ESSAYS ON PRODUCT VARIETY IN RETAIL OPERATIONS

by

SEYED ALIREZA YAZDANI TABAEI

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Student: Seyed Alireza Yazdani Tabaei

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This dissertation has been accepted and approved in partial fulfillment of the requirements for the Doctor of Philosophy degree in the Department of Operations and Business Analytics by:

Eren Çil	Co-Chair
Michael Pangburn	Co-Chair
Saeed Piri	Core Member
Bruce McGough	Institutional Representative

and

Janet Woodruff-Borden	Vice Provost and Dean of Graduate School

Original approval signatures are on file with the University of Oregon Graduate School.

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DISSERTATION ABSTRACT

Seyed Alireza Yazdani Tabaei

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Thanks to technological advances in the past few decades, firms find product variety a more viable and hopefully a more profitable strategy than before. In this two-essay dissertation, I employ analytical models to investigate the effects of emerging operations concerning product variety on firm profits and consumer surplus. In my first essay, I analyze a two-stage game to study product-design and price competition between two mass-customizing firms that serve consumers with varying tastes. By comparing equilibrium results in settings with and without mass customization, I establish that competition with customization may lead to lower profits and consumer surplus. In my second essay, I study sample boxes which potentially create value by helping consumers resolve their uncertainties regarding different product varieties more efficiently. I show that when a firm offers a sample box, consumers obtain equal or higher net expected surplus while the firm's expected profit may decrease. I also show that a firm can reverse the potential adverse profit impact of selling sample boxes by introducing an optimally specified future credit.

This dissertation includes previously unpublished co-authored material.

CURRICULUM VITAE

NAME OF AUTHOR: Seyed Alireza Yazdani Tabaei

GRADUATE AND UNDERGRADUATE SCHOOLS ATTENDED:

University of Oregon, Eugene, OR

Sharif University of Technology

DEGREES AWARDED:

Doctor of Philosophy, Operations and Business Analytics, 2019, University of Oregon

Master of Business Administration, 2013, Sharif University of Technology

Bachelor of Science, Industrial Engineering, 2006, Sharif University of Technology

AREAS OF SPECIAL INTEREST:

Retail Operations, Pricing Models, Service Operations

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To women of my life,

Elnaz, who lived on the dark side of my student life, and

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CHAPTER I

INTRODUCTION

Technological advances in the recent decades have enabled firms in almost every product and service industry to offer increased varieties. Firms provide high varieties to achieve a better overall match between the characteristics of their offerings and consumers' heterogeneous preferences. Although this improved match often results in higher levels of consumers' willingness to pay, higher profits do not necessarily ensue. This is due to the costs firms incur to increase variety, costs consumers incur to learn their valuations for different variants, and competitive pressures that can potentially yield less product differentiation among industry players. This research is centered on the implications of emerging operations regarding product variety on firm profits and consumers surplus.

In my first essay, a previously unpublished co-authored work with Dr. Eren Çil and Dr. Michael Pangburn, we study product-design and price competition between two mass-customizing firms that serve consumers with varying tastes and finite reservation prices. Mass customization provides a mechanism by which firms can better target a broad scope of consumer preferences and thus, in so doing, potentially increase their profits. We contribute to the sparse literature on mass customization by analyzing a two-stage non-cooperative game between two firms serving a Hotelling linear city. By comparing symmetric equilibrium results in settings with and without mass customization, we find that customization changes the nature of competition. We show that mass customizers earn higher equilibrium profits when consumers' fit sensitivity either significantly or only slightly exceeds the product valuation. Conversely, traditional firms are better off when facing moderate fit sensitivity. We also establish that competition with mass customization may lead to lower profits and consumer surplus. This cautionary finding is of relevance to practicing managers, and also suggests that regulators should closely evaluate whether to facilitate industry investments in customization technologies, due to potential negative social welfare implications.

In my second essay, also a previously unpublished co-authored work with Dr. Eren Çil and Dr. Michael Pangburn, we study the practice of firms offering product 'sample boxes,' which have become a mechanism to facilitate consumers' learning of their product valuations. A sample box, consisting of a set of product varieties within a specific category, can provide an efficient method for consumers to resolve their valuation uncertainties. In the absence of sample boxes, consumers may need to ultimately try multiple product variations via sequential trials to discover their preferred variants. We establish that when a firm offers a sample box, its informational value implies an optimal price premium relative to the prices of individual products. Despite this price premium, we prove that consumers obtain higher net expected surplus, while the firm's expected profit may decrease. From the firm's perspective, the potential disadvantage of encouraging seller-induced learning via sample boxes is that low-valuation consumers may avoid successive purchases after the (early) discovery of their product valuations. We prove that, by including a future credit with the purchase of a sample box, a firm increases expected profits relative to the baseline case of not offering a sample box. The future credit effectively ties a consumer's purchase of the sample box to a subsequent purchase of a product. All sample box buyers pay the premium price for this set of purchases before learning, yet only customers with high valuations (realized after trying the sample box) would have paid for a full product in the absence of a future credit.

CHAPTER II

ESSAY 1: WHAT IF HOTELLING'S FIRMS CAN MASS CUSTOMIZE?

This work is coauthored with Prof. Eren Çil and Prof. Michael Pangburn, and submitted to the Decision Sciences journal.

Introduction

Mass customization is an operational mechanism by which firms can efficiently tailor their product attributes to diverse consumer preferences, and in so doing, potentially increase profit. Perhaps for this reason the adoption of mass customization (which will be also referred to as MC henceforth) has become a common business practice. Many companies have employed flexible and integrated processes to provide individually designed goods and services to customers (Da Silveira et al., 2001). Current examples of such offerings include, but are not limited to, NIKEiD shoes, Zale jewelry, Starbucks drinks, TaylorMade golf clubs, Ventana bicycles, and Oakley sunglasses. Fogliatto et al. (2012) address several studies on successful applications of mass customization in different industries. Two decades ago, mass customization was viewed as a novel competitive weapon (Pine, 1993), but today mass customizers increasingly confront competitors employing the same practice. For example, in the eyewear business, Oakley and Ray-Ban target wide ranges of customer preferences via their customizable sunglasses. Similarly, Nike, Zale, Starbucks, TaylorMade, and Ventana are not the sole mass customizers in their industries. In this study, we shed light on how the availability of a mass customization technology affects the nature of competition between firms. By carefully addressing the dynamics in such a setting and deriving the symmetric equilibrium outcomes, we prove that customization technology may ultimately force competitors to make decisions that are detrimental to all parties in the market.

According to Pine (1993), the goal of mass customization is to produce enough variety such that nearly everyone finds exactly what they want. There are, however, both internal and external forces that limit a firm's ability to serve diverse customers' tastes. From an internal perspective, it is costly for a firm to expand its variety range. A broader scope of mass customization requires higher investments in organizational capabilities to elicit differentiated customer needs and greater degrees of product modularity, process flexibility, and logistics complexity to fulfill those needs (Zipkin, 2001; Salvador et al., 2009; Engel et al., 2017). The external forces include product segmentation concerns (Jiang, 2000) and price competition (Syam and Kumar, 2006) due to other firms' products in the market. Given these internal and external forces, a mass customizer may choose to serve a limited range of customer preferences rather than span the full taste spectrum. We next provide three industry examples of mass customizers targeting distinct customer preference segments with corresponding product ranges.

The bicycle industry provides an instance of firms with customization capabilities competing in a market that exhibits a spectrum of preferences between on-road and off-road extremes. *Parlee Cycles* allows its customers to customize their *road bikes* vis-à-vis fit, feel, components, details, and look. The output can range from a very light and agile smooth-road bike to one with wider tires and disk brakes fitting relatively rugged paths. Yet, *Parlee Cycles* does not go all the way to manufacture mountain bikes, as extending into the other category is costly and time-consuming, according to *Parlee*'s Tom Rodi.[1] On the other hand, *Ventana*

[1] https://www.enve.com/en/journal/builder-profile-parlee-cycles/

allows customization within the scope of mountain bikes, ranging from lightweight rigid frames to heavier full-suspension options.[2]

As a second example of firms competing via mass customization with distinct and limited customized varieties, consider *Nike* and *Vans*, two competitors in the shoe industry. Both firms allow the customers to design their own shoes by customizing material, fit, pattern, and color, resulting in myriad possible outcomes. Despite the similarity in terms of the customization process these firms adopt, they target dissimilar segments in a market where preferences range between performance and street-style extremes. In particular, *Nike* has a performance emphasis, whereas *Vans* has a street-style orientation. At the same time, both firms wish to serve customers with more mainstream tastes, i.e., those near the middle of the taste spectrum.

A third example of competing firms that match their custom offerings to dissimilar ranges of horizontally differentiated customers can be found in the sunglasses industry. Both *Oakley* and *Ray-Ban* offer customized sunglasses to a population with varying tastes ranging from sports-focused to fashion-led designs. Offerings within *Oakley*'s customization line fall within the outdoor-sporty to casual range, omitting fashion-oriented styles. In contrast, *Ray-Ban*'s custom offerings span a spectrum from somewhat sporty designs (albeit less so than *Oakley*) to decidedly fashion-led styles.

We model the competition between the two mass-customizing firms as a location-then-price game. Firms choose the ranges of their customized offerings in the first stage, and prices in the second stage. We treat the customization-range design decision to be more long-term than pricing due to the complexities associated with redesigning the range of product attributes. For instance, when deciding

[2] https://www.cyclemonkey.com/ventana-usa-0

on style options, Nike must consider sourcing, logistics, storage, assembly line flexibility, worker expertise, and advertising implications. We use a linear-city type framework to model the firms' design decisions and consumers' taste preferences. Location models are often used to study competitive markets with differentiated products (Hotelling, 1929). In classic location models, each product is assumed to be positioned at a single point along consumers' taste dispersion range. To purchase a certain product, a customer incurs not only the selling price, but also the *fit cost*—the cost of mismatch between their ideal taste and the position of the product. With mass customization a firm can eliminate each consumer's fit cost by designing the customization range to cover their ideal taste. Notably, not only does the fit cost magnitude matter, but also its relation to consumers' valuation. For example, when customers' fit costs are less than their valuation, they may desire even a product completely dissimilar to their ideal taste. However for some products, the fit concern may be sufficiently important that consumers would not desire a product with characteristics completely dissimilar to their ideal preferences. We define *fit sensitive* products as those for which the fit cost may exceed the product valuation, at least for some customers. In this essay, I study the competition between two mass customizing firms with a primary focus on fit sensitive products. By deriving the symmetric equilibrium outcomes and contrasting them with a baseline case without mass customization, I obtain the following three main findings.

First, we show that equilibrium profits decrease due to mass customization under a range of market characteristics that we will delineate later. Thus, while a monopolist can leverage customization to charge premium prices and boost profit, we prove that competitive pressures can reverse those potential benefits when firms have access to a mass customization technology. Therefore, firms may find it beneficial to enter markets with standardized products rather than markets with customized products. We also show that the profit decreases resulting from mass customization can coincide with a reduction in consumer surplus as well, implying a type of "lose-lose" market outcome. It is interesting that, under competition, both profits and consumer surplus can decrease, especially given that one might expect consumers should be better off when obtaining products better matched to their tastes. A regulatory implication of this finding is that legislation supporting consumer-customized products and services should be viewed with significant caution, given the potential negative social welfare impacts.

A second finding is that, in equilibrium, it is *not* the case that firms' product ranges have a monotonic relationship to consumers' sensitivity to product fit. As consumers become more sensitive to product fit, one might expect that mass customizers would leverage their flexibility to expand the scope of their offerings and serve ideal products to more customers. We confirm that this happens in competition only if consumers have sufficient sensitivity to product fit. However, when consumers are not particularly sensitive to taste fit, an *increase* in consumers' product-fit sensitivity will result in a *narrower* product-customization range. Because of this non-monotonic relationship between fit sensitivity and the customization range, we advise managers caution when assessing shifts in consumers' fit sensitivity. Specifically, managers should not operate under the assumption that markets with less (greater) sensitivity to product-fit concerns warrant less (greater) product-customization scopes.

A third major finding is that, for fit sensitive products, mass customizers' profit functions are first decreasing and then increasing in the market's fit sensitivity, whereas traditional firms' profits develop in a reverse order. Both with and without customization, not only do we show profits to be non-monotonic in the fit sensitivity, but also we find that the same is true for equilibrium prices and consumers' surplus. For mass customizers, given the aforementioned non-monotonicity, levels of fit sensitivity that are either slightly or significantly greater than customers' valuation can lead to higher prices (relative to prices emerging when fit sensitivity moderately exceed consumers' valuation). On the contrary, non-customizing firms enjoy a reduced level of competition (i.e., higher prices) in the intermediate ranges of fit sensitivity.

The remainder of this essay is organized as follows. We review the related literature in Section 2.2, and present our model in Section 2.3. In Section 2.4, we describe the monopoly outcomes and analytically characterize the competitive equilibrium structures over distinct fit-sensitivity ranges. In Section 2.5, we consider a series of extensive numerical tests to investigate whether there is any profitable deviation from the characterized equilibrium outcomes. In Section 2.6, we discuss the impact of market conditions on the evolution of equilibrium structures and compare these structures to the equilibrium outcomes in the absence of MC technology. Section 2.7 concludes our study. Proof details of propositions and corollaries are within Appendix A. We provide the supporting lemmas along with their proofs in Appendix B.

Literature Review

Variations of the Hotelling's (1929) linear city model have been employed to study diverse forms of location and price competition. In the classic setting, each firm positions and prices a single product, and consumers with diverse tastes decide which firm to purchase from. Numerous forms of competition between single-product firms appear in the literature—overviewed by Archibald et al. (1986) and Martin (1993). With linear travel costs, d'Aspremont et al. (1979) show that no pure strategy location-then-price equilibrium exists, if consumers are forced to a purchase. However, Economides (1984) shows that when consumers' reservation prices are bounded (i.e., they can opt out), equilibria do exist. We likewise allow for bounded reservation price in our analyses.

As we consider a range of customized products, our work relates more specifically to the literature on competition between firms with multiple products in horizontally differentiated markets. With product variety becoming a more viable strategy due to technological advances, a natural extension of Hotelling models has been to allow each firm to offer multiple products. One research stream in this vein (e.g., Nalebuff, 2004; Peitz, 2008) treats product variety as a means to serve customers' different needs and analyzes firms' bundling motivations. In this setting, a firm's product line is not dispersed along a single dimension, but more suitably captured by a multi-dimensional Hotelling model. Shao et al. (2014) also use a two-dimensional Hotelling model to study two retailers each carrying a manufacturer's two products. In their model, products are exogenously located at the two extremes of a Hotelling line, whereas the brand differentiation between the two retailers is represented using a secondary Hotelling dimension. In contrast to the mentioned studies, we allow competing firms to endogenously design their product lines along a single characteristic dimension. This modeling approach accounts for a situation where a firm's portfolio includes some products that have more similar attributes than others to the competitor's offerings. We assume that these products are substitutable and a consumer chooses (at most) one product. Similarly assuming a continuum of potential *locations* for substitutable offerings, Martinez-Giralt and

Neven (1988) consider two firms each possibly offering two products tailored for different horizontal market segments. They find that firms do not increase the product variety (by offering a second product), since doing so would intensify price competition. Conversely, we find that increasing variety via customization is not always detrimental to firm profits; competing mass customizers face a mitigated price competition and in some ranges of fit sensitivity enjoy higher profits, compared to traditional competitors.

In contrast with a strategy offering a few distinct products, mass customization implies the ability to serve a spectrum of consumer tastes. A few prior studies have analyzed competition between a provider of a continuous range of options versus a set of discrete alternatives. Balasubramanian (1998) analyzes the price competition between a direct marketer (that can be viewed as a mass customizer) and multiple fixed retailers located equidistantly on a circle's circumference. He concludes that the direct marketer may optimally target a subset of the market, even when targeting the entire market is costless. Alptekinoğlu and Corbett (2008), Mendelson and Parlaktürk (2008b), and Xia and Rajagopalan (2009a) study the competition between a mass customizer with infinite variety spanning all consumer tastes and a mass producer with a finite set of products. We, however, look at the competition between two mass customizers, while additionally considering their endogenous product range decisions (i.e., locations defining the endpoints of the mass customization scope).

There are few studies addressing competition between firms with mass customization capability. Ulph and Vulkan (2000) analyze two competing firms that can offer customized products and set discriminatory prices. They assume competing firms' customization ranges to be anchored at the taste extremes, where standard products are located. Each firm then extends its product range towards the center to span an interval of locations. Ulph and Vulkan (2000) find a Prisoner's Dilemma type equilibrium result, with firms adopting both mass customization and price discrimination despite realizing lower profits. One key difference between our model and Ulph and Vulkan (2000) is that we consider a customization cost that is contingent on the flexibility of the product line, whereas the cost of customization is absent in their model. This provides a zero-sum situation in the trade-off between firm profits and consumer surplus in Ulph and Vulkan (2000). However in our model, firm profits and consumer surplus may simultaneously decline due to the concurrent presence of competitive pressures and customization costs. Unlike Ulph and Vulkan (2000) who restrict each firm to providing either a standard or a customized product, Syam and Kumar (2006) allow each competing firm to offer a customized product *besides* its standard product. They consider two market segments with distinct fit sensitivities. Mendelson and Parlaktürk (2008a) consider *base products* that are located at the taste extremes, but model mass customization by assuming that each firm has a mechanism by which it can reduce consumers' travel (or, fit) cost. Because the firms in that model do not choose a *customization scope*, they engage in head-to-head competition. The conclusion is that customization only helps firms with a relative advantage in cost or quality. Syam et al. (2005) study a competition between two firms that can mass customize two attributes of a product, and show that the firms choose at most one (and the same) attribute to customize. They focus on the question of *which* attribute firms should customize, assuming that firms offer a full range of attribute options. On the other hand, our focus is on understanding *to what extent* firms should use customization to serve heterogeneous customer tastes, while allowing each firm to distinctly define its product range. Loginova and Wang

(2011) provide a model similar to Syam et al. (2005), but focus on the role of quality asymmetries in customization competition. Xia and Rajagopalan (2009b) analyze a competitive market extended in both vertical and horizontal dimensions, wherein consistent with Shao et al. (2014), the horizontal dimension represents the preference for the brands exogenously located at the extreme ends, and the vertical dimension represents the product characteristic space. Compared to the provided stream of literature, our model is less restrictive on location choices along the taste spectrum, allowing each firm to choose both endpoints of its customization range. In other words, we study a setting in which a firm's customized offerings may include some products that are more similar to those from a competitor, while other offerings are quite distinct. To the extent that customization is used to create distinct products, it may potentially mitigate head-to-head competition. On the other hand, customization also has the potential to blur product distinction and thus yield an intensified competition effect.

The Hotelling linear city model we employ in this essay has been widely adopted to study customer preference heterogeneity in markets with outlying tastes. An alternative approach to study customer preference heterogeneity is Salop's (1979) circular representation. Using a circular model, Dewan et al. (2003) and Alexandrov (2008) study competition between two firms, each serving a continuous scope of customer preferences at a cost which increases in this scope. While in Dewan et al. (2003) firms set discriminatory prices along their offerings range, in Alexandrov (2008) firms provide self-customizable products at a flat price, consistent with our model. Dewan et al. (2003) find that customization increases consumer surplus without intensifying price competition, whereas we show that customization may result in a decrease in consumer surplus. Alexandrov (2008) show that, with more differentiation in the market (as a result of higher fit costs), the optimal product scopes may increase and thus profits can decrease. Even though we also show that increased fit costs may reduce profits, the driver in our model is not higher costs associated with greater offering scopes, but rather the intensified price competition due to less distinct offerings in equilibrium. Cavusoglu et al. (2007) locate two competitors at opposite sides of a circle, and allow each to have multiple customization scopes. In this setting, they find that customization hurts firms' profits (unless its cost is very low), and as in the above study, they show that consumers benefit. They also show that below a particular cost threshold, reductions to the customization cost do not yield increases to the firms' customization ranges. Given a circular model of consumer tastes (location), consumers cannot be viewed as having mainstream versus outlying tastes. Therefore, results from this modeling alternative are not necessarily generalizable to linear-city markets, in which there are some consumers with central tastes, surrounded by those with outlying tastes. As we will show, there are distinct competitive dynamics associated with mainstream and outlying taste consumers.

Model

We consider two firms that compete to serve customers who are heterogeneous in their tastes. Every consumer has an ideal taste, identified by a taste location $x \in [0, 1]$. Therefore, given a market of consumers with heterogeneous tastes, we consider a range of consumer taste locations uniformly spread over the unit interval $[0, 1]$. We refer to this linear market as the "taste spectrum." The adoption of a one-dimensional spectrum to represent the product space corresponding to preference heterogeneity is common in the literature. Jiang et al. (2006) find product variants with one-dimensional specifications to fit best into this modeling abstraction. Lancaster (1990) and Lee and Staelin (2000) suggest that when firms customize multiple attributes, a single summary dimension can approximate product differentiation as long as we can describe a product variant in terms of relative weights of two extreme characteristics. Referring to our earlier examples, this one-dimensional abstraction yields product ranges from: (i) for bikes, on-road to off-road designs, (ii) for sneakers, performance to street-style designs, and (iii) for sunglasses, sports-oriented to fashion-led designs.

Customers have a finite reservation price V for their ideal product, and incur a *fit cost* t per unit distance between their ideal taste and the purchased product. Each customer is in the market to purchase at most one unit of product, if doing so would yield higher utility than would their outside option. Without loss of generality, we treat the outside option utility as zero; if the outside option were to yield positive utility, our model could accommodate that by lowering the reservation price by V correspondingly. For simplicity, we assume customers are small relative to the size of the market, which is normalized to 1. In contrast to the standard Hotelling model, we allow each firm to offer a range of mass-customized products covering a continuous segment of the taste spectrum. An analogous use of a continuous spectrum was first adopted by Mussa and Rosen (1978), albeit for the purpose of representing vertically differentiated products. Subsequently, Ulph and Vulkan (2000), Dewan et al. (2003), and Alptekinoğlu and Corbett (2010) extend the idea into the horizontal product space, representing *partial* customization as an interval of offerings catered to a subset of the entire taste spectrum in the market. We do not impose the requirement that the firms' product ranges be anchored at the market endpoints, nor do we require that they be non-overlapping.

We model the strategic interaction between the firms as a two-stage location-then-price game, where profit-maximizing firms make simultaneous decisions at each stage. Referring to one firm as firm A and and the other one as firm B, we denote the *mass customization scopes* of firms A and B by $a = (a_1, a_2)$ and $b = (b_1, b_2)$, respectively.[3] The firms incur a mass customization cost of c per unit length of their respective product customization ranges. We assume that MC technology fixed investment costs are sunk at the stage each firm decides on its MC scope. After the firms choose their MC scopes, each sets a uniform price for the products in its corresponding MC scope. We denote the prices set by firms A and B by p_A and p_B, respectively. Our focus on uniform pricing strategies is consistent with common practice and prior research (e.g., Syam et al. (2005), Syam and Kumar (2006), Alptekinoğlu and Corbett (2008)). With slight abuse of notation, we denote the pricing strategy profile for the second-stage subgame as (p_A, p_B) for any given location decisions of the firms. Hence, the entire location-then-price strategy profile can be summarized as $(a_1, a_2, b_1, b_2, p_A, p_B)$, where the first two pairs refer to the location decisions of the firms, and the last pair refers to the pricing decisions of the firms.

Once the firms finalize their decisions, the utility of a customer located at $x \in [0, 1]$ from buying a product at y offered by firm i can be written as

$$u(x, y) = V - p_i - t|x - y|,$$

[3]One can imagine a more general setting under which each firm offers multiple customization scopes. We will discuss in subsection 2.4 that under a wide range of MC cost assumptions firms will not adopt a multi-interval customization strategy in equilibrium.

where $i \in \{A, B\}$. Each customer buys the product that delivers the highest utility, or refrains from purchasing if doing so yields negative utility. We assume that the customers favor purchasing over not purchasing when they are indifferent. For a purchased product, we use the term *delivered price* to refer to the sum of the product price and the fit cost, that is, fit sensitivity t weighted by the mismatch measure (i.e., $|x - y|$ above). For consumers purchasing a product matching their ideal taste, there is no fit-related cost and so the delivered price is simply the product price.

We next describe the firms' profit functions for any combination of their decisions regarding product customization ranges and prices. To this end, we use the above utility function to determine the set of customers buying from each firm. Once the market coverage of each firm is determined, the profit is simply a firm's selling price times its market coverage minus its customization cost. Despite the simple profit structure, it is difficult to express the profit function for all possible combinations of firm decisions, because determining the consequent piecewise-linear market segmentation structure requires considering multiple cases to characterize the outcome of the duopoly competition. We highlight one of these cases in Figure 1, which shows the market captured by each firm for a representative set of prices and customization ranges. The horizontal axis in Figure 1 represents the taste spectrum, and the vertical axis shows the (delivered) price. Note that this figure does not depict an equilibrium outcome, but provides an illustration of a representative (general) outcome.

Based on the price and product portfolio decisions considered in Figure 1, firm A captures the market extended from $\alpha_1 = a_1 - \frac{V - p_A}{t}$, the location of the indifferent consumer between opting out and purchasing from firm A, to $m = \frac{a_2 + b_1}{2} + \frac{p_B - p_A}{2t}$, the location of the indifferent consumer between A and B. Firm B captures the market

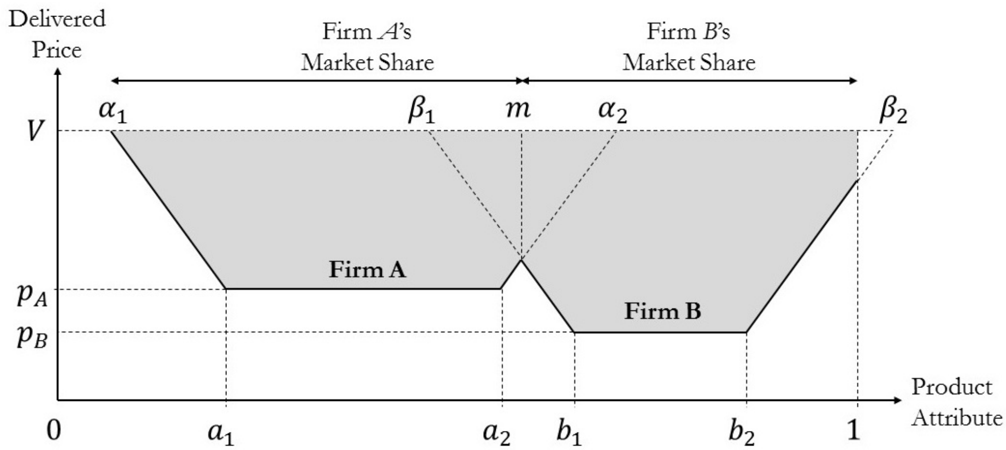


FIGURE 1. Market territories of two firms with arbitrary location and price choices

Firm A sets its range of offerings from a_1 to a_2 and charges the price p_A. Firm B customizes through $[b_1, b_2]$ and sets its price at p_B. Consequently, firm A attracts the market share ranging from α_1 to m, and firm B serves the market share bounded by m and 1.

extended from m to 1. Firm A's decisions result in a *gap*, a range of unserved consumers, to the left of α_1. On the other hand, the consumer located on the right edge of the market obtains positive utility from firm B, since $\beta_2 = b_2 + \frac{V-p_B}{t}$ is greater than 1. Between $\beta_1 = b_1 - \frac{V-p_B}{t}$ and $\alpha_2 = a_2 + \frac{V-p_A}{t}$, consumers obtain positive utility from either firm, but choose the one that gives them a higher utility. Particularly, consumers within $[\beta_1, m]$ choose firm A, while those within $[m, \alpha_2]$ purchase from firm B. Thus, in this specific example, the profits of firm A and firm B are characterized as $p_A(m - \alpha_1) - c(a_2 - a_1)$ and $p_B(1 - m) - c(b_2 - b_1)$, respectively.

Figure 1 also provides insights about the utility of the customers. Through the intervals $[a_1, a_2]$ and $[b_1, b_2]$, consumers do not incur any fit cost. Hence, the delivered price they pay only includes the firms' selling prices, p_A and p_B. However, for the consumers outside the mass customization ranges, the delivered price linearly increases at rate t as the level of mismatch between a consumer's ideal product and a firm's offering grows. Since the utility of a consumer is the difference between the reservation price, V, and the delivered price they pay, the shaded area in Figure 1 represents the total consumer surplus for the firms' decisions considered in the figure.

In Figure 1, we consider the case where each firm's market share excludes its competitor's customization scope. We can alternatively imagine a situation, as illustrated in Figure 2, where firms' choices result in one firm's market share expanding over the rival's MC scope. In Figure 2(i), firm A attracts some of the consumers who would otherwise get a perfectly fit product from firm B, i.e., when $b_1 < a_2 + \frac{p_B - p_A}{t} < b_2$. We refer to this phenomenon as *partial undercutting.* In Figure 2(ii), firm A sets its price and locations in a way to capture all of B's market, i.e., when $\alpha_1 < \beta_1$, $\alpha_2 > \beta_2$, and $p_A < p_B$. In this situation we say that firm A *fully undercuts* firm B.

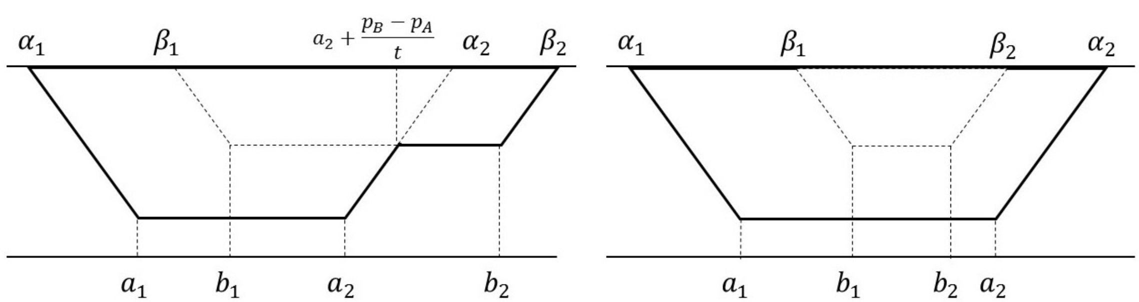


FIGURE 2. Firm A undercutting firm B

(i) partial undercutting on the left, at least some of the consumers who are provided a perfectly matched product from firm B, prefer purchasing from firm A. (ii) full undercutting on the right, all the potential customers of firm B find firm A's offerings more attractive.

Equilibrium Analysis

In this section, we characterize the equilibrium outcome of the competition between two firms with mass customization (MC) capabilities. Both of the firms ultimately aim at choosing the range of their product portfolios and the prices that maximize their profits in a heterogeneous market with taste-sensitive consumers. To better understand how the availability of MC technology alters the nature of duopoly competition, we first study a market served by a monopolist with MC capability. By analyzing both monopoly and duopoly settings, we find that the *form* of the resulting optimal decisions and the equilibrium outcomes primarily depend on the relative magnitudes of consumers' reservation price and their taste mismatch cost—defined by V/t. As the V/t ratio plays a crucial role in our analysis, we denote it as ρ and refer to it as the *value-fit ratio*. Although the unit MC cost c does not affect the structures of the market outcomes, it stays as an important market characteristic because it determines whether a firm chooses to exercise its MC capability.

Monopoly Outcomes

When there is only a monopolist in the market, we can derive its optimal pricing and product portfolio decisions as formally stated in the following proposition. In Proposition 2.4.1, we show how the firm's optimal decisions relates to the reservation price V, the fit sensitivity t, and the mass customization cost c.

