AN ANAYLSIS OF EUREKA MATH CURRICULUM FOR COMMON CORE ALIGNMENT AND DEVELOPMENT OF CONCEPTUAL UNDERSTANDING

by

NICOLE WHITEHURST

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Approved: Shlomo Libeskind

Shlomo Libeskind

Math Education has been a major topic of debate within the education sphere for years now, especially since the introduction of the Common Core State Standards. These standards have been met with mixed reviews. This research begins by analyzing the standards concerning fractions in third, fourth, and fifth grade and determining how it fits with the four domains of mathematical understanding these.

It then goes on to an in-depth look at Eureka Math curriculum concerning fractions looking at both its alignment to the Common Core State Standards and its contribution to developing conceptual understanding, one of the four domains of mathematical understanding. Eureka Math was chosen, because it is a free curriculum available for download online, which makes it a lucrative resource for teachers.

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Introduction

Anyone who has been involved in elementary, middle, or high school education, especially in the last ten to fifteen years, can tell you that math education is a major topic of debate. With that being said, obviously, I am not going to be the only one to research this topic. There are many people who have researched this before me, and I am sure there are many people who will do so after me. Regardless of that, I am passionate about math education, and providing students with the best math education possible. With my research, I hope to add to the conversation about this topic, get experience critically analyzing math curriculum, and obtain a better overall understanding of the Common Core State Standards regarding mathematics

This research will focus specifically on fractions in third, fourth, and fifth grade. In choosing this domain, I focused on late elementary school, because my goal is to be a teacher for these age groups. I then narrowed down my scope to just focus on fractions, because fractions are one of the most important topics introduced in these grades. This concept is important, because it directly relates to decimals, percentages, ratios, and proportions, which "are often used in real-life contexts as students and adults" (Francis 491). Because of this, a student's understanding of fractions can directly affect his or her understanding of these other concepts as well.

In addition to looking at the Common Core State Standards related to fractions, I will also be providing an in-depth analysis of the Eureka Math curriculum concerning this topic. Eureka Math is math curriculum that was developed by Great Minds and is available to download for free online. These materials also are known for their alignment to the Common Core State Standards for Mathematics. I chose this curriculum because as a free resource, it has the potential to be very beneficial to teachers. In order for it to be beneficial though it needs to be high quality. This math curriculum has been downloaded in every state, and “California is currently the state with the highest print orders” (McAfee). More information on this curriculum, its development, and its content is included later.

Types of Mathematical Understanding

In math education, there are four domains of math comprehension: declarative, procedural, conceptual, and problem-solving.

Declarative knowledge "involves memorization and automaticity in areas such as number recognition, counting, recall of basic facts, reading and writing fractions, telling time, and many other mathematical tasks" (Hudson, 12). Basically, this means that having declarative knowledge of math is being able to recall facts correctly without hesitation. A specific example of this would be, in kindergarten, learning to count from one to ten.

Another type of knowledge and domain of comprehension is procedural. Procedural knowledge refers to the ability to recall and perform "mathematical tasks that require the student to follow a series of sequential steps in order to solve the problem" (Hudson, 12). If students have procedural knowledge or procedural comprehension, then they are able to complete these tasks with limited hesitation or mistakes. An example of this sort of knowledge would be being able to correctly perform long division.

Next is conceptual comprehension. A conceptual understanding is one in which the method and the meaning of a concept is understood. The method refers to the algorithm or procedure used in a situation, and the meaning is the justification for the method, often expressed through theorems, axioms, diagrams, or general explanation (Hallett 2012). The meaning goes beyond just knowing these theorems axioms, diagrams, and explanations to how these ideas are connected to one another (Hallett 2010, 396). With that being said, student who possess conceptual comprehension are able to justify or explain different mathematical concepts or procedures beyond just describing the steps needed to produce a certain result. As an example, take 45-26. It is seen that 6 is greater than 5, so typically, you would "borrow" from the 4, turning it into a 3 and the 5 into a 15. A better word to use would be "regroup" because someone with a conceptual understanding would know that you are taking one group of ten from the four tens you have and turning it into ten ones instead then adding to the five ones you already have. Below is a figure containing the abstract representation on the left and the visual representation to the right in which the visual representation shows what is happening in the regrouping step of the abstract representation.

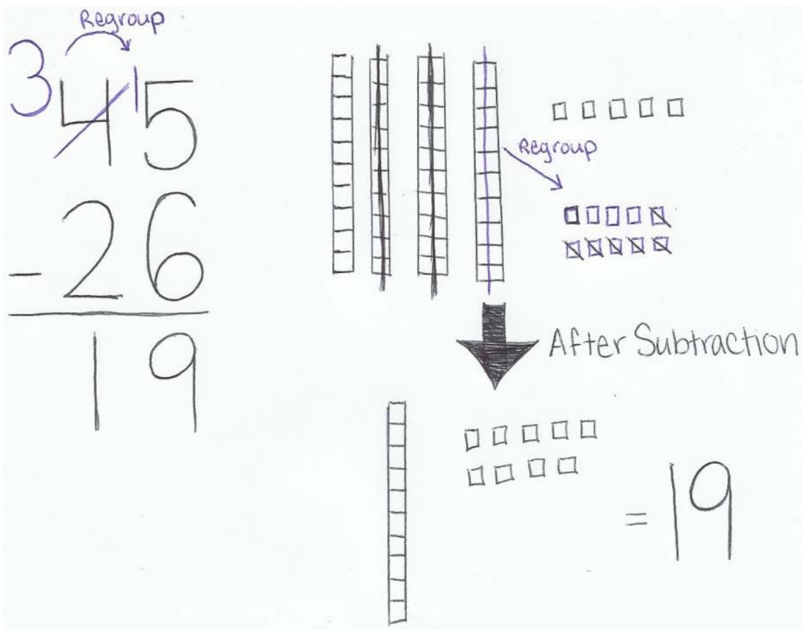


Figure 1: Abstract and Visual Representation of Two-Digit Subtraction with Regrouping

If the person did not have a conceptual understanding of this topic, it is much more likely that they would just perform the procedure without being about to explain why or how it works.

Lastly, the fourth domain of math comprehension is problem-solving comprehension. Problem-solving comprehension calls for combining “concepts, factual knowledge, and procedural strategies in real-world contexts” (Hudson, 12). In other words, if students have problem-solving comprehension of math, they are able to use declarative, conceptual, and procedural knowledge to interpret and solve a variety of problems in many different contexts. An example of a problem that would require problem-solving to solve would be something like, “I am buying 5 boxes of cereal at $3.75 per box. I also have a coupon for 10% off. How much will it cost me to buy my cereal boxes after using my coupon?” We use conceptual knowledge to interpret the problem and create a plan to solve the problem. We then use procedural and declarative knowledge to perform the necessary steps established previously. Then, by combining that knowledge, we are able to find the answer to the problem.

Ideally, a student should develop mathematical comprehension in all four domains, because this development would result in the most well-rounded mathematical understanding.

Common Core State Standards

In 2008, former Arizona governor Janet Napolitano began working on a project that would eventually become the Common Core Initiative. With a team “composed of commissioners of education, governors, corporate chief executive officers and recognized experts in higher education” (Bidwell), she developed these standards with the intended purpose of “ensuring that all students graduate from high school with the skills and knowledge necessary to succeed in college, career, and life, regardless of where they live” (“About the Standards”). She and her team did so by developing “a clear and consistent framework for educators” (“About the Standards”). This framework is not a standard pedagogy, but rather it is a set of skills or concepts that students should know by a certain level in their education. By only providing guidelines, it was the responsibility of curriculum developers, teachers, and administrators to create curriculum that would satisfy the standards. Because there was little to no instruction concerning curriculum and teaching, the implementation of the standards were not as successful as it could have been.

The general response from teachers concerning the Common Core is that they like the idea, but not the implementation (Fighting for Common Core). The Common Core demands a change in math education for which the teachers are not fully prepared. Teachers agree that there needs to be a change in the current teaching and Common Core could be part of it, but are unsure how to facilitate that change and use the Common Core effectively (Williams).

There are two types of standards outlines by the Common Core. Firstly, there are the Standards for Mathematical Practice. These eight ideas are the “varieties of expertise that mathematics educators at all levels should seek to develop in their students" ("Standards for Mathematical Practice"). In addition to the standards for mathematical practice, there are also the standards for mathematical content. These standards cover what topics should be mastered in a specific grade, which means there are different standards for each grade level. Within in each grade, they are organized such that there is a large goal then the standards that help accomplish that goal. As mentioned previously, this project will focus on fraction education in third, fourth, and fifth grade. The analysis of these standards will be to determine which of the four domains of mathematical comprehension that standard falls under. Ideally, in order to develop a well-rounded math education, all of the domains should be represented within the standards.

Also, it should be mentioned that the authors and creators of the common core standards may have had a certain domain that the different standards should fall into, but when or not the student develops that intended understanding relies heavily on the instructor, the curriculum being used, and the individual student.

Standards for Mathematical Practice

The first Standard for Mathematical Practice states, "Make sense of problems and persevere in solving them," which means "Mathematically proficient students can explain correspondence between equations, verbal descriptions, table and graphs or draw diagrams of important features and relationships, graph data, and search for regularity and trends" ("Standards for Mathematical Practice"). In order to interpret and solve a variety of problems and thus meet this standard, the student must have sufficient, relevant declarative, procedural, and conceptual understanding. For that reason, this standard is promoting a problem solving understanding of mathematics.

The second Standard for Mathematical Practice is “Reason abstractly and quantitatively” (“Standards for Mathematical Practice”). In doing this, student should be able to “bring together two complementary abilities… the ability to decontextualize… and the ability to contextualize" (“Standards for Mathematical Practice”), which means students are using a procedural understanding to manipulate quantities and equations while using a conceptual understanding to use context to determine what is being represented and expressed by these quantities and equations. Since this is combining multiple areas of understanding, this standard is promoting a problem solving understanding as well.

The third Standard for Mathematical Practice is “Construct viable arguments and critique the reasoning of others,” and it is described saying “Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments” (“Standards for Mathematical Practice”). In these types of situations, students must have declarative knowledge to recall different definitions and facts that are relevant to the problem. Then, they must reasoning conceptually to determine how these facts and definition can and should be used. Depending on the nature of the problem, the student may have to perform calculations, requiring procedural knowledge, in order to solidify their argument. The second part of this standard is being able to critique the arguments of others. Just as they need sufficient understanding of the relevant information to develop their own arguments, they also need it to critique other's arguments. Just like the previous two standards, this standard is also promoting a problem solving understanding of mathematics, because in order to meet this standard, a student must be able to use procedural, conceptual, and declarative knowledge.

The fourth Standard for Mathematical Practice is “Model with Mathematics” which means “mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.” (“Standards for Mathematical Practice”). This idea centers on being able to take previous knowledge and apply it to real world problems. In order to successfully do so, the student must have the necessary declarative, procedural, and conceptual understandings. Since it requires proficient understanding of those different types of knowledge, this standard is also working toward a problem solving understanding.

The fifth Standard for Mathematical Practice is “Using appropriate tools strategically” (“Standards for Mathematical Practice”). This standard goes beyond just being able to use a ruler, protractor, or calculator correctly; it means students are able to “make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations” (“Standards for Mathematical Practice”). Therefore, in order to meet this standard, students need to not only be able to use the tools, which is procedural knowledge, but they must also know how they are beneficial or not, which uses conceptual knowledge. Since this is combining multiple areas of mathematical knowledge, the purpose of this standard is to continue to develop problem solving understanding, just like the previous standards.

The sixth Standard for Mathematical Practice is “Attend to Precision” (“Standards for Mathematical Practice”). Some examples of attention to precision are “clear definitions,” "meaning of symbols," "specifying units of measure and labeling axes," and "calculate accurately and efficiently" (“Standards for Mathematical Practice”). These eight standards are called the Standards for Mathematical Practice, and this standard is focused on how to accurately and appropriately express situations in a mathematical context. If I had to classify this standard in one of the four domains of mathematical understanding, I would say it connects to problem solving. This is not quite problem solving in the same way that the other standards were, but the description discusses using declarative, procedural, and conceptual understanding.

The seventh Standard for Mathematical Practice is “Look for and make use of structure” (“Standards for Mathematical Practice”). In this standard, students are describing and using patterns to make broader generalizations, which is problem solving understanding as well. This is because the student must have proficient understanding of concepts, procedures, and declarative knowledge in order to make accurate generalizations and conclusions.

