Minus the Math Anxiety: Breaking Down the Barriers Between Students and Mathematics

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The public's opinion on mathematics is overwhelmingly negative. Millions of individuals suffer from a diagnosable condition called math anxiety. There are, however, ways to combat this issue. Evidence-based practices such as explicit instruction have been shown to decrease the load on working memory, something math anxiety limits. By using explicit instruction and other evidence-based practices and high-leverage practices we can reduce math anxiety and increase math achievement. Another compounding issue is the public perceived lack of relevancy of mathematics in schools. This issue can be alleviated by teaching discrete mathematics, a branch of mathematics that has close ties to computer science which is essential in our current society. Teaching python coding concurrently with discrete mathematics only increases the connection to "the real world". Additionally, teaching about diverse individuals in computer science and mathematics, along with other methods of reducing math anxiety and alternative testing methods can further decrease, or make more manageable, the prevalence of math-related anxiety.

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Introduction

Background

The construct of math anxiety was first introduced in 1957 by Dreger and Aiken who said that "Many persons report in clinical sessions and in academic classes that they are emotionally disturbed in the presence of mathematics. It has been the writers' experience that reactions of this nature to mathematical and arithmetical materials constitute a major field of emotionality in academic situations" [A1]. However, people have been expressing math anxiety for centuries. A verse from the sixteenth century says "Multiplication is vexation ... and practice drives me mad". Mathematics education has come a long way since then but math anxiety still exists and is prevalent in our society. Math anxiety is a diagnosable condition characterized by feelings of frustration and anxiety that limit the individual's capability to interact with mathematics in a meaningful way. It has been defined as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" [A2].

Math anxiety seems to consist of multiple components. The two components are worry and emotionality. "Worry is the cognitive component of anxiety, consisting of self-deprecatory thoughts about one's performance. Emotionality is the affective component of anxiety, including feelings of nervousness, tension, and unpleasant physiological reactions to testing situations" [A3]. While the affective part of math anxiety effects the actual performance of the student with math anxiety, the cognitive part influences their daily life and career path; an individual with math anxiety is less likely to pursue careers that require extensive mathematics education. Furthermore, studies have shown that math anxiety increases with age and that women are more likely to suffer math anxiety than men [A4].

One consequence of math anxiety is its limiting effect on working memory. Working memory plays a vital role in mathematical cognition. Working memory capacity is the range of information that an individual can process at the same time to perform complex tasks, and selectively shines on relevant information from one moment to another. Learning new information requires a substantial amount of working memory, something that, if you have math anxiety, is more difficult to procure. For individuals with disabilities or conditions that further limit working memory, the process of learning more information is significantly harder. In addition to limiting working memory, the symptoms of math anxiety include panic, paranoia, passive behavior, and a lack of confidence. Studies have also found that there exists an inverse relationship between math anxiety and self-efficacy [A5]. Here we define self-efficacy as in individual's confidence in their ability to perform mathematics, something which directly impacts motivation to pursue mathematics [A6]. Math anxiety is pervasive, studies have shown it affects anywhere from 6% to 68% of the population depending on the study [A7].

An Ode to Discrete Mathematics

Discrete mathematics emerged as a separate area of study in the 1960s. An essential part of the mathematics behind computer science, it includes combinatorics, graph theory, counting, probability, and more. Unlike other areas of mathematics discrete mathematics allows students to very quickly answer the question "When am I going to use this in the real world?" by using non-trivial "real world" questions that are interesting and challenging. While high school curriculums cover some discrete mathematics, its coverage is limited and depends highly on which school district you are looking at, as the United States has no national or even state-wide curricula. Elements of discrete mathematics are however in the NCTM standards. Discrete mathematics is also featured in many middle school and high school mathematics competitions. While other areas of mathematics like algebra and geometry are done by rote memorization, discrete mathematics requires creative, flexible thinking and problem-solving. In the words of Susan Picker:

> *An important aspect of discrete mathematics is, that it enables students to see mathematics more accurately, as having many problems which are still unsolved. With only a small amount of technical background students can understand enough about a class of problems such as graph colouring to understand that no algorithm exists which will always guarantee the most efficient colouring. It means that students can stand at the cutting edge of mathematical knowledge with the real possibility of making a contribution; something they could not have previously believed.*
>
> *There are a number of other student beliefs about mathematics that discrete mathematics seriously challenges. Such widely held beliefs as mathematics problems can only have one right answer, that doing maths means memorizing formulas and giving them back in the right order at the right time, that most math problems can be solved within minutes, and that mathematics can only be created by geniuses, are disproved in a discrete mathematics class where there may be many answers to a problem, there may be no formula to follow, students may work on a problem over several days, and where students may see something that the teacher has never seen that way before [A8].*

Incorporating Python

Under normal circumstances, discrete mathematics is taught in college after students have had some experience with programming. Discrete mathematics is, however, often taught abstractly and disjointed, without the use of programming. This can make some concepts more difficult to grasp. By incorporating a language such as Python, whose syntax is relatively intuitive and has its similarities to mathematical notation and algorithmic pseudocode, we can teach students abstract concepts in practical ways. Python is a simple but powerful object-oriented programming language used by novices and experts alike. Best of all, it's free. Using Python allows students to study the mathematics not only theoretically, but practically. It also helps answer the question of "When will we use this in real life?" as knowledge of computers is considered essential in this day and age. This is not just a one way relationship, learning discrete mathematics can help to improve coding skills as experts often use discrete mathematics knowledge to approach the design of a problem.

Motivation

I have been a teacher's assistant and tutor for 8 years and in that time I've seen a lot of students with math anxiety. Being able to help them understand math is one of my greatest joys. I created this curriculum as a way to connect to them as I believe that teaching discrete mathematics using effective educational practices may be the key to turning attitudes towards math from fear to understanding. Furthermore, I believe that discrete mathematics it in itself important to teach not only in college but in high school as well. There is a whole other world of mathematics that a large majority of students never get to see because they don't go into computer science. However, discrete mathematics can be very disjoint and hard to understand. This is why I believe it is important to enact these high-leverage and evidence-based practices. I also believe that adding computer science to the curriculum can help the scaffolding of the course to be clearer and more defined.

Educational Practices

Research to Practice Gap

One of the systemic problems that exists in education is the gap between research and practice. While a key goal of research is the improvement of practice, little of said research makes its way into teaching practices in the classroom. This means that, despite the proven effectiveness of the evidence-based and high-leverage practices mentioned here, the practices are not widely implemented. Four conceptual issues, as identified by Broekkamp and van Hout-Wolters, constitute the gap between research and practice [B9]. The first is that educational research yields only few conclusive results. This can be due to a myriad of factors including the complexity of education, the marginal amount of research compared to other applied disciplines, the narrow or one-sided focus of researchers, etc. The second problem is that educational research yields only few practical results. This could be due to inconclusive results or, if unambiguous evidence exists, the results often have no practical value. Other times evidence relates to problems that are too insignificant or too remote from the context of interest. Thirdly, practitioners believe that educational research is not conclusive or practical, an issue that is fed by the previous two issues mentioned. A study by Gore and Gitlin found that teachers dismissed research as they found it to be not "practical, contextual, credible, or accessible" [B10]. Finally, the fourth issue they found was that practitioners make only little (appropriate) use of educational research. This could be partly due to their mistrust of research but could also be caused by the limited fulfillment of the following two conditions. First, that practitioners " - at least some of them - should be trained to access information about research results (e.g., through reading research articles), and to critically evaluate and test them. Furthermore, practitioners should receive support from their organizations for actively using research in the form of time, money, assistance, and ultimately, collaboration" [B9]. The limited fulfillment can also explain the frequent sloppy and inappropriate use of research by practitioners.

Another article views the gap differently. Vanderlinde and van Braak pose that there are two opposing types of knowledge at play here: research-based knowledge, i.e. what is published in scientific journals, and pedagogical knowledge, i.e. the knowledge used in classrooms [B11]. The problem there is the tension formed by the practitioner asking for solutions to current and ongoing issues while the research seeks new knowledge. McIntyre expands on this, saying that another contrasting feature is that practitioners are looking for practical knowledge whereas researchers are prioritizing other values such as clarity and coherence. Furthermore, research must be, to have any value, abstract, theoretical, or generalized while teachers need research that enables them to solve the unique and context-rich problems they are facing in the classroom everyday [B12].

While it is beyond the scope of this thesis to solve this systemic of a problem, possible solutions can be touched on. First, Broekkamp and van Hout-Wolters propose four possible solution models. One possible solution is the Research Development Diffusion Model, where "practice-oriented research expands on fundamental research" [B9]. The model assigns a central role to mediators who make decisions on what to include, discard, or adapt from research for practitioners. The Evidence-Based Practice model describes the "systematic application of research results in educational practice" [B9]. Unlike the RDD model which is diverse in what it includes, the EBP model is concerned only with empirical evidence for effective teaching methods. The model of Boundary-Crossing Practices concerns cooperation between different professional domains towards a common task. Finally, the model of Knowledge Communities believes in intense collaboration between different professional domains, that they profit from each other's expertise and generate new knowledge.

McIntyre suggests that it would be helpful if research is intelligently planned from the beginning with the intention of informing practitioners of ways to adjust their practices [B12]. One interpretation of this, is the suggested use of high-leverage practices. High-leverage practices are "practices that are essential to effective teaching and fundamental to supporting student learning" [B13] They are essential practices designed provide a clear focus for teacher preparation. Despite their proven effectiveness, these practices are not widely used. There are multiple possible explanations for why this is:

1. *Teacher preparation largely occurs in settings that are removed from the practice of teaching, and most coursework emphasizes reflection, investigation, and knowing about practice rather than directly focusing on the use of practices in classrooms (Ball et al., 2009)*
2. *Most teacher education programs have very limited connections between coursework and field*

experiences. When this fragmentation occurs between theory and practice, practice primarily focuses on the 'conceptual underpinnings of teaching as opposed to the concrete practices new teachers may need to enact when they begin teaching – practice is not at the core of the curriculum' (Grossman et al., 2009, p.275).

3. The specific practices that PSTs learn to use are largely left to chance. This occurs primarily because teacher educators expect that practices will be learned during field experiences, the component of their programs over which they have the least control (Grossman & McDonald, 2008). PSTs thus learn practices that they happen to encounter in field settings, and these may or may not be effective practices.

