COMBATING MATH ANXIETY THROUGH INTERDISIPLINARY STUDIES

by

CHLOE ANN MILLER

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Approved: *Jennifer Ruef, Ph.D.*
Primary Thesis Advisor

Many agree that one of the most anxiety-provoking subjects in school is mathematics. As a future high school math teacher, I'm specifically interested in who is being affected by math anxiety and how we combat this and get students interested and excited about math. My thesis explores math anxiety through varying lenses focusing on historically and currently underrepresented and marginalized groups (women, People of Color, and English language learners) within mathematics. While centering those students who are more likely to experience math anxiety, we can think about potential solutions to help combat math anxiety. In my own experience struggling with math anxiety, what has been most impactful for me in getting interested and feeling confident in my math classes was when I could find connections to other subjects that I was passionate about. In my thesis, I look at interdisciplinary studies as a potential combatant of math anxiety. My thesis includes interdisciplinary lesson plans which combine math with the subjects of ecology, art, and literature. This project will give educators (and myself) a much-needed resource by sharing who math anxiety impacts, reasons why, and providing examples of how we can incorporate students' interests in other subjects to get them excited about math.

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Background

What is Math Anxiety?

Math anxiety is a term that has been used in countless studies to mean many different ideas and is measured in many different ways. Dr. Ann Dowker has explored this history of math anxiety over the past 60 years. She uses the definition of Frank Richardson and Richard Suinn, who state that math anxiety is “a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in… ordinary life and academic situations” (Dowker et al., 2016, p. 1). Dowker also cites the idea of math anxiety being two dimensional which comes from Allan Wigfield and Judith Meece. Wigfield and Meece define a *cognitive* and *affective dimension* of math anxiety. The cognitive dimension is “labeled as ‘worry’, refers to concern about one’s performance and the consequences of failure” and the affective dimension is “labeled as ‘emotionality’ and refers to nervousness and tension in testing situations and respective autonomic reactions” (Dowker et al., 2016, p. 2).

Sian Beilock has also done extensive research in the field of math anxiety, also calling it performance pressure, or choking under pressure. In her article “Math Anxiety: Can Teachers Help Students Reduce It?”, she describes people who suffer math anxiety as “People who feel tension, apprehension, and fear of situations involving math” (Beilock & Willingham, 2014, p. 29). In her article “More on the Fragility of Performance: Choking Under Pressure in Mathematical Problem Solving,” she defines performance pressure as a desire to perform to the best of one’s ability where there is a high degree of personally felt importance (Beilock et al., 2004). She then describes choking as “performing more poorly than expected given one’s skill level” (Beilock et al., 2004, p. 584). In the remainder of the article, Beilock looks at other studies where there is increased pressure to perform other tasks, such as golfing and soccer. She finds that "training under conditions that prompted attention to the component processes of execution enabled performers to adapt to the type of attentional focus that often occurs under pressure" (Beilock et al., 2004, p. 585). Her study provides a great example of comparing other types of performance, in this case athletics, to mathematics and shows its usefulness. This thesis will compare another subject that tends to provoke anxiety in students, language acquisition, to mathematics in order to help students who are learning a new language alongside having to learn mathematics in that new language.

In this thesis, I adopt both Richardson and Suinn's definition with Beilock's definition, specifically in the context of education. I choose to only speak to academic situations because in this thesis the objective is to create a curriculum for teachers which will help combat math anxiety for their students. I also choose to make my definition more general and broader with the feelings one might experience since there are many different ways in which different students might choose to express anxiety. Limiting the type of feeling one might experience may limit the type of student I write my definition for, as I want to include students of any race, gender, sexuality, bodily ability, etc. My definition of math anxiety is the tension, apprehension, fear, hate/anger, sadness, and other detrimental feelings which are caused by mathematical situations in an academic context.

Working Memory: What effect does math anxiety have on the brain?

Math anxiety has many effects on the brain, but one effect that has been explored the most is working memory. Beilock describes working memory as a mental scratch pad, "it's what allows you to keep several things in mind simultaneously, and to manipulate them in order to think and solve problems" (Beilock & Willingham, 2014, p. 29). Dowker explains, "Anxious people are likely to have intrusive thoughts about how badly they are doing, which may distract attention from the task or problem at hand and overload working memory resources" (Dowker et al., 2016, p. 4). Other literature also supports this idea of impacts on working memory, "the literature now supports a clear generalization concerning the important positive relationship between the complexity of arithmetic or math problems and the demand on working memory for problem solving" (Ashcraft & Krause, 2007, p. 243). Ashcraft & Krause (2007) explains that the complexity of a math problem comes from two different ideas: (1) numerical values being manipulated and (2) the total number of steps required for problem-solving. By increasing the complexity of the problem, the working memory is already taxed, and the addition of intrusive thoughts only adds to the taxing of the working memory.

Beilock's research in "More on the Fragility of Performance" explores two main theories to explain why people choke under pressure (2004). The first is self-focus/explicit monitoring theories which say that the pressure to perform increases anxiety and self-consciousness leading to increased attention and focus on the step-by-step processes involved in any particular skill. While increased attention and focus might seem like a positive, the argument here is that "Attention to performance at such a component-specific level is thought to disrupt the proceduralized or automated processes of high-level skills" (Beilock et al., 2004, p. 584). The second is distraction theories which say that pressure to perform takes over the working memory with thoughts about the situation and its importance. With this distraction, working memory then has to compete with its usual focus on executing the task.

When thinking about the cognitive effects that math anxiety has, this thesis will focus on how math anxiety impedes working memory. And when working memory is occupied with this anxiety, it makes it harder to do the mathematical task at hand. Increasing the difficulty of the task then leads students to adopt a low sense of self-math identity , have lower success and performance, and even hate math (Necka, 2015).

Stereotype Threat: How does math anxiety impact underrepresented or marginalized students?

There are many impacts on underrepresented and marginalized groups within the field of mathematics. One such effect is explained by stereotype threat. Claude Steele and Joshua Aronson define stereotype threat as "being at risk of confirming, as a self-characteristic, a negative stereotype about one's group" (Steele & Aronson, 1995, p. 797). In Steele and Aronson's research, this stereotype threat is applied to intellectual test performances of African Americans. They claim that a lifetime of exposure to a negative image of ability, a stereotype, will lead to internalized inferiority anxiety. This threat diverts attention to worries not relevant to the task at hand and causes the person to become self-conscious which interferes with cognitive functioning and memory. In other words, this fear of proving the stereotype to be true can interfere with processes such as working memory and cause lower performance.

This stereotype threat can also be applied to women (Spencer et al., 1999). Dowker states that "One possible explanation for greater mathematics anxiety in females than males is stereotype threat. Stereotype threat occurs in situations where people feel at risk of confirming a negative stereotype about a group to which they belong. In the domain of mathematics anxiety, this usually refers to females being reminded of the stereotype that males are better at mathematics than females, though it can also occur with regard to other stereotypes." (Dowker et al., 2016, p. 8).

This thesis will further explore stereotype threats as well as other impacts that underrepresented and marginalized groups in mathematics might experience. Focusing on these types of students is important because they make up a large portion of those who face math anxiety in schools.

What Has Been Done Before: What are possible solutions to math anxiety?

Dowker also addresses previous solutions and ways that math anxiety has been combated. These solutions included having those experiencing math anxiety write about their worries before an academic performance, which showed to have “significantly improved performance compared to a control condition” (Dowker et al., 2016, p. 10). This idea of a writing exercise is also expressed in the research of Beilock, who states, “such writing may not be appropriate for young students” (Beilock & Willingham, 2014, p. 32). Addressing the concern of Beilock for younger students who may not be able to write as well, David Rufo’s article “Math Hater”, tells the story of an elementary teacher who noticed one student who considered herself a math hater use art therapy to express her math anxiety (2017). This student used comics which consisted of primarily pictures with some captions/words that the student did know to be able to better express her math anxiety. Another possible solution Dowker describes is “sustained exposure to mathematical stimuli" and the “study showed that a relatively short and intensive one-on-one cognitive tutoring could remediate mathematics anxiety through modulation of neural functions” (Dowker et al., 2016, p. 11). This solution, while promising, seems to be resource-intensive, especially in a classroom with one teacher who is most likely not going to be able to individually tutor all their students. Beilock also talks about the idea of sustained exposure through a focus on the development of fundamental skills which she suggests can be better promoted through discussion of math at home (Beilock & Willingham, 2014). Already one can see the interdisciplinary nature of this solution of writing and drawing as a form of anxiety relief (although not explicitly stated in any of the research as interdisciplinary).

In Beilock's article "Math Anxiety: Can Teachers Help Students Reduce It?", she gives many other solutions including a focus on teacher training, changing types of assessment, and thinking carefully about how teachers should address students' anxiety (2014). She explains that teacher training is especially important since the higher a teacher's math anxiety, the lower their student's achievement is. Beilock also explains that math anxiety linked with poor performance is more likely to take place when students take timed tests, she argues allowing more time in assessments can reduce anxiety for students. And when thinking about ways that teachers address the anxiety of their students, she recommends not to console them but rather acknowledge their difficulty and express confidence in their capability.

A Possible Solution to Math Anxiety: What are Interdisciplinary Studies?

This thesis will focus on interdisciplinary studies as a possible solution to math anxiety. As with math anxiety, interdisciplinary studies have many different definitions. Some definitions are relatively simple and concrete, one such definition which comes from a set of interdisciplinary lesson plans says interdisciplinary studies "combine mathematics with other subjects such as art, science, history and many more" (Interdisciplinary Lesson Plans(authors), YEAR, p number). Other definitions are much more nuanced, for example in the book, Interdisciplinary Mathematics Education: The State of the Art and Beyond, the authors suggest many differing definitions of varying degrees of what it means to be disciplinary (2019). The book explains that "'interdisciplinary' mathematics involves various sorts of conjunction of mathematics with other knowledge in problem solving and inquiry" (Doig et al., 2019, p. 14). The authors go on to explain what it means to be trans-disciplinary (implying transcendence due to the seeming disappearance of marked disciplines) and meta-disciplinary (where one might be able to figure out where math is relevant and explain why). Another definition of interdisciplinary studies is briefly mentioned in a study conducted by Elizabeth Post and colleagues (2022) which states, "The use of interdisciplinary education to apply quantitative reasoning across the disciplines is another best practice for reducing math anxiety" (p. 38). The authors argue that the reiteration of mathematics in courses related to a student's primary field of study allows for exposure to real-world applications. In turn, this helps students see the value of mathematics and quantitative reasoning in their professions. "When this happens, they are more likely to engage with the material because it is more meaningful to them" (Post et al., 2022, p. 42). I argue that these engagements and meaningful experiences with mathematics are essential to combating math anxiety.

This thesis will focus on interdisciplinary studies as any overlap or combination of two or more disciplines, one of which will be mathematical in some capacity. This capacity is very generalized in this definition since the math objectives of the lesson plans will come from the mathematical content standards provided by the state of Oregon, and these standards consist of varying types of mathematics from specific knowledge of algebraic computations to logical explanations of reasoning, and more (ODE, 2021).

Motivation

Math anxiety is a big problem for students and educators alike to overcome. There are lots of studies that identify this problem, and most don't have any concrete solutions (Campbell 2005, Barroso 2021, Ramirez et al. 2013). There is little research that uses interdisciplinary lesson plans to combat math anxiety. This project will serve as a pilot study to inspire others to do further research in the realms of math anxiety, interdisciplinary studies, and curriculum. This project is also of significant importance to me and my prospects. As someone who struggles with math anxiety and plans to be a future educator, I want to find creative and exciting solutions and lesson plans for my future students. Math is an applicable skill for everyone and yet mathematics education is either seen as only for those pursuing careers in STEM or as strictly academic and not relevant to the world outside of school. As a believer in the applicability of mathematics, I wanted my thesis to be in the applicable format of lesson plans that I or other teachers could use as a guide or inspiration for their own lesson plans.

