

ESTIMATING PROBABILITY RISK PREFERENCES: A
LORENZ CURVE BASED PROBABILITY WEIGHTING
FUNCTION APPROACH

by

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A THESIS

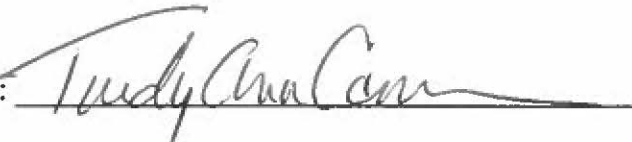
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Weighting Function Approach

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The standard parameterizations of the probability weighting function confound the estimation of its fixed point and its shape as well as control its curvature with a single parameter. We derive a three-parameter probability weighting function based on Lorenz curves. This parameterization allows for independent estimation of the fixed point and for separate curvature estimates of the “bulge” and the “sag.” We then test our probability weighting function in an experimental setting and analyze which factors influence individuals’ probabilistic risk attitudes. The probability weighting function of our sample, in aggregate, follows the dominant empirical pattern of an inverse-S shape. As an individual’s numeracy increases though, the curvature of her probability weighting function decreases. The fixed point differs with gender, with whether an individual is liquidity constrained, and with numeracy. Our sample of individuals does not appear to display more sensitivity to probability changes within the region of the bulge relative to probability changes within the region of the sag. Therefore, a single curvature parameter appears to be sufficient to characterize a heterogeneous probability weighting function in this choice context.

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I. Introduction

Probability Estimation versus Probability Weighting

Individuals tend to overestimate vastly the probability of rare events such as nuclear accidents, plane crashes, or terrorist attacks and to underestimate the probability of common events such as heart disease or car accidents [Armantier (2006); Hakes and Viscusi (2004); Lichtenstein et al. (1978); and Viscusi, Hakes, and Carlin (1997)]. These types of distortions of objective probabilities into probability estimates occur because we perceive events as more or less likely to occur than they actually are. When the probabilities of possible consequences are unknown, decisions must be made under uncertainty.

Distortions can occur even when a decision maker has full awareness of the objective probability distribution. For example, people purchase lottery tickets even though most know that the chances of winning are extremely low [Burns, Chui, and Wu (2010)]. Without a notion of distorting probabilities into decision weights, it is difficult to reconcile the facts that people buy lottery tickets and insurance. Decision weights measure the attention or importance given to outcomes, and not merely the perceived likelihood that the outcomes will occur. When the probabilities of possible consequences are actually known, decisions must be made under risk. Decision-making under uncertainty contains the subset of decision-making under risk. Given an uncertain decision, an agent can distort the unknown probabilities into probability estimates and then distort those probability estimates

into decision weights. We focus on probability weighting of decisions under risk and not probability estimation of decisions under uncertainty. This example from Richard Zeckhauser, reported in Kahneman and Tversky (1979), illustrates individuals' nonlinear weighting of probabilities:

Suppose you are compelled to play Russian roulette, but are given the opportunity to purchase the removal of one bullet from the loaded gun. Would you pay as much to reduce the number of bullets from four to three as you would to reduce the number of bullets from one to zero?

In general, people would pay more to reduce the number of bullets from one to zero, ensuring certain survival, than from four to three. The psychological impact of a change in probability from $1/6$ to 0 is greater than a change from $4/6$ to $3/6$. Conversely, most people would pay more to reduce the number of bullets from six to five, avoiding certain death, than from four to three. Whereas human instinct dictates that the reduction of bullets from one to zero has greater value than the reduction of bullets from four to three, economic theory dictates the opposite conclusion, where the value of money is reduced by the considerable probability that one will not live to enjoy it (i.e., the marginal utility of assets is greater in life than in death). A rational agent would weight each outcome by equal probability but would be willing to pay more for life-saving measures as the probability of her death increases. Tversky and Kahneman (1992) established a psychological hypothesis for this distortion: individuals are sensitive to a reference point and become less sensitive to changes in probability as they move away from that reference point. The endpoints of the probability domain, 1 and 0 , act as reference points because they represent the certainty that a particular event will or will not happen.

The Probability Weighting Function

The psychological literature has considered the *probability weighting function*, a model of probability distortion, since the studies of Preston and Baratta (1948) and Edwards (1954). Renewed interest in the function emerged from the economics literature with the work of Handa (1977) and Kahneman and Tversky (1979). The probability weighting function $w : [0, 1] \rightarrow [0, 1]$ takes the objective probability p as its argument and distorts it into a transformed probability $w(p)$, which better describes the psychological consequences of risky events. The function increases on $[0, 1]$ and restricts the endpoints to $w(0) = 0$ and $w(1) = 1$. If, for a given agent, $w(p) = p$ for the entire probability domain, then that agent assigns weights to all events equal to their corresponding objective probabilities. This case of linear probability weighting represents a baseline case against which all probability distortions will be compared. Therefore, the identity line accompanies each plot of the weighting function.

From experimental testing on the properties of the weighting function, the dominant result is that $w(\cdot)$ has a “regression” relative to the identity line, a fixed point near $p = 1/3$, and an inverse-S shape [Abdellaoui (2000); Abdellaoui, Bleichrodt, and L’Haridon (2007); Bleichrodt and Pinto (2000); Camerer and Ho (1994); Fox and Tversky (1995); Gonzalez and Wu (1999); Kahneman and Tversky (1979); Tversky and Kahneman (1992); and Wu and Gonzalez (1996, 1998)]. A regression of $w(\cdot)$ relative to the identity line means that people tend to overweight small probabilities, $w(p) > p$, and underweight the rest of the probability domain, $w(p) < p$. The point at which the weighting function

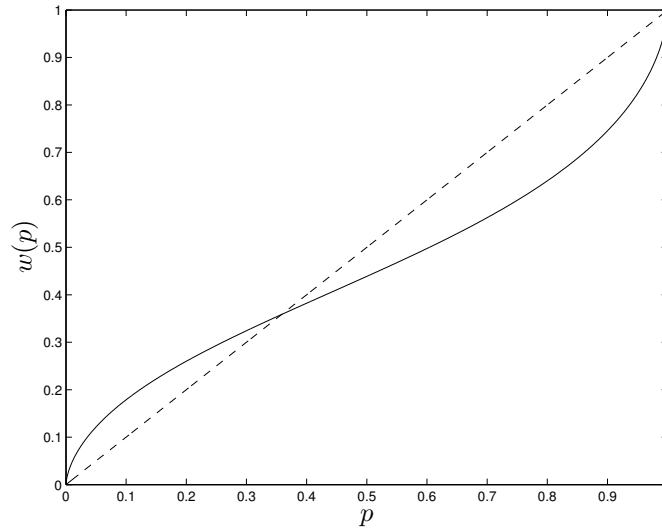


Figure 1: The Tversky and Kahneman (1992) Probability Weighting Function

The graph plots $w(\cdot)$ with $\gamma = 0.65$. In the background lies the identity function $w(p) = p$, which is a 45° line.

transitions from overweighting to underweighting, $w(p) = p$, is called the fixed point. A fixed point of $p = 1/3$ reflects the observation that the region of overweighting covers only one-third of the probability domain, whereas the region of underweighting covers the remaining two-thirds. An inverse-S shape for the probability weighting function means that $w(\cdot)$ begins as concave and then transitions into convexity. The change in curvature reflects people's diminishing sensitivity to probability changes as they move away from either endpoint [Gonzalez and Wu (1999)]. An inverse-S-shaped weighting function could lie completely above or below the identity line, and a regressive weighting function need not start as concave and end as convex; hence, an inverse-S shape and regression relative to the identity line are logically independent properties. However, the region of overweighting (underweighting) usually does correspond to the region of concavity (convexity). Under this condition, the fixed point of the probability weighting function also acts as its inflection

point.

Over the course of numerous attempts to characterize the probability weighting function, researchers have offered several simple parameterizations of its functional form. All of the common parameterizations of $w(\cdot)$ capture its three main qualitative properties. The standard inverse-S-shaped probability weighting function, introduced by Tversky and Kahneman (1992), is a function governed by a single parameter, γ , which controls both the curvature and the elevation of the function:

$$w_{TK}(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{1/\gamma}}; \text{ if } 0.3 \leq \gamma \leq 1 \quad (1)$$

Although this specification is common, a normative case for adopting it has not been made [Cavagnaro et al. (2013)]. The LinLog two-parameter specification resembles Tversky and Kahneman's one-parameter version. The LinLog specification is based on the assumption of a linear relationship between the log of the weighting odds and the log of probability odds:

$$\ln\left(\frac{w(p)}{1-w(p)}\right) = \gamma \ln\left(\frac{p}{1-p}\right) + \ln(\delta)$$

Some rearrangement yields,

$$w(p)_{LL} = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}; \text{ if } 0 < \gamma, \delta \quad (2)$$

Goldstein and Einhorn (1987) originally used this functional form, but not explicitly as a probability weighting function. Gonzalez and Wu (1999) argued for its appeal because it captures two properties to be discussed below: attractiveness and discriminability. In

contrast, Prelec (1998) derives his two-parameter probability weighting function based on the three axioms of compound invariance, subproportionality, and diagonal concavity:

$$w(p)_{PII} = \exp(-\delta(-\ln p)^\gamma); \text{ if } 0 < \gamma \leq 1, 0 < \delta \quad (3)$$

The one-parameter Prelec function is obtained by setting δ to unity:

$$w(p)_{PI} = \exp(-(-\ln p)^\gamma); \text{ if } 0 < \gamma \leq 1 \quad (4)$$

Each of these specifications of the probability weighting function nests the most simple case of linear weighting, $w(p) = p$, when $\gamma, \delta = 1$.

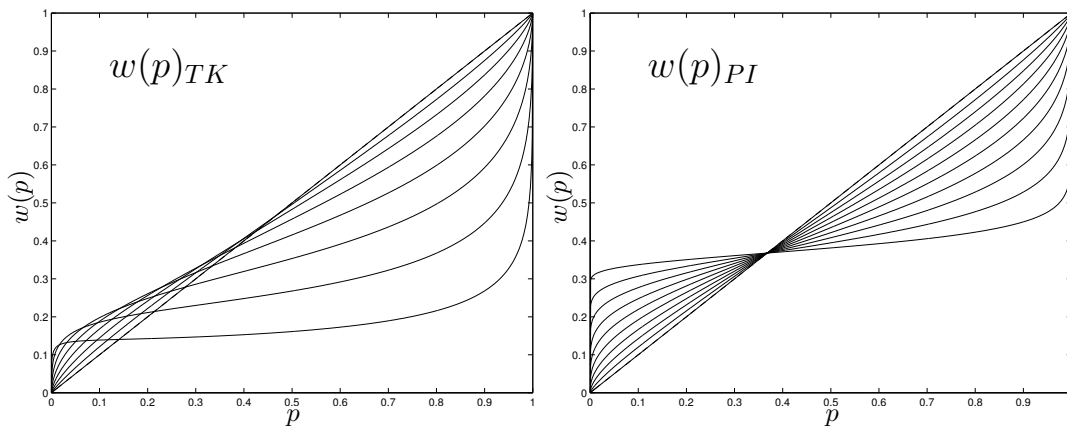


Figure 2: Commonly Used One-Parameter Probability Weighting Functions

Two families of functions have been commonly used as probability weighting functions. The two functions are plotted for a range of parameter values. Left: the Tversky and Kahneman (1992) function for $\gamma = 0.2$ to 1, in increments of 0.1. Right: the Prelec-1 function for $\gamma = 0.1$ to 1, in increments of 0.1.

Decision-Making Under Risk

So far we have distinguished between two types of probability distortions: probability estimation and probability weighting. We have also reviewed the common ways in

which researchers model probability weighting. Probability distortion plays an important role in several strands of research, including visual frequency estimation, frequency estimation based on memory, confidence rating, signal detection theory, decision-making under risk, and decision-making under uncertainty.¹ Our study of probability distortions will take place in the context of decision-making under risk.

Decisions under risk involve evaluating one or more gambles with known outcomes and probabilities. A simple gamble $\mathbf{g} = (y_1, p_1; y_2, p_2; \dots; y_N, p_N)$ is a probability distribution, where y_n is the monetary or non-monetary payoff from the n^{th} outcome with probability p_n of occurring. The first theoretical attempt to evaluate these gambles was formulated by Daniel Bernoulli (1738) and then axiomatized by von Neumann and Morgenstern (1944). Their approach to decision-making under risk derives from axioms of rational behavior and is known as expected utility theory (EUT). The experiments we use in our illustration involve only two-outcome gambles with positive payoffs. Thus, we will restrict our discussion to this case. Suppose an individual can choose either gamble \mathbf{g}_A , with a 50% chance of \$200 and a 50% chance of only \$100, or gamble \mathbf{g}_B , with a 20% chance of \$800 and an 80% change of only \$10. Under EUT, the individual would choose the gamble with the larger expected utility:

$$U(\mathbf{g}_A) = .5 \cdot u(W + \$200) + .5 \cdot u(W + \$100) \text{ and}$$

$$U(\mathbf{g}_B) = .2 \cdot u(W + \$800) + .8 \cdot u(W + \$10),$$

where W is current wealth and $u(\cdot)$ is an increasing utility function exhibiting either

¹See Zhang and Maloney (2012) for a review of the applications of probability weighting functions.

concavity or convexity. Under EUT, individuals weight outcomes by their corresponding objective probabilities, implying a probability weighting function of $w(p) = p$ for all $p \in [0, 1]$.