4. Effective instructional practices that are critical to the success of beginning teachers are seldom systematically taught during teacher preparation through closely aligned coursework and field experiences (Forzani, 2014). [B14]

Changes to teacher preparation were proposed in 2010 by the National Council for Accreditation of Teacher Education [B15]. A work group convened by the Council for Exceptional Children developed several high-leverage practices intended to be paired with culturally responsive teaching [B16].

What are Evidence-Based and High Leverage Practices?

According to one article entitled *Unraveling Evidence-Based Practices in Special Education* by Brian G Cook and Sara Cothren Cook, evidence-based practices are practices "that are supported by multiple, high-quality studies that utilize research designs from which causality can be inferred and that demonstrate meaningful effects on student outcomes" [B17]. Evidence-based practices are effective, they create a real difference in student life and performance. It is important to note that no one evidence-based practice will work for everyone. Evidence-based practices originated in the 1990s in the field of medicine. The identification of evidence-based practices soon spread to other field such as nursing, psychology, agriculture, and education. They represent what has the most potential for improving student performance, especially for struggling students who require as effective of instruction as is possible to succeed. The problem with evidence-based practice is a problem previously discussed, teachers have a noticeable lack of faith in research which leads to a lack of implementation of evidence-based practices. What these teachers might not necessarily know is that evidence-based practices go through an evidence-based review which examines the research design, quality of research studies, quantity of research studies, and magnitude of effect [B17].

Teachers currently face higher demands to prepare a diverse range of students for the high levels of performance required of them by colleges or careers. It is essential that whatever time teachers have is spent using the most effective practices possible. However, teachers are often entering schools without sufficient initial preparation for them to have the skills to enact those practices [B18]. Therein lies the problem that high-leverage practices are trying to resolve. Researchers believe that by incorporating these 22 educational practices across four areas (collaboration, assessment, social/emotional/behavioral, and instruction) into teacher preparation and education [B16]. 1-3 are collaboration, 4-6 are assessment, 7-10 are social/emotional/behavioral, and 11-22 are instruction.

Table 1: High-Leverage Practices for K-12 Special Education Teachers

1. Collaborate with professionals to increase student success
2. Organize and facilitate effective meetings with professionals and families
3. Collaborate with families to support student learning and secure needed services
4. Use multiple sources of information to develop a comprehensive understanding of a student's strengths and needs
5. Interpret and communicate assessment information with stakeholders to collaboratively design and implement educational programs
6. Use student assessment data, analyze instructional practices, and make necessary adjustments that improve student outcomes
7. Establish a consistent, organized, and respectful learning environment
8. Teachers provide positive and constructive feedback to guide student's learning and behavior (behavior focus)
9. Teach social behaviors
10. Conduct functional behavioral assessments to develop individual student behavior support plans
11. Identify and prioritize long- and short-term learning goals
12. Systematically design instruction toward a specific learning goal
13. Adapt curriculum tasks and materials for specific learning goals
14. Teach cognitive and metacognitive strategies to support learning and independence
15. Provide scaffolded supports
16. Use explicit instruction
17. Use flexible grouping
18. Use strategies to promote active student engagement
19. Use assistive and instructional technologies
20. Teach students to maintain and generalize new learning across time and settings
21. Provide intensive instruction
22. Teachers provide positive and constructive feedback to guide students' learning and behavior (learning focus)

Explicit Instruction

Explicit Instruction is a highly structured, systematic method that breaks instruction down into small steps. It is direct and uses clear, concise, and consistent language both in designing and delivering instruction. Explicit instruction uses scaffolds, a series of supports that are lessened over time as the students gain independent mastery of the subject in question. Rosenshine (1987) characterized explicit instruction as "a systematic method of teaching with emphasis on proceeding in small steps, checking for student understanding, and achieving active and successful participation by all students" [B19]. Reviews on the effectiveness of explicit instruction include Jere Brophy's and Thomas Good's chapter in the *Handbook of Research on Teaching* entitled "Teacher Behavior and Student Achievment" which reviewed dozens of studies [B20], N.L Gages' and Margaret C Needels' article "Process-Product Research on Teaching: A Review of Criticisms" in *The Elementary School Journal* [B21], and Barak Rosenshine's and Robert Stevens' chapter in the *Handbook of Research on Teaching* entitled "Teaching Functions". Between 2008 and 2009, three government reports were released that recommended the use of explicit instruction: a report by the National Mathematics Advisory Panel in 2008, a 2008 report by the Institute of Education Sciences, and a report in 2009 by the National Center for Education Evaluation and Regional Assistance. Meta-analyses have been conducted in the field of special education by Sharon Vaughn, Russell Gersten, and David J Chard [B22]; and H. Lee Swanson and Maureen Hoskyn [B23], among others.

There are 5 essential components of explicit instruction; explicit instruction should segment complex skills, use modeling and "think-alouds" to draw attention to important content features, use systematically faded supports and prompts, provide frequent opportunities for students to respond and receive feedback, and create purposeful practice opportunities. Other important and commonly used components of explicit instruction include selecting critical content, using logic to sequence skills, ensuring students have the requisite prerequisites and background knowledge necessary for success, presenting a wide range of examples and non-examples, and maintaining a brisk pace [B24]. Though explicit instruction has been viewed through many different lenses and talked about in many different forms, the essential components remain constant. The reasoning on why efficient instruction is so effective is not constant. Researchers have argued that explicit instruction is effective because it reduces the cognitive load and resulting stress on working memory [B25], while others contend that the teaching behaviors align with several applied behavior analysis principles [B26]. Regardless of the reasoning, the truth remains evident, explicit instruction is effective for both special and general education purposes.

Explicit instruction can be taught in many formats. One such format is the explicit instruction cycle. It consists of seven phases: curriculum-based assessment, planning, advance organizer, demonstration, guided practice, independent practice, and maintenance. CBA has evolved over the last few decades as a measurement system that uses data regarding student performance and progress to make data-based decisions which helps to design efficient and effective education. "CBA helps to pinpoint curricular objectives that act as the criteria for the identification of instructional targets and for the assessment of students' skills upon entry into an academic curriculum." [B27] Curriculum-based assessment (or measurement) was originally developed for special education as a way of testing the effectiveness of a intervention model called data-based program modification (DBPM). The University of Minnesota's Institute for Research on Learning Disabilities ran a 6 year long study which ultimately generated a set of criteria which established the technical adequacy, treatment validity, and logistic feasibility of the measures [B28].

CBA is technically adequate, time efficient, and easy to teach. It can be used to improve individual instruction programs, predict performance, screen for at-risk students, enhance instructional planning, reduce bias in assessments, and more [B29]. There are also different kinds of CBAs which accomplish different things. The three types of curriculum-based measurements are survey CBAs, untimed focused CBAs, and timed focused CBAs (also called probes). A survey CBA's purpose is to measure a wide range of concepts and skills. This provides the instructor is a comprehensive view of the student's mathematical ability. It can also help pinpoint individual strengths and weaknesses. Focused CBAs are designed to test very specific concepts or skills. They aim to represent all problem types of a specific skill area in order for the instructor to understand a student's level of understanding of a certain topic area. Untimed focused CBAs allow as much time as needed to complete whereas timed focused CBAs are given a more limited time frame, usually one minute. Probes (timed focused CBAs) aim to develop and measure declarative knowledge by testing fluency and recall [B30].

The next phase of the explicit teaching cycle is the planning phase. There are two key principles that go into planning the lesson: data-based decision making and instructional alignment. Data-based decision making entails using the information and data gathered during the curriculum-based assessment to inform the decisions made in planning the lesson. There are multiple types that teachers can draw on to make data-based decisions: input data, output data, process data, and context data. Input data is data on student or teacher characteristics. Outcome data is the data regarding student achievement and well being. Process data is data about instruction and types of assessments. Context data is on the school culture, building, or the curriculum. Most commonly, teachers use outcome data to make decisions [B32]. Depending on the type of CBA used, you will learn different outcome data. For example, survey CBAs help make decisions about the appropriate place in the curriculum and how to group students according to instructional needs. Regardless of the type or purpose of the CBA, an important step in data-based decision making is, when looking at outcome data, determining whether or not the student in question has met the criteria for the rate of accuracy.

The other key principle to take into consideration when planning lessons is instructional alignment. "Instructional alignment exists when there is a match in the stimulus conditions of intended outcomes, instructional processes, and instructional assessment" [B31]. This means that what a teacher is wanting to teach must match the activities or worksheets the students complete. There are three components of instructional alignment: objectives, instruction, and evaluation. Teachers should decide, when planning the lesson, what the core objectives of that lesson are. This lets them choose worksheets and activities that follow the same objectives. In keeping instruction in mind, they should take care in choosing the correct instructional strategies to match the context of the lessons. Teachers should also share the objectives with their students as students can gain autonomy by knowing the rationale of a particular activity or lesson. The final component of instructional alignment is evaluation. Evaluations are important for a myriad of reasons, one of which is enhancing student motivation by letting students track their progress. It is essential that evaluations cover material and objectives explicitly learned in class [B33].

The next step in the cycle to consider is the advance organizer. The purpose of an advance organizer is to link new ideas to existing knowledge and thereby enable students to learn new ideas and information. "It is designed to cue the relevant prior knowledge of a learner and it is usually presented at a higher level of abstraction, generality, and inclusiveness than that of the planned lesson" [B34]. This manifests as teachers asking questions, presenting an outline, or linking new knowledge to previously learned material. Advance organizers can take many forms, whether they be charts and diagrams, or concept maps. There are two types of advance organizers: expository and comparative. "Expository organisers function to provide the learner a conceptual framework for unfamiliar material, and comparative organisers are used when the knowledge to be acquired is relatively familiar to the learner" [B34]. Additionally, there are three phases of advance organizer teaching. The first stage is presentation of the advance organizer and consists of introducing the goals of the lesson, the topic, and the materials required. It is important that it provides the framework for the learning that later takes place. The next step is the presentation of the actual material, paying special attention to the ordering of the material and what it means to the students. Student participation is important at this juncture. The last stage consists of the teacher asking questions to cement the knowledge and extend the thinking of the students.

Also included in advance organizer teaching are some necessary components. One such component is review. An advance organizer should include review because it allows teachers to check that the students know the necessary prerequisite knowledge and prepares the students for success in the oncoming lesson. An important question to ask is what skills, knowledge, or concepts are needed for the students to obtain success in the lesson. It is helpful to consult a scope and sequence chart to determine the concepts necessary to review. The review should be brief and engage all students in order to get an accurate view of each student's preparedness for the lesson. The teacher should be continuously checking student progress, providing feedback, and assessing whether to move onto the lesson or continue review. Another such component to include in advance organizer teaching is stating the lesson objective and linking it to prior knowledge. The statement of the lesson objective should be brief and contain simple language. One last component to include is the development of relevance. It has been shown that when students know why they are learning a new skill, motivation and learned are elevated [B30].

