THE HISTORY AND APPLICATIONS OF CALCULUS

by

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The subject of calculus covers a multitude of complex topics and labyrinthine worlds, but the commonality of studying rates of continuous change exists in all the subfields. Before the initial discoveries of calculus, math was very discrete. Often only standstill and countable objects could be studied which ruled out the examinations of millions of scenarios. Nowadays, it is argued that every single thing in the universe is constantly in motion (relative to some point), meaning there is no such thing as standstill. From this perspective, calculus can be applied to every single thing in the universe, and this beautiful idea can be exemplified throughout the existing diverse and frequent applications of calculus in modern-day society. There exists proof of ancient discoveries pioneering this advanced, modern subject of calculus; during those ancient times, mathematicians did not know that they were about to drastically change the world. These initial mathematical discoveries created a mass of extreme, influential curiosity within the subject of calculus and inspired future mathematicians to keep expanding it. Without calculus and its historical origins, we would have a hardly-advanced, primitive civilization today and extremely far less utilizations of mathematics. By starting with investigating the origins of calculus and then analyzing these roots in the modern applications, one can appreciate the marvelous growth, expansion, and essentiality of the subject.

DEDICATION

I would like to dedicate this thesis to my family.
My mom, dad, and brother are the ones who inspired
and encouraged me to get to this point.
They are the reason that I am here today,
publishing my first paper.

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PREFACE

This thesis was written in hopes of being intelligible to non-mathematicians or non-calculus lovers. While doing my research, I often ran into journals and articles that I could only understand due to my experience studying mathematics. Of course, having an audience of mathematicians or calculus lovers makes calculations much easier to describe, but I want to investigate the reason that individuals have an existing subject to love in the first place. I chose to focus on the branch of calculus because I think the versatility of the subject is easier to understand than other branches without much math experience. The subject is also one of few truly universal disciplines and one of the most generalizable branches of mathematics.

Another thing I noticed while doing my research is that a lot of mathematical papers include proofs and discussions of certain theories, but these proofs often have roots of assumed postulates or already proven lemmas. These journals are great for education purposes, but I want to peg the question: how do you know if these initial calculations or assumptions are even accurate? In any subject, one cannot completely understand or should not completely trust an idea without investigating its history and who developed it (or contributed to its development).

To emphasize the importance of the history of a subject, imagine this scenario: you take a math class that provides course materials and textbooks decided by the teacher, who has a credible degree. These materials supply you with formulas and equations that conveniently work with the homework problems assigned to you. Doesn't this sound like a regular class? Wouldn't you automatically trust your teacher and your textbooks to be accurate?

Now imagine that teacher was playing a cruel joke on you and wanted to see if they could waste a semester of your time and money. Would you get fooled? How often do you take your math textbook and look up the history behind the subject, the author, the theorems, and try to understand where they are coming from? In any math class, I have found that it is rare to get the complete background of the subject. The histories of mathematical subjects are taught far less often than the actual mathematical content, but that does not mean that they are less important to learn. Without the founders, contributors, constant growth and expansion of the subject over the past several centuries, calculus would not be the subject it is now and our society would be drastically and unfavorably different.

To support these ideas, I set up a little experiment. I asked my friends to recall some kind of mathematical formula or idea that they learned in high school; the most common responses I received were the Pythagorean theorem or the quadratic formula. For my friends who could recall one, I then asked them, how do you know that theorem or formula works? Most of the answers I got were along the lines of, “I learned it in class” or “Because my teacher told me”. Not a single person could tell me why it is an accurate theorem or where it came from; they just had trust in their teachers and textbooks. The beauties of mathematical theorems and formulas get dulled when they are simply supplied and forced to be memorized. Understanding the background and history of calculus, or any subject for that matter, makes the subject that much more magnificent.

Alongside the history, the applications of a subject are extremely crucial to grasp when trying to understand the significance of it. Imagine another classroom scenario in which a professor teaches you masses of information and puts you through all this rigorous work, just for them to tell you that you will never come across this topic or use that information in your life. Wouldn’t that take away from one’s interest and/or the importance of the subject? The realms of calculus are quite literally limitless, and the subject’s generalizability is critical in highlighting the subject’s importance and can clearly accentuate its brilliance.