In contrast to EUT (and its characterization of how a rational agent should gamble), prospect theory attempts to describe how an agent actually gambles. Tversky and Kahneman's (1992) cumulative prospect theory (CPT) introduces some important deviations from expected utility theory. Prospect theory is still regarded as the best description of how people evaluate risk in experimental settings [Barberis (2013)]. Research over the last decade has attempted to apply prospect theory to various economic settings. In fields where attitudes toward risk play a central role, such as in finance and insurance, CPT has clear applications. For example, can we explain the cross section of average returns using a model in which investors evaluate risk in a psychologically more realistic way? Behavioral finance has been a highly fruitful line of inquiry in the recent literature. Prospect theory has been applied to answer questions related to the endowment effect, industrial organization, labor supply, and consumption-savings decisions. Kahneman and Tversky's (1979) original formulation of prospect theory evaluates the gambles in our example as

$$V(\mathbf{g}_A) = w(.5) \cdot v(\$200) + w(.5) \cdot v(\$100)$$

$$V(\mathbf{g}_B) = w(.2) \cdot v(\$800) + w(.8) \cdot v(\$10),$$

where $v(\cdot)$, termed the value function, is increasing and $v(0) = 0$. The value function describes how an individual values any possible payoff. This original prospect theory formulation poses two problems: it may violate stochastic dominance and it implies that the

decision weight depends only on the probability of the outcome and not on the outcome itself.

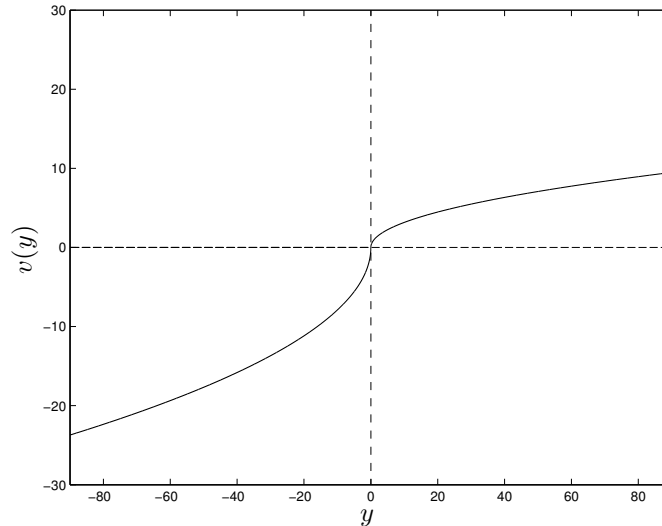


Figure 3: The Tversky and Kahneman (1992) Value Function

A plot of the value function proposed by Tversky and Kahneman (1992), where $v(y) = y^\kappa$ when $y < 0$ and $v(y) = -\lambda(-y)^\kappa$ when $y \geq 0$ and y is the dollar gain or loss. Here $\kappa = 0.5$ and $\lambda = 2.5$.

To resolve the theoretical limitations of original prospect theory, a decision-making theory must incorporate rank dependence. CPT is the most popular of the so-called rank dependent models, which acknowledge that the attention given to an outcome depends not only on the probability of the outcome but also the favorability of the outcome in comparison to the other possible outcomes in the gamble [Diecidue and Wakker (2001)]. Rank dependent models order outcomes in the gamble $g = (y_1, p_1; y_2, p_2; \dots; y_N, p_N)$ from best to worst: $y_1 > y_2 > \dots > y_n$. Suppose we own a gamble $g_C = (\$80, 1/5; \$60, 1/5; \$40, 1/5; \$20, 1/5; \$0, 1/5)$. It would be good news to receive information that the gamble will have a payoff more than \$20. The probability of receiving more than \$20, $3/5$, is the good-news probability or rank of the \$20 outcome. The rank of a specific outcome y_i ,

$r_i = p_1 + \dots + p_{i-1}$, describes the probability of the gamble yielding an outcome ranked better than y_i . The ranks of outcomes \$80, \$60, \$40, \$20, and \$0 are 0, 1/5, 2/5, 3/5, and 4/5, respectively—the lower the rank, the better the outcome. Rank dependent models apply the probability weighting function to the cumulative probability (i.e., the rank) rather than to the outcome probability. The decision weight π_i of outcome y_i is the difference between the weighted rank of the next best alternative outcome, y_{i+1} , and the weighted rank of the outcome, y_i :

$$\pi_i = w(p_1 + \dots + p_i) - w(p_1 + \dots + p_{i-1}) = w(r_{i+1}) - w(r_i) \quad (5)$$

In particular, $\pi_1 = w(p_1)$. The decision weight is the marginal contribution, measured in $w(\cdot)$ units, of the outcome probability to the rank. The decision weight π_i incorporates both the outcome probability, p_i , and the outcome’s relative favorability within the gamble.

Whereas the probability weighting function describes how people weight cumulative probabilities in their minds, the *decision weight* describes how people weight outcomes in their decision-making under risk. The divergence between the two concepts stems from the fact that people weight each outcome not just by its weighted cumulative probability, but also by how its weighted cumulative probability compares to the weighted cumulative probability of the next best outcome. The probability weighting function $w(\cdot)$ is a “good-news” weighting function because it transforms the ranks.² Under CPT, people evaluate

²See Wakker (2008) for a review of rank dependence.

gambles as

$$V(\mathbf{g}) = \sum_{i=1}^n \pi_i \cdot v(y_i) \tag{6}$$

Returning to our original example, under CPT, the individual would choose between gambles \mathbf{g}_A and \mathbf{g}_B according to the largest expected prospect value of $V(\mathbf{g}_A)$ and $V(\mathbf{g}_B)$:

$$V(\mathbf{g}_A) = w(.5) \cdot v(\$200) + [1 - w(.5)] \cdot v(\$100)$$

$$V(\mathbf{g}_B) = w(.2) \cdot v(\$800) + [1 - w(.2)] \cdot v(\$10)$$

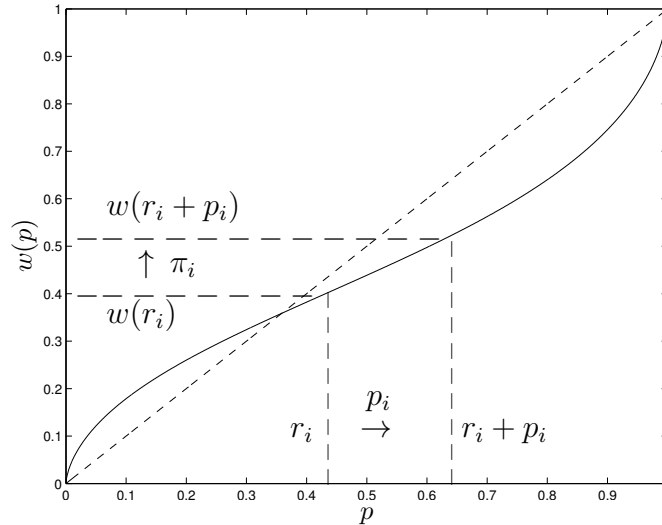


Figure 4: Marginal Weight Contribution of Outcome Probability p_i to Rank r_i

Here we see the interaction between the weighting function $w(\cdot)$ and the decision weight π under rank dependence. The rank for outcome y_i is $r_i = p_1 + \dots + p_{i-1}$.

Prospect theory captures four generalizations of EUT: reference dependence, loss aversion, diminishing sensitivity, and probability weighting. Both the utility function and the value function increase in the preference an individual has for an outcome. The curvature of each function represents an individual's risk preferences over that domain of

outcomes. Reference dependence suggests that people make decisions based on *changes* in attributes, as opposed to their absolute magnitudes. In CPT people derive prospect value from gains and losses, measured relative to some reference point, typically the status quo, rather than from absolute levels of wealth. This implies that the argument of $v(\cdot)$ is y rather than $W + y$. Loss aversion means that people are more sensitive to losses than to gains of the same magnitude. The value function generates this property by being steeper in the region of losses than in the region of gains. People tend to be risk averse with respect to gains and risk loving with respect to losses. Diminishing sensitivity to changes in wealth suggests that the value function is concave in the region of gains and convex in the region of losses. In contrast, any utility function imposes the same curvature for all wealth levels, so that risk preferences cannot switch between risk aversion and risk affinity. Unlike EUT, CPT allows for a nonlinear probability weighting function and this function can differ between losses and gains.³ Our experiment contains only gains, however, so our modeling does not need to incorporate loss aversion, diminishing sensitivity to losses, or a loss-specific probability weighting function.

Further psychological interpretations of the probability weighting function derive from the CPT framework. One person's weighting function $w_1(\cdot)$ is said to be more elevated than another's $w_2(\cdot)$ if $w_1(p) \geq w_2(p)$ for all $p \in [0, 1]$ with at least one point of strict inequality. In the context of CPT with two-outcome gambles that pay only gains, a weighting function that is more elevated will assign greater weight to the more favorable outcome. Gonzalez and Wu (1999) interpret an interpersonal difference in elevation between weight-

³See Barberis (2013) for a review of prospect theory.

ing functions as relative attractiveness. One person finds betting on the chance domain more attractive than a second person. Elevation can differ between chance domains as well as across individuals. An individual may prefer to bet on stock prices rather than on the outcomes of baseball games, for example, holding constant the chance of winning. Gonzalez and Wu (1999) link the curvature of the weighting function, implied by diminishing sensitivity, to the psychological concept of discriminability. Recall that individuals exhibit sensitivity to the probability changes near the two endpoints of the probability domain (i.e., 0 and 1), and this sensitivity to probability changes diminishes as they move away from the endpoints. A person with a EUT weighting function of $w(p) = p$ for all $p \in [0, 1]$ exhibits a perfect ability to discriminate a 1% probability change. Conversely, a person with a near horizontal weighting function and extreme steepness near the two endpoints exhibits low ability to discriminate a 1% probability change.

Although Gonzalez and Wu's (1999) psychological interpretation characterizes the weighting function as a whole, it does not adequately interpret the psychological difference between the concave overweighted region (i.e., the "bulge") and the convex underweighted region (i.e., the "sag"). Wakker (1994, 2008) and Diecidue and Wakker (2001) prove that convexity in a weighting function reflects probabilistic risk aversion or pessimism. Likewise, they prove that concavity in a weighting function reflects probabilistic risk affinity or optimism. The decision weight for an outcome, which depends on the weighting function's differences, will be affected by the slope rather than absolute level of $w(\cdot)$. Hence, a change in the slope of the decision weight implies a change in the curvature (i.e., the slope of decision weight's slopes) of the weighting function. Pessimism is present if worsening

the rank r of outcome x increases its decision weight (i.e., the worst outcomes receive relatively more decision weight than less bad outcomes). Specifically, $\pi(p) = w(r+p) - w(r)$ is decreasing in r if and only if $w(\cdot)$ is convex. Optimism is present if improving the rank r increases the decision weight (i.e., the best outcomes receive relatively more decision weight than less good outcomes). Analogously, $\pi(p) = w(r+p) - w(r)$ is increasing in r if and only if $w(\cdot)$ is concave.

Recall that the weighting function transforms the cumulative probability (i.e., rank) of each outcome. A large cumulative probability for outcome x_i , where its rank $r_i = p_1 + \dots + p_{i-1}$, corresponds with an unfavorable outcome relative to other outcomes. There is a large probability that the gamble will yield a better outcome than x_i since r_i is large, which means x_i has a relatively bad rank compared to other outcomes. A small cumulative probability for outcome x_i corresponds with a favorable outcome relative to other outcomes. There is a small probability that the gamble will yield a better outcome than x_i since r_i is small, which means x_i has a relatively good rank compared to other outcomes. Therefore, as the rank of an outcome worsens (i.e., the cumulative probability increases), its decision weight is determined by a region of the weighting function that is farther away from $w(0) = 0$ and closer to $w(1) = 1$.

Concavity of $w(\cdot)$, or optimism, for small cumulative probabilities implies that the most favorable outcomes are overweighted. Within the favorable outcome domain of small cumulative probabilities, shifting to better-ranked outcomes, according to optimism, increases the decision weight. Convexity of $w(\cdot)$, or pessimism, for large cumulative probabilities implies that the least favorable outcomes are also overweighted. Within the unfavor-

able outcome domain of large cumulative probabilities, shifting to worse-ranked outcomes, according to pessimism, increases the decision weight. Little attention or weight is given to intermediate outcomes [Quiggin (1982) and Weber (1994)]. Choices in our framework are discrete and not continuous, so the absolute level of the weighting function does impact probabilistic risk preferences. Overweighting (underweighting) intensifies the degree of optimism (pessimism). The inflection point of the weighting function separates the probability domain into a region of probabilistic risk affinity and a region of probabilistic risk aversion.

The common parameterizations of the probability weighting function do not permit independent estimation of the fixed point. An elevated weighting function indicates a higher fixed point. A higher fixed point implies an elevated weighting function holding constant the curvature parameter, under the common parameterizations. Hence, the elevation parameter controls the fixed point, but not independently from the other parameters. The Prelec-1 weighting function has its fixed point at $1/e$. The Tversky and Kahneman (1992) single-parameter weighting function necessarily confounds the estimation of the fixed point and curvature. The formula for the Prelec-2 fixed point is given by

$$p^* = \exp(-\delta^{1/(1-\gamma)}) \text{ with } 0 < \gamma \leq 1, 0 < \delta$$

Hence, the fixed point collapses to $1/e$ when $\delta = 1$. The formula for the fixed point of the LinLog weighting function is given by

$$p^* = \frac{\delta^{1/(1-\gamma)}}{1 + \delta^{1/(1-\gamma)}} \text{ with } 0 < \gamma, \delta$$

Both of the common two-parameter probability weighting functions thus have fixed points that depend on both parameters. This dependence prevents clear hypothesis testing of the location of the fixed point. We introduce a three-parameter probability weighting function that does allow for independent estimation of the fixed point; the fixed point depends on its own parameter. In fact, our alternative model allows for the case that an individual's probability weighting function may never cross the identity line within the interior of the probability domain. To the extent that the region of concavity (convexity) corresponds to the region of overweighting (underweighting), the fixed point acts as the inflection point for $w(\cdot)$. Pinning down the location of the fixed point has theoretical value for fields such as economics and psychology because this point approximates where probabilistic risk attitudes transition from risk affinity to risk aversion along the probability domain.

All of the common weighting function parameterizations, furthermore, contain only one curvature parameter. Given that the typical empirical probability weighting function with an interior fixed point has an inverse-S shape, with two distinct regions, a single curvature parameter may be overly restrictive. A natural concern with the empirical estimation of these models is whether the estimated shape may be an artifact of the functional form in one region and not fully reflective of actual probability risk preferences throughout the domain. Our alternative function estimates the bulge to the left of the fixed point independently from the sag to the right of the fixed point. That is, we allow for separate estimates of the degree of optimism and pessimism across the probability domain. Individuals may significantly overweight the most favorable outcomes more than they overweight the most unfavorable outcomes, or vice versa.