The eighth Standard for Mathematical Practice is “Look for and express regularity in repeated reasoning” (“Standards for Mathematical Practice”). This standard is similar to the seventh standard. This means that students are paying attention to the information, making generalizations, and then applying those generalizations to appropriate problems (“Standards for Mathematical Practice”). Overall, this standard calls for the student to apply their declarative or procedural knowledge and apply it conceptually to solve problems. In order to do this, the student must have a problem solving understanding of mathematics.

After analyzing all eight Standards of Mathematical Practice, I have found that all eight are working toward a problem solving understanding of mathematics. Initially, I noted that it is important that the standards reflect the necessity for proficiency in all the domains of mathematical understanding. By having all of these standards contribute to a problem solving understanding, they are actually calling for proficiency in all domains of mathematical understanding, because truly having a problem solving understanding of mathematics requires application of procedural, declarative, and conceptual understandings. The Standards for Mathematical Practice are meant to be fully developed over the course of one entire math education, not just in a single grade level. One of the main goals of math education is for students to develop a well-rounded understanding of math that include declarative, procedural, and conceptual understanding in order to solve and reason about real world problems. Therefore, having overarching standards that focus entirely on problem-solving comprehension aligns with the overall goal of the Common Core State Standards.

Standards for Mathematical Content

The focus of this section will be on the standards surrounding fractions in third, fourth, and fifth grade. Just as the previous section analyzed the Standards of Mathematical Practice in terms of what domain of mathematical understanding they work to develop, this section will do the same thing for the Standards for Mathematical Practice concerning fractions.

Third Grade

The overall goal of education regarding fractions in third grade is to "develop understanding of fractions as numbers" ("Number & Operations—Fractions."). This is important, because up until this point, students have only been working with positive whole numbers. Therefore, developing number sense concerning fractions is important for future in-depth understanding.

The chart below includes all of the Common Core State Standards concerning fractions for third grade and how I have categorized them in the four domains of mathematical understanding. Following the chart are the reasons for each classification.

Common Core State Standards For Mathematical Content: Third Grade		
Common Core State Standard Code	***Standard***	***Type of Understanding***
3.NF.A.1	Understand a fraction *1/b* as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size *1/b*.	Conceptual
3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram.	Conceptual/Procedural
3.NF.A.2.A	Represent a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size *1/b* and that the endpoint of the part based at 0 locates the number *1/b* on the number line.	Conceptual/Procedural
3.NF.A.2.B	Represent a fraction *a/b* on a number line diagram by marking off a lengths *1/b* from 0. Recognize that the resulting	Conceptual/Procedural

	interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.	
3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.	Conceptual
3.NF.A.3.A	Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.	Declarative
3.NF.A.3.B	Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.	Conceptual/Procedural
3.NF.A.3.C	Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.	Conceptual/Procedural
3.NF.A.3.D	Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions,	Conceptual

Table 1: Common Core State Standards for Mathematical Content: Third Grade

This table lists the Common Core State Standards for Mathematical Content concerning fractions in third grade, and it categorizes the standards based on the four areas of mathematical understanding.

The standard 3.NF.A.1 promotes a conceptual understanding, because it emphasizes that importance of knowing what fractions actually represent rather than just introducing the notation and calling it a fraction.

The next standard, 3.NF.3.A.2, connects to both a conceptual and procedural understanding. It is conceptual, because it is continuing to develop the idea of a fraction as a number by placing it on a number line, something that most students have probably had exposure to at some point, but only with whole numbers. It is procedural in the fact that representing fractions on a number is something done in a step by step process.

The next two standards are substandards of 3.NF.3.A.2, and they give more clarity to the larger standard. Both of these have a more procedural emphasis, but there is some conceptual understanding embedded in them. Their procedural emphasis comes from the reason why the larger standard is procedural: plotting fractions on number lines has a heavy emphasis on procedure. They also include a conceptual focus because the call for student to understand what the number line represents.

Following those two standards is 3.NF.3.A.3, which discusses fraction equivalence. This standards falls under conceptual understanding because it calls for students to reason based upon size. This is conceptual because in order to do this, student must understand what it means to be a fraction in order to determine its size compared to another fraction.

The last four standards for third grade are substandards of 3.NF.3.A.3; so, they are all concerning equivalent fractions and comparing fractions.

The first substandard has students understand that fractions that are the same size or are located at the same point on a number line are equivalent. This would contribute to a declarative understanding, because this is just understanding a fact. The first fact is that if two fractions are the same size, then they are equal. The second fact is that if two fractions land on the same point on a number line, they are equal. There have been other standards that are similar to this that I have categorized as conceptual or procedural. I decided on declarative because fulfilling this standard means that students have been given two means of determining if fractions are equivalent without emphasizing what it means to be equivalent.

The next substandard calls for students to identify equivalent fractions. This substandard is conceptual, but will later become procedural when multiplication of fractions is introduced. In third grade, students are creating these equivalent fractions through visual or concrete models rather than the procedure of multiply a fraction *a/b* by and fraction *n/n* and getting *(a*n)/(b*n)*. Using visual or concrete models support a conceptual understanding because then the student is learning that in order for two fractions to be equivalent, they must take up the same amount of the whole.

The third substandard is concerning representing whole numbers as fractions. Similar to other standards for this grade level, it is both conceptual and procedural. Expressing whole numbers as fractions is a procedural and eventually declarative skill. Take the fraction *1/b* and the whole number *a*, to find the equivalent fraction of *a* with a denominator of *b*, take $\frac{a \times b}{b}$. This can be introduced conceptually by having students use number lines or other models to determine it. The second part of this standard has students recognize fractions that are equivalent to whole numbers. This is procedural and eventually declarative for the same reason the first part was. Similar to the first part as well, this could be presented conceptually by having student represent it on a number line or another model and then reason about a pattern for determining these fractions.

The last substandard for this larger standard and the last standard for third grade is about comparing fractions with either the same numerator, which is the number above the fraction line (*a* in the fraction *a/b*), or the same denominator, which is the number below the fraction line (*b* in the fraction *a/b*). This standard supports a conceptual understanding, because the standard has students reasoning based on their size. Just as the initial standard for which this is a substandard, in order to reason about the size, the student must understand what the numerator and the denominator of a fraction mean and how to represent a fraction, which is a conceptual skill.

Fourth Grade

There are two areas of emphasis in fourth grade fraction education. Firstly, students are expected to “extend understanding of fraction equivalence and ordering” ("Number & Operations—Fractions."). Also, they are expected to “build fraction from unit fractions” ("Number & Operations—Fractions.").

Just like the analysis of the third grade standard, I have included a chart that includes the standards for fourth grade fraction education and the domain in which I have categorized them. In the list of standards concerning fractions, Common Core includes three standard that work on the concept “understand[ing] decimal notation for fractions, and compare fractions” ("Number & Operations—Fractions."). I have chosen to omit these standards from my analysis, because I do not believe they are relevant to the overall goal of this research.

Common Core State Standard for Mathematical Content: Fourth Grade		
Common Core State Standard Code	***Standard***	***Type of Understanding***
4.NF.A.1	Explain why a fraction *a/b* is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts	Conceptual

	differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	
4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Conceptual/ Procedural
4.NF.B.3	Understand a fraction *a/b* with *a > 1* as a sum of fractions *1/b*.	Conceptual
4.NF.B.3.A	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	Conceptual
4.NF.B.3.B	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions	Procedural
4.NF.B.3.C	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	Procedural
4.NF.B.3.D	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	Problem-Solving
4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	Problem-Solving
4.NF.B.4.A	Understand a fraction *a/b* as a multiple of *1/b*.	Conceptual
4.NF.B.4.B	Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a	Conceptual

	whole number.	
4.NF.B.4.C	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.	Problem-Solving

Table 2: Common Core State Standards for Mathematical Content: Fourth Grade

This table lists the Common Core Standard for Mathematical Content concerning fractions in fourth grade, and it categorizes the standards based on the four areas of mathematical understanding.

The first standard is beginning to combine standard 3.NF.A.3.B and the procedure for finding equivalent fractions. This standard is contributing to a conceptual understanding because the students are supposed to explain this fact by first using a visual model; therefore, they are being shown that what it means to have the fraction *(n*a)/(n*b)*, versus just presenting them as equivalent and not explaining why.

The next standard has students comparing fractions with different numerators or denominators. This standard has multiple parts that connect to different domains of mathematical understanding. Firstly, finding common denominators or common numerators is a procedural idea based on the previous standard. The standard also says that students can compare using a benchmark fraction. A benchmark fraction is a fraction that is considered more convenient and more commonly used. The text, *A Problem Solving Approach to Mathematics* gives the example "if a student had 59 correct answers out of 80 questions, the student answered 59/80 of the questions correctly, which is approximately 60/80 or 3/4" (Billstein 283). Using an appropriate benchmark fraction to compare is a conceptual skill, because it requires the student to not only choose an appropriate fraction, but also compare that fraction to two other fractions. The next part of this standard is “recognize that comparisons are valid only when the two fractions refer to the same whole” (“Number & Operations—Fractions.”). This skill is conceptual, because it requires students to understand what a fraction truly represents beyond just the notation of the fraction. Finally, the standard calls for students to show their comparisons using >,=, or <, which is a procedural skill, because deciding what symbol to use is a procedure.

Where the previous two standards focused on equivalence and comparison, this standard, 4.NF.B.3, begins the discussion of building fractions from unit fractions. The first standard in this section has student recognize that the fraction *a/b* is a multiple of the fraction *1/b*. This connected to a conceptual understanding and it is building on the third grade standard 3.NF.A.1. This standard is continuing to build and understanding of what fractions represent and what fractions are.

The previous standard has four substandards connected to it. The first of these is understanding what it means to add and subtract fractions. This connects to conceptual understanding because it is presented saying that addition and subtraction is “joining and separating parts referring to the same whole” (“Number & Operations—Fractions.”). This is conceptual, because the students are expected to understand not only the concepts of addition and subtraction but in order to add fractions, the wholes of those fractions must be the same.

The next substandard is about rewriting fractions as the composition of smaller fractions. I have classified this as procedural and conceptual. It is procedural, because the process of dividing a fraction into smaller parts with the same denominator is a procedure; there are steps and a limited number of combinations. The conceptual part comes in when the students are ask to justify their decompositions, because the student must reason what it means to be a fraction in order to justify.

The next substandard, 4.NF.B.3.C, is the beginning of adding and subtracting mixed numbers, which are numbers that contain a whole number part and a fractional part. An example of a mixed number would be one and a half ($1\frac{1}{2}$) or three and fourth fifths ($3\frac{4}{5}$). Since students already know what it means to add and subtract fraction and what it means to have an equivalent fraction, this is a procedural application of that conceptual understanding. The student will be able to follow the steps to add fractions: replace mixed number with equivalent fraction and add/subtract as you know previously.

The final substandard for this larger standard has students solving word problems involving addition and subtraction of fractions and mixed numbers. This is contributing to a problem-solving understanding, because the student will be interpreting problems and using their conceptual understanding to determine how the problem can or should be done, then use procedural and declarative knowledge to solve the problem in the specific context.

After that is standard 4.NF.B.4, which calls for students to use their knowledge multiplication in order to generalize how to multiply a fraction by a whole number. This builds problem-solving understanding because the students are supposed to reason with information they currently have to develop a theory or procedure in this certain context.

That standard then has three substandards; the first of which has students recognizing that the fraction *a/b* is a multiple of the fraction of *1/b*. This idea is conceptual, because it has students looking at the fraction in a new way using concepts with which they are already familiar. From multiplication, it should have been shown that *a*b* represents *a* copies of *b* and that *a*b* is a multiple of *b*; they also know that the fraction *a/b* represents *a* pieces that are size *1/b*. Now, this standard calls for them to use these two concepts to develop a new conceptual understanding of non-unit fractions.

The second substandard concerning multiplying fractions by whole numbers is closely related to the first. In this standard, students are called to extend this way of viewing non-unit fraction that was developed previously and use it determine how to multiply any fraction by any whole number. Just like the previous standard was conceptual, this one is as well. The students are using concepts they know to generalize these concepts further.