Proposition 2.4.1. *A monopolist with MC capability will optimally behave in one of the following three ways, depending on the ranges of parameters.*

- *When $\rho < 1$ and $c \geq V - \frac{V^2}{2t}$, do not mass customize and set a price to cover the partial market of size ρ.*

- *When $\rho \geq 1$ and $c \geq \frac{t}{2}$, do not mass customize, locate at the center, and price at $V - \frac{t}{2}$ to cover the entire market without leaving positive utility for the consumers with extreme tastes (located at the extremes of the taste spectrum).*

- *Outside the ranges indicated above, mass customize along the entire market and price at V.*

Proposition 2.4.1 corroborates the intuition that when customization cost (c) is high, a monopolist does not take advantage of its MC capability at all. In that case, when $\rho < 1$, due to consumers' relatively low willingness-to-pay the firm charges an (interior optimal) price that attracts a subset of the market. In contrast, at larger ρ, consumers' high valuation or tolerance to taste-mismatch justifies capturing the entire market. If the MC cost c is low, then the monopolist takes advantage of its MC capability by customizing over the entire taste spectrum, providing every customer with a perfectly fit offering. In this case, price equals the reservation price, yielding zero consumer surplus. It is evident from these results that no combination of parameters will yield a partial mass customization structure. This result follows from MC cost being linearly related to the customization scope. To understand this, consider a monopolist providing a partial customization scope. An additive change in the customization scope (say by ϵ), and thus in the MC cost (by $c\epsilon$), requires an appropriate linear adjustment in price (by $t\epsilon/2$) to yield the same market share for the firm. Since this adjustment implies a linear change in profit, a monopolist's profit maximization problem has a bang-bang solution achieved at either of the customization scope boundaries, which are 0 and 1.

Proposition 2.4.1 also confirms the intuition that as customers become more sensitive to taste fit (t), MC remains optimal over a broader range of MC costs. If we fix the fit sensitivity t, increasing V implies that the firm can justify practicing MC at higher costs through passing these costs over to more customers with higher valuations. Thus, full MC is expected through broader ranges of c with an increase in V, as long as the alternative is no MC and partial market coverage.

Competition Outcomes

We now consider two competing firms with MC capabilities and analyze the resulting equilibrium structures. Following the standard approach in the literature studying competition between firms with identical costs, we focus on *symmetric* equilibria. A symmetric equilibrium of the game is an equilibrium where firms' MC ranges are symmetric around the market midpoint, i.e., $a_1 = 1 - b_2$ and $a_2 = 1 - b_1$. In such location-symmetric profiles, we do not impose symmetry on the prices, but rather show that the resulting second-stage price equilibria entail equal (symmetric) prices. Furthermore, while conducting our equilibrium analyses for the entire game, we consider unilateral deviations corresponding to asymmetric location choices. Although accounting for asymmetric unilateral deviations complicates our analysis, after delineating an important property of a plausible symmetric equilibrium in this subsection, we reduce the set of equilibrium candidates to one possibility in the following subsections. Then in Section 2.5, we numerically show there is no profitable deviation from the characterized equilibrium candidate.

In order to understand the implications of MC technology on the nature of competition, we must compare the equilibrium outcomes in the MC competition to those resulting from the competition between *single-product* firms, i.e., lacking MC technology. Hinloopen and Van Marrewijk (1999) and Pazgal et al. (2016) derive the symmetric equilibrium outcomes of the competition between two single-product firms, and show that there is no equilibrium beyond $\rho = 7/8$. Given that we wish to contrast our MC setting results with those for single-product firms, we focus on characterizing the subgame perfect Nash equilibria (SPNE) over the range $\rho \in (0, 7/8)$. Within this range, fit sensitivity is sufficiently significant that a consumer with an extreme taste does not purchase a product at the opposite extreme of the taste spectrum. As we will discuss in Section 2.5, although establishing the existence of equilibrium is analytically intractable in the game we study, we numerically show that $\rho \in (0, 7/8)$ is sufficient for the emergence of unique symmetric location-price equilibrium.

In this subsection, we show that in the symmetric equilibrium outcomes where firms choose to offer customization, they will serve the entire market, while leaving no surplus at the taste extremes. The standard approach to derive the SPNE would be to characterize the price equilibrium for any first-stage decisions of the firms. Typically, that equilibrium is obtained from the intersection of the first order conditions of the firms' second-stage (pricing) problems. However, via Proposition 2.4.2, we will show that a profitable first-stage (location) deviation is always possible if the second-stage (price) equilibrium is obtained from the first order conditions. Therefore, if an equilibrium exists in our problem, then it must correspond to one of the many corner solutions of the pricing subgame. In particular, Proposition 2.4.2 proves that the emerging price equilibrium in SPNE should ensure that customers with extreme tastes will opt to purchase while realizing zero net utility, i.e., firms A and B should respectively charge $p_L(a_1) = V - a_1 t$ and $p_L(1 - b_2) = V - (1 - b_2)t$. Therefore, if a location-symmetric profile yields a price equilibrium different from $(p_L(a_1), p_L(1 - b_2))$, it is not a SPNE.

Proposition 2.4.2. *For $\rho < 7/8$, if firms do customize in any symmetric equilibrium, they leave zero utility at the extremes of the taste spectrum, while serving the entire market.*

Proposition 2.4.2 allows us to focus on only the following three strategy profile structures, which we denote as W, $\underline{\mathrm{W}}$, and $\overline{\mathrm{W}}$ structures. We do *not* limit the definition of these structures to location-symmetric profiles, so that we may leverage them while analyzing location-asymmetric profiles corresponding to a firm's unilateral location deviation. W, $\underline{\text{W}}$, and $\overline{\text{W}}$ structures are defined as the following and illustrated in Figure 3.

W-structure (*middle consumer obtaining positive utility but not a perfect match*): Firms serve the entire market and leave zero utility at the extremes of the taste spectrum. Furthermore, firms compete in the middle in a way that, fixing the locations, an infinitesimal price change by either firm will result in neither a gap of unserved consumers in the middle, nor any sort of undercutting (as illustrated in Figure 2). Under this structure, $\beta_1 < m < \alpha_2$, $a_2 + \frac{p_B - p_A}{t} < b_1$, and $b_1 - \frac{p_A - p_B}{t} > a_2$.

$\underline{\textbf{W}}$**-structure** (*middle consumer obtaining a perfect match from both firms at the same price*): Firms serve the entire market and leave zero utility at the extremes of the taste spectrum. Furthermore, both firms offer customized products to the indifferent consumer between them. As this structure results in the continuity of the mass customization scopes in the middle, $a_2 = b_1$ and $p_A = p_B$.

$\overline{\textbf{W}}$**-structure**(*middle consumer obtaining zero utility*): Firms serve the entire market and leave zero utility at the extremes of the taste spectrum as well as for the indifferent consumer between them. This structure holds when $\alpha_2 = \beta_1$.

Note that the distinction among the defined structures above relates to how central tastes are served in competition. Put another way, the competition hinges upon those consumers with more central tastes, who can enjoy relatively low-cost access to either firm's product ranges. The following proposition further constrains the set of solution structures.

Proposition 2.4.3. *For (i) $c > \frac{t}{4}$, and (ii) $c < \frac{t}{4}$, there exists no symmetric $\overline{W}$-structure SPNE where the firms customize.*

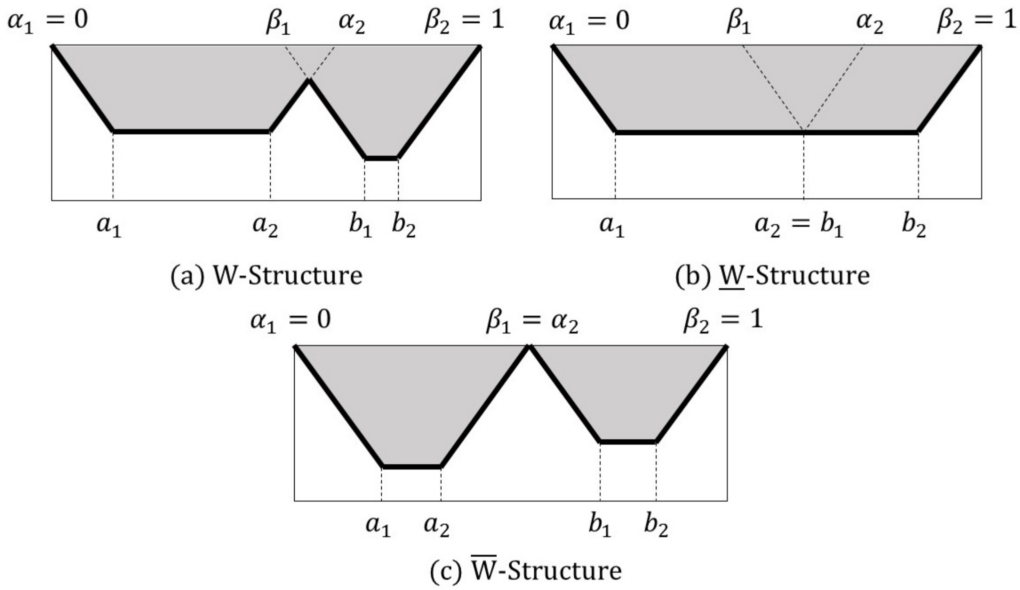


FIGURE 3. W, $\underline{\mathrm{W}}$, and $\overline{\mathrm{W}}$ structures

In equilibrium the entire market is served, but the consumers with extreme tastes receive zero utility. (a) A small change in either firm's price results in neither a gap in market coverage nor price undercutting. (b) Firms set the same prices and MC scopes are continuous in the middle. (c) A price increase by either firm results in a market coverage gap in the middle.

Proposition 2.4.3 shows that the $\overline{\mathrm{W}}$-structure is not pertinent if $c \neq t/4$. But when $c = t/4$, either firm can obtain equivalent profits as it gradually shrinks its customization scope and decreases its price to maintain zero utility at the market edges. Eventually, either firm can imitate a single-product firm located at a market quartile (1/4 or 3/4), maintaining a $\overline{\mathrm{W}}$-structure. Therefore, if there exists a symmetric $\overline{\mathrm{W}}$-structure equilibrium where firms customize, there also exist infinite other $\overline{\mathrm{W}}$-structure equilibria including the one in which firms do not customize.

As a result of the above discussion, we do not deem the $\overline{\mathrm{W}}$-structure significant in our further analyses, for two reasons. First, there is no meaningful range of MC costs over which we could obtain a $\overline{\mathrm{W}}$-structure equilibrium where firms customize. Second, even for the single value of MC cost that makes a $\overline{\mathrm{W}}$-structure equilibrium feasible, we have a profit-equivalent equilibrium where firms do not customize. It is evident that, at the same level of prices, $\overline{\text{W}}$-structure characterizes a more efficient outcome for the firms than W or $\underline{\text{W}}$ structure. However, a bilateral transition from W or $\underline{\text{W}}$ structure to $\overline{\text{W}}$-structure is unstable, as each firm would have an incentive to unilaterally deviate from the resulting outcome. In fact, via Proposition 2.4.3, we show that when location and price decisions are made in separate stages, the non-cooperative symmetric equilibrium of the game is confined to W or $\underline{\text{W}}$ structure. In the following subsections, we will show that the possible emergence of these two equilibrium structures hinges upon the value of $\rho \in (0, 7/8)$. More specifically, we will show that the equilibrium structure changes twice through this interval, once at $\rho = \frac{1}{2}$ and again at $\rho = \frac{3}{4}$. Thus, we define three value-fit ratio intervals we refer to as: *high*, *intermediate*, and *low*. At a given level of V, these ranges are respectively translated into markets with *weak*, *moderate*, and *strong* fit sensitivity. At high levels of value-fit ratio, where consumers can tolerate significant taste discrepancies, we will show that W-structure characterizes the equilibrium outcome. At intermediate levels of ρ, (an interior form of) $\underline{\text{W}}$-structure emerges. When consumers' tolerance for taste mismatch is low, a boundary form of $\underline{\text{W}}$-structure occurs in equilibrium.

Markets with Weak *Fit Sensitivity*

Proposition 2.4.4 below characterizes the W-structure equilibrium candidate when $\rho \in (3/4, 7/8)$. Proposition 2.4.5 provides the necessary conditions, in terms of reservation price, fit sensitivity, and MC cost, for having a W-structure equilibrium. As we mentioned at the beginning of Subsection 2.4, we consider asymmetric unilateral location deviations in our analyses as it can be seen from the proof of the following propositions.

Proposition 2.4.4. *For $\rho < 1$, the only feasible symmetric W-structure equilibrium with customizing firms is $(a_1^*, a_2^*, 1 - a_2^*, 1 - a_1^*, p^*, p^*)$, where:*

$$a_1^* = \frac{1}{9}(2 + 3\rho - 4\sqrt{3\rho - 2}) \tag{2.1}$$

$$a_2^* = \frac{2}{9}(4 - 3\rho + \sqrt{3\rho - 2}) \tag{2.2}$$

$$p^* = p_L(a_1^*) = \frac{2}{9}t(-1 + 3\rho + 2\sqrt{3\rho - 2}) \tag{2.3}$$

Proposition 2.4.5. *Given $\rho < 7/8$, the necessary conditions for having a symmetric W-structure equilibrium are:*

$$\rho > 3/4 \text{ , and } c \leq \begin{cases} \frac{147V^2 - 98Vt - 149t^2 + 2\sqrt{\frac{t}{3V-2t}}(189V^2 - 328Vt + 143t^2)}{432(V-2t)} & \frac{3}{4} < \rho \leq \frac{41}{49} \\ \frac{t}{4} & \frac{41}{49} < \rho < \frac{7}{8} \end{cases}$$

The above propositions have two major implications. First, W-structure, as just characterized, can only emerge beyond $\rho = 3/4$, with ρ not exceeding $7/8$. Second, by combining these findings with our next result that shows $\underline{\text{W}}$-structure can emerge only when $\rho \leq 3/4$, we establish that the W-structure is the *only* equilibrium candidate in the range $3/4 < \rho < 7/8$. An important property of the characterized W-structure equilibrium is that each firm is on the verge of undercutting, which is defined as setting a price to capture at least some of the consumers who are provided perfect matches from the competitor. More specifically, each firm's profit in equilibrium equals the supremum of profits obtained from partially and fully price-undercutting the competitor, with fixed locations. This property is used to explain some of the results in section 2.6.

We provide an illustration of the characterized W-structure equilibrium candidate in Figure 4. Note that as we decrease ρ below 7/8, competing mass customizers charge lower prices and move their MC scopes towards the center to present a less distinct portfolio from their competitor and grant a higher utility to the midpoint consumer. At the lower bound of this range, where $\rho = 3/4$, the MC scopes join in the center to provide the midpoint consumer with their ideal product.

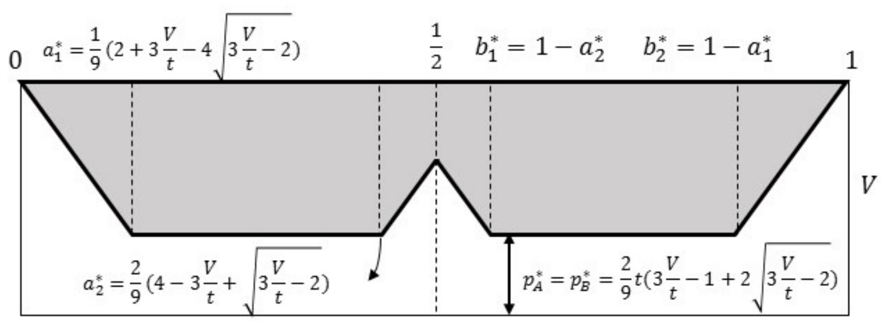


FIGURE 4. Equilibrium outcome through $\frac{3}{4} < \rho < \frac{7}{8}$

Firms set the specified locations and prices to characterize a W-structure.

Markets with Moderate *Fit Sensitivity*

We next show that as we decrease ρ below 3/4, given small enough values of c, the firms start to increase prices, keeping the continuous band of MC in the middle to form a $\underline{\text{W}}$-structure. As ρ decreases through the intermediate range, the firms also expand their MC scopes towards the edges. Eventually at $\rho = 1/2$, the firms collectively extend their MC scopes over the entire market, and charge the maximum sensible price, V, for their customized products. Propositions 2.4.6 and 2.4.7 below illuminate the characteristics of the $\underline{\text{W}}$-structure equilibrium candidate through $\rho \in (1/2, 3/4]$, as well as the conditions on the MC cost making the $\underline{\text{W}}$-

structure equilibrium viable. Similar to our previous analysis, we obtain the results in the following propositions accounting for asymmetrical location deviations.

Proposition 2.4.6. *For $\rho < 1$, the only feasible symmetric $\underline{\text{W}}$-structure equilibrium with customizing firms is $(\underline{a_1}, \frac{1}{2}, \frac{1}{2}, 1 - \underline{a_1}, \underline{p}, \underline{p})$, where $\underline{p} < V$, $\underline{a_1} = \rho - \frac{1}{2}$, and $\underline{p} = p_L(\underline{a_1}) = \frac{t}{2}$.*

Proposition 2.4.7. *The necessary conditions for having the $\underline{\text{W}}$-Structure equilibrium are:*

$$\frac{1}{2} < \rho \leq \frac{3}{4}, \text{ and } c \leq \begin{cases} \frac{3t^2 - 8tV + 6V^2}{4t - 4V} & \frac{1}{2} \leq \rho \leq \frac{3}{5} \\ \frac{3t + V}{16} & \frac{3}{5} < \rho \leq \frac{3}{4} \end{cases}$$

We establish two results via the above propositions. First, $\underline{\text{W}}$-structure (with prices less than V) does not emerge as equilibrium in any other range of ρ up to 7/8. Second, the characterized $\underline{\text{W}}$-structure equilibrium is the only plausible structure within the intermediate range of ρ. The reason is that we have already confined the emergence of W-structure to $\rho > 3/4$, and we will next show that the boundary form of $\underline{\text{W}}$-structure (with prices equal to V) can only emerge when $\rho \leq 1/2$. Figure 5 provides an illustration of the characterized $\underline{\text{W}}$-structure equilibrium candidate, where each firm mass customizes all the way to the center of the market and charges the price $t/2$.

Markets with Strong *Fit Sensitivity*

We showed that the $\underline{\text{W}}$-structure equilibrium candidates through the intermediate range of ρ reach a boundary level at the lower bound of this range, when the equilibrium prices hit V. We next establish, via Proposition 2.4.8, that

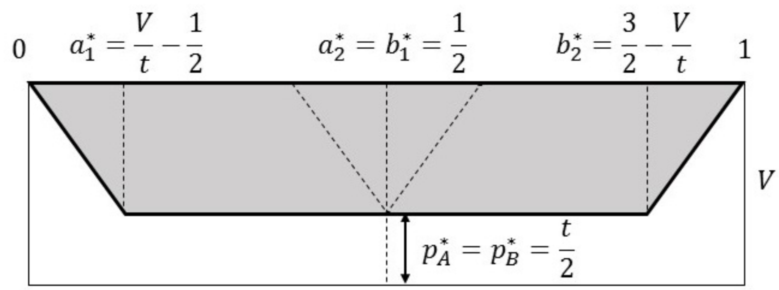


FIGURE 5. Equilibrium outcome through $\frac{1}{2} < \rho \leq \frac{3}{4}$

Firms set the specified locations and prices to characterize a $\underline{\text{W}}$-structure.

this boundary case of $\underline{\text{W}}$-structure remains the steady equilibrium outcome when $\rho \leq 1/2$ and c is small enough.

Proposition 2.4.8. *$0 < \rho \leq 1/2$ and $c \leq V - \frac{V^2}{t}$ are necessary for the existence of a symmetric equilibrium where each firm mass customizes from one market edge all the way to the center and charges the full price V.*

Proposition 2.4.8 establishes that $0 < \rho \leq 1/2$ is the only range in which the demonstrated equilibrium type may exist. Furthermore, no other MC equilibrium structure can exist in the specified ρ range. We also verify that, when ρ is small and c is large, competing firms do not mass customize at all, but instead imitate the product portfolio and pricing decisions of single-product monopolists. Offering customized products becomes a viable option only when the mass customization cost c is less than $V - V^2/t$. This result is analogous to our finding from Proposition 2.4.1; in the (boundary-form) $\underline{\text{W}}$-structure equilibrium candidate characterized in Proposition 2.4.8, each firm behaves equivalently to a monopolist that confines itself to half the market when the customization cost is small enough to let the firms extend their offerings.

Multiple Customization Scopes

We have assumed thus far that if the limits of a firm's product portfolio span some range $[a_1, a_2]$, then customers are permitted to order any customized product variant between a_1 and a_2. Effectively, this means that the firm does not leave customization "gaps" within the range of its product portfolio. A relevant question that we consider in this subsection is the potential for a firm to choose to leave one or more such gaps within its customization range, yielding a set of disjoint customization intervals, e.g., $[a_1, a_2^1] \cup [a_1^2, a_2^2] \cup \ldots \cup [a_1^n, a_2]$.

We assume, as above, that the cost associated with offering consumers a range of product varieties is dictated by the extremes of that variety range, i.e., $c(a_2 - a_1)$. Given this cost structure, if there is a benefit to the firm from product-line (customization) gaps, that benefit must be pricing and demand related. To analyze whether this is potentially the case, we next consider an equilibrium candidate solution, say for firm A, where the firm leaves product-range gaps, and show that deviating to a corresponding no-gap solution is equally profitable.

Proposition 2.4.9. *Assume a strategy profile wherein at least one firm, say firm A, has gaps within its customization range $[a_1, a_2]$, and an MC cost $c(a_2 - a_1)$. Then, there is an equi-profit equilibrium wherein firm A offers a contiguous customization range within $[a_1, a_2]$.*

This result shows that if a firm incurs the cost entailed by supporting customization to the product-range limits, then instituting gaps yields no benefit. Potentially, in practice, it is possible that gaps in the customization range could increase costs associated with delivering variety. As an example, consider Coca-Cola Freestyle machines that let users create their own drinks by mixing flavors of Coca-Cola branded products. If certain combinations were restricted, then not only implementing those restrictions with the user-interface would be costly, but also communicating such restrictions to customers could slow the (average) order process. Building upon the logic of the prior result, the following corollary formalizes the intuition that if customization gaps *increase* cost, then the presence of such gaps is not supported in equilibrium.

Corollary 2.4.10. *If instituting customization gaps within the product customization range* $[a_1, a_2]$ *yields an MC cost exceeding* $c(a_2 - a_1)$*, then the firm can increase its profit by deviating to a contiguous customization range within* $[a_1, a_2]$*.*

Another scenario to consider is the possibility that instituting customization gaps might decrease the firm's aggregate costs. For example, one might assume that a firm's costs relate to the sum of the widths of disjoint customization ranges. But, such an assumption is problematic. Consider in this case that if a firm were to locate customization intervals with spacing of $1/N$ and width of $1/N^2$, then as $N \to \infty$ the costs tend to zero (because the sum of interval widths converges to zero) yet the firm effectively covers the entire feasible product range. To control for that problem, potentially the cost model could be augmented by adding interval-specific fixed costs or other cost nonlinearities. However, given our focus on mass customization, we have assumed that a firm can adapt the product to customers' tastes, if they fall within the firm's product-portfolio range, at zero incremental costs.

Numerical Study

In the previous section, we reduced the set of equilibrium candidates to one possibility. That outcome corresponds to one of the structures characterized in propositions 2.4.4, 2.4.6, and 2.4.8, depending on the level of the value-fit ratio.

In this section, we investigate whether there is any profitable deviation from this remaining equilibrium candidate. To do so, we analyze asymmetric location profiles by considering unilateral bi-variate location deviations by one firm (while fixing its competitor's locations). After each deviation, we calculate the profits using the price equilibrium emerging from the second stage of the game. We then compare the deviating firm's profit before and after deviation. Through the results of this extensive numeric study, we find that, first, there is no profitable deviation from the derived candidates in propositions 2.4.4, 2.4.6, and 2.4.8, and second, the parameter ranges specified in propositions 2.4.5, 2.4.7, and 2.4.8 suffice for the existence of the characterized equilibria.

Our numerical study is designed as follows. We allow each of V, t, and c to vary in 0.01 increments within the $[0, 1]$ interval—thus considering one million possibilities in total. At each level of parameters, Proposition 2.4.4, 2.4.6, or 2.4.8 gives us the characterization of the unique symmetric equilibrium candidate, $(a_1^*, a_2^*, b_1^*, b_2^*, p_L(a_1^*), p_L(a_1^*))$, if a candidate exists at all. We then check for profitable deviations following the steps below. We consider the location and price increment, which we denote as τ, to be 0.0001.

1. At each level of (V, t, c), let a_2 take the values in $\{0, \tau, 2\tau, ..., 1\}$, and for each a_2, let a_1 take the values in $\{0, \tau, 2\tau, ..., a_2\}$.

2. Given the set of location variables $a = (a_1, a_2)$ and $b = (b_1^*, b_2^*)$, let p_B take the values in $\{0, \tau, 2\tau, ..., V\}$. At each level of p_B search for the best-response price of firm A, p_A, that maximizes its profit.

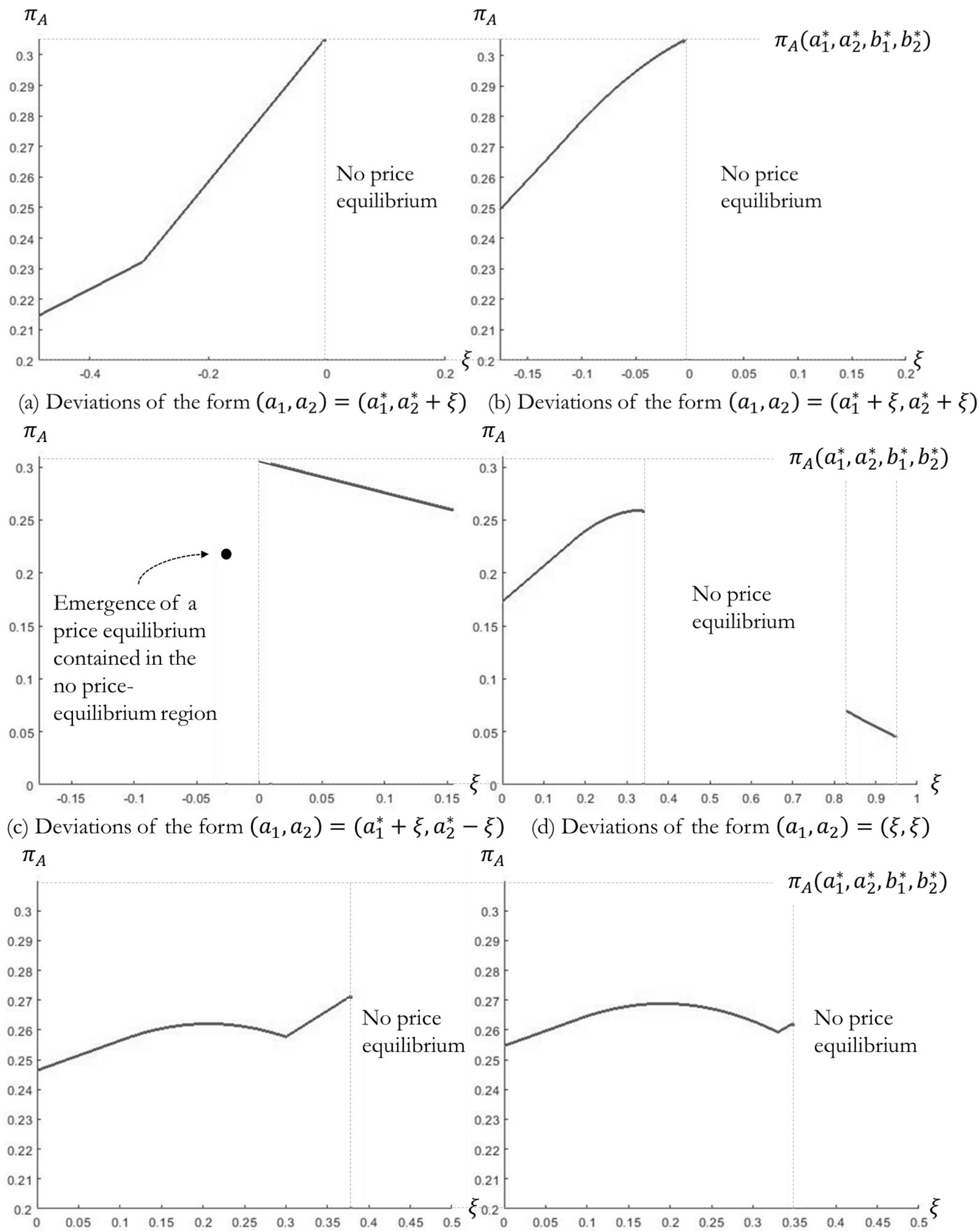
3. Given the set of location variables $a = (a_1, a_2)$ and $b = (b_1^*, b_2^*)$, let p_A take the values in $\{0, \tau, 2\tau, ..., V\}$. At each level of p_A search for the best-response price of firm B, p_B, that maximizes its profit.

4. Given the firms' locations, find the intersection of the price best response curves obtained from steps 2 and 3, yielding the price equilibrium $(p'_A, p'_B) \equiv (p_A^*(a_1, a_2, b_1^*, b_2^*),\ p_B^*(a_1, a_2, b_1^*, b_2^*))$. If the curves do not intersect, there is no price equilibrium.

5. Compute each firm's profit given $(a_1, a_2, b_1^*, b_2^*, p'_A, p'_B)$.

The above numeric approach considers all possible unilateral bi-variate location deviations from the equilibrium candidates analytically established in section 2.4 (with the given increment $\tau = 0.0001$). The bi-variate location decisions by a firm imply numerous types of possible deviation options. Considering a few of such deviation structures in Figure 6, we illustrate the significant irregularities of the profit surface, including non-monotonic, non-concave, and non-continuously differentiable regions. Besides these complexities, we observe regions of location deviations where the profit function is undefined due to the lack of a price equilibrium. The main driver of the irregularities demonstrated in Figure 6 is the complexity of the second-stage pricing game. Facing price best response functions with multiple pieces (vis-à-vis various location choices) as well as discontinuities due to partial or full undercutting makes the determination of the price equilibrium analytically challenging.

Each of the plots illustrated in Figure 6 focus on a specific type of location deviation when $c = 0.1$, $V = 0.85$, and $t = 1$—implying a high value-fit ratio. In all the plots, we investigate the profit dominance of the proposed equilibrium outcome which corresponds to W-structure as described in Proposition 2.4.4. Plots (a), (b),



(a) Deviations of the form $(a_1, a_2) = (a_1^*, a_2^* + \xi)$ (b) Deviations of the form $(a_1, a_2) = (a_1^* + \xi, a_2^* + \xi)$

(c) Deviations of the form $(a_1, a_2) = (a_1^* + \xi, a_2^* - \xi)$ (d) Deviations of the form $(a_1, a_2) = (\xi, \xi)$

(e) Deviations of the form $(a_1, a_2) = (\xi, 0.3)$ (f) Deviations of the form $(a_1, a_2) = (\xi, 0.33)$

FIGURE 6. Firm A's profit after deviating from (a_1^*, a_2^*) given (b_1^*, b_2^*) at $V = 0.85$, $t = 1$, and $c = 0.1$

and (c) in Figure 8 address the deviation types where firm A changes its location from (a_1^*, a_2^*) to $(a_1^*, a_2^*+\xi)$, $(a_1^*+\xi, a_2^*+\xi)$, and $(a_1^*+\xi, a_2^*-\xi)$, respectively. In plot (a), we fix a_1 at a_1^* and let a_2 vary around a_2^*. In plot (b), fixing firm A's customization scope, we simultaneously shift both a_1 and a_2 to the left or right. In plot (c), we expand or shrink firm A's customization scope around its midpoint. As we see from these plots, firm A's profit is maximized at $\xi = 0$, which corresponds to the firm's profit when it chooses the location (a_1^*, a_2^*). This confirms that the deviations from our proposed equilibrium candidate are not profitable. In the first three plots we focus on deviations revolving around (a_1^*, a_2^*). We next consider more global deviation types in plots (d), (e), and (f) in Figure 6. These three plots address deviation types where firm A changes its location from (a_1^*, a_2^*) to (ξ, ξ), $(\xi, 0.3)$, and $(\xi, 0.33)$, respectively. In plot (d), we consider the deviations in which firm A does not customize. In plots (e) and (f) we arbitrarily fix a_2 at 0.3 and 0.33, respectively, and let a_1 vary. Since the locations considered in plots (d), (e), and (f) exclude the equilibrium locations, the corresponding profits are strictly lower than the equilibrium profit.

