

Marriage Markets in Developing Countries

by

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DISSERTATION ABSTRACT

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This dissertation studies, using the tools of dynamic macroeconomics, marriage markets in developing countries. The goal is to understand how the marriage market affects marital fertility, female labor supply and parents' human capital investment in girls.

Chapter 1 provides the rationale for considering marriage markets in developing countries. It also presents an overview of the three research chapters. Chapter 2 develops an intergenerational model with gender bias in female education and dynamic marriage market. The model features skill-based positive assortative matching (PAM) and accounts for the gender-specific skill imbalance observed in developing countries. Within a household, spouses work in the labor market and decide about consumption, fertility and children's education. We show how the equilibrium fertility distribution depends on different types of households that arise from marriage market matching and differences in fertility outcomes based on the quality-quantity tradeoff and parental skill levels. We estimate the model using Indian data, numerically derive the steady state and establish its local stability. Based on simulation results, the model does a good job of replicating the observed skill ratios.

Chapter 3 builds on the model of chapter 2. The model is used to develop several policy-relevant results. An increase in marital sorting – as has been observed in India over time – worsens income inequality, and the gender bias in education and income. Elimination of gender bias as well as exogenous increases in returns to education and skilled-labor productivity contribute toward gender equality. Whereas gender-neutral subsidies are ineffective, the subsidies to poorer households aimed towards encouraging female higher education reduces the gender gap in education, labor supply and income. Dynamic policy analysis reveals that it takes 2 generations to reduce the gender gap in education by one-third. We conclude that gender-targeted policy can significantly weaken taste-based gender discrimination against female higher education.

Chapter 4, joint work with Shankha Chakraborty, adapts the previous framework to better

suit marriage markets in developing countries. A large percentage of marriages occur through family connections (“consensual arranged”) that prioritize economic security and cultural values. Our framework captures the central tenet of these arranged marriages: parental decision to invest in girls’ education is influenced by expectations of their marriage market outcome. We construct an intergenerational model with two-stage arranged-marriage market search model, which rationalizes parents’ subjective gains from marrying off their offspring. The theoretical model is loosely calibrated to Indian data. Preliminary results indicate that there are significant returns to girls’ education in the marriage market. In the future, we plan to extend the framework to identify the “social returns” of female education, considering its effect on marriage formation, marital fertility, labor supply and intergenerational education transmission.

This dissertation includes previously unpublished coauthored material.

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To my parents, who instilled into me the transformative power of education

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CHAPTER 1

INTRODUCTION

Marriage markets in developing countries differ from developed countries in many ways. Families often play an active role in marriage decisions, leading to “consensual arranged” marriages based on family connections, as opposed to “self-selected” marriages, which is a norm in developed countries. Moreover, marital decisions are often subject to cultural norms which give precedence to men over women. These patriarchal norms have resulted into marriage markets characterized by – transfer of resources from bride’s families to groom’s families at the time of marriage (the institution of dowry), lower age at marriage of women, more educated men than women, lower levels of bride’s education compared to groom.

Existing theories of marriage market do not quite fit into these characteristics of marriage market (with the exception of dowry). This dissertation focuses on formalizing the differences found in developing countries, as marital decisions have implications for human capital investments, marital fertility, women’s labor supply decisions, intergenerational transmission of education and human development. These models are then used to conduct policy analysis and highlight the important role played by marriage markets in economic decisions.

Chapter 2 models an overlapping generations model with taste-based gender discrimination against girls’ education and dynamic marriage market. Matching in the marriage market is exogenous, has positive assortative matching (PAM) on education levels, and accounts for the skill imbalances between men and women. The model is then estimated using Indian data, and simulation results indicate that the model fairly replicates skill levels and fertility distribution observed in Indian data.

Chapter 3 extends the model developed in chapter 2 to study dynamic evolution of the economy and comparative statics. We document that an increase in PAM – an empirical regularity – worsens inter-household income inequality. We use the model for policy analyses, and observe that education subsidies need to be targeted towards girls’ education for them to be effective against deeply-entrenched gender norms against female education. Policy dynamics enables us to comment on the time required for policy to take full effect.

Chapter 4, jointly developed with Shankha Chakraborty, adapts the framework from chapter 2 to include an important feature of arranged marriage – parental decision to invest in girls’

education is influenced by expectations of their marriage market outcome. To rationalize parents' subjective gains from marrying off their offspring, we construct an overlapping generations model with two-stage arranged-marriage market search process. Our quantitative analysis indicates presence of significant returns to female education in the marriage market.

CHAPTER 2

MARITAL SORTING AND GENDER BIAS

2.1 Introduction

Despite substantial progress towards gender equality, large gaps persist in developing countries. The sources of gender inequality are varied too. For example, they can take the form of active discrimination within the household or in the labor market, or they can emerge from market frictions that constrain women's opportunities. Various aspects of gender inequality such as income, opportunity, labor market participation, sex ratio, education and health have been extensively studied, and policy measures to address them have also been documented in the literature (see Fernández et al. (2022) for a recent review of reasons behind various forms of gender gaps and policy proposals to reduce these gaps). Yet active forms of gender bias within the household have not received the attention they deserve in the literature. These biases have their roots in patriarchal norms that are defined by “a set of predominant kinship and family structures and beliefs that give precedence to men over women, sons over daughters, fathers over mothers, husbands over wives” (Seymour, 2019).

To understand the impact of any policy measure on gender equality, a framework that addresses the gender bias within households is essential. The objective of this chapter is to construct such a theoretical framework, incorporating gender bias due to taste-based discrimination against girls' education à la Becker (1957). The subsequent chapter proposes policy solutions, in the form of education subsidies, to mitigate this bias. The theoretical framework in the chapter takes the form of a dynamic marriage market model. Participants in the marriage market differ in age, education levels, religion, social status, race, ethnicity, and so on. Despite these differences, they tend to marry those with similar characteristics, an outcome referred to as positive assortative matching, or PAM (Becker, 1973, 1974). We focus on PAM by education level, similar to the economics literature, for example, Fernández et al.' (2005) work on developed and developing countries.

In this chapter, we construct an overlapping generations model where parents make a discrete educational investment in their children and are biased against the education of girls, deriving less utility from educating them compared to boys. It is important to note that we abstract from other forms of gender bias, such as labor market discrimination against women and patriarchal norms which restrict female labor market participation. Parental

educational investment leads to children being skilled, while all children receive some basic compulsory education free of cost. Furthermore, we assume an exogenous matching of agents in the marriage market, resulting in different types of households. This matching process – we call it mechanical matching – exhibits PAM based on skill levels. Depending on the skill level of husband and wife, the matching results in four types of households: high-skilled, two types of mixed-skilled and low-skilled (where both are unskilled). Spouses in a household work in the labor market, and along with making consumption and fertility decisions, decide about their children’s education.

In a dynamic general equilibrium, the evolution of the economy is determined by – a) the ratio of skilled men to total men, b) the ratio of skilled women to total women, and c) the ratio of skilled to total labor. We estimate household parameters by the generalized method of moments (GMM) using Indian data and calibrate parameters for both the marriage market and the production sector of the economy. Our analysis reveals a unique economically meaningful, locally stable steady state characterized by empirically plausible skill ratios. Based on the simulated fertility distribution, our computational exercise does a promising job of explaining the observed fertility gap among various types of households. Additionally, the numerical results validate the presence of quality-quantity tradeoff.

Relative to the literature surveyed later, the main contribution of this chapter is a theoretical model that introduces taste-based gender discrimination and produces a skill imbalance between men and women in equilibrium. The model thus captures a main aspect of developing countries like India. Yin (2022) is the only other work we are aware of that analyzes this aspect, but there are important differences. Yin (2022) does not consider the importance of skilled labor scarcity relative to unskilled labor for human capital investment. However, Khanna (2023) has shown that general equilibrium labor market effects suppress returns to education by 6.6 percentage points for India, which reduces investment in skills differently for boys and girls. Not accounting for this margin leads Yin (2022) to conclude that “intrinsic son preference” (similar to taste-based gender discrimination in our model) does not play a significant role in the skewed gender ratio in skills. Once the labor market effects are considered, which our model does, it is clear that taste-based discrimination remains an important source of gender bias. We also extend the numerical analysis to study the policy dynamics, presented in the next chapter, which is absent from their work.

The rest of the chapter is organized as follows. The next section presents a brief review of the

literature. Section 3 elaborates on the model, the next section discusses some quantitative analysis using Indian data and the final section concludes.

2.2 Literature Review

To the best of our knowledge, this is the first paper which constructs a dynamic general equilibrium model of marriage market for developing countries. The chapter also proposes a novel matching method, labeled as mechanical matching process, to account for skill imbalance in the marriage market in these regions. Our policy analysis of dynamic effects of education subsidies is a novel contribution to the extensive literature on policies for gender equality. Moreover, Fernández and Rogerson (2001) have documented that exogenous increase in PAM leads to higher inter-household income inequality, and we extend this finding to the developing world.

The economics literature has theoretically and empirically established assortative matching as an important characteristics of marriage market, beginning with pioneering contributions by Becker (1973, 1974). He proposed the theory of assortative mating in the marriage market for the first time. When agents in the marriage market differ on one ordered characteristic, he showed that the total output from the marriage market was maximized – under the assumption of transferable utility – when there existed perfect PAM of agents on most of the characteristics including education¹ Subsequently, there have been numerous studies extending Becker’s model of the marriage market.² Choo and Siow (2006) and Siow (2015) empirically examine Becker’s theory of PAM and conclude that it is indeed a characteristic of the US marriage market. Quite a few studies establish the same for developed and developing countries (e.g. Fernández et al., 2005; Greenwood et al., 2014; Chiappori, Dias, and Meghir, 2020; Chiappori, Costa-Dias, et al., 2020). Relevant to our focus on the Indian marriage market, Borkotoky and Gupta (2016) analyze India’s marriage cohort from 1964 to 2006 and conclude that marital sorting on education levels has increased while Kashyap et al.

¹The exception was wage rates of males and females, which had negative assortative mating. It arose due to substitutability of time of husband and wife in household production function, thus gains from marriage were maximized by division of labor. We abstract away from household production and consider only market production.

²Chiappori (2020) provides a review of the recent literature on the marriage market, with the focus on microeconomic literature.

(2015) project that educationally homogamous marriages will increase from 50.6% in 2005 to 68.9% in 2050. This clearly indicates that there exists marital sorting along educational attainment (and it is going to increase). This chapter uses the proven fact of educational homogamy in the marriage market to model the household-level heterogeneity.

This chapter is related to the existing macroeconomic literature on marriage sorting, which is mainly focused on developed economies (Aiyagari et al., 2000; Fernández and Rogerson, 2001; Fernández, 2002; Greenwood et al., 2003; Fernández et al., 2005; Greenwood et al., 2014; Greenwood et al., 2016; Caucutt et al., 2021), with Yin (2022) being an exception in its focus on marital sorting for India. In the context of developed countries, Fernández and Rogerson (2001) develop a theoretical model of exogenous marital sorting along education levels, and using US data, they find that an exogenous increase in sorting leads to higher income inequality. We extend this theoretical framework to developing countries, which have gender imbalance in skills, by developing a novel marriage market matching process exhibiting PAM, which we call the mechanical matching process.³ The recent macroeconomic literature on marriage market takes computational approach (Aiyagari et al., 2000; Greenwood et al., 2003; Greenwood et al., 2016; Caucutt et al., 2021; Yin, 2022) whereas our work is primarily based on an analytically-tractable model.

The literature on the Indian marriage market has documented the marriage practices in India as being endogamous (i.e. men and women with similar characteristics tend to marry), and within-caste, the caste system being a system of social stratification (Driver, 1984; Deolalikar and Rao, 1998; Kashyap et al., 2015; Borkotoky and Gupta, 2016). The literature has largely focused on age of women at marriage, the caste system and the dowry system (Caldwell et al., 1983; Bloom and Reddy, 1986; Rao, 1993; Deolalikar and Rao, 1998; Anderson, 2003; Anderson, 2007; Banerjee et al., 2013; Chiplunkar and Weaver, 2023; Goel and Barua, 2021). Anukriti and Dasgupta (2017) provide a review of the literature on marriage markets in developing countries and the literature also largely focuses on age at marriage, marriage payments and PAM. Our work is an attempt to fill vacuum that exists in regard to studying the linkages among marriage market, income distribution and fertility distribution in the context of developing countries and particularly in the Indian context.

As we model gender bias in female education in developing countries with parents' taste-

³We endogenize the fertility decisions as well, unlike in Fernández and Rogerson (2001).

based gender bias against girls' education, our work is related to literature on taste-based discrimination. Since Becker's (1957) pioneering work, there has been a lot of research on taste-based gender discrimination, mainly focused on labor market (e.g. Riach and Rich, 2002; Bertrand and Mullainathan, 2004; Charles and Guryan, 2008; Price and Wolfers, 2010; Guryan and Charles, 2013; Blau and Kahn, 2017; Lane, 2019). In the context of developing countries, there exist empirical literature, largely focused on son-biased fertility preferences in India (e.g. Rahman et al., 1992; Chen and Drèze, 1992; Jayachandran and Kuziemko, 2011; Barcellos et al., 2014; Jayachandran and Pande, 2017; Jayachandran, 2023). To the best of our knowledge, only Lahiri and Self (2007) and Yin (2022) formally model gender bias in education as an outcome of subjective household preferences in developing countries. We follow a similar approach and assume that the gender bias is an outcome of taste-based discrimination which can result from prejudice against girls' education due to cultural and social norms. While Lahiri and Self (2007) focus on the role played by dowry and lack of co-ordination between sons' and daughters-in-law's households for gender bias in education with a partial equilibrium model, Yin (2022) is focused on explaining economic factors such as dowry, old-age support by sons, that bias the sex ratio towards males. In comparison, this chapter constructs a dynamic general equilibrium model with gender bias in education, and the subsequent chapter studies dynamics of education policy that alleviates its effects.

2.3 The Model

We present an intergenerational model with an initial distribution of skilled and unskilled men and women. In each period, agents match up exogenously in a "marriage market" and the matching process showcases PAM based on skill levels. Spouses in a household work in the labor market and decide about consumption, fertility and children's education. We assume that households discriminate against girls' higher education due to taste-based gender bias which may arise from cultural and social norms. Households' choices regarding fertility and children's education determine the distribution of skilled and unskilled workforce in the next generation. Dynamic evolution of the economy is fully characterized by: a) the ratio of skilled men to total men, b) the ratio of skilled women to total women, and c) the ratio of skilled to total labor.⁴ Fernández and Rogerson (2001) have also constructed a model

⁴While men supply a unit labor, women may not do so due to child-rearing responsibilities in this model. The "household" subsection of the model deals with the labor supply decisions

of an economy with exogenous PAM, but our model differs from it as we model fertility decisions and gender differences in human capital investment decisions of households. And importantly, they do not theoretically solve for household decision problem and assume existence of optimal solution at the household level whereas our model, presented below, solves the household optimization problem, and then we calibrate the model for numerical analysis.

Marriage Market

Consider population N_t at time t , with skilled and unskilled workers, denoted respectively, by N_{st} and N_{ut} . Superscripts m and f denote male and female respectively. Thus,

$$N_t = N_{st} + N_{ut} \tag{2.1}$$

$$N_t^m = N_{st}^m + N_{ut}^m \tag{2.2}$$

$$N_t^f = N_{st}^f + N_{ut}^f. \tag{2.3}$$

We assume that men and women are equally numerous i.e. $N_t^m = N_t^f$. The skill level is equivalent to education level: to fix ideas, suppose a skilled person is one with more than 10 years of education and an unskilled person is one with 10 years of education or below.

We assume that a fraction α of marriages is based on random matching and the rest $(1 - \alpha)$ occurs through a mechanical matching process which exhibits PAM, discussed below. The random matching parameter α is assumed to be exogenous and time-invariant. The two types of matching result in four types of households, *high-skilled* (denoted by subscript h) where both spouses are skilled, *mixed-skilled-1* (denoted by subscript $m1$) where a skilled woman is matched with an unskilled man, *mixed-skilled-2* (denoted by subscript $m2$) where a skilled man is matched with an unskilled woman, and *low-skilled* (denoted by subscript l) where both spouses are unskilled. The gender education imbalance in developing countries means there is a higher supply of skilled men than skilled women ($N_{st}^m > N_{st}^f$).⁵ Our model produces this, leading to more mixed-marriages of *mixed-skilled-2* than *mixed-skilled-1* in

in detail.

⁵For India, for example, the values of the ratios of skilled to total males and skilled to total females for India are 38.64% and 24.41% respectively, based on the Demographic and Health Survey (DHS) of 2005-06 for India.

equilibrium.

Let λ_h , λ_{m1} , λ_{m2} and λ_l be the fractions of high-skilled, mixed-skilled-1, mixed-skilled-2 and low-skilled households respectively, where,

$$\lambda_h + \lambda_{m1} + \lambda_{m2} + \lambda_l = 1 \quad (2.4)$$

Let p_t^m be the probability than a man is skilled. It is, at time t , given by,

$$p_t^m = \frac{N_{st}^m}{N_t^m}. \quad (2.5)$$

Let p_t^f be the probability than a woman is skilled. It is, at time t , given by,

$$p_t^f = \frac{N_{st}^f}{N_t^f}. \quad (2.6)$$

Now consider the mechanical matching process that leads to PAM. Skilled female matches only with skill male (shown in figure 2.1) and unskilled male matches only with unskilled female (shown in figure 2.1). This leaves some skilled men and some unskilled women unmatched due to gender imbalance in education. We assume the remaining skilled men match with the remaining unskilled women.⁶ We show the process with the help of figure 2.1.

We use superscript mm to denote matching arising from mechanical matching process i.e. λ_j^{mm} is the fraction of j -type marriage which arises from mechanical matching process, where $j \in \{h, m2, l\}$ and $\lambda_{ht}^{mm} + \lambda_{m2t}^{mm} + \lambda_{lt}^{mm} = 1$. Given the definition of mechanical matching process,

$$\lambda_{ht}^{mm} = \frac{N_{st}^f}{N_t^f} = p_t^f \quad (2.7)$$

$$\lambda_{lt}^{mm} = \frac{N_{ut}^m}{N_t^m} = 1 - p_t^m \quad (2.8)$$

$$\lambda_{m2t}^{mm} = 1 - \lambda_{ht}^{mm} - \lambda_{lt}^{mm} = p_t^m - p_t^f. \quad (2.9)$$

⁶Even though the matching process results in some matches not exhibiting PAM, their percentage is small. In the next section on quantitative analysis, the steady-state results indicate that more than 90% of the mechanical matches exhibit PAM.

Given the random matching parameter α , p_t^m and p_t^f , we will have the following fractions of matches for each type of household:

$$\lambda_{ht} = \alpha p_t^m p_t^f + (1 - \alpha) \lambda_{ht}^{mm} \quad (2.10)$$

$$\lambda_{m1t} = \alpha p_t^f (1 - p_t^m) \quad (2.11)$$

$$\lambda_{m2t} = \alpha p_t^m (1 - p_t^f) + (1 - \alpha) \lambda_{m2t}^{mm} \quad (2.12)$$

$$\lambda_{lt} = \alpha (1 - p_t^m) (1 - p_t^f) + (1 - \alpha) \lambda_{lt}^{mm}. \quad (2.13)$$

λ_{ht} is derived as follows: fraction α is the probability that agents are matched randomly, and $p_t^m p_t^f$ is the probability that a skilled man will be matched with a skilled woman. This gives us the first element $\alpha p_t^m p_t^f$ of equation (2.10). As $(1 - \alpha)$ is the probability of the mechanical matching, the fraction $(1 - \alpha) \lambda_{ht}^{mm}$ constitutes the second part of equation (2.10). Other fractions are derived similarly, with the fraction λ_{m1} consisting of the matching arising from random matching only as the mechanical matching process does not give rise to a match between a skilled female and an unskilled male.

Using equations (2.7), (2.8) and (2.9) in the above set of equations, we can rewrite the fractions of matches as follows:

$$\lambda_{ht} = \alpha p_t^m p_t^f + (1 - \alpha) p_t^f \quad (2.14)$$

$$\lambda_{m1t} = \alpha p_t^f (1 - p_t^m) \quad (2.15)$$

$$\lambda_{m2t} = \alpha p_t^m (1 - p_t^f) + (1 - \alpha) (p_t^m - p_t^f) \quad (2.16)$$

$$\lambda_{lt} = \alpha (1 - p_t^m) (1 - p_t^f) + (1 - \alpha) (1 - p_t^m). \quad (2.17)$$

We have verified that $\lambda_{ht} + \lambda_{m1t} + \lambda_{m2t} + \lambda_{lt} = 1$.

Production

Let β_t be the fraction of skilled labor (denoted by L_{st}) to total labor (denoted by L_t) at time t ,

$$\beta_t = \frac{L_{st}}{L_t}. \quad (2.18)$$

The production function is of the constant returns to scale (CRS) form, using skilled and unskilled labor to produce the unique consumption good:

$$F(L_s, L_u) = L_u F\left(\frac{L_s}{L_u}, 1\right) = L_u F\left(\frac{\beta}{1-\beta}, 1\right) = L_u f(\beta), \quad (2.19)$$

where, $f'(\beta) > 0, f''(\beta) < 0$.

We assume competitive labor market and hence the equilibrium skilled and unskilled wage rates equal to the marginal products of skilled and unskilled labor respectively,

$$w_s(\beta_t) = (1 - \beta_t)^2 f'(\beta_t) \quad (w'_s < 0) \quad (2.20)$$

$$w_u(\beta_t) = f(\beta_t) - \beta_t(1 - \beta_t)f'(\beta_t) \quad (w'_u > 0). \quad (2.21)$$

The skilled wage rate is decreasing in β and the unskilled wage rate is increasing in β . As β increases, the relative scarcity of skilled labor compared to unskilled labor decreases, which results in decrease in the equilibrium skilled wage rate and an increase in the equilibrium unskilled wage rate.

Household

The household decides on consumption, fertility and children's education subject to household income and time constraint arising out of child-rearing for female.⁷ The household has differential taste-based preference for sons' and daughters' education, which may be a result of long-standing beliefs and cultural norms. Following Lahiri and Self (2007) and Yin (2022), we model the gender bias in education as an outcome of private decisions at the household level.

