lambda_2 = 0$ and the condition of non-negative savings is not binding, this is simply the Euler equation. Similarly, the first order condition with respect to τ_t implies

$$\begin{aligned} & \beta E_t c_{2t+1}^{-1} (-x_t \omega_t E_t r_{t+1} + \gamma(x_t)\tau_t x_t E_t \omega_{t+1}) \\ & - \lambda_2 x_t \omega_t + \lambda_3 - \lambda_4 = 0 \end{aligned} \quad (\text{III.17})$$

Finally, when the requirements that savings be non-negative and that $\tau_t \in [0, 1]$ are non-binding, the latter condition becomes simply an arbitrage condition between returns to physical and human capital.

The optimization problem implies that the following set of conditions must hold in equilibrium

$$c_{1t} \leq (1 - \tau_t)x_t\omega_t \quad (\text{III.18})$$

$$c_{1t}^{-1} \geq \beta E_t c_{2t+1}^{-1} E_t r_{t+1} \quad (\text{III.19})$$

$$\tau_t \beta E_t c_{2t+1}^{-1} (-x_t w_t E_t r_{t+1} + \gamma(x_t) x_t E_t w_{t+1}) = \tau_t x_t w_t (c_{1t}^{-1} - \beta E_t c_{2t+1}^{-1} E_t r_{t+1}) \quad (\text{III.20})$$

$$\beta E_t c_{2t+1}^{-1} (-x_t w_t E_t r_{t+1} + \gamma(x_t) x_t E_t w_{t+1}) \leq x_t w_t (c_{1t}^{-1} - \beta E_t c_{2t+1}^{-1} E_t r_{t+1}) \quad (\text{III.21})$$

In addition, the equilibrium must satisfy the lifetime budget constraint

$$c_{2t} = ((1 - \tau_{t-1})w_{t-1}x_{t-1} - c_{1t-1})r_t + x_t w_t \quad (\text{III.22})$$

Aggregate capital $K_{t+1} = L_{t+1}k_{t+1}$ accumulates through aggregate household savings S_t . The equations governing capital accumulation are:

$$Q_t = (1 - \tau_{t-1})w_{t-1} - \tilde{c}_{1t-1} \quad (\text{III.23})$$

$$(2 - \tau_t)k_t = Q_t \quad (\text{III.24})$$

where \tilde{c}_{1t} is the ratio of 1st period consumption to human capital, and Q_t is the ratio of aggregate physical to human capital. Both Q_t and x_t are pre-determined, while all other variables, including both τ_t and k_t , are non-predetermined or “free” variables.

III.4.1 Steady states

To proceed, I will first characterize the steady states of the equivalent deterministic system. There are two possible steady states, one where $\tau = 0$ and the other with $\tau > 0$ and constant growth in human capital. In order for the steady state to have $\tau = 0$, it must be the case that savings is positive, or that the Euler equation (III.19) holds with equality. This implies that

$$c_{2t} = \beta c_{1t}(1 - \delta_k + r_t) \quad (\text{III.25})$$

Using equation (III.22), c_{1l} may be expressed in terms of wages and interest rates in the steady state

$$c_{1l} = \frac{w_l x_l}{1 + \beta} \left(1 + \frac{1}{r_l} \right) \quad (\text{III.26})$$

This implies that capital per effective worker in the lower steady state is determined by

$$2k_l = \frac{\beta}{1 + \beta} (1 - \alpha) k_l^\alpha - \frac{(1 - \alpha) k_l^\alpha}{(1 + \beta) \alpha k_l^{\alpha-1}} \quad (\text{III.27})$$

In order to satisfy the first order conditions, the steady state must also satisfy

$$-r_l + \gamma(x_l) \leq 0 \quad (\text{III.28})$$

Since for the lower steady state, human capital does not have a particular value, there is actually a continuum of steady states such that $\tau = 0$, indexed by their value of the human capital stock x_l . The inequality (III.28) points to a bifurcation in the dynamic system depending on initial levels of human capital, which I will explore in greater detail in the next section. In steady states with $\tau > 0$, the arbitrage condition, equation (III.21) will bind. In addition, human capital will grow, so that if sufficient time elapses, $\gamma(x_t) \rightarrow \lambda/2$. In this case, the steady state is determined by the following conditions:

$$\alpha k_h^{\alpha-1} = \hat{\gamma} \quad (\text{III.29})$$

$$(2 - \tau_h)(1 + \hat{\gamma}\tau_h)k_h = \frac{\beta}{1 + \beta} (1 - \tau_h)(1 - \alpha)k_h^\alpha - \frac{(1 - \alpha)k_h^\alpha(1 + \hat{\gamma}\tau_h)}{(1 + \beta)\alpha k_h^{\alpha-1}} \quad (\text{III.30})$$

where $\hat{\gamma} = \lambda/2$.

III.4.2 Perfect foresight equilibrium

Consider now the perfect foresight dynamic system, where I assume for simplicity that physical capital fully depreciates each period, or $\delta_k = 1$. Any equilibrium must satisfy the following set of equations:

$$k_{t+1} \leq \frac{\alpha k_t^\alpha}{\gamma(x_t)} \quad (\text{III.31})$$

$$(2 - \tau_t)x_t k_t = \frac{\beta}{1 + \beta}(1 - \tau_{t-1})x_{t-1}(1 - \alpha)k_{t-1}^\alpha - \frac{x_t(1 - \alpha)k_t}{\alpha(1 + \beta)} \quad (\text{III.32})$$

$$\tau_t \in [0, 1] \quad (\text{III.33})$$

since the Euler equation must bind in a perfect foresight equilibrium when capital full depreciates. Equations (III.31) and (III.32) imply

$$k_t \leq k_{t-1}^* \quad (\text{III.34})$$

$$k_t = \frac{1}{2 - \tau_t + \frac{1 - \alpha}{\alpha(1 + \beta)}} \frac{\beta}{1 + \beta} \frac{(1 - \tau_{t-1})(1 - \alpha)k_{t-1}^\alpha}{1 + \gamma(x_{t-1})\tau_{t-1}} \quad (\text{III.35})$$

where $k_{t-1}^* \equiv \alpha k_{t-1}^\alpha / \gamma(x_{t-1})$. The inequality (III.31) is equivalent to the condition that the ratio of future wages to interest rates must equal or exceed the ratio of present wages to the return of investment in human capital $\gamma(x_t)$. Now, define $k_{l,t-1}$ as the value of k_t when $\tau_t = 0$, and $k_{h,t-1}$ as the value of k_t when $\tau_t = 1$. That is, $k_{l,t-1}$ and $k_{h,t-1}$ are lower and upper bounds on k_t such that the capital accumulation equation will still be satisfied. There are then three distinct cases to consider: $k_{l,t-1} < k_{t-1}^* < k_{h,t-1}$, $k_{t-1}^* < k_{l,t-1}$, and $k_{h,t-1} < k_{t-1}^*$. In the first case, there are two possible solutions, with $\tau_t = 0$, where the capital accumulation equation would determine k_t , and $\tau_t > 0$ where τ_t and k_t are jointly determined. In the second case, no solution exists under our assumptions that the Euler equation holds with equality, given that τ_t is an element of the unit interval. The only

solution possible in this case will imply that the positive savings constraint will be binding. When physical capital fully depreciates, and there is zero savings, wages will be zero in the next period. This would not be on an equilibrium path under perfect foresight. Finally, for the last case, the unique solution implies $\tau_t = 0$.

Proposition III.1. *There exists a threshold value of human capital, \bar{x} , such that when the initial value of human capital $x_0 < \bar{x}$, there is a unique perfect foresight equilibrium path, with $\tau_t = 0$ for all periods t . This path converges to the lower steady state.*

Proof. The proposition can only hold if for all periods, $k_{t-1}^* > k_{h,t-1}$ or

$$\frac{1}{1 + \frac{1-\alpha}{\alpha(1+\beta)}} \frac{\beta}{1 + \beta} \frac{(1 - \tau_{t-1})(1 - \alpha)k_{t-1}^\alpha}{1 + \gamma(x_{t-1})\tau_{t-1}} < \frac{\alpha k_{t-1}^\alpha}{\gamma(x_{t-1})}$$

This may be solved for τ_{t-1} . In doing so, I obtain

$$\tau_{t-1} > \frac{(1 - \alpha)\beta\gamma(x_{t-1}) - (1 + \alpha\beta)}{(1 + \beta)\gamma(x_{t-1})}$$

This would always be true if $1 + \alpha\beta > (1 - \alpha)\beta\gamma(x_{t-1})$, or when

$$\gamma(x_{t-1}) < \frac{1 + \alpha\beta}{(1 - \alpha)\beta} = \gamma(\bar{x})$$

If the initial value of human capital x_0 satisfies $x_0 < \bar{x}$, then for $t = 1$, $\tau_t = 0$ in turn implying no growth in human capital. Thus, $\tau_t = 0$ for all future periods. This implies that effective physical capital evolves according to the following equation:

$$k_t = \frac{1}{2 + \frac{1-\alpha}{\alpha(1+\beta)}} \frac{\beta}{1 + \beta} (1 - \alpha)k_{t-1}^{\alpha-1} \quad (\text{III.36})$$

Effective physical capital will be increasing when the derivative of the right-hand

side of the equation exceeds 1. The curve defined by equation (III.36) will cross the 45-degree line when

$$\frac{1}{2 + \frac{1-\alpha}{\alpha(1+\beta)}} \frac{\beta}{1+\beta} (1-\alpha)\alpha k_{t-1}^{\alpha-1} = 1$$

which will be satisfied for $k_{t-1} = k_l$, or at the lower steady state. Effective physical capital will be increasing for $k_t < k_l$ and decreasing for $k_t > k_l$. \square

I now consider the dynamic system in the case where the initial level of human capital exceeds the threshold value \bar{x} . If $k_{h,t-1} > k_{t-1}^* > k_{l,t-1}$, then $k_t = k_{t-1}^*$. In this case, the dynamic system may be written solely in terms of x and τ by eliminating k_t using the latter equality.

$$\tau_t = 2 + \frac{1-\alpha}{\alpha(1+\beta)} - \frac{\beta}{1+\beta} \frac{(1-\tau_{t-1})(1-\alpha)\gamma(x_{t-1})}{\alpha(1+\gamma(x_{t-1})\tau_{t-1})} \quad (\text{III.37})$$

$$x_t = (1 + \gamma(x_{t-1})\tau_{t-1})x_{t-1} \quad (\text{III.38})$$

In order to evaluate local stability, I linearize equations (III.31-III.32) around the upper steady state. Since $\lim_{x_t \rightarrow \infty} \gamma'(x_t) = 0$, then I may simply set $\gamma(x_t) = \hat{\gamma}$ and linearize around k_h, τ_h . I define $\hat{\tau}_t = \log(\tau_t/\tau_h)$, $\hat{k}_t = \log(k_t/k_h)$ and obtain

$$\hat{k}_{t+1} = \alpha \hat{k}_t \quad (\text{III.39})$$

$$\begin{aligned} \hat{\tau}_{t+1} = & \frac{\beta}{1+\beta} \frac{(1-\alpha)k_h^\alpha(1+\hat{\gamma})}{k_h(1+\hat{\gamma}\tau_h)^2} \hat{\tau}_t \\ & + \left(\frac{\alpha}{\tau_h} \left(2 - \tau_h + \frac{1-\alpha}{\alpha(1+\beta)} \right) - \frac{\beta}{1+\beta} \frac{(1-\tau_h)\alpha(1-\alpha)k_h^\alpha}{\tau_h k_h(1+\hat{\gamma}\tau_h)} \right) \hat{k}_t \end{aligned} \quad (\text{III.40})$$

This linear difference equation system will be saddle-point stable, if

$$\begin{aligned}\mathcal{B}_1 &= \left(\frac{\alpha}{\tau_h} \left(2 - \tau_h + \frac{1 - \alpha}{\alpha(1 + \beta)} \right) - \frac{\beta}{1 + \beta} \frac{(1 - \tau_h)\alpha(1 - \alpha)k_h^\alpha}{\tau_h k_h (1 + \hat{\gamma}\tau_h)} \right) \\ \mathcal{B}_2 &= \frac{\beta}{1 + \beta} \frac{(1 - \alpha)k_h^\alpha(1 + \hat{\gamma})}{k_h(1 + \hat{\gamma}\tau_h)^2}\end{aligned}\quad (\text{III.41})$$

Using the steady state conditions, I may show that $\mathcal{B}_1 = 0$. The linear difference equation system will be saddle-point stable provided that $\alpha < 1$ and $\mathcal{B}_2 > 1$. The capital accumulation equation implies that

$$\frac{\beta}{1 + \beta} \frac{(1 - \tau)(1 - \alpha)k^\alpha}{1 + \hat{\gamma}\tau} > (2 - \tau)k$$

which in turn implies that

$$(1 - \tau)\mathcal{B}_2 > \frac{(2 - \tau)(1 + \hat{\gamma})}{1 + \hat{\gamma}\tau} > 2 - \tau > 1$$

The saddle-path stability condition is thus satisfied. Furthermore, one can show that the stable manifold corresponds to \hat{k}_t , while the unstable manifold corresponds to $\hat{\tau}_t$. Therefore, convergence to the upper steady state will only be possible when $\tau_t = \tau_h$.

The value of \mathcal{B}_2 implies that in a neighborhood of the steady state, τ_t will be increasing if $\tau_t > \tau_h$. Provided that the same behavior holds under the non-linear dynamic system, the training ratio τ_t will increase until it exceeds the threshold referred to in the proof above, at which point $k_{t-1}^* > k_{h,t-1}$, and the only possible solution will be $\tau_t = 0$. Transition to the upper steady state will be seen to require both $\gamma(x_t) = \hat{\gamma}$ and $\tau_t = \tau_h$.

The diagram below shows the relationship between τ_t and τ_{t-1} taking x_{t-1} as given. This is obviously not a complete picture of the dynamics, as x_t also depends on τ_{t-1} and will affect the evolution of the training ratio τ in subsequent periods.

However, it allows us to begin to comprehend the rather complex dynamics of this model.

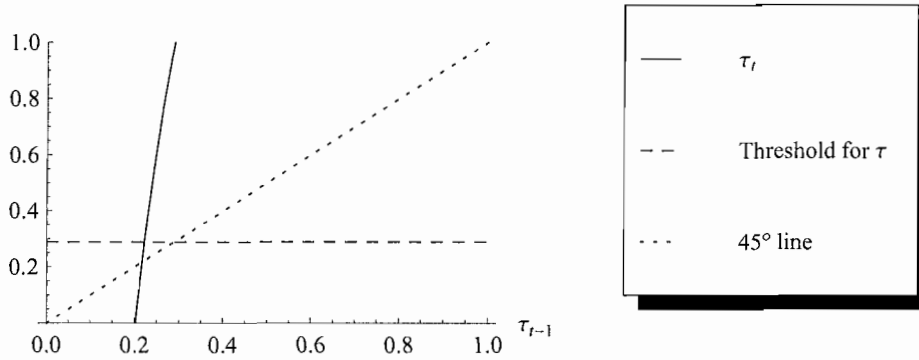


Fig. III.1: Dynamics of human capital investment: plot of τ_t for $x_{t-1} = 100$, $\alpha = 0.36$, $\beta = 0.98$, $\lambda = 50$.

First of all, it may be easily shown that τ_t is increasing in τ_{t-1} and decreasing in the level of human capital stock x_{t-1} . This may be understood when we recall that τ_{t-1} will increase the human capital inherited by the young in period t . This increase in the level of human capital increases incentives to invest in human capital, as returns to this investment are increasing in the inherited level of knowledge. On the other hand, higher levels of human capital in the previous generation increases their wages and thus their physical capital accumulation. These higher levels of physical capital increase wages, reducing incentives for investing in training. In particular, if last period's wages are sufficiently large relative to the return to human capital, then it will not be optimal for households to invest in human capital. This will be the case when $k_{h,t-1} < k_{t-1}^*$. If τ_{t-1} , the proportion of time spent in training in the previous period, increases, then labor income will decrease, reducing the aggregate capital stock. In addition, this will increase human capital x_t , further reducing k_t . If there is a sufficient increase in τ_{t-1} , then this will cause households in period t to not invest in human capital.

Proposition III.2. *For every initial level of human capital with $x_t > \bar{x}$, there exists a unique sequence of values of τ_t such that the economy converges to the upper steady state.*

Proof. First, consider the case where $\gamma(x_t) = \hat{\gamma}$. Then the mapping between τ_t and τ_{t-1} will be independent of any other variables:

$$\tau_t = 2 + \frac{1 - \alpha}{\alpha(1 + \beta)} - \frac{\beta}{1 + \beta} \frac{(1 - \tau_{t-1})(1 - \alpha)\hat{\gamma}}{\alpha(1 + \hat{\gamma}\tau_{t-1})} \quad (\text{III.42})$$

Let τ^* be the fixed point of this mapping. The capital accumulation equation for the upper steady state implies that

$$2 - \tau_h + \frac{1 - \alpha}{\alpha(1 + \beta)} = \frac{\beta}{1 + \beta} \frac{(1 - \tau_h)(1 - \alpha)k_h^{\alpha-1}}{1 + \hat{\gamma}\tau_h} \quad (\text{III.43})$$

This equation, combined with equation (III.29), implies that $\tau_h = \tau^*$. The evolution of effective physical capital will follow $k_{t+1} = \alpha k_t^\alpha / \hat{\gamma}$, where the steady state k_h is globally stable. Therefore, so long as the initial training ratio $\tau_0 = \tau_h$, the economy will converge to the upper steady state.