Discussion of Equilibrium Structures

In this section, we focus on the progression of equilibrium structures, under MC competition, as the value-fit ratio varies. As we will show, the customization scopes, prices, profits, and consumer surplus can relate non-monotonically to ρ. We also compare the MC equilibrium structures with the outcomes of the duopoly between single-product firms, previously studied by Hinloopen and Van Marrewijk (1999) and Pazgal et al. (2016). The comparison allows us to discuss the implications of MC on both profits and consumer surplus.

As we saw from the results demonstrated in propositions 2.4.4, 2.4.6, and 2.4.8, the equilibrium locations as well as the ratio of prices to consumers' reservation price (V) are functions of $\rho = V/t$ only, independent of V and t separately. Therefore, we will focus on the effects of fit sensitivity and associated equilibrium dynamics by subsequently normalizing all prices by V.[4] We equivalently normalize the unit MC cost as c/V, and denote this ratio as $\tilde{c}$. When we subsequently discuss distinct values of ρ, we set any particular ρ level via a suitable adjustment to the fit cost parameter t.

We first focus on the location decisions formalized in the previous section. The following corollary highlights that the equilibrium MC ranges relate non-monotonically to consumers' value-fit ratio. The corollary's three points follow directly from propositions 2.4.4, 2.4.6, and 2.4.8.

Corollary 2.6.1. *As consumers' value-fit ratio increases from 0 to $\frac{7}{8}$, the firm's equilibrium MC scopes evolve as follows.*

i. Through low levels of value-fit ratio ($\rho \in [0, \frac{1}{2}]$), each firm's customization range is fixed at $\frac{1}{2}$.

ii. At intermediate levels of value-fit ratio ($\rho \in [\frac{1}{2}, \frac{3}{4}]$), firms choose decreasing customization ranges that unify at the center of the market.

iii. At high levels of value-fit ratio ($\rho \in [\frac{3}{4}, \frac{7}{8})$), firms choose increasing customization ranges that shrink away from the center.

From the above corollary, we see that the firms' pricing and product portfolio decisions in equilibrium are independent of the value-fit ratio, ρ, as long as $\rho \leq \frac{1}{2}$. In

[4] We could alternatively normalize using t, with similar results; we thus focus on normalizing by V to avoid redundancy.

the specified range, we note that the firms mimic the decisions of a mass-customizing monopolist by charging the reservation price and providing perfect matches to all of their customers. Hence, one can view the market outcome for these low levels of value-fit ratios as if competing firms act like local monopolists. Once the value-fit ratio exceeds $\frac{1}{2}$, the firms abandon their local monopolist roles by shrinking their customization scopes. The firms, specifically, withdraw their product portfolios from the edges of the taste spectrum at the intermediate levels of ρ. As the value-fit ratio increases, customers become more tolerant for taste mismatches, and thus each firm is more likely to lose the customers in the middle of the taste spectrum to its competitor. In order to avoid this possibility of losing mainstream customers, firms end up offering customized products for customers with milder tastes while letting more outlying customers travel.

The two diagonally-patterned trapezoids in Figure 7 illustrate the customizing firms' equilibrium outcomes as the value-fit ratio (ρ) increases from an intermediate level to a high level. In this figure and those that follow, we set the unit MC cost below the lowest level of the continuous piecewise upperbound specified by propositions 2.4.5, 2.4.7, and 2.4.8. At such MC costs, we ensure the existence of equilibrium outcomes, which enables comparisons across different levels of ρ. Through the intermediate range (shown in the figure as ρ increases from 3/5 to 3/4), we observe that the customization scopes decrease in ρ, as we explained above. This finding is consistent with prior research (Dewan et al., 2003). However, above $\rho = \frac{3}{4}$, this trend *reverses*, evidencing a more complex dynamic. To be specific, when the value-fit ratio is above 3/4, each firm imposes a price-undercutting threat on its competitor (explained in subsection 2.4) to effectively compete for the entire market. As consumers become more tolerant to product misfit (i.e., as ρ increases), the price undercutting threat becomes more credible. In order to protect their hinterlands against this threat, the firms provide perfect matches for more outlying customers, while separating their MC scopes in the middle. Figure 7 also illustrates that single-product firms respond differently (from mass customizers) to an increase in the value-fit ratio. Specifically, as ρ exceeds 3/4, single-product firms begin to shift from the market quartile locations 1/4 and 3/4 towards the center, thus yielding more intense competition and subsequently leaving more surplus to the customers.

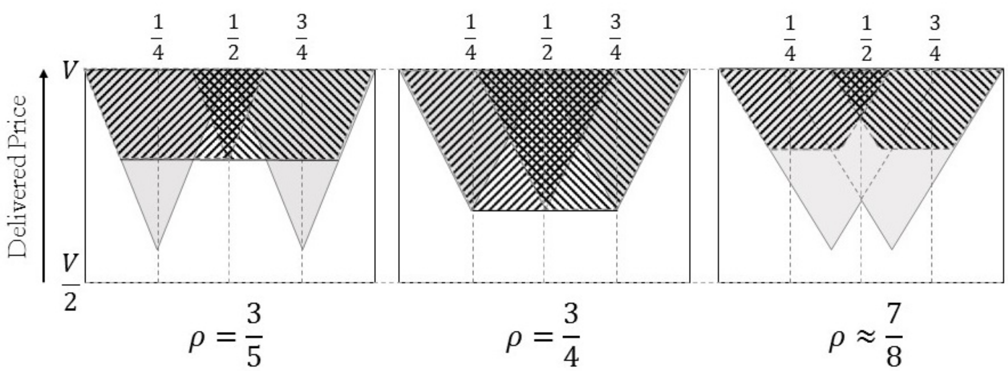


FIGURE 7. The evolution of equilibrium structures through $\frac{3}{5} \leq \rho < \frac{7}{8}$

The shaded triangles represent the equilibrium outcomes in the absence of MC, and the hashed trapezoids represent the MC equilibria. As ρ increases through $[1/2, 3/4]$, mass customizers decrease their MC ranges, and single-product firms consistently locate at the market quartiles. As ρ increases through $[3/4, 7/8)$, mass customizers expand their MC scopes and move back from the center, while single product firms approach the center.

As elaborated thus far, competing mass customizers adopt different product design strategies from competing traditional (single-product) firms. The following corollary shows that MC competition and single-product competition exhibit contrasting evolution of prices, profits and consumer surplus against ρ.

Corollary 2.6.2. *As the value-fit ratio increases from 0 to $\frac{7}{8}$, mass customizers' prices and profits develop non-monotonically, and in opposite directions from single-product firms' prices and profits.*

i. At low levels of value-fit ratio ($\rho \in [0, \frac{1}{2}]$), mass customizers and single-product firms charge constant *prices. The profits of the mass customizers are* increasing *and the profits of single-product firms are constant in ρ.*

ii. At intermediate levels of value-fit ratio ($\rho \in [\frac{1}{2}, \frac{3}{4}]$), mass customizers have increasing *prices and profits in ρ. Single-product firms' prices and profits* decrease *in ρ.*

iii. At high levels of value-fit ratio ($\rho \in [\frac{3}{4}, \frac{7}{8})$), mass customizers have decreasing *prices and profits in ρ. Single-product firms' prices and profits* increase *in ρ.*

Figure 8 illustrates the equilibrium prices and profits (on the left), in addition to the corresponding location decisions and aggregate consumer surplus (on the right). Under MC competition, we see that as ρ increases (i.e., as the fit sensitivity t decreases), both prices and profits initially follow a downward trend but subsequently increase. On the contrary, single-product competition is more intense at extreme levels of the fit sensitivity. Accordingly, mass customizers benefit from extreme (high or low) levels of the value-fit ratio, but single-product firms' profits reach their peak at moderate levels of ρ. We also note that MC competition almost always yields a higher price outcome than single-product competition does, except when $\rho = \frac{3}{4}$. While Martinez-Giralt and Neven (1988) show that increasing variety through offering additional distinct products intensifies price competition (leading to lower prices), we show that mass customizers can exploit customers' willingness to pay for their ideal products and charge higher prices compared with traditional firms. Interestingly, the value-fit ratio level leading the same equilibrium prices yields the lowest level of profits for the MC firms, whereas the single-product firms enjoy their maximum profits at $\rho = \frac{3}{4}$.

When $\rho < \frac{1}{2}$ (i.e., at relatively high values of t), mass customizers gain higher profits and leave less surplus for consumers than single-product firms, because customers are sensitive to product fit issues. As ρ increases within $[\frac{1}{2}, \frac{3}{4}]$, MC capability induces competition for consumers with mainstream tastes, yielding lower profits. In contrast, single-product firms turn out to compete without infringing directly on their competitor's market and by charging a price inversely related to the fit sensitivity. Hence, in these intermediate levels of the value-to-fit ratio, single-product firms benefit from the lowering of the fit sensitivity.

Once ρ exceeds 3/4, for both forms of competition, the progressions of prices and profits reverse. Beyond this point, single-product firms compete more aggressively in the middle, approaching the center and decreasing prices in ρ. On the contrary, mass customizers shift their MC scopes away from the center and charge higher prices. The observed difference in behaviors is rooted in the fact that single-product firms mainly compete for mainstream customers, but mass customizers compete for the entire market due to the credible price-undercutting threat (as we discussed earlier in this section). Facing the threat of being undercut, mass customizers become attentive to the outlying customers residing in their hinterlands, and exert less pressure on their competitor in the middle region. The outcome is more distinct product portfolios resulting in a less head-to-head competition and increased prices and profits.

Finally, the following corollary highlights the combination of conditions, considering ranges for both ρ and $\tilde{c}$, for which the availability of MC technology is detrimental to profits and/or consumer surplus.

Corollary 2.6.3. *There exist nonempty regions of value-fit ratio and customization cost where the following situations occur.*

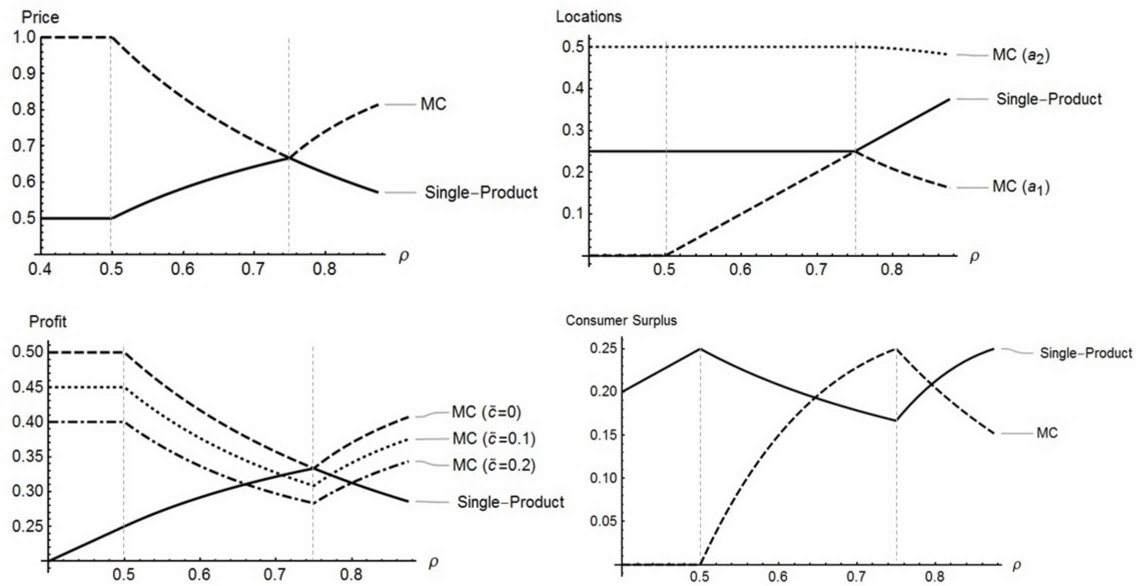


FIGURE 8. The progression of equilibrium prices, locations, profits, and consumer surplus through $\rho < \frac{7}{8}$

i. Profits are lower in MC duopoly than in single-product duopoly. That is when the following inequalities hold and the upperbound on $\tilde{c}$ *is derived from propositions 2.4.5 and 2.4.7.*

$$\tilde{c} > \frac{3-4\rho}{8\rho(1-\rho)} \text{ , and } \tilde{c} > \frac{2+5\rho-12\rho^2+2(7-8\rho)\sqrt{3\rho-2}}{12\rho(3\rho^2-8\rho+4)}$$

ii. Both profits and consumer surplus are lower in MC duopoly than in single-product duopoly. That is when in addition to the above conditions, $\rho < 1 - \sqrt{2}/4 \approx 0.65$ *or* $\rho \gtrapprox 0.80$.

The entire shaded region in Figure 9 reveals the ranges of $\tilde{c}$ and ρ for which equilibrium profits are lower in the MC duopoly than in the single-product duopoly. The region is bounded on the top by the $\tilde{c}$ threshold (derived from propositions 2.4.5 and 2.4.7), beyond which MC equilibria do not exist. To interpret the region (of profit decrease with MC), we call attention to the profit plots in Figure 8. It is evident that at $\tilde{c} = 0$ no level of ρ leads to a profit decrease with MC adoption. As $\tilde{c}$ increases, adopting MC results in profit decreases within greater ranges of ρ. This pattern is reflected in Figure 9. Furthermore, the darker shaded areas in the figure represent subsets of the discussed region where consumer surplus also diminishes. Since the equilibrium structures (prices and locations) are independent of the MC cost and consumer surplus is dictated by the equilibrium structures, consumer surplus is not affected by the level of $\tilde{c}$ as long as MC equilibrium exists.

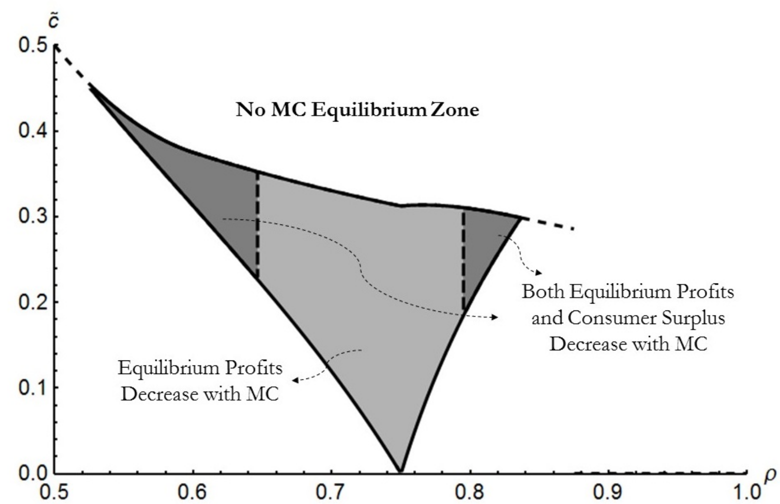


FIGURE 9. Parameter ranges where MC reduces profit and consumer surplus

The entire shaded region represents ranges of ρ and $\tilde{c}$ where firm profits are lower in equilibrium with MC than in equilibrium without MC. In the darker shaded areas, both consumer surplus and firm profits diminish with MC.

Mass customization has been hailed as a mechanism for creating economic value (Pine, 1993), by reducing the mismatch between consumers tastes and product designs. However, we have shown that while a cost-efficient MC technology can always benefit a monopolist, it can reduce competing firms' profits. Moreover, we see that under certain market circumstances, not only do profits decrease, but also consumer surplus decreases. The reason lies in the fact that MC equilibrium structures are independent of customization costs. We show that MC costs determine whether a firm utilizes the technology, but do not drive equilibrium prices or customization scopes. Instead, price competition drives the equilibrium structures.

When ρ is relatively low, firms pressure their competitor while extending customization scopes to the mid-market of consumer tastes. When ρ is high enough, we have shown that firms protect their market shares by customizing exactly to the extent beyond which the competitor undercuts. The MC scopes, thus, are not influenced by MC cost. If the customization cost is zero, mass customizers enjoy higher profits through premium prices, but as the customization cost increases, firms fail to resolve the cost inefficiencies through restructuring their pricing or product design decisions and may lose profit. Therefore, the extent of surplus transfer to consumers is not affected by the MC cost either. Syam and Kumar (2006) show that offering customized products in addition to standard products may intensify price competition but will improve profits. We reach a contrasting conclusion when firms should choose between offering either customized or standard products, but not both. Furthermore, contrasting with Dewan et al. (2003) and Cavusoglu et al. (2007), we establish the detrimental effect of MC on consumer surplus.

Conclusion

In this essay we have employed a Hotelling-type framework to study the location-then-price competition between two firms with mass customization (MC) capabilities. Each firm incurs an MC cost proportional to the breadth of its offerings. Consumers have uniformly heterogeneous tastes for product characteristics and a constant finite valuation for a perfectly matched product. To purchase a misfit product, each consumer incurs a linear-to-distance fit cost. We show that the structures of both the monopoly outcome and competitive equilibria depend on the proportion of customer valuation to fit sensitivity. MC costs influence the customization scope of neither a monopolist nor a competing firm, but determine whether a firm will opt to offer customization. We also contrast the equilibrium results for competing mass customizers with those for single-product firms. Our analyses yield three main conclusions that not only add to the academic literature on mass customization, but also suggest caution to practicing managers.

First, customization scopes in equilibrium do not monotonically decrease as consumers become more tolerant to product mismatch. Managers should therefore be aware that market trends might imply contrasting product line expansion strategies at different times. To this point, a firm's response to a decrease in fit sensitivity should be to contract its customization scope only if the sensitivity is beyond a threshold. In contrast, below that threshold the firm's response should be to expand the customization scope. This threshold therefore represents a turning point in the trends relating the customization scope and consumers' taste sensitivity. Failing to recognize the existence of this turning point, managers may jump to wrong conclusions based on past market trends within a particular region.

Second, in an MC duopoly, firm profits are maximized at extreme levels of market's sensitivity to fit. Therefore, if mass customizers can marginally influence consumers' attitudes through marketing activities, moderation is not a beneficial strategy. A more beneficial tactic would be to promote customers' sensitivity to purchasing ideal products, *if* customers are already sensitive enough. When, however, the market is such that consumers are relatively tolerant of taste discrepancies, then competing MC firms' profits would increase if consumers place even less weight on taste mismatch. In contrast, when single-product firms compete, they achieve maximum profits at moderate levels of consumers' sensitivity to fit.

Third, we show that equilibrium prices in the MC duopoly are always higher than those in the single-product duopoly. However, positive MC costs might result in lower profits for mass customizers. Therefore, as we have explored, market conditions dictate when firms would find it advantageous to compete offering customization rather than standard products. Moreover, there are certain market conditions under which neither firms nor consumers benefit from the availability of MC technology. A related implication of this finding is that regulators should evaluate the social welfare impacts of MC technology when deciding whether to facilitate MC investments within industry.

Bridge to Next Chapter

In this chapter, we considered a competition in terms of product line design and pricing between two firms with mass customization capabilities. Via flexible processes and technologies, each firm is capable of providing myriads of possibilities that match a continuous range of consumers' heterogeneous tastes. We discuss the effect of the mass customization technology on firm profits and consumer surplus at different levels of consumers' valuation and fit sensitivity. In the next chapter, we consider a different form of product variety, that is a firm offering a few distinct products. Unlike the first essay which considers a priori known customer valuations for a firm's offerings, in the upcoming essay, customers do not learn their valuations for a product until they consume it. In this setting, we study selling a box of sample-size products as a tool of seller-induced learning. We also study the common pricing tactic of offering a future credit along with a sample box.

CHAPTER III

ESSAY 2: SAMPLE BOXES FOR RETAIL PRODUCTS

This work is coauthored with Prof. Eren Çil and Prof. Michael Pangburn, and submitted to the Management Science journal.

Introduction

Buyers often have uncertain *a priori* product valuations for firms' offerings. In order to resolve this uncertainty and discover their preferred products, consumers may need to ultimately try multiple product variations, via sequential trials. A trending alternative technique for facilitating consumers' discovery of product valuations is for firms to offer *sample boxes*. A typical sample box includes a set of product varieties within a specific product category. Sample boxes are prevalent in both online and brick-and-mortar businesses. A prominent example in the online setting is Amazon, which has recently been offering sample boxes in such product categories as coffee, hair and skin care, sports nutrition, men's grooming, and pet treats. Similarly, big-box retailers such as Target and Walmart offer sample boxes in multiple product categories including fragrances, cosmetics, and skin care products. Offering sample boxes is also a common practice that has been adopted by smaller businesses; online examples include Verdant Tea's loose tea, Bubble Bandit's dishwasher detergent, and Master of Malt's whiskey sample boxes. The sample box concept may apply in different contexts under different names. For example, in a service context, wine and beer sampler "flights" similarly facilitate consumers' discovery of their preferences over multiple product varieties.

An important common characteristic in the above product examples is that consumers find it difficult or costly to fully assess their values prior to consumption. Nelson (1970) refers to these products as *experience goods*, for which consumers cannot obtain full information before purchase. Faced with multiple varieties of such products, the availability of a sample box allows consumers to avoid a potentially protracted search process. When a consumer must follow a search process, Weitzman (1979) has considered the trade-off between engaging in further exploration versus settling upon the best currently known product, identifying a threshold that defines a consumer's optimal stopping rule. Naturally, consumers continue such a search process in the hope of discovering a product variety that is preferable to what they already have. The downside risk is the possibility of trying potentially less preferred product varieties, in which case there is a negative impact on present consumption, and the imputed cost of thereby delaying the consumption of a hitherto preferred variety. The optimal stopping threshold for the search process, sometimes referred to as the *switchpoint*, equates a consumer's expected utility from further exploration with the best currently known product value.

As an alternative to the sequential search process described above, a sample box allows consumers to efficiently discover their valuations over multiple product varieties. In the absence of a sample box, a consumer faces the previously-described and potentially protracted search process to discover a desirable variety. In this case, the consumer may ultimately (and even optimally) settle upon a less-than-ideal variety, due to the cost associated with a sequential search process over full-size products. On the other hand, given the option of purchasing a sample box, the consumer can efficiently resolve the uncertainty for the sampled products. Thus, the consumer will identify the most preferred variety, and achieve this benefit within a compressed timeframe.

In this study, we analyze the potential benefits of offering sample boxes for a firm serving consumers facing value uncertainty. In particular, and consistent with the examples above, we assume the firm's products are consumable experience goods that a buyer may purchase repeatedly. Consumers have heterogeneous product valuations and are forward-looking. In each shopping period, they rationally choose a variety to purchase (if they decide to purchase at all), accounting for the learning implications of the purchase on their decisions in the ensuing periods. Facing these forward-looking consumers, the firm sets product prices to maximize profits. We first study the firm's pricing problem when consumers go through the self-discovery process—i.e., in the absence of a sample box. We then consider the alternative of offering a sample box along with its associated optimal price, and study its impact on expected profits. We also investigate the common tactic of offering a future price discount to sample box buyers. For example, in the case of Amazon.com, purchasing a sample box typically yields a *future credit* that the customer can apply to a subsequent purchase of any of the products featured within the box. Analyzing this future-credit tactic, we consider both its profit and consumer-surplus implications.

We prove that a sample box is an effective mechanism that can yield considerable value under a wide range of market settings. We establish that the informational value of a sample box yields an optimal price *premium* relative to the prices of individual products—considering equivalent net sizes. Despite this price premium, we also prove that consumers obtain equal or higher net expected surplus, while the firm's expected profit may decrease. The gain in consumer surplus is possible because the aforementioned price premium is more than offset by the expected learning benefit—i.e., avoiding potential successive purchases of suboptimal products. From the firm's perspective, the potential disadvantage of encouraging seller-induced learning via sample boxes is that some consumers avoid successive purchases after discovering their product valuations.

Our analyses also establish the benefit of including a future credit with the purchase of a sample box. We prove that by optimally specifying the future-credit level, a firm increases expected profits relative to the baseline case of not offering a sample box. The future credit effectively ties a consumer's purchase of the sample box to a subsequent purchase of a product. The firm can thereby leverage consumers' uncertainty to charge a more significant price premium for this bundle—compared to the alternative of a sample box with no future credit. This price premium is collected by the firm on all sample box buyers, more of whom would forego purchasing in the second period if no future credit were offered.

The remainder of this essay is organized as follows. We review the related literature in Section 3.2, and present our model in Section 3.3. In Section 3.4, we describe the consumers' optimal policy when they follow a sequential search process to discover their valuations. Then we characterize the firm's optimal pricing decision given consumers' search policy. In Section 3.5, we study the problem when the firm elects to offer a sample box in parallel with the individual product variants. We consider two pricing schemes: offering a sample box either *with* or *without* future credit. In Section 3.6 we provide the results regarding the impact of sample boxes on a firm's profitability. Section 3.7 concludes our study. Proof details of the propositions and corollaries are within Appendix C.

Literature Review

This study builds upon several research streams in the literature. Firstly, our work contributes to the literature on pricing of experience goods. That literature stream predominantly studies the (dynamic) pricing of an experience good when consumers face *a priori* uncertainty regarding a product's quality (see, for example, Shapiro, 1983; Bergemann and Välimäki, 2006; Yu et al., 2015; Chen and Jiang, 2016; Jiang and Yang, 2018). We extend this stream by studying the pricing decisions for a multi-product monopolist that serves heterogeneous customers who explore different product options over time. Thus, our study is also differentiated from the existing literature on the pricing of experience goods in competitive settings (see, for example, Villas-Boas, 2004a, 2006; Jing, 2015; Galbreth and Ghosh, 2017).

When no sample box is offered, we study the rational decision of a consumer who can sequentially try distinct varieties of an experience good. Therefore, our paper relates to the literature on *sequential* search for the best alternative and the optimal stopping point (Lippman and McCall, 1976; McCall et al., 2008). This stream centers on consumers' *search policy*, considering the impact of consumers' discoveries in the earlier stages on their choices in the later stages (Kohn and Shavell, 1974; Weitzman, 1979; Wolinsky, 1986). In that sense, it diverges from the contemporaneous research stream on the *prior theory of search* (Stigler, 1961, 1962; Nelson, 1970), in which consumers decide on the number of searches prior to the search process. The search processes described by Weitzman (1979) and Wolinsky (1986) consider options that are only observed but not enjoyed, when evaluated. In contrast, we consider a search process for experience goods, consistent with Kohn and Shavell (1974), where consumption occurs as part of the search. Despite this similarity, we consider a limited set of products, unlike Kohn and Shavell (1974).

Although there is extensive literature (reviewed by Ratchford, 2009) on the effect of consumers' search for price on cross-firm price dispersion, there is sparse literature on optimal pricing under consumers' sequential search for the best alternative. Cachon et al. (2008) study the effects of search cost on equilibrium prices, assortments, and profits of competing multi-product firms. In contrast, we study pricing decisions for a multi-product monopolist facing consumers who may opt to experience multiple products over time before settling upon their preferred option. Najafi et al. (2017) study the dynamic pricing decision of a firm that offers vertically differentiated products. In their model, consumers explore a limited set of options according to a sequence decided by the firm. In the setting we consider, product values are identically distributed (as opposed to assuming vertically differentiated products), and consumers may choose their individual search orderings.

We assume that consuming a product variant allows customers to perfectly resolve their valuation uncertainties for the variant. In an alternative setting, Lippman and McCardle (1991) study a problem in which acquiring an item does not fully resolve a decision maker's valuation uncertainty, but updates the prior distribution of valuation. Branco et al. (2012) and Ke et al. (2016) consider models in which consumers undergo a continuous costly search to gradually learn the characteristics of products. While their studies consider customers' evaluations before purchase (Hirshleifer, 1973), our model is better suited to the learning process via purchasing experience goods.

We consider a scenario in which the firm grants sample box buyers a future credit which they can apply to a subsequent purchase of a full-size product. Thus, our study relates to the literature on behavior-based price discrimination, i.e., setting prices that depend on a customer's purchasing history. Consistent with our model, Cremer (1984) studies a firm that serves heterogeneous customers with ex-ante unknown valuations, and sets discriminatory prices for first-time buyers (in period 1) and returning buyers (in period 2). In a similar setting, Jing (2011b) centers on comparing relative merits of different pricing strategies. Bhargava and Chen (2012) study a firm's incentive for spot selling to informed customers versus advance selling to uninformed customers who have dissimilar prior beliefs regarding their valuations. Unlike our model, the three aforementioned papers consider a firm that supplies a *single* product to the market for *two* periods. We find new insights when customers sequentially investigate a firm's multiple products over an infinite time horizon. For example, Cremer (1984) concludes that charging a premium price (followed by a lower price) improves the firm's profit, whereas we show that this result is not necessarily generalizable to a multi-product setting. Furthermore, in contrast with models of behavior-based pricing of durable goods (e.g., Fudenberg and Tirole, 1998; Villas-Boas, 2004b), we consider consumable non-durable products.

We also contribute to the literature on consumers' pre-purchase sampling of goods. Papers in this vein predominantly study the use of *free* samples as a tool by which consumers learn their valuations for a specific product (see, for example, Jain et al., 1995; Heiman et al., 2001; Wang and Zhang, 2009). On the contrary, we study the pricing of sample boxes, given that the selling of sample boxes is now common practice in retailing. The literature (for example, Bawa and Shoemaker, 2004; Li and Yi, 2017) highlights that a potential drawback of free samples is the so-called cannibalization effect, which is the reduction of paid purchases due to consumers' substituting free samples for full-size products. A firm can avoid this cannibalization loss by offering an optimally priced sample box. We find, however, there remains the so-called *acceleration effect*, which can be positive or negative depending on the firm's pricing strategy. In particular, sampling may accelerate a consumer's settling upon either a firm's product or an outside option, and thus can be a double-edged sword.

Our work also relates to the literature on product bundling. Stremersch and Tellis (2002) review the early research developments in this field and propose a classification of bundling strategies with respect to focus (product or price) and form (pure or mixed). Also, Venkatesh and Mahajan (2009) summarize the implications from a large body of stylized models that concern bundling. We forego such detailed reviews, but call attention to the understudied potential of a bundle to impact consumers' search efforts in a monopolistic setting (Harris and Blair, 2006). Guiltinan (1987) views the search benefit of a bundle as the reduced cost of assembling the complementary components. We consider a setting with substitutable products, in which the bundle helps consumers settle upon the best option more efficiently. Chhabra et al. (2014) investigate consumers' sequential inspections of several (durable) products. In their model, the seller can induce learning, in the absence of which consumers receive only a noisy signal of the true value of an option after inspection. By contrast, we consider experience goods, for which customers resolve their valuations only after consumption. Consistent with Geng et al. (2005), Chhabra et al. (2014) show that bundling information goods (or services) is profitable when consumers' values for future goods do not decrease too quickly. In the experience-good setting, however, we show that a sample box may, depending on the firm's pricing strategy, either increase or decrease profits over the entire range of time discount factors.

Model

In this study, we consider a monopolist selling two products that are substitutable variants of a consumable experience good. The firm serves consumers who are heterogeneous in their valuations. Consumers face uncertainty regarding product valuations, and thus realize their valuation for any given product variant only after consuming it. Denoting each consumer's valuation for product $i \in \{1, 2\}$ by V_i, we assume that V_1 and V_2 are independent and identically distributed random variables that follow the probability density function $f(\cdot)$ and the cumulative probability function $F(\cdot)$ on the support $[0, 1]$. We denote the mean of the distribution by $\mu = \int_0^1 v dF(v)$.