The last standard for fourth grade, 4.NF.B.4.C, has students solving word problems that require multiplication between a fraction and a whole number. Just as with the previous standard involving word problems, this standard is contributing to a problem-solving understanding of mathematics. In order to fulfill this standard, students must interpret the problem and develop a strategy to solve the problem using their conceptual, procedural, and declarative knowledge.

Fifth Grade

In the fifth grade, there are two main focuses. The first is “Use equivalent fractions as a strategy to add and subtract fractions” (“Number & Operations—Fractions.”). This concept was actually explored somewhat during fourth grade though the formal procedure was not introduced. The next focus is using what is already known about multiplication and division in order to extend that understanding to include fractions. This, like the previous focus, was discussed partial in fourth grade; the difference being that in fourth grade there was just multiplication be a fraction and a whole number. In this case, the understanding will extend to include multiplying two fractions together.

Just as with the fourth grade standards, I decided to omit a few standards, because they did not seem relevant enough to rest of standards, and fractions education in general. The first omitted standard was 5.NF.B.4.B, which has students finding the area of a triangle using unit squares. The next standards I chose not to include were 5.NF.B.5 and 5.NF.B.5.A. The first one was omitted because all it said was “Interpret multiplication as scaling (resizing), by:” (“Number & Operations—Fractions.”). There is not really anything to interpret with that; so, instead I just added it onto it substandards. The next one was a substandard of that, and it called for students to reason about the size of the product based upon the factors that were being multiplied. This is a good estimating concept to understand and skill to use, but it has a massive reach beyond just fraction education. Therefore, for sake of time and effort, I chose not to include it.

Common Core State Standards For Mathematical Content: Fifth Grade		
Common Core State Standard Code	***Standard***	***Type of Understanding***
5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators	Procedural
5.NF.A.2	Solve word problems involving addition and subtraction of	Problem-Solving

	fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	
5.NF.A.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	Declarative/Problem-Solving
5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.	Problem-Solving
5.NF.B.4.A	Interpret the product $(a/b)\times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.	Conceptual
5.NF.B.5.B	Interpret multiplication as scaling (resizing) by: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	Conceptual

5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers	Problem-Solving
5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions	Conceptual
5.NF.B.7.A	Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.	Conceptual/Procedural
5.NF.B.7.B	Interpret division of a whole number by a unit fraction, and compute such quotients.	Conceptual/Procedural
5.NF.B.7.C	Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.	Problem-Solving

Table 3: Common Core State Standards for Mathematical Content: Fifth Grade

This table lists the Common Core Standard for Mathematical Content concerning fractions in fifth grade, and it categorizes the standards based on the four areas of mathematical understanding.

The first standard is discussing addition and subtraction of fractions with different denominators. This is contributing to a procedural understanding of mathematics, because since students already know what it means to add and subtract fraction and what it means to have an equivalent fraction, this is a procedural application of that conceptual understanding. The student will be able to follow the steps to add fractions: replace fractions with equivalent fractions and add/subtract as they know previously.

The next fifth grade standard has student solving word problems including addition and subtraction of fractions. Just as with the two other standards that called for solving word problems, this standard is building a problem-solving understanding of mathematics. With these problems, students are interpreting situations and developing strategies based on what they current understand and are able to do.

After that standard, there is a shift from addition and subtraction to multiplication and division. The standard 5.NF.A.3 begins this transition and it comes in two parts. The first part is understanding that a fraction can be seen as "division of the numerator by the denominator" ("Number & Operations—Fractions."). This is declarative knowledge, because students may be shown a proof or reasoning, but overall it is just a fact that needs to be known. The second part of this standard has students answer word problems, and for the same reason as the other standards like it, this part is connected to a problem-solving understanding.

The next standard is similar to one that was presented in fourth grade. To meet this standard, students are expected to use their understanding of multiplying fractions by whole numbers and multiplication in general to multiply fractions by other fractions or mixed numbers. Just as standard 4.NF.B.4 contributed to a problem-solving understanding, so does this one, because the students are expanding concepts based or all of their previous knowledge.

The next fifth grade standard, 5.NF.B.4.A, introduces a new way of viewing multiplication of a number and a fraction. Since it is introduce a new way of considering this concept, it is building conceptual understanding

After that, comes 5.NF.B.5.B, which has students reason about how multiplication by a fraction affects the product. This is conceptual understanding because it has student develop a new understanding based on the concepts they already understand

The next standard also has students solving word problems; therefore, like similar standards, it is contributing to a problem-solving understanding of mathematics for the same reasoning as the other standards.

The next standard, 5.NF.B.7, is similar to other standards that have been previously discussed. It has students "Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions" ("Number & Operations—Fractions."). Just like similar standards, it is connected to a problem-solving understanding. This is because, like with the other standards, students are expected to use all over their knowledge to determine a generalization for this concept.

The previous standard has three substandards that are all concerning division of fractions. The first of which is dividing a unit fraction by a non-zero whole number. This idea is both conceptual and procedural. Since the students are asked to actually perform the operation, the standard has a connection to the procedural. It is also conceptual, because the students are not only asked to compute, but also determine what it means for a fraction to be divided by a non-zero whole number.

The next substandard which is dividing a non-zero whole number by a unit fractions. For the same reasons as the previous standard, this standard connects to both a procedural and conceptual understanding.

The focus of the last fifth grade standard has been discussed multiple times throughout this analysis: solving word problems. Just as with all of those standards, it is connected to a problem-solving understanding. In fulfilling this standard, students are using all of their relevant knowledge to solve the problems.

Conclusion

Number of Standards in Each Domain of Mathematical Understanding				
	Declarative	***Procedural***	***Conceptual***	***Problem-Solving***
Third Grade	1	5	8	0
Fourth Grade	0	3	6	3
Fifth Grade	1	3	5	5

Table 4: Number of Standards in Each Domain of Mathematical Understanding

This table notes how many standards in each grade fell into each domain of mathematical understanding. Note: The totals for each grade is greater than the number of standards, because there were instances where a standard would fall under multiple domains.

The table above shows the breakdown of how many standards fell into each domain of the mathematical understanding. Although there is a lack of standards connected to declarative understanding, the other domains are represented fairly equally. This means that the Common Core State Standards have developed a framework for a well-rounded understanding of fractions. The next step is to determine if the curriculum reflects that.

Eureka Math

The Eureka Math curriculum was developed by Great Minds, a Washington D.C. based non-profit, in 2013. The mission statement of this group is "Ensure that all students regardless of their circumstance receive a content-rich education in the full range of liberal arts and sciences, including English, mathematics, history, the arts, science, and foreign languages" ("Frequently Asked Questions"). The math curriculum was developed in partnership with the New York State Education Department, the same department that developed the EngageNY curriculum, another well-known, newer curriculum. Great Minds describes their development team for their math curriculum saying, "A team of more than 75 master teachers and mathematicians work together to create and continuously improve *Eureka Math*" ("Who We Are: Our Team").

The curriculum is composed of free, downloadable pdf's, and those pdf's are also compiled into textbooks and workbooks that can be purchased. Since it is available for free, it is difficult to track how many schools use the materials, but materials have been downloaded in all fifty states. In a study done by the RAND Corporation, a research organization with locations around the world, "Eureka Math is now the most widely used math curriculum in the United States" where "of 1,168 teachers responding to a nationwide survey, 57% of elementary school teachers and 47% of secondary teachers said they use either Eureka Math, or the version of this curriculum developed for EngageNY.org" (Great Minds).

Just as with the Common Core State Standards, there have been positive and negative responses to this curriculum and others like it. The majority of the responses and reviews I have found have been parents explaining how difficult and unnecessary some of the processes or activities are within the curriculum. There has been very little professional or academic analysis of this curriculum.

To restate what I mentioned during the section on the Common Core, it is important to note that although the curriculum may promote a certain type of understanding, whether or not the student gains that understanding relies heavily on the instructor and the individual student. With that being said, this is simply an analysis of the curriculum and not an instructional guide on teaching these concepts.

Common Core Alignment

Third Grade

Topic A covers lessons one through four, and it focuses on the fact that when discussing fractions, the whole must be divided into equal parts. It also discusses how, for a whole divided into *b* equal pieces, one piece represents *1/b* of the whole. This concept aligns directly with Common Core State Standard for Mathematical Content 3.NF.A.1, which states that student will "Understand a fraction *1/b* as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction *a/b* as the quantity formed by parts of size *1/b*" ("Number & Operations—Fractions").

This unit fully teaches this standard. Every lesson discusses dividing wholes into equal parts and the idea of unit fractions, which are "fractions with numerator 1" ("Grade 3, Module 5: Teacher Edition," 7), meaning the unit fraction for a whole divided into *b* equal pieces would have the unit fraction *1/b*. Lessons two and three, although they do not formally introduce fraction notation for non-unit fractions, do introduce the idea of non-unit fractions. The problem and homework set in lesson two have students saying how many pieces are shaded and how many pieces are in the whole. An example is pictured below

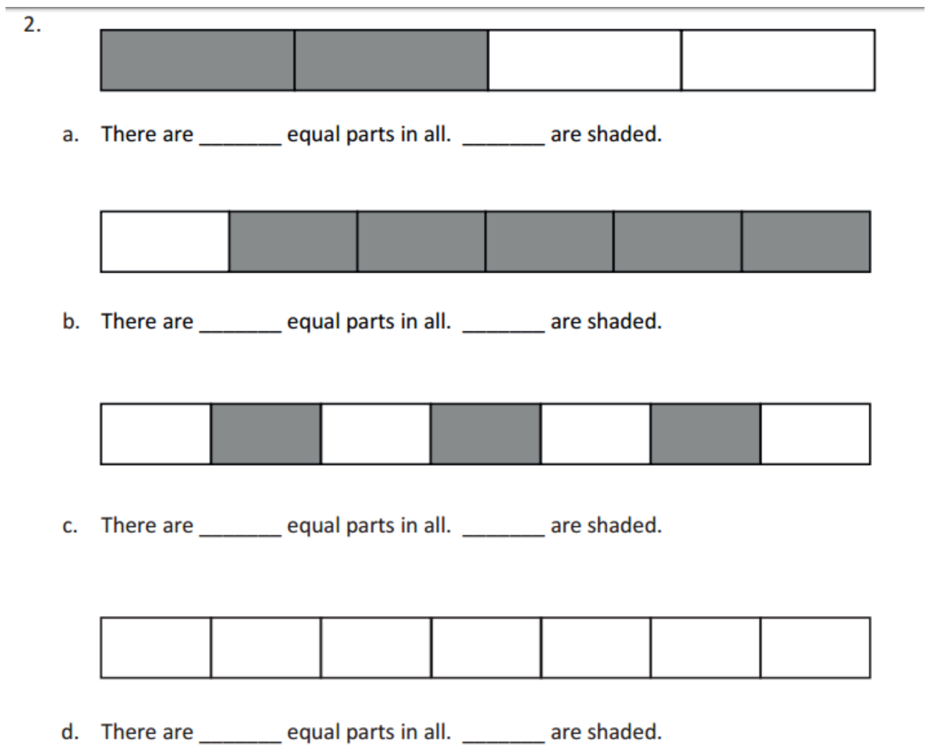


Figure 2: Problem from Lesson Two in the Third Grade Unit

This problem has students naming how many parts are shaded and how many parts are in the whole ("Grade 3, Module 5: Teacher Edition," 25).

Also, in lesson three's concept development, which is the equivalent to the lecture for the lesson, is dedicated to having student count and describe non-unit fractions.

The second topic, Topic B, covers chapters five through nine and attempts to fulfill two Common Core standards. The first is 3.NF.A.1 which states "Understand a fraction *1/b* as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction *a/b* as the quantity formed by parts of size *1/b*" ("Number & Operations – Fractions"), and the second is 3.NF.A.3.C which says "Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers." ("Number & Operations – Fractions").

Just as in Topic A, this section satisfies standard 3.NF.A.1. Lessons five and nine revisit the importance of equal parts and understanding what the whole is, which satisfies the first part of the standard. Lesson six emphasizes non-unit fractions and expressing them in fraction form, which satisfies the second part of the standard. Because of this, the second section meets this standard.

As for the second standard, 3.NF.A.3.C, this concept is not explicit taught in this section. Fraction greater than one are discussed and there is a brief mentions of a whole being three thirds, but the materials do not explicitly teach the concept in this section. For that reason, I would say that this section does not fully meet that standard

The third section, Topic C, contains lessons ten through thirteen and covers "comparing unit fractions and specifying the whole" ("Grade 3, Module 5: Teacher Edition," 109). The lessons in this section attempts to meet five standards.