Altogether, I want to shed light on the intelligence of the pioneers of calculus, examine the regional expansion of mathematical creativity, and explore key developments and applications of calculus today. Both accurately telling the history and clearly conveying the applicability of calculus is pivotal. The main reason for disinterest in calculus seems to be the disbelief of its necessity or importance. Other subjects in mathematics such as statistics or algebra may be more universally accepted or recognized as important. For example, statistics is often used by businesses to maximize performance, and individuals often use algebra in their everyday lives when completing tasks like adding up groceries or bills. No one I know (or at least no one I have met yet) can confidently tell me "Calculus is cool" or "Calculus is important" and elaborate on their reasoning. By the end of this paper, I want you, reader, to be able to say those things with confidence.

WHAT IS CALCULUS?

The first step to understanding the importance of calculus and all of its complexities is to fully comprehend its purpose. To put it simply, calculus is the branch of mathematics that was developed to study rates of change. Math used to be limited to studying certain discrete quantities independently, but the development of calculus made it so that mathematicians were able to study continuous quantities in relation to other reference variables, such as time. The term "calculus" used to be just a general term for any mathematical calculations, and not a distinct branch of mathematics; it was not until certain key developments were discovered and accepted that the subject became a concrete definition and field of study.

As long as there is an object that is moving or changing, we can calculate the rate of that change using calculus. It is interesting because today, scientists have demonstrated that any existing object or being on the Earth possesses some kind of mass, which therefore exerts some capacity of gravitational force or movement. This aspect is one of the many that makes Calculus so unique; since it is possible to prove that no object in the universe is completely standstill (when relative to something else), we can calculate and analyze the changing behavior of every single thing in the universe. The possibilities are quite literally endless.

The subject of calculus focuses on many different areas, such as continuity, limits, derivatives, integrals, and infinite series. Defining all of these will not necessarily be useful to people who do not study math, but seeing how some of these areas are used and applied will make it understandable. The two major areas of calculus are differential calculus and integral calculus. To put it simply, differential calculus solves for the rate of change of a known quantitative variable, while integral calculus solves for a quantity when the rate of change is known. Differential calculus deals with movement like slopes and velocities, and integral calculus deals with total values like areas and volumes. These are major themes throughout this paper and understanding the differences will make certain ideas much easier to understand. I will be discussing calculus in the Euclidean space, but there are many other spaces in which calculus is crucially relatable and/or applicable. To make things simpler, I will also only mainly be looking at functions with one input variable, x, but there is a whole other realm of multivariable calculus in which multiple inputs like x and y are calculated.

The way we use the subject of calculus today and even the name of the subject itself is all due to the contributions of many mathematicians over the past 3000 years. The recurring message I want to emphasize is that it is so important to know about the intelligent founders, contributors, and the increasing applications of calculus to prove the subject's integrity and necessity. Where did calculus come from? How is it used today? These questions are extremely important in guiding one's understanding of the subject. There is concrete evidence that calculus has been around for milleniums, and there are constantly new discoveries being made about its origins. For this reason, I will not be making any definitive comments about the subject's exact origins. However, I will stay true to thoroughly exploring the history of notable, influential contributors and obvious, significant applications.

THE HISTORY OF CALCULUS

Calculus has been around for centuries and the history of the subject is almost as complex as the subject itself. I believe the easiest and clearest way to go through this timeline, as many historical timelines showcase, is to highlight some of the prominent mathematicians and their contributions to modern day calculus. There are a multitude of mathematicians in the past generations that have contributed to the invention of calculus, but there were a distinct few who had particularly significant contributions to modern-day calculus. This does not necessarily mean that any of the mathematicians were smarter or better than others, but there were certain works that were more publicized, analyzed, preserved, and then accepted. The exact timeline of calculus is still being constructed and trying to cover all of it may lose your attention, so I want to cover some of the obvious major developments that led to today's understanding and applicability of the subject.

Starting off with some of the oldest origins, there is evidence of mathematicians discovering certain mathematical calculations, theorems, or even just possible theories that later inspired complete ideas dating back millenniums. Calculus did not become a popular, challenged idea until around the 17th century, but there were many noteworthy precursors to the subject dating back to ancient and medieval times. These discoveries are difficult to follow along a concrete timeline, but they are very deserving of attention and credit for modern calculations. Origins of calculus discoveries can be traced back to the Ancient Histories of Greece, Egypt, and China, and Medieval India, Middle East, and Europe. The ancient precursors in Greece are the most evidence-supported and they also seem to be the most influential to later discoveries and modern expansions of the subject.