There are four types of households, resulting from the marriage market, denoted by subscript j ($j \in (h, m1, m2, l)$). The household utility function is given by (2.22a), where we assume that the household derives utility from consumption (c), number of children (n) and children's quality (q , with the subscripts b and g for boys and girls respectively). Parameter ϕ captures taste-based gender discrimination against girls' education. When $\phi < 1$, the household is assumed to derive less utility from daughters' education as compared to sons'

⁷We assume that the time constraint is relevant only for female and not for male, a reasonable assumption in the context of developing countries.

education. This is a simple way to introduce gender bias and we shall assume hereon that $\phi \in (0,1]$. The household utility maximization problem for each type of household is

$$\begin{aligned} & \underset{c_{jt}, n_{jt}, \hat{a}_{bjt}, \hat{a}_{gjt}}{\text{maximize}} && U_{jt} = \ln c_{jt} + \gamma \ln n_{jt} + \eta(\ln q_{bjt} + \phi \ln q_{gjt}) \end{aligned} \quad (2.22a)$$

$$\text{subject to} \quad l_{wjt} + \tau n_{jt} = 1 \text{ (female time constraint),} \quad (2.22b)$$

$$c_{jt} + 0.5(r_{bjt} + r_{gjt})n_{jt}v_{jt} = w_{jt}^m + w_{jt}^f l_{wjt} \text{ (budget constraint),} \quad (2.22c)$$

$$q_{bjt} = r_{bjt}w_{st+1} + (1 - r_{bjt})w_{ut+1} \text{ (boys' quality),} \quad (2.22d)$$

$$q_{gjt} = r_{gjt}w_{st+1} + (1 - r_{gjt})w_{ut+1} \text{ (girls' quality),} \quad (2.22e)$$

$$r_{bjt} = \frac{\bar{a} - \hat{a}_{bjt}}{\bar{a} - \underline{a}}, \quad (2.22f)$$

$$r_{gjt} = \frac{\bar{a} - \hat{a}_{gjt}}{\bar{a} - \underline{a}}, \quad (2.22g)$$

taking as given wages $\{w_{st}, w_{ut}, w_{st+1}, w_{ut+1}\}$. The constraints are explained next.

Constraint (2.22b) denotes the time constraint for the mother. Her total time is normalized to 1 and l_w is the time spent on market work ($0 \leq l_w \leq 1$)⁸. Parameter τ is the time cost associated with child rearing (per child), assumed to be time-invariant and $\tau \in (0, 1)$.⁹

Budget constraint ((2.22c)) has household income ($w_{jt}^m + w_{jt}^f l_{wjt}$) on the right side, with superscripts m and f for male and female respectively and $w_{jt}^m, w_{jt}^f \in (w_{st}, w_{ut})$ depending on j . On the left side are consumption expenses and children's education expenses. v_{jt} is the cost of education per child which varies with the type of household, and it is assumed to be growing at an exogenously determined rate g with v_{j0} as the initial value i.e. $v_{jt} = v_{j0}(1+g)^t$. The fraction of boys and girls who receive education are denoted by r_b ($0 \leq r_b \leq 1$) and r_g

⁸The constraint on l_w is not explicitly taken into account while numerically solving the optimization problem.

⁹We assume that value of τ does not differ across households, whereas in reality, for a given n , τ may differ across households. In developing countries, low-skilled work like agricultural labor can often be carried out by females while simultaneously caring for children, leading to lower τ . This may lead to higher labor supply for such households than skilled-households, but we do not account for this margin.

($0 \leq r_g \leq 1$) respectively. We assume that skill levels and education are equivalent to each other. The decision to invest in education of children means investing in higher education i.e. education for more than 10 years, leading to a skilled individual whereas the decision not to invest in their education means they receive up to 10 years of education, resulting in an unskilled individual. We assume that education up to 10 years is free of cost.

Constraints (2.22d) and (2.22e) denote boys' and girls' quality.¹⁰ They are driven by the future wages, either skilled or unskilled wage rate (w_{ut+1} or w_{st+1}) and educational investment decisions of the household, which determine r_b and r_g . Last two constraints¹¹, (2.22f) and (2.22g), are based on the assumption that educational investment decision depends on the ability of each child, independently drawn from uniform distribution with support $[\underline{a}, \bar{a}]$. And there's a cut-off ability \hat{a} , which parents choose, below which parents decide not to educate their children. Fractions r_b and r_g are therefore determined by the ability cut-offs \hat{a}_b and \hat{a}_g respectively, resulting in equations (2.22f) and (2.22g). Thus, the educational investment decisions by parents are equivalent to choosing the ability-cutoffs for boys and girls.¹²

The solution to the optimization problem is described in detail in appendix A. This leads to the following results:

$$n_{jt} = \frac{(w_{jt}^m + w_{jt}^f)(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)}{(1 + \gamma)(\tau w_{jt}^f(w_{st+1} - w_{ut+1}) - v_{jt}w_{ut+1})} \quad (2.23)$$

$$\hat{a}_{bjt} = \frac{v_{jt}(\bar{a}w_{st+1} - \underline{a}w_{ut+1})(\gamma + \eta - \phi\eta) - 2\eta(w_{st+1} - w_{ut+1})(\tau w_{jt}^f(\bar{a} - \underline{a}) + \bar{a}v_{jt})}{v_{jt}(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)} \quad (2.24)$$

$$\hat{a}_{gjt} = \frac{v_{jt}(\bar{a}w_{st+1} - \underline{a}w_{ut+1})(\gamma - \eta + \phi\eta) - 2\eta\phi(w_{st+1} - w_{ut+1})(\tau w_{jt}^f(\bar{a} - \underline{a}) + \bar{a}v_{jt})}{v_{jt}(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)}. \quad (2.25)$$

¹⁰They are not constraints per se, but are equations written separately for ease of exposition.

¹¹They are not constraints per se, but are equations written separately for ease of exposition.

¹²Ability is not essential to the model and the household can directly choose r_b and r_g . In the absence of ability, the household's choice to educate some boys or girls will give rise to another type of discrimination, one based on rationing due to budget constraints. We prefer to keep ability as this involves discrimination only based on gender bias.

Using (2.22f) and (2.22g), along with (2.24) and (2.25) lead to the following results for r_{bjt} and r_{gjt} :

$$r_{bjt} = \frac{2\eta\tau w_{jt}^f}{v_{jt}(\gamma - \eta - \phi\eta)} - \frac{(\gamma + \eta - \phi\eta)w_{ut+1}}{(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)} \quad (2.26)$$

$$r_{gjt} = \frac{2\eta\phi\tau w_{jt}^f}{v_{jt}(\gamma - \eta - \phi\eta)} - \frac{(\gamma - \eta + \phi\eta)w_{ut+1}}{(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)}. \quad (2.27)$$

From equation (2.23), we see that as wife's wage (w^f) increases, fertility (n) decreases¹³ and as cost of education (v) increases, fertility (n) increases. This is due to the quality-quantity tradeoff: the increase in the woman's wage translated into lower fertility as her opportunity cost of child-rearing increases and increase in cost of education leads to increase in quantity at the cost of quality i.e. education. From equations (2.26) and (2.27), we see that as wife's wage (w^f) increases, r 's increase (i.e. children's quality increases). Also, as cost of education (v) increases, r 's decrease (i.e. children's quality decreases). These outcomes are also consistent with the quantity-quality tradeoff in fertility where increase in woman's wage translates into increase in quality and increase in the cost of education lowers quality.

Depending on the household type, an agent is either skilled or unskilled. Due to the quality-quantity tradeoff, as female income changes based on her skill level depending on the household type, it leads to different fertility outcome. Male income, based on his skill level, also contributes to this outcome. The resultant fertility distribution is thus an outcome of different types of households arising from the marriage market matching.

We may encounter cases where constraints with boundary conditions become binding, for example, $r_{bht} = 1$ or $r_{glt} = 0$. We discuss such a case, which involves $0 < r_{bjt} < 1, r_{gjt} = 0$.¹⁴ The solution to the optimization problem is as follows.

¹³The derivative dn_{jt}/dw_{jt}^f is negative.

¹⁴In the simulation exercise later in section 2.4, this case is encountered for $j=1$ i.e. for low-type households, at a steady state, $r_{gt} = 0$. And in the optimization problem described above, we substitute $r_{bjt} = 0$ and households don't choose \hat{a}_{gjt} anymore. Rest of the problem doesn't change.

$$n_{jt} = \frac{(w_{jt}^m + w_{jt}^f)(w_{st+1} - w_{ut+1})(\gamma - \eta)}{(1 + \gamma)(\tau w_{jt}^f(w_{st+1} - w_{ut+1}) - 0.5v_{jt}w_{ut+1})} \quad (2.28)$$

$$\hat{a}_{bjt} = \frac{\gamma v_{jt}(\bar{a}w_{st+1} - \underline{a}w_{ut+1}) - 2\eta(w_{st+1} - w_{ut+1})(\tau w_{jt}^f(\bar{a} - \underline{a}) + 0.5\bar{a}v_{jt})}{v_{jt}(w_{st+1} - w_{ut+1})(\gamma - \eta)}. \quad (2.29)$$

Using (2.22f) and (2.29) lead to the following equation for r_{bjt} :

$$r_{bjt} = \frac{2\eta\tau w_{jt}^f}{v_{jt}(\gamma - \eta)} - \frac{\gamma w_{ut+1}}{(w_{st+1} - w_{ut+1})(\gamma - \eta)}. \quad (2.30)$$

Dynamic Equilibrium

Given initial values for ratio of skilled to total males (p_0^m) and ratio of skilled to total females (p_0^f); a competitive equilibrium is a sequence of educational investment, fertility, female labor supply and consumption decisions for each type of household i.e. for $j \in \{h, m1, m2, l\}$, $\{\hat{a}_{bjt}, \hat{a}_{gjt}, n_{jt}, l_{wjt}, c_{jt}\}_{t=0}^{\infty}$; prices $\{w_{st}, w_{ut}\}_{t=0}^{\infty}$; fraction of skilled to total males, fraction of skilled to total females and ratio of skilled to total labor $\{p_t^m, p_t^f, \beta_t\}_{t=0}^{\infty}$ and fractions of each type of household $\{\lambda_{ht}, \lambda_{m1t}, \lambda_{m2t}, \lambda_{lt}\}_{t=0}^{\infty}$ such that: w_{st} , and w_{ut} are given by equations (2.20) and (2.21);

$$\begin{aligned} \beta_t &= \frac{L_{st}}{L_t} \\ &= \frac{0.5N_t((1 + l_{wht})\lambda_{ht} + l_{wm1t}\lambda_{m1t} + \lambda_{m2t})}{0.5N_t \sum_j (1 + l_{wjt})\lambda_{jt}} \\ &= \frac{(1 + l_{wht})\lambda_{ht} + l_{wm1t}\lambda_{m1t} + \lambda_{m2t}}{\sum_j (1 + l_{wjt})\lambda_{jt}} \\ &\equiv G(p_t^m, p_t^f, \beta_{t+1}) \end{aligned} \quad (2.31)$$

where β_t is derived from the ratio of skilled to total labor supply¹⁵, skilled labor supply coming from high-, and both mixed-skilled households and we can write β_t as a function of $p_t^m, p_t^f, \beta_{t+1}$ as l_{wjt} depends on β_t and β_{t+1} (explained below); fractions of each type of

¹⁵We multiply N_t by 0.5 in numerator and denominator as the number of households is half of the population.

household given by equations (2.14), (2.15), (2.16) and (2.17); $n_{jt}, \hat{a}_{bjt}, \hat{a}_{gjt}$ are given by equations (2.23), (2.24) and (2.25) respectively (or by (2.28) and (2.29) as the case maybe); l_{wjt} and c_{jt} are derived from household constraints as follows:

$$l_{wjt}(\beta_t, \beta_{t+1}) = 1 - \tau n_{jt}(\beta_t, \beta_{t+1}) \quad (2.32)$$

$$c_{jt}(\beta_t, \beta_{t+1}) = w_{jt}^m(\beta_t) + w_{jt}^f(\beta_t) l_{wjt}(\beta_t, \beta_{t+1}) - 0.5 \left(r_{bjt}(\hat{a}_{bjt}(\beta_t, \beta_{t+1})) + r_{gjt}(\hat{a}_{gjt}(\beta_t, \beta_{t+1})) \right) n_{jt} v_{jt} \quad (2.33)$$

where w_{jt}^m and w_{jt}^f are the wage rates for male and female respectively which depend on the type of household¹⁶; and the evolution of p_t^m , p_t^f and β_t as follows:

$$\begin{aligned} p_{t+1}^m &= \frac{N_{st+1}^m}{N_{t+1}^m} \\ &= \frac{\sum_j n_{jt}(\beta_t, \beta_{t+1}) r_{bjt}(\hat{a}_{bjt}(\beta_t, \beta_{t+1})) \lambda_{jt}(p_t^m, p_t^f)}{\sum_j n_{jt}(\beta_t, \beta_{t+1}) \lambda_{jt}(p_t^m, p_t^f)} \\ &\equiv J(p_t^m, p_t^f, \beta_t, \beta_{t+1}) \end{aligned} \quad (2.34)$$

where the ratio of skilled males arises from the fertility and educational investment decisions of boys for each type of household, and

$$\begin{aligned} p_{t+1}^f &= \frac{N_{st+1}^f}{N_{t+1}^f} \\ &= \frac{\sum_j n_{jt}(\beta_t, \beta_{t+1}) r_{gjt}(\hat{a}_{gjt}(\beta_t, \beta_{t+1})) \lambda_{jt}(p_t^m, p_t^f)}{\sum_j n_{jt}(\beta_t, \beta_{t+1}) \lambda_{jt}(p_t^m, p_t^f)} \\ &\equiv H(p_t^m, p_t^f, \beta_t, \beta_{t+1}) \end{aligned} \quad (2.35)$$

where the ratio of skilled females arises from educational investment decisions of girls along with fertility decisions for each type of household, and

$$\begin{aligned} \beta_{t+1} &= \frac{(1 + l_{wht+1}(\beta_{t+1}, \beta_{t+2})) \lambda_{ht+1} + l_{wm1t+1}(\beta_{t+1}, \beta_{t+2}) \lambda_{m1t+1} + \lambda_{m2t+1}}{\sum_j (1 + l_{wjt+1}(\beta_{t+1}, \beta_{t+2})) \lambda_{jt+1}(p_{t+1}^m, p_{t+1}^f)} \\ &\equiv G(p_{t+1}^m, p_{t+1}^f, \beta_{t+2}). \end{aligned} \quad (2.36)$$

¹⁶Recall that if $j = h$, $w^m = w_s$ and $w^f = w_s$; if $j = m1$, $w^m = w_u$ and $w^f = w_s$; if $j = m2$, $w^m = w_s$ and $w^f = w_u$; and if $j = l$, $w^m = w_u$ and $w^f = w_u$.

Steady State

We assume that the economy starts with the initial fraction of skilled to total males and skilled to total females, p_0^m and p_0^f . A steady state of the economy is given by the fixed points of equations (2.34), (2.35) and (2.36). At steady state, $p_t^m = p_{t+1}^m = \bar{p}^m$, $p_t^f = p_{t+1}^f = \bar{p}^f$ and $\beta_t = \beta_{t+1} = \beta_{t+2} = \bar{\beta}$ solve $\bar{p}^f = H(\bar{p}^m, \bar{p}^f, \bar{\beta}, \bar{\beta})$, $\bar{p}^m = J(\bar{p}^m, \bar{p}^f, \bar{\beta}, \bar{\beta})$ and $\bar{\beta} = G(\bar{p}^m, \bar{p}^f, \bar{\beta})$. Thus,

$$\bar{p}^f = \frac{\sum_j n_j(\bar{p}^m, \bar{p}^f, \bar{\beta}) r_{gj}(\bar{a}_{gj}(\bar{p}^m, \bar{p}^f, \bar{\beta})) \lambda_j(\bar{p}^m, \bar{p}^f)}{\sum_j n_j(\bar{p}^m, \bar{p}^f, \bar{\beta}) \lambda_j(\bar{p}^m, \bar{p}^f)} \quad (2.37)$$

$$\bar{p}^m = \frac{\sum_j n_j(\bar{p}^m, \bar{p}^f, \bar{\beta}) r_{bj}(\bar{a}_{bj}(\bar{p}^m, \bar{p}^f, \bar{\beta})) \lambda_j(\bar{p}^m, \bar{p}^f)}{\sum_j n_j(\bar{p}^m, \bar{p}^f, \bar{\beta}) \lambda_j(\bar{p}^m, \bar{p}^f)} \quad (2.38)$$

$$\bar{\beta} = \frac{(1 + l_{wh}(\bar{p}^m, \bar{p}^f, \bar{\beta})) \lambda_h(\bar{p}^m, \bar{p}^f) + l_{wm1}(\bar{p}^m, \bar{p}^f, \bar{\beta}) \lambda_{m1}(\bar{p}^m, \bar{p}^f) + \lambda_{m2}(\bar{p}^m, \bar{p}^f)}{\sum_j (1 + l_{wj}(\bar{p}^m, \bar{p}^f, \bar{\beta})) \lambda_j(\bar{p}^m, \bar{p}^f)}. \quad (2.39)$$

We will study (non) uniqueness of steady states in the next section using specific parameter values. For stability, we will linearize equations (2.35), (2.34) and (2.36) at (each) steady state and numerically establish stability local to the steady state. Linearization of equations (2.37), (2.38) and (2.39) yields the following system:

$$p_{t+1}^f - \bar{p}^f = H_{p^m}(\bar{p}^m, \bar{p}^f, \bar{\beta})(p_t^m - \bar{p}^m) + H_{p^f}(\bar{p}^m, \bar{p}^f, \bar{\beta})(p_t^f - \bar{p}^f) + H_{\beta}(\bar{p}^m, \bar{p}^f, \bar{\beta})(\beta_t - \bar{\beta}) \quad (2.40)$$

$$p_{t+1}^m - \bar{p}^m = J_{p^m}(\bar{p}^m, \bar{p}^f, \bar{\beta})(p_t^m - \bar{p}^m) + J_{p^f}(\bar{p}^m, \bar{p}^f, \bar{\beta})(p_t^f - \bar{p}^f) + J_{\beta}(\bar{p}^m, \bar{p}^f, \bar{\beta})(\beta_t - \bar{\beta}) \quad (2.41)$$

$$\beta_{t+1} - \bar{\beta} = G_{p^m}(\bar{p}^m, \bar{p}^f, \bar{\beta})(p_t^m - \bar{p}^m) + G_{p^f}(\bar{p}^m, \bar{p}^f, \bar{\beta})(p_t^f - \bar{p}^f) + G_{\beta}(\bar{p}^m, \bar{p}^f, \bar{\beta})(\beta_t - \bar{\beta}) \quad (2.42)$$

The steady state is locally stable if at least two eigenvalues of the matrix

$$A = \begin{bmatrix} H_{p^m}(\bar{p}^m, \bar{p}^f, \bar{\beta}) & H_{p^f}(\bar{p}^m, \bar{p}^f, \bar{\beta}) & H_{\beta}(\bar{p}^m, \bar{p}^f, \bar{\beta}) \\ J_{p^m}(\bar{p}^m, \bar{p}^f, \bar{\beta}) & J_{p^f}(\bar{p}^m, \bar{p}^f, \bar{\beta}) & J_{\beta}(\bar{p}^m, \bar{p}^f, \bar{\beta}) \\ G_{p^m}(\bar{p}^m, \bar{p}^f, \bar{\beta}) & G_{p^f}(\bar{p}^m, \bar{p}^f, \bar{\beta}) & G_{\beta}(\bar{p}^m, \bar{p}^f, \bar{\beta}) \end{bmatrix} \quad (2.43)$$

are less than 1 in absolute value.

2.4 Some Quantitative Analysis

For the marriage market model described above, the mechanical matching process exhibits PAM along skill level. The focus of this section is to replicate the steady state values observed in Indian data. We achieve this by calibrating some parameters of the model to Indian data, estimating other parameters by generalized method of moments (GMM) and by simulating the model at steady state.

To fix the random matching parameter α , we use the fraction of mixed-skilled-1 households at the steady state, which is modified from the equation (2.15):

$$\lambda_{m1}^- = \alpha p^f (1 - p^m)$$

Using the Demographic and Health Survey (DHS) of 2005-06 for India, we obtain the value of $\lambda_{m1}^- = 2.93\%$, $p^m = 19.71\%$ and $p^f = 11.28\%$, from which we calculate $\alpha = 0.3234$.

Next, we calibrate the elasticity of substitution between skilled and unskilled labor for the aggregate production function. To do so, we assume the following CES (constant elasticity of substitution) specification:

$$Y_t = A_t [A_s \mu L_{st}^\rho + A_u (1 - \mu) L_{ut}^\rho]^{(1/\rho)} \quad (2.44)$$

where Y_t is the output at time t , A_t is the total factor productivity (TFP), and we assume that it grows at the same rate as that of v_{jt} , i.e. g , with A_0 being the initial value i.e. $A_t = A_0(1 + g)^t$, A_s and A_u are factor productivities associated with skilled and unskilled labor respectively. The elasticity of substitution between skilled and unskilled labor is $1/(1 - \rho)$. Jerzmanowski and Tamura (2022) estimate the elasticity to be between 1.7 and 2.6 for the world and Behar (2010) estimate it to be about 2 for various categorization of skill levels into skilled and unskilled labor for developing countries. We choose $\rho = 0.5$, which

implies the elasticity of 2. The TFP (A_0) provides a degree of freedom and we calibrate $A_0 = 4.5$, which allows us to solve the model and obtain the steady state of the economy.

Parameters in the utility function of the household, γ, η, ϕ, τ and initial costs of education for each type of household (v_{h0}, v_{m10}, v_{m20} and v_{l0}), a total of 8 parameters, are estimated by the generalized method of moments (GMM)/minimum distance estimation (MDE) procedure. We have 4 data moments from fertility values for each type of households, derived from the the DHS of 2005-06, mentioned in table 2.1. The values are: $n_h = 2.066, n_{m1} = 2.2465, n_{m2} = 3.0752$ and $n_l = 3.5291$. We have 8 data moments, fractions of boys and girls of each type of households from the India Human development Survey-II (IHDS-II), 2011-12 database (Desai et al., 2018a). IHDS - II survey data contains information about (ever) married women reporting the educational details for their parents and parents-in-law. This information is used to calculate fractions of boys and girls who receive higher education for each type of household.¹⁷ For each individual, we calculate the type of the household their parents belonged to, using parents' education data. Then the data about individual's education is aggregated for each type of household for each gender. This leads to 8 data moments, mentioned in table 2.1.

Values of r_{bl} and r_{gl} are derived from using the observed values for other fractions as well as values for \bar{p}_m and \bar{p}_f calculated from the DHS data. Equations (2.37) and (2.38) make it clear that r_{bl} and r_{gl} get fixed (to 7.21% and 0.57% respectively in this case) when values for other variables are chosen.¹⁸

We have 12 data moments, of which 9 data moments are targeted (n_h and all r 's) and 8 parameters are estimated using GMM/MDE (mentioned in table 2.1). Thus the model is over-identified, and the choice of these data moments leads to the minimum distance.¹⁹ The predicted values for fertility for both the mixed- and low-skilled households, whose data

¹⁷It is to be noted that this information is not necessarily representative as we do not have a complete picture of a household education decisions for its children to compute the fractions. We have data about parents' education for some adults (from the survey households) which are used to calculate the fractions. There is not other data source, to our knowledge, which can be used to calculate the data moments.