There are several strategies involved in the next phase of the explicit teaching cycle, demonstration. During demonstration, it is important to incorporate teacher modeling. Teacher models should use clear, consistent, and concise language and be unambiguous in their explanations. They should show the students what they are learning and how to apply it [B35]. Critical aspects of teacher modeling includes the use of clear language, inclusion of student involvement, and sufficient number of examples to promote real understanding. When determining the number of examples, teachers should examine the difficulty of the task, the knowledge brought to the classroom by students, and student response to instruction. One way to implement teacher modeling is by using think-alouds. "The think-aloud is a technique by which the individual voices her or his thoughts during the performance of a task" [B36]. It is as simple an idea as it sounds, someone explaining their actions out loud while doing a task. Think-alouds help guide students through making critical connections in their thinking and generalizing the information learned to other applications. They "provide students with an opportunity to witness the expert thinking that is typically hidden and thus an abstract concept to the students" [B37].

Another important strategy is maximizing student engagement. One definition of student engagement says that student engagement is multifaceted and is defined under three aspects: behavioral engagement, emotional engagement, and cognitive engagement. Behavioral engagement "draws on the idea of participation" [B38]; it is considered essential for obtaining a positive educational outcomes and includes the participation in academic, social, and extracurricular activities. Emotional engagement is all about creating ties to the school and supplying the motivation to complete academic tasks. It encompasses reactions, both positive and negative, to a myriad of factors including teachers, academics, classmates and the school itself. Finally, cognitive engagement "draws on the idea of investment" [B38] and incorporates the motivation to exert the necessary effort to master complex skills and ideas. The three types of engagement are interrelated and constitute a dynamic process that has its influences on learning and achievement. Another definition of student engagement is as follows: "the student's psychological investment in and effort directed toward learning, understanding, or mastering the knowledge, skills, or crafts that academic work is intended to promote" [B39].

An article on student engagement identified four motivational constructs which represent major constructs in the field of achievement motivation. The first construct, belongingness, is about the necessity of establishing meaningful connections and relationships to others. Belongingness theory says that humans have a "pervasive drive to form and maintain at least a minimum of lasting, positive, and significant interpersonal relationships" [B40]. When applied to education and school achievement, the premise of belongingness theory is that "caring and supportive relationships facilitate student engagement and other adaptive school behaviors" [B41]. Instructional practices that offer opportunities to foster belongingness are demonstrating and encouraging mutual respect and teaching students to work together in a productive manner. The next construct is competence, the necessity for humans to meet one's goals successfully and be successful in their interaction with their environment. One theory of competence is self-efficacy. Self-efficacy means a person's belief that they can be successful when completing a certain task or achieving a certain goal. Studies show that "students who feel efficacious about learning tend to be competent and engaged and are likely to set learning goals, use effective learning strategies, monitor comprehension, evaluate goal progress, and create supportive environments" [B42]. Teachers can help increase competence by showcasing that mistakes provide information, thusly encouraging student persistence. The use of scaffolding, feedback, and appropriately challenging assignments can also help foster feelings of competence [B43].

Autonomy, the third construct, is a psychological need. It is a need to behave and make decisions and choices according to one's own values and interests. As for the effectiveness of nurturing autonomy to increase student engagement, one study showed that "teachers' autonomy support was an even better predictor of students' classroom engagement than was students' own engagement during an earlier class" [B44]. Strategies important for nurturing autonomy include using language that isn't controlling, providing rationales for tasks, acknowledging and respecting student perspectives, and allowing students to debate and ask questions freely. One study said that teachers who were autonomy supportive "(a) adopt the students' perspective; (b) welcome students' thoughts, feelings, and behaviors; and (c) support students' motivational development and capacity for autonomous self-regulation" [B45]. The final construct, meaningfulness, is the only one of the constructs that is not considered a basic human need. When learning is meaningful, it involves the development of interest or the appreciation of the topic being learned. To foster meaningfulness, one study recommends building on the prior knowledge of students, emphasizing your own interest in the topic, invoke universal human experiences, and offer opportunities for students to think complexly and participate in conversations with fellow peers to achieve shared understandings. [B43]

One additional strategy to promote student engagement is to provide opportunities to respond (OTR). Effective delivery is characterized by three aspects: type of delivery, rate of presentation, and method of response. OTRs are a subset of teacher questioning which features two types of questions: fact and higher order (cognitive). Fact questions require mere regurgitation of information, while higher order questions require independent thinking. Various modes of responding can also be provided. Students could respond individually or by choral responding (as a group). Another strategy that promotes student engagement is providing feedback. This has the added benefit of also increasing student achievement. Feedback can be positive or negative and timely feedback helps to cement concepts in student minds and correct any errors that arise. Feedback can also help regulate student behavior. While feedback can be positive or negative, the ratio between the two should heavily favor positive feedback. Increasing positive feedback has been shown to increase productivity and accuracy. A myriad studies concluded that the ratio of positive to negative feedback should be at least 4:1 [B37]. Additional effective strategies for questioning include creating a classroom culture open to dialogue; using both preplanned and emerging questions; selecting an appropriate level of questions based on learners' need; avoiding trick questions and those that require only a yes or no response; phrasing questions carefully, concisely, and clearly; addressing questions to the group or to individuals randomly; using sufficient wait time; responding to answers given by students; deliberately framing questions to promote student interest; and using questions to identify learning objectives for follow-up self-study. [B46]

The next stage in the explicit instruction cycle is guided practice. Here we make use of a practice called scaffolding. Scaffolding supports means to "provide temporary assistance to students so they can successfully complete tasks that they cannot yet do independently and with a high rate of success" [B47]. The concept behind guided practice is to start with a high level of support and to slowly decrease support as the students become more capable of independently solving the problems until you can withdraw support completely without a loss of success. Worked solutions is a primary example of this strategy. They are "matched carefully to students' current understanding of learning objectives and are systematically faded to include fewer supports as students learn to solve the problems independently" [B48]. In addition to algebraic problems and skills, scaffolding can also be used to teach higher level cognitive strategies. Above mentioned think-alouds are a scaffold that can be used to teach such skills and strategies. It is essential that during this process of scaffolding, that feedback, both positive and constructive, is frequently given. Student engagement is also crucial at this stage.

Especially during the instruction of higher level cognitive skills, but also generally, it is important to consider the load on students' working memory. Working memory is a "processing resource of limited capacity, involved in the preservation of information while processing the same or other information. Tasks that measure [working memory] assess an individual's ability to maintain task-relevant information in an active state and to regulate controlled processing" [B49]. The capacity of working memory is typically described as the amount of information that an individual can process while also performing complex tasks. The functioning of working memory is akin to a "spotlight" that shines selectively on relevant information from one moment to the next. Ineffective functioning of working memory can lead to overloading of information and forgetfulness. The use of clear, concise, and consistent language as well as activation of prior knowledge, scaffolding instructional supports, frequent review, and sufficient practice time with visual aids and feedback, when well integrated, are ideal for facilitating working memory [B50]. Guided practice, with adequate use of visual aids, is one way to facilitate working memory.

As students reach independency in their ability to complete problems using a certain skill or strategy, you can transition into independent practice. The goal of independent practice is to "develop unitization of the strategy, that is, the blending of elements of the strategy into a single, unified whole" [B51]. It also aims to decontextualize learning so the strategy can be applied easily and with little working memory. You want to build and maintain skill proficiency. It is important at this stage to try to limit prompting, with the intention of using the students' performance at this stage to assess their understanding of the topic or skill. Independent practice can take many forms including homework, in-class worksheets, quizzes, etc.. It is important that independent practice, regardless of form be distributed, i.e. take place over a longer period of time. This helps to cement skills, entering them into long-term memories. The practice that takes place when learning is occurring, during demonstrations and guided practice, is called massed practice and is equally as important. The goal for independent practice is to move on when 80% of students are accurate 80+% of the time. If students are not achieving that accuracy level, it may be necessary to reteach the specified skill.

The last stage in the explicit instruction cycle is maintenance, which refers to the student's continued ability to accurately complete mathematics problems related to a certain skill or strategy. It is the stage in which distributed practice takes place. It is also the stage where you make data-based decisions to change, adjust the curriculum, or reteach a certain skill or strategy. Data-based decision making should also occur at significant stages during instruction. These stages include when to move from review to demonstration, when to move from demonstrations to guided practice, and when to move from guide practice to independent practice. Data collected should not be only acquired from tests but rather come from four different places: input data is data about demographics of the student population, outcome data is data such as student test scores and well-being, process data is data about the quality of instruction, and context data is about policies and resources. Good data-based decision making should use data from all four types.

Data on their own, however, do not provide judgement, interpretation, or basis for action; they need to be transformed to be useful. This process of transforming data should always begin with a clear purpose as to why it's being sought. This makes it possible to know what data should be collected. After data is collected, the analyzing of the data can begin. Analyzing consists of "contextualizing, categorizing, calculating, connecting, and/or summarizing the data in a way that meets the purpose"[B52]. The next step is to interpret the data. This consists of trying to make sense of what the data means and their implications for the future and any action that is taken. Once all of that is clear, then you can take action on the data. Overall, the steps in data-based decision making should be to 1.) Using data, determine the instructional needs of individual students, 2.) Define the SMART (Specific, Measurable, Attainable, Realistic, Time-bound) and challenging goals, 3.) Based on the information in Steps 1 and 2, decide which instruction approach is most promising to achieve those goals, and 4.) Execute the strategy proposed in Step 3. [B53]. For data-based decision making that takes place you can just use data to decide whether or not to move on. This four-step process is designed for the maintenance stage of the explicit instruction teaching cycle.

Logistics

Diverse Individuals in Mathematics & Computer Science

"Despite decades of effort to increase the participation of women and people from underrepresented minority groups (URM) in STEM majors and careers, and despite the increasing diversification of the US as a whole, participation in STEM majors and careers remains stubbornly male and white" [C56]. There are a myriad of possible reasons as to why this is the case but one possible reason is that they don't see women and URM in STEM careers. One potential solution is to provide students with a list of women and URM in the field of mathematics now and in the past. The following are examples of some diverse individuals in computer science and mathematics.