Zeno of Elea (c. 490 - 425 BC) was a Greek Philosopher who was extremely influential in motivating mathematicians throughout the decades to discover new ideas. He was very well known for his paradoxes, which challenged the existing mathematical ideas at the time and also inspired future mathematicians. There was one specific paradox that influenced discoveries of infinitesimals, infinite series, limits, and several other dimensions of calculus. This paradox says that a man running a race cannot ever reach the finish line. Zeno's argument says there are infinite halfway points to reach; once the runner starts the race and gets to the first halfway point, he then has to get to the halfway point of his remaining path. This continues to go on and on and he will always have another remaining half of some distance to run, which means he would never actually finish the race; this half distance may be infinitely small but it never quite reaches zero (Alper & Bridger, 1997).

If you want to test this paradox yourself, choose any random distance of a race path (it obviously has to be greater than 0 m). If you enter this number in a calculator and divide it by two over and over again, you will never get exactly 0 (without rounding). This paradox stumped lots of mathematicians, which led to a motivation to find a solution, which later was found to be calculus. Zeno's paradoxes challenged many existing mathematical theories and inspired new discoveries surrounding derivatives, integrals and limits.

Greek astronomer and mathematician Eudoxus of Cnidus (c. 408 - 305 BC) made a critical discovery that is now recognized as one of the forerunners of calculus; this is known as the method of exhaustion. Eudoxus figured out a way to calculate or best approximate the area of a shape (e.g., a circle) by inscribing it with a polygon with an increasing number of sides (e.g., a 16-gon as opposed to a hexagon). At the time, there were only existing calculations for areas of polygons but not curved shapes, so using what was available to make new discoveries was impressive and innovative. This method that Eudoxus discovered to "exhaust" areas and volumes was a key precursor to calculus. This method did not become super popular during the time Eudoxus was alive, but another mathematician took it and ran with it, inspiring future generations to continue its expansion.

Archimedes (c. 287 - 212 BC) was a famous Greek mathematician, physicist, and inventor who is also greatly attributed to initial discoveries of modern calculus. He used the method of exhaustion, which Eudoxus discovered hundreds of years prior, to create a mass of new discoveries. Using that method, he additionally discovered how to calculate things like areas under parabolas, surface areas and volumes of spheres. He was one of the first mathematicians to construct a tangent to a curve. He also actually was the first mathematician to calculate an accurate approximation of π, which he did in the process of finding the area of a circle. Archimedes made many incredible mathematical discoveries, but his expansion of the method of exhaustion was particularly pivotal to modern calculus.

Zeno's paradox of finite values seeming to have infinite addends, and the works of Eudoxus and Archimedes that can analyze curves, come together to form a key theme of calculus. Their discoveries were precursors to the foundation of modern calculus where one can calculate the area under a curve by inscribing it with a finite but seemingly infinite number of rectangles, and find the sums of the areas of those rectangles. While doing this, they inspired generations of mathematicians to challenge the ideas of finite vs. infinite series which introduced a new idea of modern calculus called limits. Limits are key tools in integral and differential calculus; they are defined as the value that a function or its output approaches as the input approaches a certain value.

Although the two more recent Greek mathematicians aimed to calculate similar problems, Eudoxus's discoveries seem to resemble integral calculus because of areas while Archimedes used these discoveries to facilitate new methods that can resemble differential calculus because of the tangents. Eudoxus and Archimedes did not put together the fact that their discoveries of areas under curves and slopes of tangents are actually inverses of each other, but this idea was discovered in the 17th century by mathematicians Gottfried Leibniz and Isaac Newton and later established as the fundamental theorem of calculus (Lerner, 2001). Some of the ideas developed in the ancient and medieval times are technically not considered fully developed principles of calculus, but these were definitely notable precursors to the subject. There were many contributing, intelligent mathematicians throughout the years but it wasn't until the development period in the 17th century that the subject became closer to a "fully developed" mathematical discipline. These roots of calculus can be seen all across the world over many millenniums, especially clearly in Ancient Greek.

So, we know the discourse about the initial discovery or foundation of the subject of calculus is still not completely figured out, but we know that the origins of certain ideas within the range of calculus can be traced back to ancient and medieval time periods. Noting that these findings were original, existent, and very inspirational to the later eras, we can now look at the discoveries of some of the more focused areas of the newfound subject of calculus that started to come about in the 17th century.