¹⁸The reason to derive r_{bl} and r_{gl} is based on numerical constraints as choosing other fractions either led to negative values.

¹⁹Its value is 0.0132.

moments were not targeted (out-of-sample predictions), are also tabulated in table 2.1 and we can see from the last column that the exercise seems to be doing a reasonable job in predicting the major part of observed fertility distribution.²⁰

From the GMM estimation results in table 2.2, data moments in table 2.1 as well as equations (2.26) and (2.27), it can be seen that parameters for costs of education are identified by average levels of education for each type of household (given wife’s wage rate). Parameters γ and τ , due to the way they enter the utility function and the female time constraint in the household model respectively, are primarily identified by fertility for high-skilled household. Parameter ϕ (by definition) targets difference in education levels for boys’ and girls’ for all households, whereas parameter η is identified separately by variable differences in education levels for each type of household.

We fix the value of μ by calculating the ratio of skilled to unskilled wage rate using equations (2.20), (2.21), and (2.44), and it is given by,

$$\frac{w_{st}}{w_{ut}} = \frac{A_s \mu}{A_u (1 - \mu)} \left(\frac{\beta_t}{1 - \beta_t} \right)^{(\rho-1)} \quad (2.45)$$

where β_t is the ratio of skilled to total labor and $\beta_t/(1 - \beta_t)$ gives the ratio of skilled to unskilled labor. From equation (2.45), to calibrate μ , we need the following: value of ratio of skilled to unskilled wage rate, value of $\bar{\beta}$, A_s and A_u . We fix $A_s = A_u = 1$. We refer to education data from India Human development Survey (IHDS), 2005 (Desai et al., 2018b) and use the rates of return to education from Agrawal (2012). The ratio of skilled to unskilled wage rate is then calculated to be 2.4426. We also need the value of $\bar{\beta}$, which we derive from the DHS of 2005-06.²¹ From the data, we calculate the value of $\bar{\beta}$ to be 0.1825. These values result in $\mu = 0.5357$.

Table 2.2 provides a summary of the parameters, along with the data source and methodology used to derive their values. A subset of parameters, those from the marriage market

²⁰The sum of squared errors for the out-of-sample predictions is 1.8468, minimum among the various combinations of targeted data moments.

²¹We first derive the female labor supply value for each type of household by substituting $\tau = 0.15$ (from the GMM estimation) and n_j for each household in the equation $l_{wj} + \tau n_j = 1$. We also calculate fractions for each type of households and then use equation (2.39) to calculate the value of $\bar{\beta}$.

and production sections of the model, have been calibrated or taken from the literature (parameters α , ρ and μ). Parameters from the household section of the model have been estimated using the GMM procedure, already explained above.

Lastly, ability is drawn from uniform distribution with support $[0,2]$ i.e we assume $\underline{a} = 0$ and $\bar{a} = 2$.²²

We parameterize the model with above values and solve for steady state(s) numerically, by assuming different initial values for p^m, p^f and β . We solve a system of 3 non-linear equations (2.37), (2.38) and (2.39) to derive steady state(s). We obtain different steady states of which all but one are economically meaningless.²³ Therefore, we focus on the economically meaningful steady state, which is discussed next.

Table 2.3 presents the results of computational exercise along with the observed values, calculated from the DHS, 2005-06 and IHDS-II, 2011-12. The model produces steady state fraction of skilled males (20.11%), fraction of skilled females (14.42%) and fraction of skilled to total labor (18.89%) which are close to the observed values (19.71%, 11.76% and 18.26% respectively). Skill premium ratio²⁴, 2.3906, also replicates the empirical value (2.4426) very closely. The simulation results indicate that the marriage market does exhibit sorting along skill-levels where the combined fraction of high- and low-skilled marriages (86.86%) exceeding the combined fraction of marriages from both the mixed-skilled marriages (13.15%). The comparison between the simulated and empirical values for various fractions of households is as follows: the value of fraction of high-skilled households is 10.7% and 8.36%, for mixed-skilled-1 households it is 3.73% and 2.93%, for mixed-skilled-2 households it is 9.42% and 11.66%, and for low-skilled households it is 76.16% and 77.05% respectively. The simulated values for all types of households are close to the observed values. The steady state values of other variables are given in the second column of table 2.3.

The average fertility from the numerical analysis (2.6376) is reasonably close to the observed

²²As already noted in section 2.3, use of ability is optional and the choice of ability distribution parameters does not affect the numerical results.

²³In discarded steady state(s), economic variables such as female labor supply, fractions of types of households, result into negative numbers.

²⁴Skill premium ratio is the ratio of skilled to unskilled wage rate.

average fertility (3.3163) for India in 2005-06. Comparing the simulated and observed fertility distribution, we see that the computed fertility values for mixed-skilled-1 and low-skilled households (1.5413, 2.6328) are lower than the observed values (2.2465, 3.5291) and for mixed-skilled-2 household, it (3.7251) is higher than the observed value (3.0752). For high-skilled households, the simulated values (2.0963) is close to the empirical value (2.066).²⁵ We cannot have a tighter fertility distribution, as we have unit elasticity of substitution among consumption, fertility and children's quality for analytical-tractability whereas to achieve a higher tradeoff among these variables and to produce a better fertility distribution, a higher absolute value of elasticity may be required. Also, simulated value of fertility for mixed-skilled-1 household (1.5413) and low-skilled household (2.6328) are respectively lower than those of the high-skilled household (2.0963) and mixed-skilled-2 household (3.7251) as the husband's income effect dominates for the high- and mixed-skilled-2 household leading to higher fertility.

From the table, we see that all types of households invest in higher education of boys (the fractions being 87.91%, 66.44%, 60.93% and 4.06%), and all but the low-skilled households invest in higher education of girls (the fractions being 75.78%, 55.94%, 50.85% and 0%) and the simulated values are reasonably close to the observed values.²⁶ The quality-quantity tradeoff is also evident from the fertility distribution, the female labor supply values and the fractions of boys and girls getting higher education. For example, for high-skilled households, the quantity (i.e. fertility) is low, female labor supply is high and the "quality" of children i.e. fractions of boys and girls getting higher education is high (87.91% and 75.78%) whereas for low-skilled households, the quantity (i.e. fertility) is high, the female labor supply is low and the "quality" of children i.e. fractions of boys and girls getting higher education is low (4.06% and 0% respectively). Overall, the simulation exercise seems to be doing a decent job of explaining the observed fertility gap between various types of households.

We numerically validate the stability of the steady state and all three eigenvalues have absolute values less than 1, which means the steady state is locally stable.²⁷ As the model has

²⁵As we target n_h as a data moment for GMM estimation, it is no surprising to have simulated fertility value so close to the empirical value for the high-skilled household.

²⁶As we use 8 data moments of fractions of boys and girls with higher education for GMM estimation of parameters, this is an expected outcome.

²⁷The eigenvalues obtained are $-0.0238, 0.0875 + 0.3649i$ and $0.0875 - 0.3649i$.

two initial values (p_0^m and p_0^f) and three stable eigenvalues, the equilibrium is indeterminate.

The model does a promising job of replicating most part of the fertility distribution observed in practice, using model of an economy which exhibits PAM along education levels in the marriage market and skill imbalance in gender (more skilled men than skilled women) generated from human capital investment choices of different households. As already stated, we cannot have a tighter fertility distribution due to husband's income effect and assumption of log-utility which implies unit elasticity of substitution among consumption, fertility and children's human capital.

2.5 Conclusion

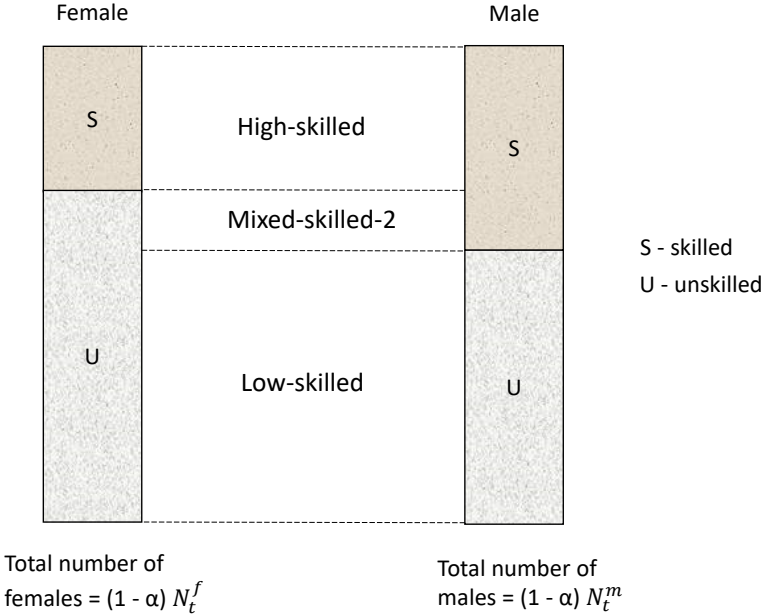
We developed a theoretical model of an economy with a novel marriage market matching exhibiting positive assortative matching based on skills, household decision problem consisting of fertility and children's education decisions, production with skilled and unskilled labor as inputs. Households discriminated against girls' higher education as they derived less utility from educating them compared to boys due to taste-based gender bias. We then characterized the dynamic equilibrium for such an economy, with evolution of the economy being driven by the ratio of skilled to total males, the ratio of skilled to total females and the ratio of skilled to total labor. The steady state behavior of the economy was also constructed.

The generalized method of moments was used for estimating the model using Indian data. We then numerically solved the model to derive the steady state. The numerical analysis of the steady state established that it is locally stable. The following steady state ratios were simulated, which are close to the observed values, indicated in brackets: the fraction of skilled males to total males = 0.2011 (0.1971), the fraction of skilled females to total females = 0.1442 (0.1128) and the fraction of skilled to total labor = 0.1889 (0.1825). The numerical values of various fractions of household types confirm that there is marital sorting based on skill levels in the Indian marriage market. The simulated fertility distribution indicates that the model seems to be doing an encouraging job of replicating the fertility distribution observed in practice. As skill-based sorting creates households with different income levels, it leads to different fertility decisions as a result of quality-quantity tradeoff. This gives rise to the resultant fertility distribution.

The model presented in the chapter is extended in the subsequent chapters to study dynamic evolution of the economy. The model is also used to analyze the effect on economy of cultural/preference changes such as changes in the spousal preferences resulting in higher marital sorting, changes in the returns to skills. We also suitably modify the model in the next chapter to test effectiveness of various policy experiments, such as gender-neutral and gender-targeted education subsidies, to neutralize gender bias.

2.6 Figures

Figure 2.1
Mechanical Matching



2.7 Tables

Table 2.1
Data Moments

Variable	Observed Value	Targeted? Yes/No	Predicted Value	Difference
n_h	2.066	Yes	2.066	0.0
r_{bh}	92.04%	Yes	95.11%	-0.0307
r_{gh}	85.95%	Yes	82.63%	0.0332
r_{bm1}	70.54%	Yes	73.36%	-0.0282
r_{gm1}	65.59%	Yes	62.54%	0.0305
r_{bm2}	70.42%	Yes	63.85%	0.0657
r_{gm2}	46.63%	Yes	53.74%	-0.0711
r_{bl}	7.21%	Yes	6.8%	0.0041
r_{gl}	0.57%	Yes	1.02%	-0.0045
n_{m1}	2.2465	No	1.506	0.7405
n_{m2}	3.0752	No	3.7446	-0.6694
n_l	3.5291	No	2.3918	0.9221

Table 2.2
Parameters

Parameter	Value	Main Data Source	Procedure
γ	0.2495	DHS, IHDS-II	GMM
η	0.0518	DHS, IHDS-II	GMM
ϕ	0.9241	DHS, IHDS-II	GMM
τ	0.15	DHS, IHDS-II	GMM
v_h	0.1787	DHS, IHDS-II	GMM
v_{m1}	0.1991	DHS, IHDS-II	GMM
v_{m2}	0.0858	DHS, IHDS-II	GMM
v_l	0.1252	DHS, IHDS-II	GMM
α	0.3234	DHS	Calibration
ρ	0.5	-	Literature
μ	0.5357	DHS, IHDS	Calibration

Table 2.3
Comparison: Observed and Simulated Values

Variable	Observed Values	Simulated Values
Fraction of skilled to total males (\bar{p}^m)	19.71%	20.11%
Fraction of skilled to total females (\bar{p}^f)	11.28%	14.42%
Fraction of skilled to total labor ($\bar{\beta}$)	18.25%	18.89%
Skilled wage rate	-	3.6104
Unskilled wage rate	-	1.5103
Skill premium ratio	2.4426	2.3906
Fraction of high-skilled households	8.36%	10.7%
Fraction of mixed-skilled-1 households	2.93%	3.73%
Fraction of mixed-skilled-2 households	11.66%	9.42%
Fraction of low-skilled households	77.05%	76.16%
Fertility (high-skilled household)	2.066	2.0963
Fertility (mixed-skilled-1 household)	2.2465	1.5413
Fertility (mixed-skilled-2 household)	3.0752	3.7251
Fertility (low-skilled household)	3.5291	2.6328
Average fertility	3.3163	2.6376
Female labor supply (high-skilled household)	0.6901	0.6856
Female labor supply (mixed-skilled-1 household)	0.663	0.7688
Female labor supply (mixed-skilled-2 household)	0.5387	0.4412
Female labor supply (low-skilled household)	0.4706	0.6051
Fraction of boys getting higher education (high-skilled households)	92.04%	87.91%
Fraction of girls getting higher education (high-skilled households)	85.95%	75.78%
Fraction of boys getting higher education (mixed-skilled-1 household)	70.54%	66.44%
Fraction of girls getting higher education (mixed-skilled-1 household)	65.59%	55.94%
Fraction of boys getting higher education (mixed-skilled-2 household)	70.42%	60.93%
Fraction of girls getting higher education (mixed-skilled-2 household)	46.63%	50.85%
Fraction of boys getting higher education (low-skilled household)	7.21%	4.06%
Fraction of girls getting higher education (low-skilled household)	0.57%	0%
Ratio of skilled females to skilled males	0.5724	0.7171
Ratio of average female to male labor supply	0.5025	0.6044
Ratio of average female to male income	0.4762	0.5831

CHAPTER 3

GENDER BIAS AND MARRIAGE MARKET: EXOGENOUS CHANGES, DYNAMICS AND EDUCATION POLICY

3.1 Introduction

The chapter studies the general equilibrium effects of gender-based policies aimed at increasing women's human capital, in the context of a model where parents discriminate against girls. We also study dynamics and understand the effect of increase in positive assortative matching (PAM), which is empirically established by the literature (e.g. Borkotoky and Gupta, 2016; Kashyap et al., 2015), on income inequality under endogenous fertility – a novel contribution to the literature.

In chapter 2, we constructed a theoretical model of the economy and marriage market with taste-based gender bias of households toward girls' higher education. This led to higher supply of skilled males than females in the steady state, when we simulated the model using Indian data. In this chapter, we first analyze dynamic evolution of the economy. The model is then used for comparative statics and dynamics analysis to understand what factors can weaken the taste-based gender bias. The analysis is also used to investigate the effect of changes to PAM on income inequality. Policy experiments are then conducted which take the form of education subsidies to low-skilled households for girls' higher education.

The policy analysis is motivated by "Kanyashree Prakalpa" (KP), a scheme introduced by the government of the state of West Bengal, India (Dutta and Sen, 2020). The KP scheme aims to reduce the under-age marriage and adolescent dropouts among girls from poorer households using conditional cash transfer (CCT). The cash transfer is conditional upon girls remaining both unmarried and pursuing education till age 18. The objective of the policy is to encourage parents to invest in girls' education. Our policy experiment also aims to provide subsidies to poorer households for girls' education prior to marriage. Our policy intervention draws from the list of policies being implemented worldwide presented by Fernández et al. (2022) to address gender gaps across various dimensions. The list includes policies targeted toward accumulation of women's human capital such as increasing government budget expenditures, focusing on imparting demand-driven vocational training skills to women, targeted education policies throughout all levels of girls' education. We focus on targeted policy toward tertiary education of girls from poorer households.

Dynamic analysis reveals that the numerically-established steady state from chapter 2 is locally stable and indeterminate. Simulations reveal that it takes the economy 3 generations, i.e. about 75 years,¹ to reach the steady state when it starts either 10% above or below it. Moreover, graphical analysis establishes that the economy follows spiral sink path of convergence.

We show that an exogenous increase in PAM leads to decrease in the fraction of skilled females, as only low-skilled households do not invest in girls' education. This results in higher skill premium ratio, increasing income of households with at least one skilled member (i.e. high-, mixed-skilled-1 and mixed-skilled-2 households). Inter-household inequality worsens as a result. Ratio of skilled females to skilled males (which is also the ratio of girls to boys getting higher education in the steady state) decreases, leading us to conclude that increase in PAM leads to worsening of gender inequality for girls and women. And it takes 4 generations, i.e. about 100 years, for the economy to converge to the new steady state. Results from the rest of comparative statics and dynamics are discussed next.

Equal educational investment in boys and girls by parents – which could be a result of exogenous cultural forces – improves measures of gender equality on the dimensions of skills, income and labor supply. Increase in investment in girls' education is accompanied by decreased fertility owing to quality-quantity tradeoff. Similar effects are observed for exogenous increase in returns to education, and the economy adjusts to the new steady state in 3 time periods. Increase in skilled-labor productivity can lead to higher human capital investment and weakening of gender bias whereas unskilled-labor productivity can result in reduced human capital investment. Exogenous decrease in the cost of education for low-skilled household affects boys more favorably than girls, exacerbating gender bias in skills.

A policy experiment is carried out wherein low-skilled households receive education subsidies for girls' higher education. This increases the fraction of girls receiving higher education by almost 2 percentage points, a significant increase due to the large proportion of low-skilled households. The fractions of skilled females and labor supply increase, and inter-household income inequality reduces as well. The skilled female to male ratio rises by approximately 10%, a sizable increase which bridges the gap by one-third towards achieving gender equality.

¹Each generation is about 25 years in the model.

Moreover, other measures of gender equality, such as labor supply and income, show some improvements. In comparison, the gender-neutral subsidies of the same magnitude fails to neutralize the gender discrimination. Additionally, the gender-targeted policy effects are robust to addition of son-biased fertility preferences towards son in the form of an exogenous parameter. Dynamic analysis of gender-targeted policy leads us to conclude that it takes 2 generations for the economy to reach the new steady state. In the context of slowly-changing cultural norms, the results underscore the need for targeted policy intervention to neutralize active gender discrimination within the household.

The rest of the chapter is organized as follows. The next section presents a brief review of the literature focused on policies to address gender inequality. We then discuss dynamic analysis, followed by comparative statics and dynamics. Policy analysis is presented next, and the final section concludes.

3.2 Literature Review

We contribute to the extensive literature focused on policy measures to alleviate various forms of gender bias (e.g. Ameratunga Kring (2017), Jain-Chandra et al. (2018), Das and Kotikula (2019), De Paz Nieves and Muller (2021), Fernández et al. (2022) provide some recent examples). In particular, Fernández et al. (2022) note that policies to reduce gender inequality are difficult to implement, as they target institutions resulting from long-standing beliefs and social norms, but these institutions can still respond to incentives. We also attempt to address social institution of gender discrimination by analyzing a policy which incentivizes households to change their behavior.

There are multiple studies which have focused on the success of conditional cash transfers/education subsidies in alleviating various forms of gender bias for developing countries (e.g. Todd and Wolpin, 2006; Sinha and Yoong, 2009; Baird et al., 2011; De Brauw and Hoddinott, 2011; Saavedra and García, 2012; Duflo et al., 2015; Powell-Jackson et al., 2015; Scarlato et al., 2016; Anukriti, 2018; Hahn et al., 2018; Yin, 2022; Sen and Thamarapani, 2022; Biswas et al., 2023). We add to this literature by analyzing the dynamic general equilibrium effects of education subsidies to poorer households for girls' higher education and document large positive gains to achieve a more gender equal society along the dimensions of

education, skills, labor supply and income. Additionally, we investigate the timing required for the policy to take full effect, which is a novel contribution to the literature.

Multiple policy briefs also document how various types of market-oriented skill development programs focusing on young women can reduce the labor force participation gap between men and women and how it can contribute to gender equality (e.g. World Bank, 2016; International Labor Organization, 2020; Najjumba et al., 2021). Our results can be seen as supplementing this literature because higher education subsidy can take the form of skill development program which can enable women to perform skilled jobs in the economy.

3.3 Dynamic Analysis

In this section, we add dynamic analysis to the quantitative analysis presented in chapter 2 and trace out time path for the economy to reach the steady state. From equations (2.34), (2.35) and (2.36), it is clear that we have a non-linear dynamical system. The steady state values obtained from simulations were $\bar{p}^m = 0.2011$, $\bar{p}^f = 0.1442$ and $\bar{\beta} = 0.1889$. The eigenvalues obtained were -0.0238 , $0.0875 + 0.3649i$ and $0.0875 - 0.3649i$. As the modulus of all eigenvalues is less than 1, the steady state is locally stable. With two initial values (p_0^m and p_0^f) and three eigenvalues with modulus less than 1, the steady state is indeterminate. As we have complex eigenvalues, we expect a spiral sink path in steady-state diagrams for p_t^m , p_t^f and β_t . The steady-state diagrams are presented later in the section.

Linearized equations (2.40), (2.41) and (2.42) are used to solve for the dynamic equilibrium local to the steady state. We start below the steady state with a deviation of 10% from steady state values for p^m , p^f and β , i.e $p_0^m = 0.181$, $p_0^f = 0.1298$ and $\beta_0 = 0.17$. Time paths for p_t^m , p_t^f and β_t are depicted in figures 3.1, 3.2 and 3.3. The gap between the deviated value and the steady state value reduces to less than 10% in 3 time periods. Each generation can be considered to be of about 25 years. This means it takes a period of 75 years for the economy to converge to the steady state when it starts at 10% deviation below it i.e. 0.02011 percentage point difference for p^m and 0.01442 point difference for p^f .²

²When we compare the values of p_t^m , p_t^f and β_t derived from the linearized system of equations with the actual values calculated from equations (2.31), (2.32) and (2.33); the gap between actual and linearized values reduces to less than 10% in 2 time periods, indicating

The steady-state diagrams for p_t^m, p_t^f and β_t are depicted in figures 3.4, 3.5 and 3.6. From the steady-state diagrams, we can see that the economy indeed follows a spiral sink path. The spiral path can be attributed to asymmetry arising due to gender inequality in the model along 2 dimensions: higher skilled males than skilled females ($p^m > p^f$) and higher male labor supply than female labor supply ³ When we simulate the economy by removing one imbalance, it does not lead to monotonic convergence.⁴ ⁵ We argue that gender asymmetry along both the dimensions together contribute to spiral sink path for the economy.