An Exploration of Myths Regarding Women, BIPOC, and English Language Learners within Mathematics

We (as educators) all want the magical cure to math anxiety, a set list of rules we can follow to eliminate it from our classrooms. And while I do address a numbered list of strategies that help combat this math anxiety it won't be a permanent or all-purpose cure. And even though this list may be what you feel is the “meat” of this exploration section, it really has no real significance without knowing who we are focusing on when we talk about “math-anxious students.” Without understanding who math anxiety is impacting we won’t make any real significant change or impact for our students. According to the National Council of Teachers of Mathematics (NCTM), “mathematical teaching practices are possible only when school mathematics programs have in place a commitment to access and equity” (2014, p. 59).

Understanding these students means also breaking the stereotypes we have against students and not blaming our students for the anxiety they may be facing. These stereotypes can then lead to students experiencing stereotype threat, as mentioned in the background. In other words, we want to challenge deficit mindsets, those that put the student or family at blame for their anxiety or performance. Having these deficit mindsets is harmful because it affects how teachers see and treat their students, leading them to teach in ways that are actually more harmful than helpful for students experiencing math anxiety (NCTM, 2014). These deficit mindsets are everywhere, especially in research regarding math anxiety. An article that summarizes this point concisely states “This insistence on deficit thinking, the need to locate a shortcoming within an individual or community rather than to find causes of suffering in oppressive violent systems, is one fundamental reason that most mainstream research remains damaged-centred in its gaze on Indigenous peoples" (Jacob et al., 2021, p. 137).

In this section, we will take a look at three groups that I believe are more likely to experience math anxiety: women, Black, Indigenous, and People of Color (BIPOC), and English language learners (ELLs). This does not mean that these groups alone experience math anxiety, but rather they may be typically underrepresented or historically marginalized within the mathematics community leading to inequities within their education as well as stereotypes that both contribute to student anxiety. I first will give a brief overview of the marginalization and underrepresentation within the group and then directly address common myths that teachers and administrators hold on how we ought to teach these groups based on harmful stereotypes and deficit mindsets. These myths not only have harmful effects on how educators teach and treat these students, but they also are harmful for the students themselves as they exacerbate the anxiety these students experience by provoking stereotype threats. I begin by focusing on women in mathematics, which is an important part of my own lived experience. I then will address the marginalization and myths associated with BIPOC and second-language learners.

Women in Mathematics: Myth Busting

Women have historically been deemed insignificant to mathematical discovery. All the famous mathematicians that students are typically exposed to in math classes are male: Euclid, Euler, Gauss, Einstein, Newton, Riemann, Hilbert, Pythagoras, Archimedes… Our society recognizes these men as the sole developers of mathematics, celebrating them in books, films, and textbooks alike. The lack of representation within the mathematics community only perpetuates the stereotypes about the role of gender and mathematical ability, or the ideas that:

men are just inherently better at math, math is unfeminine, women are better at words and writing and should focus on those pursuits, and women struggle with math more and therefore don't need to worry about it for their futures. These stereotypes, while not perhaps explicitly spoken (although they very well could be), are perpetuated and circulated within our classrooms, our schools, and our communities. And these stereotypes lead us to myths about how educators ought to treat women within mathematics education. Let's address a few myths head-on and see what we find.

Myth 1: Women just aren't interested in mathematics which is why they may struggle and decide not to continue with it.

This myth has to do with the expectations we hold for women within mathematics and is based on the historical discrimination against women within mathematics (discussed more in the following myth). According to Sheila Tobias, author of Overcoming Math Anxiety, there has been a shift from the idea of a discriminatory difference that accounts for women's participation in mathematics to the idea of "choice behavior". She explains that "Girls' choices, as already noted, are linked to what they perceive to be the value of mathematics in their lives. But they are also linked to girls' expectations of how well they will do if they were to continue in math" (Tobias, 1993, p 80). In other words, the perception that women have of themselves within the context of mathematics is directly linked to whether or not they decide to continue in it. This idea of self-math is also associated with math anxiety. Elizabeth Necka, a program director at the National Institute on Aging, studied this association and concluded that "In sum, the current work demonstrates that including math in one's sense of self – self-math overlap – predicts reduced math anxiety and a decoupling of the association between math ability and math anxiety” (Necka et al., 2015, p.11). With this knowledge that women’s choice to continue in math is associated with the perceived value of math in their lives which is also further associated with math anxiety levels, it seems clear that the problem isn’t simply that women aren’t interested in math. Rather, this problem with women not continuing with mathematics is much more nuanced. If educators truly want math to be an inclusive space, then they need to encourage women instead of deeming their shortcomings as “acceptable.” Educators need to challenge their previous expectations of women in their math classes to be the same as their male peers.

Myth 2: Most/All important contributions to mathematics were made by men, so it’s natural that math classes will focus on them rather than women.

It’s clear to see that women aren’t celebrated as much in mathematics as men. In order to combat the myth, we need to understand why this is the case. First and foremost, it’s important to acknowledge the marginalization and discrimination against women within the mathematics community. According to Tobias, “The fact is, in most cultures (not just our own), women have been unwelcomed as serious intellectuals bearing skills that men want to believe are theirs alone. Once women are excluded, men are able to convince themselves that women are less talented than they are” (Tobias, 1993, p. 85). So, it’s only natural that our celebrations of mathematics have been male dominated. What’s not natural is that math education continues to create exclusionary environments for women. There have always been women in mathematics who have made important contributions. A few great names to search include Hypatia of Alexandria, Sofya Kovalevskaya, Emmy Noether, Katherine Johnson, Dorothy Vaughan, and Mary Jackson just to name a few. As Paul C. Gorski, founder of the Equity Literacy Institute and EdChange, argues “Simplistic instructional strategies, absent a commitment to more robust institutional change, are no threat to inequities" (Gorski, 2017, p. 34). While Gorski here is talking about the inequities that students in poverty face, this principle is applicable to all inequities faced. In other words, if we continue to teach about the mathematical contributions of men because it's simple and it's what's been done before, we aren't making any real change and we're letting mathematics classrooms become an inaccessible and exclusionary space.

BIPOC in Mathematics

There are also many stereotypes associated with people of color and math ability, and sometimes these stereotypes can be perceived as complementary but have underlying connotations that are harmful. One such stereotype is that Asians are good at math, or more generally the idea of the model minority, that Asians have more grit, work harder, and are therefore more likely to succeed in American society than other marginalized/minority groups. While this might seem like a positive stereotype, the connotations and effects on math anxiety are the same as stereotypes associated with Black people and math performance, for example that Black people are bad at math. In both scenarios, stereotype threat can be triggered, increasing the pressure to perform well so that the stereotype is either upheld (Asians are good at math, therefore I should be able to be good at math) or to prove that stereotype wrong (I don't want people to think Black people are bad at math, let me show them I'm good). Either way both of these are going to have an effect on the student's anxiety and pressure to perform well, potentially leading to "choking under the pressure" as mentioned in the background. These stereotypes can then inform our teaching practices, again only exacerbating the anxiety our students face ((Beilock et al., 2004).

Myth 1: Racism and segregation is a thing of the past in schools so therefore the opportunities for everyone are equal.

It may be easy to think that segregation in education and more generally racism are things that were long-lost history only read about in textbooks, especially as a young, White teacher. While there has been progress made in terms of racism within schools, there are still significant achievement gaps between BIPOC students and their White peers. Claude Steele and Joshua Aronson, who study stereotype threats among African Americans, explain that "Gaps in school achievement and retention rates between White and Black Americans at all levels of schooling have been strikingly persistent in American society" (Steele & Aronson, 1995, p. 798). We can currently still see the effects of racism and segregation within our schools, meaning that this is not a problem of the past, but a very current and real problem.

It's also important to realize the difference between equality and equity within schools. Equality may appear to exist within schools given that everyone has the right to free public education. Equality, or equal treatment differs from *equity*, which means that everyone receives the resources they need to be successful in school. These resources will look different for different students. NCTM explains "Equity in school mathematics outcomes is often conflated with equality of inputs. Providing all students the same curricular materials, the same methods of teaching, the same amount of instructional time, and the same school-based supports for learning is different from ensuring that all students, regardless of background characteristics, have the same likelihood of achieving meaningful outcomes" (2014, p. 60)

Therefore, we need to change our goal of equality in education to be equity in education, so all students are given the opportunity and access to resources to succeed.

Myth 2: The BIPOC students who do well in math have only their grit and determination to thank for it.

Duckworth explains that grit is a combination of passion and perseverance applied toward achieving long-term goals (2020). Grit mindset might seem complimentary but is actually problematic. It shifts our focus away from larger structural issues of racism within education to the individual. While this may seem like a beneficial focus on the individual it can quickly turn into a deficit mindset of the individual, such as blaming an individual on their performance due to their lack of grittiness. Gorski argues that educators need to “understand that educational disparities result not from mindset mismatches or cultural deficiencies, but from inequities. Eliminating disparities requires us to eliminate inequities rather than change students’ mindsets or cultures” (Gorski, 2017, p. 25). By only attributing success to grit, means missing out on a bigger, more structural view of how BIPOC communities are marginalized within mathematics.

Myth 3: Most/All important contributions to mathematics were made by White people, so it’s natural that math classes will focus on them.

We can debunk this myth in the same ways we debunked our myth about men who supposedly make all the contributions in mathematics. First, by acknowledging the marginalization and discrimination against BIPOC within the mathematics community. And secondly, by troubling our own biases and stereotypes we hold by doing the research and finding the people who were lost in history to the exclusionary textbooks, classrooms, and generally the field of mathematics. A few names I recommend searching up include Benjamin Banneker, Gladys West, Mary G. Ross, Abu Kamil, and so many more. The following websites are also great resources: http://www.ams.org/about-us/edi-community, https://indigenousmathematicians.org/, https://www.nps.gov/people/mary-g-ross.htm.

English Language Learners in Mathematics

Many have probably heard the idea that math is a language, one with its own symbols and conventions. And for some this idea makes math seem more feasible, yet others might find the idea of learning a new language is just another anxiety trigger. Whether you think of math as a language or not, we can agree that learning any new language has its own challenges and can be another potential anxiety trigger for students. So, we can imagine that having to learn a new language alongside mathematics can be double the anxiety. In this section, I will focus on English language learners (ELLs) within math classrooms. There are many stereotypes associated with ELLs and their mathematics ability: that they need easier math, need to be separated from peers, etc. From these stereotypes, teachers can derive many assumptions for how to teach ELLs math. Let's investigate a few common ideas that math teachers may believe.

Myth 1: Language and math are separate disciplines that shouldn't be learned together.

This way of teaching is still common in how mathematics is being taught, by acting like math is its own language, and this language is "universal" so therefore it should be understood by everyone. This argument quickly breaks down when we consider the level of in-depth thought and reasoning that is involved with mathematics. Once students reach calculus, they are potentially engaging in proofs which require students to be able to explain why and how certain ideas work. And in almost any level, math is full of specific vocabulary and grammatical structures. Research-based approaches for teaching math for English language learners have noticed the importance of using math classrooms as a space to practice language skills. Kathryn

Chval and Óscar Chávez (both professors at the University of Missouri-Columbia) explain, "researchers in bilingual education argue that the development of English as a second language in school can be better achieved when students use it while working to understand one another's meanings during discussions or while solving problems" (2012, p. 261). NCTM also argues that "Students who are not fluent in English can learn the language of mathematics at grade level or beyond at the same time that they are learning English when appropriate instructional strategies are used" (2014, p. 63). Math and language are interconnected and inseparable, therefore they need to be taught in conjunction with one another.

Not only should language and math learning be connected, but the many aspects of students' identities should be respected, considered, and learned within mathematics teaching. NCTM explains that "Effective mathematics instruction leverages students' culture, conditions, and language to support and enhance mathematics learning" (2014, p. 63). This idea is known as social justice mathematics, where students are the problem posers and community change-makers and teachers are the creators of mathematical curriculum that challenges societal inequalities with a goal of making mathematics an inclusive space for all (Conway et al., 2023).

Myth 2: ELLs should be allowed to work by themselves and/or participate as a spectator so they aren't put on the spot or made uncomfortable.