Now consider the case where $\gamma(x_t) < \hat{\gamma}$. For simplicity, I shall initially consider the case where $\gamma(x_t) = \hat{\gamma}$ for $x_t \geq \tilde{x}$. Convergence to the upper steady state then requires that $\tau_t = \tau_h$ when human capital lies above the threshold \tilde{x} . If human capital reaches this threshold in period T , then training in period $T - 1$ must satisfy:

$$\tau_h = 2 + \frac{1 - \alpha}{\alpha(1 + \beta)} - \frac{\beta}{1 + \beta} \frac{1 - \alpha}{\alpha} \frac{(1 - \tau_{T-1})\gamma(x_{T-1})}{1 + \gamma(x_{T-1})\tau_{T-1}} \quad (\text{III.44})$$

$$x_{T-1}(1 + \gamma(x_{T-1})\tau_{T-1}) \geq \tilde{x} \quad (\text{III.45})$$

By backwards induction, one can construct a sequence of such conditions that must be satisfied by any path that allows for convergence to the upper steady state. In

particular:

$$\tau_{T-j} = \frac{\tau_{T-1} + \frac{\beta}{1+\beta} \frac{1-\alpha}{\alpha} \gamma(x_{T-j}) - 2 - \frac{1-\alpha}{\alpha(1+\beta)}}{\gamma(x_{T-j}) \left(2 + \frac{1-\alpha}{\alpha} - \tau_{T-j+1}\right)}$$

$$x_{T-j}(1 + \gamma(x_{T-j})) \frac{\beta}{1+\beta} \frac{1-\alpha}{\alpha} \frac{1}{2 + \frac{1-\alpha}{\alpha} - \tau_{T-j+1}} \geq x_{T-j+1}$$

This in turn implies that for each initial value of human capital x_0 , there exists an initial value of training τ_0 and a unique sequence $\{\tau_t, x_t\}_{t=0}^T$ such that the economy converges to the upper steady state. The level of effective physical capital is determined by $k_t = k_{t-1}^*$. The initial value of effective physical capital is determined by x_0 , τ_0 , and the initial level of aggregate physical capital. As $\tilde{x} \rightarrow \infty$, the sequence of τ_t and x_t becomes an infinite sequence which converges to the upper steady state asymptotically. \square

Thus, under perfect foresight, we can establish both that no path will exist between the two steady states, and that a path to the upper steady state will exist for every value of $x_t > \bar{x}$. However, this path requires a unique value of τ_0 for a given value of x_0 . This single condition for convergence to the steady state obtains from the fact that in the perfect foresight equilibrium, the evolution of human capital and the training rate τ_t are independent of effective physical capital. But, if we introduce aggregate productivity shocks, then training will depend on not only past training and human capital, but also on past and expected effective physical capital.

III.5 Adaptive Learning

In this section, I consider a slight modification to the model. In particular, I assume that agents form their expectations of wages and interest rates by least squares learning, and that there is a lower limit on both savings rates and training

rates. That is, every agent will make some minimum level of investment in both human and physical capital in the first period of life, and therefore, $c_{1t} \leq (1 - \bar{\phi})(1 - \tau_t)x_t w_t$ and $\tau_t \geq \bar{\tau}$. This modification is made both for tractability in the model, but also to allow for the possibility that agents use a combination of optimization and rule of thumb in choosing their investments in physical and human capital. Furthermore, in order to assure that the lower steady state still exists and to allow for potential depreciation of human capital, I assume that $x_t = x_{t-1}(1 + \gamma(x_{t-1})(\tau_{t-1} - \bar{\tau}_d))$. For simplicity in what follows, I assume that $\bar{\tau}_d = \bar{\tau}$. These assumptions imply

$$c_{1t}^{-1} \geq \beta E_t^* c_{2t+1}^{-1} E_t^* r_{t+1} \quad (\text{III.46a})$$

$$\begin{aligned} & \beta E_t^* c_{2t+1}^{-1} (-w_t E_t^* r_{t+1} + \gamma(x_t) E_t^* w_{t+1}) \\ & \leq (1 - \bar{\phi}) w_t (c_{1t}^{-1} - \beta E_t^* c_{2t+1}^{-1} E_t^* r_{t+1}) \end{aligned} \quad (\text{III.46b})$$

In order to simplify the learning algorithms, I assume that $E_t^* c_{2t+1} = (E^* c_{2t+1})^{-1}$ and that $E_t^* w_{t+1} / E_t^* r_{t+1} = (1 - \alpha) E_t^* k_{t+1} / \alpha$.¹ In addition, I define $\tilde{c}_{1t} = c_{1t} / x_t$. Under the assumption that the Euler equation binds, the first order conditions and the capital accumulation equation imply that

$$k_t \geq \left(\frac{\gamma(x_t)}{\alpha \epsilon_t^\sigma} k_{t+1}^e \right)^{1/\alpha} \quad (\text{III.47a})$$

$$k_t = \frac{1}{2 - \tau_t} \left(\frac{(1 - \tau_{t-1})(1 - \alpha) k_{t-1}^\alpha \epsilon_{t-1}^\sigma - \tilde{c}_{1t-1}}{1 + \gamma(x_{t-1})(\tau_{t-1} - \bar{\tau})} \right) \quad (\text{III.47b})$$

In order to simplify the notation, I define

$$k_t^* = \left(\frac{\gamma(x_t)}{\alpha \epsilon_t^\sigma} k_{t+1}^e \right)^{1/\alpha} \quad (\text{III.48})$$

¹ E_{t+1}^* is an operator that can denote either rational or nonrational expectations

The same analysis as under perfect foresight applies, where the interior solution will only exist if $k_{l,t-1} \leq k_t^* \leq k_{h,t-1}$, where only k_t^* is defined differently from the previous section. The ratio of aggregate physical to aggregate human capital, which I denote by Q_t , is equal to $k_{h,t-1}$. The requirement for the existence of the interior solution then implies an upper and lower bound on the value of expected physical capital k_{t+1}^e , where these bounds are functions of Q_t , human capital x_t , and the productivity shock ϵ_t . Intuitively, a high value of expected physical capital leads to an increase in training, due to high expected wages. But when expected physical capital increases enough, the optimal level of τ_t will no longer be feasible, due to a binding savings constraint. Similarly, for low values of k_{t+1}^e , training will decline, until the solution of $\tau_t = 0$ is optimal. I note that while under perfect foresight, the existence of the interior solution also implied existence of a corner solution with $\tau_t = 0$, that is not the case under either rational expectations or adaptive learning.

In order to ensure that the individual's savings constraint is satisfied as well as the Euler equation, the following inequality must also hold

$$\begin{aligned} \frac{1}{1+\beta} \left((1-\tau_t)(1-\alpha)k_t^\alpha \epsilon_t^\sigma + (1+\gamma(x_t)\tau_t) \frac{1-\alpha}{\alpha} k_{t+1}^e \right) \\ \leq (1-\bar{\phi})(1-\tau_t)(1-\alpha)k_t^\alpha \epsilon_t^\sigma \end{aligned}$$

The above inequality may be solved for k_t :

$$k_t \geq \left(\frac{1}{1+\beta} \frac{1+\gamma(x_t)\tau_t}{\left(1-\bar{\phi}-\frac{1}{1+\beta}\right)(1-\tau_t)\epsilon_t^\sigma} \frac{k_{t+1}^e}{\alpha} \right)^{1/\alpha} \quad (\text{III.49})$$

This inequality may be interpreted as the minimum level of physical capital such that for the given training ratio, the individual's savings constraint is satisfied.

When the savings constraint is in fact violated, then consumption will be determined by the binding savings constraint. This savings constraint, combined with equation (III.46b) and the capital accumulation equation, will determine values of τ_t and k_t . In particular, by combining the savings constraint with (III.46b) I obtain:

$$\begin{aligned} \beta E_t^* c_{2t+1} (-w_t x_t E_t^* r_{t+1} + x_t \gamma(x_t) E_t^* w_{t+1}) \\ \leq (1 - \tau_t)^{-1} - \beta E_t^* c_{2t+1}^{-1} E_t^* r_{t+1} (1 - \bar{\phi}) x_t w_t \end{aligned}$$

This may be simplified to

$$\begin{aligned} \beta(1 - \tau_t) \left(\gamma(x_t) \frac{1 - \alpha}{\alpha} k_{t+1}^e - \bar{\phi}(1 - \alpha) k_t^\alpha \epsilon_t^\sigma \right) \\ \leq \bar{\phi}(1 - \tau_t)(1 - \alpha) k_t^\alpha \epsilon_t^\sigma + (1 + \gamma(x_t)\tau_t) \frac{1 - \alpha}{\alpha} k_{t+1}^e \end{aligned}$$

The above expression may be solved for k_t :

$$k_t \geq \left(\frac{(\beta\gamma(x_t) - 1 - \tau_t(1 + \beta)\gamma(x_t)) \frac{1 - \alpha}{\alpha} k_{t+1}^e}{\bar{\phi}(1 + \beta)(1 - \tau_t)(1 - \alpha)\epsilon_t^\sigma} \right)^{1/\alpha} \quad (\text{III.50})$$

Due to the highly nonlinear characteristics of this model, I may not easily solve for the rational expectations equilibria, and then use those to determine appropriate learning rules for the agents in their formation of expectations for effective physical capital. However, for large enough values of human capital, $\gamma(x_t) \approx \hat{\gamma}$. In the interior equilibrium, where the Euler equation is binding and $\tau_t > 0$, the evolution of effective physical capital in the REE will be closely approximated by:

$$k_t = \left(\frac{\hat{\gamma}}{\alpha \epsilon_t^\sigma} E_t k_{t+1} \right)^{1/\alpha} \quad (\text{III.51})$$

Using the method of undetermined coefficients, I posit a solution of the form:

$$k_t = A_0 k_{t-1}^{a_1} \epsilon_{t-1}^{a_2} \epsilon_t^{a_3} \quad (\text{III.52})$$

With no serial correlation for ϵ_t , $E_t k_{t+1} = A_0 k_t^{a_1} \epsilon_t^{a_2}$, or

$$E_t^* k_{t+1} = A_0^{1+a_1} k_{t-1}^{a_1^2} \epsilon_{t-1}^{a_1 a_2} \epsilon_t^{a_1 a_3 + a_2}$$

if agents' perceived law of motion is given by equation (III.52). Substituting into equation (III.51) gives the corresponding actual law of motion

$$k_t = \left(\frac{\hat{\gamma}}{\alpha \epsilon_t^\sigma} A_0^{1+a_1} k_{t-1}^{a_1^2} \epsilon_{t-1}^{a_1 a_2} \epsilon_t^{a_1 a_3 + a_2} \right)^{1/\alpha}$$

This implicitly defines the mapping from the perceived law of motion (PLM) to the actual law of motion (ALM)

$$T \begin{pmatrix} A_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \left(\frac{\hat{\gamma}}{\alpha} A_0^{1+a_1} \right)^{1/\alpha} \\ a_1^2 / \alpha \\ a_1 a_2 / \alpha \\ (a_1 a_3 + a_2) / \alpha \end{pmatrix} \quad (\text{III.53})$$

Any rational expectations equilibrium must be a fixed point of the latter mapping.

The fixed points are

$$A_0 = \left(\frac{\alpha}{\hat{\gamma}} \right)^{1/(1-\alpha)}, a_1 = a_2 = 0, a_3 = -\frac{\sigma}{\alpha} \quad (\text{III.54})$$

$$A_0 = \frac{\alpha}{\hat{\gamma}}, a_1 = \alpha, a_2 = \sigma, a_3 \text{ is arbitrary} \quad (\text{III.55})$$

In the low-training equilibrium, where $\tau_t = \bar{\tau}$ in all periods, the evolution of effective physical capital is determined solely by the capital accumulation and Euler equations:

$$k_t = \frac{1}{2 - \bar{\tau}} \left(\frac{\beta}{1 + \beta} (1 - \bar{\tau})(1 - \alpha) k_{t-1}^\alpha \epsilon_{t-1}^\sigma - \frac{1}{1 + \beta} \frac{1 - \alpha}{\alpha} k_t^e \right) \quad (\text{III.56})$$

so that effective physical capital is now predetermined. There is a unique rational expectations equilibrium for the above dynamic system, which takes the form

$$k_t = A_0 k_{t-1}^\alpha \epsilon_{t-1}^\sigma$$

where

$$A_0 = \frac{\alpha\beta(1 - \bar{\tau})(1 - \alpha)}{1 + \alpha + (2 - \bar{\tau})\alpha\beta}$$

This equilibrium will only exist when human capital lies below a threshold given by

$$\gamma(x_t) \leq \frac{1 + \alpha(1 - \bar{\tau}) + \alpha\beta(2 - \bar{\tau})}{\beta(1 - \alpha)}$$

Having established the rational expectations equilibria in a neighborhood of each steady state, the next step is to establish whether these REE are expectationally stable or E-stable. That is, I shall consider the stability of the REE under a learning rule corresponding to the relevant perceived law of motion (PLM) in which the parameters are adjusted slowly in the direction of the implied ALM parameters.

Proposition III.3. *In the high-training equilibrium, where $\gamma(x_t) \rightarrow \hat{\gamma}$, both REE are E-unstable. In the low-training equilibrium, the REE is E-stable.*

Proof. First, consider the REE in the high-training equilibrium. E-stability will be

satisfied when the parameters of the REE correspond to a stable steady state of the differential equation

$$\frac{d}{d\mathcal{T}} \begin{pmatrix} A_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = T \begin{pmatrix} A_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} - \begin{pmatrix} A_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

In the case where $a_1 = a_2 = 0$ under the REE, it will be E-stable when $\alpha^{-1} < 1$, and E-unstable when $\alpha^{-1} > 1$. The latter case will always hold when $0 < \alpha < 1$. In the case where $a_1 = \alpha$ and $a_2 = \sigma$, the equilibrium will be E-unstable if $(1 + \alpha)/\alpha > 1$ which once again will be satisfied for $\alpha < 1$.

Now, consider the REE in the low-training equilibrium. The T-mapping between the PLM and the ALM is given by

$$T(A_0) = \frac{1 - \alpha}{(2 - \bar{\tau})(1 + \beta)} \left(\beta(1 - \bar{\tau}) - \frac{A_0}{\alpha} \right)$$

E-stability will be satisfied if $T'(A_0) < 1$, satisfied for the same condition on α as before. □

In a sequential approach to fully characterizing the dynamics of the full non-linear model under learning, I next consider linearizations around both the upper and the lower steady states. These linearizations, unlike the dynamic systems analyzed in this section, allow for small deviations from $\tau_t = \bar{\tau}$ in the no-training equilibrium and $\gamma(x_t) = \hat{\gamma}$ in the interior equilibrium.