Muhammad ibn Musa al-Khwarizimi

The words algebra and algorithm stem from one man, Muhammad ibn Musa al-Khwarizimi, an Persian polymath born around A.D. 780 in what is now Uzbekistan. Al-Khwarizimi accomplished most of his work between 813 and 833, part of which was spent working in Baghdad in the House of Wisdom. He was commissioned by the caliph (spiritual head of Islam) to do research and translate the treatises of Greek mathematicians. Al-Khwarizimi's work on elementary algebra in 820 included the writing of a book entitled "The Compendious Book on Calculation by Completion and Balancing", which was where the term algebra derives from. The term algorithm originates from the title of another of his books which was on Hindu-Arabic numerals and arithmetic entitled "Al-Khwarizimi Concerning the Hindu Art of Reckoning" and written in 820. Al-Khwarizimi also wrote a another book entitled "The Image of the Earth" which is based on *Geography* of Ptolemy but used improved values. In his book "The Compendious Book on Calculation by Completion and Balancing" Al-Khwarizimi talks about two mathematic operations: al-jabr which means restoring or completion and al-muqabala which means balancing. Al-jabr is the process that consists of removing negatives (whether they be units, roots, or squares) by adding the same quantity to each side. Ex.

$$x^2 = 40x - 4x^2$$

becomes

$$5x^2 = 40x$$

Al-muqabala is about taking all of the terms of the same type and bringing them to one side of the equation. Ex.

$$50 + 3x + x^2 = 29 + 10x$$

becomes

$$21 + x^2 = 7x$$

"The Compendious Book on Calculation by Completion and Balancing" also features lattice multiplication, something Al-Khwarizimi is credited with developing. [C57, C58]

Winifred "Tim" Alice Asprey

Asprey was born on April 8th, 1917 and died on October 19th, 2007. She was an American mathematician and computer scientist. Asprey was one of only 200 or so women to earn a PHD from an American university during the 1940's. She was responsible for bringing Vassar College its first computer in 1967 making it only the second college in the United States to acquire a IBM System/360 computer. As an undergraduate, Asprey attended Vassar College, where she met Grace Hopper also known as the "First Lady of Computing". She then taught at a myriad of different private schools in New York City and Chicago before obtaining her masters in 1942 and doctorate in 1945 from the University of Iowa. After which, Asprey returned to Vassar college as a professor, where she stayed for 38 years. There, she campaigned tirelessly for Vassar College to have its own computer, succeeding in 1967. Alongside her work with Vassar college, Asprey conducted research at IBM. Asprey retired from Vassar in 1982 and in 1989 Vassar's Asprey Advanced Computation Laboratory was named after her. [C59, C60]

Enriqueta González Baz y de la Vega

González Baz was born on September 22nd in Mexico City. She studied to be a teacher at Escuela número 8 for women before being sent by her father to Escuela Doméstica for domestic studies. There, González

Baz caught the attention of one of her teachers, Elena Picazo de Murray, who encouraged González Baz to pursue higher education. In 1994, at age 29, she became the first women to earn a degree in mathematics from the National Autonomous University of Mexico where she wrote her thesis on Special Functions (i.e. Bessel, Gama, and Legender). González Baz completed her postgraduate work in Philadelphia at Bryn Mawr College. González Baz went on to teach in various secondary schools including the National Preparatory School. Additionally, she was a researcher at the Institute and taught mathematics at the Faculty of Sciences. González Baz was also one of five founding women who founded the Mexican Mathematical society. in 1943 She died on December 22$^{\text{nd}}$, 2002. [C61, C62]

Farida Bedwei

Farida Nana Efua Bedwei is a software engineer from Ghana. She was originally born in Lagos, Nigeria where she was diagnosed with cerebral palsy a day after she was born. Farida Bedwei is the cofounder of a financial technology company called Logiciel. She is also known for her accomplishments in the fields of software architecture and deploying mobile services. She has won several awards including the Winner of Most Influential Women in Business and Government Award Financial Sector. She is also a member of The Girls in ICT Committee, a group dedicated to encourage the increase of women in IT careers. Farida Bedwei also created a superhero named Karmzah who has cerebral palsy and gets her power from her crutches. [C63]

Manjul Bhargava

Born to an Indian family, his mother Mira Bhargava was his mathematics first teacher. Manjul Bhargava was valedictorian of his high school before earning his BA at Harvard. He completed his doctoral dissertation at Princeton where he teaches today. Manjul Bhargava has earned over 10 awards and prizes including the Field's Medal which is one of the highest prizes a mathematician can earn; it is often described as the Nobel Prize of Mathematics. Manjul Bhargava is also the third youngest full professor in the history of Princeton. [C64]

Anita Borg

American-born computer scientist Anita Borg got her first programming job in 1969 at age 20. She was awarded her PHD in 1981 by New York University. Borg founded the Institute for Women and Technology and the Grace Hopper Celebration of Women in Computing. Her goal was to have 50% representation for women in computing by 2020. The first Grace Hopper Celebration of Women in Computing took place in Washington DC in June of 1994 and brought together over 500 women in computer science. Anita Borg won many awards and wa recognized by Bill Clinton who appointed her to the Presidential Commission on the Advancement of Women and Minorities in Science, Engineering, and Technology. [C65]

Kimberly Bryant

Kimberly Bryant was born in Memphis, Tennessee in 1967. A scholarship student to Vanderbilt University she majored in Electrical Engineering with minors in Computer Science and Mathematics. Bryant is best known for her founding of Black Girls Code in 2011. Inspired to create a better life for her daughter who was interested in coding Bryant hopes that Black Girls Code will help young women from minority populations to stay in STEM. Black Girls code teaches computer programming to young women via after school and summer programs. The nonprofit organization has a goal of teaching one million black girls to code by the year 2040. [C66]

Stephanie Castillo

Stephanie Castillo is the executive director of Latin Girls Code. Latin Girls Code is a Chicago based program which seeks to provide education and resources on technology to interested young Latina women, from ages 7-17. Seeking to teach not only technology but also entrepreneurial skills, the group holds workshops, hackathons, and week-long programs. In addition to teaching, Castillo also provides advice as an immigration advisor as many of the young women who are interested in technology are also undocumented. [C67]

Lynn Conway

Lynn Conway was born in Mount Vernon, New York in 1938. As a child she was shy and experienced gender dysphoria. Conway entered MIT in 1955 but dropped out due to a failed gender transition in 1957-58. Lynn Conway worked for several years as an electronics technician before attending Columbia University's School of Engineering and Applied Science, earning degrees in 1963. Recruited by IBM, Conway worked on an architecture team designing an advanced supercomputer. On hearing of Harry Benjamin's work on gender transitions Conway sought out transitioning for the second time. On hearing of the intent to transition, Lynn Conway was fired from IBM. Upon completing her transition in 1968, Lynn Conway began a new life working at several different tech companies including Computer Applications, Inc, Memorex, Xerox PARC, and DARPA. Conway even served as a visiting associate professor at MIT. After going public with her story in 2000, Conway became a transgender activist. 52 years after being fired from IBM, IBM apologized and warded Conway with the rare IBM Lifetime Achievement Award. [C68]

Marian Croak

Marian Croak grew up in New York City. She attended Princeton University before obtaining her doctoral degree from University of Southern California in 1982. Directly after graduating from USC she began work at Bells Labs, part of AT&T. After over 30 years working at AT&T, Marian Croak left AT&T to join Google as their Vice President for Engineering. One remarkable thing about her is that she holds more than 200 patents. More than 100 of those patents have to do with Voice over IP which has to do with the movement of voice communications and other multimedia sessions over the internet. Marian Croak was inducted into the Women in Technology Hall of Fame in 2013. [C69]

Shakuntala Devi

Shakuntala Devi is an Indian mathematician also known as the "Human Computer". Her father worked in the circus but quit when he realized what she could do. Devi could memorize large numbers, something her father discovered when teaching her a card trick at the tender age of three. Her father took her on road shows to exhibit her unusual gift for calculation. Devi toured Europe in 1950 and showcased her abilities in New York in 1976. In 1977 Devi calculated the 23rd root of a 201-digit number in 50 seconds at Southern Methodist University. The answer was 546,372,891 which was confirmed by the UNIVAC 1101 in the US Bureau of Standards. A special program had to be made in order to perform such a large calculation, and the speed at which the calculation was done was less than the time it took for Devi to do the same. In 1980 she made the Guinness Book of World Records when she calculated the multiplication of two 13-digit numbers, 7,686,369,774,870 and 2,465,099,745,779. The numbers, which were picked at random by the Department of Computing at Imperial College London multiply to 18,947,668,177,995,426,462,773,730, an answer she gave in 28 seconds. In 1988 Devi agreed to be studied by Arthur Jensen, a professor of educational psychology in California. Shakuntala Devi calculated the seventh root of 170,859,375 and cube root of 61,629,875 before Jensen could even write down the problems in his notebook. Note: the answers are 15 and 395 respectively. Devi also wrote a book on homosexuality despite not being one herself in which she advocates for total acceptance of homosexuals in India. [C70]

Annie Easley

Annie Easley was born before the Civil Rights Movement which means that options for African-Americans were extremely limited. Despite this her mother encouraged her to get an education. Easley graduated valedictorian from Holy Family High School then went on to major in pharmacy at Xavier University. In 1955 Annie Easley applied for and was hired by the NACA (National Advisory Committee for Aeronautics) as a "computer". Of the 2500 employees at NACA, she was one of only four African-American women. In the 1970s she helped other female and minority students to pursue careers in STEM during college career days and in 1977 received her Bachelors in Mathematics from Cleveland State. At NASA she was also held a position as an Equal Employment Opportunity counselor where she fought discrimination. Easley worked at NASA for 34 years. In 2015, after her death in 2011 she was inducted into the Glenn Research Hall of Fame. A crater on the moon was named after Easley in February of the year 2021.