Finally, we test our three-parameter function in a rich setting that seems not to have received adequate attention in the literature. Most studies concerning decision-making under risk emphasize the utility or value function component $v(\cdot)$. The few studies that do focus on the weighting function, $w(\cdot)$ often exclude heterogeneity in preferences. That is, all subjects are constrained to have approximately the same underlying preferences and the same weighting function. However, the representative individual's probability weighting function may not adequately capture each individual's weighting function. We generalize the probability weighting function to allow for heterogeneity across various sociodemographic variables. The available observable sociodemographic variables may be proxies for unobservable factors that influence probability weighting behavior. We will test whether groups formed on the basis of a number of sociodemographic variables exhibit systematic differences in the parameter values of their estimated probability weighting functions.

II. Literature Review

Parameteric and Nonparametric Methods

Any parametric model of decision-making under risk necessarily makes at least three assumptions about functional form for the components of the model: (1) the utility or value function, $v(\cdot)$; (2) the probability weighting function, $w(\cdot)$; and (3) the combination rule (e.g., expected utility theory, original prospect theory, or cumulative prospect theory) that combines the first two components, $w(p_1) \cdot v(y_1) + [1 - w(p_1)] \cdot v(y_2)$. The challenge in identifying these forms stems from the fact that they cannot be estimated directly. Instead, they must be inferred from people's observed choice behavior. The standard technique of assuming specific functional forms for $v(\cdot)$ and $w(\cdot)$ has the limitation that there is no way to assess independently the fit of $v(\cdot)$ and $w(\cdot)$.⁴ Stott (2006) finds that the performance of any standard one- and two-parameter weighting function depends on assumptions about the other component functions in cumulative prospect theory, including the value function and choice function. When the surrounding functions within which the weighting function is embedded have a worse fit, the extra flexibility provided by the parameter in the weighting function may play a compensating role.

The methodological limitation of parametric methods, that there is no way to assess the fit of each component function separately, has driven some researchers to explore non-parametric specifications for the probability weighting function. Nonparametric methods make no particular assumption about the forms of $v(\cdot)$ and $w(\cdot)$ other than the combina-

⁴For examples of the standard technique see Camerer and Ho (1994), Hey and Orme (1994), Tversky and Fox (1995), and Tversky and Kahneman (1992).

tion rule, so that each component function can be assessed separately. Both parametric and nonparametric methods can estimate the component functions at either the aggregate level or the individual level. For an aggregate weighting function, the data are averaged across participants according to a single-agent stochastic choice model, suppressing any heterogeneity in preferences. A representative agent model can be generalized to allow for heterogeneity in preferences by varying parameters systematically with observable variables. The weighting function will still be an aggregate weighting function, but an aggregate function for each differentiated observable group rather than an aggregate of the whole sample. Alternatively, estimation of individual weighting functions allows for each individual to have a separate weighting function. Depending on whether the estimation uses parametric or nonparametric methods, individual weighting functions are reflected by either unique parameter estimates for each individual or separate point estimates for each individual. Few studies actually test the weighting function at the individual level because this type of test requires an inordinate amount of data on the choice behavior of each individual.

Studies using nonparametric methods adopt one of two main research strategies. The first strategy obtains information about the qualitative properties of the weighting function by testing simple preferences conditions. These studies test the weighting function at the aggregate level. Currim and Sarin (1989) confirm underweighting, subproportionality, and convexity of $w(\cdot)$ in the aggregate. Wu and Gonzalez (1996, 1998) find aggregate behavior consistent with an inverse-S-shaped weighting function. The second strategy consists of eliciting point estimates for each individual's probability weighting function throughout the probability interval. Wakker and Deneffe (1996) first propose a trade-off method to

elicit preference functions nonparametrically. Gonzalez and Wu (1999), Abdellaoui (2000), and Bleichrodt and Pinto (2000) all follow this second strategy. Their results demonstrate significant heterogeneity in the weighting function at the individual level that is not likely to be explained purely as noise.

Bleichrodt and Pinto (2000) confirm the robustness of nonlinear probability weighting in a medical context. Participants are assigned a hypothetical disease and they have to choose between disease treatments before it was known which disease they actually have. Outcomes of treatments are expressed in terms of expected remaining life-years. The non-parametric studies have limited sample sizes because they elicit, for each individual, many choices for each probability level and for each outcome level. The study by Gonzalez and Wu (1999) elicits $w_i(\cdot)$ at eleven probability levels but has a sample size of only ten individuals. The studies by Abdellaoui (2000) and Bleichrodt and Pinto (2000) have sample sizes of forty and forty-nine, but elicit $w_i(\cdot)$ at only five probability levels. Although these studies do confirm that preference heterogeneity exists, the samples are too homogeneous (i.e., all participants were graduate students or undergraduates in economics) and too small to characterize how risk attitudes may vary systematically with observed characteristics of individuals across the general population.

Nonparametric studies typically constrain the number of subjects in the study too severely to permit a full characterization of the distribution of risk preferences in the population. However, these studies have taught us that any parametric study concerning systematic heterogeneity in $w_i(\cdot)$ across individuals $i = 1, 2, \dots, n$ or groups $j = 1, 2, \dots, J$ must employ a fairly flexible parametric form. Little is known about the distribution of risk

preferences in the population.

The Distribution of Probability Risk Preferences

Harbaugh, Krause, and Vesterlund (2002), using the Prelec-2 weighting function, investigate how risk attitudes change with age. They find that children's choices are actually consistent with the underweighting of low-probability events and the overweighting of high-probability ones (i.e., the opposite of the pattern predicted by cumulative prospect theory). This tendency diminishes with age until the point that adults appear to use the objective probability when evaluating risk prospects (i.e., the EUT weighting function).

Fehr-Duda, de Gennaro, and Schubert (2006) use the LinLog two-parameter probability function to study the relationship between gender and risk-taking behavior. They find that women tend to underweight large probabilities of gains more strongly than men do. Women appear to be more risk-averse than men when decisions are framed in investment terms. Bruhin, Fehr-Duda, and Epper (2010) use the same probability weighting function as Fehr-Duda et al. (2006) and find two classes of individuals in their data sets. Eighty percent of the subjects exhibit significant departures from linear probability weighting, consistent with CPT, and twenty percent of the subjects weight probabilities approximately linearly, consistent with EUT. The only variable that consistently affects behavioral parameters between the groups is an indicator for gender. Within the study, women have a substantially lower value of γ (i.e., a $w(\cdot)$ that is flatter in the region of medium probabilities and steeper near the ends) than men. They conclude that women tend to be less sensitive to changes in probability than men.

The above papers have been experimental studies, but Donkers, Melenberg, and Van Soest (2001) analyze a large, representative set of Dutch survey data that contains information on respondents' risk attitudes as well as background information on each person. Their semiparametric model does not specify $v(\cdot)$ or $w(\cdot)$, but makes a minimal assumption about the choice function $P(V(\mathbf{g}_R), V(\mathbf{g}_S)|x)$, a function that indicates the probability that an individual will choose the risky gamble over the safe gamble. The choice function reflects the empirical finding that decision-making has a random component. They assume that the expected prospect values, $V(\mathbf{g}_R)$ and $V(\mathbf{g}_S)$, depend on an index, $x'\beta$, where x is a vector of observable individual characteristics. Vector β is the vector of parameter values to be estimated. They do not specify how the index influences each component function of $V(\cdot)$. Specifically, they assume a higher index value $x'\beta$ implies a larger value of $P(V(\mathbf{g}_R), V(\mathbf{g}_S)|x)$, which they argue is an indication of greater risk aversion. The semiparametric estimation results show a significant relationship between the subjects' estimated degree of risk aversion and their age, gender, education level, and income.

The generality of the semiparametric model, in that it identifies only the choice function $P(V(\mathbf{g}_R), V(\mathbf{g}_S)|x)$ rather than its component functions, prevents an interpretation of the effects of differences in risk attitudes on $v(\cdot)$ or $w(\cdot)$, more generally. Moreover, the semiparametric estimation results indicate that a single index (i.e., one systematically varying parameter) is too restrictive. Hence, Donkers et al. (2001) form a parametric model with a first index, $x'\beta_v$, to control for variation in the value function, and a second index, $x'\beta_w$, to control for variation in a Prelec-1 probability weighting function. They find that the probability weighting function varies systematically with gender, age, income, education,

wealth, and marital and employment status. The estimated index, $x'\beta$, in the semiparametric model of the Donkers et al. (2001) paper closely resembles the estimated value of the differentiated index, $x'\beta_v$, in the value function rather than the estimated index, $x'\beta_w$, in the probability weighting function. The semiparametric estimation suggests two key conclusions for any parametric study concerning heterogeneity in the weighting function: (1) parametric models of decision-making under risk should embody at least two indexes, and (2) heterogeneity in $w(\cdot)$ should be permitted to be distinct from heterogeneity in $v(\cdot)$.

Best Parametric Form for the Probability Weighting Function

Another related line of research endeavors to find the “best” parametric form for $w(\cdot)$. The trade-off pits the desire for a parsimonious form against the desire for a good fit to the data. Studies within this line of research essentially set up a “horse race” between the common probability weighting functions [Abdellaoui (2000), Bleichrodt and Pinto (2000), Cavagnaro et al. (2013), Gonzalez and Wu (1999), Stott (2006), and Zhang and Maloney (2012)]. For example, Abdellaoui (2000), Bleichrodt and Pinto (2000), and Gonzalez and Wu (1999) elicit their nonparametric point estimates of each individual’s probability weighting function and use nonlinear least squares estimation to fit the standard parametric weighting functions to these point estimates. All three of the studies also fit parametric functions to the median point estimates (i.e., the median certainty equivalent for each probability level) using nonlinear least squares. These parametric estimates on the median data characterize the aggregate probability weighting function in sense that the central tendency of the sample’s choices describes the behavior of a representative agent. Bleichrodt and

Pinto (2000) and Gonzalez and Wu (1999) find that standard two-parameter functions fit the data much better than the standard one-parameter functions at the individual participant level, but only slightly better at the aggregate level. Gonzalez and Wu (1999) favor the two-parameter LinLog function for its psychological interpretation. Stott (2006) favors the Prelec-1 function, but only when paired with a power value function $v(y) = y^\lambda$ and a Logit choice function. Cavagnaro et al. (2013) find heterogeneity in the best two-parameter form; some individuals are best described by a LinLog function while other are best described by a Prelec-2 form. Each of these studies discriminates among the standard small set of one- and two-parameter functional forms. They do not consider any three-parameter forms and cannot discriminate among the set of all possible parametric forms.

Two-parameter models of $w(\cdot)$ control elevation and curvature. The empirical evidence suggests that $w(\cdot)$ may have two separate regions: an overweighted concave curve (i.e., the “bulge”) and, after crossing the diagonal, an underweighted convex curve (i.e., the “sag”). The one- and two-parameter models combine all of the curvature information into a single parameter γ . However, it may be the case that the bulge and sag have independent magnitudes. In that case, an additional parameter would be needed. According to Cavagnaro et al. (2013), they would have strictly favored the LinLog model but some exaggerated overweighting of low probabilities by a subset of participants required that the analysis resort to the Prelec-2 form for some individuals. Perhaps the need for two separate two-parameter functions to best describe the individual probability weighting curvature indicates that the individual data are best captured by an even more general parametric form. Our three-parameter specification of $w(\cdot)$ allows for independent estimation of the bulge

and the sag as well as direct estimation of the fixed point. This additional flexibility answers the question of whether individuals are more sensitive to probability changes for lower- versus higher-cumulative-probability events.

Aggregate choice data seem to be well-characterized by a one-parameter functional form for $w(\cdot)$. At the individual level, however, a more-flexible form is needed to capture heterogeneity in the shape and specification of $w(\cdot)$. Although our study does not seek to elicit the probability weighting function at the individual level, it does permit the representative agent's weighting function to vary across sociodemographic groups. These observable individual characteristics might be correlated with unobservable characteristics that influence individual risk preferences. Hence, our systematically varying weighting function might require the same degree of flexibility as a weighting function $w_i(\cdot)$ that varies across each individual. Testing our specification of $w(\cdot)$ at the group level for heterogeneity will (1) add to the relatively sparse literature on the distribution of risk attitudes and (2) identify the types of environments where a more-general parameterization of the weighting function may be appropriate.

III. Framework

Derivation of a Weighting Function Based on Lorenz Curve Forms

In welfare economics, a Lorenz curve is a cumulative plot of the share of income or wealth that is owned by the cumulative segment of the population. Lorenz curves have applications in describing problems in ecology, biodiversity, and physics. A Lorenz curve takes the form:

$$w^* = 1 - (1 - p^*)^{1-1/k} \text{ if } 1 < k$$

The restriction on parameter k ensures convexity.

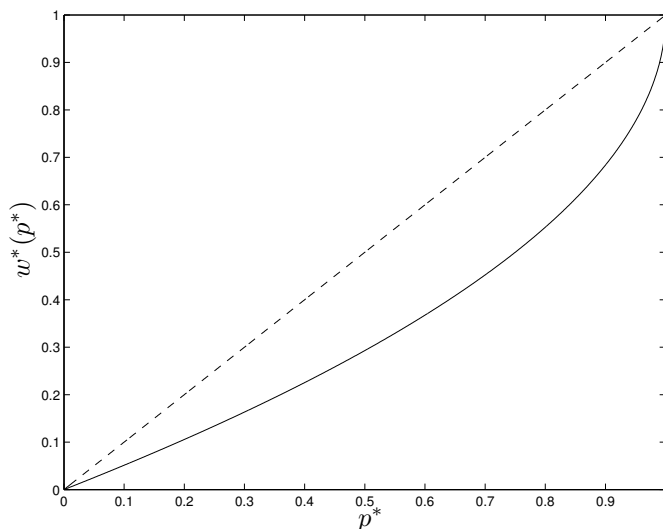


Figure 5: A Lorenz Curve with $k = 2$

Lopes (1984) argues for the intuitive value of Lorenz curves in capturing the psychological features of decision-making under risk. She proposes that even if presented with a gamble in terms of outcome probabilities, people process probabilities in terms of their cumulative probabilities. The cumulative character of Lorenz curves allows for representing

gambles in these cumulative terms. For a given gamble, she would order outcomes from worst to best. Then she plots the cumulative probability of receiving a particular outcome or worse on the x-axis and the cumulative proportion of expected value of a particular outcome or worse on the y-axis. The plot resembles a Lorenz curve. The plot can be seen as the cumulative proportion of the expected value accruing to cumulative segments of the population if the gamble is independently repeated for a large number of individuals.

