# Equilibrium Solutions for 2-Dimensional Non-axisymmetric Disks

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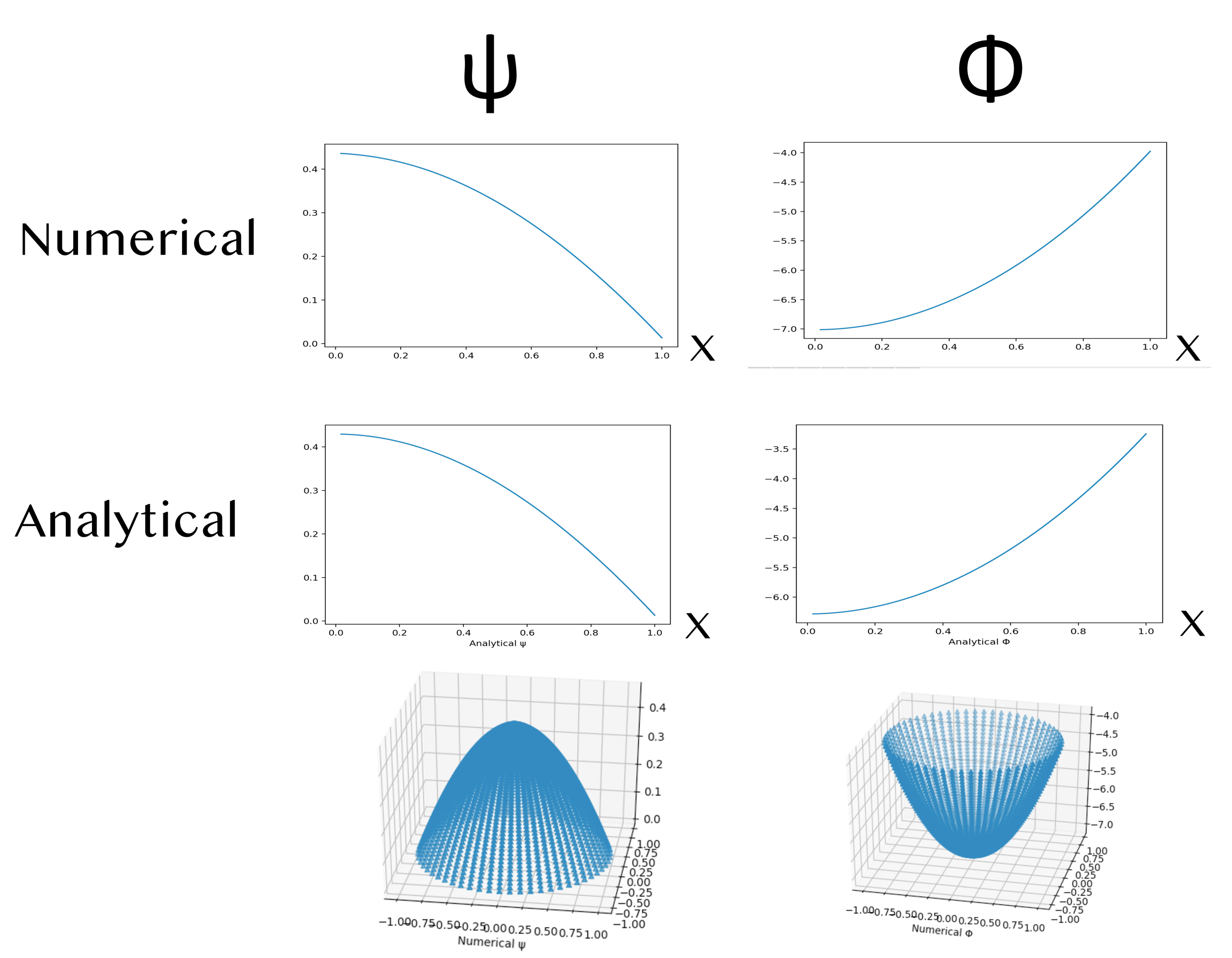
## Introduction

Self-gravitating disks arise in binary star systems and other systems where tidal forces play a large role such as in the Earth-moon system. These disks are governed by a Scalar Momentum Equation (SME) and a partial differential equation describing hydrodynamic flow within the disk (Stream Function Equation). We solve these equations using a self-consistent field approach.