Other Individuals

- Elizebeth Friedman
- Jorge Galicia Urrutia
- Sophie Germaine
- Laura Gomez I.
- Evelyn Granville
- Frank Greene
- Jon Hall
- Pamela Harris
- Piper Herron
- Grace Hopper
- Mary Horton Ann
- Emma Iwao Haruka
- Mary Jackson
- Katherine Johnson G.
- Brian Katz
- Autumn Kent
- Sal Khan
- Sofia Kovalevskaya
- Peter Landin
- Ada Lovelace
- Yukihiro Matsumoto
- Maryam Mirzakhani
- Victor Neumann-Lara
- Emmy Noether
- Maria Ong
- Radia Perlman
- Srinivasa Ramanujan
- Mina Rees
- John Resig
- Adriana Salerno
- Nicholas Saunderson
- Mary Somerville
- Christoper Strachey
- Audrey Tang
- Terence Tao
- Valerie Thomas
- Alan Turing
- John Urschel
- Dorothy Vaughan
- Gladys West
- Sophie Wilson
- Edith Windsor
- Lixia Zhang
- Miguel de Icaza
- Jesus de Loera A.
- Juana de la Cruz Ines

Methods of Reducing Math Anxiety

It has been shown in several studies that there exists a direct relationship between math anxiety and math performance. One way to alleviate math anxiety is by expressive writing. Expressive writing is the process where students write freely for 10-15 minutes about their thoughts and feelings regarding an upcoming mathematics exam (or other important stressor). Studies have shown it reduces negative thoughts and feelings in depressed individuals as well as having other physical and psychological benefits for all parties, whether they be depressed individuals, college students, or both. Studies have also shown that expressive writing can increase the availability of working memory, something which persons with math anxiety have a limited amount of [C71]. Expressive writing may also help with emotion regulation and distancing of oneself from their sources of stress.

Other studies have shown that "psychological techniques emphasizing self regulation, emotional control, and reappraisal of physiological threat responses hold promise" [C72]. Retrieval practice has also been found to decrease math anxiety. Retrieval practice is frequent, occurring at least once a week, quizzes and practice tests that accounted for little or no change to a student's grade. An important component of retrieval practice is automatic feedback. In one study, 72% of students surveyed said that they experienced less anxiety towards tests because of retrieval practice, 22% said that it made no difference and 6% said that they were more nervous [C73].

Another study advocated for digital game-based learning (DGBL) as a method of alleviating math anxiety. An article entitled *Effects of anxiety levels on learning performance and gaming performance in digital game-based learning* said that "Unlike learning performance, where learners with high anxiety demonstrated worse learning performance than those with low anxiety, the former and later demonstrated similar gaming performance [C74]. One such game that been shown to help reduce anxiety is the game Kahoot!. A literature review done of over 70 studies noted that 70% of studies that mentioned student anxiety noticed a reduction in anxiety due to playing Kahoot!. Only one study reported that it caused anxiety [C75].

Testing

An alternative to conventional testing which can cause test anxiety as well as math anxiety, is dynamic testing. Dynamic testing is designed to "assess developing or yet-to-develop abilities that are the products of underlying, but often unrecognized, cognitive capacities" [C76]. Dynamic testing consists of, according to article entitled *Learning Potential and Anxious Tendency*, allowing students to ask for and receive hints on their tests with the caveat of being docked points for doing so [C77]. In this way, the bias in educational testing towards individuals with anxious tendencies can be at least partially alleviated. This is one option for decreasing math anxiety.

Another option to decrease math anxiety is collaborative testing as shown by multiple studies [C78]. Collaborative assessments have been shown to promote critical thinking, improve test performance, reduce test anxiety, support long-term retention of the material, and more. There are, of course, drawbacks to collaborative testing, it is difficult to assess individual performance as well as cheating. The benefits are however are an increase in understanding, performance, and confidence.

Content

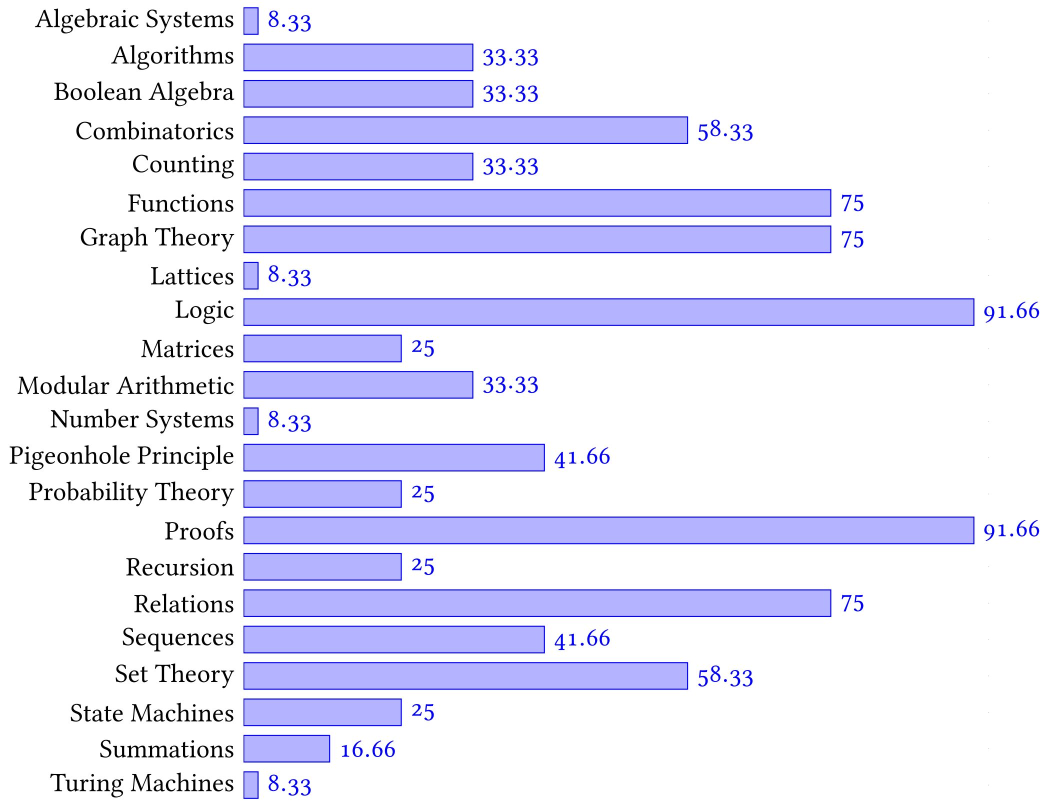
Mathematics Topics

Table 2: Table of Topics Covered in Example Syllabi

	HU[D79]	FTU[D80]	LU[D81]	UV[D82]	KAU[D83]	DLSU[D84]	ACC[D85]	AU[D86]	IVC[D87]	DSC[D88]	NYU[D89]	VTU[D90]
algebraic systems								x				
algorithms			x		x	x	x					
boolean algebra			x				x			x		x
combinatorics	x	x	x			x		x	x			x
counting	x					x		x	x			
functions	x	x	x		x	x	x	x			x	x
graph theory	x	x			x	x		x	x	x	x	x
lattices								x				
logic	x	x	x	x	x	x	x	x	x	x	x	
matrices			x		x						x	
modular arithmetic			x				x			x		x
number systems		x										
pigeonhole principle	x						x	x			x	x
probability theory	x			x								x
proofs	x	x	x	x	x	x	x	x		x	x	x
recursion					x		x		x			
relations				x	x	x	x	x	x	x	x	x
sequences		x	x		x	x				x		
set theory	x	x		x	x	x	x		x			
state machines	x			x	x							
summations		x			x							
turing machines					x							

In an effort to understand the importance of individual topics covered under the broad category of discrete mathematics I surveyed 12 different college syllabi to see in what frequency select topics were covered. The result is the table pictured above. To better illustrate my findings I also created a bar chart that lists the percentage coverage of each topic by the syllabi aforementioned. The result is illustrated on the next page. The colleges that I chose were Harvard University, Florida Tech University, Liberty University, University of Virginia, King Abdulaziz University, De La Salle University, Austin Community College, Anna University, Imperial Valley College, Daytona State College, New York University, and Visvesvaraya Technological University. These colleges and universities were chosen by virtue of their discovery whilst conducting research.

Figure 1: Bar Chart of Topics Covered in Example Syllabi



The topics that are covered here are, for the most part, broad categories. They give useful information as to what broad categories should be covered but are of little help in deciding what individual concepts or ideas should be addressed in the curriculum. To get a better idea of what topics are covered in textbooks I obtained and extracted data from 32 different discrete mathematics textbooks and noted what topics were covered from a list of more specific topics. The result is shown on the next two pages. The list of textbooks is as follows:

· *Guide to Discrete Mathematics: An Accessible Introduction to the History, Theory, Logic and Applications* by Gerard O'Regan

· *Discrete Mathematics and Its Applications* by Kenneth H. Rosen

· *Discrete and Combinatorial Mathematics: an Applied Approach* by Grimaldi

· *Discrete Mathematics with Applications* by Susanna S. Epp

· *Discrete Mathematical Structures with Applications to Computer Science* by Jean-Paul Tremblay and R Manohar

· *Applied Discrete Mathematics* by William Shoaff

· *Introduction to Discrete Mathematics via Logic and Proof* by Calvin Jongsma

· *Mathematics for Computer Science* by Eric Lehman, et al

· *Theory and Problems of Discrete Mathematics* by Seymour Lipschutz and Marc Lipson

· *A Beginner's Guide to Discrete Mathematics* by W. D. Wallis

· *Discrete Mathematics Using a Computer* by John O'donnell

· *DISCRETE MATHEMATICS : An Open Introduction* by Oscar Levin

· *Discrete Mathematics* by Jean H. Gallier

· *DISCRETE MATH WORKBOOK : A Companion Manual Using Python* by Sergei Kurgalin

· *Essential Discrete Mathematics for Computer Science* by Todd Feil and Joan Krone

· *Handbook of Discrete and Combinatorial Mathematics* by Kenneth H. Rosen

· *Foundations of Discrete Mathematics* by K.D. Joshi

· *Discrete Mathematics with Graph Theory* by Edgar G Goodaire and Micheal M Parmenter

· *Discrete Mathematics* by Martin Aigner

· *Foundations of Discrete Mathematics* by Albert D Polimeni and H Joseph Straight

· *Discrete Mathematics* by K. A. Ross and C. R. B. Wright

· *Mathematics : A Discrete Introduction* by Edward R. Schneinerman

· *Discrete Mathematics* by James Hein

· *Discrete Mathematics* by Nomran L. Biggs

· *An Introduction to Discrete Mathematics* by Steven Roman

· *Discrete Mathematics for New Technology* by Rowan Garnier and John Taylor

· *Discrete Mathematics* by R. K. Bisht and H. S. Dhami

· *Discrete Mathematics for Computer Science* by Kenneth P. Bogart, et al

· *Discrete Mathematical Structures : Theory and Applications* by D. S. Malik and M. K. Sen