The diagonal of a Lopes-Lorenz curve corresponds to a “sure thing,” a gamble with 100% probability of a single payoff. Hence, the expected value of each outcome or worse accrues evenly across the cumulative population segments in repeated trials. The degree of convexity, or deviation from the diagonal, indicates the amount of probability dispersion across outcomes, and it captures cases where the expected value accrues unevenly across members of the population. In a Lopes-Lorenz curve, the low end of the distribution corresponds with the worst outcomes. She argues that just as welfare economists emphasize the condition of those who are least well-off in judging inequality, decision-makers have increased decision weights for outcomes at the low end of the distribution in judging risk. Her argument for “the distributional model” is equivalent to an argument for what we now call rank dependence.⁵

In the same spirit as Lopes (1984) argues for the intuitive value of Lorenz curves in modeling rank dependence, we argue for the intuitive value of Lorenz curves in modeling the probability weighting function. The Lorenz curve has some properties that make it

⁵As a brief aside, Lopes orders outcomes and cumulative probabilities from worst to best, whereas modern rank dependence orders them from best to worst.

attractive for modeling the sag portion of the probability weighting function. Mirroring the axes of the Lorenz curve will enable some functional form to be adapted to model the bulge portion of the probability weighting function as well. The attractive properties of Lorenz curves for our purposes include,

1. w^* nest the linear case: $w^* = 1 - (1 - p) = p$ when $k = 1$
2. w^* is continuous on $[0, 1]$ with range $[0, 1]$.
3. w^* is increasing.
4. $w^*(0) = 0$ and $w^*(1) = 1$.
5. w^* is convex
6. w^* lies at or below the diagonal.
7. w^* transforms cumulative quantities

Within prospect theory, the diagonal serves as a benchmark case of EUT against which probability overweighting or underweighting may be measured. Within welfare economics, the diagonal corresponds to the Lorenz curve of a society in which everybody receives the same income and hence serves as a benchmark case against which actual income distributions may be measured. All of the properties listed above map nicely into the apparent empirical properties of portions of $w(\cdot)$. The region of $w(\cdot)$ to the right of its fixed point lies below the diagonal and is convex.

However unlike a Lorenz curve, $w(\cdot)$ contains a second region that lies above the diagonal and is concave. The fact that the Lorenz curve captures only the portion of the probability weighting function from the fixed point to $p = 1$ actually works to our advantage. If we mirror each axis of a typical Lorenz curve, we can achieve the desired properties

for the region of $w(\cdot)$ to the left of the fixed point. We can construct the overall weighting function in a piecewise fashion by stitching together a mirrored Lorenz curve to the left of the fixed point and typical Lorenz curve to its right. The fixed point will become its own parameter. Allowing the Lorenz-type curvature parameter to differ to the left and right of the fixed point will permit separate estimation of the bulge and the sag. Other parameterizations cannot easily be adapted to separate the estimation of bulge from the sag because it is not straightforward how to subdivide $w(\cdot)$ at the fixed point before that fixed point is defined.

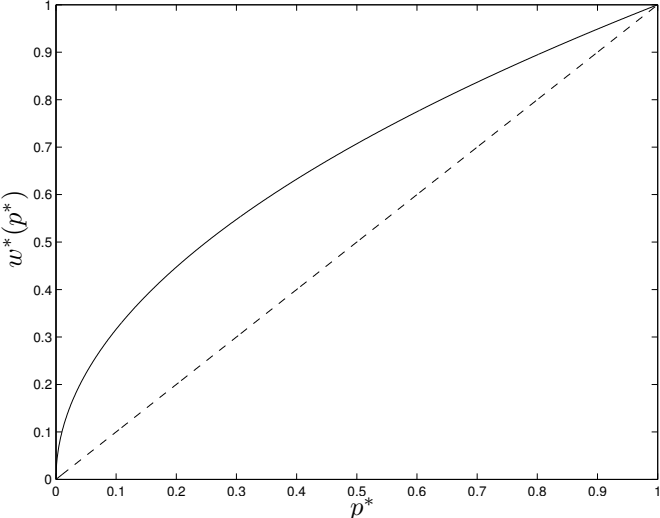


Figure 6: A Lorenz Curve Mirrored at Each Axis

We transform w^* from a curve that lies below the diagonal of a unit square into a single-crossing function that might be suitable for describing $w(\cdot)$. Intuitively, we divide the weighting function into a piecewise function that transitions at the fixed point m^* . We first consider the section of the piecewise function beyond m^* . This involves a change in

the domain of the function so that w^* is defined on $[m^*, 1]$ instead of $[0, 1]$.

$$w^* = \frac{w - m^*}{1 - m^*}, p^* = \frac{p - m^*}{1 - m^*} \quad (7)$$

where w represents our weighting function and p its cumulative probability. Substituting the expressions for w^* and p^* in terms of w and p into the Lorenz curve model produces:

$$\frac{w - m^*}{1 - m^*} = 1 - \left(1 - \frac{p - m^*}{1 - m^*}\right)^{1-1/k} \quad \text{if } p \geq m^* \quad (8)$$

Rearrangement yields a function that describes the upper region of our piecewise function:

$$w = 1 - (1 - m^*)\left(1 - \frac{p - m^*}{1 - m^*}\right)^{1-1/k} \quad \text{if } p \geq m^* \quad (9)$$

Next we consider the lower region of the piecewise function. We use a mirrored version of the same shape. Replacing p^* with $(1 - p^*)$ and w^* with $(1 - w^*)$ will mirror each axis. Now the underweighted convex curve becomes an overweighted concave curve:

$$(1 - w^*) = 1 - (1 - (1 - p^*))^{1-1/k} \quad (10)$$

$$w^* = (p^*)^{1-1/k} \quad (11)$$

This time we change each variable so that the curve is defined on $[0, m^*]$ instead of $[0, 1]$:

$$w^* = \frac{w}{m^*}, p^* = \frac{p}{m^*} \quad (12)$$

These expressions for w^* and p^* in terms of w and p substituted into equation (11) produces:

$$\frac{w}{m^*} = \left(\frac{p}{m^*}\right)^{1-1/k} \quad (13)$$

$$w = m^* \left(\frac{p}{m^*}\right)^{1-1/k} \quad \text{if } p < m^* \quad (14)$$

Our piecewise function now has one overweighted concave mirrored Lorenz curve to the left of the fixed point, m^* , and one underweighted convex Lorenz curve to its right:

$$w = \begin{cases} m[\frac{p}{m^*}]^{1-1/k} & \text{if } p < m^* \\ 1 - (1 - m^*)(1 - \frac{p-m}{1-m^*})^{1-1/k} & \text{if } p \geq m^* \end{cases} \quad (15)$$

This function remains incomplete for two reasons. First, the magnitudes of the parameters are constrained, which will complicate their estimation. The fixed point parameter, m^* , is constrained to the probability domain, $[0, 1]$. Hence, we set $m^* = \exp(m)/(1 + \exp(m))$ and estimate the unconstrained parameter m instead of m^* . Lorenz curves typically constrain the parameter k to $(1, \infty)$, or equivalently they constrain the exponent $1 - 1/k$ to the interval $(0, 1)$, so that the Lorenz curve cannot become either linear or concave. While this constraint holds for Lorenz curves used in welfare economics, the constraint does not apply to Lorenz curves used in probability weighting. In fact, the constraint is overly restrictive for our purposes. When $k \rightarrow \infty$, or equivalently when $1 - 1/k = 1$, our weighting function becomes linear. When $k < 0$, or equivalently when $1 - 1/k > 1$ the function becomes S-shaped. The typical empirical weighting function follows an inverse-S shape. However, several studies have found cases in which the weighting function does follow a linear shape or even an S-shape [Barron and Erev (2003); Bleichrodt (2001); Goeree, Holt, and Pfaffrey (2002, 2003); Henrich and McElreath (2002); Humphrey and Verschoor (2004); Jullien and Salanié (2000); Loomes (1991); Luce (1996); and Mosteller and Nogee (1951)]. Therefore, we replace the exponent $1 - 1/k$ with $s^* = \exp(s)$. The constrained parameter, s^* now rests in the proper range of $(0, \infty)$, and we estimate the unconstrained

parameter, s .

The second limitation of the new function so far is that it specifies only one shape parameter for both sides of the fixed point m^* . One motivation for this new specification is to allow independent estimation of the bulge and the sag. We then solve this problem by permitting the shape parameter s to be scaled by a factor of a^* , but only beyond the fixed point m^* . We actually estimate the logarithm of this factor $a = \ln(a^*)$ to constrain the scale factor to be positive. Alternatively, if we scaled the constrained shape parameter s^* by a factor of a^* , or equivalently added the unconstrained parameter a to the unconstrained parameter s , then certain values for a would yield a perverse M-shape or W-shape that touches the diagonal once but never crosses. Multiplying the shape parameter s by a strictly positive factor a^* prevents the possibility of this nonsensical pattern. Incorporating these generalizations yields our three-parameter Lorenz-type curve specification of the probability weighting function:

$$w_{LC}(p|s, m, a) = \begin{cases} m^* \left[\frac{p}{m^*} \right]^{\exp(s)} & \text{if } p < m^* \\ 1 - (1 - m^*) \left[1 - \frac{p - m^*}{1 - m^*} \right]^{\exp(sa^*)} & \text{if } p \geq m^* \end{cases} \quad (16)$$

where $m^* = \exp(m)/(1 + \exp(m))$ and $a^* = \exp(a)$.

Properties of Weighting Function Based on Lorenz Curve Forms

This section provides intuition for how the parameters of our Lorenz curve probability weighting function control its form. To demonstrate its flexibility, we first compare it with the most flexible of the common weighting functions, the two-parameter models. The fixed point has empirical value because it partitions where overweighting versus under-

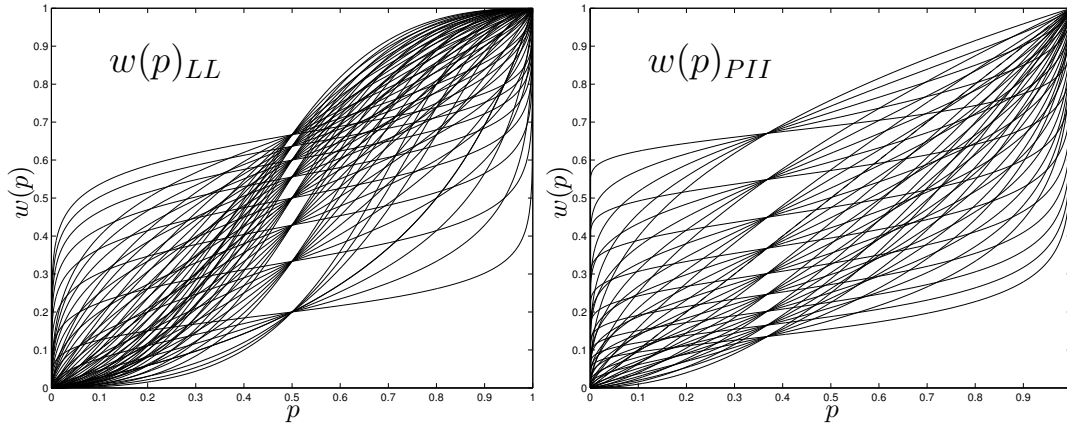


Figure 7: Commonly Used Two-Parameter Probability Weighting Functions

Two families of two-parameter weighting functions. Left: The LinLog function for $\gamma, \delta = 0.25$ to 2, both in increments of 0.25. Right: The Prelec-2 function for $\gamma = 0.2$ to 1, $\delta = 0.4$ to 2, both in increments of 0.2.

weighting occurs along the probability domain. If the fixed point equals the inflection point, then it also identifies where the transition from probabilistic risk affinity (i.e., concavity) to probabilistic risk aversion (i.e., convexity) occurs. Unlike its one-parameter counterpart which constrains the fixed point always to be equal to $1/e$, the Prelec-2 function allows the fixed point to vary. The LinLog form allows for flexibility in the estimation of the fixed point as well. However, both functions vary the location of the fixed point primarily as an artifact of varying the elevation of the entire curve.

Likewise, our function allows the fixed point to vary. As the fixed point parameter increases, so does the elevation of the entire curvature, holding fixed the shape parameters s and a . This implies that our fixed point parameter m^* corresponds approximately with the elevation parameter δ of two-parameter models, when we hold fixed parameters s and a .

Our Lorenz curve function can accommodate forms with no interior fixed point at

all. We estimate the underlying parameter m , which determines the fixed point by $m^* = \exp(m)/(1 + \exp(m))$. The function does not permit the values $m^* = 0$ or $m^* = 1$. However, if no interior fixed point appears to exist, the function, in the limit as $m \rightarrow -\infty$ (or equivalently $m^* \rightarrow 0$), converges to a single Lorenz-type curve:

$$w(p|s, m = -\infty, a) = 1 - (1 - p)^{\exp(sa^*)}$$

In the limit as $m \rightarrow \infty$ (or equivalently $m^* \rightarrow 1$), our function yields the formula:

$$w(p|s, m = \infty, a) = p^{\exp(s)}$$

If no interior fixed point exists, then the two curves degenerate into just one or the other. That is, the agent overweights or underweights the entire probability domain.