III.5.1 E-stability in the interior equilibrium

In the interior equilibrium, the first order conditions imply that $w_t = \tilde{w}(k_{t+1}^e)$, or

$$\tilde{w}(k_{t+1}^e) = (1 - \alpha) (k_t)^\alpha \epsilon_t^\sigma \quad (\text{III.57})$$

If I define $z_t = x_t^{-1}$, redefining the function $\gamma(\cdot)$ so that it is in terms of z_t , then equation (III.57) implies

$$\tau_t = 2 - Q_t \left(\frac{\alpha \epsilon_t^\sigma}{\gamma(z_t) k_{t+1}^e} \right)^{1/\alpha} = g(Q_t, \epsilon_t, z_t, k_{t+1}^e) \quad (\text{III.58})$$

The complete dynamic system will be described by the capital accumulation equation

$$Q_{t+1} = \frac{\beta}{1 + \beta} \frac{(1 - \tau_t)(1 - \alpha) \left(\frac{Q_t}{2 - \tau_t} \right)^\alpha \epsilon_t^\sigma}{1 + \gamma(z_t) \tau_t} - \frac{1}{1 + \beta} \frac{1 - \alpha}{\alpha} k_{t+1}^e = f(Q_t, \tau_t, z_t, \epsilon_t, k_{t+1}^e)$$

the accumulation equation for human capital (III.7), and the relationship between Q_t , effective physical capital k_t , and the proportion of time spent in training τ_t , or

$$k_{t+1} = Q_{t+1} / (2 - \tau_{t+1}) \quad (\text{III.59})$$

The conditions for the steady state of this system, under which $\epsilon_t = 1$, are identical to conditions for the upper steady state of the perfect foresight system, with $Q_h = (2 - \tau_h) k_h$. In order solve for the rational expectations equilibrium, I log-linearize the system around the steady state. For the purposes of this linearization, I define $\hat{k}_{h,t} = \log(k_t/k_h)$, $\hat{Q}_{h,t} = \log(Q_t/Q_h)$, $\hat{\tau}_{h,t} = \log(\tau_t/\tau_h)$,

$\hat{\epsilon}_t = \log(\epsilon_t)$, and obtain:

$$\hat{k}_{h,t+1} = \hat{Q}_{h,t+1} + \frac{\tau_h}{2 - \tau_h} \hat{\tau}_{h,t+1} \quad (\text{III.60})$$

$$Q_h \hat{Q}_{h,t+1} = Q_h f_Q \hat{Q}_{h,t} + \tau_h f_\tau \hat{\tau}_{h,t} + f_\epsilon \hat{\epsilon}_t + f_z z_t + k_h f_k E_t \hat{k}_{h,t+1} \quad (\text{III.61})$$

$$\tau_h \hat{\tau}_{h,t} = Q_h g_Q \hat{Q}_{h,t} + g_\epsilon \hat{\epsilon}_t + g_z z_t + k_h g_k E_t \hat{k}_{h,t+1} \quad (\text{III.62})$$

$$z_t = \frac{z_{t-1}}{1 + \bar{\gamma} \tau_h} \quad (\text{III.63})$$

where

$$f_{var} = \frac{\partial f(Q_h, \tau_h, 0, 1, k_h)}{\partial var}$$

$$g_{var} = \frac{\partial g(Q_h, 1, 0, k_h)}{\partial var}$$

for each variable var . By using equation (III.62) to substitute for $\tau_{h,t}$, the linear system becomes

$$Q_h \hat{Q}_{h,t+1} = (Q_h f_Q + f_\tau Q_h g_Q) \hat{Q}_{h,t} + (f_\epsilon + f_\tau g_\epsilon) \hat{\epsilon}_t + (f_z + f_\tau g_z) z_t$$

$$+ (f_\tau k_h g_k + k_h f_k) E_t \hat{k}_{h,t+1} \quad (\text{III.64})$$

$$\hat{k}_{h,t} = (1 + k_h g_Q) \hat{Q}_{h,t} + \frac{g_\epsilon}{2 - \tau_h} \hat{\epsilon}_t + \frac{g_z}{2 - \tau_h} z_t + \frac{k_h g_k}{2 - \tau_h} E_t \hat{k}_{h,t+1} \quad (\text{III.65})$$

$$z_t = \frac{z_{t-1}}{1 + \bar{\gamma} \tau_h} \quad (\text{III.66})$$

This may be written in matrix form as

$$\begin{pmatrix} a_{11} & a_{12} & 0 & 0 \\ 0 & a_{22} & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{Q}_{h,t+1} \\ \hat{\epsilon}_t \\ z_{t+1} \\ \hat{k}_{h,t} \end{pmatrix} = \begin{pmatrix} d_{11} & 0 & d_{13} & 0 \\ d_{21} & 0 & d_{23} & 0 \\ 0 & 0 & d_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{Q}_{h,t} \\ \hat{\epsilon}_{t-1} \\ z_t \\ \hat{k}_{h,t-1} \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{pmatrix} E_t \hat{k}_{h,t+1} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \hat{v}_t \quad (\text{III.67})$$

By premultiplying by the matrix of $\{a_{ij}\}$, I obtain

$$\begin{pmatrix} \hat{Q}_{h,t+1} \\ \hat{\epsilon}_t \\ z_{t+1} \\ \hat{k}_{h,t} \end{pmatrix} = \begin{pmatrix} \delta_{11} & 0 & \delta_{13} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_{33} & 0 \\ \delta_{41} & 0 & \delta_{43} & 0 \end{pmatrix} \begin{pmatrix} \hat{Q}_{h,t} \\ \hat{\epsilon}_{t-1} \\ z_t \\ \hat{k}_{h,t-1} \end{pmatrix} + \begin{pmatrix} \beta_1 \\ 0 \\ 0 \\ \beta_4 \end{pmatrix} E_t \hat{k}_{h,t+1} + \begin{pmatrix} \zeta_1 \\ 1 \\ 0 \\ \zeta_4 \end{pmatrix} \hat{v}_t \quad (\text{III.68})$$

The rational expectations equilibrium for the linear system will then be in the form:

$$\hat{Q}_{h,t+1} = a_1 \hat{Q}_{h,t} + a_2 z_t + a_3 \hat{v}_t$$

$$\hat{k}_{h,t} = b_1 \hat{Q}_{h,t} + b_2 z_t + b_3 \hat{v}_t$$

$$z_t = \delta_{33} z_{t-1}$$

$$\hat{\epsilon}_t = \hat{v}_t$$

If the PLM for $\hat{k}_{h,t}$ takes the same form as under the REE, then the T-mapping between the PLM and the ALM is given by

$$\begin{aligned} T_{b_1} &= \delta_{41} + \beta_4 b_1 \\ T_{b_2} &= \delta_{43} + \beta_4 b_2 \delta_{33} \\ T_{b_3} &= \zeta_4 \end{aligned}$$

In the REE, the values of b_1, b_2, b_3 are given by the fixed point of the T-map, with $a_1 = \delta_{11} + \beta_1 b_1$, $a_2 = \delta_{13} + \beta_1 b_2 \delta_{33}$, $a_4 = \zeta_1$. This REE will be E-stable if the Jacobian of the T-map evaluated at the REE has eigenvalues that are strictly less than unity. But, I find that the eigenvalues are β_4, δ_{33} and ζ_4 . Evaluating $\beta_4 = (k_h g_k) / (2 - \tau_h) = \alpha^{-1}$, which exceeds unity provided that $\alpha \in [0, 1]$. Therefore, the conditions for E-stability are not satisfied. Furthermore, real time learning shows that the upper steady state will not be stable under learning, and that the coefficients b_1, b_2, b_3 will not converge to their REE values.

III.5.2 E-Stability in the no-training equilibrium

In a neighborhood of the lower steady state, the condition that $w_t > \tilde{w}(k_{t+1}^e)$ will hold, so that $\tau_t = \tau_{t-1} = 0$. Defining $\hat{k}_t = \log(k_t/k_l)$, log-linearizing around the lower steady state gives

$$\hat{k}_{t+1} = \frac{1}{(2 - \bar{\tau})(1 + \beta)} \left(\beta(1 - \bar{\tau})(1 - \alpha)k_l^{\alpha-1}(\alpha\hat{k}_t + \sigma\hat{\epsilon}_t) - \frac{1 - \alpha}{\alpha}\hat{k}_{t+1}^e \right)$$

This linear equation has the associated REE

$$\hat{k}_{t+1} = \frac{\frac{1}{(2 - \bar{\tau})(1 + \beta)}\beta(1 - \bar{\tau})(1 - \alpha)k_l^{\alpha-1}}{1 + \frac{1}{(2 - \bar{\tau})(1 + \beta)}\frac{1 - \alpha}{\alpha}}(\alpha\hat{k}_t + \sigma\hat{\epsilon}_t)$$

If agents have the PLM that $\hat{k}_{t+1} = a_0 + a_1\hat{k}_t + a_2\hat{\epsilon}_t$, then the fixed point of the T-map

$$\begin{aligned} T_{a_0} &= -\frac{1}{(2-\bar{\tau})(1+\beta)} \frac{1-\alpha}{\alpha} a_0 \\ T_{a_1} &= \frac{1}{(2-\bar{\tau})(1+\beta)} \left(\beta(1-\bar{\tau})(1-\alpha)\alpha k_l^{\alpha-1} - \frac{1-\alpha}{\alpha} a_1 \right) \\ T_{a_2} &= \frac{1}{(2-\bar{\tau})(1+\beta)} \left(\beta(1-\bar{\tau})(1-\alpha)\alpha k_l^{\alpha-1}\sigma - \frac{1-\alpha}{\alpha} a_2 \right) \end{aligned} \quad (\text{III.69})$$

corresponds to the REE. This fixed point will be E-stable if the eigenvalues of $DT(\bar{a}_0, \bar{a}_1)$ have real parts less than 1, or

$$-\frac{1}{(2-\bar{\tau})(1+\beta)} \frac{1-\alpha}{\alpha} < 1$$

which will be satisfied for $\beta > 0$ and $0 < \alpha < 1$. Finally, the equilibrium is also be stable in simulations of real-time learning.

III.5.3 Stability under learning of non-linear model

Let us consider stability under learning for the full nonlinear model of the interior equilibrium, in the case where initial human capital is such that $\gamma(x_t) \approx \hat{\gamma}$. The learning rule of agents is assumed to take the form described in the previous section for this equilibrium. In this section, I shall describe the results of a series of simulations of the non-linear model under these assumptions. Figure III.2 shows the results for the first of these simulations, where there are no productivity shocks, but where initial effective physical capital is slightly above its upper steady state value. Learning has a constant gain of $\psi_t = 0.1$, and the lower bounds on training and savings are set to $\bar{\tau} = 0.05$, $\bar{\phi} = 0.05$. One can observe that effective physical capital very quickly leaves a neighborhood of the upper steady state, and then oscillates

around the steady state value through the duration of the simulations. This is due to switching in economy between a binding savings constraint and periods in which $\tau_t = \bar{\tau}$. The initial condition causes expected effective physical capital to exceed the steady state value, in turn leading to increases in time invested in training, until the savings constraint becomes binding. Then, when minimum savings leads to a decline in aggregate physical capital, and expected physical capital, this eventually leads to a return to the interior solution for τ_t and then to $\tau_t = \bar{\tau}$. Finally, the latter result causes sharp increases in Q_t until the economy returns to a binding constraint, and the cycle repeats.

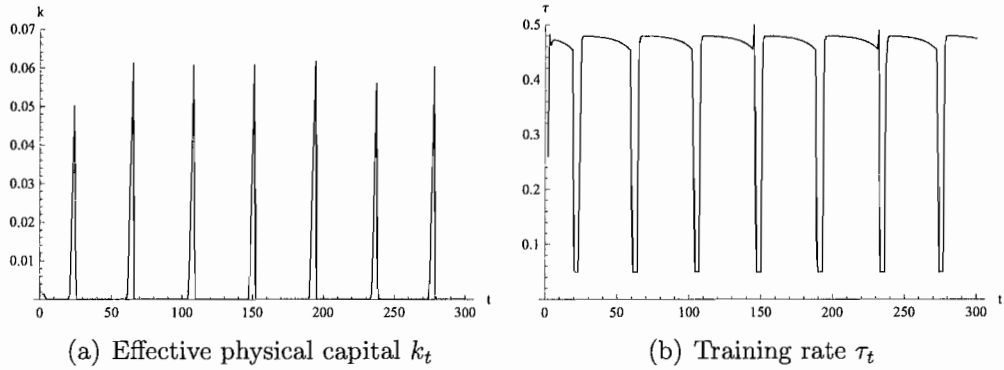


Fig. III.2: Time paths of effective physical capital and training, where $\epsilon_t = 1$, $\tau_0 = \tau_h$, $k_0 > k_h$, $\tau_h = 0.26$, $k_h = 0.0013$

I then introduce an exogenous shock ϵ_t where

$$\ln(\epsilon_t) \sim N(0, \sigma_\epsilon)$$

The effect of these productivity shocks is similar to that of small deviations from the steady state value of k , in that it leads to oscillations between a binding savings constraint and $\tau_t = \bar{\tau}$. Increasing σ_ϵ will increase both the mean and the standard deviation of effective physical capital k_t , due largely to the potential for greater increases in k_t during periods when $\tau_t = \bar{\tau}$. This behavior is unaffected by varying

the value of the minimum training and savings rates – this simply changes the amplitude of the said oscillations.

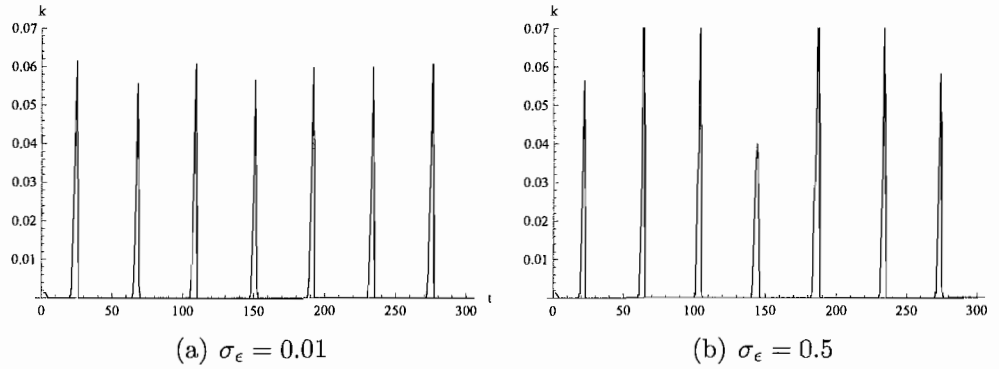


Fig. III.3: Oscillations of effective physical capital

III.6 Transitions from Lower Steady State

In a first effort to understand the dynamics of escape paths from the lower steady state, suppose that agents have the perceived law of motion:

$$k_{t+1} = A_0 k_t^\alpha \epsilon_t^\sigma \quad (\text{III.70})$$

where A_0 has the same value as in the REE of the low-training equilibrium. Since in the interior equilibrium, with $\tau_t > \bar{\tau}$, effective physical capital is determined jointly with the training rate and is not predetermined. Therefore $k_{t+1}^e = A_0 (k_t^e)^\alpha \epsilon_t^\sigma$:

$$k_{t+1}^e = A_0 (A_0 k_{t-1}^\alpha \epsilon_{t-1}^\sigma)^\alpha \epsilon_t^\sigma$$

If agents know the true value of all parameters of the model, then simulations show $\tau_t = \bar{\tau}$ for all periods, regardless of the value of σ_ϵ . This result holds provided that $\gamma(x_t)$ is sufficiently small.

Therefore, I propose introducing an additional stochastic shock, to households' expectations of wages, so that $E_t w_{t+1} = (1 + \xi_t)(1 - \alpha)(k_{t+1}^e)^\alpha$, where ξ_t is assumed to be i.i.d. If x_t is close to the threshold referred to above, then a small positive shock x_{it} will have the same effect. The postulated perceived law of motion implies that the evolution of τ_t will be independent of current productivity shocks. As a result, training rates increase until the lower bound on savings becomes binding. The only shock which impacts τ_t is the expectational shock. Provided the expectational shock has a small standard deviation, then the economy will remain at high levels of training.

Now, suppose that agents do not know the exact values of parameters, but that they assume the law of motion for effective physical capital takes the form:

$$k_t = e^{a_0} k_{t-1}^{a_1} \epsilon_{t-1}^{a_2} \epsilon_t^{a_3} \quad (\text{III.71})$$

where the values of the parameters are estimated by regressing $\ln(k_t)$ on $(\ln(k_{t-1}), \ln(\epsilon_{t-1}), \ln(\epsilon_t))'$. The initial deviations of the estimated parameters from their REE values have an identical effect initially to the expectational shocks ξ_t , in that they allow the economy to escape from the minimum training equilibrium. Furthermore, least squares learning of the parameters also ensures that τ_t will be affected by levels of effective physical capital and by the aggregate productivity shock. As a result, in the simulations, we observe oscillations, similar to those observed when the economy started at the high steady state.

Figure (III.5) shows the evolution over time of the estimated coefficients for the case of constant gain. The oscillations of τ_t and k_t inevitably perturb the values of these coefficients, so that they do not, even for the case of decreasing gain, converge to their values in either of the rational expectations equilibria identified for

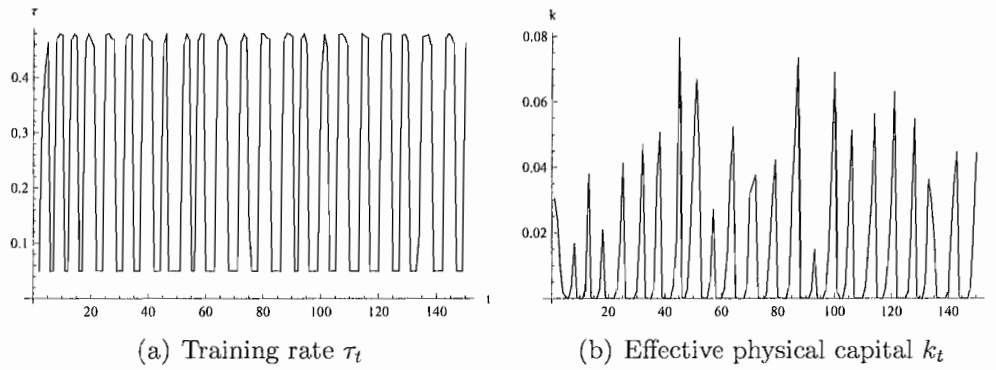


Fig. III.4: Oscillations of capital, training for $\sigma_\epsilon = 0.5$ and $k_0 = k_t$

this model. This behavior is particularly evident for a_1 and a_3 , the coefficients on effective physical capital and white noise terms respectively. If we use $\psi_t = t^{-1}$, then the values of a_0 and a_2 return to their values under the REE for minimum training, while $a_1 > \alpha$ in the limit, and $a_3 < 0$, so that neither return to their original values.