· *Discrete Mathematical Structures* by Bernard Kolman, et al

· *Logic and Discrete Mathematics* by Winfried K. Grassmann and Jean-Paul Tremblay

Table 3: Table of Topics Covered in Example Textbooks

Topic	[D91]	[D92]	[D93]	[D94]	[D95]	[D96]	[D97]	[D98]	[D99]	[D100]	[D101]	[D102]	[D103]	[D104]	[D105]	[D106]
Algebraic Structures					x											x
Algorithm Efficiency			x	x												
Binomial Theorem & Coefficients		x	x	x						x		x	x		x	
Boolean Algebra		x	x	x	x	x	x		x	x	x			x	x	x
Circuits		x		x						x	x			x	x	
Coding Theory	x		x													x
Combinatorics														x		
Computability & Decidability	x															x
Congruences	x	x										x				x
Counting		x	x	x		x			x	x		x	x		x	x
Cryptography	x	x				x				x			x		x	x
Financial Math	x					x		x								
Determinants	x														x	
Directed Graphs								x	x				x	x		x
Discrete Structures					x					x						x
Equivalences		x	x	x	x	x	x	x			x		x			
Euler Trails & Circuits		x	x				x		x	x		x	x		x	
Fields	x		x													x
Finite-State Machines	x	x	x	x	x			x	x							
Functions	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x
Gaussian Elimination	x														x	
Generating Functions			x					x				x				x
Graph Coloring	x	x	x				x		x			x				x
Groups & Semigroups			x		x				x							x
Hamiltonian Paths & Cycles	x	x	x		x		x		x	x		x	x			
Inclusion & Exclusion			x					x		x						x
Induction	x	x		x		x	x	x		x	x		x		x	
Infinite Sets							x	x		x						
Isomorphisms			x	x				x	x							
Lattices	x				x		x						x			x
Mathematics History	x								x							
Matrices	x								x	x					x	x
Modular Arithmetic		x	x	x		x		x		x						
Networks								x								x
Permutations & Combinations	x	x	x			x	x									x
Pigeonhole Principle		x	x	x		x		x		x			x			
Planar Graphs		x	x				x	x	x			x	x			x
Posets	x				x		x		x							x
Predicate Calculus	x				x											
Prime Numbers	x	x													x	x
Probability	x	x	x	x				x		x						x
Proofs	x	x		x		x	x	x		x	x	x			x	x
Propositional Calculus	x								x							
Recurrence Relations		x	x	x			x					x		x	x	x
Recursion	x	x		x	x	x	x				x		x		x	
Relations	x	x	x	x	x				x	x	x		x	x	x	x
Rings	x		x													x
RSA Encryption	x		x					x		x			x		x	
Sequences	x					x		x				x				x
Set Theory	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Sums						x		x		x						x
Trees		x	x	x	x				x	x	x	x			x	x
Truth Tables	x		x		x						x	x				
Turing Machines	x	x				x			x					x		
Vector Spaces	x															x
Venn Diagrams			x		x	x				x		x				
Well-Ordering Principle			x					x		x						

Note: The top row of the table lists the citations of the textbooks.

Table 4: Table of Topics Covered in Example Textbooks Part 2

Topic	[D107]	[D108]	[D109]	[D110]	[D111]	[D112]	[D113]	[D114]	[D115]	[D116]	[D117]	[D118]	[D119]	[D120]	[D121]	[D122]
Algebraic Structures	x			x			x				x	x				
Algorithm Efficiency		x	x				x	x								
Binomial Theorem & Coefficients		x				x		x		x		x	x	x		
Boolean Algebra	x		x	x	x	x					x	x		x	x	x
Circuits				x					x		x	x		x		
Coding Theory																
Combinatorics									x							
Computability & Decidability																
Congruences										x				x		
Counting	x	x	x		x		x	x		x		x	x	x	x	
Cryptography			x			x							x			
Financial Math																
Determinants	x															
Directed Graphs			x	x						x	x					
Discrete Structures							x					x			x	
Equivalences		x		x		x	x		x	x	x	x	x			x
Euler Trails & Circuits		x				x			x	x		x	x		x	
Fields	x			x				x				x				
Finite-State Machines									x	x		x		x	x	
Functions	x	x		x	x	x	x	x	x	x	x	x			x	x
Gaussian Elimination											x					
Generating Functions		x	x					x								
Graph Coloring		x		x		x		x				x	x	x	x	
Groups & Semigroups	x			x		x		x			x	x			x	
Hamiltonian Paths & Cycles		x							x			x	x		x	
Inclusion & Exclusion	x		x		x					x						
Induction		x		x	x				x	x			x	x	x	x
Infinite Sets					x		x					x				
Isomorphisms		x		x	x	x		x	x		x			x		
Lattices						x						x			x	
Mathematics History														x		
Matrices	x				x					x	x			x		
Modular Arithmetic			x			x		x		x			x			
Networks	x		x		x			x	x		x	x		x		
Permutations & Combinations		x	x	x		x		x	x	x		x	x	x	x	
Pigeonhole Principle					x			x	x	x		x		x	x	
Planar Graphs		x		x		x			x		x	x	x	x		
Posets					x	x						x		x	x	
Predicate Calculus					x		x									x
Prime Numbers		x												x		
Probability						x				x		x	x	x		
Proofs		x			x	x	x		x	x	x	x	x	x		x
Propositional Calculus					x		x									x
Recurrence Relations	x	x	x				x		x	x		x	x	x		x
Recursion		x			x			x		x			x			x
Relations		x		x	x	x			x		x	x		x	x	x
Rings	x			x				x				x				
RSA Encryption						x							x			
Sequences					x									x	x	x
Set Theory	x	x		x	x	x	x	x	x	x	x	x		x	x	x
Sums																x
Trees		x	x	x	x	x		x	x	x		x	x	x	x	x
Truth Tables									x	x	x		x			x
Turing Machines																
Vector Spaces	x															
Venn Diagrams										x		x				
Well-Ordering Principle												x				

As with the data regarding the discrete mathematics syllabi, I constructed a bar chart that lists the percentage coverage of each topic by the 32 textbooks I collected. The result is on the next page. The next logical step after conducting research is analyzing and deciding how to use the data. After much deliberation I made the decision to include a topic covered by the syllabi if at least 30% of the syllabi cover it. Likewise I will include the more specific topics that pass the 30% coverage by discrete mathematics textbooks. This means that the broad categories I will include in my curriculum are algorithms, boolean algebra, combinatorics, counting, functions, graph theory, logic, modular arithmetic, pigeonhole principle, proofs, sequences, and set theory. The more specific topics I plan on covering include:

· Binomial Theorem & Coefficients

· Boolean Algebra

· Circuits

· Counting

· Cryptography

· Equivalences

· Euler Trails & Circuits

· Finite-State Machines

· Functions

· Graph Coloring

· Groups & Semigroups

· Hamiltonian Paths & Cycles

· Induction

· Isomorphisms

· Modular Arithmetic

· Networks

· Permutations & Combinations

· Pigeonhole Principle

· Planar Graphs

· Posets

· Probability

· Proofs

· Recurrence Relations

· Recursion

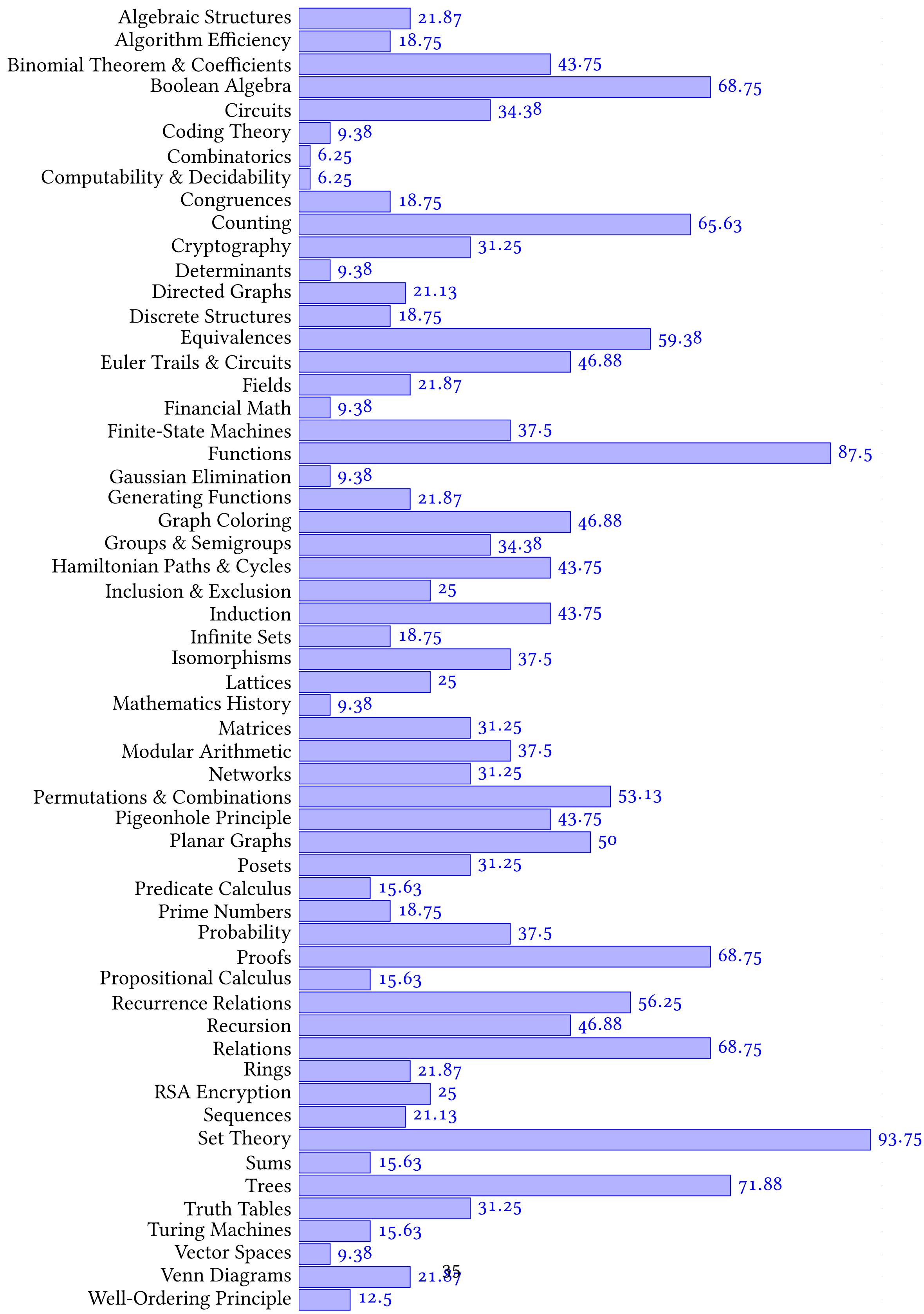
· Relations

· Set Theory

· Trees

· Truth Tables

Figure 2: Bar Chart of Topics Covered in Example Textbooks



Computer Science Topics

Based on reviews, the computer science textbook I chose was *Bite-Size Python: An Introduction to Python Programming* by April Speight. I chose this book because it was marketed towards beginners and used clear, consistent, and concise language to explain topics. It also was written by a female person of color and the graphics in the book are of diverse individuals. The chapters in the book are listed below. While I want to use this book as my main textbook to work out of, in order to cover more complex topics, it can't be my only textbook. The second textbook I chose was also designed to be easy for beginners to grasp as well as being open sourced. The textbook is *Think Python: How to Think Like a Computer Scientist* by Allen Downey. In addition to the topics covered by *Bite-Size Python* it also covers interface design, recursion, iteration, tuples, data structures, files, classes and objects, inheritance, more debugging, and analysis of algorithms.