We now prove that the fixed point m^* also acts an inflection point for $w(\cdot)$. The sign of the second derivative of a function determine its curvature. When $p \in (0, m^*)$, the sign of the second derivative equals,

$$\begin{aligned} \text{sign}\left[\frac{\partial^2}{\partial p \partial p} w(p|s, m, a)\right] &= \text{sign}\left[\frac{m^* \exp(s)(1 - \exp(s))\left(\frac{p}{m^*}\right)^{\exp(s)}}{p^2}\right] \\ &= \text{sign}[s] \end{aligned}$$

When $p \in (m^*, 1)$, the sign of the second derivative equals,

$$\begin{aligned} \text{sign}\left[\frac{\partial^2}{\partial p \partial p} w(p|s, m, a)\right] &= \text{sign}\left[\frac{\exp(a^*s)(\exp(a^*s) - 1)\left(1 - \frac{p-m^*}{1-m^*}\right)^{\exp(a^*s)-2}}{1 - m^*}\right] \\ &= \text{sign}[-a^*s] \\ &= -\text{sign}[s], \text{ since } a^* > 0 \end{aligned}$$

Only when $s = 0$, the linear case, does the weighting function have the same curvature on both sides of the fixed point. In general, the sign of curvature is opposite on either side of the fixed point. Thus, the fixed point m^* acts as the point at which curvature changes, the inflection point.

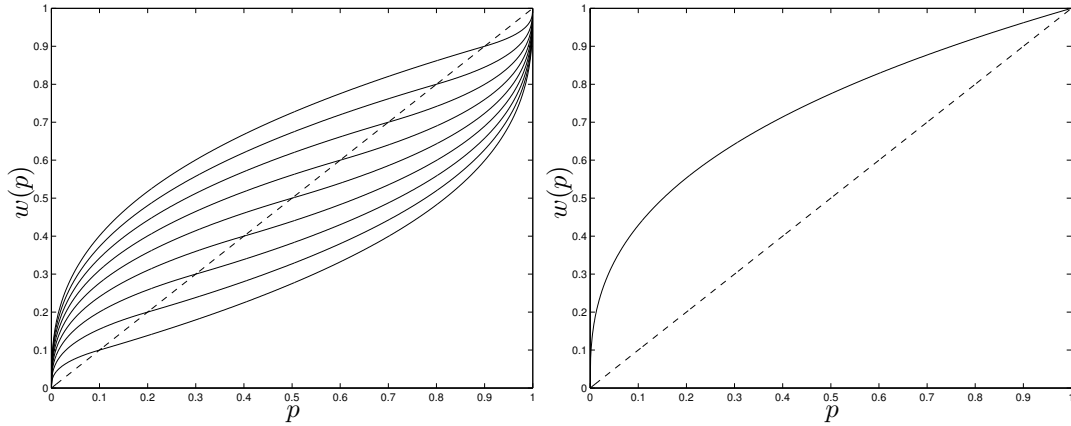


Figure 8: The Effect of Shifting the Fixed Point Parameter m , fixing s and a

Left: m varies so that $m^* = 0.1$ to 0.9 in increments of 0.1 . Right: The limiting case of m or when $m^* = 1$. Parameters $s = -1$ and $a = 0$ are held fixed.

Parameter s regulates the overall shape of the function. A negative sign on the parameter s implies that the function will follow the predominant empirical pattern of an inverse-S shape. A positive sign implies an atypical S-shape. If parameter s equals zero, then the function degenerates to linear probability weighting. The greater the magnitude of the shape parameter, the greater the degree of curvature. The bulge becomes more concave and the sag becomes more convex. Hence, our overall shape parameter s corresponds roughly with the curvature parameter γ of the LinLog and Prelec-2 functions, holding fixed the parameters m and a .

As the shape parameter approaches its limit (i.e., either $s \rightarrow \infty$ or $s \rightarrow -\infty$), the

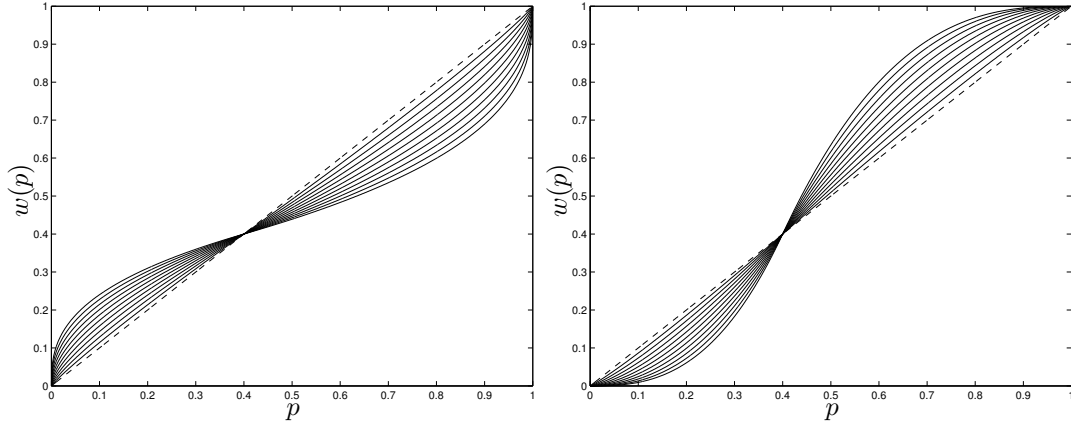


Figure 9: The Effect of Shifting the First Shape Parameter s , fixing m and a

Left: $s = -1$ to -0.1 , in increments of 0.1 . Right: $s =$ from 0.1 to 1 , in increments of 0.1 .

Parameters $m = 0.4$ and $a = 0$ are held fixed.

curve degenerates to a step function:

$$w(p|s = \infty, m, a) = \begin{cases} 0 & \text{if } p < m^* \\ 1 & \text{if } p \geq m^* \end{cases}$$

$$w(p|s = -\infty, m, a) = m^*$$

So far this Lorenz-curve-based weighting function resembles the two-parameter models that govern the elevation and curvature of $w(\cdot)$. Unlike the other functions though, our function independently estimates the fixed point. It does not conflate the estimation of the shape and the fixed point. An additional source of flexibility in our function comes from the third parameter.

Unlike other parameterizations that commingle the estimation of the two regions, our parameterization disentangles the estimation. This occurs through the interaction of the second shape parameter, a , with the first shape parameter, s , along only one side of the fixed point. The second shape parameter acts as an amplifier or dampener of the curvature

to the right of the fixed point relative to its left. Since the second shape parameter, a , acts

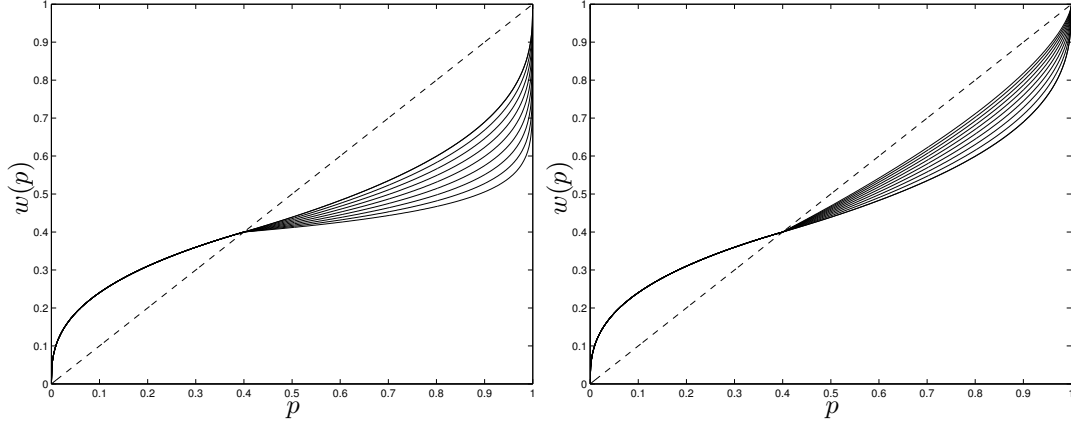


Figure 10: The Effect of Shifting the Second Shape Parameter a , fixing m and s

Left: Sag amplification with $a = 0$ to 0.8 , in increments of 0.1 . Right: Sag dampening with $a = -1$ to 0 , in increments of 0.1 . Parameters $m = 0.4$ and $s = -1$ are held fixed.

as an amplifier-dampener of curvature only beyond $p = m^*$, the remaining curvature is governed entirely by the first shape parameter, s . Hence, the first shape parameter governs not only the overall curvature, but also the relative amplification or dampening of the left curve. Therefore, given any shift in the first shape parameter, we could adjust the second shape parameter so that the right curve remains in position and only the left curve changes. Positive (negative) values of a imply an amplification (dampening) of right-side curvature relative to overall curvature. A value of $a = 0$ implies no differences in curvature between the two sides.

Finally, we consider the limiting cases of the second shape parameter. In the case of extreme right side flatness, $a \rightarrow -\infty$, the right side converge to the diagonal line:

$$w(p|s, m, a = -\infty) = \begin{cases} m^* \left[\frac{p}{m^*}\right]^{\exp(s)} & \text{if } p < m^* \\ p & \text{if } p \geq m^* \end{cases}$$

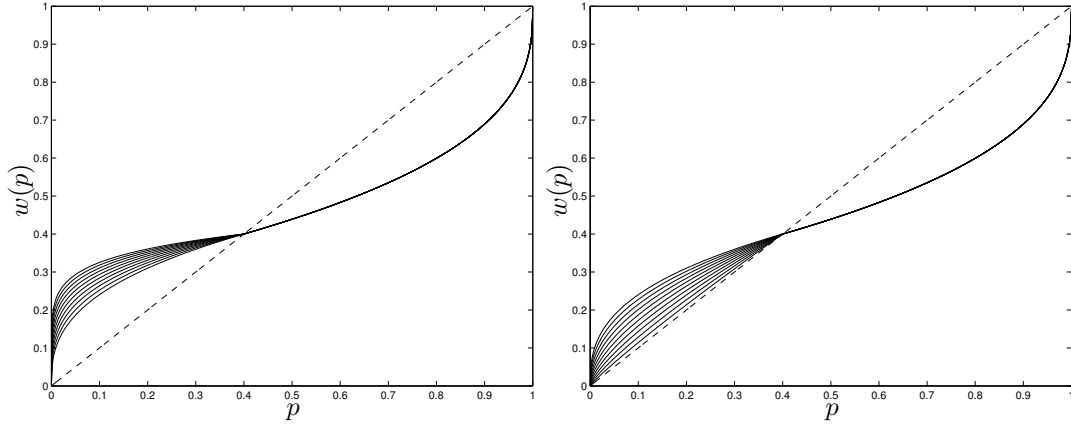


Figure 11: Amplification and Dampening along the Left Curve

Left: Bulge amplification with $s = -1.9$ to -1 , in increments of 0.1 . Right: Bulge dampening with $s = -1$ to -0.1 in increments of 0.1 . Parameter a is adjusted in a corresponding way so that the right curve remains fixed. Parameter $m = 0.4$ is held fixed.

The case of extreme right side curvature, $a \rightarrow \infty$, depends on the first shape parameter. The right side will converge to either m^* , 1 , or the diagonal line. We may notice from these limits that the amplification the right portion of the curve can never extend below the fixed point m^* . This condition comes from the property that a Lorenz curve is monotonically increasing on $[0, 1]$:

$$w(p|s < 0, m, a = \infty) = \begin{cases} m^* \left[\frac{p}{m^*} \right]^{\exp(s)} & \text{if } p < m^* \\ p & \text{if } p \geq m^* \end{cases}$$

$$w(p|s = 0, m, a = \infty) = p$$

$$w(p|s > 0, m, a = \infty) = \begin{cases} m^* \left[\frac{p}{m^*} \right]^{\exp(s)} & \text{if } p < m^* \\ 1 & \text{if } p \geq m^* \end{cases}$$

Compared to the options for the probability weighting function in the existing literature, this function makes it possible to detect whether some portions of the probability

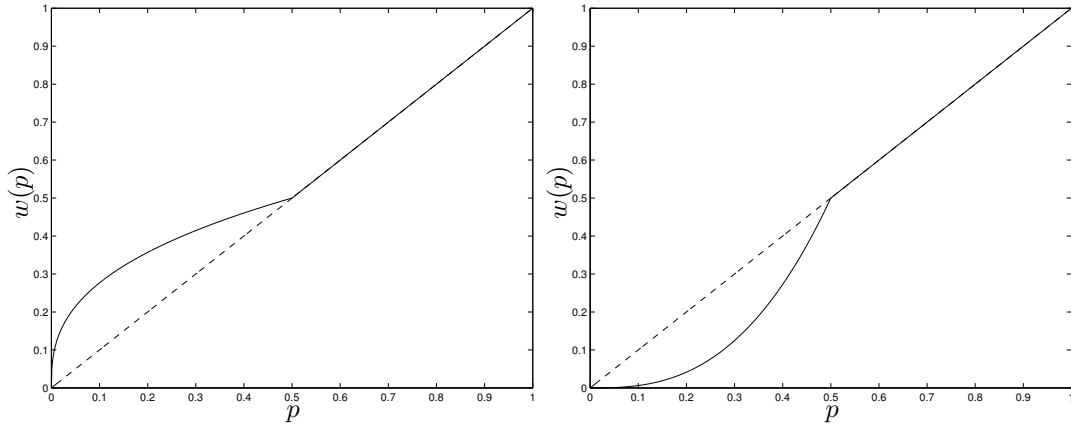


Figure 12: The Limiting Functional Form as the Second Shape Parameter $a \rightarrow -\infty$

Left: $m^* = 0.5, s = -1, a = -\infty$. Right: $m^* = 0.5, s = 1, a = -\infty$.

weighting function estimated in other applications may simply be artifacts of the functional form that best fits the rest of the probability domain. In particular, we are interested in whether the interesting curvature features of estimated probability weighting functions are real or whether they merely accommodate the best-fitting parameters of some misspecified weighting function.