This myth has to do with changing the expectations of ELLs and making it so they have less opportunities to learn than their peers. To debunk this myth, we need to see how giving ELLs' opportunities to learn in the classroom affects their performance in math. Opportunity to learn (OTL) is defined as "students' access to and engagement in the academic content" (Abedi & Herman, 2010, p. 726). Jamal Abedi and Joan Herman, authors of Assessing English

Language Learners' Opportunity to Learn Mathematics: Issues and Limitations, studied this very idea and concluded "Students who experience more difficulty may report lower levels of OTL because they do not understand or thus recognize topics that have been addressed in class. Such differential levels of OTL may be a major factor contributing to the substantial performance gap between ELL and non-ELL students" (Abedi & Herman, 2010, p. 724). In other words, ELLs who are struggling in classrooms are not being given opportunities to learn because it is assumed they are not going to understand the topics of the class, or that they will be made uncomfortable if they participate. The authors claim that because of this lack of opportunity, ELLs are falling behind their non-ELL peers.

Now that we understand that opportunities to learn are crucial for ELL students, we also need to consider how do we give these opportunities in ways that don't embarrass students or make them uncomfortable. Chval and Chávez give great ideas for how to do this including, "carefully selected partnerships, recognizing that some students who dominated partnerships would not help ELLs gain confidence in group activities" (Chval & Chávez , 2012, p. 264). They also discuss the connection of language with mathematical representations as a way to support ELL students to have more structure and confidence to share during whole class discussions, "for example, by displaying tasks on the SMART Board while the ELLs shared their solution strategies" (Chval & Chávez, 2012, p. 264). Again, this demonstrates the importance of math and language needing to be disciplines that are learned together as a support for ELL students.

A Note on Intersectionality

It is also important to consider the intersectionality of these identities. Intersectionality is a term coined by Kimberlee Crenshaw in 1989 within the field of Critical Race Theory. The word is used to make the intersections of identities easier to discuss and understand. “Intersectionality has given many advocates a way to frame their circumstances and to fight for their visibility and inclusion” (Crenshaw, 2015, p. 2). Besides this, intersectionality can help frame certain intersections of identities that hold power in society, for example a White man, whose first language is English still experiences intersectionality but isn’t being marginalized because of these identities but rather is experiencing power and privilege. Within this thesis, intersectionality is only briefly mentioned but is something that I challenge the reader to explore further on their own. As a starting point, consider the following: How might intersectionality influence the myths and stereotypes mentioned above?

Teaching Strategies to Combat Math Anxiety

As promised, a numbered list of teaching strategies to combat math anxiety follows. Many of these strategies have been in my own experience tutoring students and to others in the field of math education. This list is not all encompassing and is not perfect. These strategies should be thought of in the context of the previous section about the marginalization of students within mathematics as well as your own experience. Without this contextualization this list is just another set of simplistic teaching strategies that will make no real change in our pursuit to combat math anxiety.

1. **Show belief in students through positive reinforcement.**

Once a student reaches high school, they have already had nine years or more of math classes and teachers. They already have lived experience, whether good or bad, in learning math. And while it is important to acknowledge this lived experience (see Teaching Strategy 7), educators also have to potentially combat the negative experiences of their students. In my own experience, when I reached just middle school math, I had already been told by teachers that I was not good at math, that I did not belong in advanced classes, and that I should try to be good at something else. On the other hand, I also had good experiences with math, my mother made games and puzzles out of my homework. It always seems easier to remember the bad experiences over the good ones. Beilock explains, "Of course, there are many sources from which negativity about math could develop—ranging from parents to the media. But, clearly, information about positive and negative aspects of math can be found in the classroom, and it seems, at least at first glance, that not only do kids pick up on this negativity, but it also carries implications for their math achievement across the school year" (Beilock & Willingham, 2014, p.

31). The point is that we (educators) need to believe in our students' ability to do math. We have to counteract and challenge all the previous people who did not.

2. Quality over quantity.

It is easy to get sucked into an everyday regimented math class, to stick with the ways math has been taught, especially with our current agendas and lists of standards that are expected to be learned by the end of the year. Through my example lesson plans I will show that it is possible to fulfil teaching agendas and stick to standards while still giving time for students and giving high quality problems. According to Steven Reinhart, a middle school math teacher,

> Good discussions take time; at first, I was uncomfortable in taking so much time to discuss a single question or problem. The urge to simply tell my students and move on for the sake of expedience was considerable. Eventually, I began to see the value in what I now refer to as a 'less is more' philosophy. I now believe that all students learn more when I pose a high quality problem and give them the necessary time to investigate, process their thoughts, and reflect on and defend their findings (Reinhart, 2000, p. 479).

This same idea has been reiterated by NCTM who explains the importance in "providing students with practice on a small number of problems, 'spacing' or distributing these over time, and including feedback on student performance support learning outcomes" (2014, p. 45).

3. Build trust in intuition.

Intuition for mathematicians is considered an essential. Historically, when mathematicians made new discoveries and inventions, they used the generate-and-test procedure (what many today call guess-and-check). And through this procedure, mathematicians were able to generate new and innovative mathematics. This same feeling of discovery should be incorporated into math classrooms. Ard Van Moer, author of the article Logic and Intuition in Mathematics and Mathematics Education explains

> If mathematics is taught as a purely deductive science and pupils and students are told only how they should perform certain dull and mindless activities, then the students will perceive mathematics as an impersonal science. In this case, students are not given the opportunity to experience the kick of mathematical invention by themselves (Moer, 2007, p. 178).

Yet as Van Moer explained, in math classrooms it is rare to hear any discussion of intuition's importance. It is especially important considering its connections to math anxiety. According to Sheila Tobias, author of Overcoming Math Anxiety, "Math-anxious people seem to have little or no faith in their own intuition" (1993, p. 66). By building student's trust in their intuition, educators can help combat the initial anxious thoughts students have when using their intuition.

4. **Never pick up the pencil (do not give students the answers).**

While giving an answer might make one think they are reducing the anxiety of their students, it is not helping students learn for themselves. And this missed opportunity robs them of a sense of fulfillment that comes from understanding and working through a problem in its entirety. Ana Torres, a sixth-grade math teacher also mentions the importance of allowing for opportunities for students to trust not just their own intellect, but their peers as well (Ruef & Torres, 2020, p. 724). This again builds on the idea of building trust in student's own intuition as well as the intuition and ideas of their peers. Interaction with peers is also a critical point in combating math anxiety and overall mathematics learning. The job of teachers is to not give answers but find ways to facilitate conversation among students. According to NCTM, "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (2014, p. 10). When a teacher picks up a pencil and gives an answer this not only takes away from their individual though processes but also meaningful mathematical discourse among peers.

Not giving answers is also helpful for teachers so they can figure out the reasoning and ideas of their students. Particularly when a student explains how they came to an incorrect solution can this be extremely useful, since then the teacher can know exactly where the breakdown in the math occurred. Reinhart explains, "By listening to them, I not only give them the opportunity to develop deep understanding but also am able to develop true insights into what they know and how they think" (Reinhart, 2000, p. 57). By simply not picking up the pencil and showing students the answer, we allow them to experience deep thinking and collaboration skills.

5. **Ask Questions.**

Asking questions is a great alternative to not giving answers. When a student is stuck, instead of giving a straight answer, a teacher can give a hint in the form of a question. Asking the student to consider something they may have not thought about before, but still allowing them to think and understand for themselves. Asking questions also lets the teacher know how stuck or how much help that student actually needs on the problem. Reinhart explains in his own classroom, "To find out what a student understands, I ask process questions that require the student to reflect, analyze, and explain his or her thinking and reasoning" (Reinhart, 2000, p. 55). In other words, these questions need to have a direct purpose for students. NCTM explains that "Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships" (2014, p. 10).

6. Patience

I argue patience is one of the most important skills any type of teacher must be able to utilize in their classroom. But it is especially important when working with math anxious students who may need more time to combat their first initial anxious thoughts before being able to actually consider and think about the task at hand. After a prompt, giving wait-time is critical to helping all the students in the classroom, whether they are anxious or not. Reinhart explains, “Even very capable students can begin to doubt their abilities, and many eventually stop thinking about my questions altogether. Increasing wait time to five seconds or longer can result in more and better responses” (Reinhart, 2000, p. 55). It is also especially important to consider wait-time and patience when asking questions. NCTM explains that after asking purposeful questions teachers should be “Allowing sufficient wait time so that more students can formulate and offer responses” (2014, p. 41).

7. Give students outlets to express their anxiety (math autobiographies, journaling, drawing/art, etc.).

This strategy may seem like a waste of time to some, but actually can prevent time being wasted on anxious thoughts when problem solving. Starting off a class period with a quick five-minute time to reflect in a form of a student’s choosing can let students refocus their minds to get ready for math learning. Not only is this a potential outlet for math anxiety, but really any other anxieties students may carry with them into the classroom that may distract them from learning. There are many case studies that have tried this outlet of writing or drawing as a way to combat math anxiety. One such example is in the case of Danielle an elementary student of David Rufo who used self-administered art therapy and comic drawing as way to combat her anxiety and “hate” towards math. Her use of art as a way to vent, eventually lead her to become a math helper in the classroom (Rufo, 2017).

Another example is from Tobias’ Overcoming Math Anxiety where she describes clinics run by Dr. Frances Rosamond at Cornell University, “she insists that they keep a journal in which they freely ‘vent’ what they’re feeling about her class and about mathematics generally” (Tobias, 1993, p. 234). Dr. Rosamond keeps the journals of her students and reads them as a way to get to know her students better, a great way for her to assess what anxieties her students hold. While this may be beneficial for some classrooms, it may also be useful to keep students journals private so they can fully express their feelings without feeling limited.

And for some students, this could look like communication with others, allowing times to vent and feel heard by their peers and by the teacher. Giving students a few minutes to share with the person sitting next to them how they are feeling and be able to console one another.

8. Remove timed tests, find other means of assessment

There are numerous studies that have proven timed tests have negative effects on student’s performance levels. One of these researchers is Beilock who has studied specifically the negative affect on math anxiety, “Research has shown that math anxiety is more strongly linked to poor performance when students take a timed test” (Beilock & Willingham, 2014, p. 31). NCTM also warns against the use of timed tests explaining “The premature and overuse of such tests may hinder students’ mathematical proficiency and lower their confidence in themselves as learners of mathematics” (2014, p. 45).

If we claim we want to combat math anxiety and we know that timed tests are a potential trigger for math anxiety, then why wouldn’t we want to find other means of assessment? Many educators may argue that this is just the way that math has been historically taught and has been in place for centuries. (I argue that this is exactly the reason we ought to change.) In the previous section we combated myths and problems associated with the historical marginalization of students within mathematics. If we claim that this discrimination is something we want to change then we cannot keep the current system we have in place.

The following is a discussion of some historical and current context of the racism in place within standardized timed testing. I hope through this discussion we can see why we must change the system we have in place for something that is a more equitable means of assessment. Wayne Au, author of Testing for Whiteness? How High-Stakes, Standardized Tests Promote Racism, Undercut Diversity, and Undermine Multicultural Education explains that "the lineage of high-stakes, standardized testing can be traced directly back to intelligence (or IQ) testing and the racism, classism, and sexism of the eugenics movement of the early 1900s" (Au, 2020, p. 100). IQ testing has historically had a wide score gap between White people and the BIPOC community, and many White supremacists use these gaps to claim the inferiority of intelligence of the BIPOC community. Instead, we have to consider what other factors determine one's success on IQ testing, for example the individual's overall health. This can be affected by the nutrients (or lack thereof) within food, "Typically, racial and ethnic minority groups--defined here as Black or African American, Hispanic, Asian, and American Indian/Alaska natives --experience diet-related disparities, and consequently tend to have poorer nutrient profiles and dietary behaviors and patterns relative to whites" (Satia, 2009, p. 2). Health is also affected by access to healthcare, according to a study titled Residential Segregation and Availability of Primary Care Physicians they found that African Americans and Hispanics are less likely to have access to primary health care providers and hospitals due to residential segregation. And even if health care resources are available in nearby communities, they may not be geographically accessible by public transportation and "residents of these poor inner-city neighborhoods may not feel welcomed in more affluent neighboring communities" (Gaskin et. al., 2012, p. 2369). Health is also affected by the person's physical environment they live in,

> "Communities of color therefore end up concentrated in areas that face greater environmental harms and are more vulnerable to natural disasters, while the forces of residential segregation create systemic barriers that make it more difficult for individuals to move to less environmentally harmful areas" (Berkovitz, 2020, p. 3).