The 17th century is known to be part of the Scientific Revolution, and there were two mathematicians who contributed to the drastic changes in scientific thought during that time period. Mathematicians Isaac Newton and Gottfried Wilhelm Leibniz were extremely influential in moving calculus towards a stable, concrete mathematical discipline in their generation and are considered two of the original founders or major contributors of the modern subject of calculus. Before these two, the term ‘calculus’ was seen as any branch or body of mathematics that performs calculations; calculus is the Latin word for “pebble” and its mathematical relation only came about because Romans used to use pebbles to count and add quantities together. The two mathematicians worked to develop new, significant calculations and unequaled concepts to make calculus become a more distinct, popular term and subject. No matter the original discoveries, it is safe to say that the subject of calculus that we heavily rely on today was based around the discoveries of Newton and Leibniz in the 17th century.

Isaac Newton (1642-1727) was an English mathematician, astronomer, physicist, and author who made many outstanding mathematical discoveries. His most influential discoveries include his method of fluxions and the fundamental theorem of calculus. Newton created ‘fluents’ describing some variables that are changing over that period of time, which he labeled as x and y, and he created calculations for the ‘fluxions’, which are the instantaneous rates of changes of those variables, which he labeled as $\dot{x}$ and $\dot{y}$. This was a shock to the 17th century as math was formerly very fixed and only linear functions were able to be calculated; Newton created a method that analyzes the gradient or slope of a curved function in which the x and y values are always changing, rather than a linear function graphing the relationships of x and y that make a straight line. This method of fluxions is now known as differentiation or solving for derivatives, which are the rates of change of a function with respect to another variable. Newton calculated a function that solves for the derivative of y with respect to x. In this function, once you have the derivative, you plug in an x value to find the rate of change on any curve on any point. Using this, Newton then came to the conclusion that differentiation and integration are inverse processes, meaning if you differentiate a function then integrate it, you will get the original function; this led to the discovery of what we now call the fundamental theorem of calculus and also inspired many other topics in math like infinitesimals and applications within other subjects like physics and astronomy.

The fundamental theorem of calculus was a revolutionary discovery stating that differentiation and integration are inverse methods, and Newton was not the only one to have this epiphany. Gottfried Wilhelm Leibniz (1646-1716) was a German mathematician, writer, and scientist who made many contributions to multiple different fields like physics, philosophy, and law, but let's focus on his mathematical contributions. Leibniz is credited for helping pioneer many mathematical concepts such as the dot notation for multiplication, parentheses for sectioning algebraic expressions, and the colon symbol representing division or ratios (Kreiling, 1968). When looking specifically at the branch of calculus, Leibniz took a metaphysical approach to the subject and was motivated to find a way to explain the inverse relationship of integrals and derivatives; he also independently discovered this inverse relationship and actually published his findings before Newton. The difference is that Leibniz had a much clearer notation; he created the notation for integration, $\int f(x)\,dx$, and for differentiation, $\frac{dy}{dx}$. Although lots of people relied on Newton's work as he had loyal followers (Kreiling, 1968), modern calculus heavily relies on Leibniz's notations rather than Newton's; Leibniz created notations that could be generalized but Newton's notations seemed to only make sense to him. Putting the controversy aside, both mathematicians made the incredible discovery that integrals can solve for areas of functions and derivatives can solve for the rate of change, or in other words, a function's output with respect to its input. Their work was revolutionary in the time of the 17th century all the way up to modern times. It introduced ideas that are now known as infinitesimals, infinite series, and limits although the two never discovered limits; differential and integral calculus showed that we could apply a certain calculation an infinite number of times and still get a finite number or that we could finally solve constantly changing rates and/or functions involving infinitely small variables.

Isaac Newton and Gottfried Leibniz both independently discovered and developed notations for the fundamental theorem of calculus and published their works around the same time; Newton seemed hesitant and wanted to make sure his work was accurate while Leibniz seemed very eager to publish his work. So, Leibniz actually published his work first but Newton claimed to have thought of them first. This led to the two mathematicians accusing each other of plagiarizing or stealing the other's work and this feud lasted until they both died. There is still controversy around the topic, and different articles you read will tell you a different founder of the subject of calculus, but there is proof that they made independent discoveries (Bingham, 1971). Putting that aside, Newton and Leibniz were both extremely, independently influential to the foundation of this new branch of mathematics and were undoubtedly inspirational to future generations and even other subjects. The discoveries made by these two mathematicians were pivotal in the continuing curiosities, analyses, and foundations of calculus in later generations; without these two, modern calculus would not be what it is today.