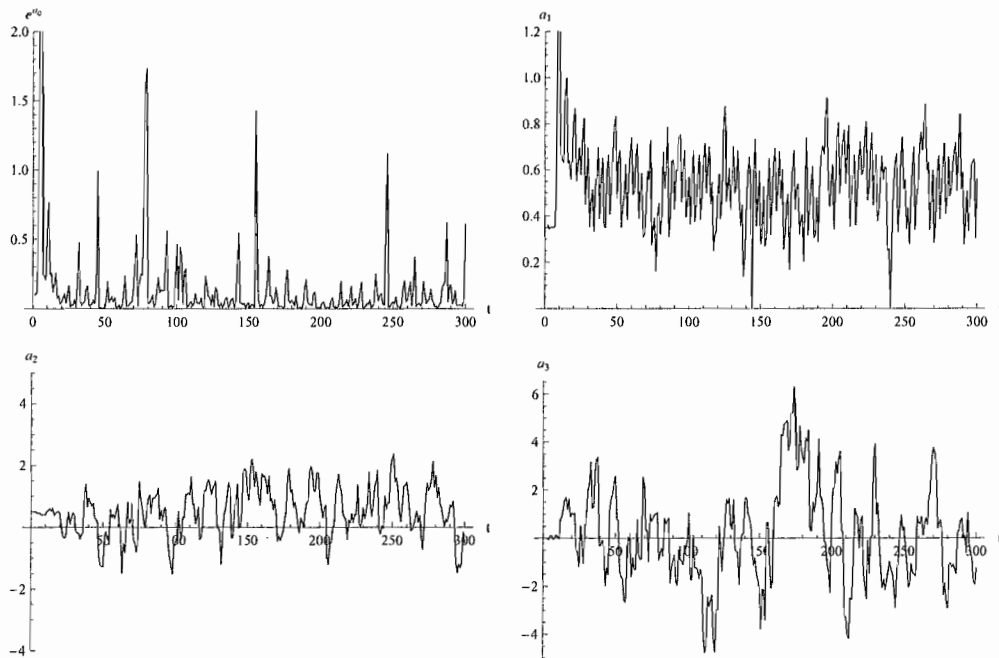


Fig. III.5: Variation in estimated coefficients, for $\psi_t = 0.07$

Therefore, shocks to expectations, either in the form of an i.i.d. stochastic shock like ξ_t , or shocks resulting from the effect of the aggregate productivity shock

on learned parameters, allow the economy to escape the low-training productivity trap. In addition, sufficiently large shocks to expectations, as we observed in the case of constant gain learning of key parameters, permit agents to oscillate from one generation to another, between maximum investment in human capital and physical capital. Provided that the variance of the stochastic shocks is large enough to permit this oscillation to take place, the size of the oscillations will be independent of ϵ_t , and will only depend on $\bar{\phi}$ and $\bar{\tau}$, the minimum savings and training rates. As seen in figure (III.6), the oscillation in training rates permits the model to capture both growth cycles and a long-run positive trend in aggregate output.

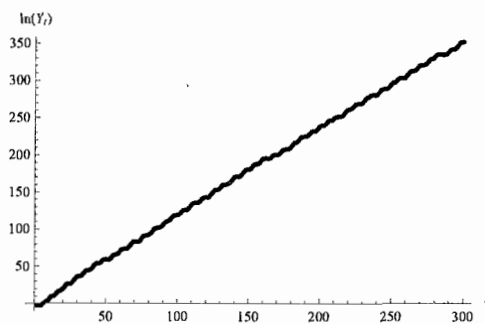


Fig. III.6: Aggregate output

III.7 Conclusion

In this chapter, I have considered the stability and transition dynamics of the Azariadis Drazen model of economic growth and human capital accumulation under adaptive learning. I found that like the perfect foresight model, the no-training equilibrium, in its linearized form, was stable around its steady state under learning. However, the interior equilibrium was not E-stable or stable under learning. Adaptive learning or expectational shocks allow the economy to escape from the low-training equilibrium, and generate endogenous oscillations in both human capital and physical capital investment. The model thus captures both endogenous

growth cycles and long-run positive trends in output as a result of the escape from the poverty trap. It clearly establishes the role of expectations in the escape path. Transitions out of the poverty trap are no longer deterministic, as they are in much of unified growth theory. This may provide a possible answer to the question: What accounts for the sudden switch to sustained growth in some countries, while stagnation has continued in others?

Two questions remain. Why does a path to upper steady state exist under perfect foresight, while the same does not appear to exist under learning? Why does adaptive learning generate endogenous oscillations rather than a smooth transition path? First, I remind the reader that the existence of the perfect foresight path was dependent on the independence of the evolution of training from effective physical capital, and on the choice of a unique initial training rate given initial human capital. The dynamics of τ_t under rational expectations and learning will depend also on both lagged and expected physical capital, and may depend on current productivity shocks. In addition, just as the perfect foresight path was dependent on an initial choice of τ_t , the same holds true under learning, and the mechanism for escape from the low-training steady state may not assure that this condition is satisfied.

Recall that under adaptive learning, when $w_t > \tilde{w}(k_{t+1}^e)$, because of positive productivity shocks, $\tau_t = \bar{\tau}$. Returning to the question of social learning vs. adaptive learning, I note that under genetic algorithm learning, fitness of τ_t and the savings rate is evaluated using wage and interest rates to calculate the lifetime utility that would have been obtained had a particular training or savings rate been in use in the previous period. The instability of the system under least squares learning derives at least in part from the ability of optimizing agents to react to productivity shocks by substituting investment in physical capital for investment in

human capital. They may do this without any negative effect on human capital, since the accumulation of human capital by previous generations has a permanent effect on human capital, provided that training equals or exceeds its minimum value $\bar{\tau}$. Switching in whether human or physical capital has the highest returns results in oscillations in training rather than the smooth increases in average training rates observed under social learning. This frequency of oscillations is increased by the one-time choice of agents whether or not to invest in physical/human capital. Increasing agents' lifespan above two periods could allow them to smooth their physical capital investments, particularly if physical capital investments occur more frequently than those in human capital.

Understanding the differences between social and econometric learning is important not only within the context of growth theory and escape paths from poverty traps, but more broadly in the literature on bounded rationality. While in the case of most macroeconomic models, least squares learning and social learning generate similar results, there have been a few examples identified in the two approaches generated different results. In particular, Arifovic et al (2007), in a New Keynesian model of monetary policy, found that equilibria which were unstable under recursive learning were in fact stable under social learning. As yet, there is no general principle allowing us to identify models in which the learning dynamics will be different under the two algorithms.

Social learning differs from econometric learning in the greater bounds placed on rationality and in the heterogeneity of agents. It then remains to determine whether the distinction between the application of social learning and least squares learning to the Azariadis Drazen model is a consequence solely of the greater bounds placed on rationality under social learning, or of the heterogeneity of agents allowed for in genetic algorithms. One possible extension of the model introduced in

this chapter could include introducing heterogeneity into agents expectations. A convenient tool would be to make the expectation shock ξ_t idiosyncratic, and then to observe any differences in the stability of the upper steady state and in the transition dynamics. An additional possible extension would be to allow for $\bar{\tau}_d$, the minimum level of human capital investment to avoid depreciation, to depend on the existing accumulation of human capital. Thus as the stock of knowledge in economies accumulates, individuals require more and more investment in training in order to access this knowledge. This could dampen the oscillations in human capital, by introducing an additional channel through which the externality in human capital could affect individuals' decisions.

CHAPTER IV

HUMAN CAPITAL, INSTITUTIONS AND TRANSITIONS BETWEEN GROWTH REGIMES

IV.1 Introduction

The determinants of stability and economic growth are fundamental to our understanding of the process of economic development. The econometrics of economic growth has largely focused on determinants of long-run growth. However, developing countries not only face greater macroeconomic volatility, the negative effects of volatility on growth are also larger in LDCs (less developed countries). Furthermore, Montiel and Serven (2005) report that extreme volatility, or that portion of volatility generated by crises, increased in developing countries in the 1980s and the 1990s, during the very period when most developed countries were experiencing a dramatic decline in volatility. This volatility may be an important contributor to the entire process of development. Aguiar and Gopinath (2007) find that the predominance of shocks in developing countries are not transitory but instead affect the trend.

More recently, attention has begun to be given to the topology of growth. Pritchett (2000) identified phases of growth, stagnation or decline of varying length experienced by most countries. Jerzmanowski (2006) uses a Markov-switching regression to find transition probabilities between stable growth, rapid growth, stagnation and crisis and analyzes the effect of institutions on such transitions. A

set of papers, Hausmann et al (2005,2006) examines periods of rapid acceleration of growth as well as crisis episodes.

These approaches allow us to capture the nonlinear dynamics associated with growth, rather than assuming that long-run growth is identified with a single steady state, with the level of growth being determined by a set of country characteristics. In particular, the four regimes employed by Jerzmanowski capture a poverty trap equilibrium, a stable growth equilibrium, and then transitions between the two. This allows us to distinguish between countries which experience stable growth, and others which transition out of a poverty trap via miracle growth. In the specification he employs, transition probabilities vary across countries, according to a static measure of economic institutions. He finds that better institutions lead to greater persistence of both stable and miracle growth regimes. In particular, countries with the highest quality of institutions, experience a sharp increase in the likelihood of transitioning from miracle growth to stable growth. Stagnation and crisis are both much more likely in countries where property rights are poorly protected.

While Jerzmanowski (2006) makes a unique contribution to the literature, its methodology has some limitations. The specification of transition probabilities is time-invariant. In addition, the author assumes that the only variable determining probabilities is institutions. The measure of institutions used is the index of government anti-diversion policies, averaged over 1985-1995. The use of the average value of an institutional measure would be perfectly adequate were it the case that institutions were highly persistent. However, while it is the case that countries which have attained the highest quality of political and economic institutions do tend to remain there over time, I will present evidence in this chapter that there is tremendous volatility in the quality of institutions in less developed countries.

In particular, the static measure of institutions ignores the dynamic effects of wars and economic crises on transition probabilities. In the case of wars and other such disasters, Hausmann et al (2004) has found them to have a negative and significant impact on the likelihood of crisis. In the case of human capital accumulation, Azariadis and Drazen (1990) and other similar models have given us a theoretical basis for regime switching when sufficient levels of human capital have been attained. The goal of this chapter is to examine determinants of transitions between growth regimes, using a broader set of explanatory variables, including indicators of changes in political regimes.

IV.2 Existing Evidence for Regime Switching in the Process of Economic Development

IV.2.1 Theoretical literature

The theoretical literature in economic growth is rich in models characterized by multiple equilibria. This includes the seminal paper, Azariadis and Drazen (1990), where increasing social returns to scale in the accumulation of human capital permits multiple steady states. Similarly, models of the relationship between institutions and development are frequently characterized by multiple equilibria. For example, Gradstein (2007) views an equal distribution of political power as a commitment device that will permit improvements in institutions and thus economic growth. He shows the existence of two possible equilibria, one with low-quality institutions, slow growth and the other with higher quality and more egalitarian institutions, and faster growth. Tornell (1997) finds that for certain parameter values, and allowing for three property rights regimes – common property, private property, and oligarchy – all Markov perfect equilibria involve a

shift from common property to private property and back to common property. This model, along with the existence of multiple equilibria in the other models described above, seems to argue in favor of a regime-switching approach in empirical studies of economic growth.

Another more recent branch of the economic growth literature attempts to rationalize the greater volatility that we observe in developing countries. Acemoglu and Zilibotti (1997) argue that economic development is associated with a decline in volatility due to better diversification opportunities and more productive use of funds. “Lucky” countries will spend relatively less time in the stage of low growth and high volatility and may thus experience miracle growth. However, their model does predict that in the long-run, economies will converge to high and stable levels of income. Cetorelli (2002) proposes a similar model to that of Acemoglu and Zilibotti (1997), in that the probability of adverse production shocks decreases as the economy develops. His model also identifies a variety of dynamic equilibria consistent with club convergence, growth miracles and growth disasters. In particular, there is the possibility of mobility within the distribution of cross-country incomes, and thus lends itself to empirical estimation by regime switching models.

IV.2.2 Empirical Literature

There has long been a concern with reconciling results on conditional convergence in the empirical literature with theoretical models of growth generating multiple equilibria. Benhabib and Gali (1995) surveys existing models of growth with multiple equilibria, and discusses some of the empirical predictions generated by those models. They argue that some of the predictions of growth models with unique equilibria are hard to reconcile with the evidence, and that existing evidence does not rule out the existence of multiple equilibria in the data. Durlauf and

Johnson (1995) uses classification and regression tree methods, and find evidence in favor of multiple regimes in economic growth. In particular, they find that even after controlling for heterogeneity in parameters, there is a role for initial conditions in explaining variation in cross-country growth behavior. Papageorgiou and Mansanjala (2004) test the robustness of the Durlauf and Johnson results by using a CES production function. Using initial income and adult literacy as threshold variables, they find evidence of four regimes.

However, these studies only consider multiple regimes in long-run patterns of growth, and hence do not allow for transitions between steady states or between regimes. Recently, Jerzmanowski (2006) estimates a Markov-switching regression to characterize four growth regression. He employs a static measure of institutions - a 1986-1995 average over several categories of data collected by Political Risk Services - as the sole determinant of transition probabilities between growth regimes. He finds that better institutions make continued stable growth more stable, and make the stable growth regime more likely to follow a period of rapid growth. In addition, improved institutions make rapid growth more likely to follow either periods of crisis or periods of stagnation.

An alternative measure to the index of government anti-diversion used by Jerzmanowski is the variable "constraints on the chief executive" from the Polity IV dataset, frequently used as an alternative measure of property rights and protection from government expropriation. This variable varies from 1 to 7, where 7 represents the highest level of constraints on the executive branch of the government. One can observe that while there is persistence in the variable, particularly for those countries starting at the highest value, there is certainly dynamic variation in this measure of institutions across the sample period. This time-series variation in institutions, particularly as a result of political regime changes, may contribute

Tab. IV.1: Transition probability matrix for constraints of executive branch of government, sample of 89 countries from 1962-1994

Quality of institutions in 1962/1994	1	2	3	4	5	6	7	All
1	0	0.08	0.38	0	0.23	0	0.31	0.16
2	0.5	0	0.25	0.25	0	0	0	0.05
3	0	0.18	0.23	0.05	0.27	0.05	0.22	0.27
4	0	0	0	0	0.33	0.33	0.34	0.04
5	0.17	0	0.17	0	0	0.5	0.16	0.07
6	0	0	0	0	0	0	1	0.01
7	0.03	0	0.09	0	0.12	0	0.76	0.4
All	0.05	0.06	0.18	0.02	0.17	0.06	0.46	1

The entry in row i , column j , indicates the probability of moving from an institutional quality of i in 1962 to a quality of j in 1994.

significantly to switches in growth regimes, and is omitted from the Jerzmanowski specification.

Rather than looking at a full set of growth regimes, Hausmann et al (2005, 2006) analyze the determinants of periods of crisis and growth accelerations. They find that changes in political regimes as well as wars and sudden stops in capital flows can precipitate periods of crisis and/or growth accelerations. But, they find that levels of human capital accumulation and other controls for economic development do not have a significant impact on either the likelihood or the duration of accelerations or crises. Hausmann et al (2004, 2006) admit to a relatively low fit of their specification. This is partly because the events they are trying to predict are relatively rare. The authors identify only 83 episodes of accelerations for 1950-2000, using the Penn World Tables dataset. While these episodes may be more frequent than one may at first suspect, as Hausmann et al (2004) argue, they are frequently the results of relatively small shocks or changes in policy. Standard control variables in cross-country growth regressions, including

human capital, do a very poor job predicting turning points in growth.

I therefore propose estimating a Markov-switching regression, like that used in Jerzmanowski (2006), but using a broader set of possible control variables for estimation of transition probabilities, including a set of discrete variables, measuring changes in political regimes, wars, and sudden stops, as well as continuous variables, including measures of institutions and human capital accumulation. With respect to institutions, I propose using a time-varying measure of institutions, which will avoid, to some extent, the problem of endogeneity inherent in the Jerzmanowski approach, in that average quality of institutions from 1986-1995 may be the outcome of growth regimes occupied by the economy prior to 1986. In addition, considering the role that human capital accumulation plays in such transitions will contribute to the ongoing literature regarding the causal relationship between education and economic growth.