All of these racial disparities affect the health of BIPOC people and are also going to affect their IQ scores. I claim the exact same argument can be said of standardized and timed testing with added disparities within the education system such as lack of access to tutors, high quality textbooks and materials, and even certified and experienced teachers.

Within the lesson plans to follow I provide a few examples of other means of assessment, specifically the idea of project-based learning (PBL). For more information on PBL visit www.pblworks.org.

9. Make math applicable

Not every student is going to be a mathematician, some will want to be biologists, artists, journalists, nurses, policy makers, engineers, the list goes on. Students may think that math does not offer them anything for their future career choice, and the reason for this is that math is not being taught in a way that shows that it is. That is why educators need to teach math as more applicable, and I argue the best way to do this is through interdisciplinary education. There are many examples of interdisciplinary education that exists and is successful. For example, one study titled A Study Assessing the Potential of Negative Effects in Interdisciplinary Math-

Biology Instruction looks at the possible adverse effects of interdisciplinary studies within mathematics and whether interdisciplinary studies could cause math anxiety. The conclusion the study comes to is that adding math calculations to biology learning does not hinder learning and can increase disciplinary learning and data interpretation skills (Madlung et. al., 2011). Imagine if students were able to use mathematics in disciplines that they were already enthusiastic and confident in. This confidence and passion can help combat their math anxiety and help them find the applicability of mathematics to their own lives.

Making math applicable also means that it is applicable to the identities of all students. While exploring myths, I introduced the idea of incorporation of aspects of students' identities within the mathematics classroom. Another important addition to this idea is making math appliable to the community it is in. NCTM explains that "Effective teachers draw community resources to understand how they can use contexts, culture, conditions, and language to support mathematics teaching and learning" (2014, p. 65).

This also means keeping in mind the context which mathematics is being taught, for example the environments and situations of mathematical word problems. There are numerous word problems which use inappropriate and inequitable contexts for students to grapple with. Gorski gives the example of a mathematical word problem's use of the word "portage." This problem was on a standardized test used in urban Minneapolis, where many students may have never experienced camping or boating in the more rural areas of Minnesota. In other words, these students were having to manage a test "designed with somebody else's life in mind" (Gorski, 2017, p. 120). To make math appliable we need to be aware of the contexts, lives, and communities of students and their families to make the problems they solve meaningful rather than another barrier or distraction.

10. Be excited! Combat your own math anxiety!

Anxiety is contagious and is easy to spread. Specifically, if we as educators are anxious then we can potentially spread this anxiety to our students. Beilock explains this concept, "Knowledge that a teacher's math anxiety can affect her students' math achievement suggests that we also need to ensure that teachers feel fully confident in their preparation to teach math" (Beilock & Willingham, 2014, p. 31). Beilock suggests teacher training is key to giving teachers the confidence needed to teach their students, even expressing that experienced teachers will benefit from training as they "may be asked to teach new material" (Beilock & Willingham, 2014, p. 31). NCTM also agrees with this idea of teacher training explaining that "some teachers' lack of deep understanding of the context that they are expected to teach may inhibit their ability to teach meaningful, effective, and connected lesson sequences" (2014, p. 71). Without having a deep understanding and connection to mathematics, teachers can find themselves feeling not prepared and limited in what types of teaching they are capable of. In other words, without teacher training and combating a teacher's own anxieties, none of the other strategies above are going to be effective.

NCTM proposes a solution to lack of preparation and combating anxieties of teachers through collaboration. "Mathematics teachers are professionals who do not do this work in isolation. They cultivate and support a culture of professional collaboration and continual improvement, driven by an abiding sense of interdependence and collective responsibility" (2014, p. 99). This means that teachers not only want to improve themselves and their own classrooms but encourage and work with their other fellow teachers to help them combat their own math anxieties.

Overall, teachers need to be able to first focus on their own limitations and anxieties through working with their peers as well as a mind-set of continued learning, that “they are lifelong learners and doers of mathematics” (2014, p. 99).

Introduction to Interdisciplinary Lesson Plans

This next section is the bulk of the thesis, the interdisciplinary lesson plans. The lesson organizer used for these lesson plans is adapted from the companion website for High School Mathematics Lessons to Explore, Understand, and Respond to Social Injustice by Robert Q. Berry III, Basil M. Conway IV, Brian R. Lawler, and John W. Staley (2023). Included within each lesson plan are two parts. Part I contains sections on the content, context, and other organizing ideas for the lesson plan. Within the content section, I've included some essential mathematical concepts and standards for mathematical practice, related Oregon Math Standards, related strategies to combat math anxiety, and social justice issues or standards. Within the context section are some ideas for the purpose, audience, and allies for the lesson plan as well as some organizing ideas for timing and type of lesson plan. Part II contains the bulk of the lesson plan divided into three main ideas, (1) Introduction/Engagement, (2) Investigation/Exploration, and (3) Share and Discuss. Within each section has explanations and guided notes on what the teacher will do and what the student will do. After the lesson plan organizer are corresponding worksheets that are referenced within the organizer. Note that these lesson plans are meant to act as a guide as you teach these concepts, they can and should be changed and adapted as seen fit for your own classroom.

The first interdisciplinary lesson plan, titled Mathematical Ecologists, gives students the chance to explore ideas that real ecologists at the University of Oregon are exploring and connecting this to the mathematical concepts of dilations and graphing of circles. This lesson plan was inspired by my own summer research at the University of Oregon's Hallet Lab working with Dr. Madelon Case and Dr. Lauren Hallett. Within my own research, I looked at satellite imaging via coding in R (a statistical analysis software) to determine the effects of cattle watering holes on invasive vs. native species surrounding the area. As a part of my project, I had to figure out the code needed in order to scale up and down the radius I was mapping/graphing around the watering hole in R. When I had the code to do this, I was then able to extract values of invasive vs. native species within specific radii and figure out the radius of effect these cattle watering holes had on the land. The lesson plan focuses on the mathematical skills needed to dilate circles, which is essentially the same exercise I went through when coding these different radii. The maps and graphs included within this lesson plan are from the exact coding I did within R that summer. I hope that students who love ecology, plants, agriculture, coding, and cows will find this lesson plan exciting and fun!

The second interdisciplinary lesson plan titled Mathematical Literature and Art is a lesson influenced by the idea of Project Based Learning (mentioned in teaching strategy 8). This lesson plan includes a guided reading of Flatland by Edwin Abbott. My hopes for the questions within the guided reading is for teachers to use this as best suited for their classrooms. This might look like giving students time to read and discuss the questions in class, it could look like giving students smaller chapters to read or listen to the audio book at home, it could look like a combination of both of these, or something completely different! I hope that this acts as a starting point for a lesson plan that inspires students to enjoy the wonders of reading. After the reading of Flatland, I want to give students the opportunity to create their own works of mathematical literature or artwork. This lesson plan was inspired by my many experiences and classes within college. I first read Flatland in Dr. Christopher Sinclair’s class, The Aesthetics of Mathematics, where we had a final project of our choice demonstrating mathematics in any other discipline. In Dr. Ellen Eischen’s Creativity Counts class I had the opportunity to work with my peers in creating a mathematical art exhibit that was displayed in the art museum on campus.

These experiences and numerous others were what first inspired me to look towards interdisciplinary studies as a combatant of math anxiety. I hope that students who love art, literature, reading, writing, and creativity will love this opportunity to use their own unique experiences to create a project they will be proud of and remember for years to come.

Mathematical Ecologists Lesson Plan

PART I

<table>
<tr><th>CONTENT</th><th></th></tr>
<tr><td>Essential Concepts
Geometry and Measurement:
Measurement of Circles
Transformations of Circles
Solving Applied Real-World Problems and Math Modeling in Geometry
Standards for Mathematical Practice
<li value="1">Make sense of problems and persevere in solving them.
<li value="2">Reason abstractly and quantitatively.
<li value="3">Construct viable arguments and critique the reasoning of others.
<li value="5">Use appropriate tools strategically.</td><td>Oregon Math Standard(s)
HS.GM.A.2 - Verify experimentally the properties of a dilation given a center and a scale factor. Solve problems in authentic contexts involving triangles or dilations.
HS.GM.C.8 - Solve authentic modeling problems using area formulas for triangles, parallelograms, trapezoids, regular polygons, and circles.
HS.GM.C.11 - Apply concepts of density based on area and volume in authentic modeling situations. (Not currently incorporated but could be modified to do so.)</td></tr>
<tr><td colspan="2">Strategies to Combat Math Anxiety
Quality over quantity.
Ask Questions.
Patience.
Remove timed test, find other means of assessment.</td></tr>
</table>

- Make math applicable.

CONTEXT	
Purpose Give students an opportunity to learn how to calculate and graph dilations of circles within the real-world context of ecology.	
Audience High school geometry class	
Allies • University of Oregon Hallett Ecology Lab (real world ecologists/researchers) • Biology/Environmental/Science Departments within your high school	
WHEN (in unit)	
Circle one: Beginning **Middle** End Special	**Number of periods:** 5 or more
HOW	
Circle one: Mathematics Tasks **Three-Act Tasks** Project-Based Learning Other	

PART II

Introduction/Engagement:

Show Space Cowboys video https://youtu.be/iyevp6DZ2HM

Discussion Questions:

- What are some of the major ecological questions and management challenges discussed in the video?

• How can satellite imaging help answer these questions/challenges? • What questions did you have from the video (vocab, processes, etc.)? • What kinds of math might we be able to use to extract and interpret data from these satellite images? • What math do you think might be associated with ecology in general?

What will the teacher do?	**What will students do?**
• Divide students into pairs/small groups. Have them designate one person to write down ideas and another to share the groups ideas aloud to the class. • Display discussion questions. • Give students time to explore these questions, walk around the room and ask questions to get students started. • Once conversations have diminished or gone off topic shift to whole class discussion. • When students share their own questions or comments, pose the question to the entire class before giving any answers. This could also be an opportunity to go over any additional background	• Discuss with partner or small group the discussion questions. • Write down ideas of what kinds of math might be applicable. Examples of potential responses might be data analysis via statistics, geometry (shapes of plots), finding density of grasses, graphing and maps to display patterns, creating models using patterns, scale/dilation (looking at smaller areas to be applied to larger areas). • Share ideas aloud with class. • Ask questions when needing additional background information, vocabulary, and other curiosities.

information and special vocabulary needed for the lesson (for example invasive vs. native definitions). • Present with excitement an introduction to the next part of the lesson: "Today we are a part of the Hallet lab at the University of Oregon, and we are mathematical ecologists that are exploring cattle grazing effects on native and invasive species in eastern Oregon. Has anyone been to eastern Oregon? This environment is actually very similar to the one from the Space Cowboys video, with long expanses of grasses, sagebrush, and some juniper trees. In order for cattle to be able to live in such a dry environment there needs to be manufactured water sources for them, like in this picture. This is called Powerline Waterhole." • Display Figure 1 for class.	• Analyze the photo of the cattle watering hole (Figure 1) and write down observations and predictions of what else might be observed.

• "Because water is essential to life, cattle are going to spend a lot of time at and around these watering holes. What do you think might be affects that cattle have on the area around these watering holes? Use this picture to help you." • Give students time to individually write down observations from picture and other things that they think they may or may not find around cattle watering holes. Have them share out and explain their reasoning as to why they think they may find/not find certain things around watering holes. • "These are some interesting predictions/hypotheses you've come up with. These are the exact kinds of questions that the Hallet lab is trying to answer. The one question we will focus on is: how does cattle grazing effect invasive vs. native plants? Our job as mathematical ecologists, who will help	

to answer this question, is to figure out what the radius of effect is for native vs. invasive species. In other words, how far can we go from a watering hole and still see the effects that cattle have on the invasive and native species? We are going to try to find this out!”	

Investigation/Exploration:

Lesson 1:

Display Figure 2 for class. What do you notice? What do you wonder?

Discussion Questions:

- What do you notice happens to the number of invasive species as you are closer to the watering hole?
- What about farther away?
- Do you see any other patterns?
- What math/geometry could be useful here?
- How can we break up the map to better analyze the data based on our goal of finding the radius of effect?