There are noteworthy contributors after the 17th century but I want to bring emphasis to the fact that those mathematicians were often inspired by Newton and Leibniz and that a good chunk of modern calculus is based around the discoveries of these two; I included early precursors to mainly provide background and discuss how we got to the 17th century developments of these two and therefore modern calculus. The 18th and 19th century involved many mathematicians challenging or supporting Newton's and Leibniz's ideas and their analyses led to the rigorous foundations of a concrete subject called calculus. So, there were many other contributors to modern calculus, but the existing subject and its modern day applications would not be the same without the discoveries of these two and their motivations and developments foreshadowed by the Greeks, so I want to focus on highlighting these two mathematicians.

As mentioned, the Scientific Revolution time period was the main, initial reason why calculus was developed into the subject it has today. This era also seems to be the most documented and easy to follow along a timeline. Again, it is important to note that aside from whom I mentioned, there has been a mass of mathematicians who independently made discoveries about calculus all around the world. Although there were some incomplete ideas, a few slight inconsistencies, and difficult-to-preserve concrete proof, there is a multitude of evidence to show that Leibniz and Newton were not the very first ones to discover certain principles of calculus even though they may be considered as the biggest contributors today. I think it is super important to tell the history of an idea accurately but with an open mind, because making false conclusions can take away from people's deserved recognition; no matter the exact contributions or timeline, everyone I mentioned were very intellectual mathematicians who took some part in developing the subject of calculus and deserves credit and praise. I do not want to make any definite claims about the exact origins of calculus; I just want to give an approximate timeline and include important, historical details that are fascinating to investigate and are extremely useful in understanding modern calculus.

MODERN APPLICATIONS OF CALCULUS

The examples of modern day applications of calculus can be much more concrete and evident than the origins of the subject, but they are equally important; without all of the history and mathematical contributions, we would not have this modern calculus that our societies heavily rely on. There are innumerable applications of calculus existing in mathematics and a variety of other subjects. Exploring all these different subjects and calculus cores will exemplify how crucial calculus is to our society.

One of the main applications that emphasizes the importance of calculus compared to other branches, is that it is often used to check answers for those different branches of mathematics. For example, you can solve functions using algebra, but calculus will tell you if/how the function changes as the variable's input changes. Derivatives in calculus can also tell you the minimums and maximums of a function, to double check that the value solved algebraically or geometrically is accurate by checking if it's in that range. Now, let's look at some of the non-mathematical applications of calculus.

Physics

Starting off with maybe a more obvious example, calculus plays a huge role in the world of physics, especially kinematics. To put it shortly, the subject of physics studies bodies of matter and their movements through space and time. With an emphasis on those motions, calculus provides the basis for calculating the rates at which these bodies of matter move. Kinematics uses integral calculus to calculate quantities like velocity and acceleration rates. For example, if an object is moving in a straight line and we know that at at a certain time, it's position or displacement (meaning the distance from a reference point) is given by a function x, we can find the velocity and acceleration of that object with respect to time t by differentiating the function twice:

$$Velocity\ is\ the\ derivative\ of\ displacement\ with\ respect\ to\ time\ [f'(t) = \frac{dx}{dt}]$$

$$Acceleration\ is\ the\ derivative\ of\ velocity\ with\ respect\ to\ time\ [f''(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}]$$

You can also do this process backwards, which would be the integration of acceleration to get velocity and the integration of velocity to get position. There are a lot of complexities to ideas in kinematics and there is no doubt that calculus is a main, huge concept that helps solve and simplify those complex calculations. Through these and countless other examples such as fluid mechanics, electromagnetism, and thermodynamics, it is clear that integral calculus plays a very important role in the subject of physics.

Pharmacology

Another prominent example is the use of calculus in pharmacology. First, any kind of prescribed drug will be decided based on certain factors such as sex, weight, and existing medical or physical conditions. Each drug prescription and dose is modified to work for one particular person based on those changing factors, and the decisions behind these controlled doses are determined by calculus. Mathematicians and physicians work together to determine how a drug will change the state of someone's medical health over time. For example, the positive and negative effects of drugs are controlled by calculus; differential equations are used to investigate confounding factors, to optimize drug strength, and to minimize negative side effects. Scientists and doctors use the following formula to measure drug dissolution rates:

$$\frac{dC}{dt} = \frac{DS}{Vh}(C_s - C)$$

Where C is the concentration of a substance, t is time, $\frac{dC}{dt}$ is the rate of dissolution, and the other variables account for changing factors like the surface area and solubility (Shi et al., 2014). There are more complex, important parts to it that technology can solve, but it's important to acknowledge the calculus formula for its usefulness. Using this, pharmacokinetics can calculate the concentration of a drug and how that concentration changes over time; this helps physicians choose the best drug options and doses for each and any individual. Without the development of calculus, this formula and numerous others would not exist, and the field of medicine would be far less advanced.