1. What is Python?
2. Install Python
3. IDLE
4. Variables
5. Numbers
6. Strings
7. Conditionals and Control Flow
8. Lists
9. for Loops
10. while Loops
11. Functions
12. Dictionaries
13. Modules
14. Next Steps

Sample Lesson Plans

Mathematics Lessons: Truth Tables

Lesson objective: Given a statement in symbolic form, students will create a truth table with a success rate of 4/5 problems or 80%.

Advance Organizer:

Professional of the Day: (Display individual in mathematics and computer science field who is from an underrepresented minority ex. Winifred "Tim" Alice Asprey)

Review: Let's begin today's lesson by reviewing some vocabulary. Can anyone give me the definition of the word negation? *Student Response* That's right, a negation of a statement is the denial of a statement and is represented by the tilde symbol. If I were to say the statement "Math is hard", what's the negation of that statement? *Student Response* That's correct, the negation of the statement "Math is hard" is the statement "Math is easy". Now, what does the word conjunction mean? *Student Response* You are correct! Conjunction means the joining of two statements by the word *and*. What does the word disjunction mean? *Student Response* That's great! The word disjunction means the joining of two statements by the word *or*. Now, we can interpret the word or in two ways, inclusive or and exclusive or. If I said the compound statement "[*Student Name*] is at school or at home" would that be an example of an inclusive or exclusive or? That's right, the statement "[*Student Name*] is at school or at home" is an example of an exclusive or. Final vocabulary word to review. What is a conditional statement? *Student Response* You are correct, a conditional statement is a statement that can be written in the form "if p then q". [*Student Name*], can you give me an example of a conditional statement? *Student Response* Good job! [*Statement*] is a great example of a conditional statement.

Let's review turning statements into symbolic logic which is what we covered in class last week. (Display figure 3) Please translate the following compound statement into symbolic logic and write the answer on your whiteboard and hold it up.

Figure 3: Review Question: Turning Statements to Symbolic Logic

p: The grass is brown
q: It is summer
r: It is raining

If it is summer and the grass is brown then it is not raining.

Nice job! I see most of you got it correct. The correct answer is (Display following)

$$(q \wedge p) \to \sim r.$$

Objective and Link: You already learned how to turn statements into symbolic logic. Now we are going to something different. We are going to take those statements written in symbolic logic and construct truth tables using them.

Relevance: Truth tables are used to determine if two logical statements are equivalent or if the structure of a logic statement is valid.

Demonstration: Truth tables are a way of telling if a compound statement is true of false. The entries in a truth table are called truth values. Truth values represent either true or false. Often times a T is used to denote true and a F is used to denote false but you could also use a 0 to denote false and a 1 to denote true. A truth table list all possible combinations of the individual statements that make up a compound statement to determine if the statement is true or not. Let's start with the negation. Take a look at the following truth table *(Display figure 4)*

Figure 4: Truth Table for Negation of p

p	~p
T	F
F	T

Figure 5: Truth Table for Conjunction of p and q

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Look at the first column, the truth values T and F represent all the possible values that p can be. The statement p can either be true or false. The next column indicates the possible values of the negation of p. When p is true, $\tilde{p}$ is false and when p is false, $\tilde{p}$ is true. Does that make sense? Let's look at another example of a truth table. Let's look at the truth table for the conjunction of p and q. *(Display figure 5)*

Now, there are more rows here because there are more possible combinations of what values p and q can be. You could have p be true and q be true, p be true and q be false, p be false and q be true, and p be false and q be false. If you have n simple statements p, q, r, ect., there will be 2^n rows. So here we have two simple statements and so we have $2^2 = 4$ rows. Now let's look at the last column. The last column represents the values the conjunction of p and q. As you can see, only in the case where both p and q are true is the conjunction true.

Now let's construct a truth table for a disjunction, p or q. How many simple statements does the compound statement p or q have? *Student Response* That's correct, the compound statement p or q has two simple statements p and q. So how many rows does our table need to have? *Student Response* That's right! Our table should have four rows. Let's construct our truth table. I'll set up the possible values for p and q for you. *(Display figure 6)*

Our first two columns are going to be the same, it's just our last column that is going to be different. Let's fill that column. If p is true and q is true is p or q going to be true or false? *Student Response* That's correct. If p is true and q is true then p or q is true also. What about if p is true and q is false? *Student*

Figure 6: Empty Truth Table for Disjunction of p and q

p	q	$p \vee q$
T	T	-
T	F	-
F	T	-
F	F	-

Figure 7: Full Truth Table for Disjunction of p and q

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Response That's right! If p is true and q is false then p or q is true. If p is false and q is true is p or q going to be true or false? *Student Response* You all are doing great! If p is false and q is true then p or q is going to be true. Last one, if both p and q are false is p or q going to be true or false? *Student Response* Great job y'all. If both p and q are false then p or q is going to be false. Here's the finished truth table. *(Display figure 7)*

Before we get into some more complicated examples, let's go over the truth table for a conditional statement if p then q. How many rows are we going to have? *Student Response* That's correct, we are going to have four rows because we have two simple statements and $2^2 = 4$. Now let's fill in our table *(Display figure 8)*

If p and q are both true, what is the value of our truth value? Remember those are the T and F values. *Student Response* That's right, if p and q are both true then the conditional if p then q is true. What about if p is true and q is false? *Student Response* That's correct. If p is true and q is false, if p then q is false. Now, this is a tricky one, if p is false and q is true, what is the value of our truth value? *Student Response* Great

Figure 8: Empty Truth Table for Conditional If p Then q

p	q	$p \rightarrow q$
T	T	-
T	F	-
F	T	-
F	F	-

Figure 9: Demonstration Question 1

Statement: I have a diploma or I have a full-time job and no diploma.
p: I have a high school diploma
q: I have a full-time job

Figure 10: Demonstration Question 1 Empty Truth Table

p	q	$\sim p$	$q \wedge \sim p$	$p \vee (q \wedge \sim p)$
T	T	-	-	-
T	F	-	-	-
F	T	-	-	-
F	F	-	-	-

job! Yeah, if p is false and q is true then the conditional if p then q is what is called vacuously true. It's sort of like the expression "when pigs fly", it doesn't matter what you promise to do when pigs fly because pigs are never going to fly. What about our last case, if p and q are both false? *Student Response* Correct! If p and q are both false then the conditional if p then q is going to be false. Are you ready to try something trickier? Let's try this problem: *(Display figure 9)*

First let's write the statement in symbolic logic. Can I have a volunteer to come write the symbolic logic on the board? Thank you [*Student Name*]! *Student writes* Thank you, you may sit down now. What do you all think abut what [*Student Name*] wrote? Give a thumbs up if you agree and a thumbs down if you disagree. *Student Response* I see most of you with your thumbs up. In symbolic logic the statement "I have a diploma or I have a full-time job and no diploma" is $p \vee (q \wedge \sim p)$. Now let's construct our truth table. When we construct our truth table we want to break the statement into its smallest parts and build up. Let me show you what I mean. *(Display figure 10)*

So, what I did was use something very similar to PEMDAS. I started at the innermost operation, the negation of p and work my way out. That way, at each step I am only doing one operation. Let's start with column three. What is going to be the first entry in column three? Remember negation means do the opposite. *Student Response* That's correct! If p is true then the negation of p is false. Now let's do the second entry. Put your thumb up if you think the truth value that goes in the second row of column 3 should be true and put your thumb down if you think it should be false. *Student Response* Good job! It should be false

Figure 11: Demonstration Question 1 Full Truth Table

p	q	~p	$q\wedge$~p	$p \vee (q\wedge$~$p)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

Figure 12: Demonstration Question 2

Statement: I own a handgun and it is not the case that I am a criminal or a police officer.
p: I own a handgun
q: I am a criminal
r: I am a police officer
Answer:

p	q	r	$q \vee r$	~$(q \vee r)$	$p\wedge$~$(q \vee r)$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	T	F

again. Now the next entry, if p is false the negation of p is... *Student Response* Correct! It should be true. What about the last entry? Put your thumb up for true and thumb up for false. *Student Response* That's right, it should be true. Now let's move to the next column. if q is true and the negation of p is false, what is conjunction of q and not p? *Student Response* That's right, it should be false. What about the next entry in column 4? If q is false and the negation of p is false what should our truth value be? *Student Response* That's right! It should be false. *Repeat for rest of truth table* (See figure 11). As indicated in the truth table, the statement "I have a diploma or I have a full-time job and no diploma" is true under all conditions except one. The statement is false if you don't have a high school diploma or a full-time job.

Repeat for figure 12, figure 13, figure 14, and figure 15. Then repeat with new problem and decreased prompts. Decrease prompts gradually until students can complete problems without assistance.

The problems provided are sourced from a textbook entitled *Mathematics: A Practical Odyssey* [E123].