The first of these standards discusses comparing fractions with either the same denominator or numerator, understanding that fraction comparisons are only valid if the fractions connect to the same whole, and describing the comparisons with the appropriate symbol: >, =, or <. Since this section focuses on unit fractions, there were definitely more instances of comparing fractions with the same numerator, since unit fractions always have a numerator of one. For the next part of the standard, lesson eleven centers on the importance of understanding and distinguishing the whole, which directly meets part of this standard. As for the last part of the standard, lesson ten included questions in the problem set that required the students to use >, =, or < to describe the comparison including this one:

4. Use >, <, or = to compare.

a.	1 third	◯	1 fifth	b.	1 seventh	◯	1 fourth
c.	1 sixth	◯	$\frac{1}{6}$	d.	1 tenth	◯	$\frac{1}{12}$
e.	$\frac{1}{16}$	◯	1 eleventh	f.	1 whole	◯	2 halves

Figure 3: Problem from Lesson Ten in the Third Grade Unit

This problem is an example of when students must use >, =, or < to compare two fractions (“Grade 3, Module 5: Teacher Edition,” 119).

Overall, this section covered the majority of this standard, and the only thing it was missing was comparisons of fractions with the same denominator, but that is also information that has been covered in the preliminary introduction to fractions from the previous sections.

The next standard connected to this section expects students to “Understand a fraction *1/b* as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction *a/b* as the quantity formed by a parts of size *1/b*” (“Number & Operations – Fractions”). This standard was discussed in part while discussing the previous standard. To reiterate, there was no explicit instruction concerning this topic in this section. I think the curriculum as whole meets this standard, but this section by itself does not.

This section also looks to fulfill standard 3.NF.A.3.A, which relates to equivalent fractions by reasoning using size and/or place on a number line. This section does not include work with a number line but it used many visual models. In lesson eleven, students are asked not only to create visual representations of equivalent fractions, but also reason about why a fraction was greater than, less than, or equal to another fraction. Although they do not use a number line in this section, I think they still meet this standard, because they meet the overall goal, which is to have students compare fractions in a conceptual, representational way.

Overall, the section focuses more on comparing to different fractions than comparing equivalent fractions. The last two standards that are connected to this section focus on equivalent fractions. The first calls for student to create and distinguish equivalent fractions and explain why they are equivalent. The other standard has students representing whole numbers as fractions and distinguishing fractions that equal whole numbers. These ideas are never explicitly discussed, only briefly included in problem set. These problems usually have students explaining why two fractions are equivalent or why a whole number is equivalent to a fraction, but little to no work on creating equivalent fractions. Since that is such a major part of the overall goal of the standards, this section does not meet these standards.

The main topic of focus for the fourth section, Topic D, is “fractions on the number line” (“Grade 3, Module 5: Teacher Edition,” 165). This section covers lessons fourteen through nineteen, and it looks to meet three standards.

The first standard is “Understand a fraction as a number on the number line; represent fractions on a number line diagram” (“Number & Operations – Fractions”). Lessons fourteen and fifteen have an emphasis on how to represent fractions on number lines, and they oftentimes use the fraction strips that the students have used throughout the unit to develop an understanding of number lines that include fractions. The whole section works to develop what a number line with fractions means. Therefore, this section meets this standard

The next standard is one that also connects to the previous section, but not was satisfied by that section, and it deals with representing whole number as fractions and recognizing fractions that are equal to whole numbers. The focus of both lessons sixteen and seventeen is representing number greater than one on a number line that includes fractions, meaning students have to represent the whole numbers as fractions. These lessons also include discussions where the students were asked to identify fractions that represented whole numbers. Where the previous section did not have explicit instruction about representing whole numbers as fractions, this section does, and therefore, it satisfies this standard.

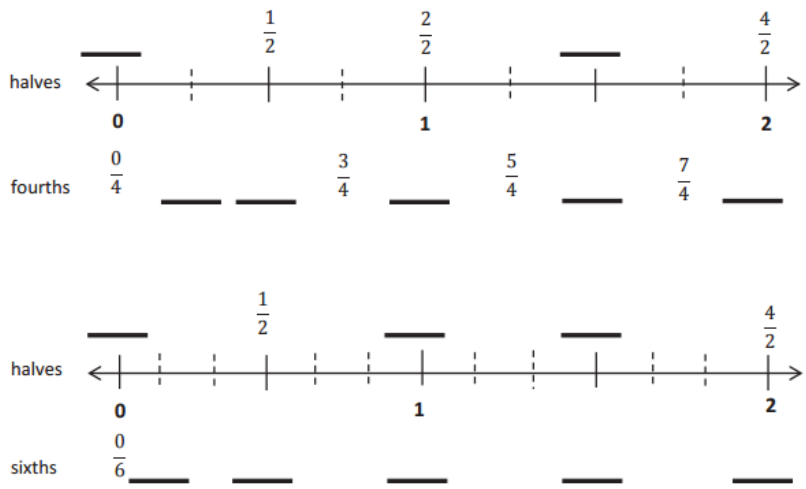
The last standard for this section is also one that was discussed in a previous section. This standard intends students to be able to “Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions” (“Number & Operations – Fractions”). The previous section was missing comparisons of fractions with the same denominator. Lesson nineteen in this section is specifically focused on this concept. Since the previous section handled the rest of this standard, this standard has now been completely met.

The next section, Topic E, has a focus on equivalent fractions, and spans from lesson twenty to lesson twenty-seven. It sets out to fulfill four standards.

The first standard is 3.NF.A.3 in which students are expected to “Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size” (“Number & Operations – Fractions”). Determining if fractions are equal and finding equivalent fractions happens in every lesson in this section; therefore, it definitely meets the first part of this standard. As for comparing fractions based on size, this does not happen much, if at all. This is to be expected in part though, since the topic of the section is on equivalence, not comparison. Overall this standard is partial fulfilled.

The next standard this section attempted to meet is “understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line” (“Number & Operations – Fractions”). Lesson twenty-one focuses on finding equivalent fractions using a number line. One problem that does this effectively is picture below:

1. Use the fractional units on the left to count up on the number line. Label the missing fractions on the blanks.



2. Use the number lines above to:
   - Color fractions equal to 1 half blue.
   - Color fractions equal to 1 yellow.
   - Color fractions equal to 3 halves green.
   - Color fractions equal to 2 red.

Figure 4: Problem from Lesson Twenty-one in the Third Grade Unit

This problem has students generating equivalent fractions using a number line by first having students fill in the missing fractions then color the fractions that are equivalent ("Grade 3, Module 5: Teacher Edition," 249).

On the other hand, lesson twenty-seven has students determining equivalence based upon concrete or pictorial representation of fractions. There are other lessons in this section that included questions in the problem set or demonstrations in the class discussions using these ideas. Because of this, this section meets this standard.

The third standard connected to this section is standard 3.NF.A.3.B, and it calls for students to create their own equivalent fractions and explain how they know it is equivalent. This is a part of multiple lessons in this section including lessons twenty-one, twenty-two, and twenty-seven. They show it through visual models in the problem sets and in the Concept Development part of the lesson. It is discussed that equivalent fractions can be found by dividing the current pieces into smaller, equal sized piece, which changes the fraction representation, but not the amount it represents. Therefore, this section does meet this standard.

The last standard this section is trying to meet is representing the fraction equivalent of whole numbers. This standard has been connected to other sections before this, but this section conveys this standard well. This concept is included in some way in at least six of the eight lessons in this section. It is included in every way from a point of discussion in lesson twenty-one, to tasks in the problem sets in lessons twenty-four, through twenty-six. Since this concept is included in many of the lessons of this section in a variety of ways, this section meets this standard.

The last section of the third grade curriculum, Topic F, covers the topic of "comparison, order, and size of fractions" ("Grade 3, Module 5: Teacher Edition," 327), and it covers lessons twenty-eight through thirty. Overall, this section attempts to meet standard 3.NF.A.3.D which calls for student to

> Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions [Number & Operations - Fractions]

This topic has been visited throughout this entire unit, especially during the third and fourth sections.

The first part of this standard is comparing fractions with the same numerator by considering their sizes. All of the Concept Development sections and problem sets focus on doing these comparisons both using visual models and number lines. This section only focuses on comparing fractions with a common numerator, and did not discuss the importance of referencing the same whole. Therefore, it did not satisfy those parts of the standard. The last part of this standard is using >, =, < correctly and justifying the response. This is done during the problem set from lesson twenty-nine. In that problem set, students are asked to compare to fractions and draw a model to represent that fraction. Since it only met half of the parts of this standard, it did not really meet the whole standard, but it should be noted that the combination of all the other sections has satisfied this standard.

Standards Discussed in Each Topic of the Third Grade Curriculum							
Topic >> Standard ∨∨	***A***	***B***	***C***	***D***	***E***	***F***	***Did the unit overall meet the standard?***
3.NF.A.1	X	X	O				YES
3.NF.A.2				X			YES
3.NF.A.2.A							YES, Explanation Below
3.NF.A.2.B							YES, Explanation Below
3.NF.A.3					/		YES, Explanation Below
3.NF.A.3.A			X		X		YES
3.NF.A.3.B			O		X		YES
3.NF.A.3.C		O	O	X	X		YES
3.NF.A.3.D			/	X*		/	YES

Table 5: Standards Discussed in Each Topic of the Third Grade Curriculum

This table notes what standard were attempted, met, or partially met in the third grade curriculum. “O” indicates standard that were attempted but not met. “X” indicates standards that were met. “/” indicates standards that were partially met. *The first part of this standard was met within Topic C and the second part was satisfied in Topic D.

Fraction education in third grade has nine standards associated with it. These materials explicitly teach six of these nine. I believe that they also meet the other three standards without specifically noting it within the lesson plans.

The first standard that is not explicit noted is 3.NFA.2.A in which students are expected to "Represent a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size *1/b* and that the endpoint of the part based at 0 locates the number *1/b* on the number line" ("Number & Operations – Fractions"). This is a substandard of another standard which has students representing fractions on a number line and understanding what the number line represents. In order to successfully do that, students must be able to define an interval and partition it the correct number of times. Since the skill expressed in standard 3.NFA.2.A is necessary to meet the larger standard that was met in the curriculum, this standard is also met in these materials.

The next standard that is not specifically designated by the materials was 3.NFA.2.B, another substandard about denoting fractions on a number line. The only difference between them is the first standard works on the interval from zero to one and this standard is not limited to one. For the same reason that the other standard was met with this curriculum, this standard was also satisfied.

The final standard has students "Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size" ("Number & Operations – Fractions"). This standard was partial met in Topic E. This was only partial met, because that section only discussed equivalence. The next part of the standard met because that standard has four substandard in which the fourth one is centered on comparing fractions by looking at their size. Therefore, the students were taught to "compare fractions by reasoning about their size" ("Number & Operations –

Fractions”). This is fulfilling the part of the standard that was missing, and thus this standard was actually met.

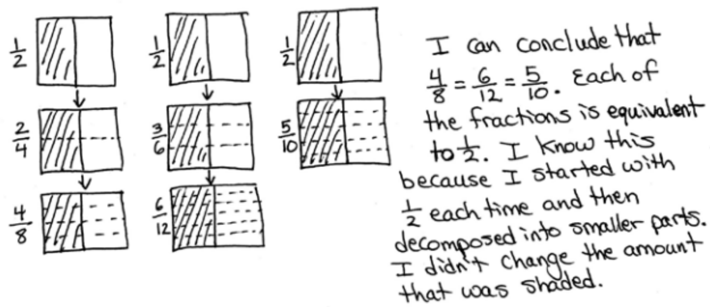
Since all nine standards associated to the third grade concerning fraction was met within this curriculum, this curriculum is aligned with the Common Core Standards

Fourth Grade

The first topic, Topic A concerning fraction in fourth grade is “decomposition and fraction equivalence” (“Grade 4, Module 5: Teacher Edition,” 15). This section covers lesson one through six and seeks out to fulfill three standards.