The nature of dynamics where the economy starts at a deviation of 10% above the steady state is similar to the case presented for the deviation of -10% below the steady state. The case of 10% deviation is discussed in appendix B.

3.4 Comparative Statics and Dynamics

In this section, we change various parameters of the model, one at a time, to understand its impact on the steady state of the economy. We are also interested in understanding whether changes to measures such as PAM, factor productivities, cost of education can neutralize taste-based gender bias against girls' higher education and time required for the same. The exogenous changes to the parameters can occur due to technological, social or cultural changes not accounted for in the model, for example, technological improvements leading to increased productivity of skilled-labor (increase in A_s). We conduct the comparative statics and comparative dynamics exercise by changing taste-based gender discrimination (parameter ϕ), increasing PAM (decreasing parameter α), by increasing returns to education (parameter μ), by increasing skill-biased TFPs (A_s and A_u), and by increasing cost of education for low-skilled household (parameter \bar{v}_l). We start by first investigating the case

that 10% deviation can be reasonably considered to be local to the steady state.

³Due to endogenous fertility and child-rearing responsibility, women supply less than a unit labor.

⁴With no gender bias in skills i.e. $p^m = p^f$, simulations result in eigenvalues $0.0783 + 0.4812i$ and $0.0783 - 0.4812i$. Both the eigenvalues have modulus less than 1, indicating locally stable steady state. But we still get a spiral sink path for the economy due to complex eigenvalues.

⁵With exogenous fertility, steady state analysis leads to these eigenvalues: -0.1742 and 0.0028 . As one eigenvalue is negative, it indicates an oscillating convergence.

when there's change in taste-based gender bias.

Effect of Change in Gender Bias

In our model, taste-based gender discrimination against girls' higher education is denoted by the parameter ϕ , and we have estimated it to be 0.9241. The taste-based bias can change due to social and cultural factors leading to more favorable conditions for girls' education leading to increase in ϕ . We consider the case where there's no gender bias i.e. $\phi = 1$ and compare it with $\phi = 0.9241$, as given in table 3.1.

Elimination of gender bias leads to equal investment in boys' and girls' higher education. Compared to the baseline case of presence of gender bias ($\phi = 0.9241$), this translates to higher investment in girls' education and reduced investment in boys' education for all types of households, as seen in table 3.1 (change of 15.29%, 17.11%, 22.87% and 3.18 percentage points for girls; change of -0.61%, -1.4%, 2.54% and -21.67% for boys of $h, m1, m2, l$ type households respectively). Skilled males' and females' ratio equalize as well (a value of 18.27%), which represents increase of 26.7% for p^f and decrease of 9.15% for p^m compared to the gender bias case. A huge increase in p^f leads to increase in the fractions of high- and mixed-skilled-1 households and decrease in the fraction of mixed-skilled-2 households (change of 25.61%, 29.49% and -48.73% for $h, m1, m2$ type households respectively compared to the case of gender bias). The fraction of low-skilled households increases slightly by 0.97% due to the counteracting effect of decrease in p^m .

Skill premium decreases owing to overall increase in the skilled workforce, and it drives down the fertility of mixed-skilled-2 and low-skilled household due to substitution effect induced by increased unskilled wage rate. Fertility for high- and mixed-skilled-1 households increase, mainly due to domination of husband's income effect. The net result is -0.62% decrease in the average fertility for the economy and increase of 2.47% in the average female labor supply. As expected, the measures of gender equality – ratio of skilled females to skilled males⁶, ratio of average female to male labor supply, and ratio of average female to male income – show improvement (increase of 39.45%, 2.47% and 9.36% respectively. Thus we can see that gender bias leads to decrease in skill ratios, increase in fertility, lowering of

⁶This ratio is exactly equal to the ratio of girls to boys getting higher education in the steady state.

female labor supply and worsening of gender equality measures.

The comparative dynamics with time path for p^f is shown in figure 3.7. The gap between the value at time t and the steady state value reduces to less than 10% in 3 time periods. This means the economy with a taste-based gender discrimination converges to the steady state of gender equality in about 75 years.

Steady-state diagram 3.8 shows that the economy follows a spiral sink path while converging to the steady state.

Effect of Increase in Marital Sorting

We now turn our attention to analyze the effects of exogenous increase in marital sorting on steady state of the economy, income inequality and gender bias. Marital sorting can change due to economic and social changes that change preferences for partners in the marriage market. Increased sorting refers to increase in the fractions of marriages of same skill level. For the marriage market described in the chapter, decrease in α translates to less random matching and more mechanical matching, resulting in net decrease in the fraction of mixed-skilled households⁷ and increase in the fraction of high and low-skilled households. In this section, our focus is on analyzing the effect of exogenous changes to marital sorting in presence of taste-based gender discrimination on steady state fractions of skilled males, females and labor; fertility distribution and income distribution.

Table 3.2 shows the numerical result of comparative statics exercise when α decreases from 0.3234 to 0.2 i.e. when marital sorting increases.⁸ As the sorting increases, the mixed-skilled families are replaced by high- and low-skilled families (as per table 3.2, fractions of mixed-skilled-1 and mixed-skilled-2 households decrease by 39.68% and 11.57%, fractions of

⁷Decrease in α creates two opposing effects for the fraction of mixed-skilled-2 households, decrease in random matching reducing the fraction and increase in mechanical matching which increases the fraction, and our numerical analysis indicates net decrease in the fraction of mixed-skilled-2 households.

⁸The calibrated value of α is 0.3234, and we choose to describe the analysis as decrease in α as empirical literature has documented that marital sorting has increased e.g. Borkotoky and Gupta (2016), Kashyap et al. (2015).

high- and low-skilled households increase by 10.93% and 1.81% respectively). As low-skilled households do not invest in girls' education whereas other households do, increase in the low-skilled households reduces the fraction of skilled females from 14.42% to 14.13%, a decrease of 2.01%, shown in table 3.2. The reduction in the fraction of skilled females comes from one more source: as α decreases, both fractions of mixed-skilled households decrease but fraction of mixed-skilled-1 households decreases more than that of mixed-skilled-2 households (decrease of 39.68% as compared to 11.57%). This is because as α decreases, the fraction of random matching decreases and the fraction of mechanical matching increases. Mixed-skilled-1 households arise only from random matching and mixed-skilled-2 households arise from both types of matching. The end result is steeper decrease in the fraction of mixed-skilled-1 matches than the fraction of mixed-skilled-2 households, which leads to decrease in fractions of skilled females and unskilled males.

The increase in fraction of skilled males, as a result, is 0.5%, an increase from 20.11% to 20.21%. The overall effect from changes in \bar{p}^f and \bar{p}^m is decrease in the fraction of skilled to total labor (by 0.74%) and increase in the skill premium ratio by 0.48%. The increased skill premium leads to higher investment in boys' and girls' education by all households (except for girls in low-skilled households), and the values are given in table 3.2. Increased investment in children's quality leads to decreased fertility for all households but for mixed-skilled-2 households⁹ and increase in female labor supply for all but mixed-skilled-2 households. Increase in skilled wage rate and decrease in unskilled wage rate, along with changes in female labor supply leads to the following changes to household incomes: +0.37%, +0.24%, +0.22% and -0.13% for $h, m1, m2$ and l -type households respectively.

The overall percentage point increase in the low-skilled households is 1.38 percentage points, which is balanced by the net decrease for high- and both the mixed-skilled households. The discussion shows that the frequency distribution of households gets skewed towards the low-skilled households. Combined with the fact about changes to income, we conclude that increased sorting leads to increased income inequality, in confirmation with the findings of existing literature. Also, as we can see from the table, measures of gender equality such as ratio of skilled females to skilled males (which is also the ratio of girls to boys getting

⁹For mixed-skilled-2 households, increase in skilled wage rate for husband leads to increase in fertility due to income effect. The substitution effect induced by decrease in unskilled wage rate attributed to wife also acts in the same direction, leading to a slight increase in fertility.

higher education in the steady state), ratio of average female to male income decrease by 2.5% and 0.58% and the ratio of average female to male labor supply doesn't change much (a slight increase of 0.15%). Therefore we conclude that increase in PAM leads to worsening of gender bias against girls and women.

The comparative dynamic analysis reveals that it takes about 4 generations for the economy to converge to the new steady state and it follows spiral sink path for convergence. The graphs from the analysis are presented in appendix C.

Effect of Increase in Returns to Education

An exogenous increase in returns to education can arise due to many factors e.g. more demand for educated workforce due to more intensive use of technology in the production process. From equation (2.45), we can see that increase in μ leads to increase in the skill premium i.e. the ratio of skilled to unskilled wage rate. We thus use increase in μ as a proxy for relative increase in returns to higher education to study its effect on the steady state.

Table 3.3 shows the numerical result of comparative statics exercise when μ increases from 0.5357 to 0.6, a 12% increase. As expected, the ratio of skill premium increases by 8.51%, leading to increase in fractions of skilled males, skilled females and skilled labor by 30.68%, 35.37% and 32.66% respectively as shown in table 3.3. This also results in increased fractions of high- and both mixed-skilled households and decrease in fraction of low-skilled households.¹⁰ Low-skilled households also increase educational investment in boys and girls due to increased returns to skills, leading to 5.27% and 0.11% of boys and girls receiving higher education.

Due to quality-quantity tradeoff, fertility of low-skilled household decreases by 0.54% as investment in quality increases. As relative decrease in unskilled wage rate is more than that of skilled wage rate as seen from the table, (decrease of 10.98% vs 3.4%), for mixed-skilled-1 household, decreased fertility owing to husband's income effect (due to decrease in unskilled wage rate) dominates increase in fertility created by substitution effect from wife's income (due to decrease in skilled wage rate), leading to net decrease in fertility by 5.6%. By similar

¹⁰The percentage change for high-, mixed-skilled-1, mixed-skilled-2 and low-skilled households are +38.97%, +24.66%, +21.13% and -9.31% respectively as per table 3.3.

argument, for mixed-skilled-2 household, increase in fertility due to substitution effect from wife's decreased income (due to decrease in unskilled wage rate) dominates the decreased fertility due to husband's income effect (due to decrease in skilled wage rate), leading to net increase in fertility by 5.21%. The overall effect on fertility is a net decrease by 0.44%. Thus we see that increase in returns to education leads to increase in human capital investment and supply of skilled labor, and decrease in fertility. Also, all the measures of gender equality such as ratio of skilled females to skilled males, ratio of average female to male labor supply and ratio of average female to male income show improvement as seen from the table. Therefore we conclude that increase in returns to education can act against the taste-based gender discrimination contributing toward gender equality.

It takes about 5 generations for the economy to converge to the new steady state for a 12% increase in returns to higher education, as per the comparative dynamics. The dynamics are depicted in appendix D.

Effect of Skill-biased Technological Change

Skill-biased technological changes in the production process can change factor productivities of skilled or unskilled labor. From the production function in equation (2.44), A_s and A_u denote skilled- and unskilled-labor productivities respectively. Change in either A_s or A_u does not affect the other parameter, unlike the case of μ , therefore these parameters denote absolute change in factor productivity. We thus use exogenous changes in A_s and A_u to analyze the effect of skill-biased technological changes to study the effect on the steady state, starting with an increase in A_s .

Table 3.4 shows the numerical result of comparative statics exercise when A_s increases from 1 to 1.1, a 10% increase. As expected, the skilled labor supply increases, reflected in increases in fractions of skilled males, skilled females and skilled labor by 30.68%, 27.95% and 25.89% respectively, shown in table 3.4. This also results in increased fractions of high- and both mixed-skilled households and decrease in fraction of low-skilled households.¹¹ Low-skilled households also increase investment in boys due to increase in skilled-labor productivity, leading to 19.95% increase in the fraction of boys getting higher education. Thus increase

¹¹The percentage change for high-, mixed-skilled-1, mixed-skilled-2 and low-skilled households are +30.75%, +19.57%, +20.06% and -7.76% respectively as per table 3.4.

in skilled-labor productivity can result in higher human capital investment. From changes to the measures of gender equality in the table, it can be seen that improvement in the skilled-labor productivity can help in achieving a more gender equal society.

The comparative dynamics caused by change in the steady state indicates that the economy converges to the new steady state in about 75 years as the skilled-labor productivity increases by 10%. The dynamics are given in appendix E.

Next, table 3.5 shows the computational result from increase in factor productivity linked to unskilled labor, i.e. A_u from 1 to 11, a 10% increase. It leads to 0.21% decrease in the fraction of skilled females, as expected but the fraction of skilled males increases by 0.4%. As \bar{p}^f decreases, it leads to decrease in the fractions of high- and mixed-skilled-1 households (by 0.28% and 0.54%), and the fraction of mixed-skilled-2 households increases by 0.96%, which is comprised of a skilled male and an unskilled female. The balancing in the marriage market requires that there is increased supply of skilled males due to increase in mixed-skilled-2 households, which contributes to the increase in \bar{p}^m . Thus increase in unskilled-labor productivity leads to decrease in the ratio of skilled females to males by 0.6%.

The economy takes 4 generations to converge to the new steady state when the unskilled labor-productivity increases by 10%, as shown by comparative dynamic analysis (discussed in appendix F).

Effect of Decrease in the Cost of Education

We now analyze the effect of exogenous decrease in the steady state cost of education of low-skilled household on the economy. For a 5% decrease in \bar{v}_l from 0.1252 to 0.11894, the results are documented in table 3.6. Decrease in the cost of education increases investment in quality by low-skilled household, leading to increase in the fraction of boys and girls getting higher education (the ratio increases by 2.18 and 0.09 percentage points), which results in increase in the fraction of skilled males and skilled labor by 6.81% and 3.76% respectively. The fraction of skilled females decreases by 1.73%, largely due to reduction in the returns to higher education leading to lower investment in girls' education by $h, m1$ and $m2$ -type households.

For the low-skilled household, quality-quantity tradeoff causes fertility to decrease by 0.52% as investment in quality increases. As increase in the skilled labor supply reduces skill premium, for the high-skilled household, increase in fertility due to substitution effect from wife's decreased income dominates the decreased fertility from husband's income effect, leading to net increase in fertility by 1.76%. For the mixed-skilled-1 household, increased fertility arising from substitution effect from wife's income adds to the increased fertility from husband's income effect, leading to increase in fertility by 2.74%. For the mixed-skilled-2 household, the opposite happens, decreased fertility arising from substitution effect from wife's income adds to the decreased fertility from husband's income effect, leading to decrease in fertility by 0.41%. The overall effect on fertility is positive, a 0.45% increase. Thus decrease in cost of education for low-skilled household leads to increase in skilled labor and net increase in fertility. And the decreased cost strengthens the gender discrimination due to reduction in human capital investment by the remaining households, as all the measures of gender equality worsen as seen from the table.

The comparative dynamic analysis leads us to conclude that the economy follows a spiral sink path to converge to the new steady state in about 5 generations, as the cost of higher education decreases by 5% for low-skilled households. The details are mentioned in appendix G.

3.5 Policy Analysis

We are interested in knowing whether policy interventions can neutralize the taste-based gender bias against girls' higher education. As seen in section (2.4), low-skilled households educate fewer girls than boys, and we conduct policy experiment of subsidizing girls' cost of higher education for the low-skilled households. We thus introduce government in this section and its role is to levy lumpsum taxes on richer households. Tax proceeds are used to subsidize girls' higher education in poorer households i.e. low-skilled households in our model. Our analysis shows that use of unconditional cash transfer fails to influence the human capital investment decisions of poorer households and the households may be incentivized by way of subsidy to tackle the gender bias. Changes to the model are presented first, followed by numerical analysis.

Household

To introduce higher education subsidies in the model, we need to alter the household decision problem. In our model, the household decides on consumption, fertility and children's education choices and it has differential preference for sons' and daughters' education. Richer households i.e. high- and both the mixed-skilled households are levied a lumpsum tax by the government and the low-skilled household receives subsidy for girls' higher education from the tax proceeds.

We first describe the household decision problem for low-skilled household i.e. $j = l$.

$$\underset{c_{jt}, n_{jt}, \widehat{a}_{bjt}, \widehat{a}_{gjt}}{\text{maximize}} \quad U_{jt} = \ln c_{jt} + \gamma \ln n_{jt} + \eta(\ln q_{bjt} + \phi \ln q_{gjt}) \quad (3.1a)$$

$$\text{subject to} \quad l_{wjt} + \tau n_{jt} = 1 \text{ (female time constraint),} \quad (3.1b)$$

$$c_{jt} + 0.5r_{bjt}n_{jt}v_{bjt} + 0.5r_{gjt}n_{jt}v_{gjt} = w_{jt}^m + w_{jt}^f l_{wjt} \text{ (budget constraint),} \quad (3.1c)$$

$$r_{bjt}w_{st+1} + (1 - r_{bjt})w_{ut+1} = q_{bjt} \text{ (boys' quality),} \quad (3.1d)$$

$$r_{gjt}w_{st+1} + (1 - r_{gjt})w_{ut+1} = q_{gjt} \text{ (girls' quality),} \quad (3.1e)$$

$$\frac{\bar{a} - \widehat{a}_{bjt}}{\bar{a} - \underline{a}} = r_{bjt}, \quad (3.1f)$$

$$\frac{\bar{a} - \widehat{a}_{gjt}}{\bar{a} - \underline{a}} = r_{gjt}, \quad (3.1g)$$

taking as given wages $\{w_{st}, w_{ut}, w_{st+1}, w_{ut+1}\}$. Here, the added notations to the budget constraint, v_{bjt} and v_{gjt} , denote the cost of education for boys and girls. Low-skilled household faces different costs for education for boys and girls as it receives subsidy for girls' higher education. v_{bjt} and v_{gjt} are assumed to be growing at the same rate as that of v_{jt} i.e. g with v_{bj0} and v_{gj0} as initial values i.e. $v_{bjt} = v_{bj0}(1 + g)^t$ and $v_{gjt} = v_{gj0}(1 + g)^t$, and from the subsidy, we get the following relation,

$$v_{glt} \leq v_{blt}. \quad (3.2)$$

As already stated in chapter 2, skill levels and education are equivalent to each other, education up to 10 years is free of cost, and the decision to invest in education of children means

investing in higher education i.e. education for more than 10 years.

Solving the optimization problem leads to the following results:

$$n_{jt} = \frac{(w_{jt}^m + w_{jt}^f)(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)}{(1 + \gamma)((\tau w_{jt}^f + \psi)(w_{st+1} - w_{ut+1}) - 0.5(v_{bjt} + v_{gjt})w_{ut+1})} \quad (3.3)$$

$$\begin{aligned} \widehat{a}_{bjt} = & \left((\bar{a}w_{st+1} - \underline{a}w_{ut+1})(v_{bjt}\gamma + v_{gjt}\eta - v_{bjt}\phi\eta) \right. \\ & \left. - 2\eta(w_{st+1} - w_{ut+1})(\tau w_{jt}^f(\bar{a} - \underline{a}) + 0.5\bar{a}(v_{bjt} + v_{gjt})) \right) / (v_{bjt}(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)) \end{aligned} \quad (3.4)$$

$$\begin{aligned} \widehat{a}_{gjt} = & \left((\bar{a}w_{st+1} - \underline{a}w_{ut+1})(v_{gjt}\gamma - v_{gjt}\eta + v_{bjt}\phi\eta) \right. \\ & \left. - 2\eta\phi(w_{st+1} - w_{ut+1})(\tau w_{jt}^f(\bar{a} - \underline{a}) + 0.5\bar{a}(v_{bjt} + v_{gjt})) \right) / (v_{gjt}(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)). \end{aligned} \quad (3.5)$$

From equation (3.3), we see that as the overall cost of education for low-skilled household decreases i.e. $0.5(v_{bjt} + v_{gjt}) < v_{lt}$, where v_{lt} is the cost of education without any subsidy, it should lead to increase in human capital investment and quality-quantity tradeoff implies decrease in fertility.

The household problem for high-, mixed-skilled-1 and mixed-skilled-2 households i.e for $j = h, m1, m2$ is described next.

$$\underset{c_{jt}, n_{jt}, \widehat{a}_{bjt}, \widehat{a}_{gjt}}{\text{maximize}} \quad U_{jt} = \ln c_{jt} + \gamma \ln n_{jt} + \eta(\ln q_{bjt} + \phi \ln q_{gjt}) \quad (3.6a)$$

$$\text{subject to} \quad l_{wjt} + \tau n_{jt} = 1 \text{ (female time constraint),} \quad (3.6b)$$

$$c_{jt} + 0.5(r_{bjt} + r_{gjt})n_{jt}v_{jt} = w_{jt}^m + w_{jt}^f l_{wjt} - \kappa_{jt} \text{ (budget constraint),} \quad (3.6c)$$

$$r_{bjt}w_{st+1} + (1 - r_{bjt})w_{ut+1} = q_{bjt} \text{ (boys' quality),} \quad (3.6d)$$

$$r_{gjt}w_{st+1} + (1 - r_{gjt})w_{ut+1} = q_{gjt} \text{ (girls' quality)}, \quad (3.6e)$$

$$\frac{\bar{a} - \widehat{a}_{bjt}}{\bar{a} - \underline{a}} = r_{bjt}, \quad (3.6f)$$

$$\frac{\bar{a} - \widehat{a}_{gjt}}{\bar{a} - \underline{a}} = r_{gjt}, \quad (3.6g)$$

taking as given wages $\{w_{st}, w_{ut}, w_{st+1}, w_{ut+1}\}$. Here, the additional notation in the budget constraint compared to the model in chapter 2 is κ_{jt} . It denotes the lumpsum tax, and it is assumed to be growing at the same rate as that of v_{jt} i.e. g with κ_{j0} as the initial value i.e. $\kappa_{jt} = \kappa_{j0}(1 + g)^t$.

The solution to the optimization problem is described below.

$$n_{jt} = \frac{(w_{jt}^m + w_{jt}^f - \kappa_{jt})(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)}{(1 + \gamma)(\tau w_{jt}^f(w_{st+1} - w_{ut+1}) - v_{jt}w_{ut+1})} \quad (3.7)$$

$$\widehat{a}_{bjt} = \frac{v_{jt}(\bar{a}w_{st+1} - \underline{a}w_{ut+1})(\gamma + \eta - \phi\eta) - 2\eta(w_{st+1} - w_{ut+1})(\tau w_{jt}^f(\bar{a} - \underline{a}) + \bar{a}v_{jt})}{v_{jt}(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)} \quad (3.8)$$

$$\widehat{a}_{gjt} = \frac{v_{jt}(\bar{a}w_{st+1} - \underline{a}w_{ut+1})(\gamma - \eta + \phi\eta) - 2\eta\phi(w_{st+1} - w_{ut+1})(\tau w_{jt}^f(\bar{a} - \underline{a}) + \bar{a}v_{jt})}{v_{jt}(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)}. \quad (3.9)$$

From equations (3.8) and (3.9), the ability cutoff equations are not affected by introduction of lumpsum tax. From equation (3.7), we can see that lumpsum tax creates income effect, wherein the reduced income leads to decrease in fertility.