COVID-19

To exaggerate this idea that calculus plays a big part in the medicine/healthcare world, let's look at one of the existing, ongoing threats that our nation is faced with: Covid-19. This pandemic and many other epidemics have horrendous effects on societies; epidemiologists use calculus to try and combat these harmful consequences. They do this by creating models that input quantitative statistics of positive covid test results to calculate the rates of which the disease spread and to estimate new population statistics after a certain amount of time. These calculus models can help epidemiologists locate the origins of certain epidemics, predict how far and fast they will spread, and control morbidity and mortality rates.

As you can see, any subject or field that aims to study something that changes or moves can use calculus to evaluate the rates of that change. Not only is calculus extremely applicable, but it simplifies a lot of concepts; the examples I gave could probably be figured out without having calculus formulas, but it would take a lot more time and energy in which we don't always have the luxury to have or simply don't want to overexert our resources.

Combining the idea that everything in the universe is always in motion with the existing evidence of discoveries made today, it is clear that there are countless applications of calculus constantly happening all over the world and have been occurring for milleniums. The applicability of the subject is recognized and utilized at the global level.

Others

The following list describes just a few more of the abundant applications of calculus that can be found in our societies today:

Engineering (Electrical, Mechanical, and Nuclear):

Electronic engineers use calculus to solve differential equations occurring in the examinations of electronic circuits, radioactive decay, and servomechanisms (Sawant, 2018). One example is the Laplace Transform; this method helps solve those differential equations by using something called the Laplace integral to convert them into algebraic equations. It is a method used to analyze and simplify calculations in system modeling, digital signal processing, and process controls; this method is crucial to many engineering fields (Sawant, 2018).

Economics:

Calculus and a few other branches of mathematics are helpful in the field of economics. Economics analyzes what societies do in times of scarcity of resources; calculus helps create mathematical models for these results and determine what the best options are to access resources, promote welfare in these times, and maximize profits based on existing quantities. For example, calculus can analyze and combine functions of costs, demands, and revenues to get a function of total production and total profit; one can differentiate this equation to get a new one that gives you an output of profit optimization values based on the input of the number of items produced (Marsitin, 2019). Differential equations like this are seen all throughout economics, and since economics studies the changes in quantitative resources in societies, we will usually always be able to calculate those rates of change.

Technology:

I would say technology or computer science is the subject most reliant on calculus out of all these listed; there are innumerable possible actions you can make with technology, and calculus accounts for most of them. Most technological devices are dynamic when used so they constantly require calculations to be most efficient to the user. For example, when you click a certain letter on your keyboard which is really just a certain section of your screen, your phone will register it and show that letter on the screen. Testing how fast this happens is an example of inputting a quantity or action to get an output quantity and calculating that rate of change; calculus is used to minimize the time that it takes and to maximize the efficiency so that as you type, the letters quickly show up on the screen. Technology, when in use, is always changing and collecting data to calculate, utilize, and optimize.

Medicine and Biology:

Calculus plays a huge role in the world of medicine, which has been revolutionary to better individuals' health and even increase life spans. Monitoring changes in medical statistics over the decades helps us find cures and therapies for specific illnesses and plan for future diseases or complications. We already analyzed one area of medicine, pharmacology, where calculus is heavily exploited. Other utilizations of calculus include the enhancement of medical images, the calculation of population growth rates and percentiles such as weight vs. age, and the modeling of tumor growth and other growth/decay issues. In all of these examples, one can see how a quantity is inputted in hopes of getting proper outputs and that process is calculated in order to optimize the results. Also, as discussed, technology heavily relies on calculus and you will find that hospitals are filled with complex technology and mathematical systems; medicine itself and the ability to practice it would be completely different without technology using calculus.