IV. Methodology

Structure of the Experiment

Holt and Laury (2002) pioneered the Multiple Price List (MPL) format, which is designed to elicit risk preferences. The format presents ten pairs of gambles. The participant then chooses her most-preferred gamble for each pair. The payoffs for the gambles are set so that Gamble B is more risky than Gamble A because the spread between payoffs is larger. The difference in expected value between Gamble A and Gamble B decreases as one moves down the table. Hence, the number of the gamble at which a person switches from the safe Gamble A column to the risky Gamble B column reveals her risk preferences. In Table 1, a risk-neutral participant would select the safe gamble in the first four decisions and the risky gamble in the last six decisions.

According to EUT, once an individual switches from preferring the safe column to preferring the risky column, it would be inconsistent for her to reverse back to the safe column in her choices. Although some studies design experiments so participants must behave consistently, we do not impose the constraint of no reversals. Reversals in the price list indicate a reversal in risk preferences. Normative expected utility theory constrains risk preferences to be the same for all outcome and probability levels. Prospect theory allows for a reversal in risk preferences within the value function and the probability weighting function. For any agent who reverses his risk preferences, we can construct a pair of gambles in which a reversal of preferences would result in an irrational decision to give up an additional certain payoff.

Some evidence also suggests that individuals respond differently to hypothetical choices involving risk than they do in real-choice contexts [Battalio, Kagel, and Jiranyakul (1990); Laury (2002, 2005); and Taylor (2013)]. The real payoffs need to be sufficiently consequential to induce any difference in behavior. However, other studies find no significant difference in risk attitudes across settings [Beattie and Loomes (1997); Camerer and Hogarth (1999); Kang et al. (2011); and Kühberger, Schulte-Mecklenbeck, and Perner (2002)]. Therefore, we explicitly allow for a systematic difference in risk attitudes between real and hypothetical choices.

For each session of ten gambles that a participant completes, all ten choices are made in either a real or a hypothetical setting. Participants who considered gambles in the real setting were informed that they would make ten choices, one of which would be randomly selected to determine their payoff. Payoffs were relatively large—as high as \$81. Those participants assigned to complete the hypothetical choices were informed that they would make ten choices, one of which would be randomly selected. They would then be shown how much their payoff would have been had the choice been real.

Procedure

The experiment was computer-based. It included a risk preferences task and a post-task section. The post-task section included a cognitive ability test and a survey that included demographic questions as well as questions about the individual's preparation in math and economics. This questionnaire also inquired about whether participants had been distracted during the experiment, whether they were liquidity constrained, their education

level and educational aspirations, the extent of their math and probability training, their academic major, and their gender.

Several studies have found a link between cognitive ability and utility risk preferences [e.g., Campitelli and Labollita (2010); Cokely and Kelley (2009); Frederick (2005); Oechssler, Roider, and Schmitz (2009); and Taylor (2013)]. We wish to see how cognitive ability might influence probability weighting behavior. Our cognitive ability test contains nine questions. The first three questions are adapted from the Cognitive Reflection Test (CRT), which was developed by Frederick (2005) to assess an individual's System 2 cognitive ability. That is, the CRT measures an individual's ability to solve problems that require effort, motivation, concentration, and the execution of learned rules. The final six ability questions are taken from the eight-item test introduced by Weller et al. (2013). The eight-item test contains two items from the CRT, which is already included in our ability test. The six unique questions from Weller et al. (2013) produce a measure of a participant's numeracy. Numeracy tests measure an individual's ability to understand and manipulate numeric and probabilistic information. We hypothesize that both cognitive ability and numeracy are factors that influence probability risk-taking behavior due to the computational complexity of the task.

Participants

We pool the data from two different experiments for our estimates. Taylor (2013) conducted the first experiment at the University of Oregon between March and April of 2011. Taylor (2014) conducted the second experiment at the University of Montana be-

tween November 2013 and March of 2014. The two experiments have some minor differences in their designs, but both follow the procedures listed above. Taylor (2014) includes the further experimental treatments of characterizing the safe-gamble column within the Multiple Price List by either uncertainty or certainty. Taylor (2013) includes only the uncertainty treatment. The additional treatments do not alter our ability to elicit risk preferences, nor do they change the fundamental design of the first experiment. Therefore, we see a strong justification to pool the data and increase the statistical power available to identify individual parameters in our new model of probability weighting behavior.⁶ Table 2 provides summary statistics.

The experiment conducted at the University of Oregon had 98 participants, while the experiment conducted at the University of Montana had 184 participants. Both experiments recruited primarily undergraduate students from a diverse array of courses and academic majors. For participants who selected the choice from the “safe” column for decision ten (e.g., choosing \$40 with certainty over \$77 with certainty), standard practice dictates that we drop them from the analysis because their choice suggests that they are confused about the task. Three participants from the University of Montana experiment were dropped based on this exclusion criteria. One participant from the University of Oregon experiment failed to seek any of the available probability information about the gambles and was dropped for that reason. The overall sample thus includes a total of 278 participants. Table 2 provides summary statistics including demographic information collected for each

⁶These data from Taylor (2013, 2014) are used with permission. Taylor will be a coauthor on any published papers which may result from this project.

participant in the estimating sample.

Stochastic Choice

Cumulative prospect theory is deterministic in its original form, but decision-making under risk is stochastic [Abdellaoui (2000); Ballinger and Wilcox (1997); Camerer (1989); Hey and Carbone (1995); Hey and Orme (1994); Loomes and Sugden (1998); and Wakker, Erev, and Weber (1994)]. Individuals occasionally change their minds when asked the same question multiple times. Reversal rates for repeated questions usually range from 10% to 30%. With that said, this stochastic behavior follows a lawful pattern in the sense that stochastic choice varies according to a probability distribution [Stott (2006)]. Therefore, to analyze decision-making under risk, we must combine the deterministic cumulative prospect theory with a stochastic choice function.

Our data feature choices between a safe gamble, g_S , and a risk gamble, g_R , so we will adopt this standard notation. Recall that prospect theory claims that people evaluate risky decisions by comparing the sizes of their expected prospect values $V(g_S)$ and $V(g_R)$. Now that we allow for stochastic choices, we say that people have some likelihood of picking gamble g_R given the alternative choice g_S :

$$f(g_R|g_S, \theta, x) = P(V(g_R), V(g_S)|x)$$

Standard functional forms for the choice function, $P(\cdot)$, include specifications generally known as constant error, probit, logit, and Luce. Each of these functions has its relative

merits, but we adopt the probit as the choice function for our participants:

$$P(V(\mathbf{g}_R), V(\mathbf{g}_S)|x) = \Phi_{0,\sigma^2}(\nabla V) \quad (17)$$

where $\Phi_{0,\sigma^2}(\nabla V)$ is the cumulative normal distribution with mean zero and variance σ^2 at point ∇V . Observed choices modeled using the choice function are linked to unobserved preferences by a latent index, ∇V , which identifies the gamble that the individual should systematically prefer (sometimes in spite of his observed choices):⁷

$$\nabla V = \frac{V(\mathbf{g}_R) - V(\mathbf{g}_S)}{\mu}$$

The latent index depends upon unobserved attributes of the alternatives or characteristics of the individual, x_i . The sign of the numerator of our index determines the gamble which the model predicts to be preferred. The denominator features a noise (error dispersion) parameter, μ , which captures decision-making errors. As the noise parameter increases, the latent index ∇V becomes less sensitive to differences in the expected prospect values and choices become essentially random [Hey and Orme (1994)]. Although not necessary, inclusion of the noise parameter within the latent index has the advantage that we can measure the extent that randomness influences decision-making.⁸

⁷See Drichoutis and Lusk (2014) for a review of the standard latent index and choice function specifications.

⁸This is in contrast to the empirically popular conditional logit choice model employed in random utility method (RUM) models.

Value Function

Stott (2006) found that when the value and choice functions have a worse fit, the extra parameter in the weighting function plays a compensating role. We want the parameter estimates of the weighting function, however, to reflect actual risk preferences, so we require a sufficiently flexible value function $v(\cdot)$ for the choice-model. Holt and Laury (2002) propose using a hybrid function that combines the common power and exponential functions:

$$v(y) = \frac{1 - \exp(-\alpha y^{1-r})}{\alpha} \quad (18)$$

This “power-expo” value function is designed to capture both increasing relative risk aversion and decreasing absolute risk aversion. That is, increasing the payoff scale should increase risk aversion, and high-stakes gambles should not imply absurd levels of risk aversion. Consider the Arrow-Pratt index of relative risk aversion corresponding to this function:

$$\frac{-v''(y)y}{v'(y)} = r + \alpha(1-r)y^{1-r}$$

This index reduces to the case of constant relative risk aversion r when $\alpha = 0$ and to the case of constant absolute risk aversion α when $r = 0$. When both parameters are positive, the more-general function exhibits both increasing relative risk aversion and decreasing absolute risk aversion [Holt and Laury (2002)]. We select this function as the value function, $v(\cdot)$, for our participants because of its flexibility and its ease of interpretation.

Scalar and Systematically-Varying Parameters

We introduced several key parameters into our model of decision-making under risk. For the choice function, we defined one noise parameter (μ). For the value function, we defined a constant absolute risk aversion parameter (α) and a constant relative risk aversion parameter (r). For the probability weighting function, we defined three parameters: the fixed point parameter (m), the first shape parameter (s), and the second shape parameter (a). We designate the vector of our unknown parameters as $\theta = (\mu, \alpha, r, m, s, a) = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$, where θ_k is the k^{th} key parameter.

If we limit each of these parameters to be simple scalars, then we implicitly assume that everyone in the sample shares the same preferences. That is, one functional form with one six-element vector of parameter values accommodates all risk preferences. To the extent that people have different preferences, the error component in the model would then be left to capture the gap between the preferences of each individual and the overall average preferences in the sample (sometimes interpreted as the preferences of the representative agent). The maximized log-likelihood value under a maintained hypothesis of homogeneous preferences will be lower than it could be, if heterogeneous preferences are permitted in the model.

With enough data, we can test the initial hypothesis of homogeneous preferences. We can replace at least some of these scalar parameters with systematically varying parameters. That is, for several $k = 1, \dots, 6$, we transform θ_k from a scalar to a J_k -sized column vector, where J_k represents the number of variables that vary within the k^{th} key parameter.

Now that some key parameters θ_k systematically vary, we need to establish how the J_k -sized row vector of individual characteristics, x'_{ki} , influences the parameter values for the i^{th} choice task, θ_{ki} , where $i = 1, \dots, n$. The subscript i indicates that these are the risk preference parameters for the individual completing the i^{th} choice task. We form an index $\theta_{ki} = x'_{ki}\theta_k$ such that the vector of individual characteristics x'_{ki} is permitted to shift the value of parameters in θ_k . The subscript k on x_{ki} indicates that the vector of individual characteristics x_{ki} may contain different variables for each key parameter θ_k .⁹

We do not have a sufficient sample size to allow all of the parameters to vary systematically at the same time. In their semiparametric estimation, recall that Donkers et al. (2001) find that just one index is too restrictive. They form a parametric model with one index, $x'\beta_v$, to control for variation in their two-parameter value function and another index, $x'\beta_w$, to control for variation in their Prelec-1 probability weighting function. We mimic their approach, but innovate by employing our more-general weighting function with more than a single parameter. We test each weighting function parameter for evidence of systematic variation across our sample.

Economic theory does not offer much guidance on which of the two power-exponential value-function parameters, if not both, might exhibit heterogeneity. We allow the constant relative risk aversion parameter to vary systematically:¹⁰ For the individual completing the

⁹If $x_{ki} = 1$ for $i = 1, \dots, n$, then $\theta_{ki} = \theta_k$, so θ_k collapses to a scalar parameter.

¹⁰Our results are robust to systematic variation in the constant absolute risk aversion parameter α , instead of the constant relative risk aversion parameter r . The estimation process has difficulty reaching convergence if both parameters are simultaneously rendered as systematically varying functions of the same short list of explanatory variables.

i^{th} individual choice task, the value function parameter values correspond to

$$r_i = x'_{ri}r, \quad \alpha_i = \alpha \quad (19)$$

We render the noise parameter to be a scalar parameter:

$$\mu_i = \mu \quad (20)$$

The parameters for the probability weighting function will be permitted to vary systematically:

$$s_i = x'_{si}s \quad (21)$$

$$m_i = x'_{mi}m \quad (22)$$

$$a_i = x'_{ai}a \quad (23)$$

Choice conditions are randomly assigned to individuals, so there can be no correlations between attributes of the individual and the level of the payoffs or the probabilities. Thus we assume that each binary choice is independent. We do not seek to net out any individual-specific fixed effects and rely on the equal number of chances for each participant to ensure equal weights on each person's preferences. Individuals who differ based on observable variables are permitted to have different risk preferences.

The vector of available observable individual characteristics, x_{ki} , may contain up to seven candidate variables. As always, if some type of theory or empirical evidence does not support the inclusion of each variable, then we risk finding spurious patterns in the data:

1. $1(\text{Hypo})_{ki}$: Indicator for whether the choice was hypothetical or real

2. $1(\text{Female})_{ki}$: Indicator for whether that participant identifies as female
3. $1(\text{Stats})_{ki}$: Indicator for whether that participant has had prior statistics training
4. $1(\text{Bus/Econ Major})_{ki}$: Indicator for whether that participant's academic major is business or economics
5. $1(\text{Liq. Constrained})_{ki}$: Indicator for whether that participant is liquidity constrained
6. CRT_{ki} : Cognitive ability (continuous index)
7. Numeracy_{ki} : Ability to understand numeric and probability information (continuous index)

The indicator variable for hypothetical choices permits us to control for (and identify) any hypothetical bias in apparent preferences that might exist when choices have no real consequences, as suggested in the literature. Cognitive ability has been tied empirically to risk preferences, so we include the CRT and Numeracy measures. However, other studies have focused on the possibility that cognitive ability impacts the utility function and not the weighting function. Eckel and Grossman (2008), and Croson and Gneezy (2009), and Taylor (2013) find that females are more risk-averse at a statistically significant level. Thus we evaluate whether a difference in risk preferences might also be manifested in how females and males weight probabilities.

Chilton and Hutchinson (2003), Green et al. (1998), and Schkade and Payne (1994) all study hypothetical willingness to pay and find that participants respond to risky choice questions as though they are puzzles to which they must construct (i.e., figure out) a solu-

tion. Participants who are familiar with the concepts of expected value or expected utility theory thus may weight probabilities as if to “solve” the decision-making puzzle. We include indicator variables for statistics training and for business or economics majors, which control for exposure to these concepts.