Consumers are in the market to maximize their expected utilities over an infinite time horizon with a discount rate of $\beta \in [0, 1)$ per period. Each consumer may purchase the firm's products repeatedly, yet can make only one purchase in each shopping period, denoted by $t = 1, 2, ...$ For simplicity, we assume individual consumers are small relative to the size of the market, which is normalized to 1. In our base model, we study a setting where the firm offers only full-size products in each shopping period. We then extend this model by allowing the firm to introduce a *sample box* consisting of the two varieties, each half the size of the full product.[1] In both models, the firm's objective is to maximize its expected profit over an infinite time horizon with a per-period discount rate β. We denote the firm's prices for the

[1]We keep the size of the sample box equal to the size of the full product so that we can focus on the value of seller-induced learning and avoid dealing with the mixed effects of learning and size. A variable size of the sample box has implications on the consumption utility consumers obtain from, and the length of time it takes to consume, the sample box, complicating the model in ways that do not serve our purpose. This assumption is also to keep consistency with the literature on seller-induced learning (see, for example, DeGraba, 1995; Jing, 2011a).

full-size products and the sample box (if offered) by p and b, respectively.[2] Motivated by common practice, we also consider the option of offering sample-box purchasers an amount of *future credit* that is applicable to a subsequent purchase. We denote the monetary value of this future credit by δ. For notational convenience, we consider the marginal cost of production to be zero.

Once the firm sets the prices for its product offerings, each customer chooses one of the following four options in each shopping period: i) purchase Product 1, ii) purchase Product 2, iii) purchase the sample box if offered, iv) not make any purchase. In each period, consumers obtain a net utility of $V_i - p$ if they purchase product $i \in \{1, 2\}$ without having a future credit. In case they are granted a future credit in the previous period, they can apply it to their current purchase to obtain the net utility $V_i - p + \delta$. On the other hand, a consumer purchasing the sample box achieves the net utility $\frac{1}{2}(V_1 + V_2) - b$. We assume that consumers exercise an outside option which gives them a utility of $\underline{u}$ when they do not make any purchases. Moreover, we assume that when a consumer is indifferent between purchasing and exercising the outside option, the consumer will purchase.[3]

Analysis: Self-Discovery

We first consider a setting in which customers must discover product values via sequential search. We will next characterize customers' optimal policy and subsequently study the firm's problem given customers' rational choice.

[2]We can show that, in all the scenarios we will study, it is optimal for the firm to set equal prices for the two full-size products.

[3]This assumption is innocuous because given prices for which consumers are indifferent between purchase options, a trivial price adjustment (e.g., \$0.01) can induce the desired behavior.

Consumer's Problem

When the firm offers only full-size products, in each period a customer can purchase either product (Product 1 or Product 2) or purchase nothing. Potentially, a consumer could purchase either or both products before settling upon their preferred option—which could be to simply refrain from further purchases. Consumers are forward looking and therefore take into account the fact that the information gleaned from a current purchase can yield more informed decisions later. Such forward-looking behavior is an integral part of customers' decision making, particularly when they are in the process of discovering their product valuations. For example, by purchasing either product in period 1, the customer assesses not only the value of that immediate consumption, but also how the resulting product-value knowledge will improve subsequent decisions.

We generically denote the purchasing decisions of a customer in period t by $a_t \in \{1, 2, \varnothing\}$, where $a_t = i$ when the customer buys Product $i \in \{1, 2\}$, and $a_t = \varnothing$ when the customer does not buy any products. Thus, the sequence $A = (a_t)_{t=1}^{\infty}$ defines the full sequence of purchase decisions for the customer. As we do not impose any structure on the purchasing sequence of a customer, the sequence $A = (a_t)_{t=1}^{\infty}$ may potentially take infinitely many forms. However, we show that the optimal purchasing decisions of a customer must follow one of the following four patterns: i) the customer purchases the same product in all periods, ii) the customer purchases different products in the first two periods and subsequently consumes the most-preferred variant, iii) the customer makes a purchase only in the first period, and iv) the customer does not consume any of the firm's products in any periods. We formally present this result in Proposition 3.4.1.

Proposition 3.4.1. *Let $A^* = (a_t^*)_{t=1}^{\infty}$ be the optimal purchasing sequence of a customer. Then, we have that A^* follows one of the four patterns:*

1. $a_t = i$ *for all* $t \geq 1$, *where* $i \in \{1, 2\}$.

2. $a_1 = i$, $a_2 = j$, *and* $a_t = \begin{cases} 1 & \text{if } v_1 = max\{v_1, v_2, p + \underline{u}\} \\ 2 & \text{if } v_2 = max\{v_1, v_2, p + \underline{u}\} \\ \varnothing & \text{if } p + \underline{u} \geq max\{v_1, v_2\} \end{cases}$ *for all* $t \geq 1$, *where* $i, j \in \{1, 2\}$, $i \neq j$.

3. $a_1 = i$ *and* $a_t = \varnothing$ *for all* $t \geq 2$.

4. $a_t = \varnothing$ *for all* $t \geq 1$.

Essentially, Proposition 3.4.1 implies that once a customer stops exploring, restarting such exploration later is suboptimal. Put another way, a customer does not benefit from postponing the trial of a product, if exploring that product has positive expected value. Therefore, for example, if a customer (optimally) makes no purchase in the first period, then rationally there will be no subsequent purchasing. Given that there are two products, all valuation assessments will therefore be completed in the first two periods. Thus, customers' purchasing decisions in and after the third period are straightforward. For a customer who has tried both products in the first two periods, purchasing decisions from period 3 are simply based on comparing the realized values for Product 1, Product 2, and the outside option, which are v_1, v_2, and $p + \underline{u}$, respectively.

In the second period, if the consumer had previously purchased Product $i \in \{1, 2\}$, then the customer's realized valuation v_i will determine whether it is preferable to try and discover the value of Product j (where $j \neq i$), or perhaps simply continue purchasing Product i. In particular, after possibly trying Product j in the second period, a customer will rationally continue to purchase Product j if $v_j \geq \nu \equiv \max\{v_i, p + \underline{u}\}$. Otherwise, the customer will continue with either Product i or the outside option, whichever yields higher surplus. The threshold $\nu \equiv \max\{v_i, p + \underline{u}\}$, which we henceforth refer to as the *on-hand value*, conveniently summarizes the value of the best hitherto evaluated alternative, including the outside option. The value of exercising the outside option, as reflected in the second argument of the maximum function, is the utility obtained from the outside option plus the value of avoiding the purchase (at price p).

We now leverage the on-hand value threshold ν to express the expected consumer utility from period 2 onward. Let $u_{ij}(p, v_i)$ denote the expected net present value (NPV) of consumer expected surplus if Product j is purchased in period 2, given that Product $i \neq j$ (with realized value v_i) was purchased in the first period.

$$u_{ij}(p, v_i) = \int_0^{\nu} (v_j - p + \beta \frac{\nu - p}{1 - \beta}) dF(v_j) + \int_{\nu}^{1} \frac{v_j - p}{1 - \beta} dF(v_j) \tag{3.1}$$

The first integral expresses the value associated with v_j outcomes less than ν, in which case the consumer will opt for ν in subsequent periods. The second integral expresses the value associated with v_j outcomes greater than ν, in which case the consumer will opt for v_j in subsequent periods. We now consider the expected consumer utility associated with the alternative second-period decision to *not* switch from Product i to Product j in the second period. The consumer will thus obtain a surplus of $\nu - p$ in each successive period. We denote the utility NPV for this alternative as $u_{i\bar{j}}(p, v_i)$, such that

$$u_{i\bar{j}}(p, v_i) = \frac{\nu - p}{1 - \beta}. \tag{3.2}$$

Recall from Proposition 3.4.1 that if the consumer does not switch to Product j in the second period, then it cannot be rational to do so subsequently.

Comparing the relative magnitudes of the expected utilities $u_{ij}(p, v_i)$ and $u_{i\bar{j}}(p, v_i)$ will determine a customer's optimal decision in the second period. We will next show, in Proposition 3.4.2, that comparing these levels conveniently reduces to a comparison between the on-hand value ν and a threshold that we refer to as the valuation *switchpoint*. We denote this critical switchpoint value as $\tilde{\nu}$. If the on-hand value exceeds this switchpoint, then the customer rationally continues with the on-hand alternative, but otherwise will (optimally) purchase the not-yet-evaluated option. Naturally, discovering the valuation of Product j in the second period will be appealing only for a customer who realizes a relatively low value for Product i.

Proposition 3.4.2. *Assume that the firm offers only the full-size products at price p. Consider a customer who has consumed Product $i \in \{1, 2\}$ in the first period with resulting valuation v_i.*

(i) It is optimal for this customer to try Product $j \neq i \in \{1, 2\}$ in the second period only if

$$\max\{v_i, p + \underline{u}\} \leq \tilde{\nu},$$

where $\tilde{\nu}$ is the unique solution to

$$\int_0^{\nu} (v_j - \nu) dF(v_j) + \int_{\nu}^{1} \frac{v_j - \nu}{1 - \beta} dF(v_j) = 0, \tag{3.3}$$

and becomes $\frac{1-\sqrt{1-\beta}}{\beta}$ when $F(v) = v$.

(ii) $\tilde{\nu}$ is an increasing function of β and converges to μ and 1 as β approaches 0 and 1, respectively.

In Proposition 3.4.2, we define the switchpoint $\tilde{\nu}$ as the unique solution to (3.3) for any general distribution $F(.)$ governing consumers' stochastic product valuations. When $F(v) = v$, we can obtain the consumer switchpoint as $\frac{1-\sqrt{1-\beta}}{\beta}$. The expression on the left-hand side of (3.3) can be interpreted as the expected discounted consumer utility when the firm offers only one product at price ν—in the absence of the outside option. Thus, the switchpoint $\tilde{\nu}$ corresponds to the price level that extracts *all* of the consumer surplus in the one-product setting. When we account for the non-zero outside option in the one-product model, the firm has to reduce its price to $\tilde{\nu} - \underline{u}$ to make sure consumers will not prefer their outside option. We also show that the switchpoint $\tilde{\nu}$ can be as low as the expected product valuation μ for low levels of discount rate β, but increases up to the highest possible valuation, 1, as consumers increasingly value future consumption (i.e., as β increases).

Having characterized the purchasing decisions of customers in the second period, we next identify the conditions under which customers purchase in the first period. As we showed in Proposition 3.4.1, customers rationally will not buy any of the firm's products in the future if they do not start trying products in the first period. Therefore, the discounted utility of a customer who does not make a purchase in the first period is simply $\underline{u}/(1-\beta)$. On the other hand, for a customer who purchases Product i, the first-period net utility of $v_i - p$ may potentially be high enough (i.e., higher than $\underline{u}$) to raise the customer's on-hand value when entering the second period. As explained earlier, if the on-hand value ν exceeds the switchpoint $\tilde{\nu}$, the customer will prefer not to explore Product j and thus obtains the discounted utility $u_{i\bar{j}}$ in period 2. Otherwise, it will be optimal to try Product j, which generates the expected discounted utility u_{ij}. Thus, we can write the discounted expected utility of a customer who purchases Product i in the first period as

$$u_i(p) = \begin{cases} \mu - p + \beta(\int_0^{\tilde{\nu}} u_{ij}(p, v_i)dF(v_i) + \int_{\tilde{\nu}}^1 u_{i\bar{j}}(p, v_i)dF(v_i)) & p \leq \tilde{\nu} - \underline{u} \\ \mu - p + \beta \int_0^1 u_{i\bar{j}}(p, v_i)dF(v_i) & p > \tilde{\nu} - \underline{u} \end{cases} \tag{3.4}$$

where $\tilde{\nu}$ is the switchpoint characterized in Proposition 3.4.2. As we next prove, if the firm chooses a price p less than $\tilde{v} - \underline{u}$ then the resulting utility (the first case in (3.4) above) dominates the benefit $\underline{u}/(1-\beta)$ that would ensue from pursuing the outside option. On the other hand, when facing a price p greater than $\tilde{v} - \underline{u}$, it is optimal for the consumer to purchase the outside option.

Proposition 3.4.3. *When the firm offers full-size products at price $p \leq \tilde{v} - \underline{u}$, consumers rationally purchase Product i in the first period and continue with their optimal purchasing decisions. If the firm charges $p > \tilde{v} - \underline{u}$, then consumers will consume the outside option (yielding utility $\underline{u}$).*

Propositions 3.4.2 and 3.4.3 evince a clear resemblance between the purchasing decisions of customers in the first two periods. Namely, in both periods 1 and 2, customers compare their on-hand values, $p + \underline{u}$ and $\max\{v_i, p + \underline{u}\}$ respectively, with the constant switchpoint $\tilde{\nu}$. This is consistent with a result by Kohn and Shavell (1974), who study consumers' purchasing policy given infinite options. They show that, when facing unlimited options, a consumer's problem remains essentially unchanged over successive periods; in each period, customers compare their on-hand value with a switchpoint representing the expected value of continuing their search over the remaining infinite set of options. Given unlimited and identically distributed purchase options, it is intuitive that the switchpoint should remain constant over time. One might expect the switchpoint to reduce in a product setting with limited alternatives, as the consumer's option-set for further exploration reduces (leaving the consumer with fewer options as the search process unfolds). If consumers were to set a switchpoint higher than $\tilde{\nu}$ in the first period, this would imply they would make a purchase in the first period even when their on-hand utility $p + \underline{u}$ is above $\tilde{\nu}$ (equivalently, when the price is above $\tilde{\nu} - \underline{u}$). They would then stop their search process after the first period since their on-hand utility would not decrease. In that case, consumers would never try the second product, reducing their search problem to a one-product version of our model for prices above $\tilde{\nu} - \underline{u}$. However, as we discussed after Proposition 3.4.2, consumers rationally choose to exercise their outside option in a one-product version of our model when the firm charges more than $\tilde{\nu} - \underline{u}$. Hence, as proven in Propositions 3.4.2 and 3.4.3, even while the set of remaining (unexplored) product alternatives declines over time, the same switchpoint remains pertinent across both the initial search periods (during which time the consumer can potentially discover the value of both products). In each of these periods, if a customer's guaranteed utility exceeds the switchpoint $\tilde{\nu}$, then the consumer rationally ceases their search process.

Combining the results in propositions 3.4.2 and 3.4.3, we illustrate customers' optimal purchasing behavior in Figure 10. (The figure illustrates the first two periods since subsequent consumer decisions are straightforward, as highlighted previously.) Consequently, when the price is attractive, meaning $p \leq \tilde{\nu} - \underline{u}$, the customer's optimal discounted expected utility, which we denote as $u(p)$, is given by:

$$u(p) = \mu - p + \beta\left(\int_0^{\tilde{\nu}} u_{ij}(p, v_i)dF(v_i) + \int_{\tilde{\nu}}^1 u_{i\bar{j}}(p, v_i)dF(v_i)\right). \tag{3.5}$$

Otherwise, when $p > \tilde{\nu} - \underline{u}$, the customer optimally obtains the discounted net utility $\frac{\underline{u}}{1-\beta}$ via the outside option. Given uniformly distributed valuations ($F(v) = v$), the above utility reduces to:

$$u(p)|_{F(v)=v} = \frac{\sqrt{1-\beta} - 1 + 3\beta(1-p) - \beta\sqrt{1-\beta} + \beta^3(p+\underline{u})^3}{3\beta(1-\beta)}. \tag{3.6}$$

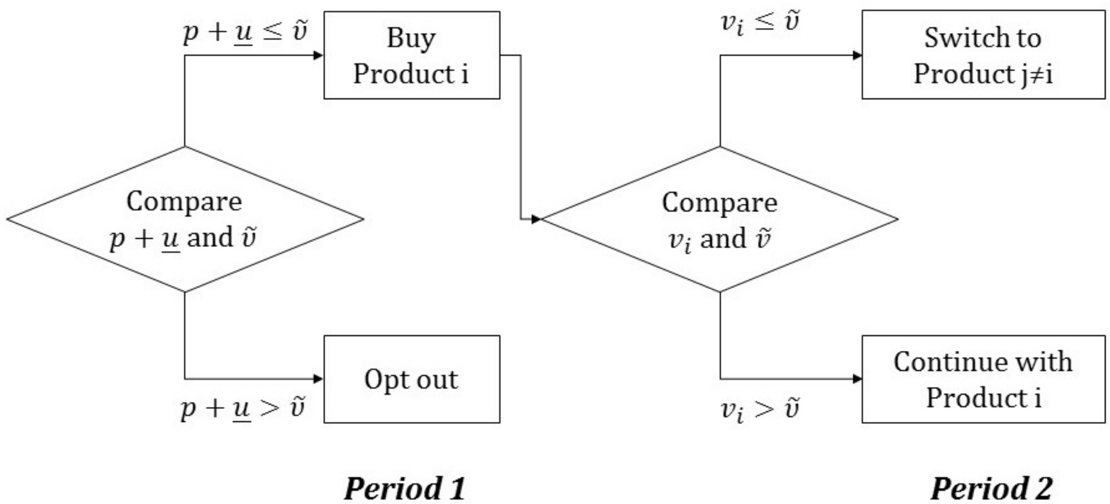


FIGURE 10. Consumer's optimal policy in the first two periods

The representative consumer makes a purchase in period 1 if the sum of the price and the outside option utility is less than a threshold, and buys the same product in period 2 if the realized valuation for that product is greater than the same threshold considered in period 1. Otherwise, the consumer buys a different product in period 2.

As illustrated in Figure 10, a notable property of the consumer's optimal policy is that although resorting to the outside option in period 2 after trying a product in period 1 is a *feasible* purchasing behavior, it is not optimal. This follows from the constancy of the switchpoint $\tilde{\nu}$. The same price level that justifies purchasing in the first period also applies in the second period, thereby supporting the trial of a different product if the first trial was not sufficiently positive. An implication of this result is that, in the second period, the comparison between the on-hand value ($\nu \equiv \max\{v_i, p + \underline{u}\}$) and the switchpoint $\tilde{\nu}$ is reduced to the comparison between v_i and $\tilde{\nu}$, since in the optimal policy a second-period purchase must follow a first-period purchase, which in turn requires $p + \underline{u} \leq \tilde{\nu}$.

Firm's Problem

Having analyzed the structure of consumers' rational purchase decisions, we next investigate the firm's corresponding optimal pricing policy. The firm aims to set, for each of the two identical products, a fixed price p, with the objective of maximizing the expected profit NPV. Given consumers' rational choice policy, we can show that it is not optimal for the firm to set different prices for the two products. From the consumer decision structure, summarized in Figure 10, we know that a consumer who makes a purchase in the first period will also rationally purchase in the second period, at which time the customer will either consume the same product again, or instead try a second product. Thus, for $p \leq \tilde{\nu} - \underline{u}$, a customer will purchase product i or j in the first two periods. If the consumer learns that *both* v_1 and v_2 are less than $p + \underline{u}$ (the likelihood of which is given by $F(p+\underline{u})^2$), then the consumer will not purchase after the second period. Otherwise, with either of the product valuations at least equal to $p + \underline{u}$, the consumer will continue purchasing the (preferred) product over time. Given that the two products are substitutable and identically priced variants from the firm's perspective, it is immaterial to the firm whether a consumer ultimately prefers variant i or j—the firm collects p per period in either case. The firm's problem of setting a product price p to maximize discounted expected profits, for the (present) case where consumers learn product values via sequential trials, is thus as follows.

$$\max_p \pi(p) = (1+\beta)pF(p+\underline{u})^2 + \frac{p}{1-\beta}\left[1 - F(p+\underline{u})^2\right] \tag{3.7}$$

$$\text{s.t.}$$

$$\text{participation: } u(p) \geq \frac{\underline{u}}{1-\beta} \Longleftrightarrow p \leq \tilde{\nu} - \underline{u}$$

$$0 \leq p \leq 1$$

The participation constraint ensures that purchasing occurs; otherwise the firm earns no profits. Given any price p that satisfies the constraint, the firm will collect p from the consumer in each of the first two periods. With probability $F(p+\underline{u})^2$, neither product will be sufficiently attractive to continue purchasing past period two, in which case the firm collects p in only the first two periods. Otherwise, with probability $1 - F(p+\underline{u})^2$, the firm collects p in every period, the present value of which is simply $\frac{p}{1-\beta}$. These two alternatives reflect the two terms of the discounted expected profit. We next establish the optimal price p^* that maximizes the firm's profits.

Proposition 3.4.4. *If $pf(p+\underline{u})$ is increasing in p,[4] and consumers follow the optimal search policy, there exists a $\tilde{\beta}(\underline{u}) \in (0,1)$ at which the specification of the firm's optimal price changes as below.*

$$p^* = \begin{cases} \tilde{\nu} - \underline{u} & \beta \leq \tilde{\beta}(\underline{u}) \\ \tilde{p}(\beta) & \beta > \tilde{\beta}(\underline{u}) \end{cases}, \tag{3.8}$$

[4]This assumption implies that the density function does not decrease too quickly at any value of p (Caminal, 2012). Throughout the essay, we consistently adopt this assumption as a sufficient condition to derive our results for the generic distribution $f(.)$. Nevertheless, many of the results would still hold for a broader family of distributions.

where $\tilde{p}(\beta)$ *solves*

$$1 - \beta^2 F(p + \underline{u})\,[F(p + \underline{u}) + 2pf(p + \underline{u})] = 0. \tag{3.9}$$

Additionally, $\tilde{p}(\beta)$ *is decreasing in* β*.*

Proposition 3.4.4 describes the firm's optimal price given that consumers undergo a sequential search process. It demonstrates that as the discount factor increases in the low-value range, the firm charges an increasing price, while leaving zero surplus beyond that of the outside option. As the discount factor passes a threshold, the firm decreases the price and increasingly grants surplus in excess of that from the outside option. Figure 11 illustrates the behavior of the optimal price.

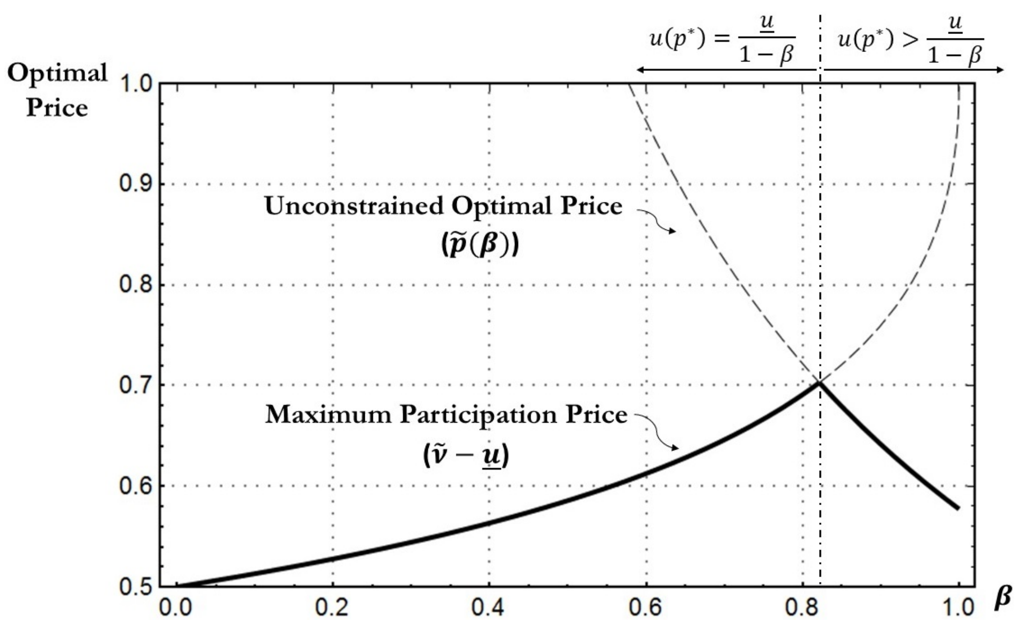


FIGURE 11. Firm's optimal price facing consumers' self-discovery process ($F(v) = v$)

When the discount factor is low the firm charges an increasing price and leaves zero consumer surplus. For large values of the discount factor, the firm sets a decreasing price and leaves a positive surplus.

We can understand the non-monotonicity of the optimal price by contrasting the two terms of the objective function in (3.7). The first term captures the profit generated by the customers who make purchases only in the first two periods. These customers leave the firm after the second period because neither of the products yield sufficiently high value. The second term corresponds to those customers whose high valuation for at least one product creates a long-term revenue stream for the firm. When the firm tries to solve the problem without the participation constraint, the first term of the profit function, attributing to low-valuation customers who leave after period 2, is maximized at the highest possible price, 1. On the other hand, the second term of the profit function is maximized at an interior price, lower than 1, due to its non-monotonicity in price. As β increases, the firm puts less weight on the revenues from the low-valuation customers who leave after trying the product in the first two periods, and more weight on the high-valuation customers who keep buying in the long run. Consequently, the unconstrained optimal price of the firm decreases—as shown in Figure 11.

Now consider the impact of the consumer participation constraint. Naturally, customers' willingness to pay for the product increases as they obtain a higher lifetime utility when future consumption becomes more valuable. The increasing plot in Figure 11 reflects this effect. On the one hand, for low values of the discount factor, customers' willingness to pay restricts the firm's desire to ideally charge a high price as described formerly. The best price the firm can set will then be the highest that customers are willing to pay. As a result, in this region customers are indifferent between buying and leaving. On the other hand, when the discount factor is high, the firm's profit-maximizing price is below customers' willingness to pay, so consumers enjoy a higher surplus from participation compared to leaving.

Corollary 3.4.5 specifies the optimal price when customer valuations follow the uniform distribution. These results are directly obtained from Proposition 3.4.4 by substituting v for $F(v)$.

Corollary 3.4.5. *If $F(v) = v$, the firm's optimal price characterized in Proposition 3.4.4 converts to*

$$p^* = \begin{cases} \frac{1-\sqrt{1-\beta}}{\beta} - \underline{u} & \beta \leq \tilde{\beta}(\underline{u}) \\ \frac{\sqrt{3+\beta^2\underline{u}^2}}{3\beta} - \frac{2\underline{u}}{3} & \beta > \tilde{\beta}(\underline{u}) \end{cases}, \tag{3.10}$$

where $\tilde{\beta}(\underline{u}) \in (0,1)$ solves $4\beta^3\underline{u}^2 + 3\beta^2(3 - 4\underline{u}) + 2\beta(3 + 2\underline{u}) - 11 = 0$.

The optimal price characterized in Corollary 3.4.5 is illustrated in Figure 11 specifically for the uniform distribution case with $\underline{u} = 0$. The trends exhibited in this illustration are also representative of the optimal price (from Proposition 3.4.4) for other families of distributions, as we discussed above. We next turn our attention to understanding the firm's optimal pricing and profits if it opts to offer a sample box.

Analysis: Sample Box

In this section, we consider the impact of the firm offering a sample box alongside the standard full-size products in its product line. The first two subsections study the problems of consumers and the firm assuming that the firm does not offer a future credit. In the subsequent two subsections we incorporate the offering of a future credit with the sample box.

Consumer's Problem

When the firm offers a sample box, consumers have an alternative opportunity to learn their valuations, in lieu of the more costly search process with full-size products. They can, simply, purchase the sample box and resolve the uncertainty over their valuations for both products in the first period. Given that the firm has two products, we assume the sample box will consist of two half-size versions of the standard products, so that the sample box serves the consumers' first-period consumption. Thus, the a priori expected value of the sample box is μ. We assume that the sample box is only offered in the first period.[5] We denote the price of the sample box by b. If a consumer purchases the sample box, we represent the resulting expected discounted utility as $u_s(b, p)$, which we express next.

$$u_s(b,p) = \mu + \frac{2\beta}{1-\beta}\int_{p+\underline{u}}^{1}\int_{0}^{v_j}(v_j - p)dF(v_i)dF(v_j) + \frac{\beta\underline{u}}{1-\beta}F(p+\underline{u})^2 - b \quad (3.11)$$

The second term in the above expression captures the expected utility of a consumer who purchases the sample box in the first period, and learns that the most-preferred product yields higher utility than the outside option. The third term captures the scenario where a consumer learns that both tried products in the sample box yield

[5]We verify that for a reasonable range of distributions, the firm does not find it optimal to offer the sample box after the first period. Note that consumers learn their valuations for both featured products after purchasing the sample box. For the sample box to be chosen over the full-size products more than once, its price should be lower than the price of the full-size products. Considering several members of the beta distribution family, including uniform ($\alpha = \beta = 1$), right-triangle ($\alpha = 2$ and $\beta = 1$), left-triangle ($\alpha = 1$ and $\beta = 2$), and bell-shaped ($\alpha = \beta = 2$), where α and β are the distribution parameters, we find that the firm would never optimally charge a lower price for the sample box than the full-size product. Furthermore, we confirm that it is not optimal for a consumer to postpone the purchase of the sample box to any period other than period 1.

less value than the outside option. Note that (3.11) can also be written as

$$u_s(b, p) = g(p) - b, \tag{3.12}$$

where, given uniformly distributed product valuations, $g(p)$ is as follows.

$$g(p)|_{F(v)=v} = \frac{3 + \beta + 2\beta(p^3 + 3p^2\underline{u} + \underline{u}^3 - 3p(1 - \underline{u}^2))}{6(1 - \beta)} \tag{3.13}$$

Firm's Problem

When it offers a sample box, the firm's objective is to jointly set the box and (full-size) product prices, b and p respectively, to maximize profit. In the first period, faced with the same a priori information, consumers will consistently opt for the same expected-value maximizing purchase alternative, i.e., purchase the box, a full-size product, or nothing—the outside option. If the representative consumer decides to purchase the sample box in the first period, then the consumer realizes both v_1 and v_2, and knowing those valuations, will decide whether to purchase either full-size product in the second and subsequent periods. If *both* product valuations lie below the value associated with pursuing the outside option (i.e., $p + \underline{u}$), then a consumer will rationally opt out. Denoting the NPV of the firm's expected profits under the sample-box selling scenario by $\pi_s(b, p)$, we can therefore express this profit function as follows.

$$\pi_s(b, p) = b + \frac{\beta p}{1 - \beta}(1 - F(p + \underline{u})^2) \tag{3.14}$$

In the second term of this profit function, $p/(1 - \beta)$ reflects the stream of purchases at price p for the consumer's preferred full-size product, beginning (potentially) in period two. The factor $1 - F(p + \underline{u})^2$ reflects the probability that this stream will occur, i.e., the likelihood of the preferred product dominating the outside option. The leading factor β simply discounts the present value from period two back to period one.

Consumers will purchase the sample box in the first period only if doing so is expected to yield higher utility than the alternative of purchasing one of the full-size products or pursuing the outside option. These purchase alternatives respectively define the *self-selection* and *participation* conditions, both of which must be met for the consumer to purchase the sample box. In other words, the expected discounted utility $u_s(b,p)$ must at least equal both $u(p)$ and $\underline{u}/(1-\beta)$. The following formulation thus represents the firm's profit-maximization problem.

$$\max_{b,p} \pi_s(b,p) \tag{3.15}$$

$$\text{s.t.}$$

$$\text{participation: } u_s(b,p) \geq \frac{\underline{u}}{1-\beta}$$

$$\text{self-selection: } u_s(b,p) \geq u(p)$$

$$0 \leq p \leq 1$$

Note that because consumers will not purchase full-size products at a price higher than the upper-support of valuations, we must have $0 \leq p \leq 1$. It is feasible, however, for the sample box price b to exceed 1, given that consumers will (rationally) be willing to pay a price premium (relative to the prices of individual products with comparable sizes) for the added informational value the sample box provides. Proposition 3.5.1 characterizes the solution of the provided constrained maximization problem for a general distribution $F(.)$ defining consumers' product valuations.