Government

Government is responsible for levying taxes and operationalizing the higher education subsidies. The government's revenues equals expenses and the balanced budget condition is given by,

$$\sum_{j=h,m1,m2} \lambda_{jt}\kappa_{jt} = \lambda_{lt}(0.5r_{glt}n_{lt}(v_{blt} - v_{glt})). \quad (3.10)$$

The left side of equation (3.10) gives government revenues from lumpsum taxes on high, mixed-skilled-1 and mixed-skilled-2 households. The right side gives the subsidy provided

to low-skilled households, the subsidy being the difference between the cost of education for boys and girls ($v_{bht} - v_{ght}$), as the low-skilled household face a different cost for girls' higher education. The difference is multiplied by the fraction of low-skilled households in the economy, λ_{lt} and the number of girls in the household ($0.5n_{lt}$) and the fraction of girls receiving the higher education among them, r_{ght} .

Numerical Analysis

Based on the new set of results from the household optimization problem, the dynamic equilibrium is modified to the extent of these new results and additionally the government balances its budget. The steady state also gets modified to the extent of the new results. We now describe the changes to steady state outcomes when the policy is implemented numerically. The steady state lumpsum tax value for high-skilled household is: $\bar{\kappa}_h = 0.00105$ ¹², and there is no tax on both the mixed-skilled households. This translate to about 0.02% of skilled labor income for the high-skilled household, and even though the percentage looks small, it is important to note that this is a permanent policy change. With the government balancing its budget¹³, and with the implementation of subsidies to low-skilled households, the cost of higher education for girls decreases by 5% from 0.1252 to 0.11894. The results are described in tables 3.7 (partial equilibrium) and 3.8 (general equilibrium).

From table 3.7 which presents the partial equilibrium effect of the policy, the subsidy translates to 2.63 percentage point increase in the fraction of girls of low-skilled households getting higher education (increase from 0% to 2.63%). As the average cost of higher education decreases due to subsidy, it frees up household resources leading to its investment in boys' higher education, and the fraction jumps by 16.26% from 4.06% to 4.72%. The decrease in average cost also leads to decrease in fertility by 0.85% for these households in confirmation with the quality-quantity tradeoff.

Table 3.8 shows the general equilibrium effect of the policy. The fraction of skilled females \bar{p}^f increase by 7.63% from 14.42% to 15.52% due to increase in girls' educational investment by low-skilled households from 0% to 1.92%. Increase in \bar{p}^f drives up the skilled labor ratio

¹²The steady state variable for lumpsum tax is $\bar{\kappa}_h = \kappa_{ht}/(1+g)^t$.

¹³Numerically, the government has a budget surplus of 7×10^{-7} .

$\bar{\beta}$, an increase of 0.74%. The fraction of skilled males \bar{p}^m decreases by 2.24% to 19.66%, driven by decrease in boys' educational investment due to decrease in skill premium (explained in the next paragraph). Increase in \bar{p}^f also contributes to increase in fractions of high- and mixed-skilled-1 households by 7.29% and 8.04%, and decrease in the fraction of mixed-skilled-2 households by 13.27%. The fraction of low-skilled households doesn't change much, an increase of 0.2%, mainly due to the effects of increase in \bar{p}^f and decrease in \bar{p}^m acting against each other.

Increase in the skilled-to-total labor ratio causes decrease in the skill premium ratio by 0.44%. The decrease is large enough to reduce human capital investment by all households except for girls in low-skilled households, which can be seen from decrease in r 's in table 3.8. As already noted, decrease in the cost of education for girls in the low-skilled households increases the fraction of girls receiving higher education from 0% to 1.92%. This general equilibrium effect is muted by 0.71 percentage points compared to the partial equilibrium effect (2.63%), due to decrease in the returns to higher education resulting from increase in the skilled labor supply. Similar effect is also observed for investment in boys' education, the fraction decreasing by 1.48% from 4.06% to 4%, and the general equilibrium effect is lower by 0.72 percentage points compared to the partial equilibrium effect (4.72%). Khanna (2023) documents that general equilibrium labor market effects suppress returns to education by 6.6 percentage points, and our results are qualitatively in line with this result.

Fertility of high- and mixed-skilled-1 households increase by 0.31% and 0.51%, primarily driven by the substitution effect from decrease in skilled wage rate attributed to wife. The substitution effect induced by increase in unskilled wage rate for wife drives down the fertility of mixed-skilled-2 and low-skilled households by 0.09% and 0.47% (for low-skilled households, the quality-quantity tradeoff plays a big role as well, already explained while discussing partial equilibrium effects). Changes to the female labor supply work in the opposite direction, leading to decrease for high- and mixed-1 households, and increase for mixed-skilled-2 and low-skilled households. Calculation of changes to income of each type of household shows that the income inequality reduces as well.¹⁴

Measures of gender equality also show encouraging results. The ratio of skilled females to

¹⁴The change in income for each type of household is: high-skilled -0.34%, mixed-skilled-1 -0.23%, mixed-skilled-2 -0.2% and low-skilled +0.28%.

skilled males (which is equal to the ratio of girls to boys getting higher education in the steady state) increases from 0.7171 to 0.7894, a large increase of 10.08%. The value of 1 for the ratio denotes the perfect gender equality in our model, and the policy is bridging the gap by almost one-third, underlining the policy success to neutralize the taste-based gender discrimination against girls' higher education. Other measures of gender equality, the ratio of average female to male labor supply and the ratio of average female to male income show increase of 0.73% and 2.54% as well, conveying the effect of the policy in countering the gender bias.

Results from dynamic analysis for \bar{p}^m , \bar{p}^f and $\bar{\beta}$ using linearized equations (2.40), (2.41) and (2.42) are shown in – figures 3.9 to 3.11 for time path, and 3.12 to 3.14 for steady-state diagram.¹⁵ Initial values p_0^m , p_0^f and β_0 come from the initial steady state i.e. when the policy is not in place. Since the equilibrium path is generally indeterminate, we make an assumption to determine a specific path the economy takes to adjust to the new steady state. We implicitly assume fertility is chosen first, and the policy announcement comes as a surprise to agents, they then decide about children's education. This means the female labor supply is chosen before the policy is in place, enabling us to use the value of β from the initial steady state as β_0 . Dynamic analysis reveals that it takes 2 generations, i.e about 50 years, for the economy to reach the new steady state where gender inequality in skills ratio is reduced by about one-third. Uncertainty regarding reduction in gender discrimination due to slowly-changing cultural norms underlines the importance of policy intervention to achieve a more gender equal society.

Steady-state diagrams in figure 3.12 to 3.14 show that the economy follows a spiral sink path toward convergence.

When the gender-targeted subsidy, studied above, is compared with the gender-neutral subsidy of the same magnitude, the gender-neutral subsidy achieves increase in \bar{p}^m as expected, but it is not effective in targeting the gender bias against girls' education. The end result is the worsening of skilled female to male ratio to 0.6871, a 4.18% decrease.

¹⁵When we compare the values of p_t^m , p_t^f and β_t derived from the linearized system of equations with the actual values calculated from equations (2.34), (2.35) and (2.36); the gap between actual and linearized values reduces to less than 10% in just 1 time period, indicating that policy changes can be considered to be local to the steady state.

To summarize, we study both the partial and general equilibrium effects arising from the gender-targeted policy intervention. Partial equilibrium effects lead to higher investment in human capital of both boys and girls for low-skilled households. The general equilibrium weakens the partial equilibrium effect as increase in the ratio of skilled females leads to decrease in the skill premium as well as returns to human capital investment dampening the investment of low-skilled households. Overall, the gender-targeted subsidy significantly reduces the gender gap in education, along with achieving decrease in fertility, increase in female labor supply and reduction in income inequality. Dynamic analysis leads us to conclude that the policy takes about 2 generations in reducing the gender gap in skills by one-third. Thus the policy experiment shows that taste-based gender bias can be countered effectively by carefully designed policy interventions.

The Case of Skewed Sex Ratio

The implicit assumption so far has been that there is no son-bias in fertility decisions of households. In this section, we conduct robustness check for the gender-targeted policy when the sex ratio is skewed towards males. The fertility preference for son is added as an exogenous parameter π , which denotes the ratio of sons to total children in a household. This will modify the budget constraint as follows:

$$c_{jt} + \pi r_{bjt} n_{jt} v_{bjt} + (1 - \pi) r_{gjt} n_{jt} v_{gjt} = w_{jt}^m + w_{jt}^f l_{wjt} - \kappa_{jt}, \quad (3.11)$$

where the educational expenditure by household is now driven by the the fractions π (for boys) and $(1 - \pi)$ (for girls). The household optimization solution is modified to this extent. Using the value of 901 girls per 1000 boys as sex ratio at birth for India in 2005-06 from the United Nations Population Fund, $\pi = 1000/1901 = 0.53$. The subsidy of similar magnitude increases the ratio of skilled females to males from 0.7128 to 0.7776, a 9.09% increase; which is close to the increase of 10.08% observed without skewed sex ratio. Thus the effect of subsidy is robust to inclusion of sex ratio in the model.

3.6 Conclusion

We focused on how to counter the taste-based gender bias against girls' higher education in this chapter. We had constructed a theoretical model of the economy and the marriage market in chapter 2. In this chapter, dynamic analysis of the economy revealed that it took about 3 generations for the economy to converge to the steady state by following a spiral sink path when it started with values of p^m, p^f and β 10% below or above the steady state.

The comparative statics and dynamics exercise was conducted to understand what factors could weaken the gender bias and also to inform how the PAM affected income inequality. We showed that exogenous increase in PAM took 4 generations to take full effect. It worsened the gender bias in education and income, and also led to more income inequality, extending the findings in the literature to developing countries. Elimination of gender bias and exogenous increase in returns to education led to decrease in fertility and more gender equal society. Analysis of increase in skilled- vs unskilled-labor productivity showed that changes to human capital investment could work in the opposite directions. Reduction in the higher educational expenditure for poorer households did not have the desired effect of weakening of gender norms as boys benefited more from it than girls.

We then conducted a distributional policy experiment to understand its effectiveness in tackling the gender bias. The government subsidized the cost of higher education of girls from the poorer households, by taxing richer households. With the simulation exercise, we concluded that the subsidy significantly reduced the gender gap in higher education, skills and income. The policy was able to bridge the gap between the perfect gender equality in the skill level and the current imbalance by one-third. The policy intervention also decreased fertility, increased female labor supply and reduced income inequality. Dynamic analysis showed that it took only about 2 generations for the economy to reduce the gender gap in skills by one-third. When these results were compared with gender-neutral policy, it could be seen that the gender-neutral subsidies were not effective in tackling the gender bias. The results show effectiveness of targeted policy as a tool for achieving a more gender equal society, women empowerment and family planning. And we can also forecast time required for policy intervention to take effect.

The model can be extended to add labor market discrimination against women. This can enable investigation of what proportion of labor market policies and household subsidies

will show the maximum effectiveness in weakening of gender discrimination, which we leave for future research. Another important topic for future research is to adapt the framework presented in the chapter to better suit marriage markets in developing countries, by modeling agents' and families' decisions in the marriage market and study its implications on female education, gender bias, fertility and female labor supply.