IV.3 Empirical Methodology and Data

Many theoretical models of the role of institutions and human capital accumulation in economic growth are characterized by two steady states, of which one is a poverty trap. In order to fully allow for the possibility of two stable states, and transitions to and from these states, a minimum of four regimes is required in the empirical specification. Growth of output per worker follows an AR(1) process

$$\hat{y}_{k,t} = \mu_{s_{kt}} + \phi_{s_{kt}} \hat{y}_{k,t-1} + \epsilon_{k,t,s_{kt}} \quad (\text{IV.1a})$$

$$\epsilon_{k,t,s_{kt}} \text{ i.i.d. } N(0, \sigma_{s_{kt}}^2) \quad (\text{IV.1b})$$

where s_{kt} is the regime in place in country k in period t . The regime affects both the coefficients on lagged growth and the variance of the stochastic shock.¹

Transition probabilities between regimes are country-specific, and are determined by a vector of variables $x_{i,t}$:

$$P(s_{kt} = i_1 | s_{k,t-1} = i_0) \equiv p_{kt}^{i_0 i_1}(x_{kt}) \quad (\text{IV.2})$$

I follow Diebold et al (1994) in specifying the transition probabilities as

$$\begin{aligned} p_{kt}^{i_0 1}(x_{k,t}) &= \frac{\exp(x'_{kt} \beta_{i_0,1})}{1 + \sum_{i_1=1}^3 \exp(x'_{it} \beta_{i_0,i_1})} & p_{kt}^{i_0 2}(x_{k,t}) &= \frac{\exp(x'_{kt} \beta_{i_0,2})}{1 + \sum_{i_1=1}^3 \exp(x'_{it} \beta_{i_0,i_1})} \\ p_{kt}^{i_0 3}(x_{k,t}) &= \frac{\exp(x'_{kt} \beta_{i_0,3})}{1 + \sum_{i_1=1}^3 \exp(x'_{it} \beta_{i_0,i_1})} & p_{kt}^{i_0 4}(x_{k,t}) &= 1 - \sum_{i_1=1}^3 p_{kt}^{i_0 i_1}(x_{kt}) \end{aligned}$$

Since the states are unobservable, parameters will be chosen to maximize the expected complete-data log likelihood, conditional upon the observed data, given below:

$$\begin{aligned} L(\mathcal{Y}_{k,T}, \mathcal{S}_{k,T} | \mathcal{X}_{k,T}; \theta) &= \sum_{k=1}^N \left\{ \sum_{i=1}^3 \rho_i [\log f(y_{k1} | s_{k1} = i, y_{k0}) + \log(\rho_i)] \right. \\ &\quad + \left(1 - \sum_{i=1}^3 \rho_i \right) \left[\log f(y_{k1} | s_{k1} = 4, y_{k0}) + \log \left(1 - \sum_{i=1}^3 \rho_i \right) \right] \\ &\quad + \sum_{t=2}^T \left[\sum_{i=1}^4 P(s_{kt} = i | \mathcal{Y}_{kT}, \mathcal{X}_{kT}) \log f(y_{kt} | s_{kt}, y_{kt-1}) \right. \\ &\quad + \sum_{i_1=1}^3 \sum_{i_0=1}^4 P(s_{kt} = i_1, s_{k,t-1} = i_0 | \mathcal{Y}_{kT}, \mathcal{X}_{kT}) \log p_{kt}^{i_0 i_1} \\ &\quad \left. \left. + \sum_{i_0=1}^4 P(s_{kt} = 4, s_{k,t-1} = i_0 | \mathcal{Y}_{kT}, \mathcal{X}_{kT}) \log \left(1 - \sum_{i_1=1}^3 p_{kt}^{i_0 i_1} \right) \right] \right\} \quad (\text{IV.3}) \end{aligned}$$

To clarify the notation, \mathcal{Y}_{kT} indicates the complete history of growth for country k ,

¹A single lag is used in the empirical specification, due to the fact that when estimating an AR(p) model with country fixed effects only one lag is significant.

through period T , while ρ_i indicates the unconditional probability of state i occurring in country k in the initial period. The vector of all parameters is given by θ . Given the AR-1 process specified for y_{kt} , the density of y_{kt} conditional on s_{kt} and $y_{k,t-1}$ is:

$$f(y_{kt}|s_{kt} = i, y_{kt-1}; \mu_i, \phi_i, \sigma_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(\frac{-(y_{kt} - \mu_i - \phi_i y_{kt-1})^2}{2\sigma_i^2}\right) \quad (\text{IV.4})$$

For the filter leading to the smoothed probabilities $P(s_{kt} = i|\mathcal{Y}_{kT}, \mathcal{X}_{kT})$ and $P(s_{kt} = i_1, s_{kt-1} = i_0|\mathcal{Y}_{kT}, \mathcal{X}_{kT})$, I would ask the reader to refer to Diebold et al (1994). Taking the smoothed probabilities as given, the first order conditions with respect to μ_i, ϕ_i and σ_i are given by

$$\mu_i = \frac{\sum_{k=1}^N \sum_{t=2}^T P(s_{kt} = i|\mathcal{Y}_{kT}, \mathcal{X}_{kT})(y_{kt} - \phi_i y_{kt-1})}{\sum_{k=1}^N \sum_{t=2}^T P(s_{kt}|\mathcal{Y}_{kT}, \mathcal{X}_{kT})} \quad (\text{IV.5})$$

$$\phi_i = \frac{\sum_{k=1}^N \sum_{t=2}^T P(s_{kt} = i|\mathcal{Y}_{kT}, \mathcal{X}_{kT})(y_{kt} - \mu_i)y_{kt-1}}{\sum_{k=1}^N \sum_{t=2}^T P(s_{kt}|\mathcal{Y}_{kT}, \mathcal{X}_{kT})y_{k,t-1}^2} \quad (\text{IV.6})$$

$$\sigma_i^2 = \frac{\sum_{k=1}^N \sum_{t=2}^T P(s_{kt} = i|\mathcal{Y}_{kT}, \mathcal{X}_{kT})(y_{kt} - \mu_i - \phi_i y_{kt-1})^2}{\sum_{k=1}^N \sum_{t=2}^T P(s_{kt}|\mathcal{Y}_{kT}, \mathcal{X}_{kT})} \quad (\text{IV.7})$$

These parameters may be calculated by estimating the smoothed probabilities given an initial parameter vector $\hat{\theta}_0$, and then updating the parameters μ_i, ϕ_i, σ_i using the above first order conditions.

The first order conditions for $\beta_{i_0 i_1}$ are nonlinear in the parameter:

$$\sum_{k=1}^N \sum_{t=2}^T x_{k,t-1} \left\{ P(s_{kt} = i_1, s_{kt-1} = i_0|\mathcal{Y}_{kT}, \mathcal{X}_{kT}) - P(s_{kt-1} = i_0|\mathcal{Y}_{kT}, \mathcal{X}_{kT})p_{kt}^{i_0 i_1} \right\} = 0 \quad (\text{IV.8})$$

The parameters are thus updated by using numerical methods to solve the above

nonlinear equations for $\beta_{i_0 i_1}$. Once a new set of parameter values is obtained, the smoothed probabilities are once again calculated, and the process is repeated, until a convergence criterion is satisfied.

The vector x_{kt} may be composed of two groups of explanatory variables: non-discrete variables, including measures of institutions and human capital accumulation, and indicator variables giving country-specific events that will influence transition probabilities. In the latter group, I include measures of wars and natural disasters, along with episodes of both economic reform and sudden stops in capital flows. In estimating the model, I start with a rather parsimonious specification, including two explanatory variables, a measure of political institutions and a measure of human capital accumulation. I will also consider more complex specifications, bearing in mind, however, the limitations imposed through the availability of data.

The Markov regime-switching specification outlined above is estimated using annual data on growth of output per worker from Penn World Tables 6.2. Initial estimations are performed using the full set of countries for which data is available for the growth rate of output per worker from 1962-1994, obtained from the Penn World Tables 6.1. The institutional measure in this case is the ICRG index. This is a measure of the quality of economic institutions across five different dimensions: government corruption, rule of law, bureaucratic quality, repudiation of government contracts, and expropriation. The measure is based on an aggregation across a number of risk factors. Individual risk assessments are made by the staff of the Political Risk Service Group. For comparability to Jerzmanowski, the average value from 1986-1995 is used. The full data set is a balanced panel for 94 countries.

The quality of political institutions is measured by the polity variable from the Polity IV dataset. This variable is calculated by subtracting an autocracy

measure from a measure of democracy. The quality of democracy is measured along several dimensions, including the competitiveness of political participation and executive recruitment, the openness of executive recruitment, and constraints on the powers of the executive branch of government. The autocracy measure looks at the same dimensions, along with regulations of political participation. Human capital accumulation is measured using total years of schooling from the Barro-Lee dataset. To make the data most comparable across countries, I use total years of schooling for the population over 25 years of age. As the data is only available at 5 year intervals from 1960-1999, I interpolate the data. There are 68 countries with data on output per worker from 1962-2003, and measures of political institutions and human capital accumulation, over this same period.

Finally, another time-varying measure of economic institutions is available, the rule of law dimension of the Economic Freedom Index, compiled by the Fraser Institute. I focus on this dimension, rather than including all of the dimensions, as some of the others are primarily the reflection of policies rather than the institutions themselves. The rule of law dimension focuses specifically on legal structure and the security of property rights. This measure of the rule of law has been less used than measures such as the ICRG which have limited time variation and are typically employed in cross-sectional growth regressions, but has the advantage of allowing for time-variation in institutions over a sample of 48 countries and 33 years.

Prior to 1995, the rule of law measure contained within the Economic Freedom Index was measured along two dimensions: security of property and support for the rule of law. These are measured using a compilation of data from the ICRG and the World Competitiveness Report. Rule of law is measured after 1995 along seven dimensions: judicial independence, impartial courts, protection of property rights, military interference in rule of law, integrity of the legal system,

legal enforcement of contracts, and regulatory restrictions on the sale of real property. The first two dimensions are measured via responses to the Global Competitiveness Report's survey questions on judicial independence and functioning of courts. Military interference and judicial integrity is based on measures from the ICRG. The last two dimension are based on the World Bank's "Doing Business" estimates.

IV.4 Results

First, I estimate the transition probabilities for the full sample, allowing for these probabilities to be affected by the ICRG index, as in Jerzmanowski (2006). The only difference between my estimation and his is that my full sample includes 94 countries, rather than 89. The AR1-parameter results, given in table(IV.2) and (IV.3), are relatively similar for the stable growth, stagnation and miracle growth states (1,2 and 4 respectively). State 3, roughly equivalent to periods of crisis, has a significantly larger long-run growth rate in the full sample. However, I note that the likelihood of being in this state is primarily driven by high volatility of economic growth.

Tab. IV.2: AR-1 parameters, full sample, control variables: ICRG

State	Constant (μ_i)	AR Coeff. (ϕ_i)	Std. Dev. (σ_i)	Long-run growth
1	1.283 (0.0061)**	0.3737(0.0008)**	2.569(0.041)**	2.05
2	-0.0841(0.0869)	0.1844(0.0225)**	5.868 (0.2856)**	-0.07
3	2.948 (0.0632)**	-0.0648 (0.0048)**	13.737 (0.345)**	2.77
4	4.8753 (0.0096)**	0.0972 (0.0028)**	4.212 (0.046)**	5.4

Standard errors in parentheses. **Significance at 5% level. State 1: stable growth, State 2: stagnation, State 3: crisis, State 4: miracle growth

Figure (IV.1) shows the transition probabilities given a particular initial state. For countries that start out in stable growth, which have a significantly high

Tab. IV.3: AR-1 parameters, Jerzmanowski sample of 89 countries, 1962-1994, control variables: ICRG

State	Constant (μ_i)	AR Coeff. (ϕ_i)	Std. Dev. (σ_i)	Long-run growth
1	1.32**	0.3761**	2.11**	2.12
2	0.1	0.1799**	4.56**	0.12
3	-1.01**	-0.0045	13.16**	-1.00
4	5.36**	0.1417**	2.71**	6.25

**Significance at 5% level, State 1: stable growth, state 2: stagnation, state 3: crisis, state 4: miracle growth

level of the ICRG index, the probability of remaining in the stable growth regime is very high, while it is close to zero for countries below that threshold. These countries are most likely to transition to the crisis state. Countries currently in stagnation and with a low quality of institutions, high probability of remaining in that state. But, for countries with high quality of institutions, there is increasing probability of transition either to stable growth, or to miracle growth. As in the case of stagnation, highest probability of remaining in crisis occurs in countries with low quality of institutions. As institutions improve, there is a sharp increase in the likelihood of miracle growth. In the case of miracle growth, countries with poor quality of institutions are most likely to move to the crisis regime. As institutions improve, countries are most likely to either remain in miracle growth, or to transition to stable growth.

But these results are entirely based on the assumption that institutions, and thus transition probabilities, are constant over time. Figures (IV.2-IV.5) give the changes in the probabilities of being in the four growth regimes, for four countries, Argentina, Bolivia, Ghana and South Africa, that experienced significant changes in their political institutions over the sample period. The polity measure for Argentina declined from -1 to -9 in 1966, and increased from -9 to 6 in 1973. This corresponds to a sharp increase, then decrease in the probability of being in the stable growth

regime. Similarly, the sharp decrease in the polity measure in Bolivia during the mid 1970's corresponds to a sharp increase in the probability of being in the stable growth regime, as opposed to stagnation. Finally, the positive and abrupt changes in polity in Ghana are associated with sharp increases in the probability of being in the stable growth regime. Similarly, there is a brief but noticeable decline in the probability of being in the stable growth regime in South Africa in 1992 during the change of regime there.

What are the consequences of imposing that the transition probabilities are determined by the averaged value, rather than the time-varying institutional measure? The institutional measure is used, in the first order condition for β_{i0i1} to weight the smoothed probabilities. When the averaged value is used, the weight is constant across time. The sole variation that can be used to explain differences in transition probabilities are cross-sectional differences in institutions. Let us then turn our attention to the two other data samples, the polity sample, and the economic freedom sample, containing data for 68 countries and 48 countries respectively. These two samples are characterized by slightly higher quality of institutions and slightly lower volatility of economic growth.

Tab. IV.4: AR-1 parameters, polity sample, control variables: polity

State	Constant (μ_i)	AR Coeff. (ϕ_i)	Std. Dev. (σ_i)	Long-run growth
1	0.8527(0.0094)	0.3712 (0.0003)	1.962(0.0270)	1.356
2	1.8697 (0.0479)	0.3944 (0.0014)	3.025(0.0959)	3.087
3	11.074 (0.0731)	-0.861 (0.0015)	15.103 (0.264)	5.95
4	-0.15 (0.0385)	0.1007 (0.0023)	6.374 (0.142)	-0.024

Standard errors in parentheses. State 1: stable growth, state 2: stagnation, state 3: crisis, state 4: miracle growth

Tables (IV.4) and (IV.5) give the AR-1 parameter estimates, by state, for the two samples. For the polity sample, the interpretation of states is no longer consistent with what it was under the full sample. The first state is similar, and

Tab. IV.5: AR-1 parameters, economic freedom sample, 4 regimes, control variables: economic freedom, average years of schooling

State	Constant (μ_i)	AR Coeff. (ϕ_i)	Std. Dev. (σ_i)	Long-run growth
1	0.8192(0.094)	0.1621(0.004)	4.657(0.527)	0.9778
2	0.7853(0.01)	0.3641(0.001)	1.935(0.08)	1.4553
3	0.1346(0.06)	0.1161(0.002)	6.5701(0.348)	0.1523
4	3.234(0.008)	0.3182(0.005)	2.8487(0.139)	4.7432

Standard errors in parentheses. State 1: slow growth, state 2: stable growth, state 3: crisis, state 4: miracle growth

roughly corresponds to the stable growth state, although its long-run growth rate and volatility are slightly lower than before. State 4 roughly corresponds to state 2 or stagnation in the full sample, with a long-run growth rate that is close to zero, and higher volatility than the stable growth rate. State 2 in the polity sample is most similar to the miracle growth state, or state 4 under the full sample, although with a lower growth rate on average, lower volatility and higher persistence. The most striking difference is in state 3. In both samples, this corresponds to the highest volatility of all the growth regimes, and is the sole to have a negative AR-1 coefficient. However, there is a dramatic increase in the intercept and AR-1 coefficient, with the result that the long-run growth rate of this state is much higher than in the full sample. Examples of this state include Ghana(1963-1966, 1968-1972), Iran(1976-1977), and Israel (1968), all of which involve significant volatility in economic growth and disruptions due to political crisis or the Six-Day War in the case of Israel.

Figure (IV.6) shows the variation in the transition probabilities with the polity index. The states can best be described as follows: state 1 corresponds to stable growth, state 2 to miracle growth, state 3 to crisis, and state 4 to stagnation. Autocratic countries, with $polity \leq 0$, will transition from stable growth to stagnation with probabilities close to 1. The most democratic countries are most

likely to remain in the stable growth regime. However, in democratic countries with autocratic tendencies, i.e. with *polity* $\simeq 0$, the probability of switching to miracle growth approaches 1. This would include countries such as South Korea and Thailand that experienced a combination of autocratic regimes and miracle growth.

Having once entered either rapid growth or stagnation, countries are most likely to remain there, regardless of the quality of their political institutions, although institutional quality will slightly increase the likelihood of remaining in the rapid growth state, and decrease the likelihood of remaining in stagnation. On the other hand, institutions do have a significant effect of transition probabilities out of the crisis state. The most autocratic countries are the most likely to remain in this state, while the most democratic countries will transition to the miracle growth regime.

Suppose now that we consider an alternative measure of institutions, the economic freedom index, which is more focused on the rule of law and the security of property than measuring the functioning of political institutions. The sample of countries is the smallest of all those I have considered, as complete data, on both economic freedom and human capital, is only available for 48 countries, from 1970-2003. Economic growth is the least volatile in this sample, while the average ICRG index is the highest. The four growth regimes roughly correspond (in numerical order) to stable growth, slow growth, stagnation and miracle growth. Stagnation in this sample corresponds to the highest volatility growth regime. The distinction between the stable growth and slow growth regimes lies primarily in reduced volatility, lower long-run growth and higher persistence. Thus, countries characterized by a high average probability of being in the slow growth regime are Japan, the countries of Western Europe, Canada, Australia and the United States. The difference in the identification in growth regimes is primarily driven by the fact

that even when using the transition probabilities as estimated in the full sample, the countries in the economic freedom sample spend under 3% of their time, on average, in the crisis state (state 3 for the full sample). The slow growth and stable growth states are both similar to the stable growth state in the full sample. An alternative approach would be to restrict the number of states for this sample to three: stable growth, stagnation, and miracle growth.