## Research Objective

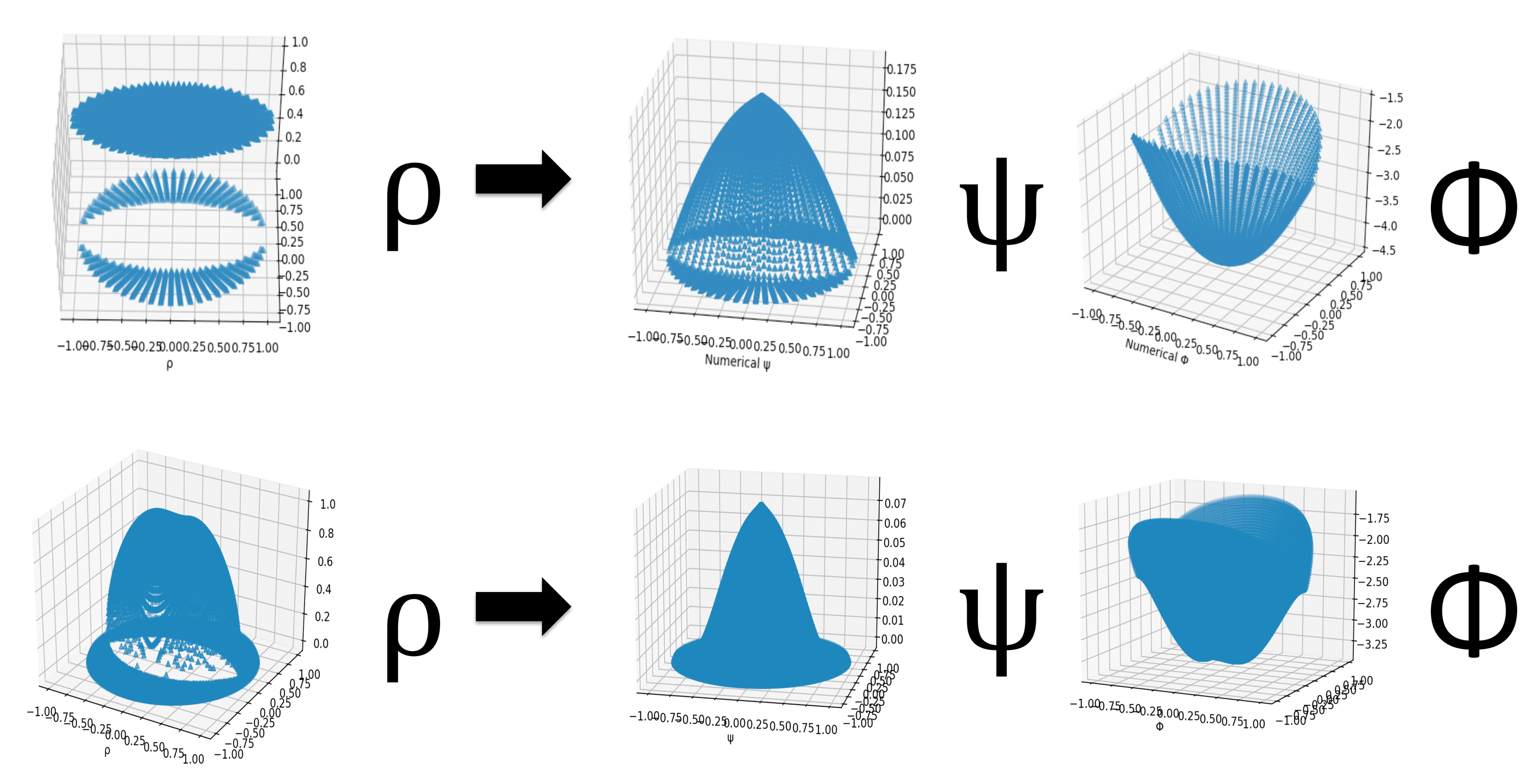
In this study we seek equilibrium (constant in time) solutions to certain non-axisymmetric disks. These disks are parameterized by axis ratio, polytropic index *n*, and vorticity terms $C_0$ and $C_I$.

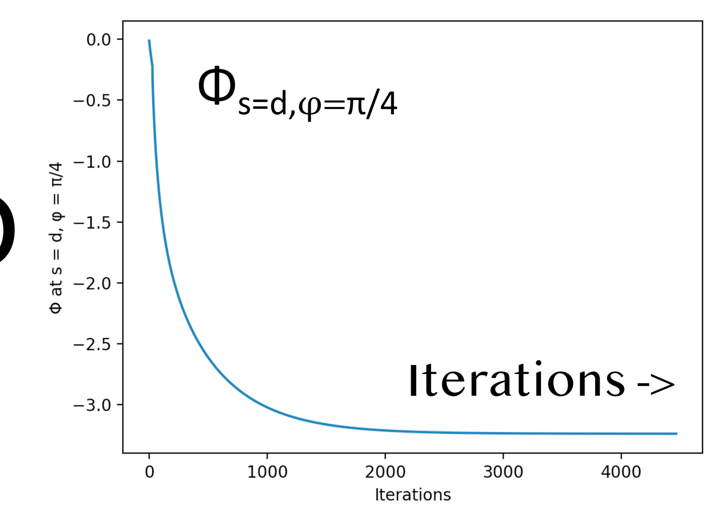
## Analytical Solutions




Comparison of numerical and analytical solutions to the relevant PDEs for the simple case of rotational symmetry and uniform denisty