Proposition 3.5.1. *If $pf(p+\underline{u})$ is increasing in p,*

(i) there exists a threshold $\hat{\beta}(\underline{u}) \in (0,1)$ such that the optimal prices (b_s^*, p_s^*) for (3.15) are given by:

$$p_s^* = \begin{cases} \tilde{\nu} - \underline{u} & \beta \leq \hat{\beta}(\underline{u}) \\ \hat{p}(\beta) & \beta > \hat{\beta}(\underline{u}) \end{cases} \quad \text{and } b_s^* = g(p_s^*) - u(p_s^*), \tag{3.16}$$

where $\hat{p}(\beta)$ solves

$$1 - \beta F(p+\underline{u})\Big[\beta(p+\underline{u}) + 2pf(p+\underline{u})\Big] = 0, \tag{3.17}$$

and $u(p)$ and $g(p)$ are obtained from (3.5) and (3.12). Furthermore, $\hat{p}(\beta)$ is decreasing in β.

(ii) $b_s^* > p_s^*$.

Proposition 3.5.1 establishes important properties of the firm's optimal prices when offering a sample box. Over the entire range of β, we find that optimal pricing keeps consumers indifferent between choosing the sample box and undertaking self-discovery. Moreover, when β is low, that indifference also applies to the alternative of the outside option. When β is high, the sample box yields a higher consumer surplus than the outside option. We also see that the optimal full-size product price is first increasing and then decreasing with β. The final result in Proposition 3.5.1 verifies that the firm optimally charges a price premium for the sample box, relative to the prices of full-size products. Figure 12 shows these properties of the optimal prices, using the uniform distribution $F(v) = v$ as an illustrative example.

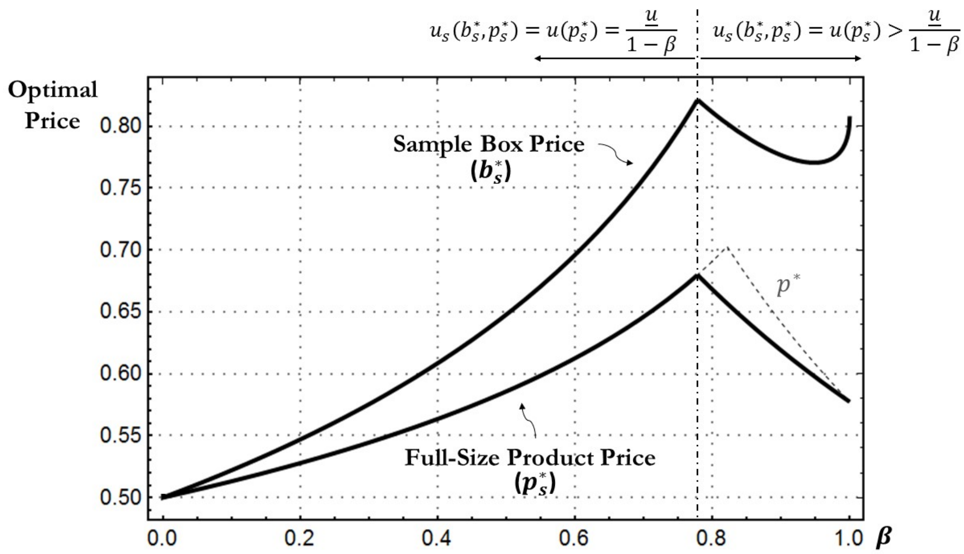


FIGURE 12. Firm's optimal price when offering a sample box $(F(v) = v)$

Over the entire range of β, the firm charges such prices to make customers indifferent between self-discovery and purchasing the sample box. When β is low, customers also obtain the same utility as if they leave. When β is high, customers strictly prefer purchasing to opting out.

Contrasting Figure 12 with Figure 11 (from Subsection 3.4) demonstrates, as β increases, a consistent non-monotonic optimal price trend for the full-size product. To understand this non-monotonic behavior, we again focus on the two-part structure of the firm's profits, which corresponds to (3.14) when the sample box is present. The first term, b, is the revenue from customers who purchase the sample box in the first period. The second term captures the profit from high-valuation customers who keep purchasing from period 2 onward. Let us for now disregard the consumer participation constraint, assuming that all customers purchase the sample box in the first period, but continue optimally, contingent on their realized valuations. We however keep the self-selection constraint in effect, implying that selecting the sample box is weakly preferred to buying the full-size product. The first term of the profit function is ever-increasing in b, but to keep the sample box preferred to self-discovery, p should increase accordingly. Therefore, the first term is maximized when p attains its highest possible value, 1. The second term is non-monotonic in p and is maximized at an interior solution. As β increases, the firm puts more weight on the revenue stream from period 2 onwards and less on the revenue from the sample box in the first period. Therefore, the optimal price of the full-size product should decrease with β. Since, facing the optimal prices, consumers are indifferent between selecting the sample box and going through self-discovery, for the full-size product, they perceive the same reservation prices to participate in the two processes. As we discussed earlier, this reservation price binds the firm's optimal price when β is low, but the unconstrained optimal price of the firm is lower than the reservation price for high values of β.

Another notable property of the optimal solution is the price premium charged for the sample box—relative to the expected consumption value it yields. This premium is attributable to the informational value of the sample box, facilitating consumers' optimal future decisions. We observe that as β approaches 0, the informational value of the sample box diminishes. As a result, the asymptotic optimal price of the sample box at $\beta = 0$ matches the optimal price of a full-size product, i.e., the expected single-period consumption utility μ.

Corollary 3.5.2 specifies the results from Proposition 3.5.1 when valuations are uniformly distributed.

Corollary 3.5.2. *When $F(v) = v$, the closed forms of the optimal prices in Proposition 3.5.1 are obtained as below.*

$$p_s^* = \begin{cases} \frac{1-\sqrt{1-\beta}}{\beta} - \underline{u} & \beta \leq \hat{\beta}(\underline{u}) \\ \frac{\sqrt{1+\frac{2}{\beta}+\underline{u}^2}-\underline{u}(1+\beta)}{2+\beta} & \beta > \hat{\beta}(\underline{u}) \end{cases} \quad \text{and } b_s^* = g(p_s^*)|_{F(v)=v} - u(p_s^*)|_{F(v)=v} \tag{3.18}$$

where $\hat{\beta}(\underline{u}) \in (0,1)$ *solves* $\beta^3 + \beta^2(4\underline{u}^2 - 4\underline{u} + 6) + \beta(5 - 4\underline{u}) - 8 = 0$*, and* $u(p)|_{F(v)=v}$ *and* $g(p)|_{F(v)=v}$ *are obtained from (3.6) and (3.13), respectively.*

The results from this corollary, represented in Figure 12, are consistent with the results established earlier for the general distribution of valuations. In addition, the figure demonstrates that with the uniform distribution, the firm starts to leave a positive surplus at a smaller β threshold when a sample box is offered, compared to when consumers engage in self-discovery.

Consumer's Problem with Future Credit

We now consider that, along with the sample box, the firm offers a future credit of value δ that can be applied towards a subsequent purchase of a full-size product. The impact of the credit on customers is that the second purchase period will be a transient low-price period (with price $p - \delta$); subsequently, a customer must outlay the full purchase price p. To reflect this change, the prior utility structure from (3.11) adjusts as follows.

$$u_c(b,p,\delta) = \mu - b + 2\beta \int_{p-\delta+\underline{u}}^{1} \int_{0}^{v_j} (v_j - (p-\delta))dF(v_i)dF(v_j) + \beta\underline{u}F(p-\delta+\underline{u})^2$$
$$+ \frac{2\beta^2}{1-\beta} \int_{p+\underline{u}}^{1} \int_{0}^{v_j} (v_j - p)dF(v_i)dF(v_j) + \frac{\beta^2\underline{u}}{1-\beta} F(p+\underline{u})^2 \qquad (3.19)$$

In the above expression, $\mu - b$ represents the consumer's net utility from purchasing the sample box in period 1. The next two terms reflect the second period surplus (thus discounted by β) associated with choosing either the reduced-price full product or the outside option, respectively. The following two terms are analogous to the second and third terms from (3.11) and express, respectively, the subsequent (i.e., the third purchase period onward) expected surplus from purchasing the preferred product or going with the outside option.

In a similar fashion to how we described the utility function (3.11) as the addition of separable functions of b and p, we can express the utility function (3.19) as

$$u_c(b, p, \delta) = -b + g_c(p, \delta). \tag{3.20}$$

Later we use this structure of the utility function to characterize the firm's optimal prices.

Firm's Problem with Future Credit

If consumers choose the sample box over the full-size product in the first period and obtain a future credit of δ, then the firm's resulting expected profit, which we denote by $\pi_c(b, p, \delta)$, is as follows.

$$\pi_c(b, p, \delta) = b + \beta(p - \delta)\big[1 - F(p - \delta + \underline{u})^2\big] + \frac{\beta^2}{1-\beta} p\big[1 - F(p + \underline{u})^2\big] \tag{3.21}$$

The first term in the above expression is the revenue the firm collects in the first period if consumers purchase the sample box. After sampling both products in the first period, a consumer may decide to make a product purchase in period 2 at the discounted price $p - \delta$. The second term above captures the corresponding revenue, which occurs provided *both* products are not dominated by the outside option (this option is preferred with probability $F(p-\delta+\underline{u})$). Similarly, the last term reflects the possibility of ongoing product purchases at the full price in period 3 onward. The firm's problem, formulated below, is to set such prices and a level of future credit to optimize its profit while ensuring that consumers choose the sample box in the first period.

$$\max_{b,p,\delta} \pi_c(b,p,\delta) \tag{3.22}$$

$$\text{s.t.}$$

$$\text{participation: } u_c(b,p,\delta) \geq \frac{\underline{u}}{1-\beta}$$

$$\text{self-selection: } u_c(b,p,\delta) \geq u(p)$$

$$0 \leq p \leq 1 \ , \ 0 \leq \delta \leq p$$

Proposition 3.5.3 characterizes the resulting optimal full product and sample box prices, denoted by b_c^* and p_c^*, and the optimal future credit level δ^*.

Proposition 3.5.3. *If $pf(p+\underline{u})$ is increasing in p,*

(i) there exists a threshold $\breve{\beta}(\underline{u}) \in (0,1)$ such that the optimal solution (b_c^*, p_c^*, δ^*) for the firm's profit maximization problem (3.22) is given by:

$$p_c^* = \delta^* = \begin{cases} \tilde{\nu} - \underline{u} & \beta \leq \breve{\beta}(\underline{u}) \\ \breve{p}(\beta) & \beta > \breve{\beta}(\underline{u}) \end{cases} \quad \textit{and } b_c^* = g_c(p_c^*, \delta^*) - u(p_c^*), \tag{3.23}$$

where $\breve{p}(\beta)$ solves

$$1 - \beta^2 F(p+\underline{u})\Big[p + \underline{u} + 2pf(p+\underline{u})\Big] = 0, \tag{3.24}$$

and $u(p)$ and $g_c(p,\delta)$ are obtained from (3.5) and (3.20), respectively. Furthermore, $\breve{p}(\beta)$ is decreasing in β.

(ii) For all $\beta \in (0,1)$, $\breve{p}(\beta) > \hat{p}(\beta)$ and $\breve{\beta}(\underline{u}) > \hat{\beta}(\underline{u})$, where $\hat{p}(\beta)$ and $\hat{\beta}(\underline{u})$ are defined in Proposition 3.5.1.

Proposition 3.5.3 characterizes the non-monotonic progression of the firm's optimal prices and future credit over the entire range of β. Figure 13 represents the formally stated characteristics of the optimal prices, using $F(v) = v$ as an illustrative example. As demonstrated, the full-size product price is first increasing and then decreasing in β. Furthermore, at any β, consumers are indifferent between choosing the sample box and pursuing a search process. Only when β is low, consumers are also indifferent between buying the sample box and consuming the outside option. For high levels of β, consumers strictly prefer the sample box over the outside option. In this range, the firm sets a higher price for the full-size product compared with the scenario in which no future credit is offered. Proposition 3.5.3 also proves that it is optimal for the firm to set the credit amount equal to the price of the full-size product.[6] Interestingly, we thus find that with optimal pricing, what might otherwise seem as a generous discount by the firm is in fact simply the optimal means to extract surplus from consumers.

To help understand the intuition behind the firm (optimally) offering a 100% price discount, we focus on the firm's profit in the second period. Assume now that the firm were to offer only a partial credit, or no future credit ($\delta < p$), and consumers are indifferent between purchasing the sample box and undertaking the search process. Given the full-size product price, increasing the future credit value has a mixed effect on the firm's profit in the second period: the firm enjoys a greater market share but yet sells at a lower effective price. However, each customer is willing

[6] Even with such a "generous" discount in effect, the problem formulation accounts for the possibility that some consumers might yet prefer the outside option, if it is sufficiently appealing and their (realized) valuations for the firms' products are both sufficiently low.

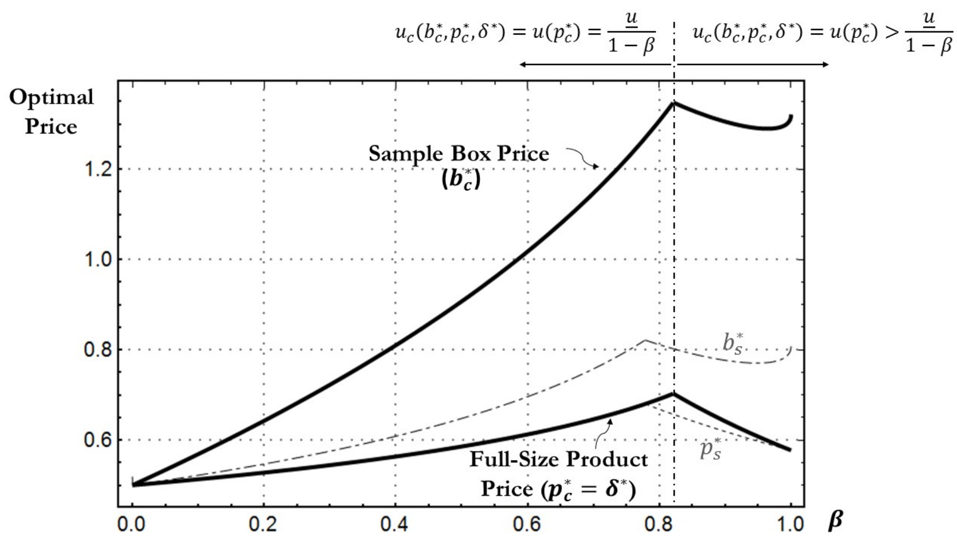


FIGURE 13. Firm's optimal price when offering a sample box with future credit ($F(v) = v$)

Over the entire range of β, the firm charges such prices to make customers indifferent between self-discovery and purchasing the sample box. When β is low, customers also obtain the same utility as if they leave. When β is high, customers strictly prefer purchasing to opting out. Also, the optimal value of the future credit equals the full-size product price.

to pay as much premium in the first period as the present value of the expected additional price discount they receive in the second period. In other words, the firm can recoup its second-period loss (due to the lower effective price) by increasing the sample box price in such a way to maintain the same consumer surplus. As a result, the net effect of increasing the future credit value, and correspondingly the sample box price, is the effect of an expanded market share in the second period. The firm benefits from an expanded market share as long as it charges a positive effective price in the second period. Thus, the firm increases the future credit value to the extent that the effective price in the second period reaches zero, that is, $\delta = p$. The mathematical counterpart to this explanation lies within case 2 of the proof of Proposition 3.5.3.

To explain the price non-monotonicity for the current scenario, notice that if we set $\delta = p$ in the profit function (3.21), we obtain a profit function structure very similar to that of (3.14)—thus begetting an analogous optimal price trend. Now to help understand why $p_c^* > p_s^*$, recall the explanation for the decreasing piece of p in Subsection 3.5. When offering future credit, the firm's revenue stream from full-size products is postponed by one period, and therefore the firm puts less weight on this revenue stream (exactly factored by β), relative to the no future credit scenario. Therefore, a higher full-size product price is optimal.

Corollary 3.5.4 provides the closed form of the results from Proposition 3.5.3 when valuations are uniformly distributed.

Corollary 3.5.4. *When $F(v) = v$, the optimal solution in Proposition 3.5.3 is obtained as below.*

$$p_c^* = p^* = \begin{cases} \frac{1-\sqrt{1-\beta}}{\beta} - \underline{u} & \beta \leq \tilde{\beta}(\underline{u}) \\ \frac{\sqrt{3+\beta^2\underline{u}^2}}{3\beta} - \frac{2\underline{u}}{3} & \beta > \tilde{\beta}(\underline{u}) \end{cases} \quad \text{and } b_c^* = g_c(p_s^*, \delta^*)|_{F(v)=v} - u(p_s^*)|_{F(v)=v} \tag{3.25}$$

where $\tilde{\beta}(\underline{u}) \in (0,1)$ solves $4\beta^3\underline{u}^2 + 3\beta^2(3-4\underline{u}) + 2\beta(3+2\underline{u}) - 11 = 0$.

Figure 13 illustrates the corollary results but is representative of the characteristics of the firm's optimal prices for the generic distribution of valuations. An additional property evident in Figure 13 for the case of uniform valuations ($F(v) = v$) is that the firm charges a price premium for the sample box when offering a future credit.

Relative Profit Gains from Sample Boxes

In this section we contrast the firm's expected profits with and without the offering of a sample box. For tractability, throughout this section we rely on the results we obtain for the uniform distribution of valuations as characterized in corollaries 3.4.5, 3.5.2, and 3.5.4. We consider a range of outside option utilities ($\underline{u}$), with $\underline{u}$ small enough to let consumers opt for the firm's products at least in period 1.[7]

Our first set of results, formally stated in Proposition 3.6.1, establishes that the firm achieves strictly greater profits when it offers a sample box coupled with a future credit compared to when no box is offered or when the box is accompanied with no future credit.

Proposition 3.6.1. *If $F(v) = v$, for any $\beta \in (0, 1)$ and $\underline{u} \in [0, \frac{1}{2})$,*

(i) $\pi_c(b_c^*, p_c^*, \delta^*) > \pi(p^*)$;

(ii) $\pi_c(b_c^*, p_c^*, \delta^*) > \pi_s(b_s^*, p_s^*)$.

To shed more light on the results provided in Proposition 3.6.1, we illustrate in Figure 14 the relative profitability of offering a sample box in the presence or absence of a future credit. The solid curves in the figure correspond to the profit ratios for $\underline{u} = 0$. For each, the shaded regions show the ratios that are obtainable given $\underline{u} > 0$, i.e., with a more attractive outside option available to consumers. The results shows that, when the outside option offers low (e.g., zero) utility, offering a sample box may diminish the firm's profit, unless the box is coupled with a future credit. The

[7] The feasible range of $\underline{u}$ for consumer participation is $\underline{u} \le \tilde{\nu} - p$. Since μ is the lower-bound of $\tilde{\nu}$, $\underline{u} < \mu$ is necessary to have a price that induces participation over the entire range of β. We consider the range $\underline{u} \in [0, 0.3]$ in Figure 14 to keep the plots uncluttered.

future credit (optimally equal to the full-size product price) makes offering a sample box profitable. At any level of $\underline{u}$, offering the future credit yields a strictly higher profit than not doing so.

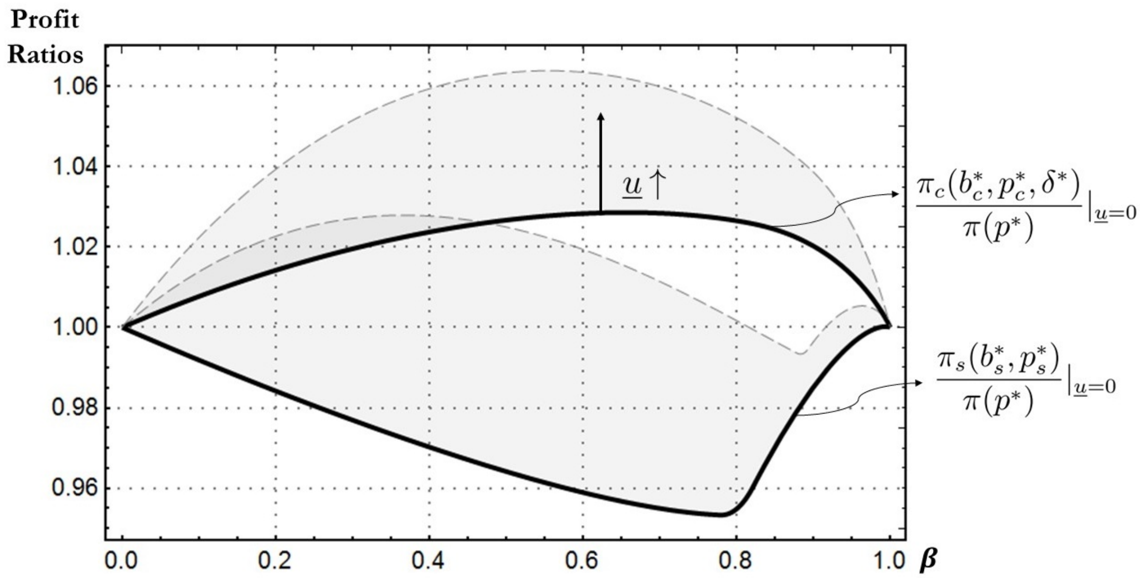


FIGURE 14. Relative profitability of offering a sample box with and without a future credit ($F(v) = v$)

Providing a sample box without a future credit may decrease a firm's profit. Offering an optimally specified future credit makes the sample box practice profitable.

It is important to understand the potential adverse effect of a sample box on the firm's profit, even with optimal pricing—given no future credit. As we showed in Proposition 3.5.1, consumers are willing to pay a price premium for the informational value of a sample box, beyond its immediate consumption value. Although this price premium would seem to stimulate the firm's profit, it can have a detrimental effect because it accelerates customers' settling upon their ideal alternative, which is, for some customers, the outside option. More specifically, following a sequential search process, a fraction of customers realize low valuations for the firm's products, and thus stop purchasing, but not before sampling the firm's product options—exactly two in a two-product setting. The firm benefits from this search inefficiency on consumers' part. Thus, the offering of a sample box yields an *acceleration effect* whereby consumers quickly discover their valuations, inducing low-valuation customers to opt out after period one. From the firm's perspective, the drawback of losing customers as early as in the second period can easily outweigh the benefit of collecting a price premium in the first period—especially when $\underline{u}$ is low.

We now consider how the expected benefits from the presence of a sample box relate to the discount factor. Earlier, in reference to Figure 12, we discussed that at low β, consumers optimally receive zero expected surplus beyond what the outside option would provide. The implication is, given that the firm may also earn less profit when offering a sample box, *neither* the firm *nor* consumers may be better off. Only at higher β levels, at which a sample box's future information value is significant, do consumers benefit. Yet, we see in Figure 14 that even at a relatively high β (e.g., β near 0.85 or 0.9) the firm itself may be unable to capture the value generated by its sample box. The practical importance of the mentioned finding is that, if a firm lacks the infrastructures to apply targeted future credits, then offering a sample box may decrease profits—even when optimally priced.

Another significant managerial implication highlighted by Figure 14 is that by offering future credit with its sample box, a firm can increase its expected profitability. Notably, even in scenarios when a sample box would otherwise be detrimental, the combination of the box and future credit mechanism is profitable. To understand how profits improve with the future credit, recall that in the absence of the credit, the firm's customer base thins as early as in the second period. As we found in Subsection 3.5, by setting the credit equal to the full-size product price, the firm regains its market potential in the second period. We showed that, although customers receive an ostensibly free product in the second period, the firm levies a compensating up-front price premium via the sample box. In other words, when offering the future credit (optimally equal to the full-size product price), the firm effectively bundles the first-period sample box with the second-period product, and sells this bundle at a premium to *all* consumers before they embark on their learning process. Observing the considerable benefits from accompanying sample boxes with future credits, one may also be interested in the implications of offering future credits when consumers follow sequential search. We can show that the firm optimally will *not* offer future credits in the absence of a sample box. The main driver of this result is the fact that when facing sequential search, the firm's optimal price induces all customers to buy in the first two periods. Therefore, future credit cannot enhance the firm's second-period market potential, which is the driver for the profit increase when a sample box is offered. Since it does not provide any further managerial insights, we omit the formal treatment of the future credit offers in the absence of a sample box.

Finally, we examine the contribution of the outside option to the relative profitability of offering a sample box. As we next show, offering a sample box with a future credit becomes relatively more profitable as $\underline{u}$ increases. This result holds for a considerable range of β even if no future credit is offered.

Proposition 3.6.2. *If $F(v) = v$,*

(i) for any $\beta \in (0,1)$ and $\underline{u} \in [0, \frac{1}{2})$ we have that $\frac{d}{d\underline{u}}(\frac{\pi_c(b_c^*, p_c^*, \delta^*)}{\pi(p^*)}) > 0$;

(ii) for any $\beta \leq \hat{\beta}(\underline{u})$ and $\underline{u} \in [0, \frac{1}{2})$ we have that $\frac{d}{d\underline{u}}(\frac{\pi_s(b_s^*, p_s^*)}{\pi(p^*)}) > 0$.

The first part of Proposition 3.6.2 establishes that, *relative* to the baseline scenario of self-discovery, the profitability of offering a sample box along with a future credit increases as the outside option becomes more attractive. It should be noted that, as we should expect, the absolute profit magnitudes in both scenarios decrease as consumers' utility from the outside option increases. Interestingly, however, when offering a sample box with a future credit, the firm experiences smaller profit declines.

Notice that, with the uniform distribution, the price of the full-size product in the "no sample box" scenario is the same as that in the "sample box with future credit" scenario. Thus, to compare the effects of the outside option utility on the firm's profits in the two scenarios, we only need to focus on the first two periods. Recall that when offering a sample box, the firm sets such prices to make consumers indifferent between the box and sequential search. As $\underline{u}$ increases, customers sacrifice the consumption of a more desirable outside option for a protracted timespan (two periods in a two-product setting) if they employ a sequential search process to discover the firm's products. By resolving consumers' uncertainties in one period, a sample box thus yields a more desirable discovery mechanism. Since an increase in $\underline{u}$, ceteris paribus, relatively favors the sample box customers, the firm forfeits more profits in the "no sample box" scenario to satisfy consumer participation constraint, that is, to keep the the two uncertainty resolution mechanisms equally attractive. The second part of Proposition 3.6.2 establishes that when β is not too large, an increase in $\underline{u}$ also increases the relative profitability of offering a sample box, even in the absence of a future credit. As we observe in Figure 15, the same effect may hold even beyond the β threshold discussed in Proposition 3.6.2.

The managerial implication of the above discussion is that, when facing a more attractive outside option, it is *often* relatively advantageous for the firm to offer a sample box. With an increase in the outside option utility, the relative advantage of offering a sample box with a future credit increases for any level of the time discount factor.

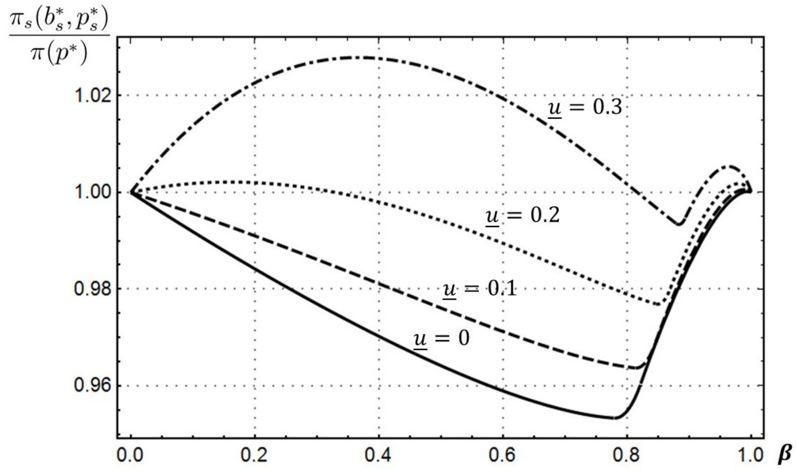


FIGURE 15. Relative profitability of offering a sample box without a future credit ($F(v) = v$)

An overall impression of an increase in $\underline{u}$ is an increase in the profitability of offering a sample box with no future credit relative to the profit with sequential search. There are exceptions in narrow ranges of β and $\underline{u}$.

Conclusion

In this study we have analyzed the potential benefits of offering a sample box for a firm that serves heterogeneous consumers with valuation uncertainty. A sample box enables consumers to resolve their valuation uncertainties over multiple product varieties in an efficient manner. Offering sample boxes has become a common practice adopted by many businesses selling experience goods, for which customers cannot obtain full information before consumption. In the absence of a sample box, customers learn their valuations for substitutable variants of a product via a sequential search process by purchasing standard (full-size) products. The benefit of a sample box from consumers' perspective is that they can settle upon their ideal variant just after consuming the sample box, without undergoing the more costly and protracted search process.

To investigate the benefits of a sample box, we first studied the baseline case with no sample box offered. In this setting, we show that a consumer's optimal policy is characterized by a value threshold that is constant over time. If, after consuming a product variant, a customer's realized valuation for that variant is above the threshold, then the customer should not explore other options; otherwise, the consumer will purchase and try another product variety. Facing rational consumers that follow the described optimal policy, the firm sets its full-size product prices to maximize expected profits. We then study a setting in which a sample box is offered. We find that the informational value of the sample box dictates a price premium for the box. We also investigate a common pricing tactic that is offering a future credit along with the sample box. A sample-box purchaser can apply the future credit (only) to a subsequent purchase of a full-size product. We show that the firm optimally sets the value of the future credit equal to the price of the full-size product. As a result, consumers who purchase the sample box in the first period optimally receive a 100% price discount on a full-size product in the second period.

Contrasting the resulting expected profits, with and without the sample-box option, our results highlight that managers may be ill-advised to offer a sample box in the absence of the future-credit mechanism. Furthermore, only when the discount factor is high enough to justify the value of learning will a sample box boost consumer surplus. Moreover, we show that by providing a future credit equal to the full-size product price, managers enhance the relative profitability of the sample box. The future credit enables the firm to recover its market share loss in the second period due to low-valuation customers who would leave otherwise after learning their valuations (via the consumption of the sample box). This second-period price discount is compensated by the premium all consumers pay for the sample box before discovering their valuations. Additionally, we find that, as the utility consumers receive from the outside option increases, the profitability of the sample box practice—with a future credit—is less compromised than when customers follow a traditional search process. Thus, it is relatively advantageous for a firm to offer a sample box when consumers have access to attractive alternative outside options in the market.

CHAPTER IV

CONCLUSION

Our study sheds light on the implications of emerging practices related to product variety on firm profits and consumer surplus under a variety of market settings. In Chapter 2, we use a Hotelling-type framework to study the location-then-price competition between two firms with mass customization capabilities. Each firm incurs a customization cost proportional to the scope of its offerings matching a range of consumers' heterogeneous tastes. We show that the structures of the competitive equilibria depend on the proportion of customer valuation to fit sensitivity, and derive three main conclusions. First, customization scopes in equilibrium do not monotonically decrease as consumers become more tolerant to product mismatch. A firm's response to a decrease in fit sensitivity should be to contract its customization scope only if the sensitivity is beyond a threshold. In contrast, below that threshold the firm's response should be to expand the customization scope. Second, customizing firms' profits are maximized at extreme levels of market's sensitivity to fit. Therefore, if mass customizers can marginally influence consumers' attitudes through marketing activities, a beneficial tactic would be to promote customers' sensitivity to purchasing ideal products, if customers are already sensitive enough, and deemphasizing fit sensitivity if they are relatively tolerant of taste discrepancies. Third, we show that equilibrium prices in the competition between customizers are always higher than those in the single-product duopoly. However, positive customization costs might result in lower profits for mass customizers. Therefore, market conditions dictate when firms would find it advantageous to compete offering customization rather than standard products. Moreover, there are certain market conditions under which neither firms nor consumers benefit from the availability of customization technology. Thus, regulators should evaluate the social welfare impacts when deciding whether to facilitate investments in customization within industry.