Tab. IV.6: AR-1 parameters, economic freedom sample, 3 regimes, control variables: economic freedom, average years of schooling

State	Constant (μ_i)	AR Coeff. (ϕ_i)	Std. Dev. (σ_i)	Long-run growth
1	0.9373(0.003)	0.3641(0.0005)	1.969(0.046)	1.474
2	0.566(0.007)	0.0788(0.002)	6.244(0.201)	0.614
3	1.449(0.002)	0.4847(0.0002)	2.795(0.053)	2.812

Standard errors in parentheses. State 1: stable growth, state 2: crisis, state 3: rapid growth

For this sample, I allow for the transition probabilities to be jointly determined by the economic freedom index and average years of schooling. Figure (IV.7) shows the effect of the rule of law on transition probabilities, when the country has the sample average level of schooling. Figure (IV.8) shows the effect of schooling for countries with the average level of economic freedom. The transition probabilities for the stable growth and slow growth regimes are unaffected by the quality of the legal system, and countries in these regimes are most likely to remain there.

However, the rule of law does significantly affect the transition probabilities for stagnation and miracle growth. Poor rule of law increases the probability of remaining in stagnation, while countries with the greatest protection of property rights are most likely to transition to stable growth. Those countries with intermediate quality of economic institutions will likely transition to miracle growth. These results are qualitatively similar to those obtained for the polity sample.

Finally, poor institutions increases the likelihood of switching from miracle growth to stable growth, while better quality of institutions increases the likelihood of remaining in this regime. Again, we see that the effect of the rule of law and the quality of political institutions is similar.

While Barro and Lee measures of human capital frequently do not have a significant impact in cross-sectional growth regressions, the average number of years of schooling does significantly affect the transition probability for every initial state except for stable growth. Countries currently in the slow growth regime will remain there if their human capital is sufficiently high, and transition to miracle growth if not. The latter result is consistent with the interpretation of miracle growth as the transition path between steady states corresponding respectively to low and high levels of investment in human capital. Countries currently in the miracle growth regime will tend to remain there, but their probability of doing so is initially increasing, then decreasing in the level of human capital. The likelihood of switching from miracle growth to stagnation is increasing in the level of schooling. Finally, countries with low levels of human capital are most likely to remain in stagnation. The transition probability to miracle growth is increasing then decreasing in the level of schooling. This is primarily because the most developed countries are most likely to return to slow growth, rather than miracle growth.

Overall, the results for the effect of the economic freedom index are similar to those for the polity measure, with the exception that the crisis regime is not populated sufficiently in the economic freedom sample. Allowing for the time variation of the economic freedom index increases the average probability of being in the miracle growth regime. Similarly, allowing for time variation in the polity measure, increases the average probability of both stagnation and miracle growth. The results contrast with cross-sectional growth regressions in that measures of

economic and political institutions are found to have similar and significant effects on patterns of growth, as does human capital accumulation.

As a further robustness check, I have also estimated the four-regime model, where the economic freedom index, the polity index, and the average number of years of schooling are all included as control variables for the transition probabilities. The results from this estimation are included in figures (IV.9-IV.11) and in table (IV.7). First of all, the ordering and the value of the AR-1 parameters are different from those in table (IV.5).² Most significant is that with the exception of the stable growth regime, most regimes see an increase in volatility. The crisis regime can be more clearly identified, due to a strongly negative long-run growth rate, while the stagnation regime also has a lower average growth rate.

Tab. IV.7: AR-1 parameters, economic freedom sample, 4 regimes, control variables: economic freedom, average years of schooling, polity

State	Constant (μ_i)	AR Coeff. (ϕ_i)	Std. Dev. (σ_i)	Long-run growth
1	3.0793(0.02)	0.271(0.0003)	4.5 (0.2814)	4.224
2	-0.9174(0.03)	0.5267(0.009)	6.012 (0.2441)	-1.938
3	0.6088(0.0319)	0.1379(0.0007)	5.576 (0.2472)	0.7062
4	1.0672(0.0024)	0.4417(0.0015)	2.3173 (0.1296)	1.912

Standard errors in parentheses, state 1: miracle growth, state 2: crisis, state 3: stagnation, state 4: stable growth

Of particular interest are the transition probabilities when we control for both political and economic institutions. For the case of stagnation, crisis, and miracle growth regimes, none of the various control variables effect transition probabilities on the margin are affected by economic/political institutions or by human capital accumulations. However, all three of the measures do have a significant effect of transitions out of stable growth. The quality of both political and economic institutions will increase the probability of remaining in the stable

²The ordering of states is arbitrary, and differences between specifications in this regard are irrelevant

growth state, and reduce the probability of transition to crisis. On the other hand, for countries at the average level of political and economic institutions, human capital accumulation will reduce the probability of remaining in stable growth and increase the probability of entering an economic crisis.

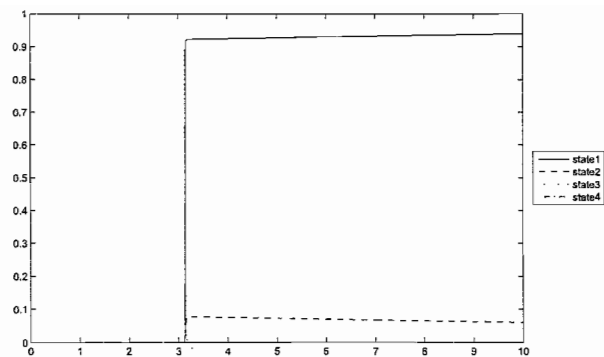
Due to the fact that the figures are based upon mean levels of other relevant variables, they do not give a complete description of the behavior of transition probabilities. That said, the sign of these marginal effects is robust to changes in the level of institutions/human capital at which the effect is evaluated. These results imply primarily that political institutions and economic institutions are substitutes in increasing the probability of remaining in the stable growth regimes, while human capital accumulation in the presence of mediocre institutions may actually reduce the stability of the stable growth regime.

IV.5 Conclusion

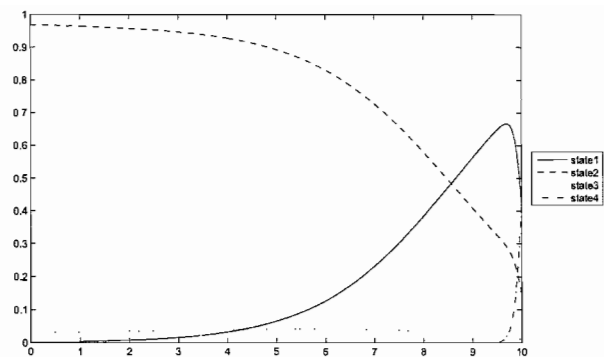
The empirical growth literature has been dominated by cross-sectional growth regressions in various forms, including increasingly sophisticated approaches to the econometric analysis, but which ignore time-wise variations in growth. In order to capture switches between growth regimes, I estimate a Markov-switching regression using cross country data on growth rates of output per worker. In the specification employed by Jerzmanowski (2006), transition probabilities are determined by a single static measure of institutions, the ICRG index. In this chapter, I explore the effects of two alternative, time-varying institutional measures – the polity and economic freedom index. In addition, my final specification allows for human capital to also affect the transition probability matrix. Theoretical models indicate that there can be significant threshold effects of both institutions

and human capital on economic growth. Convergence to a poverty trap equilibrium implies stagnation in economic growth, human capital accumulation and institutional quality, while the balanced growth equilibrium implies stable economic growth, along with high quality of institutions and human capital accumulation, both of which will increase over time. The empirical results confirm this, and in addition find that allowing for the time variation of both institutions and human capital will increase the likelihood of identifying the regime as miracle growth or stagnation rather than as stable growth. That is, the time variation of the variables determining transition probabilities permits us to more easily identify transitions between poverty trap and balanced growth equilibria.

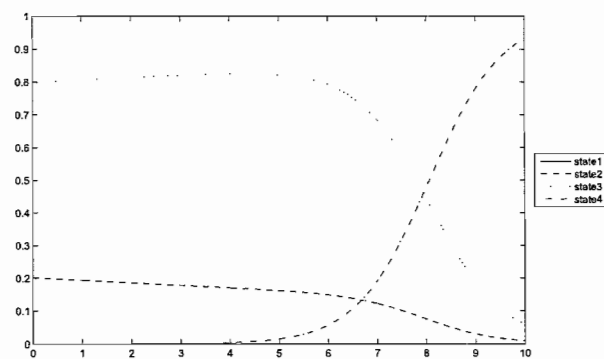
This chapter provides an initial approach to estimating time varying transition probabilities in a Markov switching model of economic growth. However, because of significant time involved in each estimation, not all specifications have been completely explored, including the inclusion of human capital along with the institutional measure in the polity sample. In addition, future work would test the appropriateness of the specification, including the number of regimes, possibly via Monte Carlo simulations. Finally, there is evidence that the AR-1 parameters, specifically the intercept and volatility terms, depend significantly on institutional and human capital measures, suggesting that the growth regimes themselves may depend on the level of development. This is certainly worthy of further investigation.



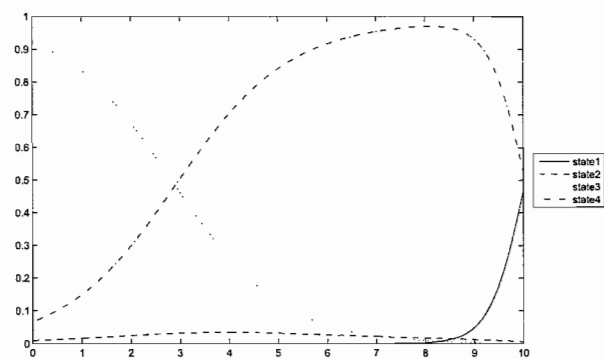
(a) Transition probabilities from state 1 (stable growth)



(b) Transition probabilities from state 2 (stagnation)



(c) Transition probabilities from state 3 (crisis)



(d) Transition probabilities from state 4 (miracle growth)

Fig. IV.1: Transition probabilities for full sample, dependence on ICRG index

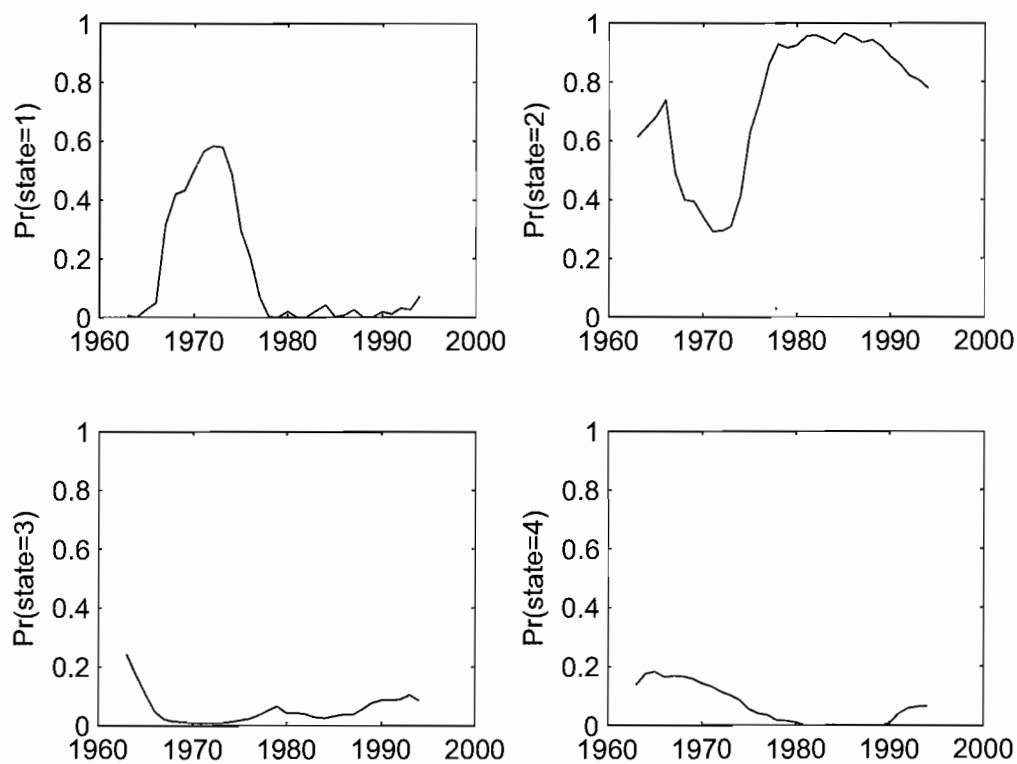


Fig. IV.2: Changes in probability of a given growth regime, Argentina, estimates from full sample (control variable = ICRG), state 1 = stable growth, state 2 = stagnation, state 3 = crisis, state 4 = miracle growth

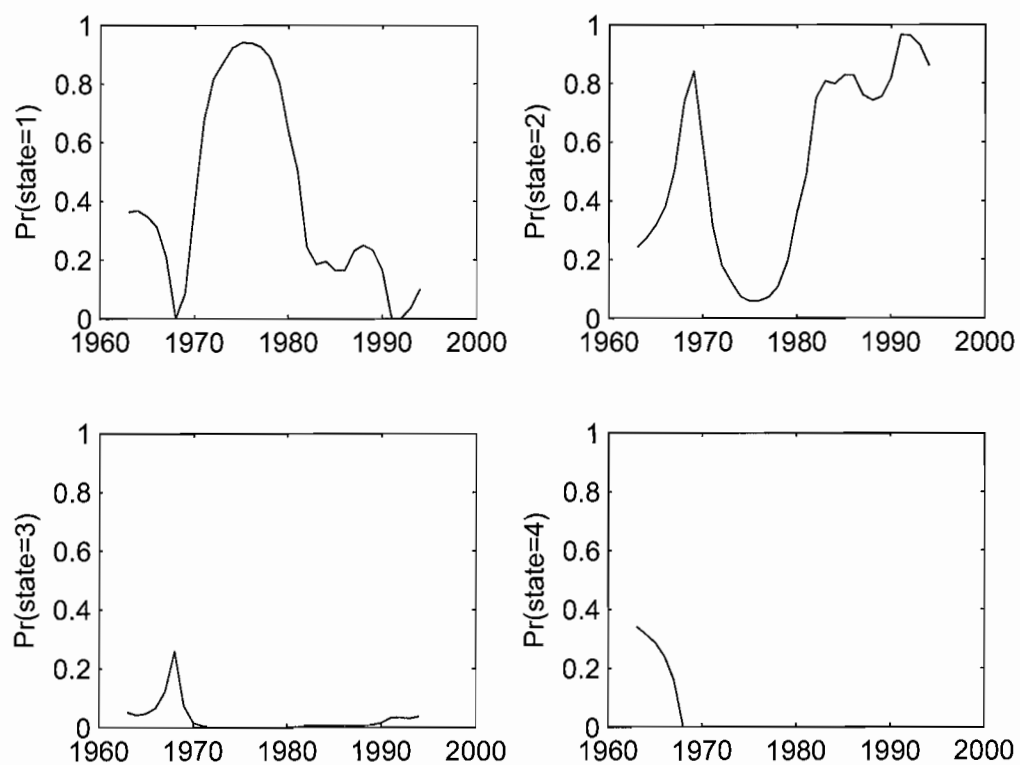


Fig. IV.3: Changes in probability of a given growth regime, Bolivia, estimates from full sample (control variable = ICRG), state 1 = stable growth, state 2 = stagnation, state 3 = crisis, state 4 = miracle growth

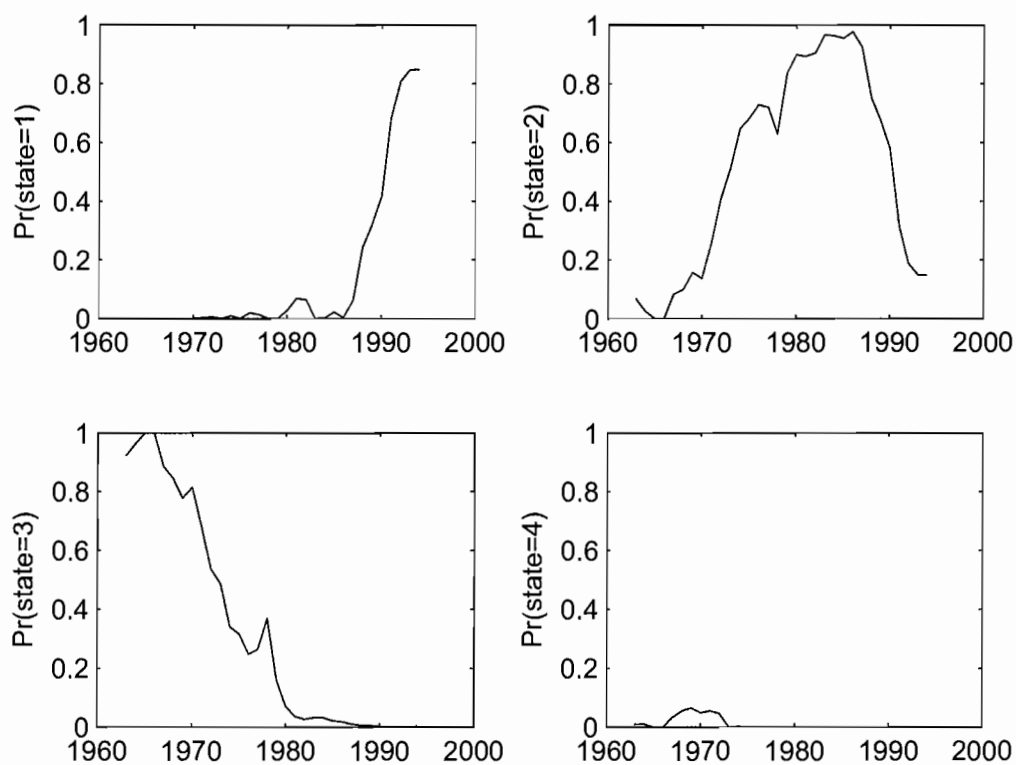


Fig. IV.4: Changes in probability of a given growth regime, Ghana, estimates from full sample (control variable = ICRG), state 1 = stable growth, state 2 = stagnation, state 3 = crisis, state 4 = miracle growth