The liquidity-constrained indicator variable differentiates between participants according to whether they lack at least \$1,000 in a liquid account to pay for an unforeseen emergency. Individuals who are more likely to face income uncertainty or to become liquidity constrained seem to exhibit higher risk aversion [Guiso and Paiella (2008)]. Our sample includes primarily college students, and therefore does not exhibit the same variability in ages as the general population. Hence we exclude the available age variable from our index specifications due to its insufficient variation across the estimating sample. Table 3 features the variance-covariance matrix for these seven candidate variables.

Maximum Likelihood Estimation

Once we have specified parametric families for the value function, the weighting function, and the choice function, we can estimate the parameters of the model by maximizing the log-likelihood of observing the actual choices, given the data. We use general function-optimizing software (MATLAB’s `fminunc`) to find the values of the unknown parameters in θ that maximize the objective function:

$$\max_{\theta} \ln L(\theta|c, x) = \max_{\theta} \sum_{i=1}^M [c_i (\ln \Phi(\nabla V)|x_i) + (1 - c_i) (\ln(1 - \Phi(\nabla V))|x_i)]$$

where $c_i = 1$ denotes the choice of the risky gamble and $c_i = 0$ denotes the choice of the safe gamble in the choice task i . The indicator variable c_i activates on the relevant version of the log-density for each risky/safe choice. The vector x_i denotes individual characteristics of the participant completing the i^{th} risky/safe choice task.

V. Results

Specifications with Homogeneous Preferences

We wish to answer two empirical questions: (1) how does the probability weighting function differ across individuals and (2) does the extent of its curvature above the fixed point differ systematically from that below? Our strategy to address question (1) is to generalize the basic parameters of the model from scalars to systematically varying parameters. Therefore, we begin with the simplest specification of using all scalar parameters as a base case against which more-complicated specifications will be compared. We then make comparisons both without, and then with, heterogeneity in the probability weighting function. The second shape parameter of our new proposed probability weighting function, a , captures information that will address the second question. We answer question (2) by comparing the fit of two specifications: one with a freely estimated second shape parameter and one which imposes the constraint of $a = 0$.

The maximized log-likelihood function will be larger, the better a given specification fits the data. In Table 4 we see that the inclusion of the second shape parameter improves the fit only negligibly, but we should formalize this conclusion. An additional parameter guarantees that the fit of the more-complicated specification will at least match, if not improve upon, the fit of the simpler specification (i.e., the new parameter could always be set to zero). Too many parameters, however, may result in overfitting. Thus, a specification should be penalized for the greater complexity of additional parameters. The likelihood ratio test captures this trade-off between goodness of fit and parsimony. The probability

distribution of the likelihood ratio test statistic is approximately χ^2 with degrees of freedom equal to $df_{alt} - df_{null}$. Hence, the more new parameters that a complicated model includes, the more likely the test will fail to reject the simple model. A likelihood ratio test performed on the homogeneous preferences specifications with a freely estimated and with a restricted to zero fails to reject the simpler (restricted) specification at any reasonable significance level:

$$D_1 = -2 \cdot \ln\left(\frac{\text{likelihood of simpler model}}{\text{likelihood of unrestricted model}}\right)$$

$$= -2 \cdot (-1343.83 + 1343.77) = 0.11$$

$$P(\chi^2(1) \geq 0.11) = 0.741$$

A specification with scalar parameters implies that aggregate preferences adequately represent the preferences of each individual. Given that previous studies conclude that a one-parameter probability weighting function fits the data quite well at the aggregate level, it is not surprising that the likelihood ratio test D_1 favors the simpler model. Whereas a likelihood ratio test evaluates the incremental improvement in the model's fit to the data from including additional parameters, a asymptotic t-test (i.e., a Wald-type test) for an individual coefficient evaluates whether an individual parameter is statistically significantly different from zero. The second shape parameter, a , is not individually significant, because its p-value is 0.687, so the shapes of the bulge and the sag do not appear to differ systematically at the aggregate level under homogeneous preferences. With that said, in Table 4, the specification with restricted parameter $a = 0$ had no statistically significant parameters, but the specification with freely estimated parameter a had statistically significant ($p < 0.05$)

parameters m and s .

We also wish to know whether the shapes of the bulge and sag within the weighting function differ when risk preferences are permitted to be heterogeneous. Heterogeneity in the value function has already been extensively explored and confirmed. The innovation in the present study is to measure the incremental gain in fit from also introducing heterogeneity into the weighting function. Therefore, we first allow for heterogeneity in the value function, then we introduce heterogeneity into the weighting function, and assess the extent of any improvement in the ability of the model to predict the observed choices in the data.

Introducing Heterogeneity into the Value Function (via r)

Table 5 reinforces the conclusions of numerous empirical studies: heterogeneity exists in the value function parameter r . Female participants and those who are more liquidity constrained are more risk averse. Business and economics majors are less risk averse. More pertinent to our current investigation, however, introducing heterogeneity into the value function parameter r in Table 5 dramatically improves the fit of the model in comparison to the fit of homogeneous preferences case shown in Table 4. A likelihood ratio test comparing the homogeneous value function specification and the heterogeneous value function specification strongly rejects the former:

$$D_2 = -2 \cdot (-1342.83 + 1323.15) = 41.35$$

$$P(\chi^2(7) \geq 41.35) \approx 0.000$$

A likelihood ratio test comparing the heterogeneous specification with and without the second shape parameter fails to reject the specification with only one shape parameter:

$$D_3 = -2 \cdot (-1323.07 + 1323.15) = 0.16$$

$$P(\chi^2(1) \geq 0.16) = 0.693$$

Just as in the homogeneous specifications, the second shape parameter does not improve the fit in the heterogeneous value function specifications. Likewise, the second shape parameter is not statistically different from zero in the heterogeneous case. The inclusion of a second shape parameter does not greatly alter the parameters estimates, but it increases the precision with which several model parameters are estimated. In Table 5, the parameters μ , α , m and s did not significantly differ from zero when parameter a is restricted, but they strongly differ from zero when parameter a is freely estimated.

Introducing Heterogeneity into the Weighting Function

Now we can measure the incremental gain in goodness of fit from generalizing to a heterogeneous weighting function. For the sake of parsimony, we trim variables from the estimated index for our value function parameter r whose coefficients did not statistically differ from zero. The coefficients for the indicator variables for gender, business or economics majors, and a liquidity constraint all remain dramatically different from zero and will be included in following specification. We repeat this process of excluding insignificant variables for each subsequent estimation.

We systematically vary each of the three parameters within the weighting function.

In Table 6, we vary both the value function parameter r and weighting function parameter s . The constant on parameter s is statistically different from zero for both the specification with and without the second shape parameter. A negative constant value for parameter s reflects the familiar result that people overweight small probabilities and underweight large probabilities. Numeracy shifts the s parameter to a statistically significant extent, with greater numeracy serving to reduce the extent of this distortion. As an individual's numeracy increases, the curvature of her probability weighting function decreases. She converges toward the linear case of probability weighting. The parameter on CRT differs from zero, and it is negative, but at only a 0.10 significance level. The sign of the CRT coefficient suggests that greater cognitive capacity worsens the probability weighting function distortion, controlling for numeracy. Both numeracy and CRT will be retained as shifters for the s parameter in subsequent specifications. The second shape parameter, a , does not statistically differ from zero in Table 6, and it does not improve the precision of parameter estimates in Table 6 as it did in Tables 4 and 5. A likelihood ratio test fails to reject the specification with a single shape parameter.

$$D_4 = -2 \cdot (-1313.12 + 1313.10) = 0.04$$

$$P(\chi^2(1) \geq 0.04) = 0.838$$

Along with the r and s parameters, we now allow the second shape parameter a to vary systematically. Table 7 reports the estimates for this specification. None of the coefficients on the variables allowed to shift a is statistically significantly different from zero—not even the constant. Between Table 6 and Table 7, we exclude the individually

insignificant variables from the first shape parameter s , which diminishes the goodness of fit. Note that the fixed point parameter m now has a large statistical significance from zero, in Table 7, with the introduction of heterogeneity in parameter s . The fixed point parameter m seems to indicate that it would improve the fit if permitted to depend on an index of at least some of the available individual characteristics.

Given that the second shape parameter a yields neither parameters significantly different from zero nor significant improvement in fit, we exclude the parameter a from our next estimation. Table 8 allows the parameters r , s , and the fixed point parameter m to vary systematically with participant characteristics. Permitting heterogeneity in parameter m across people greatly improves the fit beyond any other specification. The fixed point of the probability weighting function differs with gender, with whether an individual is liquidity constrained, and with numeracy (all to an extent that is statistically different from zero). The weak statistical significance of the coefficient on the CRT variable as a shifter for the first shape parameter, reported in Table 6, has now disappeared in Table 8. The only shift variable for the first shape parameter s with a coefficient that is significant from zero is numeracy. Thus, we have answered question (1) to the extent that our sample and tests permit. We confirm the result of Donkers et al. (2001) in finding heterogeneity of the value function to be distinct from heterogeneity of the weighting function. The influence of numeracy remains statistically significantly different from zero for the probability weighting function's shape parameter s and fixed point parameter m but not for the value function parameter r .

A likelihood ratio test can distinguish between nested models only. That is, the unre-

stricted model must contain all of the parameters and specify all of the the functional forms of the simpler model as one of its special cases under parameter restrictions. We could perform likelihood ratio tests between nested models with and without a second shape parameter as well as on the scalar parameter model versus the systematically varying r model. However, we introduced four non-nested specifications with systematic variation in different parameters: (1) r [i.e., first specification in Table 5]; (2) r and s [i.e., first specification in Table 6]; (3) r , s , and a [i.e., specification in Table 7]; (4) r , s , m [i.e., specification in Table 8]. The reason none of these specifications nests each other is because, with each successive estimation, we excluded the variables with statistically insignificant coefficients. Both the Akaike information criterion (AIC) and Bayesian information criterion (BIC) provide a criterion for model selection among non-nested models. Both measures compare the trade-off between goodness of fit and complexity of a model—the lower the score, the more preferred is the model. In comparison to AIC, BIC penalizes a model more harshly for having additional parameters. Kass and Raftery (1995) feature a table that provides guidelines on what constitutes a substantial difference in criterion values when using BIC. They consider a difference in BIC values of 6–10 to be strong evidence for selecting one model over the other models. Specification (4) dominates the other specifications when using either model selection measure, as seen in Table 9. In fact, specification (4) dominates all other specifications by a BIC score of eight or more.

From these results, we can answer question (2). One systematically varying weighting function parameter appears to be sufficient to characterize the curvature of a heterogeneous weighting function in this context. Each version of our model that allows for a

second shape parameter is rejected in favor of the single-shape parameter model. Moreover, neither the scalar nor the systematically varying second shape parameters statistically differ from zero. Notably, in the specifications without a heterogeneous probability weighting function, the presence of this additional parameter allows for other scalar parameters to become individually statistically significant.

Specification (4), found in Table 8, has all of the index variables within the fixed point parameter m^* , including some which have statistically insignificant coefficients. We perform a likelihood-ratio test for specification (4) versus a modification of specification (4) that excludes the insignificant variables. The likelihood ratio test strongly rejects the simpler model in favor of specification (4) with its additional heterogeneity:

$$D_5 = -2 \cdot (-1309.04 + 1298.51) = 21.07$$

$$P(\chi^2(4) \geq 21.07) \approx 0.000$$

Comparison of Common Specifications and Lorenz Curve Specification

Now that we have established specification (4) as the minimal sufficient model of our heterogeneous probability weighting function, we wish to assess its performance relative to the standard models. Table 10 compares the AIC criterion values of our probability weighting function to the AIC criterion values for analogous Prelec-2 and LinLog weighting functions across various specifications. The γ and δ parameters of the Prelec-2 and LinLog functions approximately correspond to the s and m parameters within our function, respectively. We compare our specifications to analogous Prelec-2 and LinLog

specifications using this correspondence. For the scalar parameter specification and the systematically varying value function parameter r specification, the standard models dominate our Lorenz curve model. However, once the specifications introduce heterogeneity into the weighting function, we see a reversal in relative performance. For the specification that varies both r and s , the Lorenz curve model strongly outperforms the Prelec-2 model with analogous heterogeneity. For our minimal sufficient specification (4) that varies r , s , and m , the Lorenz curve model outperforms both the Prelec-2 and LinLog models. We attribute this reversal to the fact that our model can estimate independently the fixed point, such that curvature estimates do not necessarily distort fixed point estimates.

VI. Conclusion

The minimal sufficient specification from our analysis includes a two-parameter weighting function as well as heterogeneity in the value and weighting functions. We have introduced a three-parameter probability weighting function based on the formula for the common Lorenz curve. We rejected the three-parameter version of our Lorenz curve weighting in favor of a simple two-parameter version. This sample of individuals do not appear to display more sensitivity to probability changes above the fixed point of the probability weighting function relative to below this point. That does not imply that our three-parameter specification may not have value in other contexts. As Gonzalez and Wu (1999) suggest, the choice domain of decision-making under risk influences the shape of the probability weighting function. In choice domains where we expect sensitivity to differ systematically along either side of the fixed point, the three-parameter version would have clear applications. Previous studies on model selection compared other one- and two-parameter specifications. Thus, our three-parameter tests indicates that the choice-set of specifications within those studies may be sufficiently general.

Our earlier discussion on the properties of the two-parameter version of the Lorenz curve weighting function illustrates that the function has the same qualitative properties as Prelec-2 and LinLog functions. However, our function has own key advantage: it allows for direct and independent estimation of the location of the weighting function's fixed point. The standard specifications of $w(\cdot)$ necessarily confound the curvature and fixed point estimates. As our results indicate, this dependence might have diminished the Prelec-

2 and LinLog's fit to the data when the weighting function parameters are allowed to be heterogeneous. A single parameter that describes the fixed point allows for straightforward hypothesis testing. Moreover, evidence of systematic variation in the location of the fixed point can now be explored.