Lesson 2:

Worksheet 1/Presentation Instructions

Lesson 3:

Worksheet 2

What will the teacher do?	What will students do?
Lesson 1 • Display Figure 2 for class. • Ask students: What do you notice? What do you wonder? • Allow for think-pair-share. • Reveal information about the map as needed: "This map was created using satellite imaging and some coding in R, a programming language typically used to analyze data through the creation of graphs. This is a map of Powerline watering hole, actually the same watering hole displayed earlier, and is located in eastern Oregon. This map is displaying the percent coverage of annual forbs and grasses (AFG) which are invasive species. Each one of these small boxes are pixels and each pixel is about 30m worth of land. The white color means there is 0% annual cover, i.e., there is bare ground here. The	**Lesson 1** • Make observations, predictions, and questions about the map individually, with a partner, and with the class. • Discuss in small groups and partners the discussion questions.

orange color means there is around 5-10% of annual cover, yellow is 10%, green is 15% or more. Our x-axis is longitude, and our y-axis is latitude." Model for students on board, PowerPoint slide, or hand out. • Have students break out in small groups and go over discussion questions.	
Lesson 2 • Give some introduction to the project worksheet and instructions: "Our job as mathematical ecologists is to develop the mathematics needed to be able to analyze the data collected from satellite imaging. To do this our primary objective is to plot graphs that will be able to show us different radii around the waterhole. The graphs we plot will then be used with coding in R to be able to extract percentages of cover from different radii. We need to come up with	**Lesson 2** • Work in small groups to create a plan to present to the class on what mathematics we predict will need to be used to accomplish the goal of how we can plot different radii around the watering hole and explain why. • Use the guided worksheet to help organize our thoughts and presentation. • Present a brief plan to the class

a plan of how we can plot different radii around the watering hole." • Have students break into small groups (3-4 students). Their job is to create a written plan of what mathematics they think they will need and what measurements, equations, and tools they will need to accomplish their plan. This is purposefully vague instructions for your students, this can allow students time to explore the problem for themselves and work on teamwork skills. Give students hints if they need more structure. Have students explore this and come up with a plan on their own of what they might need to explore this and create their own mathematics! Don't give them the answers but have them work together to come up with a rough draft of a plan they think might be able to solve this problem! You can ask them questions or give them more	

information if they get stuck, but really let them try to figure things out! Then once they have written plans, have them share them with everyone else, maybe calling attention to things you know are important. Even if they don't get the "right" answer, that's fine! Then after students have had time to "play" with these ideas, go on to the next section showing the plan. • After students have come up with their plans have them do mini presentations to the class of their ideas. • After everyone has presented make a collective list of students' ideas that will be helpful in solving the problem.	
Lesson 3 • Divide students into teams of 3-4 students to work through Worksheet 2.	**Lesson 3** • Work in small groups on Worksheet 2, where we implement an outlined plan to accomplish the goal of how we can plot different radii around the watering hole.

<table>
<tr><td>

- Give additional instruction or help for using Desmos depending on your students' needs.
- Give additional hints, instruction, help for scale factors depending on your students' needs.

</td><td>

- Use scale factors and dilation to accomplish this goal.
- Identify that to apply a scale factor to a circle we need to multiply by the radius.
- Use Desmos to graph our final product.

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Share and Discuss:

- Display Figure 5
- Display Figure 6
- What do you notice? What do you wonder?
- Reflection Activity

</td></tr>
<tr><td>

What will the teacher do?

- Display Figure 5 and Figure 6. Ask students what do they notice and what do they wonder?
- Slowly reveal information as needed: "This is a map showing annual forbs and grasses (invasive species) with the plot of our 4 original points and a 100m radius circle. There are more annuals closer to the waterhole at 100m and they decrease as you go out to 500m".

</td><td>

What will students do?

- Make observations and predictions about the graphs displayed using knowledge from the beginning of the lesson as well as the work done with dilation of circles.
- Pick one of the three options to reflect on the lesson, discuss what I learned, what I liked/disliked, and/or my own thoughts/feelings regarding math anxiety.

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</table>

• Give students 3 options to reflect on their experience with the overall lesson. Give about 5-10 minutes to do this reflection activity. (1) Have students write a quick one-page or less reflection, (2) have students draw a picture and add a few sentences of what the picture means to them within the context of reflection, or (3) have students use a laptop, phone, tablet, or other recording device to record a quick reflection. This reflection can be mathematical and based on what concepts the student learned, what the student liked/disliked about the lesson plan, or even their own thoughts/feelings regarding math anxiety. Perhaps using a What, Who, When, Where, Why, How, Impact structure. Asking questions such as: What did we accomplish? Why does what we did matter? Who does this matter to? Etc.	

Ecology Worksheet 1

Goal: We need to come up with a plan of how we can figure out the radius of effect around our watering hole. In order to do this, we need to be able to plot different radii around the watering hole. In other words, we need to create new circles of larger and larger radius around the watering hole.

Instructions: Use the guiding questions below to come up with a quick presentation on how you might go about accomplishing the goal above.

- How might we be able to plot new radii around the watering hole (of larger and larger radius)?

- What would we need in order to plot these new radii?

- What tools would you need to plot this?

Ecology Worksheet 2

Goal: Use dilations and scaling to plot different radii around the watering hole.

Instructions: Work through the following plan that shows one way of how we can plot different radii around the watering hole.

Background information:

Longitude (x-axis)	Latitude (y-axis)
-119.3705	43.55109
-119.3709	43.55123
-119.3716	43.5511
-119.3712	43.55098

Table 1: Original Plot Points of the Watering Hole

- Center = (-119.371, 43.5511)
- Approximate radius = .00035

Step 1: Initial Plotting in Desmos

- Plot the original plot points of the watering hole (from Table 1).
- Plot the graph of the approximate circle of the watering hole using the approximate radius and center provided above.
- Make observations for how close of an approximation this is to the original plot points of the watering hole and ideas of how you could make it better (Perhaps a different shape than a circle?).

Step 2: Scaling

- We want to scale our circle to find different radii, specifically the lab would like to have 100m, 200m, 300m, 400m, and 500m added to the original radius.
- To scale a circle (or any shape) we need to know how much we want to increase our original shape (through multiplication). Do we need to make it 2 times the original circle? 3 times? 4 times? The amount we want to scale by is called a **scale factor**.
- The equation of a scale factor is:

$$scale\ factor = radius\ of\ scaled\ circle\ \div\ radius\ of\ original\ circle$$

- Let's find the scale factors for the circles with radii of 100m, 200m, 300m, 400m, and 500m added to the original radius. (In other words, if we know the original radius is .00035 degrees in longitude/latitude we need to convert this into meters and expand our circle out 100m (add 100m to the original radius) and do this same step for 200-500 meters added.) You will need the following conversion:

$$100m = .00089\ degrees\ in\ longitude\ or\ latitude$$

(Hint: $radius\ of\ scaled\ circle = radius\ of\ original + added\ radius\ (converted)$.)

Step 3: Applying the Scale Factor

- Now that we have found all of our scale factors, we want to be able to apply these scales to our original circle to create new circles.
- Brainstorm: Where in our equation of a circle do you think we will multiply our scale factor by? (Feel free to ask another group questions if you get stuck here!)

Answer: The ____________! The equation of a circle is ______________________________ where (h, k) is our center and r is the radius. And we can apply the scale factor by multiplying by ____________.

- This process is called dilation, which is a type of transformation that changes the size of a figure and requires a center point and scale factor.
- Now that we know how to apply scale factor, write the equations for your new 5 circles of 100m, 200m, 300m, 400m, and 500m added radius.

Step 4: Plotting Our New Circles in Desmos

- Plot the 5 equations from the previous step in Desmos!
- First try plotting them all separately and then try to plot them all on the same plot.
- Try plotting all 5 equations as well as the original circle with the original points from Step 1.
- Play with adding colors and making the graph clear and organized so your fellow lab mates can understand it.

Associated Images for Mathematical Ecologists Lesson Plan

Figure 1: Powerline Waterhole

This image was taken by Chloe Miller in Eastern Oregon during the summer of 2022.

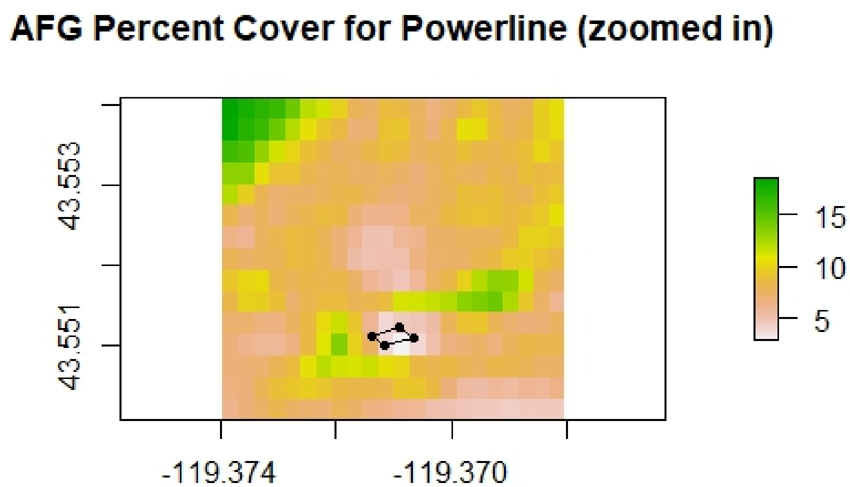

Figure 2: AFG Percent Cover for Powerline

This image was created by Chloe Miller during the Summer of 2022 using coding in R as well as information from GPS plot points and data from the Rangeland Analysis Platform.

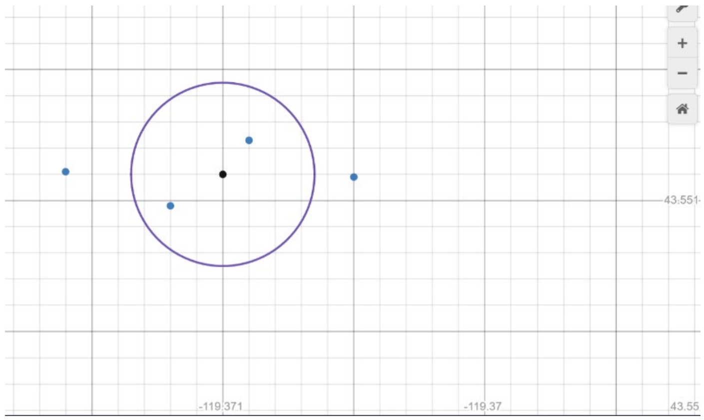

Figure 3: Plotting in Desmos Example 1

This image was created by Chloe Miller using Desmos as an example of a plot from Worksheet 2, Step 1.

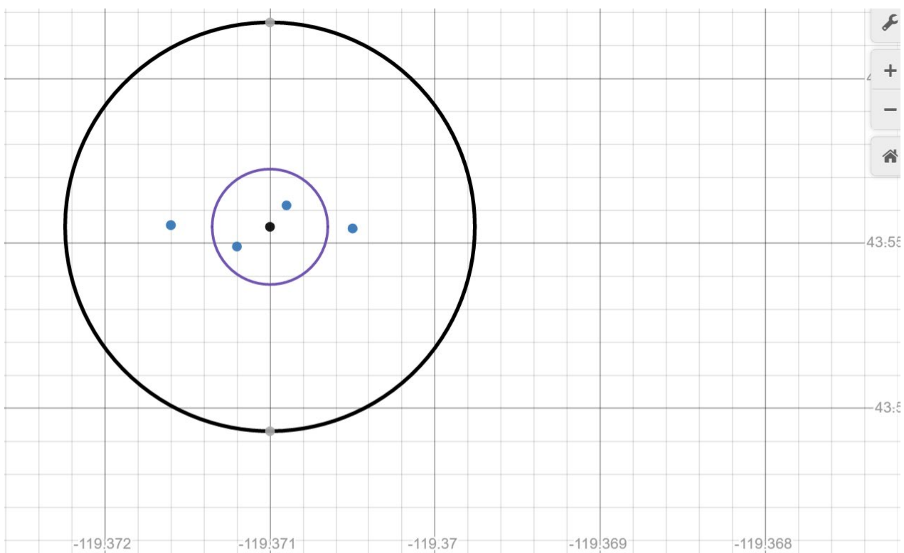

Figure 4: Plotting in Desmos Example 2

This image was created by Chloe Miller using Desmos as an example of a plot from Worksheet 2, Step 4.