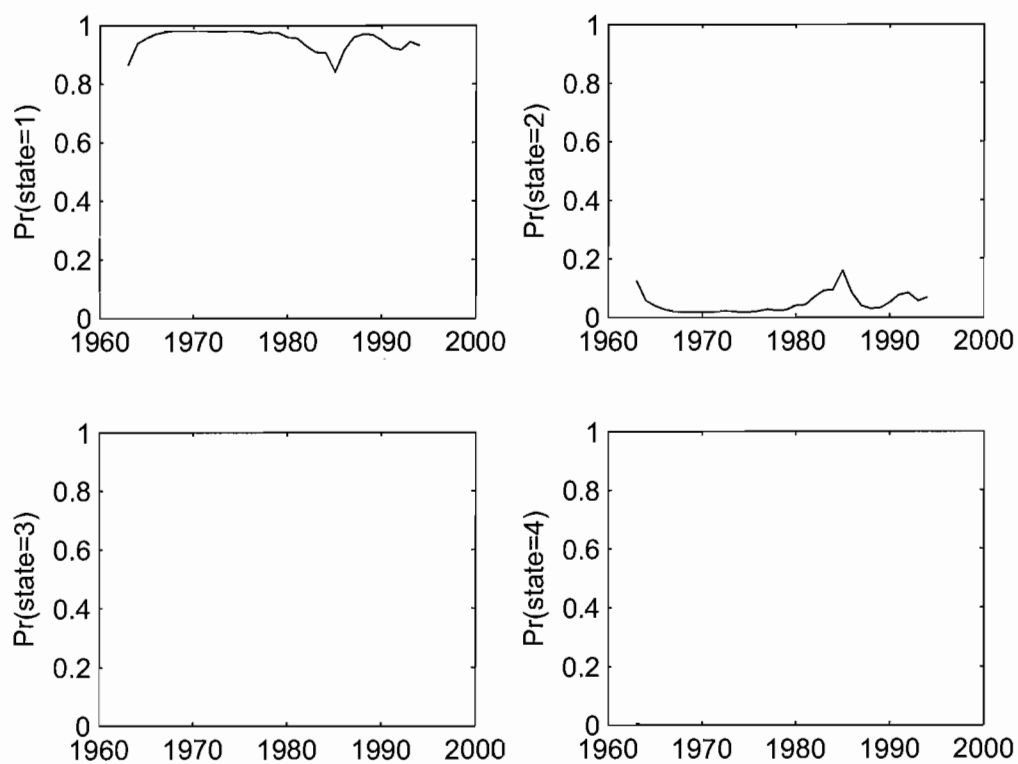
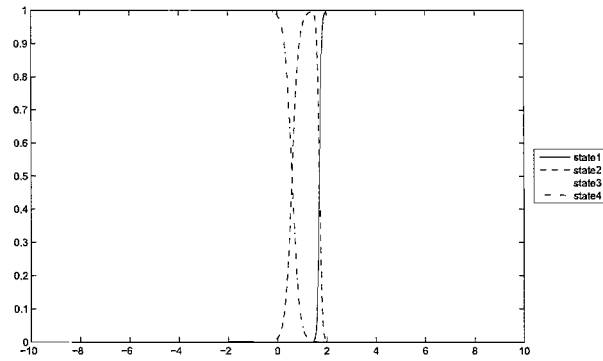
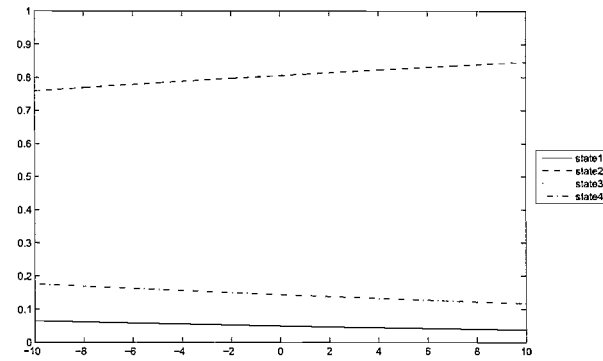


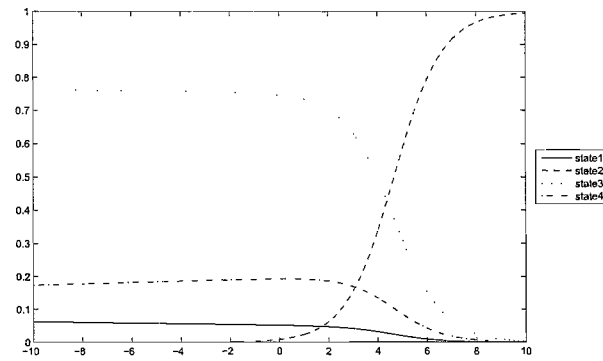
Fig. IV.5: Changes in probability of a given growth regime, South Africa, estimates from full sample (control variable = ICRG), state 1 = stable growth, state 2 = stagnation, state 3 = crisis, state 4 = miracle growth



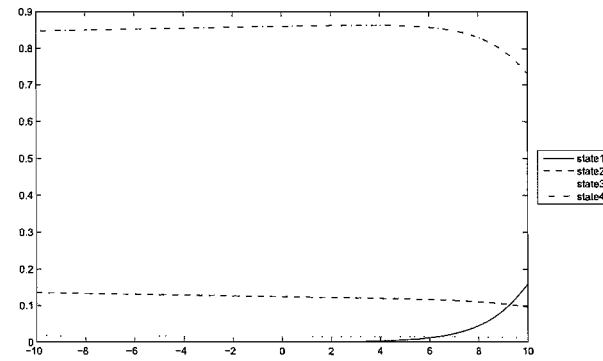
(a) Transition probabilities from state 1 (stable growth)



(b) Transition probabilities from state 2 (stagnation)

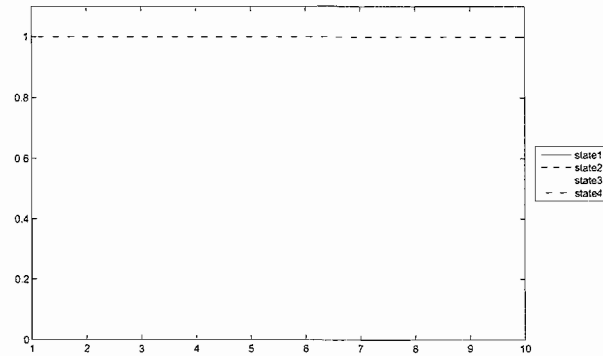


(c) Transition probabilities from state 3 (crisis)

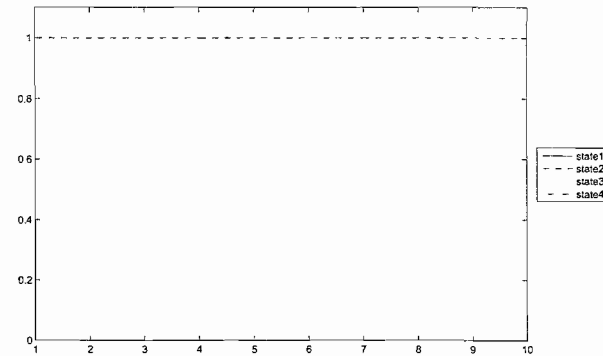


(d) Transition probabilities from state 4 (miracle growth)

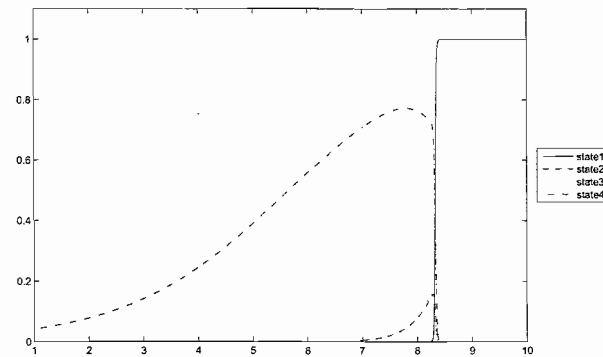
Fig. IV.6: Transition probabilities for polity sample, dependence on polity index



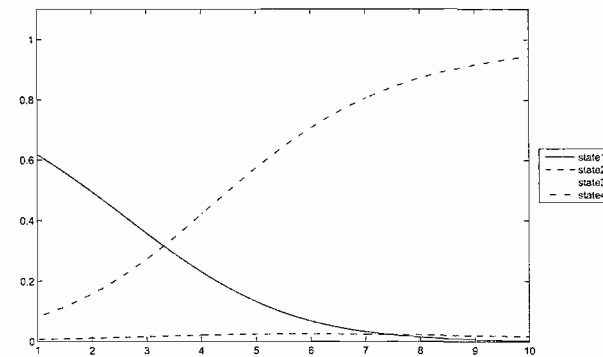
(a) Transition probabilities from state 1 (slow growth)



(b) Transition probabilities from state 2 (stable growth)

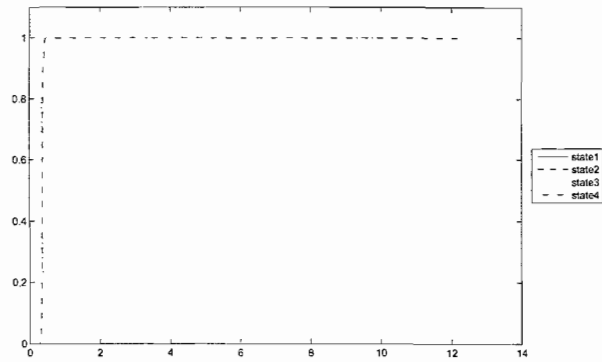


(c) Transition probabilities from state 3 (crisis)

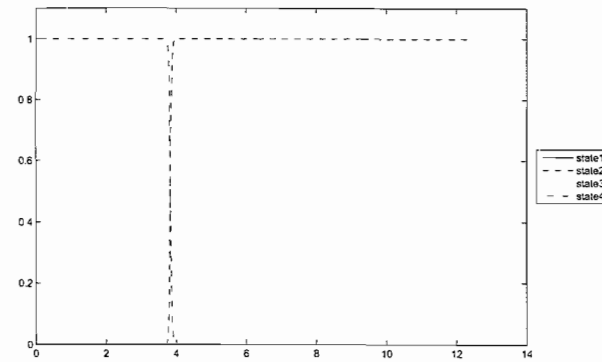


(d) Transition probabilities from state 4 (miracle growth)

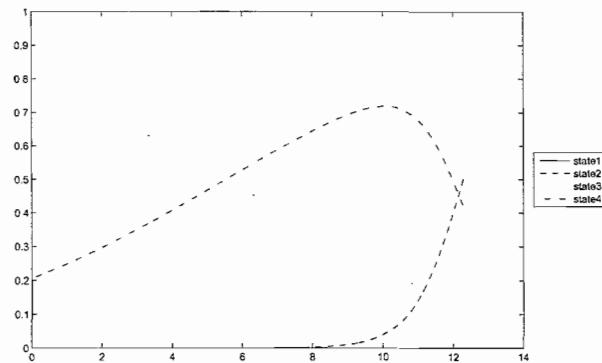
Fig. IV.7: Transition probabilities, effect of economic freedom, restricted specification, where control variables include economic freedom and schooling. Transition probability is computed for average level of schooling



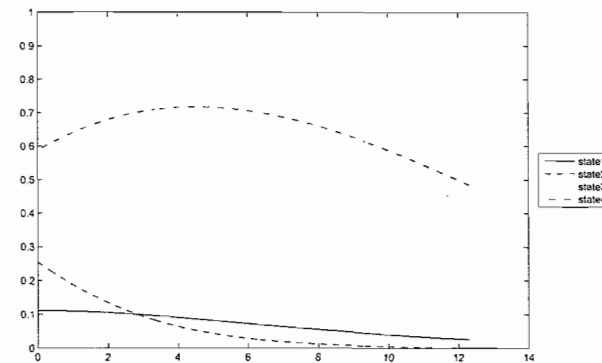
(a) Transition probabilities from state 1 (slow growth)



(b) Transition probabilities from state 2 (stable growth)

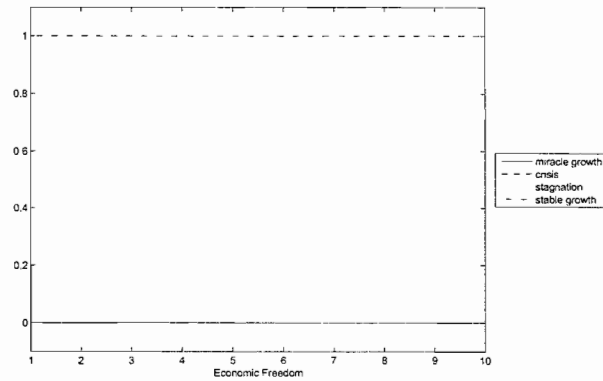


(c) Transition probabilities from state 3 (crisis)

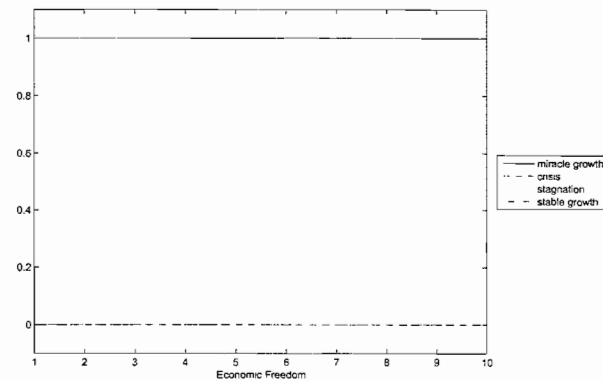


(d) Transition probabilities from state 4 (miracle growth)

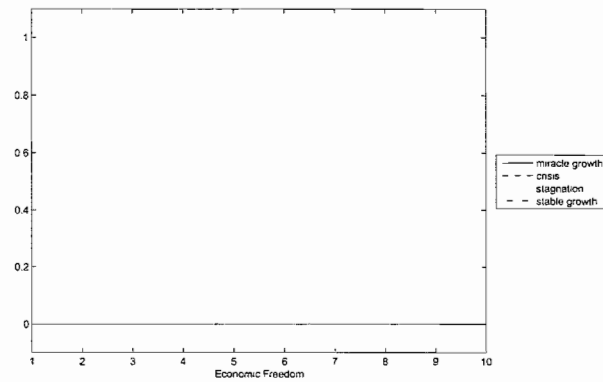
Fig. IV.8: Transition probabilities, effect of schooling, restricted specification. Probability is computed for average level of economic freedom index



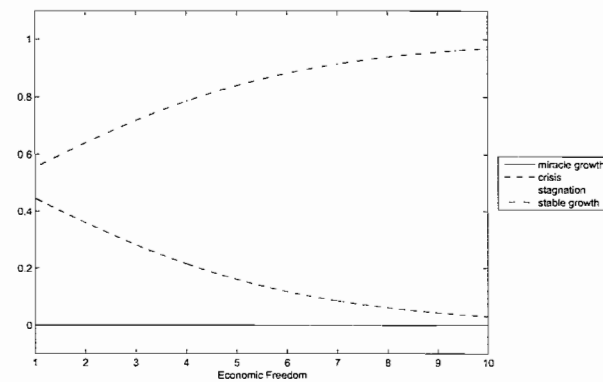
(a) Transition probabilities from state 1 (miracle growth)



(b) Transition probabilities from state 2 (crisis)

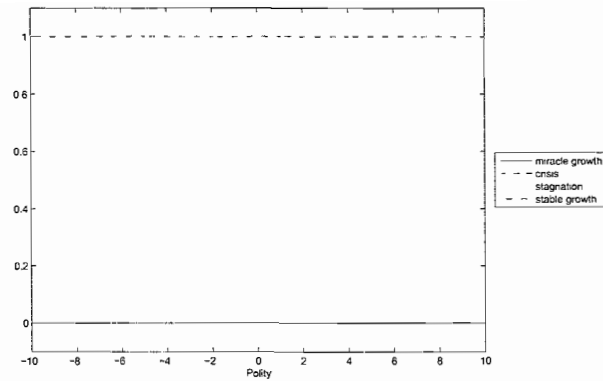


(c) Transition probabilities from state 3 (stagnation)