3.7 Figures

Figure 3.1
Time path for p^m : -10% Deviation

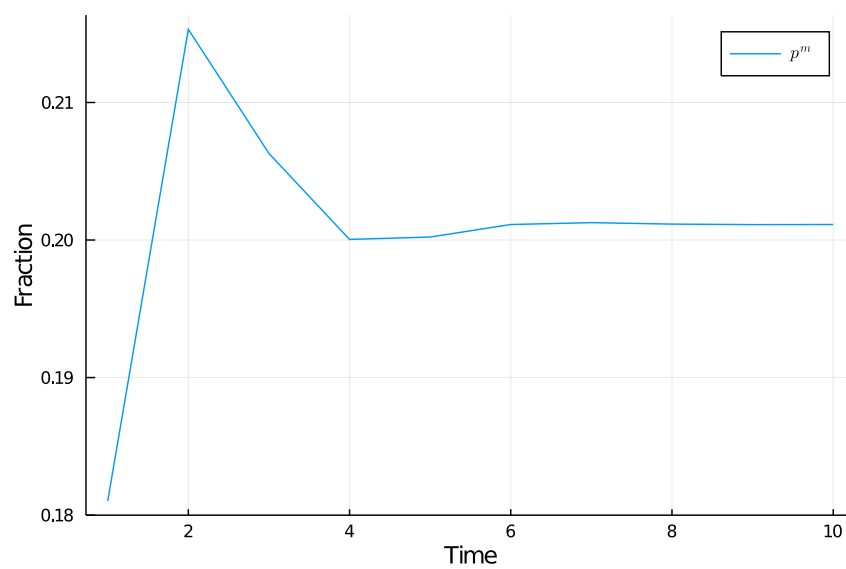


Figure 3.2
Time path for p^f : -10% Deviation

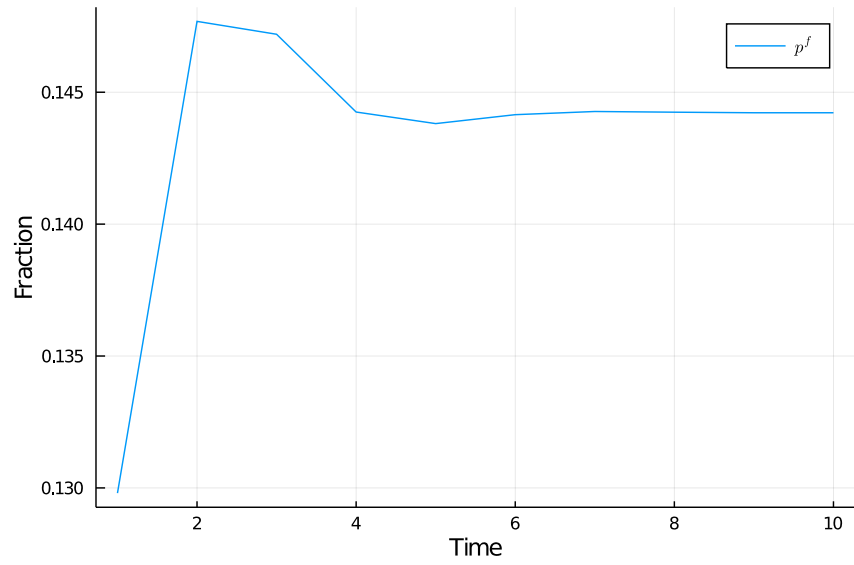


Figure 3.3
Time path for β : -10% Deviation

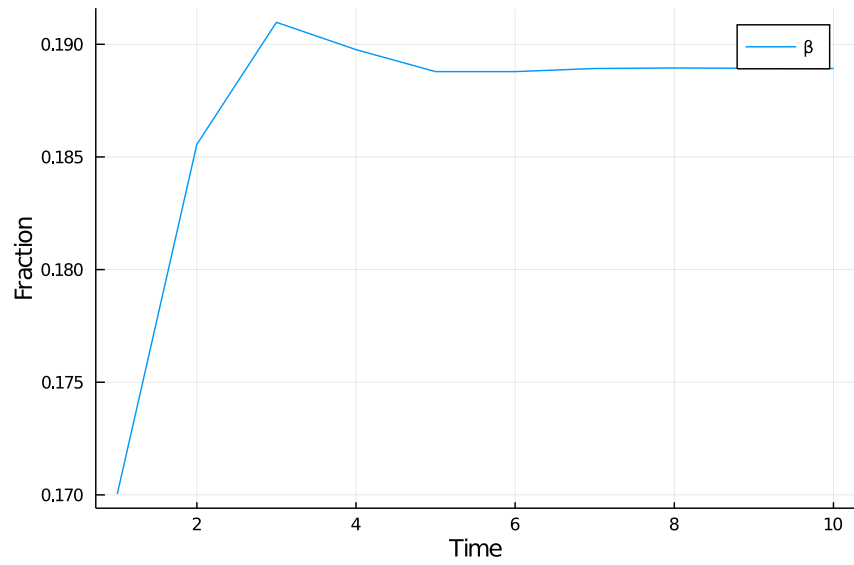


Figure 3.4
Steady-state Diagram for p^m : -10% Deviation

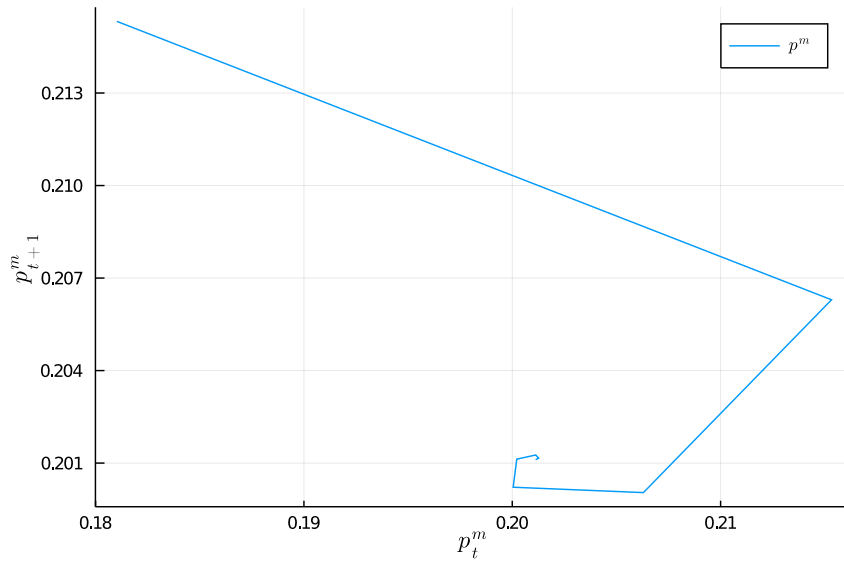


Figure 3.5
Steady-state Diagram for p^f : -10% Deviation

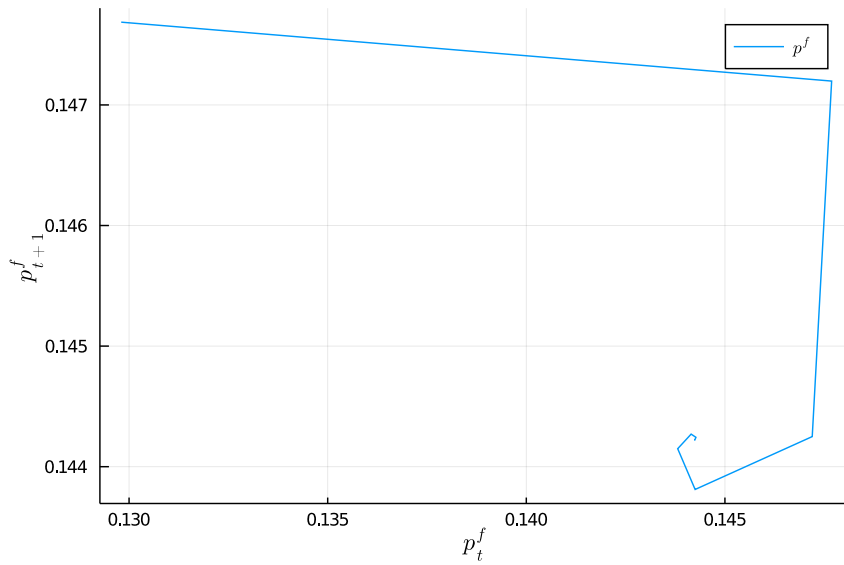


Figure 3.6
Steady-state Diagram for β : -10% Deviation

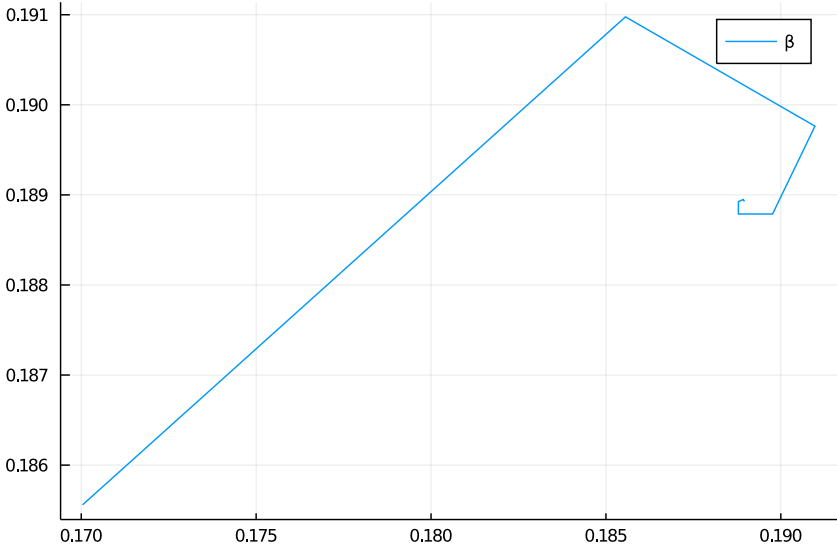


Figure 3.7
Time path for p^f : Elimination of Gender Bias

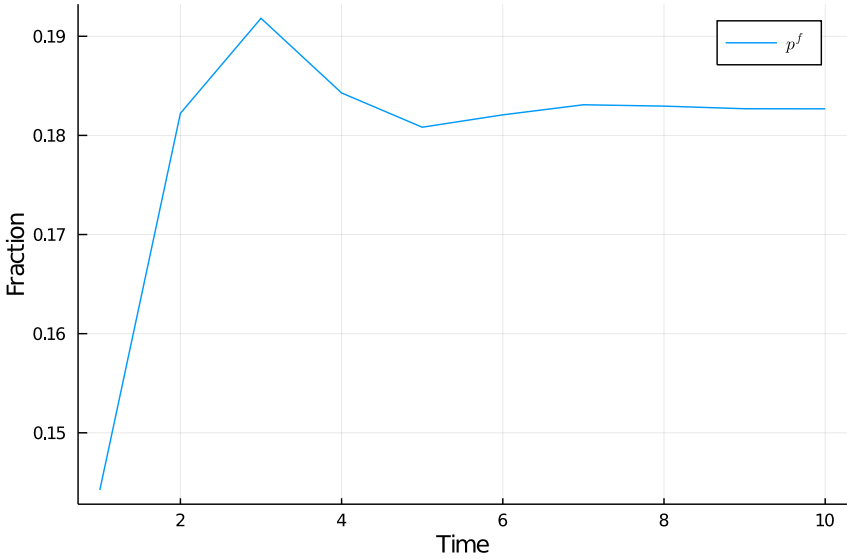


Figure 3.8
Steady-state Diagram for p^f : Elimination of Gender Bias

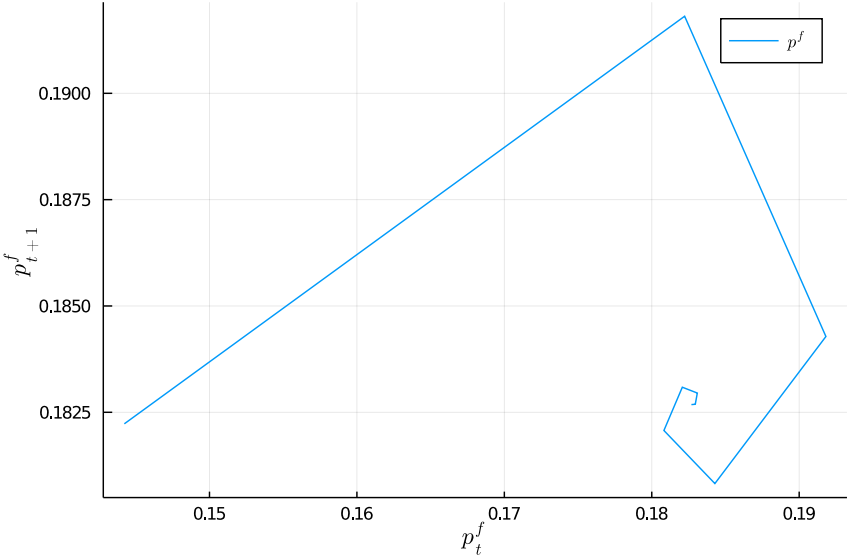


Figure 3.9
Time path for p^m : Education Policy

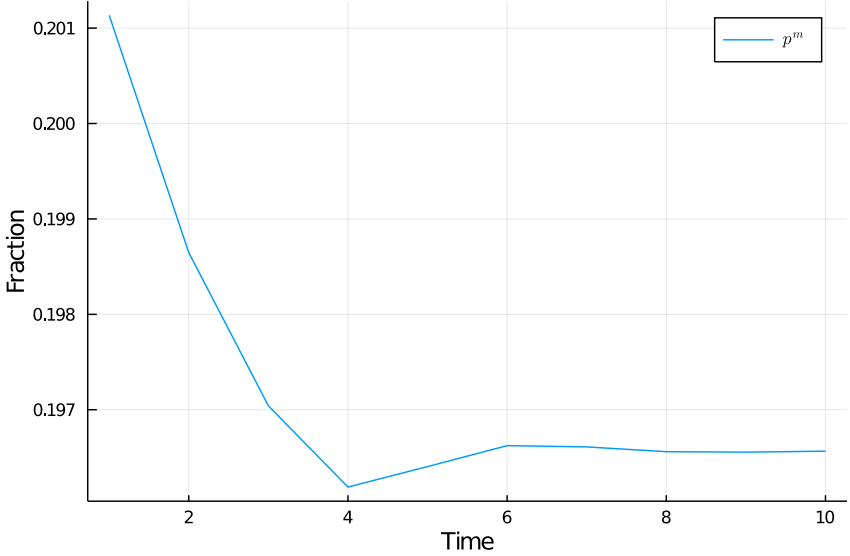


Figure 3.10
Time path for p^f : Education Policy

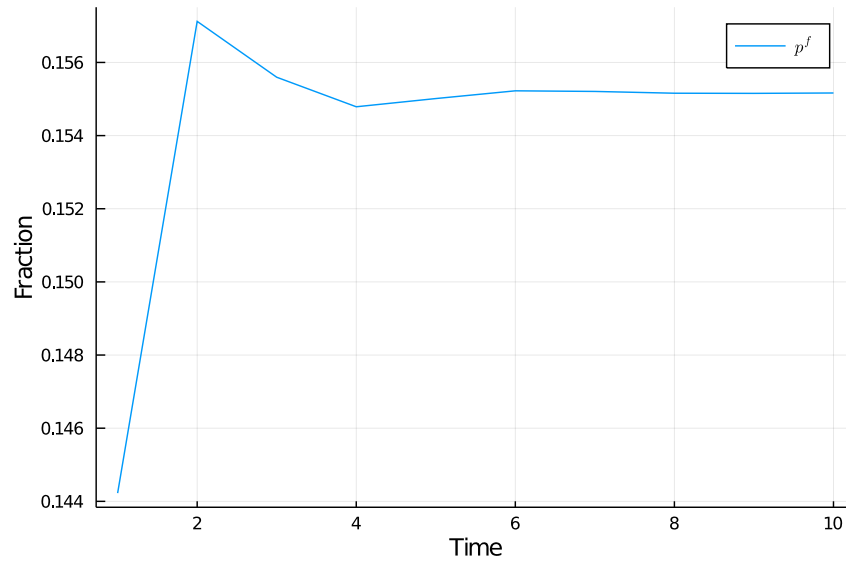


Figure 3.11
Time path for β : Education Policy

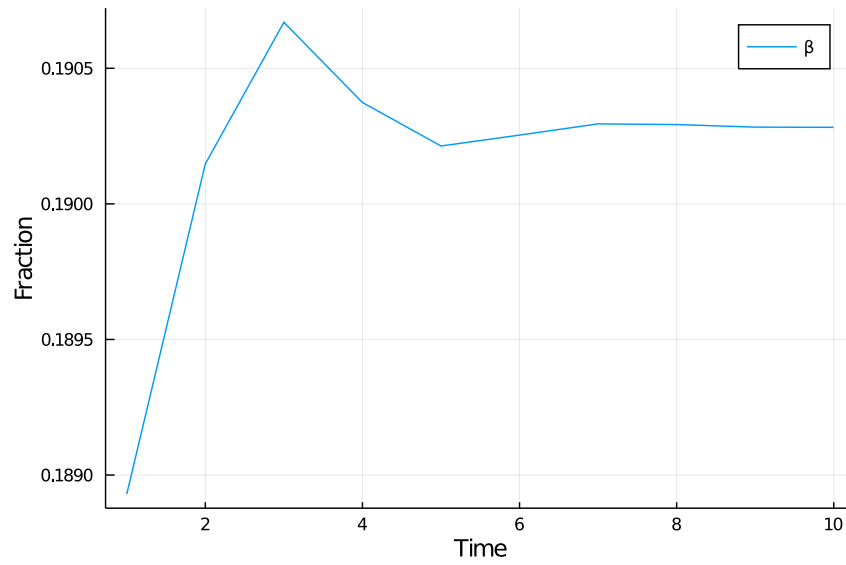


Figure 3.12
Steady-state Diagram for p^m : Education Policy

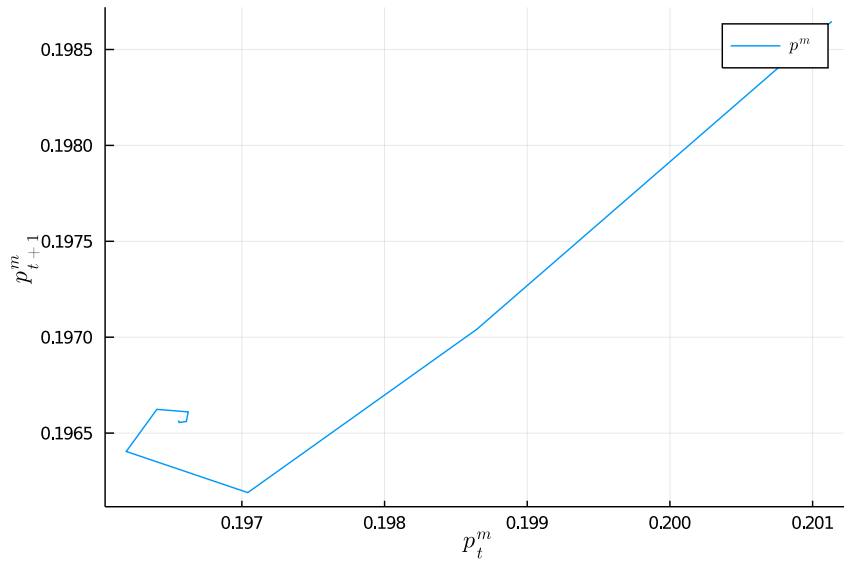


Figure 3.13
Steady-state Diagram for p^f : Education Policy

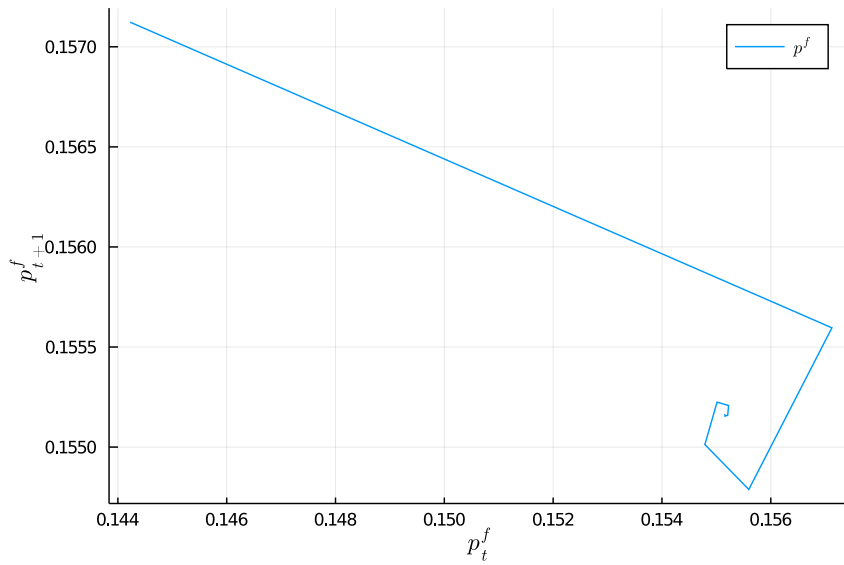
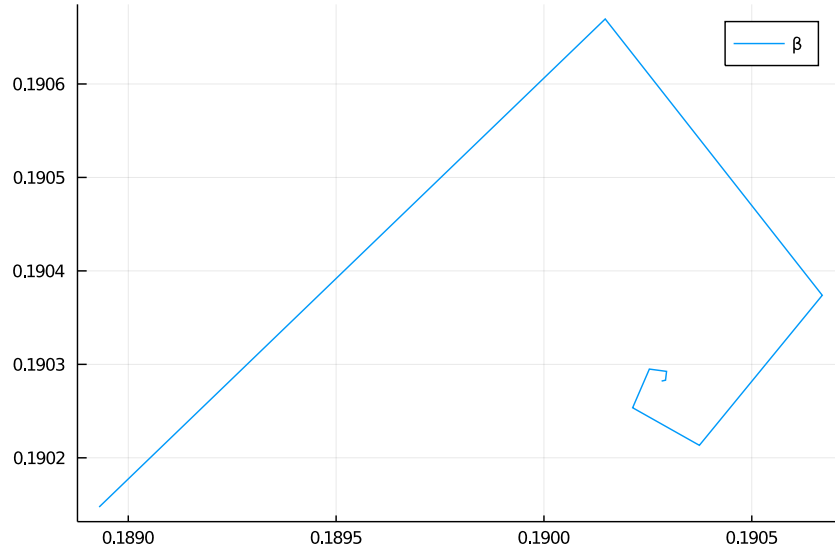


Figure 3.14
Steady-state Diagram for β : Education Policy



3.8 Tables

Table 3.1
The Effect of Elimination of Gender Bias

Variable	$\phi =$ 0.9241	$\phi =$ 1	Percentage Change
Fraction of skilled to total males (p^m)	20.11%	18.27%	-9.15%
Fraction of skilled to total females (p^f)	14.42%	18.27%	26.7%
Fraction of skilled to total labor (β)	18.89%	19.32%	2.28%
Skilled wage rate	3.6104	3.5787	-0.88%
Unskilled wage rate	1.5103	1.5178	0.5%
Skill premium ratio	2.3906	2.3579	-1.37%
Fraction of high-skilled households	10.7%	13.44%	25.61%
Fraction of mixed-skilled-1 households	3.73%	4.83%	29.49%
Fraction of mixed-skilled-2 households	9.42%	4.83%	-48.73%
Fraction of low-skilled households	76.16%	76.9%	0.97%
Fertility (high-skilled household)	2.0963	2.0625	-1.61%
Fertility (mixed-skilled-1 household)	1.5413	1.5252	-1.04%
Fertility (mixed-skilled-2 household)	3.7251	3.6181	-2.87%
Fertility (low-skilled household)	2.6328	2.6166	-0.62%
Average fertility	2.6376	2.5378	-3.78%
Female labor supply (high-skilled household)	0.6856	0.6906	0.73%
Female labor supply (mixed-skilled-1 household)	0.7688	0.7712	0.31%
Female labor supply (mixed-skilled-2 household)	0.4412	0.4573	3.65%
Female labor supply (low-skilled household)	0.6051	0.6075	0.4%
Fraction of boys getting higher education (high-skilled households)	87.91%	87.37%	-0.61%
Fraction of girls getting higher education (high-skilled households)	75.78%	87.37%	15.29%
Fraction of boys getting higher education (mixed-skilled-1 household)	66.44%	65.51%	-1.4%
Fraction of girls getting higher education (mixed-skilled-1 household)	55.94%	65.51%	17.11%
Fraction of boys getting higher education (mixed-skilled-2 household)	60.93%	62.48%	2.54%
Fraction of girls getting higher education (mixed-skilled-2 household)	50.85%	62.48%	22.87%
Fraction of boys getting higher education (low-skilled household)	4.06%	3.18%	-21.67%
Fraction of girls getting higher education (low-skilled household)	0%	3.18%	-
Ratio of skilled females to skilled males	0.7171	1	39.45%
Ratio of average female to male income	0.5831	0.6377	9.36%

Table 3.2
The Effect of Increase in Marital Sorting

Variable	$\alpha =$ 0.3234	$\alpha =$ 0.2	Percentage Change
Fraction of skilled to total males (p^m)	20.11%	20.21%	0.5%
Fraction of skilled to total females (p^f)	14.42%	14.13%	-2.01%
Fraction of skilled to total labor (β)	18.89%	18.75%	-0.74%
Skilled wage rate	3.6104	3.6216	0.31%
Unskilled wage rate	1.5103	1.5077	-0.17%
Skill premium ratio	2.3906	2.402	0.48%
Fraction of high-skilled households	10.7%	11.87%	10.93%
Fraction of mixed-skilled-1 households	3.73%	2.25%	-39.68%
Fraction of mixed-skilled-2 households	9.42%	8.33%	-11.57%
Fraction of low-skilled households	76.16%	77.54%	1.81%
Fertility (high-skilled household)	2.0963	2.089	-0.35%
Fertility (mixed-skilled-1 household)	1.5413	1.533	-0.54%
Fertility (mixed-skilled-2 household)	3.7251	3.7286	0.09%
Fertility (low-skilled household)	2.6328	2.6286	-0.16%
Average fertility	2.6376	2.6315	-0.23%
Female labor supply (high-skilled household)	0.6856	0.6867	0.16%
Female labor supply (mixed-skilled-1 household)	0.7688	0.7701	0.17%
Female labor supply (mixed-skilled-2 household)	0.4412	0.4407	-0.11%
Female labor supply (low-skilled household)	0.6051	0.6057	0.1%
Fraction of boys getting higher education (high-skilled households)	87.91%	89.55%	1.87%
Fraction of girls getting higher education (high-skilled households)	75.78%	77.34%	2.06%
Fraction of boys getting higher education (mixed-skilled-1 household)	66.44%	68.01%	2.36%
Fraction of girls getting higher education (mixed-skilled-1 household)	55.94%	57.44%	2.68%
Fraction of boys getting higher education (mixed-skilled-2 household)	60.93%	61.61%	1.12%
Fraction of girls getting higher education (mixed-skilled-2 household)	50.85%	51.52%	1.32%
Fraction of boys getting higher education (low-skilled household)	4.06%	4.64%	14.29%
Fraction of girls getting higher education (low-skilled household)	0%	0%	-
Ratio of skilled females to skilled males	0.7171	0.6992	-2.5%
Ratio of average female to male labor supply	0.6044	0.6053	0.15%
Ratio of average female to male income	0.5831	0.5797	-0.58%

Table 3.3
The Effect of Increase in Returns to Education

Variable	$\mu =$ 0.5357	$\mu =$ 0.6	Percentage Change
Fraction of skilled to total males (p^m)	20.11%	26.28%	30.68%
Fraction of skilled to total females (p^f)	14.42%	19.52%	35.37%
Fraction of skilled to total labor (β)	18.89%	25.06%	32.66%
Skilled wage rate	3.6104	3.4877	-3.4%
Unskilled wage rate	1.5103	1.3445	-10.98%
Skill premium ratio	2.3906	2.594	8.51%
Fraction of high-skilled households	10.7%	14.87%	38.97%
Fraction of mixed-skilled-1 households	3.73%	4.65%	24.66%
Fraction of mixed-skilled-2 households	9.42%	11.41%	21.13%
Fraction of low-skilled households	76.16%	69.07%	-9.31%
Fertility (high-skilled household)	2.0963	2.0349	-2.93%
Fertility (mixed-skilled-1 household)	1.5413	1.455	-5.6%
Fertility (mixed-skilled-2 household)	3.7251	3.9191	5.21%
Fertility (low-skilled household)	2.6328	2.6187	-0.54%
Average fertility	2.6376	2.6261	-0.44%
Female labor supply (high-skilled household)	0.6856	0.6948	1.34%
Female labor supply (mixed-skilled-1 household)	0.7688	0.7818	1.69%
Female labor supply (mixed-skilled-2 household)	0.4412	0.4121	-6.6%
Female labor supply (low-skilled household)	0.6051	0.6072	0.35%
Fraction of boys getting higher education (high-skilled households)	87.91%	96.31%	9.56%
Fraction of girls getting higher education (high-skilled households)	75.78%	84.24%	11.16%
Fraction of boys getting higher education (mixed-skilled-1 household)	66.44%	75.57%	13.74%
Fraction of girls getting higher education (mixed-skilled-1 household)	55.94%	65.07%	16.32%
Fraction of boys getting higher education (mixed-skilled-2 household)	60.93%	56.41%	-7.42%
Fraction of girls getting higher education (mixed-skilled-2 household)	50.85%	47.37%	-6.84%
Fraction of boys getting higher education (low-skilled household)	4.06%	5.27%	29.8%
Fraction of girls getting higher education (low-skilled household)	0%	0.11%	-
Ratio of skilled females to skilled males	0.7171	0.7429	3.6%
Ratio of average female to male labor supply	0.6044	0.6061	0.28%
Ratio of average female to male income	0.5831	0.5841	0.17%

Table 3.4
The Effect of Increase in Skilled-Labor Productivity

Variable	$A_s =$ 1	$A_s =$ 1.1	Percentage Change
Fraction of skilled to total males (p^m)	20.11%	25.3%	30.68%
Fraction of skilled to total females (p^f)	14.42%	18.45%	27.95%
Fraction of skilled to total labor (β)	18.89%	23.78%	25.89%
Skilled wage rate	3.6104	3.7666	4.33%
Unskilled wage rate	1.5103	1.6579	9.77%
Skill premium ratio	2.3906	2.2719	-4.97%
Fraction of high-skilled households	10.7%	13.99%	30.75%
Fraction of mixed-skilled-1 households	3.73%	4.46%	19.57%
Fraction of mixed-skilled-2 households	9.42%	11.31%	20.06%
Fraction of low-skilled households	76.16%	70.25%	-7.76%
Fertility (high-skilled household)	2.0963	2.128	1.51%
Fertility (mixed-skilled-1 household)	1.5413	1.5925	3.32%
Fertility (mixed-skilled-2 household)	3.7251	3.5893	-3.65%
Fertility (low-skilled household)	2.6328	2.6302	-0.1%
Average fertility	2.6376	2.6221	-0.59%
Female labor supply (high-skilled household)	0.6856	0.6808	-0.7%
Female labor supply (mixed-skilled-1 household)	0.7688	0.7611	-1%
Female labor supply (mixed-skilled-2 household)	0.4412	0.4616	4.62%
Female labor supply (low-skilled household)	0.6051	0.6055	0.07%
Fraction of boys getting higher education (high-skilled households)	87.91%	85.63%	-2.59%
Fraction of girls getting higher education (high-skilled households)	75.78%	73.16%	-3.46%
Fraction of boys getting higher education (mixed-skilled-1 household)	66.44%	63.23%	-4.83%
Fraction of girls getting higher education (mixed-skilled-1 household)	55.94%	52.46%	-6.22%
Fraction of boys getting higher education (mixed-skilled-2 household)	60.93%	67.42%	10.65%
Fraction of girls getting higher education (mixed-skilled-2 household)	50.85%	56.34%	10.8%
Fraction of boys getting higher education (low-skilled household)	4.06%	4.87%	19.95%
Fraction of girls getting higher education (low-skilled household)	0%	0%	-
Ratio of skilled females to skilled males	0.7171	0.7292	1.69%
Ratio of average female to male labor supply	0.6044	0.6067	0.38%
Ratio of average female to male income	0.5831	0.5833	0.03%

Table 3.5
The Effect of Increase in Unskilled-Labor Productivity

Variable	$A_u =$ 1	$A_u =$ 1.1	Percentage Change
Fraction of skilled to total males (\bar{p}^m)	20.11%	20.19%	0.4%
Fraction of skilled to total females (\bar{p}^f)	14.42%	14.39%	-0.21%
Fraction of skilled to total labor ($\bar{\beta}$)	18.89%	18.81%	-0.42%
Skilled wage rate	3.6104	3.8495	6.62%
Unskilled wage rate	1.5103	1.7664	16.96%
Skill premium ratio	2.3906	2.1793	-8.84%
Fraction of high-skilled households	10.7%	10.67%	-0.28%
Fraction of mixed-skilled-1 households	3.73%	3.71%	-0.54%
Fraction of mixed-skilled-2 households	9.42%	9.51%	0.96%
Fraction of low-skilled households	76.16%	76.1%	-0.08%
Fertility (high-skilled household)	2.0963	2.1677	3.41%
Fertility (mixed-skilled-1 household)	1.5413	1.6481	6.93%
Fertility (mixed-skilled-2 household)	3.7251	3.5037	-5.94%
Fertility (low-skilled household)	2.6328	2.6382	0.21%
Average fertility	2.6376	2.6335	-0.16%
Female labor supply (high-skilled household)	0.6856	0.6748	-1.58%
Female labor supply (mixed-skilled-1 household)	0.7688	0.7528	-2.08%
Female labor supply (mixed-skilled-2 household)	0.4412	0.4744	7.52%
Female labor supply (low-skilled household)	0.6051	0.6043	-0.13%
Fraction of boys getting higher education (high-skilled households)	87.91%	79.99%	-9.01%
Fraction of girls getting higher education (high-skilled households)	75.78%	67.48%	-10.95%
Fraction of boys getting higher education (mixed-skilled-1 household)	66.44%	57.1%	-14.06%
Fraction of girls getting higher education (mixed-skilled-1 household)	55.94%	46.33%	-17.18%
Fraction of boys getting higher education (mixed-skilled-2 household)	60.93%	70.09%	15.03%
Fraction of girls getting higher education (mixed-skilled-2 household)	50.85%	58.34%	14.73%
Fraction of boys getting higher education (low-skilled household)	4.06%	3.88%	-4.43%
Fraction of girls getting higher education (low-skilled household)	0%	0%	-
Ratio of skilled females to skilled males	0.7171	0.7128	-0.6%
Ratio of average female to male labor supply	0.6044	0.605	0.1%
Ratio of average female to male income	0.5831	0.5839	0.14%