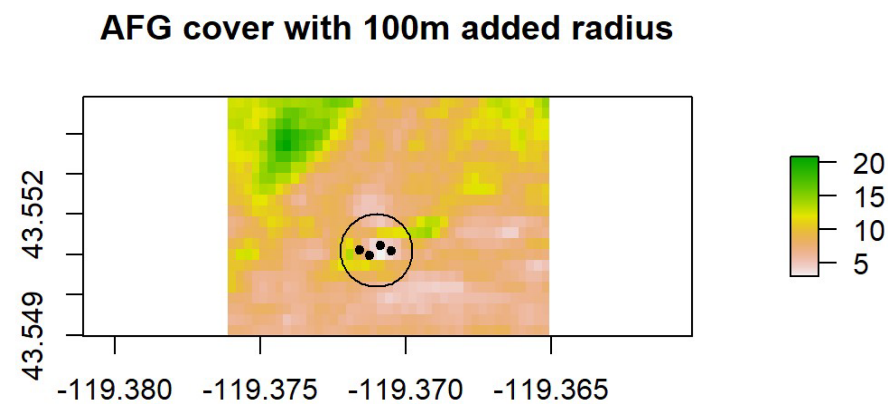

Figure 5: AFG Cover With 100m Added Radius

This image was created by Chloe Miller during the Summer of 2022 using coding in R as well as information from GPS plot points and data from the Rangeland Analysis Platform.

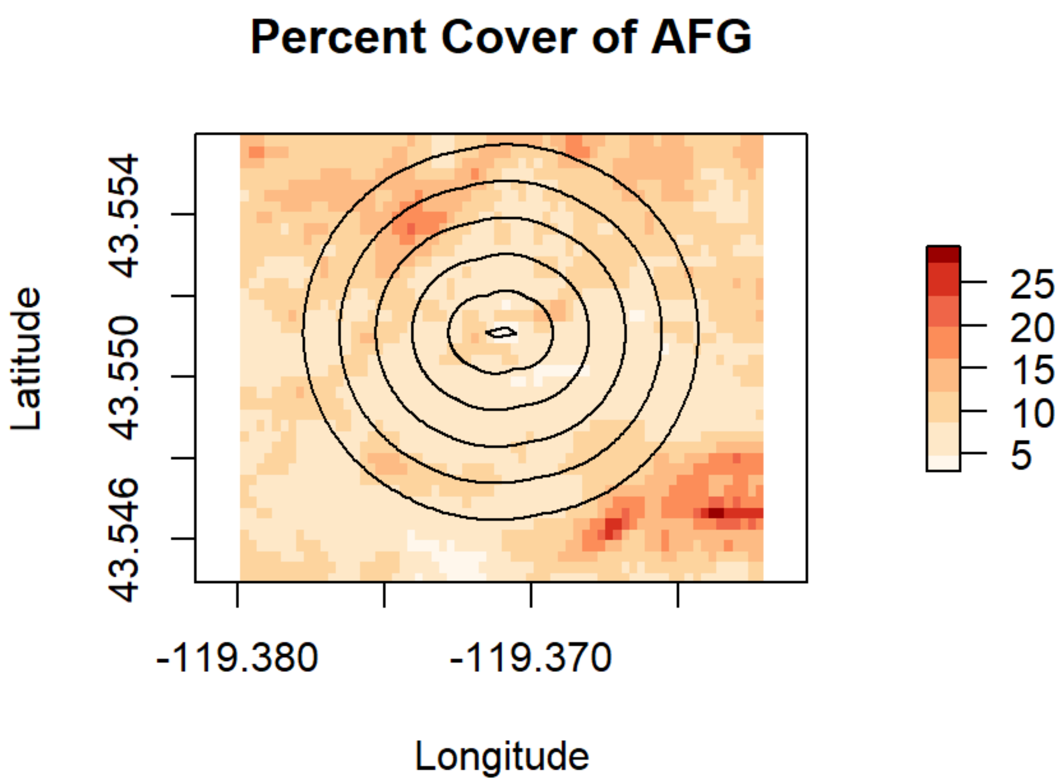

Figure 6: Percent Cover of AFG

This image was created by Chloe Miller during the Winter of 2023 using coding in R as well as information from GPS plot points and data from the Rangeland Analysis Platform.

Mathematical Literature and Art Lesson Plan

PART I

CONTENT	
Essential Concepts Geometry and Measurement: • Measurement of 2D and 3D shapes. • Solving Applied Real-World Problems and Math Modeling in Geometry **Standards for Mathematical Practice** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique reasoning of others. 8. Look for and express regularity in repeated reasoning.	**Oregon Math Standard(s)** HS.AFN.D.8 Model situations involving arithmetic patterns. Use a variety of representations such as pictures, graphs, or an explicit formula to describe the pattern. HS.GM.B.6 Justify theorems of line relationships, angles, triangles, and parallelograms; and use them to solve problems in authentic contexts. HS.GM.C.9 - Use volume and surface area formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems and apply to authentic contexts. HS.GM.C.10 Use geometric shapes, their measures, and their properties to describe real world objects, and solve

	related authentic modeling and design problems.

Strategies to Combat Math Anxiety

- Quality over quantity.
- Ask questions.
- Remove timed tests, find other means of assessment.
- Make math applicable.

Social Justice Issue or Standards (Grades 9–12 Outcomes)

- I respectfully express curiosity about the history and lived experiences of others and exchange ideas and beliefs in an open-minded way.
- I understand that diversity includes the impact of unequal power relations on the development of group identities and cultures.
- I can recognize, describe, and distinguish unfairness and injustice at different levels of society.
- I express empathy when people are excluded or mistreated because of their identities and concern when I personally experience bias.
- I will join with diverse people to plan and carry out collective action against exclusion, prejudice, and discrimination, and we will be thoughtful and creative in our actions in order to achieve our goals.

CONTEXT

Purpose

Identify and challenge social justice issues within Flatland while peaking student interest in geometry through discussion of both mathematical and social justice content.	
Audience High school geometry	
Allies • Physics teachers within your school • Language Arts, English, Second Language Acquisition groups within your school • History, Social Studies teachers within your school • Art teachers within your school	
WHEN (in unit)	
Circle one: Beginning Middle End **Special Lesson**	**Number of periods:** 11 or more
HOW	
Circle one: Mathematics Tasks Three-Act Tasks **Project-Based Learning** Other	

PART II

<table>
<tr><td colspan="2">Introduction/Engagement:
Exploration of Database of Mathematical Literature/Art
Mathematical Literature and Art List of Examples for Inspiration
(https://kasmana.people.cofc.edu/MATHFICT/?fbclid=IwAR1gEvA1k1ub1pM3KeluEH6jCCI1YeyUvR08zJmasWpxO6MJOJau9WR189M)</td></tr>
<tr><td>What will the teacher do?
Give students time to explore the database of mathematical literature and art (10-15 minutes). They can work in pairs or small groups if desired.
Have students write down at least one source they liked, and what mathematical concept was being covered in that source.
Have students present their findings to the class.
Give students the Mathematical Literature and Art List of Examples for Inspiration, potentially showing a few examples for the whole class.</td><td>What will students do?
Explore the database of mathematical literature and art with my peers.
Write down observations of what sources I enjoyed, why I enjoyed them, and what mathematical concept is being covered in the source.
Share my observations to the class.
Help to add more sources to the Mathematical Literature and Art List of Examples for Inspiration page.</td></tr>
</table>

<table>
<tr><td>Feel free to add examples students find from their exploration to the Mathematical Literature and Art List of Examples for Inspiration.</td><td></td></tr>
<tr><td colspan="2">Investigation/Exploration:
Reading of Flatland with Discussion Questions
Watch Flatland movie</td></tr>
<tr><td>What will the teacher do?
Give students time in class and/or homework to read sections of Flatland and have students write down answers to discussion questions.
For the following class go over discussion questions with students, and have students ask questions and share with others what they noticed and wondered.
This could work as a separate lesson plan or could be a jumping off point to begin other lessons as you work through the book and the mathematics associated.</td><td>What will students do?
Read Flatland.
Answer discussion questions, be prepared to discuss in class.
Ask questions. Take note of observations and wonderments.
Discuss in class with peers and teacher both social justice issues as well as mathematical content.
Watch all or parts of Flatland movie in class, paying attention to similarities and differences between the movie and the book.</td></tr>
</table>

• Watch all or parts of the Flatland movie in class, this can give students another medium to understand some of the social justice issues within Flatland as well give a visual for some of the more complex ideas of dimensionality within Flatland. To help students stay on task during the movie have them make observations of what parts are similar and/or different from the book.	
Share and Discuss: Final Project, Create your own math literature/art.	
What will the teacher do? • Remind students of the examples of math art/literature (refer to list in intro). • Give students rubric with an example project. • Use presentations as a way that students can demonstrate knowledge (another form of assessment).	**What will students do?** • Create a work of mathematical literature and art. • Follow the rubric and use the example as a starting ground for requirements. • Make a 7–10-minute presentation which addresses each criterion.

• Give students feedback on what they can do to extend their work as well as apply other mathematical concepts.	• Listen and ask questions of peers to learn about other's projects when they present.

Mathematical Literature and Art List of Examples for Inspiration

1. Large Database: https://kasmana.people.cofc.edu/MATHFICT/?fbclid=IwAR1gEvA1k1ub1pM3KeluEH6jCCI1YeyUvR08zJmasWpxO6MJOJau9WR189M

2. Poetry: https://www.maa.org/sites/default/files/images/upload_library/4/vol6/Growney/MathPoetry.html

3. Music Video: https://www.youtube.com/watch?v=yoHR8qwuqmY

4. Comics/Manga: The Manga Guide to Calculus by Hiroyuki Kojima and Shin Togami

5. Book: "The Housekeeper and the Professor" by Yoko Ogawa - This novel follows the relationship between a housekeeper and a mathematician who specializes in number theory who has a memory that only lasts for 80 minutes.

Mathematical Literature and Art: Flatland Guided Reading Questions

Preface/Introduction

- What do you know about dimensions? What is 2D? What is 3D?
- Make a list of 2D shapes and a list of 3D shapes with properties about each.

Chapter 1/Chapter 2: Settings of Flatland

- Describe Flatland.
    - How does one see a circle within Flatland?
    - What specific features do the houses in Flatland have? Why?
- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 3/Chapter 4: The Inhabitants and Women of Flatland

- Describe or make a visual representation of the class system within Flatland.
- How does this class system compare to others that you might know about from past, present, or even other works of fiction? Describe the similarities and differences between the Flatland class system and another class system of your choice. Also describe what problems you might see within this class system.
- Why are women in Flatland considered “dangerous”? What are they able to do within Flatland to their appearance?
- Do you think the current Code for laws regarding women is just/fair? Why or why not? How might you change the Code that still keeps Flatlanders safe from “the dangers of women” but might be more equitable or fair?

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 5/Chapter 6: Recognition of Figures in Flatland

- What are the three methods of recognition available for the inhabitants of Flatland? What are the pros and cons of each method? What class system(s) typically use each method?
- How does fog help the inhabitants of Flatland recognize one another?
- How does the separation of types of recognition based on class affect education? Do you think this current system is fair/just? Why or why not? How might you change this system?
- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 7: Irregular Figures in Flatland

- What does it mean to be a regular/irregular shape? (What are the properties of regular/irregular shapes?)
- How are irregular shapes treated within Flatland? Why? Do you think this is fair/just? Why or why not?
- What does the treatment of Irregular Figures remind you of? What types of comparisons might the author be trying to make to the real world?
- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 8/Chapter 9/Chapter 10: Color in Flatland

- Who is Chromatistes? What did he do?