Figure 13: Demonstration Question 3

Statement: I walk up the stairs if I want to exercise or the elevator isn't working.
p: I walk up the stairs
q: I want to exercise
r: The elevator is working
Answer:

p	q	r	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	F

Figure 14: Demonstration Question 4

$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$

Answer:

p	q	$p \rightarrow q$	$\sim q$	$(p \rightarrow q) \wedge \sim q$	$\sim p$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Figure 15: Demonstration Question 5

$[(p \rightarrow \sim q) \wedge \sim q] \rightarrow p$

Answer:

p	q	$\sim q$	$p \rightarrow \sim q$	$(p \rightarrow \sim q) \wedge \sim q$	$[(p \rightarrow \sim q) \wedge \sim q] \rightarrow p$
T	T	F	F	F	T
T	F	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F

Computer Science Lessons: Conditionals

Lesson objective: Given a statement students will create a if statement with a success rate of 4/5 problems or 80%.

Advance Organizer:

Professional of the Day: (Display individual in mathematics and computer science field who is from an underrepresented minority ex. Annie Easley)

Review: Let's begin today with some review. Last week we talked about functions. I want you to write down on your paper a function that takes no arguments but prints out your name. I'll give you a minute do do that then I'll come around and check. *Wait one minute then check answers* Good job everyone, you all wrote excellent functions. Now can I have a volunteer come up to the board and write a function that take two arguments a and b, and returns what a mod b is. Thank you, *[Student Name]* for volunteering! *Wait* What does everyone think of this function? Does the function return a mod b? *Student Response* That's correct, that is a great example of a function that returns a mod b.

Objective and Link: You already learned how to write functions. Now we are going to learn how to write functions using conditionals or if statements.

Relevance: If statements are really useful if you only want a block of code to run under certain conditions.

Demonstration: First let's talk about comparison operators. Comparison operators are similar to those used in mathematics. One example of a comparison operator is "greater than". Can anyone give me another example of a comparison operator? *Student answer* Yes, that's a good example of a comparison operator. Can anyone give me another? *Continue until you have most of the operators then display table 5*

Table 5: Comparison Operators

Operator	Name	Example
==	Equal	5 == 5
!=	Not Equal	26 != 3
>	Greater Than	100 > 67
<	Less Than	89 < 216
>=	Greater Than or Equal To	90 >= 54
<=	Less Than or Equal To	23 <= 77

[E124]

When you use a comparison operator like the ones shown, python will return a Boolean value of either True or False. In all of the examples shown, python will return True because they are all true statements. Will you all please write on your boards one example of a statement that is False using a comparison operator? *Give feedback* You can also have more complex comparisons. Some examples include: *Display figure 16*

Figure 16: Complex Comparisons

```
34 - 3 < 24 * 2$
```

```
13 % 3 == 17 - (8*2)
```

```
15 - (2*3 + 4) >= 36/4 + (5**2 - 4*7)
```

I will give you a minute to calculate whether each of these lines of code will return a True or False. *Wait* Okay, thumbs up if you think the first line of code will return the Boolean value True, thumbs down if you think it will return False. *Student Response* That's correct. The first line will return True because 31 is not equal to 48. Second line: thumbs up if it will return True. *Student Response* That's right, the second line will return True because 1 is equal to 1. What about the third line? *Student Response* Good job! The third line will return False because 5 is not greater than or equal to 6. You can also use comparison operators to compare strings. For example: *Display figure 17*

Figure 17: String Comparisons

```
sentence1 = "This string is short"
sentence2 = "This string is shorter"
print(len(sentence1) < len(sentence2))
```

```
favColor = "Blue"
color = "blue"
print(favColor == color)
```

What will that first block of code print? *[Student Name]* what do you think? Will the first block of code print True or False? *Student Response* That's correct. The first block of code will print True because the first sentence has fewer characters than the second sentence. What about the second block of code? Will it print True or False? Can I have a volunteer? Yes, *[Student Name]*? *Student response* That is right. That was a tricky question wasn't it? The second block of code will print False because a capital "B" is not the same as a lowercase "b". Comparison operators are not the only type of operators that exist. Another type of operator is a logical operator. These are very similar to what you've seen with symbolic logic. There are three logical operators They are "and", "or" and "not". *Display table 6*

Table 6: Logical Operators

Operator	Description	Example
and	Returns True if both statements are true	4 > 3 and 2 * 3 < 6
or	Return True if either statements are true	3 + 2 == 2 ** 2 + 1 or 22 < 5
not	Returns the opposite Boolean value of the statement	not 64 >= 4 ** 3

Please write on your boards what each of the examples will return. *Student response* Good job everyone. The first example will return False because the first statement is True and the second statement is False. Remember, 6 is not less than 6. The second example will return True because the first statement is True and the second statement is False. Finally, the third example will return False because the statement is True. You can use logical operators with strings as well. Now let's move on to if statements. An if statement lets you write code that will only run if certain conditions are met. If statements look like this: *Display following*

```
if some condition:
  action
```

The formatting is similar to functions, anything you want in your if statement needs to be indented. Let's look at some examples. *Display following*

```
a = 12
b = 23
if a < b:
  print("a is less than b")
```

```
dayOfTheWeek = "Tuesday"
temperature = 80
if dayOfTheWeek == "Saturday" and temperature >= 80:
  print("Let's go to the beach!")
```

For the first block of code, will the line inside the if statement run? Put your hand up if you think it will run. *Student response* That's correct. Because a is less than b, i.e. 12 is less than 23, the code inside the if statement will run and "a is less than b" will be printed. How about the second block of code? Will the code inside the if statement run or not? Hands up if you think yes. *Student response* That's correct, it will not run because the dayOfTheWeek variable is not equal to "Saturday". Now, what if we want some code to run under a certain condition but if that condition is not met we want different code to run? Here is where we use if-else statements. If-else statements look like this: *Display following*

```
if some condition:
  action
else:
  different action
```

Now let's write our own if-else statement. Say we wanted to know if a number, say, 27378 is divisible by three. If it is, we want to print out "This number is divisible by 3" and if it is not we print out "This number is not divisible by 3". How might we do that? Let's start with the if statement. How do we tell if a number is divisible by three? What code can we use to figure it out? *[Student Name]*? What do you think? *Student response* That's correct. A number is divisible by three if the number modulo 3 is equal to 0. Let's put that into code: *Display following*

```
x = 27378
if x % 3 == 0:
```

What do we need to put in the if statement? *Student response* That's correct. The code inside the if statement needs to say print("This number is divisible by 3"). *Display following*

```
x = 27378
if x % 3 == 0:
  print("This number is divisible by 3")
```

Now we need to write the else condition. With the else condition you don't have to specify a condition because the condition is the negation of the if statement condition so all we have to do is write the code that will go in the else condition. And that code is what? *Student response* That's correct, the code that goes in the else condition is print("This number is not divisible by 3") *Display following*

```
x = 27378
if x % 3 == 0:
  print("This number is divisible by 3")
else:
  print("This number is not divisible by 3")
```

Which line will print, the one inside the if statement or the one inside the else condition? Thumbs up if you think the code inside the if statement will run and thumbs down if you think the code inside the else statement will run. *Student response* That's correct, since 27378 is divisible by three, the line inside the if statement will run and "This number is divisible by 3" will print. Sometimes you need the code to have more than one condition. This is where if-elif-else statements come in. If-elif-else statements are similar to if statements and if-else statements in the way they function. The formatting of if-elif-else statements look like this: *Display following*

```
if some condition:
  action 1
elif some other condition:
  action 2
else:
  action 3
```

Python first looks at the if statement and determines whether or not to run the code inside based on whether the condition is True or False. If the condition is False, Python looks at the elif condition. If that condition is True, Python runs the code inside. If not, Python runs the code inside the else condition. Let's look at an example. Say that you want to print if a number is bigger than, equal to, or less than another number. You would use an if-elif-else statement. You have two expressions x and y. *Display following*

```
x = 12*14 - 7(8**2 + 34*3) + 1000
y = (42%13) + 14(5-13*2) + 11*3**3
```

You want to print whether x is less than, greater than, or equal to y. Let's write the if statement first. If x is less than y we want to print "x is less than y" *Display following*

```
x = 12*14 - 7(8**2 + 34*3) + 1000
y = (42%13) + 14(5-13*2) + 11*3**3
if x < y:
  print("x is less than y")
```

If x is greater than y we want to print "x is greater than y". This will be our elif condition. *Display following*

```
x = 12*14 - 7(8**2 + 34*3) + 1000
y = (42%13) + 14(5-13*2) + 11*3**3
if x < y:
  print("x is less than y")
elif x > y:
  print("x is greater than y")
```

If neither is True, i.e. x == y, we want to print "x equals y". This will be our else condition. *Display following*

```
x = 12*14 - 7(8**2 + 34*3) + 1000
y = (42%13) + 14(5-13*2) + 11*3**3
if x < y:
  print("x is less than y")
elif x > y:
  print("x is greater than y")
else:
  print("x equals y")
```

Which print statement will run? I'll give you a minute to calculate. When you are ready, write 1 on your board if you think the first print statement will run, 2 if you think the second print statement will run, and 3 if you think the last print statement will run. *Student response* The answer is that the third print statement will run because both x and y equal 6.

Repeat with new problem and decreased prompts. Decrease prompts gradually until students can complete problems without assistance.

Connection Project: Truth Tables & Conditionals

Project objective: Given sample code, students will create functions that evaluate the truth of logical operators for Boolean variables.

Project Instructions: Attached to this thesis is code I found while doing research on coding truth tables. While not perfect, it serves as the basis for a project that combines the use of truth tables with if statements and conditionals. If you were to use this project you might provide the code and have them write the conditional, biconditional, and_func, or_func, and negate functions themselves. The source for this code is: *https://codereview.stackexchange.com/questions/145465/creating-truth-table-from-a-logical-statement*

Conclusion

Math anxiety is a pervasive problem that spans centuries. There are several ways to combat this issue. Some ways to combat the issue are evidence-based practices such as explicit instruction or high leverage-practices, practices that are intended to be easily implemented and have been shown to be effective. It also helps to teach mathematics where the relevancy is clearly defined. One such branch of mathematics is called discrete mathematics. Discrete mathematics largely consists of the mathematics needed for computer science. By implementing discrete mathematics alongside computer coding lessons the mathematics is made more accessible and less anxiety inducing. Incorporating expressive writing, retrieval practice, digital game-based learning, alternative testing methods and examples of diverse individuals in the mathematics and computer science field, are further methods of reducing math anxiety. All of these methods culminated in a lesson plan on truth tables as an example of what this kind of teaching might look like. Future research would include testing to determine the effectiveness of the practices not already evidence-based.

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