In Chapter 3, we study the potential benefits of offering a sample box for a firm that serves heterogeneous consumers with valuation uncertainty. A sample box enables consumers to resolve their valuation uncertainties over multiple product varieties in an efficient manner. Offering sample boxes has become a common practice adopted by many businesses selling experience goods, for which customers cannot obtain full information before consumption. In the absence of a sample box, customers learn their valuations for substitutable variants of a product via a sequential search process by purchasing standard (full-size) products. The benefit of a sample box from consumers' perspective is that they can settle upon their ideal variant just after consuming the sample box, without undergoing the more costly and protracted search process. We find that the informational value of the sample box dictates a price premium for the box. We also investigate a common pricing tactic that is offering a future credit along with the sample box. A sample-box buyer can apply the future credit to a subsequent purchase of a full-size product. We show that the firm optimally sets the value of the future credit equal to the price of the full-size product, resulting in a 100% price discount on a consumer's purchase following the purchase of the box. Contrasting the resulting expected profits with and without the sample-box option, our results highlight that managers may be ill-advised to offer a sample box in the absence of the future-credit mechanism. Furthermore, only when consumers put enough weight on their future consumptions will a sample box boost consumer surplus. Moreover, we show that by providing a future credit equal to the full-size product price, managers enhance the relative profitability of the sample box. The future credit enables the firm to recover its market share loss in the second period due to low-valuation customers who would leave in the absence of the credit after learning their valuations (via the consumption of the sample box). This second-period price discount is compensated by the premium all consumers pay for the sample box before discovering their valuations. Additionally, we find that it is relatively advantageous for a firm to offer a sample box when consumers have access to attractive alternative outside options in the market.

APPENDIX A

ESSAY 1: PROOFS OF PROPOSITIONS AND COROLLARIES

Proof of Proposition 2.4.1. let us denote the monopolist's MC scope by d and price by p. Since a monopolist's profit from symmetrically setting its MC scope weakly dominates the profit with asymmetric location choices at the same levels of d and p, we focus only on location-symmetric MC scopes under monopoly. A mass-customizing monopolist obtains a profit of $p[d+2(V-p)/t]-cd$ if it serves the market partially, and a profit of $p-cd$ if it serves the entire market. The former expression has an interior profit-maximizing price solution, and the latter is increasing in p. Therefore, given d, the profit-maximizing price is obtained as below. Note that the first piece of this function (the interior solution) results in a market share less than one, and the second piece (the boundary solution) yields a market share of size one.

$$p^*(d) = \begin{cases} \frac{V}{2} + \frac{td}{4} & 0 \leq d \leq 2 - \frac{2V}{t} \\ V + \frac{t}{2}(d-1) & d > 2 - \frac{2V}{t} \end{cases}$$

Plugging in the optimal prices, we obtain the monopolist's profit as a function of d as shown below.

$$\pi^M(d) = \begin{cases} \frac{(dt+2V)^2}{8t} - cd & 0 \leq d \leq 2 - \frac{2V}{t} \\ V + \frac{t}{2}(d-1) - cd & d > 2 - \frac{2V}{t} \end{cases}$$

The first piece of the demonstrated profit function is convex, and the second piece is monotonic in d. Thus, we confine the profit-maximizing d candidates to 0, $2 - \frac{2V}{t}$, and 1. For $2 - \frac{2V}{t}$ to be contained in $[0, 1]$, ρ should be within $[1/2, 1]$, in which

$d = 0$ or $d = 1$ results in a higher profit. Therefore, we obtain the maximum profit at either $d = 0$ or $d = 1$. The optimal (d, p) candidates are, consequently, $(0, V/2)$ and $(1, V)$ if $\rho < 1$, as well as $(0, V - t/2)$ and $(1, V)$ if $\rho \geq 1$. Comparing the resulting profits will lead us to the three optimal monopoly outcomes stated in the proposition, depending on the parameter ranges. □

Proof of Proposition 2.4.2. From Lemma B.0.1 we know that in any symmetric equilibrium the entire market is served. Now, suppose that in a symmetric equilibrium, while mass customizing and serving the entire market, firms leaves positive utility at the extremes of the taste spectrum. Then neither firm should benefit from fully undercutting the competitor only by decreasing price. Let us refer as the *full-undercutting profit* to the supremum of all profits achieved by a firm from adopting prices that result in full undercutting, fixing all the other variables. Similarly, we refer to the supremum of such prices as the *full-undercutting price.* Since each firm's profit is increasing in its price in the full-undercutting region, the full-undercutting profit occurs at the full-undercutting price.

First, assume that firms' full-undercutting profits are strictly lower than their profits in the supposed equilibrium. Lemma B.0.2 suggests that the price equilibrium will not change by an infinitesimal increase in a_1. Since this change does not affect firm A's market share and price, but reduces its MC scope (and thus MC cost), it is a profitable deviation. Now assume that firms' profits in the supposed equilibrium equal their full-undercutting profits. In other words, either firm is on the verge of full undercutting in the supposed equilibrium. Through the four cases below we rule out this possibility as well.

Case 1: Middle consumer obtaining positive utility but not a perfect match

In this case, firm A's equilibrium price is the interior maximizer of firm A's profit which is specified as $p_A m(p_A, p_B) - c(a_2 - a_1)$, with firm A's market boundaries 0 and $m(p_A, p_B)$. Considering a similar profit function for firm B, the optimal price is derived as $p_A^* = p_B^* = t$. For $\rho < 1$, the obtained price is greater than V, the upperbound of prices each firm can reasonably charge. Thus, equilibrium cannot happen in this case.

Case 2: Middle consumer obtaining a perfect match ($a_2 = b_1 = \frac{1}{2}$)

We know that both firms are on the verge of fully undercutting their competitor. This implies equal prices in the supposed equilibrium. Defining p^* as the equilibrium price and $p_U = p^* + t(a_1 + a_2 - 1)$ as the full-undercutting price, we have:

$$\pi_A(a_1, \frac{1}{2}, \frac{1}{2}, 1 - a_1, p^*, p^*) = \lim_{p_A \to\ p_U{}^-} \pi_A(a_1, \frac{1}{2}, \frac{1}{2}, 1 - a_1, p_A, p^*) \Rightarrow p^* = t(1 - 2a_1)$$

Since $a_2 = b_1$ and equilibrium prices are equal, profit functions are continuous in a small neighborhood of the equilibrium prices. Therefore, the first order conditions can be used to yield the profit-maximizing prices, $(t(\frac{3}{4} - a_1), t(\frac{3}{4} - a_1))$. Setting these prices equal to the price obtained above will result in $a_1 = \frac{1}{4}$ and $p^* = \frac{t}{2}$. Also, to have positive utility at the edges, we need to have $\rho > \frac{3}{4}$. Next, we argue that there is a profitable location-deviation by either firm from the symmetric profile $(\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{t}{2}, \frac{t}{2})$, when $\frac{3}{4} < \rho < \frac{7}{8}$.

Assume that, given firm B's fixed locations at $(\frac{1}{2}, \frac{3}{4})$, firm A decreases a_2 to $\frac{2}{4\rho - 1} - \frac{1}{2}$, while keeping a_1 at $\frac{1}{4}$. First, we will show that at the new location profile $(p_L(\frac{1}{4}), p_L(\frac{1}{4})) = (V - \frac{t}{4}, V - \frac{t}{4})$ is the price equilibrium, and then we will show that the described deviation is profitable for firm A. By verifying the following inequalities, we show that, after the deviation, $V - \frac{t}{4}$ is the locally optimal price for either firm given the other firm's price $V - \frac{t}{4}$. Note that we need to separately consider the left derivative $(\frac{\partial_-}{\partial})$ and right derivative $(\frac{\partial_+}{\partial})$ for each firm, since decreasing and increasing price from the supposed price equilibrium result in different specifications of the profit function.

$$\frac{\partial_- \pi_A}{\partial p_A}(\frac{1}{4}, \frac{2}{4\rho - 1} - \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, V - \frac{t}{4}, V - \frac{t}{4}) \geq 0$$

$$\frac{\partial_+ \pi_A}{\partial p_A}(\frac{1}{4}, \frac{2}{4\rho - 1} - \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, V - \frac{t}{4}, V - \frac{t}{4}) \leq 0$$

$$\frac{\partial_- \pi_B}{\partial p_B}(\frac{1}{4}, \frac{2}{4\rho - 1} - \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, V - \frac{t}{4}, V - \frac{t}{4}) \geq 0$$

$$\frac{\partial_+ \pi_B}{\partial p_B}(\frac{1}{4}, \frac{2}{4\rho - 1} - \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, V - \frac{t}{4}, V - \frac{t}{4}) \leq 0$$

To show that $(V - \frac{t}{4}, V - \frac{t}{4})$ is indeed the price equilibrium after the deviation, we only need to investigate either firm's undercutting incentive in addition to the above analysis, thanks to Lemma B.0.3. As either firm decreases its price given its competitor's fixed price, there is the price threshold $V + \frac{2t^2}{4V-t} - \frac{5t}{4}$ below which partial undercutting occurs. Since the left price derivatives of firms' profits at this threshold are non-negative, part (ii) of Lemma B.0.3 implies that firms do not benefit from partially undercutting their competitor. As a firm further decreases its price (given the competitors fixed price at $V - \frac{t}{4}$), we expect another transition from partial to full undercutting at some price level. We verify that within $\frac{3}{4} < \rho < \frac{7}{8}$ each firm's full-undercutting profit is lower than the its profit when firms adopt $(p_A, p_B) = (V - \frac{t}{4}, V - \frac{t}{4})$. This is the final step to establish that the mentioned price profile is the price equilibrium after the deviation. To show that the deviation is profitable for firm A, we need to consider its profit after the deviation, $\pi_A(\frac{1}{4}, \frac{2}{4\rho-1} - \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, V - \frac{t}{4}, V - \frac{t}{4}) =$

$\frac{1}{4}[t + c(3 - \frac{8t}{4V-t})]$, and its profit before the deviation, $\pi_A(\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{t}{2}, \frac{t}{2}) = \frac{1}{4}(t-c)$. It is evident that for any positive c and $\frac{3}{4} < \rho < 1$ the former expression exceeds the latter one.

Case 3: Middle consumer receiving zero utility but making a purchase

The firms' prices are calculated as $p_L(\frac{1}{2} - a_2)$ in this case. Since each firm is on the verge of full-undercutting, we obtain $a_2 = \frac{1}{6}(5 - 4a_1 - 2\rho)$. Also, to maintain positive utility at the market edges, a_1 should be less than $\frac{1}{2} - a_2$. Therefore, $a_1 < \rho - 1$. The right-hand-side of this inequality is negative when $\rho < 1$, ruling out the possibility of a positive a_1 in this case.

Case 4: Shared MC scope

In this case, a range of consumers are offered perfectly matched products from both firms. Since each firm is on the verge of full-undercutting, prices should be equal, and thus, the second stage of the game is a Bertrand's duopoly with no price equilibrium.

Ruling out all the four cases above, we conclude that if there exists a symmetric MC Nash equilibrium within $\rho < \frac{7}{8}$, while the entire market is served, no firm leaves positive utility at the extremes of the taste spectrum. Put another way, in any symmetric MC Nash equilibrium, $(p_L(a_1), p_L(a_1))$ must be the price equilibrium, and $\alpha_2 \geq \beta_1$. $\square$

Proof of Proposition 2.4.3. We follow a proof-by-contradiction approach.

i. $c > \frac{t}{4}$

Suppose $(\bar{a_1}, \frac{1}{2} - \bar{a_1}, \frac{1}{2} + \bar{a_1}, 1 - \bar{a_1}, p_L(\bar{a_1}), p_L(\bar{a_1}))$ is a symmetric $\overline{\text{W}}$-structure Nash equilibrium with $\bar{a_1} < \frac{1}{2} - \bar{a_1} \Rightarrow \bar{a_1} < \frac{1}{4}$. For $(p_L(\bar{a_1}), p_L(\bar{a_1}))$ to be the price equilibrium we need to verify that no local price deviation is profitable, occurring when $\rho - 1 \leq \bar{a_1} \leq \rho - \frac{1}{4}$. We consider a deviation by firm A from $(\bar{a_1}, \frac{1}{2} - \bar{a_1})$ to $(\frac{1}{4}, \frac{1}{4})$.

We first argue that, if $\rho \geq \frac{1}{2}$, the price equilibrium will be $(p_L(\frac{1}{4}), p_L(\bar{a_1}))$ after such a deviation, constructing an asymmetric $\overline{\text{W}}$-structure. Considering the mentioned price profile, firm A does not have an incentive to increase its price, since from Proposition 2.4.1 a single-product monopolist optimally captures a market share of ρ, when ρ is less than 1. Also, since $\frac{\partial_- \pi_A}{\partial p_A}(\frac{1}{4}, \frac{1}{4}, \frac{1}{2} + \bar{a_1}, 1 - \bar{a_1}, p_L(\frac{1}{4}), p_L(\bar{a_1}))$ is non-negative in the region of ρ and $\bar{a_1}$ specified above, firm A does not benefit from decreasing its price below $p_L(\frac{1}{4})$ either, given firm B's price $p_L(\bar{a_1})$. Also, firm B does not benefit from charging any other price than $p_L(\bar{a_1})$, since the left and right derivatives are the same as before the deviation. We also verify that neither firm has the price-undercutting incentive after the deviation. Therefore, $(p_L(\frac{1}{4}), p_L(\bar{a_1}))$ is indeed the price equilibrium after the described when $\rho \geq \frac{1}{2}$. This deviation is profitable for firm A because, when $c > \frac{t}{4}$, the following holds.

$$\pi_A(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}+\bar{a_1}, 1-\bar{a_1}, p_L(\frac{1}{4}), p_L(\bar{a_1})) > \pi_A(\bar{a_1}, \frac{1}{2}-\bar{a_1}, \frac{1}{2}+\bar{a_1}, 1-\bar{a_1}, p_L(\bar{a_1}), p_L(\bar{a_1}))$$

Now, assuming that $\rho < \frac{1}{2}$, we examine the price equilibrium at the location profile $(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}+\bar{a_1}, 1-\bar{a_1})$. In fact, without completely characterizing the price equilibrium, we will show that firm A's equilibrium price is the single-product monopoly price, $\frac{V}{2}$, resulting in a market share of less than a half for firm A. To this end, we demonstrate that $p_L(\frac{1}{4} + \bar{a_1})$ provides a lower bound for firm B's equilibrium price. Put another way, we need to show that the first derivative in the following expression is positive.

$$\frac{\partial_{-}\pi_B}{\partial p_B}(\frac{1}{4},\frac{1}{4},\frac{1}{2}+\bar{a_1},1-\bar{a_1},\frac{V}{2},p_L(\frac{1}{4}+\bar{a_1}))$$
$$-\frac{\partial_{-}\pi_B}{\partial p_B}(\bar{a_1},\frac{1}{2}-\bar{a_1},\frac{1}{2}+\bar{a_1},1-\bar{a_1},p_L(\bar{a_1}),p_L(\bar{a_1}))=\frac{3}{8}-\frac{\rho}{4}$$

Since the above difference is always positive for $\rho < 1/2$, and the latter derivative is non-negative, the former derivative is positive. Therefore, given firm A's price $\frac{V}{2}$, the minimum optimal price firm B charges is $p_L(\frac{1}{4}+\bar{a_1})$, which guarantees that all the consumers between the firms are served and the indifferent consumer between the firms receives zero utility. In other words, in the price equilibrium firm A's single-product monopoly is not disturbed by firm B. For $c > \frac{t}{4}$, firm A's deviation to $(\frac{1}{4},\frac{1}{4})$ with $p_A = \frac{V}{2}$ and $p_B \geq p_L(\frac{1}{4}+\bar{a_1})$ is profitable.

ii. $c < \frac{t}{4}$

Again, assume that $(\bar{a_1},\frac{1}{2}-\bar{a_1},\frac{1}{2}+\bar{a_1},1-\bar{a_1},p_L(\bar{a_1}),p_L(\bar{a_1}))$ is a symmetric $\overline{\text{W}}$-structure Nash equilibeium with $\bar{a_1} < \frac{1}{2}-\bar{a_1}$. We verify that $\frac{\partial_{-}\pi_A}{\partial p_A}(\bar{a_1},\frac{1}{2}-\bar{a_1},\frac{1}{2}+\bar{a_1},1-\bar{a_1},p_L(\bar{a_1}),p_L(\bar{a_1}))$ is strictly positive when $\rho < 1$. Let us consider a deviation of firm A to $(\bar{a_1}-\epsilon,\frac{1}{2}-\bar{a_1}+\epsilon)$. We have:

$$\frac{\partial_{-}\pi_A}{\partial p_A}(\bar{a_1}-\epsilon,\frac{1}{2}-\bar{a_1}+\epsilon,\frac{1}{2}+\bar{a_1},1-\bar{a_1},p_L(\bar{a_1}-\epsilon),p_L(\bar{a_1}))$$
$$-\frac{\partial_{-}\pi_A}{\partial p_A}(\bar{a_1},\frac{1}{2}-\bar{a_1},\frac{1}{2}+\bar{a_1},1-\bar{a_1},p_L(\bar{a_1}),p_L(\bar{a_1}))=-\frac{\epsilon}{2}$$

Therefore, there exists a small enough ϵ for which the former derivative is also positive, and thus firm A has no incentive to infinitesimally decrease its price below $p_L(\bar{a_1} - \epsilon)$ after the deviation. The similar difference for the right derivatives of firm A's profit with respect to price is negative, with the derivative before the deviation being also negative. Having this result, according to Lemma B.0.3, firm A has no incentive to increase its price above $p_L(\bar{a_1} - \epsilon)$ after the deviation. The price derivatives of firm B's profit after firm A's deviation do not change, given firm A's price $p_L(\bar{a_1} - \epsilon)$. Through a similar approach as adopted in the proof of Proposition 2.4.2 (case 2), we can also rule out the possibility of either firm's undercutting after the deviation. Thus, we establish that at the location profile $(\bar{a_1} - \epsilon, \frac{1}{2} - \bar{a_1} + \epsilon, \frac{1}{2} + \bar{a_1}, 1 - \bar{a_1})$, the price equilibrium is $(p_L(\bar{a_1} - \epsilon), p_L(\bar{a_1}))$. The proposed deviation is profitable for firm A, since the following expression is positive for $c < \frac{t}{4}$.

$$\begin{aligned}\pi_A(\bar{a_1} - \epsilon, \frac{1}{2} - \bar{a_1} + \epsilon, \frac{1}{2} + \bar{a_1}, 1 - \bar{a_1}, p_L(\bar{a_1} - \epsilon), p_L(\bar{a_1}))& \\ - \pi_A(\bar{a_1}, \frac{1}{2} - \bar{a_1}, \frac{1}{2} + \bar{a_1}, 1 - \bar{a_1}, p_L(\bar{a_1}), p_L(\bar{a_1})) &= \frac{\epsilon}{2}(t - 4c)\end{aligned}$$

□

Proof of Proposition 2.4.4. Assume there exists a symmetric MC W-structure equilibrium different from the one characterized in the proposition statement. For $(p_L(a_1), p_L(a_1))$ to be price equilibrium we need:

$$\frac{\partial_- \pi_i}{\partial p_i}(a_1, a_2, 1 - a_2, 1 - a_1, p_L(a_1), p_L(a_1)) \geq 0$$

$$\frac{\partial_+ \pi_i}{\partial p_i}(a_1, a_2, 1 - a_2, 1 - a_1, p_L(a_1), p_L(a_1)) \leq 0,$$

where $i \in \{A, B\}$. As we slightly increase either firm's price given its competitor's price, we will have a gap of unserved consumers on one side of the market. On the other hand, a slight decrease in either firm's price will leave positive utility at one market edge. In both cases, the middle consumer obtains positive utility, but not a perfectly matched product. The conditions on the derivatives above, taken from different pieces of the profit function, translate into $\rho - 1 \leq a_1 \leq \rho - \frac{1}{3}$. Since $\rho < 1$ and $a_1 > 0$, we have $\rho - 1 \neq a_1$, and from Lemma B.0.4-(ii), $a_1 \neq \rho - \frac{1}{3}$. For an interior value of a_1 in the provided range, Lemma B.0.4-(i) suggests a W-structure outcome after an infinitesimal change in a_2, fixing all the other location variables and assuming that firm B will not undercut. If no firm has an undercutting motivation after deviation, Firm A's profit improves either by an ϵ-increase in a_2 when $c < \frac{p_L(a_1)}{2}$, or by an ϵ-decrease in a_2 when $c > \frac{p_L(a_1)}{2}$.[1] We conclude that, if a symmetric MC W-structure equilibrium exists, an increase in a_2, or equivalently, a decrease in b_1, given the other firm's location, should prompt the competitor's undercutting. In other words, each firm should be on the verge of undercutting in any symmetric W-structure equilibrium where firms customize. In such an equilibrium the profit of each firm should be equal to the *supremum* of the profits obtained from undercutting the competitor. We refer to this supremum as the *undercutting profit* achieved at the asymptotic *undercutting price*.

Let us first assume that, in the supposed W-structure equilibrium, each firm obtains the same profit as its partial-undercutting profit.[2] This property is

[1] In case $c = p_L(a_1)/2$, firm A can benefit from deviating to either $(a1-\epsilon, a2+\epsilon)$ or $(a1+\epsilon, a2-\epsilon)$ if $c \neq t/4$. Otherwise, we have a range of profit-equivalent equilibria evolving to an equilibrium in which firms do not customize. This evolution is similar to that explained at the end of subsection 2.4.

[2] We can alternatively begin with the assumption that each firm's equilibrium profit equals its full-undercutting profit, and subsequently, reach the same conclusion using a similar approach to what follows.

mathematically expressed via the following equality.

$$\pi_A(a_1, a_2, 1-a_2, 1-a_1, p_L(a_1), p_L(a_1)) = \pi_A(a_1, a_2, 1-a_2, 1-a_1, p_{PU}(a_2, p_L(a_1)), p_L(a_1)),$$

where $p_{PU}(a_2, p_B) = \frac{1}{2}(p_B + a_2 t)$ is the price that maximizes firm A's profit as firm A partially undercuts firm B. From the above equation, we solve a_2 in terms of a_1 as $a_2^{opt}(a_1) = a_1 - \rho + \sqrt{2(\rho - a_1)}$. The obtained specification for a_2^{opt} should satisfy $0 < a_1 < a_2^{opt} < \frac{1}{2}$ and also $1 - a_2^{opt} < a_2^{opt} + \frac{p_L - p_{PU}}{t} < 1 - a_1$, the latter conditions assuring that p_{PU} will result in partial undercutting. These conditions are simplified to the following inequality, which provides the only range of a_1 where partial undercutting is feasible and can also (optimally) provide an equal profit to what firm A obtains in the supposed W-structure equilibrium.

$$a_1 < \rho - \frac{1}{2} \tag{A.1}$$

Since there is a positive range of feasible $(a_1, a_2^{opt}(a_1))$, there are infinite profiles of the form $(a_1, a_2^{opt}(a_1), 1 - a_2^{opt}(a_1), 1 - a_1)$ at which firms are indifferent between keeping W-structure and partially undercutting the competitor; however, next we will rule out all of these equilibrium candidates in favor of only one exception. Let us pick one of these infinite location profiles, only considering that each firm's full-undercutting profit at this location set is strictly less than its profit in the supposed W-structure equilibrium. We suggest that firm A deviates to $(a_1 + \epsilon_1, a_2^{opt}(a_1) + \epsilon_2^{opt}(\epsilon_1))$, where $\epsilon_2^{opt}(\epsilon_1)$ is calculated as $\epsilon_2^{opt}(\epsilon_1) = \epsilon_1(\epsilon_1 t - 1 + \sqrt{\frac{2t}{V - a_1 t}})$ in such a way that shifting (a_1, a_2) by $(\epsilon_2^{opt}(\epsilon_1), \epsilon_1)$ to the right will keep firm B indifferent between keeping W-structure and partially undercutting firm A. If such a deviation by firm A results in W-structure again, we have the following equation.

$$\lim_{\epsilon_1 \to 0} \frac{\partial \pi_A}{\partial \epsilon_1}(a_1+\epsilon_1, a_2^{opt}(a_1)+\epsilon_2^{opt}(\epsilon_1), 1-a_2^{opt}(a_1), 1-a_1, p_L(a_1+\epsilon_1), p_L(a_1))$$
$$= 2c - \frac{t}{2} - c\sqrt{\frac{2t}{V-a_1 t}} + \sqrt{\frac{t(V-a_1 t)}{2}}$$

This expression is positive for $a_1 < \rho - \frac{1}{2}$, the range through which partial undercutting is feasible and also potentially as profitable as W-structure. As a result, firm A has an incentive to deviate as suggested, if the price equilibrium leads to W-structure at the new locations. Using a similar approach to what we followed in Lemma B.0.4, we can show that such a deviation by firm A should result in either W-structure or full undercutting. So, as long as firm B has no incentive to fully undercut, firm A has the incentive to shift to the right by $(\epsilon_1, \epsilon_2^{opt}(\epsilon_1))$. As a result, SPNE is achieved when either firm's equilibrium profit equals its partial-undercutting and full-undercutting profits, given the competitor's equilibrium locations and price. Solving the two corresponding equations for profits, we can obtain the suggested location profile in the proposition statement. □

Proof of Proposition 2.4.5. Plugging in a^* from equation 2.1 into inequality A.1 yields the necessary condition on ρ. To find the feasible range of c we consider two types of deviations, both implying single-product location profiles (at which firms do not customize).

First, consider a deviation of firm A to (a, a), given firm B's $(1-a_2^*, 1-a_1^*)$. Assume, for now, that the resulting price equilibrium forms W-structure. Firm A's best deviation of this form is obtained from the first order condition as follows.

$$\frac{\partial \pi_A}{\partial a}(a, a, 1-a_2^*, 1-a_1^*, p_L(a), p_L(a_1^*)) = 0 \Rightarrow a^* = \frac{1}{36}(1+15\rho - 2\sqrt{3\rho-2})$$

As long as a^* is less than $\bar{a} = (10 - 3\rho - 2\sqrt{3\rho - 2})/18$, or equivalently $\frac{3}{4} < \rho \leq \frac{41}{49}$, we can apply the same method as in Proposition 2.4.2 to show that W-structure is the outcome of the price equilibrium after the deviation. If however a^* exceeds $\bar{a}$, firm B finds full undercutting more profitable than maintaining W-structure. Given that within the provided range of a and ρ, W-structure is the outcome of the deviation, we can show that deviating to (a^*, a^*) is not profitable when c is at most equal to the threshold provided in the proposition statement.

Second, considering $\frac{41}{49} < \rho < \frac{7}{8}$ through which $a^* > \bar{a}$, we propose a deviation of firm A to $(\bar{a}, \bar{a})$. At the new location profile, the price equilibrium leads to W-structure, and firm B's profit equals its full-undercutting profit. As long as $c \leq \frac{t}{4}$, the described deviation is not profitable. □

Proof of Proposition 2.4.6. From Proposition 2.4.2 we know that, in any symmetric SPNE where firms customize, the price equilibrium should be such that, while all the market is served, the customers with extreme tastes receive zero utility. For $(p_L(a_1), p_L(a_1))$ to be the price equilibrium at $(a_1, \frac{1}{2}, \frac{1}{2}, 1 - a_1)$, the following conditions must hold.

$$\frac{\partial_- \pi_A}{\partial p_A}(a_1, \frac{1}{2}, \frac{1}{2}, 1 - a_1, p_L(a_1), p_L(a_1)) \geq 0$$

$$\frac{\partial_+ \pi_A}{\partial p_A}(a_1, \frac{1}{2}, \frac{1}{2}, 1 - a_1, p_L(a_1), p_L(a_1)) \leq 0$$

Note that the two derivatives above are of two different pieces of the profit function. The first derivative considers an infinitesimal price decrease (by ϵ) resulting in the market boundaries 0 and $\frac{1}{2} + \frac{\epsilon}{t}$, while the second derivative considers a price increase (by ϵ) resulting in the market boundaries α_1 and $\frac{1}{2} - \frac{\epsilon}{t}$ for firm A. The above two conditions will reduce to $\underline{a_1} \leq a_1 \leq \rho - \frac{1}{4}$, where $\underline{a_1} = \rho - \frac{1}{2}$. We follow a proof approach to show that a $\underline{W}$-structure equilibrium is possible only when $a_1 = \underline{a_1}$, ruling out the case $\underline{a_1} < a_1 \le \rho - \frac{1}{4}$.

Assume that $\underline{a_1} < a_1 \le \rho - \frac{1}{4}$. Given firm B's location, $(\frac{1}{2}, 1 - a_1)$, let us consider firm A's deviation from $(a_1, \frac{1}{2})$ to $(\hat{a_1}, \hat{a_2})$, where $\hat{a_1} = \underline{a_1} + \epsilon$, $\hat{a_2} = \rho - a_1 + \epsilon$, and $0 < \epsilon < a_1 - \underline{a_1}$. We will show that $\hat{\pi_A} = \pi_A(\hat{a_1}, \hat{a_2}, \frac{1}{2}, 1 - a_1, p_L(\hat{a_1}), p_L(a_1))$ provides a lower bound for firm A's profit after the deviation. Figure 16 illustrates the suggested location and price profile.

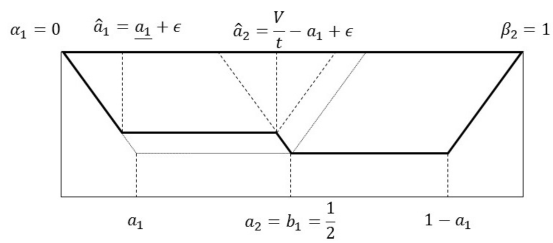


FIGURE 16. Deviation to $(\hat{a_1}, \hat{a_2})$

The market structure is a boundary case between W-structure and firm B partially undercutting A.

Let us first verify that, when $\rho - \frac{1}{2} < a_1 \le \rho - \frac{1}{4}$ and $(p_L(a_1), p_L(a_1))$ is price equilibrium at $(a_1, \frac{1}{2}, \frac{1}{2}, 1 - a_1)$, the two following properties hold.

$$\frac{\partial_- \pi_A}{\partial p_A}(\hat{a_1}, \hat{a_2}, \frac{1}{2}, 1 - a_1, p_L(\hat{a_1}), p_L(a_1)) \ge 0$$

$$\frac{\partial_- \pi_A}{\partial p_A}(\hat{a_1}, \hat{a_2}, \frac{1}{2}, 1 - a_1, p_L(a_1) + t(\hat{a_2} - \frac{1}{2}), p_L(a_1)) \ge 0$$

The first property, along with Lemma B.0.3, suggests that, for a positive but sufficiently small $\epsilon < a_1 - \underline{a_1}$, firm A has no incentive to infinitesimally decrease its price below $p_L(\hat{a_1})$ in response to firm B's $p_L(a_1)$. The second property shows that firm A does not benefit from undercutting firm B either. Following a similar approach, we conclude that firm A does not increase its price above $p_L(\hat{a_1})$ given firm B's $p_L(a_1)$. We also show that firm B does not decrease its price below $p_L(a_1)$ in response to firm A's $p_L(\hat{a_1})$. However, we make no conclusions regarding firm B's incentive to increase its price above $p_L(a_1)$ yet. In fact, we consider two cases, one implicating firm B's incentive to maintain $p_L(a_1)$ and the other to increase its price.