Our investigation also illuminates the implication that studies which constrain the weighting function to be homogeneous likely produce biased estimates of the weighting function parameters. The fixed point of our Lorenz curve function also acts as the inflection point. Therefore, the fixed point represents the point at which risk attitudes shift from probabilistic risk affinity (i.e., concavity of $w(\cdot)$) to probabilistic risk aversion (i.e., convexity of $w(\cdot)$). Our results indicate that the fixed point of the weighting function systematically varies with gender, numeracy, and whether a person is liquidity constrained. We also find that the curvature of the weighting function differs systematically across with numeracy, and variation along this dimension does not appear to be present in the value function component of the model.

A natural extension of our research is to study in detail how the weighting function differs across numeracy, similar to how Harbaugh, Krause, and Vesterlund (2002) investigate the weighting function across age. Our study has relied upon a moderately large sample size to determine whether any observable variables help to explain what would otherwise be scalar parameters. This large sample size limits the complexity of the computational questions we can ask of participants in the process of testing their levels of numeracy. Therefore, we have to impose a parametric structure to identify the weighting function. A future study could constrain the sample size, but focus on just three distinct

samples: low-numeracy individuals, moderate-numeracy individuals, and high-numeracy individuals. With smaller sample sizes and adequate compensation to individuals for their time, a subsequent study could ask the demanding volume of questions required for non-parametric estimation of the weighting function.

As a public good, this research benefits at least four sets of people: economists, psychologists, policymakers, and actuaries. A central question within the fields of economics and psychology is, “how do individuals make risky decisions?” Cumulative prospect theory, and more generally, behavioral economics combines knowledge from both fields to answer this question. Economists may be interested in understanding the distribution of probability risk preferences in the population. This study has identified several key variables that seem proxy for the probability risk preferences of an individual. More specifically, economists may be interested in testing how the fixed point of the weighting function varies across sociodemographic variables, which our specification of the weighting function does permit. A psychologist may be interested in the result that an increase in an individual’s numeracy decreases her degree of probability distortion. Policymakers, as stewards of the public interest, may be interested in understanding how various sectors of the public interpret and process probabilistic information. If low-numeracy individuals are particularly susceptible to distorting probabilities in their decision-making, policymakers may wish to devise legislation that will mitigate the harmful effects of these decisions (e.g., policies that mitigate exploitation of low-numeracy individuals by lottery organizations). Actuaries, in computing insurance premiums for various sociodemographic groups, may wish to know how observable sociodemographic groups distort probability in their decision-making.

VII. Appendix

HL	(1)	(2)	(3)	(4)
Decision	<u>Gamble A</u>		<u>Gamble B</u>	
Number	$p(\$40)$	$p(\$32)$	$p(\$77)$	$p(\$2)$
1	0.1	0.9	0.1	0.9
2	0.2	0.8	0.2	0.8
3	0.3	0.7	0.3	0.7
4	0.4	0.6	0.4	0.6
5	0.5	0.5	0.5	0.5
6	0.6	0.4	0.6	0.4
7	0.7	0.3	0.7	0.3
8	0.8	0.2	0.8	0.2
9	0.9	0.1	0.9	0.1
10	1	0	1	0

Table 1: Holt and Laury Multiple Price List

Variables	Oregon Subsample	Montana Subsample	Overall Sample
N	97	181	278
Observations	1940	1810	3750
Numeracy Score (Max: 6)	4.49	4.61	4.57
CRT Score (Max: 3)	1.27	1.09	1.15
Age ^α	21.77	21.78	21.78
1(Female)	.40	.51	.47
1(Stats)	.57	.67	.64
1(Bus/Econ Major)	.30	.29	.29
1(Years of College > 1)	.66	.47	.54
1(Income < \$10,000)	.24	.73	.56
1(Liquidity Constrained)	.29	.47	.41
1(Distracted)	.05	.08	.07

Table 2: Summary Statistics

^αThree participants did not provide a response

	1(Hypo)	1(Fem.)	1(Stats)	1(Bus/Econ)	1(Liq.)	CRT	Num.
1(Hypo)	0.250	0.006	0.015	0.009	0.009	0.005	0.006
1(Fem.)		0.248	-0.019	-0.041	-0.026	-0.057	-0.068
1(Stats)			0.236	0.025	0.018	0.027	0.041
1(Bus/Econ)				0.208	-0.005	-0.006	0.024
1(Liq.)					0.235	-0.013	-0.057
CRT						0.993	0.411
Num.							1.202

Table 3: Variance Covariance Matrix for the Seven Candidate Variables

Parameters	Specification (1) No Second Shape Parameter			Specification (2) With Second Shape Parameter		
	Estimate	Std. Err.	<i>p</i> -value	Estimate	Std. Err.	<i>p</i> -value
μ (Noise)	3.087	3.644	0.397	3.618	2.573	0.160
α (CARA)	0.025	0.024	0.301	0.022*	0.012	0.060
r (CRRA)	-0.031	0.220	0.889	-0.059	0.140	0.675
m (Fixed)	1.286	0.964	0.182	1.056***	0.401	0.008
s (Shape 1)	-0.507	0.318	0.111	-0.492**	0.228	0.031
a (Shape 2)	0			-0.254	0.656	0.699
Observations	3750			3750		
LogL	-1343.83			-1343.77		

Table 4: Estimation with All Scalar Parameters

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Variables	Specification (1) No Second Shape Parameter			Specification (2) With Second Shape Parameter		
	Estimate	Std. Err.	<i>p</i> -value	Estimate	Std. Err.	<i>p</i> -value
<i>μ</i>						
Constant	5.686	5.289	0.282	5.684**	2.754	0.039
<i>α</i>						
Constant	0.015	0.011	0.185	0.015***	0.005	0.003
<i>r</i>						
1(Hypo)	0.012	0.012	0.313	0.012	0.012	0.314
1(Female)	-0.037***	0.013	0.004	-0.037***	0.012	0.003
1(Stats)	0.011	0.012	0.387	0.011	0.012	0.382
1(Bus/Econ)	0.045***	0.015	0.004	0.044***	0.015	0.003
1(Liq. Const.)	-0.046***	0.013	0.001	-0.046***	0.012	0.000
CRT	0.010	0.006	0.119	0.010	0.006	0.105
Numeracy	-0.007	0.006	0.306	-0.007	0.006	0.265
Constant	-0.142	0.181	0.433	-0.140	0.111	0.207
<i>m</i>						
Constant	0.921	0.783	0.240	0.847***	0.005	0.000
<i>s</i>						
Constant	-0.398	0.245	0.105	-0.422***	0.183	0.021
<i>a</i>						
Constant	0			-0.230	0.571	0.687
Observations	3750			3750		
LogL	-1313.12			-1323.07		

Table 5: Estimation with Systematically-Varying Parameter *r*

*** *p* < 0.01, ** *p* < 0.05, * *p* < 0.1

Variables	Specification (1) No Second Shape Parameter			Specification (2) With Second Shape Parameter		
	Estimate	Std. Err.	<i>p</i> -value	Estimate	Std. Err.	<i>p</i> -value
<i>μ</i>						
Constant	6.504**	3.007	0.031	6.584**	3.120	0.035
<i>α</i>						
Constant	0.013***	0.004	0.002	0.012***	0.004	0.003
<i>r</i>						
1(Female)	-0.044***	0.013	0.001	-0.044***	0.012	0.001
1(Bus/Econ)	0.055***	0.018	0.002	0.055***	0.018	0.002
1(Liq. Const.)	-0.043***	0.013	0.001	-0.043***	0.013	0.001
Constant	-0.173*	0.100	0.083	-0.176*	0.102	0.086
<i>m</i>						
Constant	0.569***	0.200	0.004	0.550**	0.222	0.013
<i>s</i>						
1(Hypo)	-0.109	0.100	0.277	-0.113	0.106	0.282
1(Female)	-0.088	0.101	0.384	-0.090	0.105	0.391
1(Stats)	-0.117	0.104	0.259	-0.115	0.107	0.282
1(Bus/Econ)	0.037	0.117	0.755	0.037	0.120	0.759
1(Liq. Const.)	0.111	0.101	0.272	0.115	0.106	0.279
CRT	-0.087*	0.051	0.087	-0.089*	0.053	0.094
Numeracy	0.201***	0.053	0.000	0.208***	0.065	0.001
Constant	-1.109***	0.296	0.000	-1.146***	0.351	0.001
<i>a</i>						
Constant	0			-0.078	0.384	0.838
Observations	3750			3750		
LogL	-1313.12			-1313.10		

Table 6: Estimation with Systematically-Varying Parameters *r* and *s*

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Variables	Estimate	Std. Err.	<i>p</i> -value
<i>μ</i>			
Constant	6.060	3.749	0.106
<i>α</i>			
Constant	0.014**	0.006	0.023
<i>r</i>			
1(Female)	-0.041***	0.013	0.002
1(Bus/Econ)	0.044***	0.016	0.004
1(Liq. Const.)	-0.046***	0.012	0.000
Constant	-0.163	0.135	0.226
<i>m</i>			
Constant	0.754	0.001	0.000
<i>s</i>			
CRT	-0.094*	0.053	0.074
Numeracy	0.145***	0.047	0.002
Constant	-0.979***	0.317	0.002
<i>a</i>			
1(Hypo)	-0.263	0.434	0.545
1(Female)	-0.296	0.688	0.667
1(Stats)	0.789	0.882	0.371
1(Bus/Econ)	-1.083	0.922	0.240
1(Liq. Const.)	-0.219	0.179	0.220
CRT	0.069	0.191	0.719
Numeracy	-0.337	0.325	0.300
Constant	1.179	1.183	0.319
Observations	3750		
LogL	-1314.33		

Table 7: Estimation with Systematically-Varying Parameters *r*, *s*, and *a*

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Variables	Estimate	Std. Err.	<i>p</i> -value
<i>μ</i>			
Constant	11.719***	2.827	0.000
<i>α</i>			
Constant	0.009***	0.002	0.000
<i>r</i>			
1(Female)	-0.053***	0.016	0.001
1(Bus/Econ Major)	0.059***	0.019	0.002
1(Liq. Constrained)	-0.052***	0.014	0.000
Constant	-0.280***	0.069	0.000
<i>m</i>			
1(Hypo)	-0.473	0.595	0.426
1(Female)	2.566**	1.242	0.039
1(Stats)	4.554	2.965	0.124
1(Bus/Econ)	-2.296	1.404	0.102
1(Liq. Const.)	-1.452**	0.678	0.032
CRT	0.047	0.290	0.871
Numeracy	-1.147**	0.604	0.057
Constant	2.551**	1.074	0.018
<i>s</i>			
CRT	-0.022	0.027	0.430
Numeracy	0.181***	0.034	0.000
Constant	-0.816***	0.155	0.000
<i>a</i>			
Constant	0		
Observations	3750		
LogL	-1298.51		

Table 8: Estimation with Systematically-Varying Parameters *r*, *s*, and *m*. Fix *a* = 0

*** *p* < 0.01, ** *p* < 0.05, * *p* < 0.1

Specification	LogL	Parameters	Observations	AIC	BIC
(1) SVP: r	-1323.15	12	3750	2670.3	2745.1
(2) SVP: r & s	-1313.12	15	3750	2656.2	2749.7
(3) SVP: r , s & a	-1314.33	16	3750	2664.7	2776.8
(4) SVP: r , s & m	-1298.51	17	3750	2631.0	2736.9

Table 9: Criterion Scores among Lorenz Curve Weighting Function Specifications

Specification	Lorenz	Prelec-2	LinLog
(1) SVP: none	2697.7	2686.6	2694.0
(2) SVP: r	2670.3	2663.9	2668.4
(3) SVP: r & s	2656.2	2661.9	2655.0
(4) SVP: r , s & m	2631.0	2632.3	2637.4

Table 10: AIC among Specifications with Heterogeneous Weighting Functions

For each systematically varying parameter s in the Lorenz curve model, we varied the parameter γ in the Prelec-2 and LinLog models. Likewise, for the systematically varying parameter m in the Lorenz curve model, we varied the parameter δ in the Prelec-2 and LinLog models.

VIII. Glossary

1. **Bulge** - The region of the empirically dominant probability weighting function that is concave and overweighted.
2. **Decision weight** - How an individual weights the outcomes in a gamble. The difference between the rank of an outcome and the rank of the next best alternative outcome.
3. **Decision under risk** - Decisions with respect to a gamble with known outcomes and known probabilities.
4. **Decision under uncertainty** - Decisions with respect to a gamble with known outcomes and unknown probabilities.
5. **Expected utility theory** - A decision-making theory that describes how a rational agents should make decisions under risk.
6. **Fixed point** - The point of probability weighting function that crosses the identity line.
7. **Gamble** - A probability distribution $g = (y_1, p_1; y_2, p_2; \dots; y_N, p_N)$, where y_n is the monetary or non-monetary payoff from the n^{th} outcome with probability p_n of occurring. Outcomes are ordered from best to worst: $y_1 > y_2 > \dots > y_n$.
8. **Lorenz curve** - A cumulative plot of the share of income or wealth that is owned by the cumulative segment of the population.
9. **Probability risk affinity (optimism)** - As an outcome becomes better (i.e.,

- has smaller cumulative probability), its decision weight increases.
10. **Probability risk aversion (pessimism)** - As an outcome becomes worse (i.e., has larger cumulative probability), its decision weight increases.
 11. **Probability weighting function** - How an individual weights in her mind the cumulative probability of receiving a better outcome.
 12. **Prospect theory** - A decision-making theory that describes how people actually make decisions under risk. It generalizes expected utility theory by replacing the utility function with the value function and by replacing the objective probabilities with a probability weighting function.
 13. **Rank** - The cumulative probability of receiving a better outcome than the outcome under consideration.
 14. **Sag** - The region of the empirically dominant probability weighting function that is convex and underweighted.
 15. **Stochastic choice function** - A function that models the randomness in decision-making when comparing the expected prospect value of each gamble.
 16. **Systematically varying parameter** - A parameter that varies with more than just the constant variable.
 17. **Value function** - The function in prospect theory that describes how an individual values each possible payoff.

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