Table 3.6
The Effect of Decrease in the Cost of Education

Variable	$\bar{v}_l =$ 0.1252	$\bar{v}_l =$ 0.11894	Percentage Change
Fraction of skilled to total males (p^m)	20.11%	21.48%	6.81%
Fraction of skilled to total females (p^f)	14.42%	14.17%	-1.73%
Fraction of skilled to total labor (β)	18.89%	19.6%	3.76%
Skilled wage rate	3.6104	3.5581	-1.45%
Unskilled wage rate	1.5103	1.5228	0.83%
Skill premium ratio	2.3906	2.3366	-2.26%
Fraction of high-skilled households	10.7%	10.57%	-1.21%
Fraction of mixed-skilled-1 households	3.73%	3.6%	-3.49%
Fraction of mixed-skilled-2 households	9.42%	10.91%	15.82%
Fraction of low-skilled households	76.16%	74.92%	-1.63%
Fertility (high-skilled household)	2.0963	2.1332	1.76%
Fertility (mixed-skilled-1 household)	1.5413	1.5835	2.74%
Fertility (mixed-skilled-2 household)	3.7251	3.71	-0.41%
Fertility (low-skilled household)	2.6328	2.6192	-0.52%
Average fertility	2.6376	2.6495	0.45%
Female labor supply (high-skilled household)	0.6856	0.68	-0.82%
Female labor supply (mixed-skilled-1 household)	0.7688	0.7625	-0.82%
Female labor supply (mixed-skilled-2 household)	0.4412	0.4435	0.52%
Female labor supply (low-skilled household)	0.6051	0.6071	0.33%
Fraction of boys getting higher education (high-skilled households)	87.91%	79.97%	-9.03%
Fraction of girls getting higher education (high-skilled households)	75.78%	68.22%	-9.98%
Fraction of boys getting higher education (mixed-skilled-1 household)	66.44%	58.81%	-11.48%
Fraction of girls getting higher education (mixed-skilled-1 household)	55.94%	48.67%	-13%
Fraction of boys getting higher education (mixed-skilled-2 household)	60.93%	57.53%	-5.58%
Fraction of girls getting higher education (mixed-skilled-2 household)	50.85%	47.48%	-6.63%
Fraction of boys getting higher education (low-skilled household)	4.06%	6.24%	53.69%
Fraction of girls getting higher education (low-skilled household)	0%	0.09%	-
Ratio of skilled females to skilled males	0.7171	0.6597	-8%
Ratio of average female to male labor supply	0.6044	0.6026	-0.3%
Ratio of average female to male income	0.5831	0.5713	-2.02%

Table 3.7
The Effect of Subsidizing Higher Education: Partial Equilibrium

Variable	$\bar{v}_{lg} =$ 0.1252	$\bar{v}_{lg} =$ 0.11894	Percentage Change
Fertility (high-skilled household)	2.0963	2.096	-0.01%
Fertility (mixed-skilled-1 household)	1.5413	1.5413	0%
Fertility (mixed-skilled-2 household)	3.7251	3.7251	0%
Fertility (low-skilled household)	2.6328	2.6103	-0.85%
Average fertility	2.6376	2.6205	-0.65%
Female labor supply (high-skilled household)	0.6856	0.6856	0%
Female labor supply (mixed-skilled-1 household)	0.7688	0.7688	0%
Female labor supply (mixed-skilled-2 household)	0.4412	0.4412	0%
Female labor supply (low-skilled household)	0.6051	0.6085	0.56%
Fraction of boys getting higher education (high-skilled households)	87.91%	87.91%	0%
Fraction of girls getting higher education (high-skilled households)	75.78%	75.78%	0%
Fraction of boys getting higher education (mixed-skilled-1 household)	66.44%	66.44%	0%
Fraction of girls getting higher education (mixed-skilled-1 household)	55.94%	55.94%	0%
Fraction of boys getting higher education (mixed-skilled-2 household)	60.93%	60.93%	0%
Fraction of girls getting higher education (mixed-skilled-2 household)	50.85%	50.85%	0%
Fraction of boys getting higher education (low-skilled household)	4.06%	4.72%	16.26%
Fraction of girls getting higher education (low-skilled household)	0%	2.63%	-
Ratio of average female to male labor supply	0.6044	0.6069	0.41%

Table 3.8
The Effect of Subsidizing Higher Education: General Equilibrium

Variable	$\bar{v}_{lg} =$ 0.1252	$\bar{v}_{lg} =$ 0.11894	Percentage Change
Fraction of skilled to total males (\bar{p}^m)	20.11%	19.66%	-2.24%
Fraction of skilled to total females (\bar{p}^f)	14.42%	15.52%	7.63%
Fraction of skilled to total labor ($\bar{\beta}$)	18.89%	19.03%	0.74%
Skilled wage rate	3.6104	3.6002	-0.28%
Unskilled wage rate	1.5103	1.5127	0.16%
Skill premium ratio	2.3906	2.3801	-0.44%
Fraction of high-skilled households	10.7%	11.48%	7.29%
Fraction of mixed-skilled-1 households	3.73%	4.03%	8.04%
Fraction of mixed-skilled-2 households	9.42%	8.17%	-13.27%
Fraction of low-skilled households	76.16%	76.31%	0.2%
Fertility (high-skilled household)	2.0963	2.1028	0.31%
Fertility (mixed-skilled-1 household)	1.5413	1.5492	0.51%
Fertility (mixed-skilled-2 household)	3.7251	3.7219	-0.09%
Fertility (low-skilled household)	2.6328	2.6203	-0.47%
Average fertility	2.6376	2.6077	-1.13%
Female labor supply (high-skilled household)	0.6856	0.6846	-0.15%
Female labor supply (mixed-skilled-1 household)	0.7688	0.7676	-0.16%
Female labor supply (mixed-skilled-2 household)	0.4412	0.4417	0.11%
Female labor supply (low-skilled household)	0.6051	0.607	0.31%
Fraction of boys getting higher education (high-skilled households)	87.91%	86.39%	-1.73%
Fraction of girls getting higher education (high-skilled households)	75.78%	74.34%	-1.9%
Fraction of boys getting higher education (mixed-skilled-1 household)	66.44%	64.98%	-2.2%
Fraction of girls getting higher education (mixed-skilled-1 household)	55.94%	54.44%	-2.48%
Fraction of boys getting higher education (mixed-skilled-2 household)	60.93%	60.29%	-1.05%
Fraction of girls getting higher education (mixed-skilled-2 household)	50.85%	50.22%	-1.24%
Fraction of boys getting higher education (low-skilled household)	4.06%	4%	-1.48%
Fraction of girls getting higher education (low-skilled household)	0%	1.92%	-
Ratio of skilled females to skilled males	0.7171	0.7894	10.08%
Ratio of average female to male labor supply	0.6044	0.6088	0.73%
Ratio of average female to male income	0.5831	0.5979	2.54%

CHAPTER 4

MARRIAGE MARKET RETURNS AND WOMEN'S EDUCATION

4.1 Introduction

Throughout the developing world, marital decisions are often the result of complex tensions between family, social and individual aspirations. A large percentage of marriages occur through family connections – “consensual arranged” – that prioritize economic security and cultural values such as social norms, ethnicity and family reputation. In fact, only a small percentage of marriages are “self-selected” and “autonomous” (Das Dasgupta, 2008). Existing theories of marriage markets, including the one developed in chapter 2, do not neatly fit into this widely-prevalent marriage market.

In India, while self-selected marriages have been rising, arranged marriages continue to be the dominant form. As recently as 2018, over 90% for young couples in their twenties reported having an arranged marriage (Rukmini, 2021). Of course the institution of arranged marriage itself has changed over the years, notably in giving brides and grooms a greater say in their spousal selection. Still, families and social norms continue to exert a strong influence on the process and a better understanding of the country's persistent gender inequality requires researchers to play close attention to the particularities of that process.

This chapter takes a first step towards formalizing an arranged marriage market that embodies a specific patriarchal value system, the degree to which brides and grooms should be positively assortatively matched on observables such as education. The value system dictates that groom's parents accept a match from bride's family only if the bride is equally- or less-educated than the groom. The chapter also examines a particular aspect of patriarchal norms, one that lies at the intersection of taste-based and market-based sources of discrimination against girls. The market in question is the marriage market, and this chapter explores how parental decisions to invest in girls' education is influenced by expectations of their marriage market outcome. While we do not explicitly model the process of matching in the marriage market, this assumption captures the central feature of the institution of arranged marriage: parenting explicitly accounting for their offspring's future marriage prospects. Andrew and Adams (2022) have recently documented a significant marriage-market returns to girls' education by estimating a dynamic discrete choice model with primary data on a district in Rajasthan, an Indian state. Our model focuses on pan-India data, allowing us

to document general equilibrium effects due to education, and its decomposition into labor market and marriage market returns.

Similar to chapter 2, we construct an overlapping generations model wherein parental educational decisions are lumpy in nature, leading to two discrete levels of human capital – high-type and low-type. Agents still match up exogenously in a marriage market, with the market exhibiting positive assortative matching (PAM) on human capital levels, but there is a key departure from the earlier framework: the marriage market is subject to a patriarchal norm wherein groom’s parents do not accept a match from a more-educated bride’s parents. Spouses in a household work in the labor market and decide about consumption, fertility and children’s education. Parents derive utility from marrying off their offspring, which depend on human capital levels of children and children-in-laws. The subjective marital gains are derived by a two-stage arranged-marriage market search model by extending the framework of Chiplunkar and Weaver (2023). In the search model, subjective marital gains in each stage are divided between the groom’s and bride’s households by Nash bargaining, and dowry solves the Nash bargaining problem.¹

We characterize dynamic equilibrium of the model. The theoretical model is then loosely calibrated using Indian data. Preliminary quantitative results indicate that there are substantial returns to children’s education in the marriage market. Moreover, the returns seem to be higher for women’s education as dowry is assumed to be decreasing in women’s education in the model. Our assumption is supported by Goel and Barua (2023), who have recently estimated dowry to be decreasing in women’s education.

In the future, we plan to extend the presented framework to construct a calibrated dynamic model that will be used to identify the “social returns” of female education, taking into account its effect on marriage formation, marital fertility, labor supply and intergenerational education transmission. We also plan to use the model to study the effect of education subsidies for girls, and compare the results against those derived in previous chapters.

The rest of the chapter is organized as follows. Section 2 presents the model in detail, the

¹We use transferable utility framework to solve the Nash bargaining problem, wherein the marital gains take the form of transferable utility, and dowry determines its division between the groom’s and bride’s households in equilibrium. Chiappori (2020) presents a recent review of transferable utility framework for marriage market.

next section discusses some quantitative analysis and the final section concludes.

4.2 Model

Similar to chapter 2, we construct an intergenerational model with two discrete levels of human capital h – high-type h_H , or low-type h_L . There exist an initial distribution of these human capital levels for men and women. Agents match up exogenously in a marriage market, with a key deviation from the previous framework – the marriage market is subject to a social norm wherein groom’s parents do not accept a match from a more-educated bride’s parents. Various markets of the economy are presented next.

Marriage Market

Marriage market still exhibits positive assortative matching on education. Males and females are denoted by superscripts m and f respectively. The fraction of high-type males to total males is denoted by λ^m , similarly for females (λ^f), and in equilibrium $\lambda^m > \lambda^f$. Unlike in chapter 2, number of males can be greater than females due to son-biased sex-selective abortions. Parameter π captures these preferences, given by the sex ratio (defined as the ratio males to total population). A small fraction of males of type-L remain unmarried so that males and females are equally numerous. This modifies the fraction of males in the marriage market to –

$$\tilde{\lambda}^m = \frac{\pi}{(1 - \pi)} \lambda^m. \quad (4.1)$$

Two levels of human capital for males and females lead to the possibility of four types of matches, HH, HL, LH and LL (with the first position denoting male). The marriage market is subject to a specific gender bias: grooms and their families will not accept a marriage proposal from a bride’s family if the bride is more educated, this means LH match is not feasible. Hence the marriage market in this chapter is similar to the mechanical matching from chapter 2. Let ψ_{kt} denote the fraction of k -type match in period t , where $k \in \{HH, HL, LL\}$. The equations for each type of household are given by,

$$\psi_{HHt} = \lambda_t^f \quad (4.2)$$

$$\psi_{HLt} = \tilde{\lambda}_t^m - \lambda_t^f \quad (4.3)$$

$$\psi_{LLt} = 1 - \tilde{\lambda}_t^m. \quad (4.4)$$

It can be easily verified that $\psi_{HHt} + \psi_{HLt} + \psi_{LLt} = 1$.

Household

The framework is similar to that of chapter 2 in that parents value their own consumption, number of children and child quality in the form of human capital. Human capital still takes the discrete form, but there is a key departure from that framework. In choosing to invest in their children's education, parents take into account how that education affects their future marriage markets, by proposing a search model. While we do not explicitly model the process by which parents choose brides and grooms for their children, this assumption captures the central feature of arranged marriages: parenting explicitly takes into account future marriage market prospects.

Search Model

In this section, we extend the framework presented by Chiplunkar and Weaver (2023) with significant modification of reservation utilities for unmarried agents, which lead to different equilibrium outcomes. We develop a two-stage arranged-marriage market search model, called "early" or "late" marriage based on timing of the match-up. The types of groom and bride are denoted by subscripts i and j respectively, i.e. $i, j \in H, L$.

For the late marriage, reservation utility RU from remaining unmarried (which acts as a threat point) is:

$$RU^r = v^r + f(h^r) \text{ for } r \in \{m, f\} \text{ and } f' > 0, \quad (4.5)$$

where v^r is the (invariant) subjective disutility and $f(h^r)$ is the economic utility. Parents' payoffs for getting offspring married off in period t are given by,

$$U_{ijt} = \gamma(h_i^m)^\alpha(h_j^f)^\beta + d_{ijt} \text{ (Groom's parents)} \quad (4.6)$$

$$V_{ijt} = (1 - \gamma)(h_i^m)^\alpha(h_j^f)^\beta - d_{ijt} \text{ (Bride's parents)}, \quad (4.7)$$

where $(h_i^m)^\alpha(h_j^f)^\beta$ is the subjective marital gains, divided between the two households, with groom's household receiving share γ , and bride's household $(1 - \gamma)$. d is the dowry payment

from bride's family to groom's family, and its value is determined in equilibrium by Nash bargaining. For the late marriage (labeled by period 2), it is determined as follows:

$$\begin{aligned} \text{maximize}_{d_{ij2}} \quad & \left(\gamma(h_i^m)^\alpha(h_j^f)^\beta + d_{ij2} - v^m - f(h_i^m) \right)^\phi \\ & \left((1 - \gamma)(h_i^m)^\alpha(h_j^f)^\beta - d_{ij2} - v^f - f(h_j^f) \right)^{1-\phi}, \end{aligned} \quad (4.8)$$

where ϕ and $(1 - \phi)$ denote the bargaining power of groom's and bride's family respectively. Therefore ϕ can be seen as a "patriarchy" parameter. Solving the optimization problem leads to:

$$d_{ij2} = (\phi - \gamma)(h_i^m)^\alpha(h_j^f)^\beta + (1 - \phi)(v^m + f(h_i^m)) - \phi(v^f + f(h_j^f)) \quad (4.9)$$

$$U_{ij2} = \phi(h_i^m)^\alpha(h_j^f)^\beta + (1 - \phi)(v^m + f(h_i^m)) - \phi(v^f + f(h_j^f)) \quad (4.10)$$

$$V_{ij2} = (1 - \phi)(h_i^m)^\alpha(h_j^f)^\beta - (1 - \phi)(v^m + f(h_i^m)) + \phi(v^f + f(h_j^f)). \quad (4.11)$$

If threat points are zero, for d to be positive, we need $\phi > \gamma$ or $\phi(1 - \gamma) > \gamma(1 - \phi)$. We can think of $\phi(1 - \gamma)$ and $\gamma(1 - \phi)$ as parents' subjective valuation of gains from a son's marriage vs gains from a daughter's marriage. Higher the value of ϕ (i.e. bargaining power of groom's family), higher the probability of $d > 0$. Also higher is $(1 - \gamma)$ (i.e. parents really want to marry off their daughter), higher the possibility of $d > 0$.

It is clear from equation (4.9) that given h_j^f , $\partial d / \partial h_i^m > 0$, i.e. dowry is increasing in groom's human capital levels, conforming with the data. Similarly, dowry is found to be decreasing in bride's education levels. Hence $\partial d / \partial h_i^f < 0$ given h_i^m . For $\beta = 1 - \alpha$, and $f(h) = \delta h$, it requires:

$$\frac{h_j^f}{h_i^m} > \left(\frac{(\phi - \gamma)(1 - \alpha)}{\phi} \right)^{1/\alpha}. \quad (4.12)$$

As RHS can be less than 1, this is possible.

Let's now model the early match-up (denoted by period 1) of the search model. For the early marriage, the Nash bargaining takes the following form:

$$\text{maximize}_{d_{ij1}} \quad (U_{ij1} - EU_{i2})^\phi (V_{ij1} - EV_{j2})^{1-\phi}, \quad (4.13a)$$

where EU_{i2} and EV_{j2} are the groom's and bride's expected utility when they marry "late". Solution to the optimization problem is given below:

$$d_{ij1} = (\phi - \gamma)(h_i^m)^\alpha (h_j^f)^\beta + (1 - \phi)EU_{i2} - \phi EV_{j2} \quad (4.14)$$

$$U_{ij1} = \phi(h_i^m)^\alpha (h_j^f)^\beta + (1 - \phi)EU_{i2} - \phi EV_{j2} \quad (4.15)$$

$$V_{ij1} = (1 - \phi)(h_i^m)^\alpha (h_j^f)^\beta - (1 - \phi)EU_{i2} + \phi EV_{j2}. \quad (4.16)$$

If the fraction of high-type males to total males in period 2 is denoted by λ_2^m and similarly for females (λ_2^f), then EU_{i2} and EV_{j2} are given by,

$$EU_{i2} = \phi(h_i^m)^\alpha \overline{(h_2^f)^\beta} + (1 - \phi)(v^m + f(h_i^m)) - \phi(v^f + \overline{f(h_2^f)}) \quad (4.17)$$

$$EV_{j2} = (1 - \phi)\overline{(h_2^m)^\alpha} (h_j^f)^\beta - (1 - \phi)(v^m + \overline{f(h_2^m)}) + \phi(v^f + f(h_j^f)), \quad (4.18)$$

where

$$\overline{(h_2^f)^\beta} = \lambda_2^f (h_H^f)^\beta + (1 - \lambda_2^f) (h_L^f)^\beta \quad (4.19)$$

$$\overline{f(h_2^f)} = \lambda_2^f f(h_H^f) + (1 - \lambda_2^f) f(h_L^f) \quad (4.20)$$

$$\overline{(h_2^m)^\alpha} = \lambda_2^m (h_H^m)^\alpha + (1 - \lambda_2^m) (h_L^m)^\alpha \quad (4.21)$$

$$\overline{f(h_2^m)} = \lambda_2^m f(h_H^m) + (1 - \lambda_2^m) f(h_L^m). \quad (4.22)$$

The matching takes place early if $U_{ij1} > EU_{i2}$ and $V_{ij1} > EV_{j2}$. Using equations for utility and expected utility, for both the genders, this leads to

$$\begin{aligned} & \phi(h_i^m)^\alpha \left((h_j^f)^\beta - \overline{(h_2^f)^\beta} \right) + (1 - \phi)(h_j^f)^\beta \left((h_i^m)^\alpha - \overline{(h_2^m)^\alpha} \right) \\ & \geq \phi \left(f(h_j^f) - \overline{f(h_2^f)} \right) + (1 - \phi) \left(f(h_i^m) - \overline{f(h_2^m)} \right). \end{aligned} \quad (4.23)$$

We argue that parents of L -type bride prefer to marry her off early due to lower reservation utility for remaining unmarried (note that reservation utility depends on agent's human capital level). Thus LL match-up happens early, which requires,

$$\overline{f(h_2^m)} - f(h_L^m) > (h_L^f)^\beta \left(\overline{(h_2^m)^\alpha} - (h_L^m)^\alpha \right) \quad (4.24)$$

$$\overline{f(h_2^f)} - f(h_L^f) > (h_L^m)^\alpha \left(\overline{(h_2^f)^\beta} - (h_L^f)^\beta \right). \quad (4.25)$$

HH match-up happens in period 2 (late), and this requires

$$f(h_H^m) - \overline{f(h_2^m)} > (h_H^f)^\beta \left((h_H^m)^\alpha - \overline{(h_2^m)^\alpha} \right) \quad (4.26)$$

$$f(h_H^f) - \overline{f(h_2^f)} > (h_H^m)^\alpha \left((h_H^f)^\beta - \overline{(h_2^f)^\beta} \right). \quad (4.27)$$

As these are not contradictory, LL marrying early and HH marrying late are possible. For HL to match up early, as $\lambda_2^m = 1, \lambda_2^f = 1$, the following must hold true:

$$f(h_H^f) - f(h_L^f) > (h_H^m)^\alpha \left((h_H^f)^\beta - (h_L^f)^\beta \right). \quad (4.28)$$

Dowry should be higher for LL -pair for "late" marriage due to lower reservation utility. Using the equations for dowry, we derive

$$\phi(v^f + f(h_L^f)) \geq (1 - \phi)(v^m + f(h_L^m)). \quad (4.29)$$

Consider the following scenario: $f(h) = \delta h$, this leads to LL and HL matching up early and HH matching up late, when δ is large enough². Also, $\lambda_2^m = \lambda_2^f = 1$ and the solution takes the following form:

$$d_{HH} = (\phi - \gamma)(h_H^m)^\alpha (h_H^f)^\beta + (1 - \phi)(v^m + \delta h_H^m) - \phi(v^f + \delta h_H^f) \quad (4.30)$$

$$U_{HH} = \phi(h_H^m)^\alpha (h_H^f)^\beta + (1 - \phi)(v^m + \delta h_H^m) - \phi(v^f + \delta h_H^f) \quad (4.31)$$

$$V_{HH} = (1 - \phi)(h_i^m)^\alpha (h_j^f)^\beta - (1 - \phi)(v^m + \delta h_H^m) + \phi(v^f + \delta h_H^f), \quad (4.32)$$

$$EU_{H2} = \phi(h_H^m)^\alpha (h_H^f)^\beta + (1 - \phi)(v^m + \delta h_H^m) - \phi(v^f + \delta h_H^f) \quad (4.33)$$

$$EU_{L2} = \phi(h_L^m)^\alpha (h_H^f)^\beta + (1 - \phi)(v^m + \delta h_L^m) - \phi(v^f + \delta h_H^f) \quad (4.34)$$

$$EV_{L2} = (1 - \phi)(h_H^m)^\alpha (h_L^f)^\beta - (1 - \phi)(v^m + \delta h_H^m) + \phi(v^f + \delta h_L^f) \quad (4.35)$$

$$(4.36)$$

²Exact value of δ is determined numerically.