## Methods




Convergence of each solution is tracked both numerically and graphically in terms of the largest change in any single grid cell between iterations.

ψ = stream function, Φ = gravitational potential, ρ = mass density

We find numerical solutions to discretized versions of the Stream Function Equation and Poisson equations using Gauss-Seidel Algorithms as shown. These algorithms were implemented in python and they converge to approximate values for gravitational potential and velocity fields within the disks.

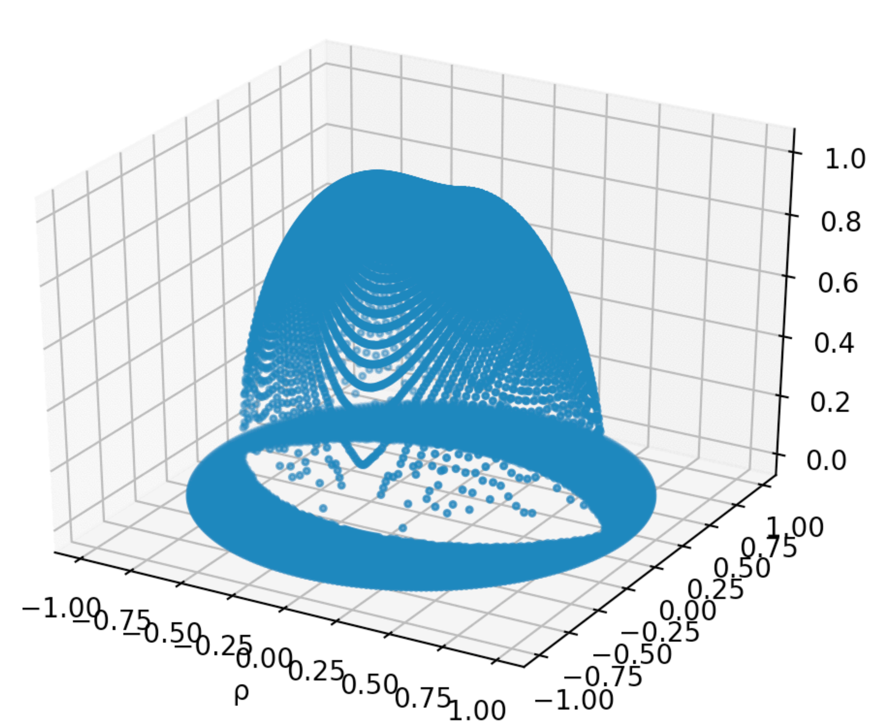
## Equilibrium Solutions

$$C_I = H + \Phi + F(\psi) + \frac{1}{2}V^2 - \frac{1}{2}\Omega^2 s^2$$

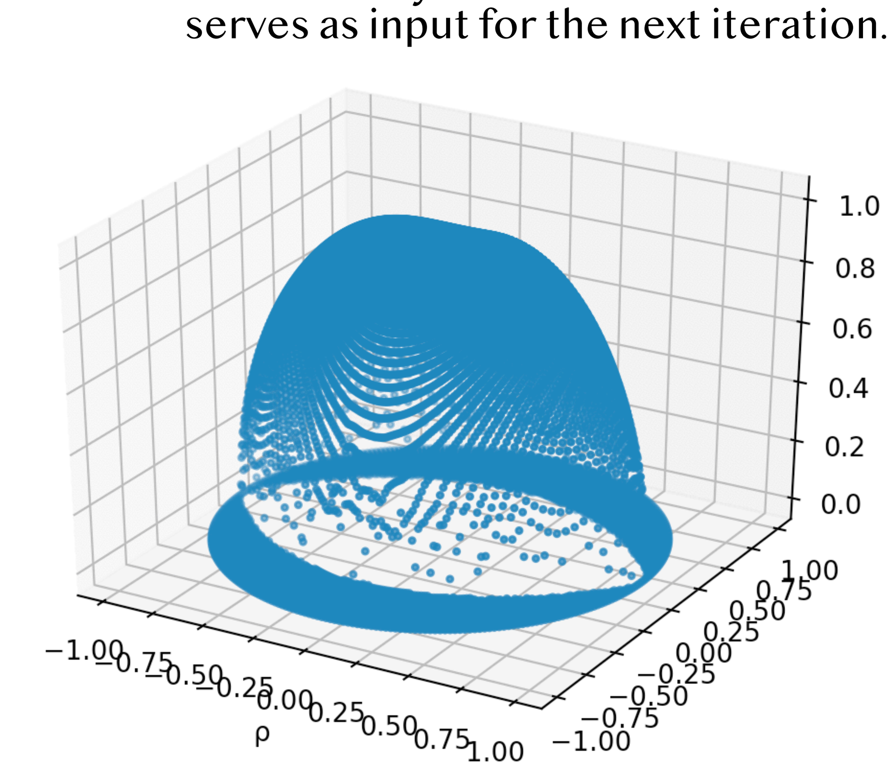
$$F(\psi) = c_0\psi + \frac{1}{2}c_1\psi$$