The first standard is 4.NF.B.3.B, and this standard expects students to “decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.” (“Number & Operations – Fractions”). This standard is discussed in five out of the six lessons in this section. Each of these sections focuses on decomposing fractions into the sum of smaller fractions and then representing that using a number sentence and/or a visual diagram. One particularly good way this was shown is in lesson six, when the information from lesson five is being reviewed, in which it is shown that the fraction $\frac{1}{2}$ can be decomposed a variety of ways:

Use area models to prove that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$, $\frac{1}{2} = \frac{3}{6} = \frac{6}{12}$, and $\frac{1}{2} = \frac{5}{10}$. What conclusion can you make about $\frac{4}{8}$, $\frac{6}{12}$, and $\frac{5}{10}$? Explain.



Note: This Application Problem builds from Lesson 5, where students decomposed unit fractions using area models to show equivalence. Consider leading a discussion with a question, such as "Why can you show $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$ on one model, $\frac{1}{2} = \frac{3}{6} = \frac{6}{12}$ on another, and $\frac{1}{2} = \frac{5}{10}$ on another?" Or perhaps lead with a question, such as "Why can't you show $\frac{1}{2} = \frac{2}{4} = \frac{5}{10}$ on the same area model?"

Figure 5: Discussion from Lesson Six in the Fourth Grade Unit

This discussion topic reviews information presented in lesson five and emphasizes that fractions can be decomposed numerous ways ("Grade 4, Module 5: Teacher Edition," 78).

Since this entire standard is discussed, this section does meet this standard.

The next standard is 4.NF.B.3.A, which calls for students to "Understand addition and subtraction of fractions as joining and separating parts referring to the same whole" ("Number & Operations – Fractions"). The section does include a lot of exposure to addition of fractions, but never discusses subtraction of fractions; therefore this section only partially meets this standard.

The last standard this section attempts to meet is 4.NF.B.4.A, which has students "understand a fraction *a/b* as a multiple of *1/b*" ("Number & Operations – Fractions"). In lesson three, the Concept Development focused on this topic using examples like $\frac{3}{4} = 3 \times \frac{1}{4}$ and $\frac{10}{8} = 10 \times \frac{1}{8}$ ("Grade 4, Module 5: Teacher Edition," 44-45). The problem set for this lesson also centers on this concept. The Concept Development in lesson four has exercises where students have to decompose fraction then write the decompositions as multiples of unit fractions. This section shows this concept and the applications of this idea; therefore, this section meets this standard.

The second section of this curriculum, Topic B, covers the topic “fraction equivalence using multiplication and division” (“Grade 4, Module 5: Teacher Edition,” 91), and contains lessons seven through eleven. This section looks to fulfill two standards.

The first standard is 4.NF.A.1, and in order to be fulfilled, students must

> Explain why a fraction *a/b* is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. [“Number & Operations – Fractions”]

Lesson seven uses a visual model to introduce this topic. After seeing a few examples of equivalent fractions using an area model, the students are prompted to make conclusion about what the multiplication does to the model and thus to the fraction. Lesson eight fulfills the second part of this standard. The problem set focuses on showing equivalent fractions and creating more equivalent fractions with visual models. Since all parts of this standard are met, this section fulfills this standard

The next standard connected to this section is 4.NF.B.3.B. This standard is met in the previous section as well and it has students “decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.” (“Number & Operations – Fractions”). This is used in lesson eleven as a method for approaching finding equivalent fractions in addition to using multiplication or division. There is no explicit instruction on this idea, but this standard was met in the previous sections. Since this section revisits it and discusses as a valid method for finding equivalent fractions, this section meets this standard.

The third topic of this unit, Topic C, is on comparing fractions and this section contains lesson twelve through fifteen. This section only has one standard of focus, 4.NF.A.2, which requires students to

> Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. [“Number & Operations – Fractions”]

This standard is actually an extension of a standard from third grade. Where third graders were only comparing fraction with either the same numerator or the same denominator, fourth graders are expected to compare any to fractions, especially those with different numerators and denominators.

Lesson twelve and thirteen are focused on using benchmark fractions to compare fractions and the concept is included in both the Concept Development and the problem set of these lessons. Therefore the first part of this standard is met. Lessons fourteen and fifteen have students finding common numerators or denominators to compare fractions, which meets another part of this standard. All of these lessons have students using the proper symbols to describe the comparison. Although this section does not explicit cover that when comparing fraction, the fractions must refer to the same whole, that concept was developed throughout third grade; so, students should have already had exposure to that concept. Therefore, this section meets this standard.

The next topic, Topic D, covers addition and subtraction of fractions. This section spans over lessons sixteen through twenty-one and focuses on three standards.

The first standard covered by this was 4.NF.B.3.A. In order to meet this standard, students must “understand addition and subtraction of fractions as joining and separating parts referring to the same whole” (“Number & Operations – Fractions”). All of the lessons in this section have students performing fraction addition and subtraction, oftentimes having them draw visual models to represent these operations. Section B included a discussion on addition of fraction; so, lesson sixteen introduces what it means to subtract fractions in its Concept Development. The problem set of this lesson includes both addition and subtraction of fractions and has student using visual models to represent these equations. Therefore this standard is met.

The next standard is 4.NF.B.3.D which has students solving word problems that requires addition and subtraction of fractions with the same denominator. Lesson nineteen focuses on solving word problems. The Concept Development goes over how to approach these problems and the problem set has the student apply that approach and solve the problems, which include both addition and subtraction problems. Since there is an entire lesson that went over this topic in detail, this section meets this standard.

The final standard that this section attempts to fulfill is 4.NF.A.1, and it states,

> Explain why a fraction *a/b* is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. [“ Number & Operations – Fractions”]

This is also a topic that is discussed in another section; so, this section just revisits this topic and shows another application of it. One lesson that uses this concept is lesson twenty when students have to “add two fractions with related units” (“Grade 4, Module 5: Teacher Edition,” 264). In this lesson, they use tape diagrams and number lines specifically:

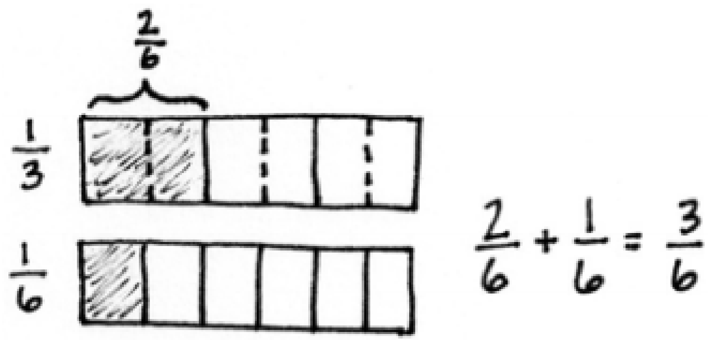


Figure 6: Tape Diagram Example from Lesson Twenty in the Fourth Grade Unit

This shows addition fractions with unlike denominators using a tape diagram (“Grade 4, Module 5: Teacher Edition,” 267).

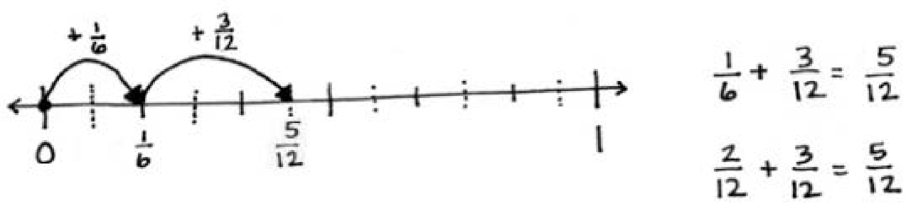


Figure 7: Number Line Example from Lesson Twenty in the Fourth Grade Unit

This shows addition of fraction with unlike denominators using a number line (“Grade 4, Module 5: Teacher Edition,” 268).

Therefore, since this section revisits this topic and often a brief explanation again, this section meets this standard.

The fifth section, Topic E, covers “extending fraction equivalence to fractions greater than 1” (“Grade 4, Module 5: Teacher Edition,” 302). This section is connected to four standards concerning fractions and extends over lessons twenty-two through twenty-eight.

The first standard is 4.NF.A.2, in which the expectation is that students

> Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions ["Number & Operations – Fractions"]

Lesson twenty-six focuses on comparing fraction greater than one using benchmark fractions. Then, lesson twenty-seven compares fractions by first finding common denominators or common numerators. When doing so, the students also have to use the proper symbols to describe the comparison. Hence, this section covers all the parts of this standard and it is fully met.

The next standard is 4.NF.A.1, and in order meet this standard, students must

> Explain why a fraction *a/b* is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. ["Number & Operations – Fractions"]

Many of the lessons in this section discuss converting mixed numbers to fractions, oftentimes using number lines or number bonds. In those lessons, they do not explicitly explain why *a/b* is equivalent to *(n*a)/(n*b)*, but this topic is discussed in previous sections. What this section does do is create another application for this concept. Because of this, this section meets this standard.

The last two standards for this section are 4.NF.B.3 which has students "understand a fraction *a/b* with *a>1* as a sum of fractions *1/b*" ("Number & Operations – Fractions"), and 4.NF.4.A, which discusses the concept that "a fraction *a/b* as a multiple of *1/b*." ("Number & Operations – Fractions"). These topics are discussed in lessons twenty-three and twenty-four when converting mixed numbers to fractions and multiplying unit fractions by whole numbers are first explained. Since this is a topic that has been discussed previously, it is not necessary to completely reteach these concepts. Just with the previous standard, their inclusion in this section allows the students to see another application of these concepts

The next section of this unit, Topic F, includes lessons twenty-nine through thirty-four, and it discusses “addition and subtraction of fraction by decomposition” (“Grade 4, Module 5: Teacher Edition,” 385). This section has one standard connected to it: 4.NF.B.3.C. This standard says that student should be able to “add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction” (“Number & Operations – Fractions”).

Lesson thirty is an introduction to addition of mixed numbers by focusing on adding a mixed number and a fraction, and lesson thirty-one is when the students are explicitly taught how to add two mixed numbers. They do this by having the students add the whole part and the fraction part separately then add the sums of these two parts:

Problem 2: Add mixed numbers when the sum of the fractional units is greater than 1 by combining like units.

T: (Write $2\frac{5}{8} + 3\frac{5}{8}$.) Right away, we see that the sum of the eighths is greater than 1.

T: The sum of the ones is…?

S: 5.

T: The sum of the eighths is…?

S: 10 eighths.

T: Take out 8 eighths to make one.

S: $1\frac{2}{8}$. → $\frac{8}{8}$ and $\frac{2}{8}$. (Record with a number bond.)

T: (Write the following.)

$$2\frac{5}{8} + 3\frac{5}{8} = 5 + \frac{10}{8}$$
$$= 5 + \frac{8}{8} + \frac{2}{8}$$
$$= 6\frac{2}{8}$$

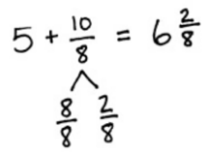


Figure 8: Instruction for Adding Mixed Numbers from Lesson Thirty-One in the Fourth Grade Unit

This figure shows how the students are taught to consider each part of the mixed number then combine the sums of those two parts (“Grade 4, Module 5: Teacher Edition,” 415).

Therefore, the first part of this standard is met.

Lesson thirty-two introduces subtraction of mixed numbers by first working with subtraction of a fraction from a mixed number. Lessons thirty-three and thirty-four then teach the concept in the Concept Development and allow the student to practice it during the problem set. Therefore, the second part of this standard is also met. Since both addition and subtraction of mixed numbers are covered in this section, this section meets that standard.

Topic G covers the idea of “repeated addition of fractions as multiplication” (“Grade 4, Module 5: Teacher Edition,” 462), and it includes lessons thirty-five through forty. This sections only has one standard connected to it, and, just like the previous section, the standard is 4.NF.B.4 in which students are expected to “Apply and extend previous understandings of multiplication to multiply a fraction by a whole number" ("Number & Operations – Fractions").

Lesson thirty-five extends the Associative Property of Multiplication, which says for quantities a, b, and c, $a \times (b \times c) = (a \times b) \times c$, to multiply a fraction by a whole number. The beginning of the Concept Development part of the lesson has the teacher explain $4 \times (3\ fifths) = (4 \times 3) fifths$. The concept this then extended where instead of writing the unit name, the fraction representation is used and the students find that $4 \times (3/5) = (4 \times 3) \times (1/5)$ ("Grade 4, Module 5: Teacher Edition," 468). Lesson thirty-six then goes on to expand on that concept further.