In case $\frac{\partial_+ \pi_B}{\partial p_B}(\hat{a_1}, \hat{a_2}, \frac{1}{2}, 1-a_1, p_L(\hat{a_1}), p_L(a_1)) \leq 0$, then $(p_L(\hat{a_1}), p_L(a_1))$ will be the price equilibrium, and firm A's profit will be $\hat{\pi_A}$. If, however, this derivative is positive, firm B's tendency to increase its price will prevent $(p_L(\hat{a_1}), p_L(a_1))$ from being the price equilibrium. But as firm B's market share shrinks, firm A faces more room to choose its optimal price, and this will lead in an improved profit of firm A compared to $\hat{\pi_A}$. Having this result, we next demonstrate that $\hat{\pi_A}$ is greater than $\pi_A(a_1, \frac{1}{2}, \frac{1}{2}, 1-a_1, p_L(a_1), p_L(a_1))$ when $\underline{a_1} < a_1 \leq \rho - \frac{1}{4}$, and thus rule out the equilibria of the form $(a_1, \frac{1}{2}, \frac{1}{2}, 1-a_1)$.

$$\hat{\pi_A} - \pi_A(a_1, \frac{1}{2}, \frac{1}{2}, 1-a_1, p_L(a_1), p_L(a_1)) = \epsilon(\frac{t}{2} + a_1 t - V - 2\epsilon t)$$

This statement will always be positive for $0 < \epsilon < a_1 - \hat{a_1}$. □

Proof of Proposition 2.4.7. Let us first find a suitable range for ρ. Considering $(\underline{a_1}, \frac{1}{2}, \frac{1}{2}, \underline{a_1}, \underline{p}, \underline{p})$ as the only candidate $\underline{\text{W}}$-structure equilibrium, $0 \leq \underline{a_1} = \rho - \frac{1}{2}$ imposes the lower-bound $\frac{1}{2}$ on ρ. Also, no firm should benefit from decreasing its price to fully undercut the other firm. Note that $p_L(1-a_2)$ is firm A's full-undercutting price—the supremum of all firm A's prices that result in the full undercutting of firm B.

$$\pi_A(\underline{a_1}, \frac{1}{2}, \frac{1}{2}, 1-\underline{a_1}, \underline{p}, \underline{p}) \geq \pi_A(\underline{a_1}, \frac{1}{2}, \frac{1}{2}, 1-\underline{a_1}, p_L(\frac{1}{2}), \underline{p})$$

This condition will also impose the upper-bound $\frac{3}{4}$ on ρ.

Now let us find a range for c. For $(\underline{a_1}, \frac{1}{2}, \frac{1}{2}, \underline{a_1}, \underline{p}, \underline{p})$ to be equilibrium, firm A should not have any profitable deviation to a new location set. Within the range $\frac{3}{5} < \rho \leq \frac{3}{4}$ our proposed deviation is that, given firm B's $(\frac{1}{2}, 1-\underline{a_1})$, firm A relocates from $(\underline{a_1}, \frac{1}{2})$ to $(\underline{a_1'}, \underline{a_1'})$, where $\underline{a_1'} = \frac{3}{4}\rho - \frac{1}{4}$. First, we will show that at $(\underline{a_1'}, \underline{a_1'}, \frac{1}{2}, 1-\underline{a_1})$ price equilibrium results in W-structure, and second, we will find the range of c for which this move is not profitable.

Using the same approach as in the proofs of propositions 2.4.2 and 2.4.6, we argue that at $(\underline{a_1'}, \underline{a_1'}, \frac{1}{2}, 1 - \underline{a_1})$, price equilibrium will result in zero utilities at the extremes of the taste spectrum. Consequently, in the range $\frac{3}{5} < \rho \leq \frac{3}{4}$ where firms adopt the prices $(p_L(\underline{a_1'}), p_L(\underline{a_1}))$,

$$\alpha_2 = 2\underline{a_1'} = \frac{3}{2}\rho - \frac{1}{2} > \beta_1 = \frac{1}{2} - \underline{a_1} = 1 - \rho$$

$$\underline{a_1'} + \frac{p_L(\underline{a_1}) - p_L(\underline{a_1'})}{t} = \frac{\rho}{2} < \underline{b_1} = \frac{1}{2}$$

$$\underline{b_1} - \frac{p_L(\underline{a_1'}) - p_L(\underline{a_1})}{t} = \frac{3}{4} - \frac{\rho}{4} > \underline{a_1'} = \frac{3\rho}{4} - \frac{1}{4}$$

From the above inequalities we conclude that as firm A deviates to $(\underline{a_1'}, \underline{a_1'})$, price equilibrium results in W-structure, and thus the left and right market boundaries of firm A are 0 and m, respectively. For firm A to have no incentive to deviate to this location set, given firm B's location, the following should hold.

$$\pi_A(\underline{a_1}, \frac{1}{2}, \frac{1}{2}, 1 - \underline{a_1}, p_L(\underline{a_1}), p_L(\underline{a_1})) \geq \pi_A(\underline{a_1'}, \underline{a_1'}, \frac{1}{2}, 1 - \underline{a_1}, p_L(\underline{a_1'}), p_L(\underline{a_1}))$$

This condition is verified when $c \leq \frac{3t+V}{16}$.

Now let us consider the range $\frac{1}{2} \leq \rho \leq \frac{3}{5}$. In this range of ρ, our proposed deviation is $(\underline{a_1''}, \underline{a_1''})$, where $\underline{a_1''} = \frac{1}{2} - \frac{\rho}{2}$. Considering the derivative of profits with respect to prices at $(\underline{a_1''}, \underline{a_1''}, \frac{1}{2}, 1-\underline{a_1}, p_L(\underline{a_1''}), p_L(\underline{a_1}))$ and ruling out the possibilities of partial and full undercutting by either firm, $(p_L(\underline{a_1''}), p_L(\underline{a_1}))$ will be the price equilibrium. At the mentioned locations and prices, $\alpha_2 = \beta_1$ and thus we will have $\overline{\text{W}}$-structure. Again, for firm A to not benefit from deviating to $(\underline{a_1''}, \underline{a_1''})$ we should have $\pi_A(\underline{a_1}, \frac{1}{2}, \frac{1}{2}, 1-\underline{a_1}, p_L(\underline{a_1}), p_L(\underline{a_1})) \geq \pi_A(\underline{a_1''}, \underline{a_1''}, \frac{1}{2}, 1-\underline{a_1}, p_L(\underline{a_1''}), p_L(\underline{a_1}))$. Solving this inequality gives us an upper-bound for c as $\frac{3t^2-8tV+6V^2}{4t-4V}$. □

Proof of Proposition 2.4.8. Consider the suggested location profile, $(a, b) = (0, \frac{1}{2}, \frac{1}{2}, 1)$. Fixing the locations, no firm can profitably increase its price above V. As we decrease price, each firm's price derivative of profit, calculated as $\frac{1}{2} - \rho$, should be non-negative. This condition determines the upper-bound on ρ.

It should be noticed that, when $\rho \leq \frac{1}{2}$, even in the absence of the competitor, no firm has an incentive to decrease its price below V, with locations fixed at the edge and the center. Thus, as the competitor draws back its market from the center, the new price equilibrium will still involve the firm setting the price V. Now for the proposed profile in the statement to be the SPNE, we require that no firm be able to profitably deviate to another location set. Our deviation candidate is the transition to the single-product monopoly outcome, without interfering with the competitor's market share. Setting the single-product monopoly price $\frac{V}{2}$ and capturing the market share $\frac{V}{t}$ yield a profit of $\frac{V^2}{2t}$, which should not exceed the original profit, $\frac{V-c}{2}$. This condition produces the upper-bound on c. □

Proof of Proposition 2.4.9. Consider the proposed multi-interval strategy profile in which firm A's offerings are represented as $[a_1, a_2^1] \cup [a_1^2, a_2^2] \cup \ldots \cup [a_1^n, a_2]$, where $a_1 \leq a_2^1 \leq a_1^2 \leq a_2^2 \leq \ldots \leq a_1^n \leq a_2 \leq b_1$ and at least one interval is of positive length.

First, we argue that, in equilibrium, firm A is not price-undercut either partially or fully. Otherwise, it could profitably deviate by eliminating the undercut portions of its product portfolio without affecting firm B's market share and thus the emerging price equilibrium. Moreover, firm A should arrange its customization intervals so as to serve all customers within $[a_1, a_2]$ in equilibrium. This is due to the linearity of the MC cost in MC scope; if c is too low, firm A should expand (at least) one of its customization intervals to serve previously unserved customers. Otherwise if c is too high, firm A should forego customization across the board. Now, if firm A integrates all its customization scopes, neither firm A's nor firm B's market boundaries will be affected. As a result, the price equilibrium does not change, and we achieve an equilibrium where firm A offers a unified customization range and both firms' profits remain constant. □

Proof of Corollary 2.4.10. Suppose, for the sake of contradiction, that firm A adopts a multi-interval strategy in equilibrium. Using the same logic as provided in Proposition 2.4.9 we argue that no firm undercuts the competitor in such an equilibrium candidate and firm A serves all the customers within $[a_1, a_2]$ in the equilibrium candidate. Consider a deviation by which firm A unifies all its customization intervals. With the equilibrium properties explained above, this deviation does not affect either firm A's or firm B's market boundaries. Thus, the price equilibrium does not change. With unaffected prices and market shares, a lower MC cost for firm A implies a profitable deviation. □

Proof of Corollaries 2.6.2 and 2.6.3. Hinloopen and Van Marrewijk (1999) and Pazgal et al. (2016) derive the normalized equilibrium prices in the single-product duopoly as below.

$$\tilde{p}^*_{SP} = \frac{p^*_{SP}}{V} = \begin{cases} \frac{1}{2} & 0 < \rho < \frac{1}{2} \\ 1 - \frac{1}{4\rho} & \frac{1}{2} \leq \rho \leq \frac{3}{4} \\ \frac{1}{2\rho} & \frac{3}{4} < \rho < \frac{7}{8} \end{cases}$$

Correspondingly, the equilibrium profits of the single-product firms are

$$\tilde{\pi}^*_{SP} = \frac{\pi^*_{SP}}{V} = \begin{cases} \frac{\rho}{2} & 0 < \rho < \frac{1}{2} \\ \frac{1}{2} - \frac{1}{8\rho} & \frac{1}{2} \leq \rho \leq \frac{3}{4} \\ \frac{1}{4\rho} & \frac{3}{4} < \rho < \frac{7}{8} \end{cases}.$$

The findings stated in the corollaries are direct implications of comparing the above results with the equilibrium results we establish for the MC duopoly. □

APPENDIX B

ESSAY 1: STATEMENTS AND PROOFS OF LEMMAS

Lemma B.0.1. *In any symmetric equilibrium where firms mass customize, the entire* $[0,1]$ *market interval is served.*

Proof of Lemma B.0.1. We follow a proof-by-contradiction approach. First, suppose that there is a gap, an interval of unserved consumers, and also there exists a consumer between the firms receiving zero utility. This implies that each firm is a local customizing monopolist that does not interfere with its competitor's market share. In this situation, there is at least one firm leaving a gap on one side of its market. According to Proposition 2.4.1, no customizing monopolist optimally behaves as described.

Now assume that, in equilibrium, there are gaps on both market ends, and the middle consumer obtains positive utility. Price equilibrium should, therefore, be the interior solution of the first order conditions, obtained as below.

$$FOC \begin{cases} \frac{\partial}{\partial p_A}[p_A(m(a,b,p_A,p_B) - \alpha_1(a,p_A))] = 0 \\ \frac{\partial}{\partial p_B}[p_B(\beta_2(b,p_B) - m(a,b,p_A,p_B))] = 0 \end{cases} \Rightarrow p_A^* = p_B^* = \frac{2V + t(1-2a_1)}{5}$$

We propose that, fixing $b = (1-a_2, 1-a_1)$, firm A shifts both a_1 and a_2 to the left by an infinitesimal δ. Since before the deviation neither firm had an incentive to undercut the competitor and the deviation augments both firms' profits, neither firm benefits from undercutting after the deviation. Hence, after the deviation we will derive an interior-type price equilibrium. In the resulting price equilibrium firm A leaves a gap to the left of its market and the indifferent consumer between the two firms obtains positive utility. The price equilibrium, calculated vis-à-vis similar first order conditions to the above, is characterized as $(p'_A, p'_B) = (p^*_A + \frac{\delta t}{5}, p^*_B + \frac{\delta t}{5})$. As a result of the proposed deviation, firm A's market share increases by:

$$[m(a_2-\delta, b, p^*_A+\frac{\delta t}{5}, p^*_A+\frac{\delta t}{5})-\alpha_1(a_1-\delta, p^*_A+\frac{\delta t}{5})]-[m(a_2, b, p^*_A, p^*_B)-\alpha_1(a_1, p^*_A)] = \frac{3\delta}{10}$$

An increase in both price and market share for firm A implies a profitable deviation.

$\square$

Lemma B.0.2. *Suppose that (p^*_A, p^*_B) is the price equilibrium at a location profile where firm A customizes and leaves positive utility at $x = 0$. (p^*_A, p^*_B) will also be the price equilibrium after firm A infinitesimally shifts a_1 to the right, if this move does not incentivize firm B to fully undercut firm A.*

Proof of Lemma B.0.2. Denote the original location profile by (a'_1, a_2, b), where $a_2 > a'_1$ and $a'_1 - \frac{V-p^*_A}{t} < 0$, and the consequent location profile by (a''_1, a_2, b), such that $a'_1 < a''_1 < a_2$, and $a''_1 - \frac{V-p^*_A}{t} \leq 0$. If (p^*_A, p^*_B) is the price equilibrium at (a'_1, a_2, b), then for any $\underline{p_A}$, $\overline{p_A}$, $\underline{p_B}$ and $\overline{p_B}$ such that $0 \leq \underline{p_A} < p^*_A < \overline{p_A} \leq V$ and $0 \leq \underline{p_B} < p^*_B < \overline{p_B} \leq V$,

$$\pi_A(a'_1, a_2, b, p^*_A, p^*_B) - \pi_A(a'_1, a_2, b, \underline{p_A}, p^*_B) \geq 0 \tag{B.1}$$

$$\pi_A(a'_1, a_2, b, p^*_A, p^*_B) - \pi_A(a'_1, a_2, b, \overline{p_A}, p^*_B) \geq 0. \tag{B.2}$$

$$\pi_B(a'_1, a_2, b, p^*_A, p^*_B) - \pi_B(a'_1, a_2, b, p^*_A, \underline{p_B}) \geq 0 \tag{B.3}$$

$$\pi_B(a'_1, a_2, b, p^*_A, p^*_B) - \pi_B(a'_1, a_2, b, p^*_A, \overline{p_B}) \geq 0. \tag{B.4}$$

Let us represent, by $\tilde{\alpha}_1(a_1, b, p_A, p_B)$ and $\tilde{\alpha}_2(a_2, b, p_A, p_B)$, respectively the left and right market boundaries of firm A. Since firm A leaves positive utility at the left market edge, then $\tilde{\alpha}_1(a_1', b, p_A^*, p_B^*) = 0$, and consequently for any $\underline{p_A} < p_A^*$, $\tilde{\alpha}_1(a_1', b, \underline{p_A}, p_B^*) = 0$. Also, if firm A's move to (a_1'', a_2) does not prompt firm B's full undercutting, $\tilde{\alpha}_1(a_1'', b, p_A^*, p_B^*) = \tilde{\alpha}_1(a_1'', b, \underline{p_A}, p_B^*) = 0$. From (B.1) we have:

$$\begin{aligned} & \pi_A(a_1', a_2, b, p_A^*, p_B^*) - \pi_A(a_1', a_2, b, \underline{p_A}, p_B^*) \geq 0 \\ \Rightarrow & [p_A^* \tilde{\alpha}_2(a_2, b, p_A^*, p_B^*) - c(a_2 - a_1')] - [\underline{p_A} \tilde{\alpha}_2(a_2, b, \underline{p_A}, p_B^*) - c(a_2 - a_1')] \geq 0 \\ \Rightarrow & [p_A^* \tilde{\alpha}_2(a_2, b, p_A^*, p_B^*) - c(a_2 - a_1'')] - [\underline{p_A} \tilde{\alpha}_2(a_2, b, \underline{p_A}, p_B^*) - c(a_2 - a_1'')] \geq 0 \end{aligned}$$

$$\Rightarrow \pi_A(a_1'', a_2, b, p_A^*, p_B^*) - \pi_A(a_1'', a_2, b, \underline{p_A}, p_B^*) \geq 0 \tag{B.5}$$

As a result, at (a_1'', a_2, b) firm A does not benefit from decreasing its price below p_A^* given firm B's price p_B^*. Now we want to show that firm A does not benefit from increasing its price either. We know that, as long as $0 < a_1 < a_2$ and firm B does not fully undercut firm A, the left market boundary of firm A, $\tilde{\alpha}_1$, is non-decreasing in a_1, fixing all the other location variables and prices. So $\pi_A(a_1, a_2, b, p_A, p_B) + c(a_2 - a_1)$ is non-increasing in a_1 given all the other variables, and consequently,

$$\pi_A(a_1', a_2, b, p_A, p_B) + c(a_2 - a_1') \geq \pi_A(a_1'', a_2, b, p_A, p_B) + c(a_2 - a_1'').$$

From (B.2) we have:

$$\begin{aligned} & \pi_A(a_1', a_2, b, p_A^*, p_B^*) - \pi_A(a_1', a_2, b, \overline{p_A}, p_B^*) && \geq 0 \\ & \quad \Rightarrow [p_A^* \tilde{\alpha}_2(a_2, b, p_A^*, p_B^*) - c(a_2 - a_1')] - \pi_A(a_1', a_2, b, \overline{p_A}, p_B^*) && \geq 0 \\ & \quad \Rightarrow [p_A^* \tilde{\alpha}_2(a_2, b, p_A^*, p_B^*) - c(a_2 - a_1')] && \\ & \quad - [\pi_A(a_1'', a_2, b, \overline{p_A}, p_B^*) + c(a_2 - a_1'') - c(a_2 - a_1')] && \geq 0 \\ & \quad \Rightarrow [p_A^* \tilde{\alpha}_2(a_2, b, p_A^*, p_B^*) - c(a_2 - a_1'')] - \pi_A(a_1'', a_2, b, \overline{p_A}, p_B^*) && \geq 0 \end{aligned}$$

$$\pi_A(a_1'', a_2, b, p_A^*, p_B^*) - \pi_A(a_1'', a_2, b, \overline{p_A}, p_B^*) \geq 0 \tag{B.6}$$

Together, (B.5) and (B.6) imply that p_A^* is the price best response to p_B^* at (a_1'', a_2, b).

Now we need to show that p_B^* is also firm B's price best response to p_A^* after firm A changes a_1' to a_1''. As long as no full undercutting occurs, for any p_A and p_B, $\pi_B(a_1', a_2, b, p_A, p_B) = \pi_B(a_1'', a_2, b, p_A, p_B)$. Given that (p_A^*, p_B^*) is the price equilibrium before firm A's move, we infer the following from B.3 and B.4.

$$\begin{aligned}
&\pi_B(a_1'', a_2, b, p_A^*, p_B^*) - \pi_B(a_1'', a_2, b, p_A^*, \underline{p_B}) \\
&\qquad = \pi_B(a_1', a_2, b, p_A^*, p_B^*) - \pi_B(a_1', a_2, b, p_A^*, \underline{p_B}) \geq 0 \\
&\pi_B(a_1'', a_2, b, p_A^*, p_B^*) - \pi_B(a_1'', a_2, b, p_A^*, \overline{p_B}) \\
&\qquad = \pi_B(a_1', a_2, b, p_A^*, p_B^*) - \pi_B(a_1', a_2, b, p_A^*, \overline{p_B}) \geq 0
\end{aligned}$$

Thus, (p_A^*, p_B^*) is the price equilibrium at (a_1'', a_2, b). □

Lemma B.0.3. *Either firm's profit is concave in its price within the following ranges:*

i. When the price does not lead to any undercutting structure.

ii. When the price leads to being partially undercut or partial undercutting. The profit is concave over the entire range if the transition (from being undercut to undercutting) is continuous, or put another way, $a_2 = b_1$.

iii. when the price leads to full undercutting.

Proof of Lemma B.0.3. Let us consider firm A's profit.

i. As long as firm A does not partially or fully undercut firm B, firm A's market boundaries are defined by $max(0, \alpha_1)$ on the left and $min(m, \alpha_2)$ on the right.

The corresponding profit is calculated as below.

$$\begin{aligned}\pi_A^i(a,b,p_A,p_B) &= p_A[\min{(m,\alpha_2)} - \max{(0,\alpha_1)}] - c(a_2-a_1) \\ &= \min(p_A m, p_A\alpha_2) - \max(0, p_A\alpha_1) - c(a_2-a_1) \\ &= \min(p_A m, p_A\alpha_2) + \min(0, -p_A\alpha_1) - c(a_2-a_1)\end{aligned}$$

Since $p_A m$, $p_A\alpha_2$, and $-p_A\alpha_1$ are all concave in p_A, the minimums and consequently the sum will also be concave in p_A.

ii. When firm A either partially undercuts firm B (as defined in section 3.3) or is partially undercut by firm B, its profit can be characterized as below:

$$\pi_A^{ii}(a,b,p_A,p_B) = \begin{cases} p_A[a_2 + \frac{p_B-p_A}{t} - \max(0,\alpha_1)] - c(a_2-a_1) & p_A \le p_B \\ p_A[b_1 - \frac{p_A-p_B}{t} - \max(0,\alpha_1)] - c(a_2-a_1) & p_A > p_B \end{cases}$$

$$= \begin{cases} p_A a_2 + \frac{p_A p_B - p_A^2}{t} + \min(0,-p_A\alpha_1) - c(a_2-a_1) & p_A \le p_B \\ p_A b_1 + \frac{p_A p_B - p_A^2}{t} + \min(0,-p_A\alpha_1) - c(a_2-a_1) & p_A \le p_B \end{cases}$$

$p_A a_2$ and $p_A b_1$ are linear in price, $\frac{p_A p_B - p_A^2}{t}$ and $\min(0, -p_A\alpha_1)$ are concave in price, and the sum of concave functions is also concave. Therefore, each of the pieces above is concave. Moreover, if the transition from one piece to another is continuous, happening when $a_2 = b_1$, we can merge the two pieces into one concave function.

iii. The profit of firm A as it fully undercuts firm B is defined as:

$$\begin{aligned}\pi_A^{iii} &= p_A[\min(1, a_2 + \frac{V - p_A}{t}) - \max(0, \alpha_1)] - c(a_2 - a_1) \\ &= \min(p_A, p_A a_2 + \frac{V p_A - p_A^2}{t}) + \min(0, -p_A \alpha_1) - c(a_2 - a_1)\end{aligned}$$

π_A^{iii} is the sum of concave functions and is concave itself.

□

Lemma B.0.4. *Suppose that at the symmetric location profile,* $(a_1, a_2, 1-a_2, 1-a_1)$*, price equilibrium yields W-structure.*

i. For $\frac{1}{3} + a_1 < \rho < 1 + a_1$*, we can find a positive* ϵ *such that, if no firm has an incentive to undercut, price equilibrium at* $(a_1, a_2 \pm \epsilon, 1 - a_2, 1 - a_1)$ *will again result in W-structure.*

ii. $a_1 \neq \rho - 1/3$

Proof of Lemma B.0.4. i. According to Lemma B.0.3, each firm's profit is concave in its own price as long as no firm partially or fully undercuts the competitor. Thus, if no firm benefits from undercutting, showing that firms have no incentive to infinitesimally deviate from $p_L(a_1)$ would suffice to prove $(p_L(a_1), p_L(a_1))$ is price equilibrium. First, given $i \in \{A, B\}$, we verify that $\frac{1}{3} + a_1 < \rho < 1 + a_1$ is sufficient for the following inequalities to hold.

$$\frac{\partial_- \pi_i}{\partial p_i}(a_1, a_2, 1 - a_2, 1 - a_1, p_L(a_1), p_L(a_1)) = \frac{1}{2}(1 + a_1 - \rho) > 0$$

$$\frac{\partial_+ \pi_i}{\partial p_i}(a_1, a_2, 1 - a_2, 1 - a_1, p_L(a_1), p_L(a_1)) = \frac{3}{2}(\frac{1}{3} + a_1 - \rho) < 0$$

Now, as we shift a_2 to the right or left by ϵ, changes in the above derivatives are calculated as below:

$$\frac{\partial_- \pi_A}{\partial p_A}(a_1, a_2 \pm \epsilon, 1 - a_2, 1 - a_1, p_A, p_L(a_1)) - \frac{\partial_- \pi_A}{\partial p_A}(a_1, a_2, 1 - a_2, 1 - a_1, p_A, p_L(a_1)) = \pm\frac{\epsilon}{2}$$

$$\frac{\partial_+ \pi_A}{\partial p_A}(a_1, a_2 \pm \epsilon, 1 - a_2, 1 - a_1, p_A, p_L(a_1)) - \frac{\partial_+ \pi_A}{\partial p_A}(a_1, a_2, 1 - a_2, 1 - a_1, p_A, p_L(a_1)) = \pm\frac{\epsilon}{2}$$

$$\frac{\partial_- \pi_B}{\partial p_B}(a_1, a_2 \pm \epsilon, 1 - a_2, 1 - a_1, p_L(a_1), p_B) - \frac{\partial_- \pi_B}{\partial p_B}(a_1, a_2, 1 - a_2, 1 - a_1, p_L(a_1), p_B) = \mp\frac{\epsilon}{2}$$

$$\frac{\partial_+ \pi_B}{\partial p_B}(a_1, a_2 \pm \epsilon, 1 - a_2, 1 - a_1, p_L(a_1), p_B) - \frac{\partial_+ \pi_B}{\partial p_B}(a_1, a_2, 1 - a_2, 1 - a_1, p_L(a_1), p_B) = \mp\frac{\epsilon}{2}$$

For a sufficiently small ϵ, thus, we conclude the provided price derivatives of profit do not change sign, and price equilibrium will still be $(p_L(a_1), p_L(a_1))$.

ii. Let us assume there exists a symmetric W-structure equilibrium at $(\rho - \frac{1}{3}, a_2, 1 - a_2, \frac{2}{3} - \rho)$. At this equilibrium no firm has a partial undercutting incentive, since each firm's profit is strictly decreasing in price within the partial-undercutting zone, when $a_2 < \frac{1}{2}$. Let us first assume that each firm's profit is greater than its full-undercutting profit (the supremum of profits achieved from fully undercutting the competitor). Since an increase in a_2 by an infinitesimal ϵ will not prompt the competitor's undercutting, the following statements verify that the price equilibrium at the new location set is obtained as $(p_1^*, p_L(a_1))$, where

$p_1^* = \frac{1}{3} + \frac{\epsilon t}{6}$ is the solution of $\frac{\partial \pi_A}{\partial p_A}(a_1, a_2 + \epsilon, 1 - a_2, 1 - a_1, p_A, p_L(a_1)) = 0$, and the market boundaries of firm A are determined by α_1 and m.

$$\frac{\partial_- \pi_B}{\partial p_B}(a_1, a_2 + \epsilon, 1 - a_2, 1 - a_1, p_1^*, p_L(a_1)) = \frac{1}{3} - \frac{5\epsilon}{12} > 0$$

$$\frac{\partial_+ \pi_B}{\partial p_B}(a_1, a_2 + \epsilon, 1 - a_2, 1 - a_1, p_1^*, p_L(a_1)) = -\frac{5\epsilon}{12} < 0$$

Considering the price equilibrium $(p_1^*, p_L(a_1))$, we conclude that the profit change as a result of shifting a_2 to the right by ϵ is positive when $c < \frac{t}{6}$. Similarly, as we decrease a_2 by ϵ, fixing a_1 and b, $(p_L(a_1), p_1^*)$ will be the price equilibrium. The deviation is profitable for $c \geq \frac{t}{6}$.

Let us, now, search for a profitable deviation when each firm's profit at the equilibrium candidate equals its full-undercutting profit. This equality occurs when $a_2 = \frac{7}{6} - \rho$. So our equilibrium candidate is now $(\hat{a_1}, \hat{a_2}, \hat{b_1}, \hat{b_2}) = (\rho - \frac{1}{3}, \frac{7}{6} - \rho, \rho - \frac{1}{6}, \frac{2}{3} - \rho)$. For $(p_L(\rho - \frac{1}{3}), p_L(\rho - \frac{1}{3}))$ to be price equilibrium at this location profile, we need $\frac{2}{3} < \rho < \frac{3}{4}$.

First, we propose, that given firm B's $(\hat{b_1}, \hat{b_2})$, firm A deviates to $(\hat{a_1} - \epsilon, \hat{a_1})$. Similar to how we evaluated the post-deviation price equilibrium above, we can show that the price equilibrium as a result of decreasing $\hat{a_1}$ to the left by ϵ is $(p_L(\hat{a_1} - \epsilon), p_1^*)$. At this profile, with all locations fixed, no firm benefits from either marginally changing its price or fully undercutting its competitor. This location deviation by firm A is always profitable when ϵ is infinitesimally small and $c < \frac{t}{3}$.

Now, we propose an alternative deviation of firm A to $(\hat{a_1}, \hat{a_1})$. Adopting the same approach as above, we can show that the new price equilibrium will be

$(p_L(\hat{a}_1), p_2^*)$, where $p_2^* = \frac{7t}{12} - \frac{V}{3}$ is the interior maximizer of firm B's profit when B's market boundaries are m and β_2, and A adopts the price $p_L(\hat{a}_1)$. Firm A's profit improves as a result of such a deviation when $c > \frac{t}{6}$.

Since, as we unite the two ranges of c above, for any value of c we can find a profitable deviation from the provided equilibrium candidate, we establish that no symmetric W-structure equilibrium exists when $a_1 = \rho - 1/3$.

$\square$

APPENDIX C

ESSAY 2: PROOFS OF PROPOSITIONS AND COROLLARIES

Proof of Proposition 3.4.1. Denote by A_r^* the reduced sequence obtained from dropping the first r elements in A^*, by $U(A)$ the discounted expected utility of following policy A, and by $\mathbb{A}$ the set of all possible sequences.

1. Suppose that the optimal policy A^* contains $a_1^* = a_2^* = i \in \{1, 2\}$. The optimality of A^* requires that, having observed v_i and p, both A_1^* and A_2^* be optimal. We have:

$$U(A_1^*|v_i, p) \geq U(A|v_i, p) \quad , \quad \forall A \in \mathbb{A}$$
$$U(A_2^*|v_i, p) \geq U(A|v_i, p) \quad , \quad \forall A \in \mathbb{A}$$

   Consequently, $A_1^* \equiv A_2^*$, and thus for all $t \geq 3$, $a_t^* = a_{t-1}^*$. Since $a_2^* = i$, $a_t^* = i$ for every $t \geq 3$.

2. It is obvious that after realizing both v_1 and v_2, not consuming the best alternative among $\{1, 2, \varnothing\}$ is suboptimal.

3. Suppose that the optimal policy A^* contains $a_1^* = i \in \{1, 2\}$ and $a_2^* = \varnothing$. Then,

$$U(A_1^*|v_i, p) \geq U(A|v_i, p), \quad \forall A \in \mathbb{A}$$

   Assume that after period 2, there exists at least one period where A^* does not advise the consumption of $\varnothing$. Denote by s the smallest period index among such periods. Then A^* suggests the consumption of $\varnothing$ in all periods prior to s except period 1. Therefore, $U(A_{s-1}^*|v_i, p) \geq U(A|v_i, p), \forall A \in \mathbb{A}$. As a result,

$A_1^* \equiv A_{s-1}^*$, requiring that $a_2^* = \varnothing$ be equal to $a_s^* \in \{1, 2\}$. Since this equality never holds, S^* should contain $\varnothing$ in all periods $t \geq 3$.

4. Assume that the optimal policy A^* contains $a_1^* = \varnothing$ and at least one period in which the consumer tries either Product 1 or Product 2. Denote by s the minimum period index among such periods. We should have $A^* = A_{s-1}^*$, and thus $a_1^* = a_s^*$. By definition of s, this equality does not hold. So if an optimal policy begins with $\varnothing$, the sequence should proceed (interminably) with $\varnothing$.