- How did the creation of color disrupt the class system? Give two examples and explain their potential implications.
- What is the Universal Color Bill? Who supported it? Who opposed it?
- What caused the use of color in Flatland to become abolished?
- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 11/Chapter 12: The Priests of Flatland

- Who are the priests of Flatland? Explain why they are not really the shape they claim to be.
- What can happen to the children of many-sided polygons and circles? Do you think this is fair/just? Why or why not?
- What is the doctrine of the Circles? How does it affect the family dynamics of Flatland?
- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 13/Chapter 14: Lineland

- Describe Lineland.
    - How does one see our main character the Square in Lineland? How does one see any shape in Lineland?
    - What are the family dynamics like within Lineland?
    - How do the inhabitants of Lineland recognize one another (how do they determine one another's length)?
    - What directions do not exist in Lineland that exist in Flatland?

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 15/Chapter 16: Intro to Spaceland

- What is the Square's grandson, the Hexagon on page 53 describing when he says, "it must be that a Square of three inches every way, moving somehow parallel to itself (but I don't see how) must make Something else (but I don't see why)) of three inches every way – and this must be represented by 3^3" (Abbott, 1884, p. 53)?
- Define space in Flatland (2D). Define space in Spaceland (3D). Feel free to use visuals to help explain. *Challenge: What is space in Lineland (1D)?*
- Think back to the Chapter 1 question when you made a list describing 3D shapes. Recall the properties and equations of a sphere and a cube.
- What do you think a "terminal point" (from page 61) is? Feel free to draw a picture to help describe.
- What is an arithmetic progression/pattern? Give an example of a situation that involves an arithmetic pattern (not mentioned within Flatland). Feel free to use visual representations to help you explain.
- What is a geometric progression/pattern? Give an example of a situation that involves a geometric pattern (not mentioned within Flatland). Feel free to use visual representations to help you explain.
- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 17/Chapter 18: Proving the Existence of Spaceland to the Square

- How did the Sphere take the tablet from the cupboard and feel the inside of the Square? Feel free to use a visual to explain.
- Why do you think the Square holds the Sphere in such reverence? How might you feel if a being from another dimension came to take you to that dimension? Would you feel fearful, excited, awe-struck, something else?
- How might have you gone about proving the existence of Spaceland to the Square? What would you do the same as the Sphere? What would you do differently?
- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 19/Chapter 20: Four dimensions and No Dimensions

- Do you think a land of 4-dimensions is possible? Why or why not?
- Describe Pointland.
    - Who lives in Pointland?
    - What directions do not exist in Pointland?
- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 21/Chapter 22: The Gospel of Three-Dimensions

- Why did the Square decide to teach his grandson rather than his own sons the Gospel of Three-Dimensions?
- The Square describes the idea of "Upward, but not Northward" which confuses both his grandson and later the president. Think of another way this might be explained to a shape

in Flatland. How might you try to convince the President of Flatland the Gospel of Three Dimensions?

- How does the story of the Square end?
- What else stood out to you in these few chapters? What did you notice? What do you wonder?
- What concluding thoughts, ideas, feelings, do you have after finishing the book?

Mathematical Literature and Art: Flatland Guided Reading Questions with Answers

This section is only for the teacher to help give ideas of what students' answers might look like or give more inspiration for what topics might need to be gone over in class.

Preface/Introduction:

- What do you know about dimensions? What is 2D? What is 3D?

Answer: Science Fiction and fantasy movies with dimensional talk (many different worlds that exist in different dimensions of the space time continuum). The eyes of humans can only see in 2D can't really perceive 3D. 2D definition: length and breadth but no depth. 3D definition: length, breadth, and depth.

- Make a list of 2D shapes and a list of 3D shapes with properties about each. Ex. 2D Shape: Square. Properties: four sides, four 90-degree angles, two sets of parallel sides.

Answer:

2D Shapes	3D Shapes
• Square, four equal sides, four 90-degree angles, two sets of parallel sides • Rectangle, two sets of equal sides, four 90-degree angles, two sets of parallel sides • Equilateral Triangle: three 60-degree angles, 3 equal sides	• Sphere, perfectly symmetrical, no edges/vertices, has radius, diameter, circumference, surface area, volume • Cube, 6 faces, 12 edges, 8 vertices, faces are all made up of squares, all sides have same length, all angles right angles

• Isosceles Triangle: two congruent base angles, two congruent sides	

Chapter 1/Chapter 2: Settings of Flatland

- Describe Flatland.
- How does one see a circle within Flatland?

Answer: A line, in fact everything within Flatland is seen as a line.

- What specific features do the houses in Flatland have? Why?

Answer: The houses are pentagons because the angles aren't as sharp as triangles or squares, so they aren't as dangerous to run into. They also have two entrances, one for men and one for women (the reason is not yet known to us).

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 3/Chapter 4: The Inhabitants and Women of Flatland

- Describe or make a visual representation of the class system within Flatland.

Answer: The general social rule of Flatland is the more sides one has the higher the class they have. Women are of the lowest standard because they are lines, then triangles, squares, pentagons, other many-sided polygons, and finally circles as the highest class.

- How does this class system compare to others that you might know about from past, present, or even other works of fiction? Describe the similarities and differences between the Flatland class system and another class system of your choice. Also describe what problems you might see within this class system.

Answer: The class system from Flatland is very similar to pre-woman suffrage and rights in the US. All women in both Flatland and in the US at the time are considered to be inherently less than man and therefore shouldn't be allowed the right to vote. And while there were still classes among men based on social rank (and specifically skin color), there was still a time when all men no matter their social rank could vote but no women could legally. A difference between Flatland and the US though is that the ranks of shapes in Flatland is very one-dimensional and not very intersectional. Their social classes are only determined by the number of sides and regularity. While in real life people are marginalized and discriminated against for many more complex and intersectional identities including but not limited to race, wealth, gender, sexuality, (dis)ability, etc.

- Why are women in Flatland considered "dangerous"? What are they able to do within Flatland to their appearance?

Answer: The women in Flatland are considered "dangerous" because they have a point at each end of them which means they are able to seemingly disappear from sight within Flatland. This means that others can easily collide with her and die.

- Do you think the current Code for laws regarding women is just/fair? Why or why not? How might you change the Code that still keeps Flatlanders safe from "the dangers of women" but might be more equitable or fair?

Answer: Instead of only having women have to have a "peace cry" have everyone have to have a "peace cry" or announce themselves when entering new rooms. Instead of having separate doors for men or women, have a detection door that makes a noise or announcement whenever someone enters the home, so others are warned when the door is in use.

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 5/Chapter 6: Recognition of Figures in Flatland

- What are the three methods of recognition available for the inhabitants of Flatland? What are the pros and cons of each method? What class system(s) typically use each method?

Answer:

6. Recognition by sense of hearing.

   Pros: Simple, can quickly identify someone without any real practice.

   Cons: Easy to mistake someone for someone else or pretend to sound like someone else.

   Who uses: Everyone, but also is deemed insufficient to use by itself.

7. Recognition by feeling.

   Pros: More accurate than hearing, can be used when first meeting someone who voice does not recognize.

   Cons: Requires some practice and training. Can't distinguish between many-sided polygons, too hard to feel the number of sides. Could cause injury if not careful.

   Who uses: Women and lower classes.

8. Recognition by sight.

   Pros: Very mathematical and considered artful. Able to distinguish many-sided polygons

   Cons: Hard to learn, complex, requires many years of education, not accessible to everyone.

   Who uses: Upper classes.

- How does fog help the inhabitants of Flatland recognize one another?

Answer: If a side is farther away from the viewer in fog you can see it recede into the fog or become less visible. For example, when viewing an Equilateral Triangle their side (i.e., the line visible) “will shade away rapidly into dimness” or “recede rapidly into the fog” (Abbott, 1992, p. 19). In comparison to a Pentagon their side “will shade away less rapidly into dimness” or “recede less rapidly into the fog” (Abbott, 1992, p. 19). (Note: Go over figures on pages 19 and 20 with the whole class!)

- How does the separation of types of recognition based on class affect education? Do you think this current system is fair/just? Why or why not? How might you change this system?

Answer: The separation of types of recognition based on class affects the educational system. It requires more education to be able to recognize by sight vs. by feeling or even by hearing. This means that the more education one has, specifically regarding mathematics, the better able someone is to determine one’s shape. It also means that lower classes might be able to be tricked more by those who have higher education since they aren’t able to recognize how many sides certain figures have at a certain extent. This seems unfair, and I would try to fix this by providing math education to everyone at Flatland to learn.

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 7: Irregular Figures in Flatland

- What does it mean to be a regular/irregular shape? (What are the properties of regular/irregular shapes?)

Answer: A regular shape means that all angles and sides have equal length. This means there exists some symmetries within these shapes. An irregular shape is one where not all angles and sides have equal length.

- How are irregular shapes treated within Flatland? Why? Do you think this is fair/just? Why or why not?

Answer: Irregular shapes are either destroyed or made to undergo therapy to change within most of Flatland. They are considered to be dangerous and less than if they are allowed to survive. While the danger of irregular shapes might be present since they may have sharp edges that are not easily expected they can bump into others and cause harm. But this system is not fair since Women also have this same danger, but they are still allowed to exist. This problem might be solved if everyone has their own "peace cries" to announce they are near to everyone around.

- What does the treatment of Irregular Figures remind you of? What types of comparisons might the author be trying to make to the real world?

Answer: The treatment of Irregular Figures reminds me of the discrimination and marginalization against people with disabilities. This is because many spaces and environments in the past (and even still today) are not made accessible for people with other (dis)abilities in mind. This is similar to Irregular Figures where Flatland society doesn't have structures or environments that are built for everyone's wellbeing, safety, and use.

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 8/Chapter 9/Chapter 10: Color in Flatland

- Who is Chromatistes? What did he do?

Answer: Chromatistes is the inventor of color within Flatland. He came up with the idea of painting one's sides as a means of recognition.

- How did the creation of color disrupt the class system? Give two examples and explain their potential implications.

Answer:

1. The creation of color within Flatland meant that everyone had equal access to recognize one another without needing specialized math education. This meant that lower classes (Triangles) were on the same grounds as upper classes (Pentagons). And therefore, the lower classes proclaimed that they should be recognized as equal and entitled to equal rights.

2. It also meant that the art of recognition by sight was almost unused expect by the very highest classes of circles and polygons.

- What is the Universal Color Bill? Who supported it? Who opposed it?

Answer: The Universal Color Bill proclaimed that everyone should submit to being painted, including the women and priests. They proposed that "in every Woman the half containing the eye and mouth should be coloured red, and the other half green. The Priests were to be painted in the same way" (Abbott, 1992, p. 29). The leaders of the Revolution supported this, that is the lower classes and even women. The Priests and upper classes opposed this, and eventually got the women and some of the middle classes on their side.

- What caused the use of color in Flatland to become abolished?

Answer: The story of triangle painting himself with the twelve colors of a Dodecagon and tricking a woman into marrying him began to get traction. And once the woman discovered the fraud, she committed suicide, which scared women and middle classes alike. Then there was a large gathering, the start to a revolt of Triangles who supported the Color Bill. The Chief Circle announced that he would accept the bill, but only if it was in the best interests of the majority. And he made a speech to counter Chromatistes, where he explained that distinctions between classes would cease with color. This scared those of the middle classes who did not want to be distinguished as lower and scared women who had heard the story of the fraudulent marriage. And eventually a battle occurred, and the Circle's military won, abolishing the Universal Color Bill once and for all.

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 11/Chapter 12: The Priests of Flatland

- Who are the priests of Flatland? Explain why they are not really the shape they claim to be.

Answer: The priests of Flatland are Circles. But these circles are not perfect circles. They are "only a Polygon with a very large number of very small sides. As the number of sides increases, a Polygon approximates to a Circle" (Abbott, 1992, p. 35). (Note: Show students this GeoGebra visual that shows the idea of adding more sides to a polygon to eventually approximate a circle: https://www.geogebra.org/m/wdrhnpgz.)

- What can happen to the children of many-sided polygons and circles? Do you think this is fair/just? Why or why not?