Epidemiology:

Epidemiology is a branch of medicine that investigates diseases and aims to promote the medical health of populations, which shows the usefulness for calculus to be applied to each individual and masses of groups. As discussed, the COVID-19 pandemic is one example of how epidemiologists use calculus to accurately make conclusions about the rates of disease spread. No matter the type or even quantity of illnesses, if there are statistics provided, calculus can be used to model disease expansions and help mitigate the global spread of these illnesses. Another example is the HIV/TB co-pandemic. People who have HIV are at higher risk of getting Tuberculosis; the co-infection demographics today are modeled by calculus and analyzed in order to prevent its expansion. To make sure they can accurately analyze the big picture while considering certain variables like confounding factors and outliers, mathematics and epidemiologists use fractional calculus (a generalization of ordinary calculus dealing with arbitrary orders) to model certain functions and systems that are nonlinear or patternless; one of those confounding factors include heredity, prior knowledge of a disease, and memory effects (Tanvi et. al, 2021). Since each of these has a large spectrum of elements that are always changing, it is important to be able to analyze nonlinear functions in epidemiology. This was one of Newton's goals; he wanted to be able to account for those changes and find the gradients of changing, nonlinear relationships.

Architecture:

Architects use calculus to build the best-quality structures while considering changing variables like quality of materials, how much stress the structure can take over time, and what sizes, shapes, and angles of materials built would optimize its durability. The Eiffel Tower was actually built utilizing calculus; architects formed a mathematical model that provided the best design plan for optimizing the output of wind resistance based on the inputs of exceedingly large height and weight of the tower (Weidman & Pinelis, 2004).

Astronomy:

Astronomers want the best photography results to facilitate accurate analyses, so they use image processing to clean up and filter images taken by space satellites and telescopes. Fractional calculus can be and is used in ways that magnify the visibility of galactic structures, restore images that have weak contrasts or details, and intensify the images and details of planetary surfaces (Sparavigna & Milligan, 2009). Calculus can also be used to calculate the rates of change of moving objects in space like planets and stars that are constantly in motion. This branch of mathematics is evidently significant in the modern subject of astronomy as it played a huge role in expanding and improving multiple areas of astrophotography and astronomy as a whole.

By comparing the modern day applications with its initial foundations, one can clearly see that calculus has evolved at an incredible rate (you can actually calculate this rate with the subject itself!) and it is absolutely incredible to see how progressive our societies have become with the popular inclusion and use of calculus. It is clear that the subject is applicable and critical in many other branches of mathematics and a variety of other subjects. Things we see around us everyday were built using calculus. The computer programming and technology that runs the world today is heavily reliant on calculus. In college, non-math majors are encouraged or even required to take a calculus class as many majors partially or heavily rely on calculus. I could go on and on about how applicable and necessary calculus is or how old-fashioned and unprogressive our world would be without it. You individually may not use it daily, but there is concrete proof that it is constantly being utilized all around you.

CONCLUSION

The history and applications of any subject are extremely important to know and should be prioritized before learning anything about the subject or being forced to memorize its principles. I remember when I took calculus classes, I was simply told to memorize derivatives and the differentiation rules, but I had no idea where these rules came from. I did not need to know the history to get a good grade in the class; I just needed to memorize those rules and apply them. This made me think, where did derivatives come from? I began to wonder and generalize that idea to the whole subject; where did calculus come from? Everyone trusts that their textbooks and educators are feeding them accurate information, but no one will know unless they discover where the information came from and/or try to apply the information to outside scenarios. The history of calculus is like a big, long proof; the complex history proves why calculus is the advanced subject it is today. The applicability of any subject is equally important to recognize; it seems pointless to learn a bunch of information that cannot ever be put to use. The history and applications of a subject can greatly, independently educate an individual, but when researched together they can be extraordinary. As soon as one can understand the limitless possibilities of what derivatives and integrals can solve, I believe it is basically impossible to believe calculus is anything but extraordinary. If one simply imagined how adverse our world would be without calculus, they could understand all of its importance and beauty. Reader, after all of this, I hope you can understand it too.

If you want to know more about the overall historical timeline of calculus rather than focusing on the prominent contributors to modern calculus, I would recommend reading the book *Calculus and Its Origins.* Author David Perkins goes into great depths about a multitude of mathematicians who independently discovered calculus, what these discoveries entailed and how they are important. There is not a lot of research on the history of calculus so I think this is a great read; on the other hand, you can find plenty of articles discussing the uses of calculus today.

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