Lesson thirty-seven uses the Distributive Property of Multiplication over Addition, which says for the quantities a, b, and c, $a \times (b + c) = a \times b + a \times c$, to show how to multiply mixed numbers by whole numbers. In this process the mixed number is separated into the sum of its whole number part and its fractional part, the whole number in the original problem is then distributed over that sum. Just as with lesson thirty-six, lesson thirty-eight expands and this concept further.

Since this section offers multiple ways of considering multiplying whole numbers by fractions and those different ways are based upon the previous understanding of multiplication, this section meets the standard.

The last section for the fourth grade curriculum, Topic H, has students looking for patterns within fractions. This sections does not cover a specific standard concerning fractions; so its analysis will be limited to if there is evidence of conceptual instruction and in the case that there is a standard that is not met in another chapter.

Standards Discussed in Each Topic of the Fourth Grade Curriculum									
Topic >> Standard ∨∨	***A***	***B***	***C***	***D***	***E***	***F***	***G***	***H***	***Did unit overall meet the standard?***
4.NF.A.1		X		X	X				YES
4.NF.A.2			X		X				YES
4.NF.B.3					X				YES
4.NF.B.3.A	/			X					YES
4.NF.B.3.B	X	X							YES
4.NF.B.3.C						X			YES
4.NF.B.3.D				X					YES
4.NF.B.4							X		YES
4.NF.B.4.A	X				X				YES
4.NF.B.4.B									YES, Explanation below
4.NF.B.4.C									YES, Explanation below

Table 6: Standards Discussed in Each Topic of the Fourth Grade Curriculum

This table notes what standard were attempted, met, or partially met in the third grade curriculum. “O” indicates standard that were attempted but not met. “X” indicates standards that were met. “/” indicates standards that were partially met.

Within the text, nine out of the eleven standards are directly connected to and fulfilled within a section. There are two standards that are not explicitly noted in the materials: 4.NF.B.4.B and 4.NF.B.4.C, which are substandards of the standard discussed in Topic G: “Apply and extend previous understandings of multiplication to multiply a fraction by a whole number” (“Number & Operations – Fractions”).

The first of these states, “Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number” (“Number & Operations – Fractions”). This topic is discussed in lessons thirty-five and thirty-six, when the Associative Property is being used. In order to use that property, the students must use this concept. Therefore, this standard is met by these materials.

The other standard expects students to be able to “Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem” (“Number & Operations – Fractions”). This goal is the focus of lessons thirty-nine and forty, where the whole Concept Development and problem set of both lessons is on solving word problems that require multiplication of a fraction by a whole number. Therefore, this standard is also met.

Since those two standards are actually met within the text, the curriculum concerning fractions for fourth grade is aligned with the Common Core State Standards, just as the third grade standards were.

Fifth Grade

The fifth grade curriculum is actually composed of two parts. The first part is focuses on addition and subtraction of fractions, and the second part of focuses on multiplication and division of fractions.

In the first part of this curriculum, the first section, Topic A, only has a fourth grade standard connected to it and serves as a review section. Therefore, this section will not be analyzed for its fifth grade Common Core State Standards alignment.

The second section of this curriculum, Topic B, discusses “making like units pictorially” (“Grade 5, Module 3: Teacher Edition.” 40) and it spans over lessons three through seven. This section is connected to two standards: 5.NF.A.1 and 5.NF.A.2.

The first of these standards expects students to “Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators” (“Number & Operations – Fractions”). Lesson three is focused on the addition with the Concept Development and problem set composed entirely of such problems. Lesson five is centered on the subtraction. Although adding and subtracting mixed numbers is not a topic that has a lesson specifically dedicated to it, it is still used in the Concept development of lessons four and six and in the problem sets of most of the lessons. Therefore, this standard is met.

The next standard connected to this section is met if students can

> "Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers" ["Number & Operations – Fractions"]

Lesson seven is focused on solving word problems that require adding and subtracting fractions; hence, this section meets this standard as well.

The next section of part one discusses "making like units numerically" ("Grade 5, Module 3: Teacher Edition." 126). This section is also connected to 5.NF.A.1 and 5.NF.A.2 and covers lessons eight through twelve.

The first standard, which was discussed in the previous section, deals with adding and subtracting fractions and mixed numbers with unlike denominators. Lessons eight through ten discuss the addition portion and include more examples of adding mixed numbers than in the previous section. Lesson eleven and twelve cover the subtraction part of the standard, and also include more examples of operating with mixed numbers. Therefore, since the unit covers both parts of this standard, the standard is met.

The second standard connected to this section was also taught previously, and it calls for students to solve word problems using addition and subtraction of fractions.

The problem sets for each of these lessons have word problems that require addition and subtraction of fractions. An example of a word problem requiring addition from the text is “Jackie brought $\frac{3}{4}$ of a gallon of iced tea to the party. Bill brought $\frac{7}{8}$ of a gallon of iced tea to the same party. How much iced tea did Jackie and Bill bring to the party?” (“Grade 5, Module 3: Teacher Edition,” 154). Another example that requires subtraction instead is “Sandy ate $\frac{1}{6}$ of a candy bar. John ate $\frac{3}{4}$ of it. How much more of the candy bar did John eat than Sandy?” (“Grade 5, Module 3: Teacher Edition,” 187). Therefore, this section also fulfills this standard.

The last section in this part of the curriculum, Topic D, discusses other applications of fraction addition and subtraction. This section is connected to the same two standards as the two previous sections, and it covers lessons thirteen through sixteen.

The first standard, concerning addition and subtraction of fractions and mixed number, connects to lesson fourteen since the topic of the lesson is adding and subtracting multiple fractions. Therefore, this section also fulfills this standard.

The second standard is about students doing word problems that use addition and subtraction of fractions. Lesson fifteen is focused on “solving multi-step word problems” (“Grade 5, Module 3: Teacher Edition.” 234). Just as with the previous section, each of the lessons within this unit has such word problems in its problem sets. Therefore, this standard is, once again, met.

The second part of the fifth grade math curriculum focuses on multiplication and division. The first topic discussed in this unit, Topic A, is “line plots of fraction measurements” (“Grade 5, Module 4: Teacher Edition,” 14). This section contains only lesson one and is not connected to any of the designated standards. Therefore, this section will not be analyzed for its Common Core alignment.

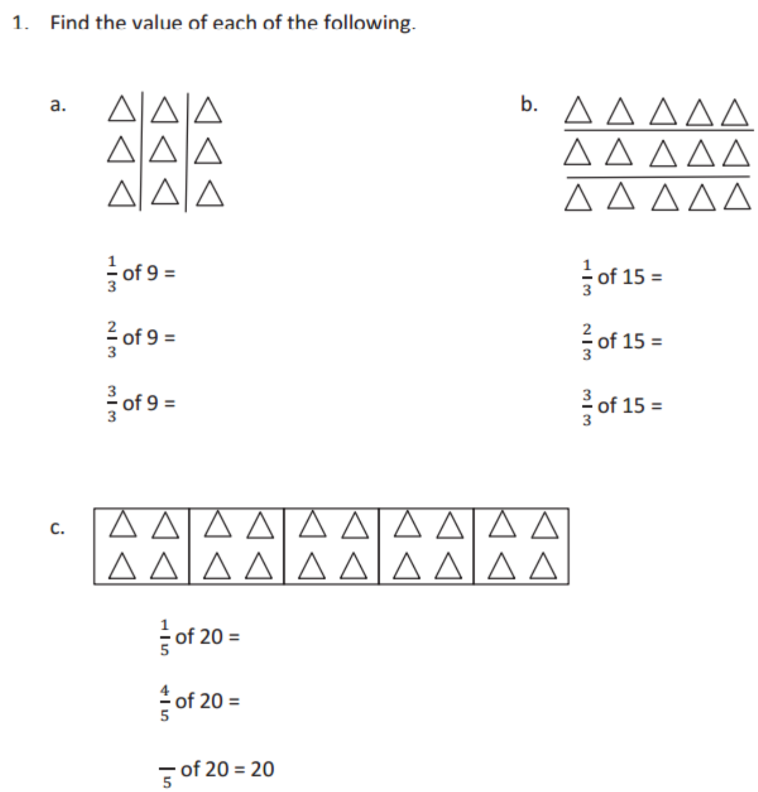
The next section in this unit, Topic B, covers the topic "fractions as division" ("Grade 5, Module 4: Teacher Edition," 27) and looks to fulfill standard 5.NF.B.3, which has students

> "interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem" ["Number & Operations – Fractions"]

This section contains lessons two through five.

Lesson two introduces the topic of fractions as division using concrete examples. The Concept Development section goes over this idea and the problem set gives the students the opportunity to practice working with this concept, including providing word problems that require division between whole numbers to be represented as a fraction or mixed number. Lesson three expands on this idea and incorporates the standard algorithm for division into the concept. Lesson four uses visual models to explain the concept. In addition to the word problems included in many of the problem sets in this section, lesson five focuses entirely on solving word problems that use this idea. Since this section explains fractions as division and includes word problems, this section meets this standard.

The next section of this unit, Topic C, focuses on "multiplication of a whole number by a fraction" ("Grade 5, Module 4: Teacher Edition," 85) and includes lessons six through nine. The standard connected with the section is 5.NF.4.A, in which students will "Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$" ("Number & Operations – Fractions"). In lessons six and seven, they use the language *(a/b)* of *q* and visual models to represent the process to ease the transition and explain the concept of *(a/b)*q=a*q/b.* An example of this from the problem set of lesson six is shown below in which students have to find the value of parts of whole numbers:


1. Find the value of each of the following.

a.

$\frac{1}{3}$ of 9 =

$\frac{2}{3}$ of 9 =

$\frac{3}{3}$ of 9 =

b.

$\frac{1}{3}$ of 15 =

$\frac{2}{3}$ of 15 =

$\frac{3}{3}$ of 15 =

c.

$\frac{1}{5}$ of 20 =

$\frac{4}{5}$ of 20 =

$\frac{}{5}$ of 20 = 20

Figure 9: Problem from Lesson Six of the Second Fifth Grade Unit

These problems represent multiplying fractions and whole numbers as taking a fraction part of the whole number ("Grade 5, Module 4: Teacher Edition," 96).

By using the models, the students see that this mathematical expression represents partitioning an amount *q* into *b* parts then taking *a* of those parts, which is the main goal of this standard. Lesson eight goes on to explain the process and why this is true by using the Commutative Property of Multiplication which states that for quantities *a* and *b, a*b=b*a*, which speaks to the later part of this standard. Therefore, this section meets this standard.

The fourth section of this unit, Topic D, discusses "fraction expressions and word problems" ("Grade 5, Module 4: Teacher Edition," 138) and contains lessons ten, eleven, and twelve. There are two standards connected to this section: 5.NF.B.4A and 5.NF.B.6

The first standard is the same as the one from the previous section which expects students to "interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations" ("Number & Operations – Fractions"). This section did not add anything else to this beyond what was discussed in the previous section. This concept is used throughout, but it is more of a review section than anything. With that being said, it does discuss this topic and emphasizes what the expression means; therefore, it does meet this standard.

The next standard calls for students to "solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem" ("Number & Operations – Fractions"). Lessons eleven and twelve focus on solving work problems involving multiplication of fractions and whole numbers, but there is nothing about mixed numbers in this section. Since this is a distinction that is made in this standard, this section only partially meets this standard.

The next section, Topic E, focuses on "multiplying a fraction by a fraction" ("Grade 5, Module 4: Teacher Edition," 197) and includes lessons thirteen through twenty. There are three standards connected to this section: 5.NF.B.4.A, 5.NF.B.6, and

5.NF.B.4.B, but, the last standard was omitted from this analysis for its indirect connection to fraction education.