Answer: The sides of a many-sided polygon or circle children are very fragile and can be easily fractured, which means that the child rarely survives. They usually have to be placed in a Circular Neo-Therapeutic Gymnasium, which is therapy to fix fractured sides and make sure they are regular in length. But if they don't get "better", they are ultimately destroyed.

- What is the doctrine of the Circles? How does it affect the family dynamics of Flatland?

Answer: The doctrine of Circles is "Attend to your Configuration". This maintains the regularity of Flatland meaning that those who do not conform to that regularity are destroyed or imprisoned. This doctrine affects the family dynamics of Flatland where a person is supposed to honor their sons and grandsons (i.e., the younger generations) because they are usually of a higher class than they are. This is in contrast to many cultures in the real world where we are taught to honor and respect our elders as they are wiser and have more knowledge/experience.

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 13/Chapter 14: Lineland

- Describe Lineland.
  - How does one see our main character the Square in Lineland? How does one see any shape in Lineland?

  Answer: The Square (and any shape for that matter) is seen as a point within Lineland.
  - What are the family dynamics like within Lineland?

  Answer: The family dynamics of Lineland is every man has two wives which are his neighbors. There are also three births every time with two girls for every boy.
  - How do the inhabitants of Lineland recognize one another (how do they determine one another's length)?

  Answer: The inhabitants of Lineland recognize one another by their sense of hearing which is much more advanced and trained than those of Flatland or even Spaceland. They make a calculation when first meeting one another by calling with one of their mouths and then the other so they can determine the space between the sounds/mouths.

- What directions do not exist in Lineland that exist in Flatland?

Answer: The directions that do not exist in Lineland is right or left (perhaps even eastward and westward).

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 15/Chapter 16: Intro to Spaceland

- What is the Square's grandson, the Hexagon on page 53 describing when he says, "it must be that a Square of three inches every way, moving somehow parallel to itself (but I don't see how) must make Something else (but I don't see why)) of three inches every way – and this must be represented by 3^3" (Abbott, 1884, p. 53)?

Answer: He is describing a cube of three-dimensions which doesn't exist in Flatland or can't be seen in Flatland. 3^3 would be the volume of that cube (i.e., the space within the cube).

- Define space in Flatland (2D). Define space in Spaceland (3D). Feel free to use visuals to help explain. *Challenge: What is space in Lineland (1D)?*

Answer: Space in Flatland (2D) is area of a shape or space. For a square its length multiplied by width. For a triangle it is half of the base multiplied by the height. Space in Spaceland (3D) is volume. For a cube it is length multiplied by width multiplied by height. Space in Lineland (1D) is length, a singular measurement that doesn't require calculation in our world, but rather can be measured using a ruler.

- Think back to the Chapter 1 question when you made a list describing 3D shapes. Recall the properties and equations of a sphere and a cube.

Answer:

1. Equations/Properties of Sphere:

Volume = $\frac{4}{3}\pi * r^3$ (where r is the radius)

Surface Area = $4\pi r^2$ (where r is the radius)

Other Properties: Perfectly symmetrical, no edges or vertices

2. Equations/Properties of Cube:

Volume = a^3 (where a is the length of a side)

Surface Area = $6a^2$ (where a is the length of a side)

Other Properties: Angles of cube are all right angles, have 6 faces, 12 edges, and 8 vertices

- What do you think a "terminal point" (from page 61) is? Feel free to draw a picture to help describe.

Note: This may be a good opportunity to explain terminal points on a unit circle to the class.

- What is an arithmetic progression/pattern? Give an example of a situation that involves an arithmetic pattern (not mentioned within Flatland). Feel free to use visual representations to help you explain.

Answer: An arithmetic progression/pattern is when each number in the pattern differs by a constant amount (i.e., there is a constant difference between terms). An example of this would be 10,8,6,4,2,0, -2, … which is constantly decreasing by two every time. (Note: This could be a chance to go over arithmetic progression/patterns in more depth.)

- What is a geometric progression/pattern? Give an example of a situation that involves a geometric pattern (not mentioned within Flatland). Feel free to use visual representations to help you explain.

Answer: A geometric progression/pattern is when each number in the pattern differs by a common ratio. An example of this would be 20, 10, 5, 2.5, 1.25… where you divide by 2 to get the next term. (Note: This could be a chance to go over geometric progression/patterns in more depth.)

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 17/Chapter 18: Proving the Existence of Spaceland to the Square

- How did the Sphere take the tablet from the cupboard and feel the inside of the Square? Feel free to use a visual to explain.

Answer: The Sphere was able to take the tablet from the cupboard by rising above Flatland and into Spaceland, very similar to when the Square rose out of Lineland and into Flatland with the King of Lineland who simply saw him disappear.

- Why do you think the Square holds the Sphere in such reverence? How might you feel if a being from another dimension came to take you to that dimension? Would you feel fearful, excited, awe-struck, something else?
- How might have you gone about proving the existence of Spaceland to the Square? What would you do the same as the Sphere? What would you do differently?
- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 19/Chapter 20: Four dimensions and no dimensions

- Do you think a land of 4-dimensions is possible? Why or why not?
- Describe Pointland.
  - Who lives in Pointland?

  Answer: The only resident of Pointland is the King of Pointland.

    - What directions do not exist in Pointland?

  Answer: There is no length, width, or breadth in Pointland but is only a Point which contains the King of Pointland.

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

Chapter 21/Chapter 22: The Gospel of Three-Dimensions

- Why did the Square decide to teach his grandson over his own sons the Gospel of Three-Dimensions?

Answer: The Square decided to attempt to teach his grandson over his own sons the Gospel of Three-Dimensions for a few reasons (1) his grandson had already been discussing ideas of three dimensionality and had been very curious and intelligent in the matter (not yet completely sullied by the world and those around him), (2) his sons were not as skilled in mathematics and were also of good standing where they may not be as inclined to believe him or might even turn him in.

- The Square describes the idea of "Upward, but not Northward" which confuses both his grandson and later the president. Think of another way this might be explained to a shape in Flatland. How might you try to convince the President of Flatland the Gospel of Three Dimensions?
- How does the story of the Square end?

Answer: The Square proclaims the Gospel of Three-Dimensions to the President of Flatland and is subsequently thrown in jail and writes his story (the one we are reading on tablets).

- What else stood out to you in these few chapters? What did you notice? What do you wonder?

- What concluding thoughts, ideas, feelings, do you have after finishing the book?

Mathematical Literature and Art Lesson Plan Instructions/Rubric

Instructions: Create your own mathematical art or literature. You can also choose to create compilations or edits of previously made mathematical art or literature.

Requirements/Rubric:

- Main Idea: Create a 7-10 min presentation about mathematical literature/art you create based on the content of the class.
- Need to create something unique. There needs to be at least one element of your project that comes from you, this could be combining or editing something that exists, creating your very own or using someone else's idea and adding a twist.
- Proper citations! Need to give credit to those whose work you are using or who inspired you.
- Needs to cover either something gone over in class or an extension of something related to the subject of the class.
- Accuracy of mathematics presented.
- Needs to have the following components:
    - Relevant historical context. Where does the math come from? Who created this? Why did they create this?
    - Equation or process of computation fully explained. What are the relevant equations to this concept? What does each variable mean? When would you use this equation? Provide a real contextual example of the equation (perhaps with actual numbers)?

- Mathematical diagram, table, or graph fully explained. This can be something you create, or something created by someone else. What are the relevant diagrams, tables, or graphs to this concept?
    - For a diagram make sure there are labels and explain all labels. Why is the diagram helpful? What might make the diagram better?
    - For a table find and explain patterns observed. Why is this data helpful or what is it used for? How might you make the table into a graph or other visualization?
    - For a graph explain all axes and keys. What is the story the graph is trying to tell? What might make the graph better?

Mathematical Literature and Art Lesson Plan Example

Note that this example is fairly simple and requires potentially less substantial mathematical content, but this could be used to show the organization of the project to students.

Pi by Chloe Miller

Chocolate,

Peach,

Banana Cream,

Plum,

Marionberry,

Peanut Butter and Coconut Cream,

Apple,

and Strawberry Rhubarb.

Pies I like to eat!

Explanation of creative project: This poem uses the digits of pi (3.14159265) as the number of syllables per line. So, for the first line Chocolate is 3 syllables, the second Peach is 1 syllable, so on and so forth. The poem is about the different types of pies that exist.

Historical Context: Archimedes of Syracuse was first credited with calculating pi. This calculation of pi is not the actual number of pi, but rather an approximation. He approximated the calculation of pi by approximating the area of a circle by using the Pythagorean Theorem to calculate the areas of two regular polygons. Another similar approach was done by a Chinese mathematician Zu Chongzhi where he calculated the ratio of the circumference of a circle to its diameter. A fun fact is that the Greek letter π wasn't used until the 1700s while Archimedes was alive in 287-212 BC.

Equation: We know that $C = 2\pi r$ where C is the circumference, and r is the radius of a circle. So, to approximate pi we can rearrange the equation by dividing each side by $2r$. This gets us $C \div 2r = \pi$. One could then use measurement tools to measure the distance of the circumference and the radius to plug into the equation to approximate the value of pi.

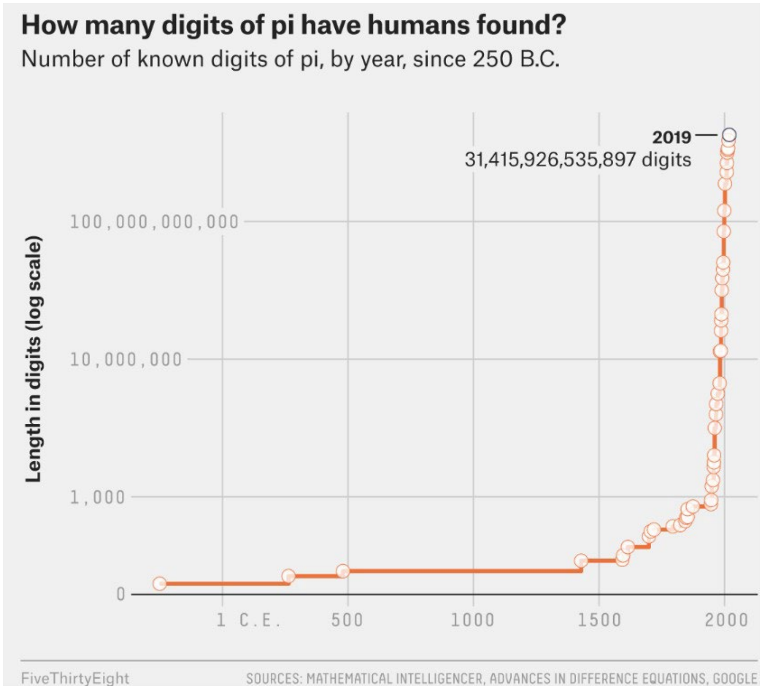

Figure 7: How Many Digits of Pi Have Humans Found?

Source is from Mathematical Intelligencer, Advances in Different Equations.

This graph displays time on the x-axis starting with 250 B.C. and ending in 2019. The y-axis is the number of digits of pi which have been calculated and is in log scale because it allows the comparison of very small data to very large data. The graph is telling us how many digits of pi have been calculated out over time, specifically showing the substantial increase between 1500 and 2000 when the invention of calculators and computers made calculating digits of pi easier and quicker than calculating by hand. To make this graph better we could not include the little overlapping white dots since it's hard to tell what the purpose is or how many there are. We could also make a better x-axis which makes it clear the starting point in time instead of knowing it's sometime before 1 C.E.

Example Bibliography

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Conclusion

Math anxiety is a big problem that both teachers and students face. This thesis addresses the issues of who is impacted by math anxiety and what the potential solutions of math anxiety are with the primary solution being interdisciplinary studies. And finally, I ended the thesis with actual lesson plans that can be incorporated into a high school math classroom that interact with subjects like ecology, art, and literature in order to get students excited and passionate about mathematics. My hope is that this thesis will be directly applicable to my own future as a high school math teacher and to inspire others to research the impacts of math anxiety. Future research should explore more impacts and implications of math anxiety on marginalized and underrepresented groups within mathematics, including groups not addressed within this thesis. Research should also look at other types of interdisciplinary lessons with more variety of disciplines such as music, medicine, engineering, etc.

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