The disk is ultimately governed by the Scalar Momentum Equation.

Applying appropriate boundary conditions and numerical solutions (see above) we obtain values for $C_I$ and Ω. A new density distribution from these values serves as input for the next iteration.



Final density distribution
$C_I$ = -2.84
Ω = 0.77

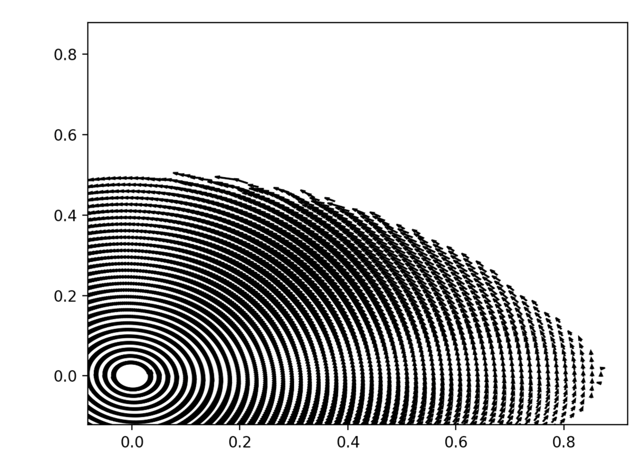


Final density distribution
$C_I$ = -3.22
Ω = 0.70

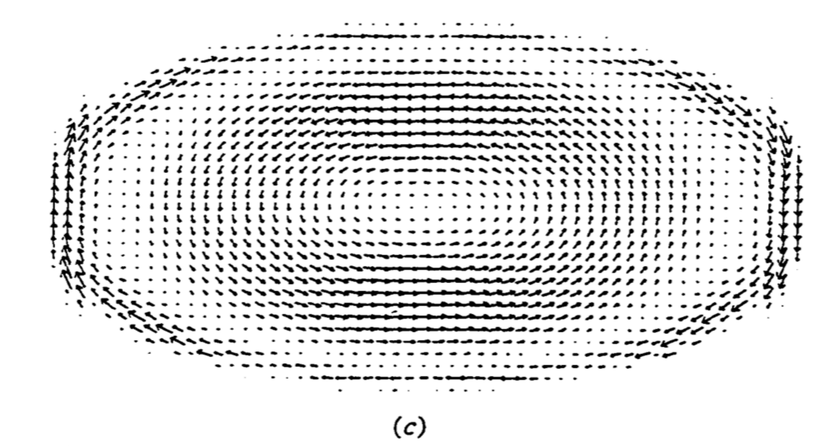
Final density distribution for equilibrium solutions which have converged successfully. Many sets of parameters do not correspond to physical systems, so only a handful of converged models have been found so far.

## Conclusions and Future Research

Parameters for our converged solution were taken from Andalib's 1993 dissertation. So far the resulting density and velocity fields do not match Andlib's results.



Velocity fields for model D16: TOP: Converged Solution. BOTTOM: Plot from Andalib (1993).



Present research is focused on testing and refining the procedure for finding equilibrium solutions, with the goal of analyzing more complex systems.

## Glossary

Ω = frame rotation rate
ψ = stream function
Φ = gravitational potential
ρ = mass density
H = enthalpy
$c_0$, $c_1$ = SFE parameters
s, φ = radial, angular coordinates
$C_I$ = integration value

## References

1. Andalib, S. W. (1998). *The Structure and Stability of Selected, 2-D Self- Gravitating Systems.*
2. Imamura, James (2020). Personal communications.

## Acknowledgments

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