The first standard connected to this section has been discussed and was met in the previous two sections. Lesson thirteen expands on the idea from the previous section in that the previous section only has students multiplying fractions by whole numbers, but in the Concept Development and the problem set for this lesson, students are multiplying unit fractions with other unit fractions using the same description of taking *a* parts of *q* divided into *b* parts. The Concept Development part of the lesson presents the situation “Jan has $\frac{1}{3}$ a pan of crispy rice treats. She sends $\frac{1}{4}$ of the treats to school with her children. What fraction of a pan of crispy rice treats does Jan send to school?” (“Grade 5, Module 4: Teacher Edition,” 203). This area model is then produced to represent the problem:

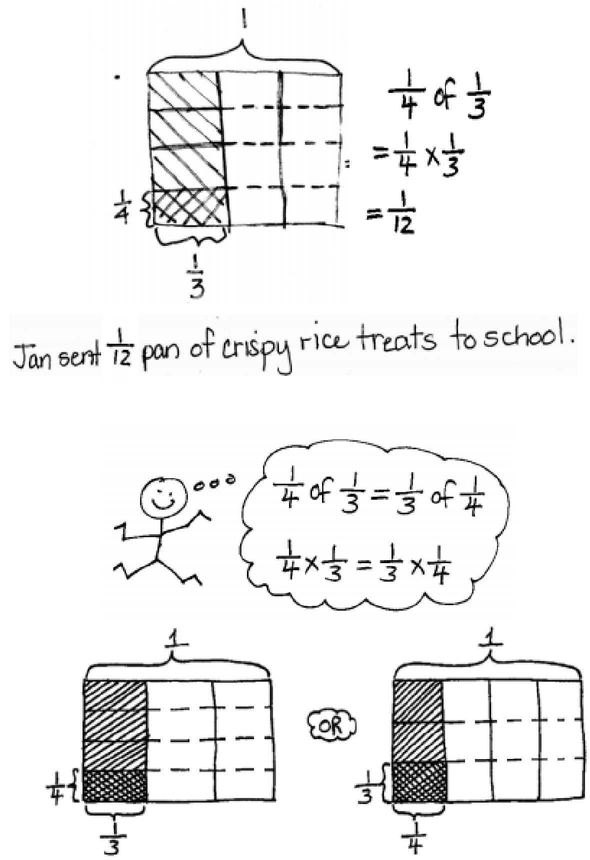


Figure 10: Area Model from Lesson Thirteen from the Second Fifth Grade Unit

This representation shows multiplying two fractions by using an area model, which is similar to what was used when multiplying a fraction by a whole number ("Grade 5, Module 4: Teacher Edition," 203).

The next lesson then expands this concept further by multiplying unit fractions by non-unit fractions. Lesson fifteen continues to fill in this topic by focusing on multiplication between two non-unit fractions. Therefore since it does develop this idea fully, this standard is met.

The next standard was discussed previously as well, during Topic D, but it was only partially met. The previous section only works on multiplying fractions by whole numbers, but lessons thirteen through fifteen introduce multiplying two fractions together, and does so using real world contexts, in particular using the idea of a mother send a fraction of a pan of crispy rice treats to school, which is an example used throughout those lessons. Lesson sixteen focuses entirely on solving word problems using multiplication of two fractions. Lesson twenty looks at conversion problems, which require students to use fraction multiplication. Some of these problems use mixed numbers and many of the examples given during the Concept Development portion use mixed numbers ("Grade 5, Module 4: Teacher Edition," 301-4). Therefore since this section expands this idea first introduced in the previous section and includes mixed numbers, this standard is now completely met.

The fifth section in this unit, Topic F, discusses "multiplication with fractions and decimals as scaling and word problems" ("Grade 5, Module 4: Teacher Edition," 311). This section contains lessons twenty-one through twenty-four and is connected to standards 5.B.NF.5 and 5.B.NF.6. The first standard was omitted because it did not stand alone very well, but since it did have a substandard that I am analyzing, I will be looking to see if the standard 5.B.NF.5.B is met within this section.

Standard 5.B.NF.6 was partially met during Topic D then fully met in Topic E. It expects students to "solve real world problems involving multiplication of fractions and mixed numbers" ("Number & Operations – Fractions"). The objective of lesson twenty-four is to "solve word problems using fraction and decimal multiplication" ("Grade 5, Module 4: Teacher Edition," 354). The problem set for this section includes many real world problems that require multiplication of fractions and mixed numbers; therefore, this section also meets this standard.

Although it is not specifically connected to this section, standard 5.B.NF.5.B states that students will be able to

> "Interpret multiplication as scaling (resizing), by explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a/b* = (n × a)/(n × b) to the effect of multiplying *a/b* by 1" ["Number & Operations – Fractions"]

In lesson twenty-one, the focus is multiplying fractions by fractions equivalent to one, which meets the last part of this standard. Lesson twenty-two and then uses this understanding to compare the products of fractions less than and greater than one where part of the Concept development section is spent reasoning why the product would by greater than or less then the given number. This then satisfies the first part of this standard; therefore, this section meets this standard.

Now that multiplication of fractions has been thoroughly discussed, Topic G moves on to "division of fractions and decimal fractions" ("Grade 5, Module 4: Teacher Edition," 369). This section includes lesson twenty-five through thirty-one. It focuses on standard 5.NF.B.7, which expects students to "Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions" ("Number & Operations – Fractions").

One part of this standard is dividing whole numbers by fractions. Lesson twenty-five is centered on this concept. The lesson introduces this concept by presenting division as splitting into equal sized groups, the only difference now being that the size of the groups are fractions.

The next part is dividing unit fractions by whole numbers. Just as in lesson twenty-five, lesson twenty-six introduces the topic by using the same context that division was used in previously: separating amounts into equal groups. The difference in this case is that the amounts that are being split are fractions. This is a good visual representation of this concept:

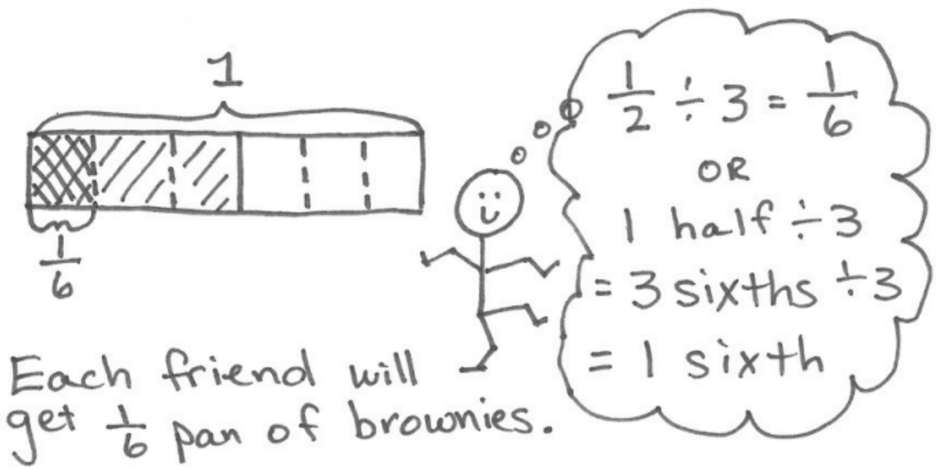


Figure 11: Visual Representation from Lesson 26 from the Second Fifth Grade Unit

This represents dividing fractions by whole number using the previous understanding of division as splitting into equal groups ("Grade 5, Module 4: Teacher Edition," 392).

This model is a good representation, because it shows one half as part of a whole, which makes it easier to determine the quotient later. It also builds what is already understood about division. In division, you are separating into equal parts; so, to divide one half by three, you can divide one half into three equal parts.

Lesson twenty-seven then does a good job of supplementing these concepts by having students solve word problems that involve fraction division. In these problems, sometimes, the fraction is the divisor, the number dividing the other number, and other times, it is the dividend, the number being divided. Because of this, the students have to determine this before finding the solution.

Because this section addresses dividing fractions by whole numbers, dividing whole numbers by fractions, and distinguishing when to use each one in such a way that the previous knowledge of division was used, this section meets this standard.

The last section of this unit, Topic H discusses “interpretations of numerical expressions” (“Grade 5, Module 4: Teacher Edition,” 475), but it is not connected to a standard relating to fractions. Therefore, this section will not be included in the analysis unless it is needed to show that another standard was or was not included in the materials.

Standard Discussed in Each Topic of the Fifth Grade Curriculum														
	Part One				*Part Two*									
Topic >> Standard ∨∨	***A***	***B***	***C***	***D***	***A***	***B***	***C***	***D***	***E***	***F***	***G***	***H***	***Did the unit overall meet the standard?***
5.NF.A.1		X	X	X									YES
5.NF.A.2		X	X	X									YES
5.NF.B.3						X							YES
5.NF.B.4													YES, Explanation below
5.NF.B.4.A							X	X	X				YES
5.NF.B.5.B										X			YES
5.NF.B.6								/	X	X			YES
5.NF.B.7											X		YES
5.NF.B.7.A													YES, Explanation below
5.NF.B.7.B													YES, Explanation below
5.NF.B.7.C													YES, Explanation below

Table 7: Standards Discussed in Each Topic of the Fifth Grade Curriculum

This table notes what standard were attempted, met, or partially met in the third grade curriculum. “X” indicates standards that were met. “/” indicates standards that were partially met.

There were four standards that were not specifically connected to a certain topic. Those standards were 5.NF.B.4, 5.NF.B.7.A, 5.NF.B.7.B, and 5.NF.B.7.C.

The first of these standards states that students should “apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction” (“Number & Operations – Fractions”). When fraction multiplication was discussed in lesson eight, the introduce it by reminding students that multiplication is repeated addition and that the same logic applies when looking at multiplying fractions by whole numbers (“Grade 5, Module 4: Teacher Edition,” 115-6). This is using the student previous understanding of multiplication to extend that definition to include multiplying fractions by whole numbers; thus, this standard is met within the unit.

The next two standards that are not connected to a specific section are 5.NF.B.7.A and 5.NF.B.7.B which expects students to "interpret division of a unit fraction by a non-zero whole number, and compute such quotients” (“Number & Operations – Fractions”), and “Interpret division of a whole number by a unit fraction, and compute such quotients” (“Number & Operations – Fractions”). These standards are met in lessons twenty-five through twenty-seven where the focus of the two of these lesson is on these two topics and the third lesson works on deciphering which one to use. Therefore, both these standards are met as well.

The last standard that was not designated by a specific section is 5.NF.B.7.C, in which students have to be able to “Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem” (“Number & Operations – Fractions”). Lesson twenty-seven focuses on “[solving] problems involving fraction division" ("Grade 5, Module 4: Teacher Edition," 405), and the problem set for this section is composed entirely of real world problems that require fraction division: both dividing a fraction by a whole number and dividing a whole number by a fraction. Therefore, this standard is also met within this unit.

With that last standard, all the fifth grade standards concerning fractions are met within this unit; therefore, this unit is fully aligned with the common core.

Conceptual Understanding

After examining all the units concerning fractions for third, fourth and fifth grade, all of the standards have been met; so, the Eureka Math curriculum about fraction aligns completely with the Common Core State Standards regarding fractions. The next question is, do the materials present this information in a way that promotes conceptual understanding? From the categorization of each of all of the standards, it was found that the majority of the standards required some form of conceptual understanding. In third grade, there were eight out of nine standards that promoted conceptual understanding. In fourth grade, there were nine out of eleven standards, including the standards that contributed to a problem-solving understanding. In fifth grade, ten out of eleven standards required some sort of conceptual understand, again including the standards that developed a problem-solving understanding. Therefore, since the Common Core supports a conceptual understanding and these materials are aligned with the Common Core State Standards, these materials should also promote a conceptual understanding.

To determine the degree to which these materials develop a conceptual understanding, I will be looking at what research-based practices that emphasize a conceptual understanding are used and how frequently they are used.

Researched-Based Practices

One major practice that is frequently used to foster conceptual understanding is the concrete-representational-abstract approach or CRA approach. Hudson and Miller cite this approach to be "one of the best ways to promote conceptual understanding of mathematics" (Hudson, 64). In this practice, instruction begins with physical manipulatives showing the concept. As understanding develops and students become fluent in concretely applying the concept, there is a transition to the representational portion in which concepts are being represented through two-dimensional imagery. Once students are able to use that representation sufficiently, there is a transition to the abstract in which the concept, now that the students understand what is being represented, is represented using traditional mathematical notation.

A study conducted by Elham Kazemi and Deborah Stipek found that mistakes "provide opportunities to reconceptualize a problem, explore contradictions in solutions, or pursue alternate strategies" (Kazemi, 64). Using errors as something to promote reevaluation can encourage conceptual understanding. These mistakes can be either intentionally done by the instructor or on accident by the students. Regardless of who makes the mistake, the opportunity for reevaluation is still present. This practice of using mistakes as learning opportunities fosters a conceptual understanding, because in these situations, students not only have to explain what is incorrect but also correct the mistake. By doing so, they are exhibiting their understanding of how different concepts do or do not apply depending on the context.