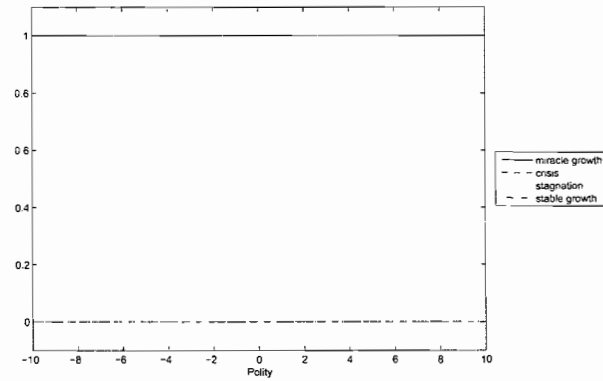


(d) Transition probabilities from state 4 (stable growth)

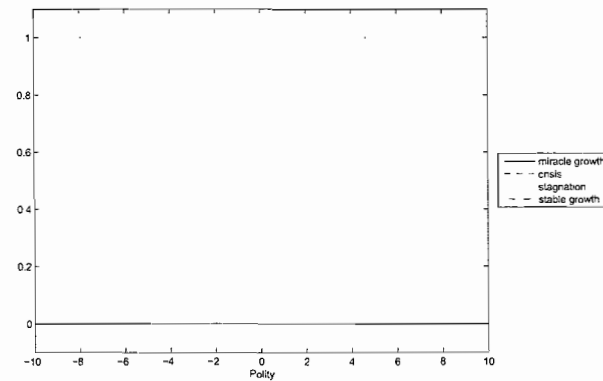
Fig. IV.9: Transition probabilities, effect of economic freedom, full specification. Probability is computed for average level of schooling, and average level of polity index



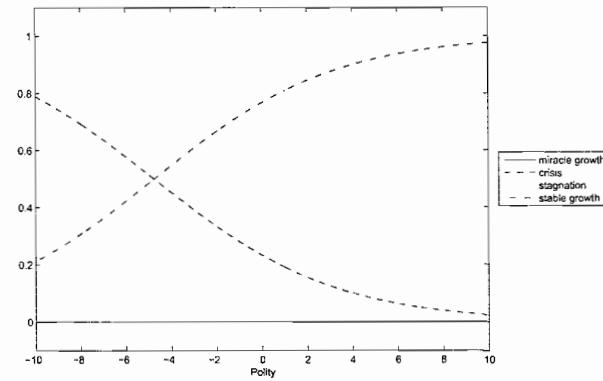
(a) Transition probabilities from state 1 (miracle growth)



(b) Transition probabilities from state 2 (crisis)

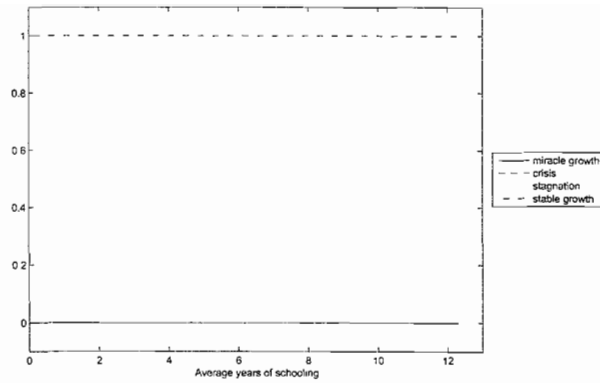


(c) Transition probabilities from state 3 (stagnation)

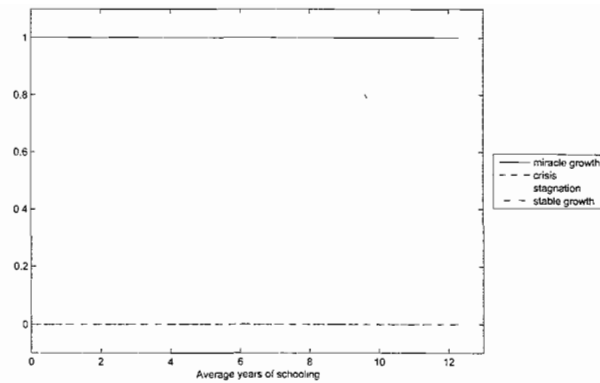


(d) Transition probabilities from state 4 (stable growth)

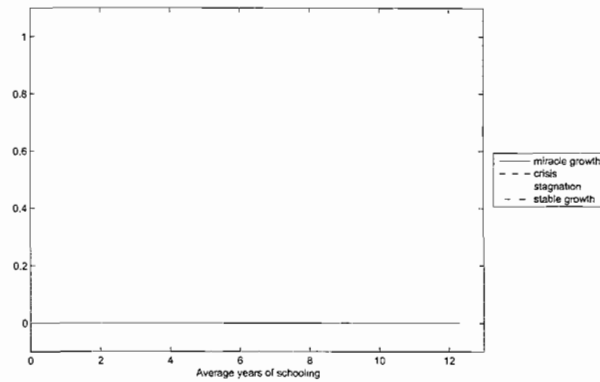
Fig. IV.10: Transition probabilities, economic freedom sample, effect of polity, for average level of schooling, and average level of economic freedom index



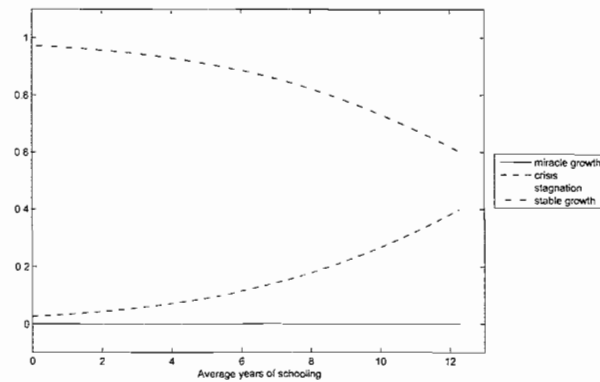
(a) Transition probabilities from state 1 (miracle growth)



(b) Transition probabilities from state 2 (crisis)



(c) Transition probabilities from state 3 (stagnation)



(d) Transition probabilities from state 4 (stable growth)

Fig. IV.11: Transition probabilities, effect of schooling, full specification, for economic freedom sample. Probability is computed at the average level of economic freedom and polity indices

CHAPTER V

CONCLUSION

This dissertation has focused on the extent to which models characterized by multiple equilibria can explain both the income gap between developed and developing countries, and the volatility in growth typically displayed in developing economies. Of particular interest were the roles that institutions and human capital may play in the formation of multiple equilibria, as well as the conditions which allow for escapes from poverty traps. These questions were treated in three essays. The first two essays broadly explored the role that economic institutions and human capital externalities play in the formation of poverty traps and in transitions out of such traps. The final essay examined the importance of time variation of institutions and human capital accumulation in determining transitions between growth regimes, using Markov regime switching regressions.

The first essay considered the role of the quality of institutions, and the ability of the government to commit to institutional reform over an infinite horizon. In marked contrast to previous models in this literature, I found that variations in the quality of institutions will not necessarily generate poverty traps. However, as one might expect, the steady state level of legal infrastructure, or protection against appropriation, is increasing in the return to appropriation, while the level of effective protection of output is decreasing. If returns to appropriation vary systematically across countries, and are monotonically increasing in the level of

development, then protection of output will increase as countries develop. In the case where the government is able to commit to reform, there is a single steady state. The poverty trap result, for a given return to appropriation, is only obtained when governments are incapable of committing to future policy, and thus may overinvest in property rights.

The second essay focused on the global dynamics of models with multiple equilibria. Under perfect foresight, in a model of economic growth with human capital externalities, there is no possible transition path between the poverty trap and the upper steady state. Furthermore, simply allowing for rational expectations and stochastic shocks to aggregate productivity may not be sufficient to force countries out of the poverty trap, as such productivity shocks may generate further investment in physical rather than human capital. However, either expectational shocks or adaptive learning, or a combination thereof, may increase perceived returns to human capital sufficiently to allow the economy to escape poverty. This essay, in addition to illustrating significant differences between dynamics under learning and under perfect foresight, also points to significant differences between econometric learning and social learning which merit further exploration.

Finally, the third essay analyzed the role of time variation in institutions and human capital accumulation on transitions between economic growth regimes, and thus in explaining differences in patterns of economic growth across countries. In contrast to the results of most cross-country growth regressions, I find that measures of both political institutions and human capital accumulation have a significant effect on transitions between growth regimes. In particular, countries with the highest quality of both political and economic institutions are most likely to remain in a stable growth regime. The improvement in fit when using dynamic measures of institutions as opposed to the more typical static measures, suggests

that both economic and political institutions may not be as stable, particularly in developing countries, as previously thought.

The third essay in fact provides to some extent an empirical test of the first two essays. The first essay proposed that there may exist a continuum of steady states, even under commitment, indexed by the return to appropriation. Countries with the lowest return to appropriation have higher incomes, due both to spending less resources on law enforcement and to have higher effective protection of output. Furthermore, if countries with high institutional quality also are more likely to commit to reform programs, then such countries are more likely to escape the poverty trap and move toward stable growth. These results are captured at least in part in the empirical results of the third essay. Similarly, the second essay indicates that a dramatic increase in human capital will frequently be associated with a transition out of the poverty trap, but that the simultaneous growth in human capital and output will not necessarily continue in the future. Once again, the results of the third essay do indicate a significant role of human capital accumulation in growth transitions. Both the empirical and theoretical results of this dissertation indicate the importance of not ignoring the role that nonlinear dynamics may play in helping us to explain differences in growth patterns across countries. Furthermore, I anticipate that the theoretical work in this dissertation may form the foundation for further research on the role of expectations in the process of economic growth, and on the effect of a lack of commitment to reform on economic outcomes.

REFERENCES

- ACEMOGLU, D., AND J. A. ROBINSON (2007): "Persistence of Power, Elites and Institutions," *NBER Working Paper Series*, (12108).
- ACEMOGLU, D., AND F. ZILIBOTTI (1997): "Was Prometheus Unbound by Chance? Risk, Diversification, and Growth," *Journal of Political Economy*, 105(4), 709 – 751.
- AGUIAR, M., AND G. GOPINATH (2007): "Emerging Market Business Cycles: The Cycle is the Trend," *Journal of Political Economy*, 115(1), 69–102.
- ARIFOVIC, J., J. BULLARD, AND J. DUFFY (1997): "The Transition from Stagnation to Growth: An Adaptive Learning Approach," *Journal of Economic Growth*, 2, 185–209.
- ARIFOVIC, J., J. BULLARD, AND O. KOSTYSHYNA (2007): "Social Learning and Monetary Policy Rules," *Federal Reserve Bank of St. Louis Working Paper Series*, 2007-007.
- AZARIADIS, C., AND A. DRAZEN (1990): "Threshold Externalities in Economic Development," *The Quarterly Journal of Economics*, 105(2), 501–526.
- AZARIADIS, C., AND J. STACHURSKI (2003): "A Forward Projection of the Cross-Country Income Distribution," *Institute of Economic Research, Kyoto University, Discussion paper series*, 570.
- BAZDRESCH, S., AND A. WERNER (2005): "Regime switching models for the Mexican Peso," *Journal of International Economics*, 65(1), 185–201.
- BENHABIB, J., AND J. GALI (1995): "On Growth and Indeterminacy: Some Theory and Evidence," *Carnegie-Rochester Conference Series on Public Policy*, 43, 163–211.
- BENIGNO, P., AND M. WOODFORD (2006): "Optimal taxation in an RBC model: A linear-quadratic approach," *Journal of Economic Dynamics and Control*, 30, 1445–1489.
- CETORELLI, N. (2002): "Could Prometheus be bound again? A contribution to the convergence controversy," *Journal of Economic Dynamics and Control*, 27, 29–50.

- CHARI, V., L. J. CHRISTIANO, AND P. J. KEHOE (1994): "Optimal Fiscal Policy in a Business Cycle Model," *The Journal of Political Economy*, 102(4), 617–652.
- COHEN, D., AND M. SOTO (2007): "Growth and human capital: good data, good results," *Journal of Economic Growth*, 12, 51–76.
- DEBORTOLI, D., AND R. C. NUNES (2007): "Loose Commitment," *FRB International Finance Discussion Paper*, 916.
- DEMSETZ, H. (1967): "Toward a Theory of Property Rights," *American Economic Review*, (2), 347–359.
- DIEBOLD, F. X., J.-H. LEE, AND G. C. WEINBACH (1994): *Regime switching with time-varying transition probabilities* chap. 10, pp. 283–302. Oxford University Press.
- DURLAUF, S. N., AND P. JOHNSON (1995): "Multiple Regimes and Cross-Country Growth Behavior," *Journal of Applied Econometrics*, 10(4), 365–84.
- EVANS, G. W., AND S. HONKAPOHJA (2001): *Learning and Expectations in Macroeconomics*. Princeton University Press, Princeton and Oxford.
- EVANS, G. W., S. HONKAPOHJA, AND P. ROMER (1998): "Growth Cycles," *American Economic Review*, 88(3), 495–515.
- GALOR, O., AND D. TSIDDON (1997): "The Distribution of Human Capital and Economic Growth," *Journal of Economic Growth*, 2(1), 93–124.
- GLAESER, E. L., R. LA PORTA, F. LOPEZ-DE SILANES, AND A. SHLEIFER (2004): "Do Institutions Cause Growth?," *Journal of Economic Growth*, 9, 271–303.
- GOODFRIEND, M., AND J. MCDERMOTT (1995): "Early Development," *American Economic Review*, 85(1), 116–133.
- GRADSTEIN, M. (2004): "Governance and growth," *Journal of Development Economics*, 73, 505–518.
- (2007): "Institutional Traps and Economic Growth," *CEPR Discussion Papers*, 6414.
- GROSSMAN, H., AND M. KIM (1995): "Swords or Plowshares? A Theory of the Security of Claims to Property," *Journal of Political Economy*, 103, 1275–1288.
- HALL, R. E., AND C. I. JONES (1999): "Why Do Some Countries Produce So Much More Output Per Worker Than Others?," *Quarterly Journal of Economics*, 114(1), 83–116.

- HAMILTON, J. (1989): "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica*, 57, 357–384.
- HAUSMANN, R., L. PRITCHETT, AND D. RODRIK (2005): "Growth Acceleration," *Journal of Economic Growth*, 10, 303–329.
- HAUSMANN, R., F. RODRIGUEZ, AND R. WAGNER (2006): "Growth Collapses," *Center for International Development, Harvard University, Working Paper*, (136).
- HOFF, K., AND J. E. STIGLITZ (2002): "After the Big Bang? Obstacles to the Emergence of the Rule of Law in Post-Communist Societies," *World Bank Policy Research Working Paper Series*, 2934.
- JERZMANOWSKI, M. (2006): "Empirics of hills, plateaus, mountains and plains: A Markov-switching approach to growth," *Journal of Development Economics*, 81, 357–385.
- KIM, C. J., AND C. R. NELSON (1999): *State-Space Models with Regime Switching*. MIT Press, Cambridge, Massachusetts.
- KLEIN, P., P. KRUSELL, AND J.-V. RIOS-RULL (2004): "Time Consistent Public Expenditures," *CEPR Discussion Papers*, 4582.
- KLEIN, P., AND J.-V. RIOS-RULL (2003): "Time Consistent Optimal Fiscal Policy," *International Economic Review*, 44(4), 1217–1245.
- KYDLAND, F. E., AND E. C. PRESCOTT (1977): "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 85, 473–491.
- LIPSET, S. M. (1960): *Political Man: The Social Bases of Politics*. Anchor Books, Garden City, NJ.
- MANSANJALA, W. H., AND C. PAPAGEORGIOU (2004): "The Solow Model with CES Technology: Nonlinearities and Parameter Heterogeneity," *Journal of Applied Econometrics*, 19, 171–201.
- MONTIEL, P., AND L. SERVEN (2006): "Macroeconomic Stability in Developing Countries: How Much is Enough?," *The World Bank Research Observer*, 21(2), 151–178.
- NORTH, D. (1990): *Institutions, Institutional Change and Economic Performance*. Cambridge University Press, Cambridge, UK.
- OGUS, A. (2004): "The Importance of Legal Infrastructure for Regulation and Deregulation in Developing Countries," *Centre on Regulation and Competition*, 65.

- ORTIGUEIRA, S., AND J. PEREIRA (2007): "Markov-Perfect Optimal Fiscal Policy: The Case of Unbalanced Budgets," Working Paper.
- PRITCHETT, L. (2000): "Understanding Patterns of Economic Growth: Searching for Hills among Plateaus, Mountains, and Plains," *World Bank Economic Review*, 14(2), 221–50.
- RIECHMANN, T. (1999): "Learning and behavioral stability: An economic interpretation of genetic algorithms," *Journal of Evolutionary Economics*, 9, 225–242.
- SACHS, J. D. (2001): "Tropical Underdevelopment," *NBER Working Paper Series*.
- (2003): "Institutions Don't Rule: Direct Effects of Geography on Per Capita Income," *NBER Working Paper Series*.
- TORNELL, A. (1997): "Economic Growth and Decline with Endogenous Property Rights," *Journal of Economic Growth*, 2, 219–250.
- WOODFORD, M. (1999): "Commentary : how should monetary policy be conducted in an era of price stability?," *Proceedings*, pp. 277–316.
- ZAK, P. J. (2002): "Institutions, Property Rights, and Growth," *Louvain Economic Review*, 68(1-2), 55–73.