$\square$

Proof of Proposition 3.4.2. (i) The customer is indifferent between trying Product j in the second period and continuing with the on-hand alternative only if $u_{ij}(p, v_i) = u_{i\bar{j}}(p, v_i)$. From equations 3.1 and 3.2 we have:

$$\begin{aligned}
&\int_0^{\nu} (v_j - p + \beta\frac{\nu - p}{1-\beta})dF(v_j) + \int_{\nu}^{1} \frac{v_j - p}{1-\beta}dF(v_j) = \frac{\nu - p}{1-\beta} \\
\iff &\int_0^{\nu} (v_j - p - (1-\beta)\frac{\nu - p}{1-\beta})dF(v_j) + \int_{\nu}^{1} (\frac{v_j - p}{1-\beta} - \frac{\nu - p}{1-\beta})dF(v_j) = 0 \\
\iff &\int_0^{\nu} (v_j - \nu)dF(v_j) + \int_{\nu}^{1} \frac{v_j - \nu}{1-\beta}dF(v_j) = 0.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
&\frac{d}{d\nu}(\int_0^{\nu} (v_j - \nu)dF(v_j) + \int_{\nu}^{1} \frac{v_j - \nu}{1-\beta}dF(v_j)) \\
=&\int_0^{\nu} \frac{d}{d\nu}(v_j - \nu)dF(v_j) + \int_{\nu}^{1} \frac{d}{d\nu}(\frac{v_j - \nu}{1-\beta})dF(v_j) \\
=&-F(\nu) - \frac{1}{1-\beta}(1 - F(\nu)) < 0
\end{aligned}$$

Therefore, the left-hand side of the stated equation in the proposition is strictly decreasing. We can easily verify that when $\nu = 0$ the expression is positive and when $\nu = 1$ it is negative. Thus, there is a unique solution to the equation on $\nu \in [0, 1]$.

(ii) For any $\nu \in [0, 1)$ we have:

$$\frac{\partial}{\partial \beta}\left(\int_0^{\nu}(v_j - \nu)dF(v_j) + \int_{\nu}^{1}\frac{v_j - \nu}{1-\beta}dF(v_j)\right) = \frac{1}{1-\beta^2}\int_{\nu}^{1}(v_j - \nu)dF(v_j) > 0$$

Since the left-hand side of the equation in the proposition statement is strictly decreasing in β, $\tilde{\nu}$ is increasing in β.

Now we look at the asymptotic values of $\tilde{\nu}$.

$$\lim_{\beta \to 0^+}\left(\int_0^{\nu}(v_j - \nu)dF(v_j) + \int_{\nu}^{1}\frac{v_j - \nu}{1-\beta}dF(v_j)\right) = \mu - \nu = 0$$
$$\Rightarrow \lim_{\beta \to 0^+}\tilde{\nu} = \mu$$

To derive an upper limit for $\tilde{\nu}$ we first verify that, when $\tilde{\nu} = 1$,

$$\lim_{\beta \to 1^-}\left(\int_0^{1}(v_j - 1)dF(v_j) + \int_{1}^{1}\frac{v_j - 1}{1-\beta}dF(v_j)\right) = 0.$$

We can also verify that no other asymptotic value of $\tilde{\nu}$ solves the equation as β approaches 1:

$$\forall \tilde{\nu} \in [0, 1), \quad \lim_{\beta \to 1^-}\left(\int_0^{\tilde{\nu}}(v_j - \tilde{\nu})dF(v_j) + \int_{\tilde{\nu}}^{1}\frac{v_j - \tilde{\nu}}{1-\beta}dF(v_j)\right) = +\infty.$$

□

Proof of Proposition 3.4.3. Let us first consider the region where $p \leq \tilde{\nu} - \underline{u}$. From Specification 3.4 we have:

$$\begin{aligned}
\frac{d}{dp}u_i(p) = -1 + \beta u_{ij}(p, p+\underline{u})f(p+\underline{u}) + \beta \int_0^{p+\underline{u}} \frac{\partial}{\partial p} u_{ij}(p, v_i) dF(v_i) \\
- \beta u_{ij}(p, p+\underline{u})f(p+\underline{u}) + \beta \int_{p+\underline{u}}^{\tilde{\nu}} \frac{\partial}{\partial p} u_{ij}(p, v_i) dF(v_i) + \beta \int_{\tilde{\nu}}^{1} \frac{\partial}{\partial p} u_{i\bar{j}}(p, v_i) dF(v_i) \\
= -1 + \beta \int_0^{p+\underline{u}} \frac{\partial}{\partial p} u_{ij}(p, v_i) dF(v_i) \\
+ \beta \int_{p+\underline{u}}^{\tilde{\nu}} \frac{\partial}{\partial p} u_{ij}(p, v_i) dF(v_i) + \beta \int_{\tilde{\nu}}^{1} \frac{\partial}{\partial p} u_{i\bar{j}}(p, v_i) dF(v_i)
\end{aligned}$$

When $v_i \leq p + \underline{u}$, and thus $\nu = p + \underline{u}$, we can calculate a consumer's second period utility from switching to product j as follows.

$$\begin{aligned}
u_{ij}(p, v_i) &= \int_0^{p+\underline{u}} (v_j - p + \beta \frac{\underline{u}}{1-\beta}) dF(v_j) + \int_{p+\underline{u}}^{1} \frac{v_j - p}{1-\beta} dF(v_j) \\
\Rightarrow \frac{\partial}{\partial p} u_{ij}(p, v_i) &= (\underline{u} + \beta \frac{\underline{u}}{1-\beta}) f(p+\underline{u}) \\
&\quad - \int_0^{p+\underline{u}} dF(v_j) - \frac{\underline{u}}{1-\beta} f(p+\underline{u}) - \frac{1}{1-\beta} \int_{p+\underline{u}}^{1} dF(v_j) \\
&= - \int_0^{p+\underline{u}} dF(v_j) - \frac{1}{1-\beta} \int_{p+\underline{u}}^{1} dF(v_j)
\end{aligned}$$

For $v_i > p + \underline{u}$ and thus $\nu = v_i$ we next assess a consumer's second period utility from switching to product j.

$$\begin{aligned}
u_{ij}(p, v_i) &= \int_0^{v_i} (v_j - p + \beta \frac{v_i - p}{1-\beta}) dF(v_j) + \int_{v_i}^{1} \frac{v_j - p}{1-\beta} dF(v_j) \\
\Rightarrow \frac{\partial}{\partial p} u_{ij}(p, v_i) &= - \int_0^{v_i} (1 + \frac{\beta}{1-\beta}) dF(v_j) - \int_{v_i}^{1} \frac{1}{1+\beta} dF(v_j)
\end{aligned}$$

For the consumer's second period utility from repeatedly purchasing ν we have:

$$u_{i\bar{j}}(p, v_i) = \begin{cases} \frac{\underline{u}}{1-\beta} & p + \underline{u} \geq v_i \\ \frac{v_i - p}{1-\beta} & p + \underline{u} < v_i \end{cases} \Rightarrow \frac{\partial}{\partial p} u_{i\bar{j}}(p, v_i) = \begin{cases} 0 & p + \underline{u} \geq v_i \\ \frac{-1}{1-\beta} & p + \underline{u} < v_i \end{cases}$$

Since all the additive terms in $\frac{d}{dp}u_i(p)$ are non-positive, $u_i(p)$ is decreasing when $p \leq \tilde{\nu} - \underline{u}$. Now consider the range where $p > \tilde{\nu} - \underline{u}$.

$$\frac{d}{dp}u_i(p) = \begin{cases} -1 & p + \underline{u} \geq v_i \\ -1 - \frac{1}{1-\beta} & p + \underline{u} < v_i \end{cases}$$

We conclude that $u_i(p)$ is decreasing on the full domain of feasible prices. Finally, we need to show $u_i(\tilde{\nu} - \underline{u}) = \frac{\underline{u}}{1-\beta}$. When $v_i \leq \tilde{\nu}$, we have that $\nu = \max\{v_i, p + \underline{u}\} = \max\{v_i, \tilde{\nu}\} = \tilde{\nu}$. Thus, in the second period the consumer is indifferent between the outside option and switching to product j, obtaining the same utility from these choices. If $v_i > \tilde{\nu}$, the consumer will purchase product i in period 2 onward. Therefore the consumer's utility in period 1 is derived as follows.

$$\begin{aligned} u_i(\tilde{\nu} - \underline{u}) &= \int_0^{\tilde{\nu}} (v_i - \tilde{\nu} + \underline{u} + \beta \frac{\underline{u}}{1-\beta}) dF(v_i) + \int_{\tilde{\nu}}^1 \frac{v_i - \tilde{\nu} + \underline{u}}{1-\beta} dF(v_i) \\ &= \int_0^{\tilde{\nu}} (v_i - \tilde{\nu}) dF(v_i) + \int_{\tilde{\nu}}^1 \frac{v_i - \tilde{\nu}}{1-\beta} dF(v_i) + \frac{\underline{u}}{1-\beta} \end{aligned}$$

Since, by definition, $\tilde{\nu}$ solves $\int_0^{\nu}(v_j - \nu)dF(v_j) + \int_{\nu}^1 \frac{v_j - \nu}{1-\beta} dF(v_j) = 0$, the above expression is equal to $\underline{u}/(1-\beta)$. □

Proof of Proposition 3.4.4. From Proposition 3.4.3 we know that consumers do not make any purchases at all if the price is greater than $\tilde{\nu} - \underline{u}$. Let us for now relax this price constraint in the firm's profit maximization problem. The price that maximizes the firm's profit function in (3.7) should satisfy the first-order condition (FOC) and the second-order condition (SOC) as below.

$$\begin{aligned}
\text{FOC: } \pi'(p) &= \frac{1}{1-\beta}[1-\beta^2(F(p+\underline{u})^2 + 2pF(p+\underline{u})f(p+\underline{u}))] = 0 \\
&\Rightarrow F(p+\underline{u})\,[F(p+\underline{u}) + 2pf(p+\underline{u})] = \frac{1}{\beta^2} \\
\text{SOC: } \pi''(p) &= \frac{-2\beta^2}{1-\beta}\Big[f(p+\underline{u})^2 + F(p+\underline{u})(2f(p+\underline{u}) + pf'(p+\underline{u}))\Big] < 0
\end{aligned}$$

We verify the SOC in light of the assumption that $pf(p+\underline{u})$ is increasing in p, or equivalently, $f(p+\underline{u}) + pf'(p+\underline{u}) \geq 0$.

Contemplate the left-hand side of the FOC. Define $\tilde{h}(\beta,p) = 1-\beta^2(F(p+\underline{u})^2 + 2pF(p+\underline{u})f(p+\underline{u}))$, and denote by $\tilde{p}(\beta)$ the price that solves $\tilde{h}(\beta,p) = 0$ (and also the FOC). For any positive β and p we have:

$$\begin{cases}
\frac{\partial\tilde{h}(\beta,p)}{\partial\beta} = -2\beta\Big(F(p+\underline{u})^2 + 2pF(p+\underline{u})f(p+\underline{u}\Big) < 0 \\
\frac{\partial\tilde{h}(\beta,p)}{\partial p} = -2\beta^2\Big(pf(p+\underline{u})^2 + F(p+\underline{u})\big(2f(p+\underline{u}) + pf'(p+\underline{u})\big)\Big) < 0
\end{cases}$$

$$\Rightarrow \frac{\partial\tilde{p}(\beta)}{\partial\beta} = -\frac{\partial\tilde{h}(\beta,p)/\partial\beta}{\partial\tilde{h}(\beta,p)/\partial p} < 0$$

This means that $\tilde{p}(\beta)$, the optimal price for the unconstrained profit-maximization problem, is decreasing in β. Also, note that there exists a $\beta \in (0,1]$ where $\tilde{p}(\beta) = 1-\underline{u}$ solves the FOC. Now, let us consider $\tilde{\nu}-\underline{u}$, which is the maximum price consumers are willing to pay. Via Proposition 3.4.2 we verify that $\tilde{\nu}-\underline{u}$ is increasing in β. Also, $\tilde{\nu}-\underline{u}$ approximates to $1-\underline{u}$ as β approaches 1. Therefore, there exists a unique $\tilde{\beta}(\underline{u})$ where $\tilde{p}\big(\tilde{\beta}(\underline{u})\big) = \tilde{\nu}-\underline{u}$. Below and beyond $\tilde{\beta}$, the optimal prices are characterized by $\tilde{\nu}-\underline{u}$ and $\tilde{p}(\beta)$, respectively. □

Proof of Corollary 3.4.5. With $F(v) = v$, the first-order condition transforms into the following.

$$(p + \underline{u})(p + \underline{u} + 2p) = \frac{1}{\beta^2}$$

The only positive root of this equation is

$$p = \frac{\sqrt{3 + \beta^2 \underline{u}^2}}{3\beta} - \frac{2\underline{u}}{3}$$

which will become the interior optimal price if this price is less than $\tilde{\nu} - \underline{u}$. Otherwise, $\tilde{\nu} - \underline{u}$ maximizes the profit. We obtain $\tilde{\beta}(\underline{u})$ from setting the above price equal to $\tilde{\nu} - \underline{u}$. $\square$

Proof of Proposition 3.5.1.

(i) Consider the following Lagrangian function corresponding to the constrained profit maximization problem (3.15).

$$\mathcal{L}_s = \pi_s(b, p) + \lambda_1(u_s(b, p) - \frac{u}{1 - \beta}) + \lambda_2(u_s(b, p) - u(p)) + \lambda_3(1 - p)$$

Since the utility function (3.11) is linearly decreasing in b, the profit function (3.14) is linearly increasing in b, and there is no upper-bound on b, at least one of the participation and self-selection constraints must be binding at the optimal profit. Thus, at least one of λ_1 and λ_2 should be positive. As a result, we construct the following cases.

Case 1: $\lambda_1 > 0$. In this case the participation constraint is binding. Therefore, b should satisfy the following equality for any p.

$$u_s(b,p) = -b + g(p) = \frac{\underline{u}}{1-\beta}$$
$$\Rightarrow b = \tilde{b}(p) \equiv g(p) - \frac{\underline{u}}{1-\beta}$$
$$\Rightarrow \pi_s(\tilde{b}(p),p) = g(p) - \frac{\underline{u}}{1-\beta} + \frac{\beta p}{1-\beta}(1 - F(p+\underline{u})^2$$

From (3.11) we have

$$\frac{dg(p)}{dp} = \frac{\beta}{1-\beta}(F(p+\underline{u})^2 - 1) \Rightarrow \frac{d\pi_s(\tilde{b}(p),p)}{dp} = \frac{-2\beta}{1-\beta}pF(p+\underline{u})f(p+\underline{u}) < 0.$$

In words, as long as the box price (b) is adjusted in such a way to keep the consumer indifferent between the sample box and the outside option, the firm finds it profitable to decrease the price of the full-size product (p). The firm decreases p to the extent that the consumer will be on the verge of switching to the self-discovery process. This happens when the self-selection constraint becomes binding, and thus, $p_s^* = \tilde{\nu} - \underline{u}$.

Case 2: $\lambda_2 > 0$ **and** $\lambda_1 = 0$. In this case the self-selection constraint is binding. Therefore, b should satisfy the following equality for any p.

$$u_s(b,p) = -b + g(p) = u(p)$$
$$\Rightarrow b = \hat{b}(p) \equiv g(p) - u(p)$$

Then the profit maximization problem (over non-negative values of p) will become:

$$\max_{p} \pi_s(\hat{b}(p), p)$$

$$\text{s.t.}$$

$$u_s(\hat{b}(p), p) = u(p) \geq \frac{\underline{u}}{1-\beta} \Longleftrightarrow p \leq \tilde{\nu} - \underline{u}$$

Let us for now consider the unconstrained problem, relaxing the upper-bound on p.

$$\text{FOC: } \frac{d\pi_s(\hat{b}(p), p)}{dp} = \frac{1 - \beta F(p+\underline{u})(\beta(p+\underline{u}) + 2pf(p+\underline{u}))}{1-\beta}$$

We know that the problem does not have any corner solutions, since $p = 0$ is suboptimal for the firm, and at $p = 1$ consumers prefer the outside option to participation. The necessary condition for having an interior solution is

$$h(\beta, p) \equiv 1 - \beta F(p+\underline{u})\Big[(p+\underline{u})\beta + 2pf(p+\underline{u})\Big] = 0.$$

Since $pf(p+\underline{u})$ is increasing in p, $f(p+\underline{u}) + pf'(p+\underline{u}) \geq 0$. As a result, for any positive β we have:

$$\frac{\partial h(\beta, p)}{\partial p}$$
$$= -\beta\Big[f(p+\underline{u})\big(\beta(p+\underline{u})+2pf(p+\underline{u})\big)+F(p+\underline{u})\big(\beta+2f(p+\underline{u})+2pf'(p+\underline{u})\big)\Big] < 0$$

$$\frac{\partial h(\beta, p)}{\partial \beta} = -2F(p+\underline{u})\Big[\beta(p+\underline{u}) + pf(p+\underline{u})\Big] < 0$$

Let $\hat{p}(\beta)$ solve $h(\beta, p) = 0$. The former condition above guarantees that this solution is unique. The solution to the constrained profit maximization problem will then become $p_s^*(\beta) = \min\{\hat{p}(\beta), \tilde{\nu} - \underline{u}\}$. From the latter condition above we infer

$$\frac{d\hat{p}(\beta)}{d\beta} = -\frac{\partial h(\beta, p)/\partial \beta}{\partial h(\beta, p)/\partial p} < 0.$$

We verify that there exists a $\beta \in (0, 1]$ where $p = 1 - \underline{u}$ solves $h(\beta, p) = 0$. Also the asymptotic value of $\tilde{\nu} - \underline{u}$ is $1 - \underline{u}$ as β approaches 1. Because $\hat{p}(\beta)$ is decreasing in β and $\tilde{\nu} - \underline{u}$ is increasing in β, there exists a $\hat{\beta}(\underline{u})$ such that

$$p_s^*(\beta) = \begin{cases} \tilde{\nu} - \underline{u} & \beta \leq \hat{\beta}(\underline{u}) \\ \hat{p}(\beta) & \beta > \hat{\beta}(\underline{u}) \end{cases}.$$

Note that The first piece of the above function is the solution from case 1, whereas the latter piece is a solution where p_s^* is strictly less than $\tilde{\nu} - \underline{u}$.

(ii) Following a proof-by-contradiction approach, suppose $b_s^* \leq p_s^*$. From part (i) we know that $u_s(b_s^*, p_s^*) = u(p_s^*) \geq \frac{\underline{u}}{1-\beta}$. Also consumers realize v_i via the purchase of either the sample box or the full-size product in the first period. We now consider the discounted expected utility of a consumer at period 2, conditional on v_i. We construct the three following subcases.

(a) $v_i \geq \tilde{\nu}$: In the sequential search process, consumers continue with Product i, so do they in the sample box scenario only if i is better than j, otherwise they switch to the superior Product j.

(b) $p + \underline{u} \leq v_i < \tilde{\nu}$: In the sequential search process, consumers switch to Product j, which only happens in the sample box scenario only if $v_j > v_i$.

(c) $v_i < p + \underline{u}$: In the sequential search process consumers switch to Product j, only happening in the sample box scenario when $v_i \geq p + \underline{u}$.

In all the above cases, the discounted expected utility of a consumer in period 2 is strictly greater in the sample box scenario than the sequential search. Also, in the first period, $E[v_i] - p_s^* = \mu - p_s^* \leq E[\frac{v_1+v_2}{2} - b_s^* = \mu - b_s^*$. Therefore, $u_s(b_s^*, p_s^*) > u(p_s^*)$, which contradicts the equality of the utilities established in part (i) of the proposition.

□

Proof of Corollary 3.5.2. The first-order condition specified by equation (3.17) in Proposition 3.5.1 is written as the following, when $F(v) = v$.

$$1 - \beta^2(p + \underline{u})^2 + 2\beta p(p + \underline{u}) = 0$$

The only positive root of the above equation is $p = \frac{\sqrt{1+\frac{2}{\beta}+\underline{u}^2} - \underline{u}(1+\beta)}{2+\beta}$. Furthermore, we obtain $\hat{\beta}(\underline{u})$ from equating this root with the first piece of the optimal price in (3.15), which becomes $\frac{1-\sqrt{1-\beta}}{\beta} - \underline{u}$ for the uniform distribution. □

Proof of Proposition 3.5.3.

(i) We follow an approach similar to the one we used to prove Proposition 3.5.1. Consider the following Lagrangian function corresponding to the constrained optimization problem stated in (3.22).

$$\mathcal{L}_c = \pi_c(b, p, \delta) + \lambda_1\big(u_c(b, p, \delta) - \frac{u}{1-\beta}\big) + \lambda_2\big(u_c(b, p, \delta) - u(p)\big) + \lambda_3(1-p) + \lambda_4(p-\delta)$$

Since the utility function (3.19) is linearly decreasing in b and the profit function (3.21) is linearly increasing in b, at least one of λ_1 and λ_2 must be positive. Consequently, we consider the following two cases.

Case 1: $\lambda_1 > 0$

In this case, the participation constraint is binding. We have:

$$u_c(b,p,\delta) = -b + g_c(p,\delta) = \frac{\underline{u}}{1-\beta} \Rightarrow b = \check{b}(p,\delta) \equiv g_c(p,\delta) - \frac{\underline{u}}{1-\beta} \Rightarrow$$
$$\pi_c\big(\check{b}(p,\delta),p,\delta\big) = g_c(p,\delta) - \frac{\underline{u}}{1-\beta} + \beta(p-\delta)\big[1-F(p-\delta+\underline{u})^2\big] + \frac{p\beta^2}{1-\beta}\big[1-F(p+\underline{u})^2\big]$$

Define $\check{z}(p,\delta) = \pi_c\big(\check{b}(p,\delta),p,\delta\big)$, which is the firm's profit as a function of p and δ when the sample box price is adjusted to ensure that the participation constraint is binding. For any positive p we have:

$$\frac{\partial \check{z}(p,\delta)}{\partial p} = -2\beta\Big[\frac{\beta p F(p+\underline{u}) f(p+\underline{u})}{1-\beta} + (p-\delta)F(p-\delta+\underline{u})f(p-\delta+\underline{u})\Big] < 0$$

Therefore, as long as the participation constraint is binding, the firm has an incentive to decrease p to the extent that either the self-selection or $\delta \le p$ becomes binding. We consider the following two subcases, each corresponding to one of these binding constraints.

Subcase 1-1: self-selection constraint binding $(u_c(b,p,\delta) = u(p))$

When both participation and self-selection constraint are binding, consumers become indifferent between self-discovery and the outside option, and thus, $p = \tilde{\nu} - \underline{u}$. For any $\tilde{\nu} - \underline{u}$ we have

$$\frac{d\check{z}(\tilde{\nu}-\underline{u},\delta)}{d\delta} = 2\beta(\tilde{\nu}-\underline{u}-\delta)F(\tilde{\nu}-\delta)f(\tilde{\nu}-\delta) > 0,$$

which means that the firm benefits from increasing the future credit value until it reaches $p = \tilde{\nu} - \underline{u}$.

Subcase 1-2: full price discount ($\delta = p$)

$$\frac{d\breve{z}(p,p)}{dp} = \frac{-2p\beta}{1-\beta}F(p+\underline{u})f(p+\underline{u}) < 0,$$

which means that the firm has an incentive to decrease $p = \delta$ so far as it achieves $\tilde{\nu} - \underline{u}$. This result is similar to what we reached in Subcase 1-1.

Case 2: $\lambda_1 = 0$ **and** $\lambda_2 > 0$

When the self-selection constraint is binding, we can obtain the price of the sample box as below.

$$u_c(b,p,\delta) = u(p) \Rightarrow b = \breve{b}(p,\delta) \equiv g_c(p,\delta) - u(p)$$

Define $\breve{z}(p,\delta) = \pi_c\big(\breve{b}(p,\delta),p,\delta\big)$, which is the firm's profit as a function of p and δ when the sample box price is adjusted to ensure that the self-selection constraint is binding.

$$\begin{aligned}
\breve{z}(p,\delta) = g_c(p,\delta) - u(p) + \beta(p-\delta)\big[1 - F(p-\delta+\underline{u})^2\big] + \frac{\beta^2}{1-\beta}p\big[1 - F(p+\underline{u})^2\big] \\
\Rightarrow \frac{\partial \breve{z}(p,\delta)}{\partial \delta} = \frac{\partial g_c(p,\delta)}{\partial \delta} + \frac{\partial}{\partial \delta}\Big(\beta(p-\delta)\big[1 - F(p-\delta+\underline{u})^2\big]\Big) \\
= \beta\big[1 - F(p-\delta+\underline{u})^2\big] \\
+\beta\big[-1 + F(p-\delta+\underline{u})^2 + 2(p-\delta)F(p-\delta+\underline{u})f(p+\underline{u}-\delta)\big] \\
= 2\beta(p-\delta)F(p-\delta+\underline{u})f(p-\delta+\underline{u})
\end{aligned}$$

Therefore, for any given p and $\delta < p$, the firm benefits from increasing δ, as long as the sample box price changes in such a way to keep consumers indifferent between self-discovery and the sample box. Note that, since we keep p constant, the utility from self-discovery does not change, neither does the utility from purchasing the box. Therefore, the firm increases δ to the extent that it reaches p. Given that at the optimal solution $p = \delta$ and the self-selection constraint is binding, we rewrite the firm's profit as a function of p.

$$\pi_c\big(\breve{b}(p,p),p,p\big) = g_c(p,p) - u(p) + \frac{\beta^2}{1-\beta}p\big[1 - F(p+\underline{u})^2\big]$$
$$\text{FOC: } \frac{d\pi_c}{dp} = \frac{1}{1-\beta}\Big[1 - \beta^2 F(p+\underline{u})\big(p+\underline{u}+2pf(p+\underline{u})\big)\Big] = 0$$

Define $\breve{h}(\beta,p)$ as the expression within the brackets in the FOC above, and let $\breve{p}(\beta)$ solve $\breve{h}(\beta,p) = 0$. We skip the rest of the proof to avoid redundancy, since in the same way as we proceeded in the proofs of propositions 3.4.4 and 3.5.1, we can show that $\frac{d\breve{p}(\beta)}{d\beta} < 0$, and also there exists a $\breve{\beta}(\underline{u})$ passed which $\breve{p}(\beta)$ is the optimal price.

(ii) By definition, $\hat{p}(\beta)$ solves the equation in (3.17), and $\breve{p}(\beta)$ solves the equation in (3.24). Below, we verify that the subtraction of the left-hand side of the former equation from the left-hand side of the latter equation is strictly positive for any positive p and $\beta \in (0,1)$.

$$\Big[1-\beta^2F(p+\underline{u})\big(p+\underline{u}+2pf(p+\underline{u})\big)\Big] - \Big[1-\beta F(p+\underline{u})\big((p+\underline{u})\beta+2pf(p+\underline{u})\big)\Big]$$
$$= 2p\beta(1-\beta)F(p+\underline{u})f(p+\underline{u})$$

Thus, $1-\beta^2 F(\hat{p}(\beta)+\underline{u})\big(\hat{p}(\beta)+\underline{u}+2pf(\hat{p}(\beta)+\underline{u})\big)>0$. Since the left-hand side in (3.24) is decreasing in p, we have that $\breve{p}(\beta)>\hat{p}(\beta)$. Moreover, $\breve{\beta}(\underline{u})$ and $\hat{\beta}(\underline{u})$ are obtained from intersecting $\breve{p}(\beta)$ and $\hat{p}(\beta)$, respectively, with $\tilde{\nu}-\underline{u}$, which is increasing in β. As a result, $\breve{\beta}(\underline{u})>\hat{\beta}(\underline{u})$.

□

Proof of Corollary 3.5.4. Since for the uniform distribution of valuations $F(p+\underline{u})=p+\underline{u}$, the FOC in (3.24) will become equivalent to the FOC in (3.9). As a result, in this specific case $p_c^*=p^*$. □

Proof of Proposition 3.6.1. (i) From propositions 3.4.4 and 3.5.3, we have that $\tilde{\beta}(\underline{u})=\breve{\beta}(\underline{u})$ and $p^*=p_c^*$ when $F(v)=v$. Thus, the profit of the firm in period 3 onwards is the same between the two scenarios—no sample box and sample box with future credit. After plugging the optimal prices from corollaries 3.4.5 and 3.5.4 into the profit functions in (3.7) and (3.21) we have that, when $F(v)=v$,

$$\pi_c(b_c^*,p_c^*,\delta^*)-\pi(p^*)=\frac{2-2\sqrt{1-\beta}-\beta+2\beta^2\underline{u}^3}{6\beta}.$$

For any positive $\underline{u}$ and $\beta\in(0,1)$,

$$2>1+\sqrt{1-\beta}\Rightarrow\frac{\beta}{1+\sqrt{1-\beta}}>\frac{\beta}{2}\Rightarrow 1-\sqrt{1-\beta}>\frac{\beta}{2}$$
$$\Rightarrow\frac{2-2\sqrt{1-\beta}-\beta}{6\beta}>0\Rightarrow\frac{2-2\sqrt{1-\beta}-\beta+2\beta^2\underline{u}^3}{6\beta}>0.$$

(ii) We know that $\pi_s(b_s^*, p_s^*) = \pi_c(b_s^*, p_s^*, 0)$. Case 2 in the proof of Proposition 3.5.3 shows that,

$$\exists\, b, \delta > 0 \ \text{ s.t. } \ \pi_c(b, p_s^*, \delta) > \pi_c(b_s^*, p_s^*, 0). \tag{C.1}$$

Moreover, by definition,

$$\forall\, b, p, \delta \ \ ; \ \ \pi_c(b_c^*, p_c^*, \delta^*) \geq \pi_c(b, p, \delta) \tag{C.2}$$

Therefore, $\pi_c(b_c^*, p_c^*, \delta^*) > \pi_s(b_s^*, p_s^*)$.

□

Proof of Proposition 3.6.2. (i) From the proof of Proposition 3.6.2, we have that, when $F(v) = v$,

$$\pi_c(b_c^*, p_c^*, \delta^*) - \pi(p^*) = \frac{2 - 2\sqrt{1-\beta} - \beta + 2\beta^2 \underline{u}^3}{6\beta}$$
$$\Rightarrow \frac{d}{d\underline{u}}\Big(\pi_c(b_c^*, p_c^*, \delta^*) - \pi(p^*)\Big) > 0.$$

In addition, we we can easily verify that $\pi(p^*)$ is decreasing, and thus $\frac{\pi_c(b_c^*, p_c^*, \delta^*) - \pi(p^*)}{\pi(p^*)}$ is strictly increasing, in $\underline{u}$. Therefore, $\frac{\pi_c(b_c^*, p_c^*, \delta^*)}{\pi(p^*)}$ is also strictly increasing in $\underline{u}$.

(ii) When $\beta \leq \hat{\beta}(\underline{u})$, we have that $p^* = p_s^* = \tilde{\nu} - \underline{u}$. For $F(v) = v$ and $\beta \leq \hat{\beta}(\underline{u})$,

$$\pi_s(b_s^*, p_s^*) - \pi(p^*) = \frac{\beta(-3 + \sqrt{1-\beta})}{6(1 + \sqrt{1-\beta})^3} + \frac{\underline{u}}{(1 + \sqrt{1-\beta})^2}$$
$$\Rightarrow \frac{d}{d\underline{u}}\Big(\pi_s(b_s^*, p_s^*) - \pi(p^*)\Big) > 0.$$

In a similar fashion to the proof of part (i), we can show that $\frac{\pi_s(b_s^*, p_s^*)}{\pi(p^*)}$ is strictly increasing in $\underline{u}$ within the specified range of β.

$\square$

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