Another result of Kazemi and Stipek's research was that teachers should not only give "an explanation [consisting] of a mathematical argument" (Kazemi, 64), but also expect the same from students. Oftentimes, “students can describe the steps they took to solve a problem without explaining why the solutions work mathematically” (Kazemi, 64). This occurrence shows procedural or declarative knowledge, but by having students justifying themselves with mathematical concepts, development of conceptual understanding becomes the focus.

Also, to incorporate conceptual understanding, the lessons should include “precise math vocabulary” (Jungjohann). By using precise language, the students are developing their ability to describe math accurately and correctly identify elements of these different concepts. It is important to use precise math vocabulary early on, because “imprecise definitions of mathematically inappropriate language can lead to later misconceptions” (Doabler, 204). Of course, this should be done with discretion toward what vocabulary is appropriate for the specific grade level, but as a general rule, being as precise and specific as possible is a good thing.

In addition to the language that used, it is important that the curriculum encourages the instructor to use “explicit and systematic instruction,” which means students are provided “clear teacher demonstrations, scaffolded instruction, guided practice, academic feedback, and cumulative review” (Doabler, 201). Although it is up to the instructor as to how the materials are presented, if the materials support this type of instruction, it is more likely that the instructor will do the same. In terms of what this looks like, “clear teacher demonstrations” are the instructor fully explaining a concept and its applications while explaining their mathematical reasoning as the concept is used. “Scaffolded instruction” is where as the instruction progresses, the students become more and more independent in using that skill or applying that concept.

“Guided practice” is similar to scaffolded instruction in which there is group practice where the student gradually take on more responsibility in the completion of a task or explanation of a topic. “Academic feedback” is just as you would expect, the teacher is offering constructive feedback to the students concerning their performance or explanation. Finally, “cumulative review” is incorporating all of the relevant concepts within a unit into a review while emphasizing how the topics connect to one another.

Finally, another important aspect of the curriculum that supports conceptual understanding is the practice opportunities it provides. These practice opportunities should include both written and verbal responses. This means that students should not only be able to explain their reasoning on paper or through visual representations, but also be able to vocalize their reasoning (Doabler, 201). These practice opportunities should also include “discrimination practice” (Doabler, 204), which means students are not just robotically completing the same type of exercise, but they have to comprehend what is being asked and then decide what concept, procedure, or principle to apply. Having a variety of opportunities to practice and ways to practice fuels conceptual understanding for a similar reason as creating mathematical arguments; to complete the practice, the student must understand the concept enough to perform the task given, explain themselves, and decide whether or not the concept is applicable.

Within the Curriculum

A major tool used in this third grade curriculum is “fraction strips” which are “made from paper, used to fold and model parts of a whole” (“Grade 3, Module 5: Teacher Edition,” 8). Based on the pictures from the text (“Grade 3, Module 5: Teacher Edition,” 22), these are my fractions strips:

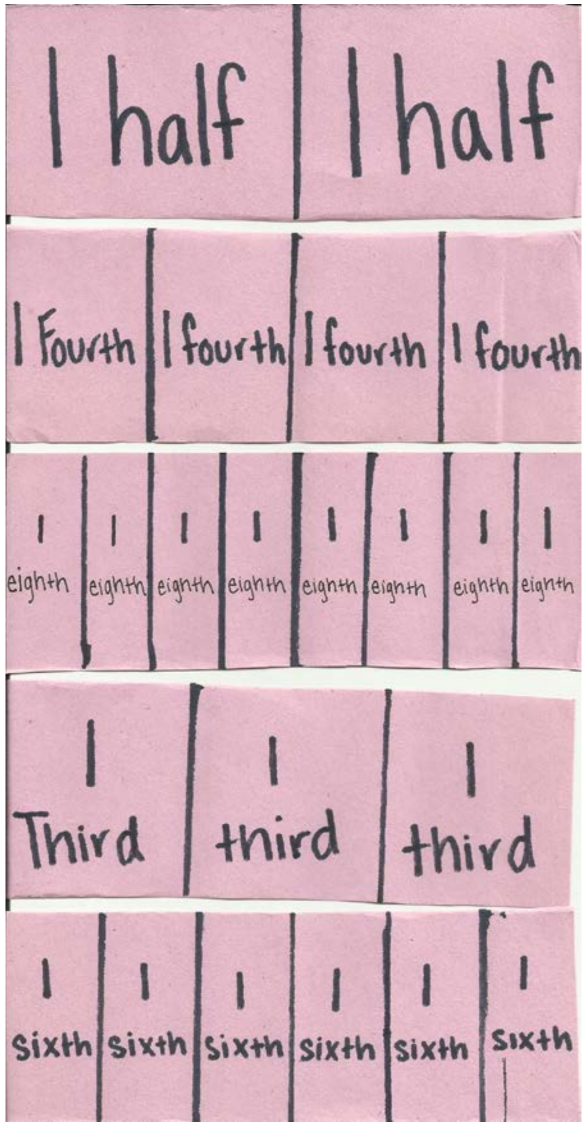


Figure 12: Fraction Strips

This is an excellent example of the concrete portion of the CRA approach, because it emphasizes what a fraction represents and gives students a physical representation of fractions. In the lessons where these are used, especially lesson one through four, the problem set often times shifts to the representation part of the CRA approach where there are images or diagram of the fractions strips that need to be divided or the students are asked to draw their own. In lesson five, after the students have become more comfortable with the concept of a fraction in general, they introduce the “fraction form” which connects the abstract portion of the CRA approach. Therefore this concept fully uses the CRA approach and is building conceptual understanding that way. In the fourth grade, beginning in the first lesson, these materials use the fraction strips then incorporate visual representations then use the abstract representation, which completes the CRA approach. It is not limited to the first lesson though, with each new major topic, they are using number lines, number bonds, and other visual representation to show what is happening then moving to the abstract. These materials use the CRA approach very thoroughly. Just as with a previous grades, the fifth grade curriculum uses the CRA approach throughout their materials.

Another method that is not outline as frequently if at all within these materials is the use of mistakes, intentional or unintentional as learning opportunities. This could be because the curriculum puts that sort of responsibility on the teachers or they could have just omitted it, but regardless of the reason, this practice was not a frequent part of the curriculum.

Another practice that is used frequently is having the students construct mathematical arguments. Within the lessons, the teacher are prompted to have students discuss with their peers about a concept or a question relevant to the objective using reasoning to back up their claim. Topics of discussion include everything from determining if a shape has been split into equal parts ("Grade 3, Module 5: Teacher Edition," 56) to comparing fractions using visual models ("Grade 3, Module 5: Teacher Edition," 126) to "what it means when two fractions are at the same point on the number line" ("Grade 3, Module 5: Teacher Edition," 246). Each lesson also includes a "Student Debrief" section in which students are encouraged to ask questions, but also teachers are given questions that can be used to fuel a discussion on the objective of the lesson. These prompts will have students show their work to the class and explain their reasoning and have the students reflect on the objective and think about how their understanding of a concept has changed. This time is a very good way to develop this mathematical reasoning skills and conceptual understanding. Like the third grade materials, the fourth grade materials prompt teacher to have their students discuss their mathematical reasoning with their peers during the lesson. These discussion points include everything from how to represent an equation using a visual representation ("Grade 4, Module 5: Teacher Edition," 23), to how multiplication and addition of fractions are related ("Grade 4, Module 5: Teacher Edition," 44), to whether a fraction is closer to one fraction or another ("Grade 4, Module 5: Teacher Edition," 161). Also, just like in the third grade materials, each lesson includes a Student Debrief section, which provides teachers with prompts to fuel class discussions. Using this practice does not stop in third and fourth grade, but it continues in the same way through the fifth grade materials: peer-to-peer discussions in the lesson and whole group discussions at the end of the lesson. This practice to promote conceptual understanding is used very frequently and effectively.

The language within this unit is not quite as precise as it could be. In the third grade curriculum, nowhere are the terms numerator and denominator introduced to the students, even though they spend two lessons comparing fractions with the same numerator. These terms are not introduced until lesson seven in the fourth grade curriculum. These are major terms associated with fractions, especially when developing the concept of a fraction; so, introducing them could have been a good thing especially as they develop a more abstract understanding of fractions. But they do introduce the terms unit fractions and equivalency early on. Overall, some important vocabulary was introduced but there is definitely room for improvement as to when to introduce these terms.

The outline for all of the lessons, regardless of which grade level, is generally the same. They begin with fluency exercises in which students are testing their declarative knowledge concerning a variety of topics. Then there is an application problem that usually is based on the concept from the previous lesson. After that is Concept Development, in which the new concept is introduced. Finally there is the problem set and the class discussion. Within these lessons, sometimes clear, explicit instruction is used, but oftentimes, only parts of it is used. Guided practices and scaffolded instruction are always used. Clear teacher demonstrations call for the teacher to take full responsibility in explaining the topic and/or performing the task, and this is not used as often as guided practices and scaffolded instruction. This curriculum shows preference to these other two instructional methods over clear teacher demonstrations. Just as the materials did not offer much in regards to using mistakes as opportunities to revisit topics or explanations, these materials also do not give a lot of information toward giving academic feedback. This could be because they cannot predict every mistake that a student makes or every success a student has. Finally, the last component of clear, explicit instruction is cumulative review. They start every lesson with a review problem that is usually related to the previous topic and the practice sets usually include a variety of problems. Overall, this practice could be used more effectively to not only review the different topics, but also reiterate the connection between all these topics.

These materials provide a lot of opportunities to practice. In all three of these grades, each lesson has a problem set that incorporates performing the task outlined in the lesson, explaining reasoning either with words or visual models, and although less frequently, discrimination problems. This supports the writing portion of this practice, and the verbal portion is also heavily emphasized. As discussed before, the students are often encouraged to discuss their reason or discuss a concept with a peer or with the whole group. The practice opportunities that are provided to the students are very effective in promoting a conceptual understanding.

Conclusion

The practices listed are just a few practices that have been seen to promote conceptual understanding, but they are some that have been used often and have been really effective. Overall, these materials incorporate a good number of research-based practices that contribute to a conceptual understanding. There are some practices that could have been used more, but the majority of these practices are used effectively and frequently. Of course, whether the students develop a conceptual understanding depends greatly on the individual teachers and the students, but these materials provide a well-develop foundation to support this type of understanding.

Conclusion

Going into this research, I knew I was not the only one concerned with math education and definitely not the only person doing research about; so, beginning this research, I had three overall goals. These three goals were accomplished through my in-depth analysis of the Common Core State Standards and the Eureka Math curriculum concerning fractions.

The first goal was to add to the discussion around math education. This does not mean that my research will be world changing or life altering, but at the very least, it offers something else to consider. While researching, I found that there was very little critical research surrounding the Eureka Math curriculum. My research only surrounded their materials concerning fractions, but seeing that the materials aligns with the Common Core State Standards and works to develop a conceptual understanding of mathematics makes it probable that other sections also do so. This idea opens up the potential for more research concerning this curriculum and other curriculum like it. This is important, not only because this curriculum is used in schools around the country, but because it available for free download making it a good resource for teachers to use to supplement the curriculum they use.

Another goal of my research was to gain experience with critically analyzing math curriculum. After reading over 1,900 pages of math curriculum and analyzing it for its Common Core alignment and for how the information is presented, I believe I have gotten quite a bit of experience. Having this experience will help me critically look at the curriculum I will use as a teacher and understand its strengths and its shortcomings, which is very important as a person in charge of shaping students overall math understanding for a full school year.

My last goal for this research was to get a better understanding of the Common Core State Standards for Mathematics. The better understanding I developed through this research began with learning the history and context of the Common Core State Standards. It then developed further as I investigated the Standards for Mathematical Practice and learned the overall goals of math education according to the Common Core. This understanding then became more specific as I looked at the Standards for Mathematical Content. I not only learned what the specific standards were, but by looking at Eureka Math, I also was able to see how they were applied and presented in a variety of contexts.

Moving forward, I hope to refine this research and use it to develop a more standardized method of examining curriculum for Common Core alignment and dedication to any of the four domains of mathematical understanding.

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