$$d_{HL} = (\phi - \gamma)(h_H^m)^\alpha (h_L^f)^\beta + (1 - \phi)EU_{H2} - \phi EV_{L2} \quad (4.37)$$

$$U_{HL} = \phi(h_H^m)^\alpha (h_L^f)^\beta + (1 - \phi)EU_{H2} - \phi EV_{L2} \quad (4.38)$$

$$V_{HL} = (1 - \phi)(h_H^m)^\alpha (h_L^f)^\beta - (1 - \phi)EU_{H2} + \phi EV_{L2}, \quad (4.39)$$

$$d_{LL} = (\phi - \gamma)(h_L^m)^\alpha (h_L^f)^\beta + (1 - \phi)EU_{L2} - \phi EV_{L2} \quad (4.40)$$

$$U_{LL} = \phi(h_L^m)^\alpha (h_L^f)^\beta + (1 - \phi)EU_{L2} - \phi EV_{L2} \quad (4.41)$$

$$V_{LL} = (1 - \phi)(h_L^m)^\alpha (h_L^f)^\beta - (1 - \phi)EU_{L2} + \phi EV_{L2}. \quad (4.42)$$

For dowry to be increasing in ϕ (the intensity of patriarchy), the sufficient condition is $\partial d_{HH}/\partial \phi > 0$. Therefore the following condition needs to be satisfied:

$$-(v^m + v^f) > \delta(h_H^m + h_H^f) - (h_H^m)^\alpha (h_H^f)^\beta. \quad (4.43)$$

Household Decision Problem

A household comprises of a man and a woman, denoted by superscripts m and f . Human capital levels are gender-neutral i.e $h_H^m = h_H^f = h_H$ and $h_L^m = h_L^f = h_L$. There are three types of households, denoted by subscript $k \in \{HH, HL, LL\}$. In period t , the household derives utility from consumption (c_{kt}), number of children (n_{kt}), educational expenditures on boys and girls (e_{kt}^m and e_{kt}^f per child respectively) and expected subjective gains from marrying off their offspring (denoted by $E_t(U_k)$ for sons and $E_t(V_k)$ for daughters). The optimization problem takes the following form:

$$\begin{aligned} & \text{maximize} && \mathbb{U}_{kt} = c_{kt} n_{kt}^\theta (e_{kt}^m)^{\gamma_b} (e_{kt}^f)^{\gamma_g} + \omega_b E_t(U_k) + \omega_g E_t(V_k) && (4.44a) \\ & c_{kt}, n_{kt}, e_{kt}^m, e_{kt}^f \end{aligned}$$

$$\text{subject to} \quad l_{wkt} + \tau n_{kt} = 1 \text{ (female time constraint),} \quad (4.44b)$$

$$c_{kt} + (\pi e_{kt}^m + (1 - \pi) e_{kt}^f + e_{0t}) n_{kt} = w_{kt}^m + w_{kt}^f l_{wkt} \text{ (budget constraint),} \quad (4.44c)$$

$$p_{kt}^m = \text{Prob}(h_{t+1}^m = h_H^m | e_{kt}^m) = 1 - \exp(-\eta_k^m e_{kt}^m) \text{ (son: H-type),} \quad (4.44d)$$

$$p_{kt}^f = \text{Prob}(h_{t+1}^f = h_H^f | e_{kt}^f) = 1 - \exp(-\eta_k^f e_{kt}^f) \quad (\text{daughter: H-type}), \quad (4.44e)$$

taking as given the wage rates for males and females $\{w_{kt}^m, w_{kt}^f\}$. Depending on k ; $w_{kt}^m, w_{kt}^f \in \{w_{Ht}, w_{Lt}\}$. With $\gamma_b > \gamma_g$, we have taste-based discrimination against girls' education, similar to chapter 2. Parameters ω_b and ω_g are the weights on $E_t(U_k)$ and $E_t(V_k)$, and these expected gains are derived from the search model of marriage market. They are expressed as follows:

$$\begin{aligned} E_t(U_k) &\equiv v_b(p_{kt}^m, h_i^m, \bar{h}_j^f) \\ &= p_{kt}^m \underbrace{\left(\frac{\psi_{HH,t+1}}{\psi_{HH,t+1} + \psi_{HL,t+1}} U_{HH} + \frac{\psi_{HL,t+1}}{\psi_{HH,t+1} + \psi_{HL,t+1}} U_{HL} \right)}_A + (1 - p_{kt}^m) \underbrace{\left(\frac{\psi_{LL,t+1}}{\psi_{LL,t+1}} U_{LL} \right)}_B \end{aligned} \quad (4.45)$$

$$\begin{aligned} E_t(V_k) &\equiv v_g(p_{kt}^f, h_j^f, \bar{h}_i^m) \\ &= p_{kt}^f \underbrace{\left(\frac{\psi_{HH,t+1}}{\psi_{HH,t+1}} V_{HH} \right)}_C + (1 - p_{kt}^f) \underbrace{\left(\frac{\psi_{HL,t+1}}{\psi_{HL,t+1} + \psi_{LL,t+1}} V_{HL} + \frac{\psi_{LL,t+1}}{\psi_{HL,t+1} + \psi_{LL,t+1}} V_{LL} \right)}_D. \end{aligned} \quad (4.46)$$

v_b and v_g (where subscripts b and g stand for boys and girls) are functions of probabilities of receiving higher education for boys and girls (p_t^m and p_t^f), and their and their partner's human capital levels (h^m and \bar{h}^f).

Constraint (4.44b) is the time constraint for mothers, with τ being the time-cost of child-rearing (per child) and l_w denoting female labor supply. Budget constraint (4.44c) has the consumption and education expenditure on the left side of the equation, and the household income on the right side. Parameter π is the sex ratio (defined here as the ratio males to total population), resulting from the son-biased fertility preference in developing countries. When $\pi = 0.5$, fertility preferences are gender-neutral and $\pi > 0.5$ indicates fertility preference towards sons. e_0 is the basic educational expenditure for each child, enabling them to be at least of L -type. The next two constraints, (4.44d) and (4.44e), characterize human capital production functions. They represent probabilities of boys and girls acquiring higher education, given educational expenditure (e^m and e^f) and parameters capturing efficiency

of human capital production (η_k^m and η_k^f). These probabilities are monotonically increasing in the educational expenditures.

Solving for the optimization problem leads to the following set of equations:

$$n_{kt} = \frac{\theta(w_{kt}^m + w_{kt}^f)}{(1 + \theta)(\tau w_{kt}^f + e_{0t} + \pi e_{kt}^m + (1 - \pi)e_{kt}^f)} \quad (4.47)$$

$$\begin{aligned} n_{kt}^\theta (e_{kt}^m)^{\gamma_b - 1} (e_{kt}^f)^{\gamma_g} [n_{kt} \gamma_b (\tau w_{kt}^f + e_{0t} + (1 - \pi)e_{kt}^f) + \pi(\gamma_b + 1)n_{kt} e_{kt}^m - \gamma_b (w_{kt}^m + w_{kt}^f)] \\ = \omega_b (A_{t+1} - B_{t+1}) \eta_k^m \exp(-\eta_k^m e_{kt}^m) \end{aligned} \quad (4.48)$$

$$\begin{aligned} n_{kt}^\theta (e_{kt}^m)^{\gamma_b} (e_{kt}^f)^{\gamma_g - 1} [n_{kt} \gamma_g (\tau w_{kt}^f + e_{0t} + \pi e_{kt}^m) + (1 - \pi)(\gamma_g + 1)n_{kt} e_{kt}^f - \gamma_g (w_{kt}^m + w_{kt}^f)] \\ = \omega_g (C_{t+1} - D_{t+1}) \eta_k^f \exp(-\eta_k^f e_{kt}^f). \end{aligned} \quad (4.49)$$

The equations are analytically intractable, and the household decision problem for c , n , e^m and e^f has to be solved numerically.

Production

We assume a linear production function, as follows:

$$Y_t = A_t (h_H L_{Ht} + h_L L_{Lt}), \quad (4.50)$$

where Y_t is output at time t , A_t is the total factor productivity (TFP) growing at an exogenous rate g , and L_{Ht} and L_{Lt} denote the labor supply of H - and L -type in period t . The labor market is competitive, and wage rates for H - and L -type are given by,

$$w_{Ht} = A_t h_H \quad (4.51)$$

$$w_{Lt} = A_t h_L. \quad (4.52)$$

Dynamic Equilibrium

Given initial fractions of high-type to total males and females (λ_0^m and λ_0^f); a competitive equilibrium is a sequence of household decisions $\{c_{kt}, n_{kt}, e_{kt}^m, e_{kt}^f\}_{t=0}^\infty$ for each type of household $k \in \{HH, HL, LL\}$ given by (4.47), (4.48) and (4.49); wage rates w_{Ht}, w_{Lt} from (4.51)

and (4.52); fractions of each types of households $\psi_{HHt}, \psi_{HLt}, \psi_{LLt}$ given by (4.1), (4.2), (4.3) and (4.4); and the evolution of λ_t^m and λ_t^f as follows:

$$\begin{aligned}\lambda_{t+1}^m &= \frac{\sum_{k=HH,HL,LL} n_{kt} P_{kt}^m(\lambda_{t+1}^m, \lambda_{t+1}^f) \psi_{kt}(\lambda_t^m, \lambda_t^f)}{\sum_{k=HH,HL,LL} n_{kt} \psi_{kt}(\lambda_t^m, \lambda_t^f)} \\ &\equiv M(\lambda_t^m, \lambda_t^f, \lambda_{t+1}^f)\end{aligned}\tag{4.53}$$

$$\begin{aligned}\lambda_{t+1}^f &= \frac{\sum_{k=HH,HL,LL} n_{kt} P_{kt}^f(\lambda_{t+1}^m, \lambda_{t+1}^f) \psi_{kt}(\lambda_t^m, \lambda_t^f)}{\sum_{k=HH,HL,LL} n_{kt} \psi_{kt}(\lambda_t^m, \lambda_t^f)} \\ &\equiv K(\lambda_t^m, \lambda_t^f, \lambda_{t+1}^m).\end{aligned}\tag{4.54}$$

Balanced Growth Path

A balanced growth path of the model is a path along which $\lambda_t^m = \lambda_{t+1}^m = \bar{\lambda}^m$ and $\lambda_t^f = \lambda_{t+1}^f = \bar{\lambda}^f$, which is given by the fixed points of equations (4.53) and (4.54).

4.3 Some Quantitative Analysis

Model Simulation

The model is loosely calibrated to match with the Indian data. The low-type agent is defined to be one with up to 10 years of education and the high-type agent has more than 10 years of education. Using the returns to education calculations from chapter 2, the human capital levels are fixed as follows: $h^H = 2.5$ and $h^L = 1$. Parameters from the search model of marriage market are calibrated based on the Rural Economic and Demographic Survey (REDS) in 1999, using the median dowry to annual household income value of 0.5. The parameter choice also conforms with the restrictions imposed by equations (4.12), (4.24), (4.25), (4.26), (4.27), (4.28), (4.29) and (4.43). The parameters are tabulated in table 4.1. Table 4.2 documents the remaining household parameters, chosen to match the ratios of H-type to total men and women ($\lambda^H = 0.1971$ and $\lambda^L = 0.1128$, from the DHS 2005-06).

Simulating the economy at the steady state using the selected parameter values lead to the results as per table 4.3. We can see from the table that the simulated economy replicates λ^m

and λ^f close to the empirical values. The fractions of various types of households also match closely with the data. The simulated fertility and human capital distributions do a fair job of replicating observed distributions, with the model constrained by the unit elasticity of substitution between fertility and human capital investment.

Comparative Statics

In this section, we try to understand the role played by the marriage market in returns to human capital investment. We compare the simulated model with the case where parents do not account for the perceived gains from their offspring's marriage, i.e. $\omega_b = \omega_g = 0^3$. From table 4.4, it can be seen that the fractions of *H*-type males and females drop from 16.76% to 4.63% (72.37% decrease) and 10.28% to 1.45% (85.89% decrease), and the ratio of *H*-type females to males also shows a significant decrease from 0.6134 to 0.3131 (a 48.96% drop). These preliminary results indicate that there are significant returns to education in the marriage market. Additionally, as dowry is decreasing in female education, these returns seem to be higher for women.

4.4 Conclusion

This chapter proposes a framework for arranged marriages by including parents' education decisions which account for children's future marriage prospects. We begin by constructing an intergenerational model with exogenous marriage market matching exhibiting PAM, similar to chapter 2. A key difference is the absence of a household with a low-type groom and a high-type bride, due to patriarchal norms. Parents derive subjective marital gains from matching their offspring, with the gains dependent on the children and their spouses education levels. To rationalize the division of gains, we propose a two-stage arranged-marriage search process, wherein the net subjective payoffs are divided between the groom's and bride's households by the Nash bargaining, with dowry determining the distribution in equilibrium.

³This would make the model similar to chapter 2, but the results are not directly comparable due to different specifications for human capital investment.

We characterize dynamic equilibrium of the model, and it is then loosely calibrated to Indian data. Quantitative analysis further reveals that absence of marriage market returns leads to a drop in the ratio of high-type to total men by 72.37% and the ratio of high-type to total women by 85.39%. As dowry is decreasing in women's education, we see that there is a larger decrease for women. Based on these results, we can conclude that there are significant returns to education in the marriage market, more so for women.

We plan to extend the present framework to derive "social returns" of women's education, taking into consideration its effect on marriage market, labor market, fertility and inter-generational human capital transmission. Our plan also includes using the model to study the effect of education subsidies for girls, and compare the results against those derived in previous chapters. While the quantitative details need to be worked out, it is likely that presence of marriage market returns will increase the effectiveness of education subsidies.

4.5 Tables

Table 4.1
Parameters: Search Model

Parameter	Value
ϕ	0.57
γ	0.45
α	0.5
β	0.5
v^m	-0.57
v^f	-0.6
δ	0.7

Table 4.2
Parameters: Household

Parameter	Value
θ	0.3
γ_b	0.05
γ_g	0.015
ω_b	1.0
ω_g	1.0
τ	0.15
e_0	0.0
π	0.5
η_{HH}^m	5.0
η_{HH}^f	5.0
η_{HL}^m	2.0
η_{HL}^f	2.0
η_{LL}^m	0.8
η_{LL}^f	0.8

Table 4.3
Comparison: Observed and Simulated Values

Variable	Observed Values	Simulated Values
Fraction of H-type to total males (λ^m)	19.71%	16.76%
Fraction of H-type to total females (λ^f)	11.28%	10.28%
Ratio of H-type females to males	0.5724	0.6134
Fraction of HH households	11.28%	10.28%
Fraction of HL households	8.43%	6.48%
Fraction of LL households	80.29%	83.24%
Fertility (HH household)	2.066	2.0471
Fertility (HL household)	3.0752	3.792
Fertility (LL household)	3.5291	2.2804
Average fertility	3.3257	2.3544
Probability for boys getting higher education (HH households)	92.04%	83.28%
Probability for girls getting higher education (HH households)	85.95%	75.98%
Probability for boys getting higher education (HL household)	70.42%	22.42%
Probability for girls getting higher education (HL household)	46.63%	9.66%
Probability for boys getting higher education (LL household)	7.21%	8.65%
Probability for girls getting higher education (LL household)	0.57%	3.08%

Table 4.4
No Utility from Offspring's Marriage

Variable	Baseline Values	$\omega_b =$ $\omega_g = 0$
Fraction of H-type to total males (λ^m)	16.76%	4.63%
Fraction of H-type to total females (λ^f)	11.28%	1.45%
Ratio of H-type females to males	0.6134	0.3131
Fraction of HH households	10.28%	1.45%
Fraction of HL households	8.43%	3.18%
Fraction of LL households	80.29%	95.37%
Fertility (HH household)	2.066	3.0328
Fertility (HL household)	3.0752	5.3074
Fertility (LL household)	3.5291	3.0328
Average fertility	3.3257	3.1051
Probability for boys getting higher education (HH households)	92.04%	45.01%
Probability for girls getting higher education (HH households)	85.95%	16.42%
Probability for boys getting higher education (HL household)	70.42%	9.12%
Probability for girls getting higher education (HL household)	46.63%	2.83%
Probability for boys getting higher education (LL household)	7.21%	3.75%
Probability for girls getting higher education (LL household)	0.57%	1.14%

CHAPTER 5

DISSERTATION CONCLUSION

This dissertation models marriage markets in developing countries. In chapter 2, we develop an intergenerational model of an economy with a novel marriage market matching exhibiting PAM, and households deciding on fertility and children's education. The model features taste-based gender discrimination against girls' education. We then characterize the dynamic equilibrium of the economy, estimate the household parameters with the generalized method of moments using Indian data and numerically solve the model to derive the steady state. Simulation results indicate that the model does a good job of replicating Indian data, especially the skill ratios.

Chapter 3 extends the work in chapter 2 by analyzing dynamics and comparative statics using Indian data. An exogenous increase in PAM takes 4 time periods to be fully effective and worsens inter-household income inequality. Comparative statics results which can lead to weakening of gender bias include increase in returns to education and skilled-labor productivity. Distributional policy experiment, when targeted towards female education, has the desired effect of reducing gender inequality in higher education, skills and income. Other notable positive effects are decrease in fertility, increase in female labor supply and reduction in inter-household income inequality. Policy dynamics leads us to conclude that it takes about 50 years for the economy to converge to the new steady state. In comparison, gender-neutral subsidies are clearly not effective in neutralizing the gender bias in education, mainly due to the fact that they disproportionately benefit boys compared to girls. The analysis underscores the importance of targeted policy interventions to achieve a more gender equal society.

Chapter 4, joint work with Shankha Chakraborty, proposes a framework for arranged marriages, with its central feature – parental education decisions accounting for their offspring's future marriage prospects. We extend the model from chapter 2, with a key difference in the marriage market: absence of a household with a low-type groom and a high-type bride, due to patriarchal norms. To model the division of gains that parents derive from matching their offspring, we propose a two-stage arranged-marriage search model. The gains are distributed by the Nash bargaining, with dowry deciding the distribution in equilibrium. We then characterize dynamic equilibrium of the economy and calibrate the model to Indian data. Further quantitative analysis reveals that there are significant returns to female education in the marriage market.

Taken together, this dissertation fills the gap in macroeconomic literature, which is largely focused on advanced economies. It is achieved by constructing models with features from developing economies such as skill imbalance in marriage market, taste-based discrimination against girls' education, lower levels of bride's education compared to groom in marriage market and institution of arranged marriage. These models are estimated and simulated using Indian data, and analyses underscore the importance of gender-targeted subsidies to counter gender bias, policy dynamics and the role played by education in the marriage market.

APPENDIX A

UTILITY MAXIMIZATION

The household utility maximization problem for each type of household ($j \in (h, m1, m2, l)$) is reproduced below.

$$\begin{aligned} & \underset{c_{jt}, n_{jt}, \hat{a}_{bjt}, \hat{a}_{gjt}}{\text{maximize}} && U_{jt} = \ln c_{jt} + \gamma \ln n_{jt} + \eta(\ln q_{bjt} + \phi \ln q_{gjt}) \end{aligned} \quad (\text{A.1a})$$

$$\text{subject to} \quad l_{wjt} + \tau n_{jt} = 1 \quad (\text{female}) \quad (\text{female time constraint}), \quad (\text{A.1b})$$

$$c_{jt} + 0.5(r_{bjt} + r_{gjt})n_{jt}v_{jt} = w_{jt}^m + w_{jt}^f l_{wjt} \quad (\text{budget constraint}), \quad (\text{A.1c})$$

$$q_{bjt} = r_{bjt}w_{st+1} + (1 - r_{bjt})w_{ut+1} \quad (\text{boys' quality}), \quad (\text{A.1d})$$

$$q_{gjt} = r_{gjt}w_{st+1} + (1 - r_{gjt})w_{ut+1} \quad (\text{girls' quality}), \quad (\text{A.1e})$$

$$r_{bjt} = \frac{\bar{a} - \hat{a}_{bjt}}{\bar{a} - \underline{a}}, \quad (\text{A.1f})$$

$$r_{gjt} = \frac{\bar{a} - \hat{a}_{gjt}}{\bar{a} - \underline{a}}. \quad (\text{A.1g})$$

Rearranging terms of equation (A.1b), we get:

$$l_{wjt} = 1 - \tau n_{jt}. \quad (\text{A.2})$$

Now, using equations (A.2), (A.1f) and (A.1g) in equation (A.1c), and rearranging, we get,

$$c_{jt} = w_{jt}^m + w_{jt}^f - n_{jt} \left(\tau w_{jt}^f + 0.5 \left(\frac{2\bar{a} - \hat{a}_{bjt} - \hat{a}_{gjt}}{\bar{a} - \underline{a}} \right) v_{jt} \right). \quad (\text{A.3})$$

Using equation (A.1f) in equation (A.1d), and simplifying, we get,

$$q_{bjt} = \frac{\bar{a} - \hat{a}_{bjt}}{\bar{a} - \underline{a}} w_{st+1} + \frac{\hat{a}_{bjt} - \underline{a}}{\bar{a} - \underline{a}} w_{ut+1}. \quad (\text{A.4})$$

Using equation (A.1g) in equation (A.1e), and simplifying, we get,

$$q_{gjt} = \frac{\bar{a} - \hat{a}_{gjt}}{\bar{a} - \underline{a}} w_{st+1} + \frac{\hat{a}_{gjt} - \underline{a}}{\bar{a} - \underline{a}} w_{ut+1}. \quad (\text{A.5})$$

Now, using equations (A.3), (A.4), and (A.5) in the objective function (i.e. equation (A.1a)), we rewrite the utility maximization problem as below:

$$\begin{aligned}
& \underset{n_{jt}, \hat{a}_{bjt}, \hat{a}_{gjt}}{\text{maximize}} \quad U_{jt} = \ln \left(w_{jt}^m + w_{jt}^f - n_{jt} \left(\tau w_{jt}^f + 0.5 \left(\frac{2\bar{a} - \hat{a}_{bjt} - \hat{a}_{gjt}}{\bar{a} - \underline{a}} \right) v_{jt} \right) \right) + \gamma \ln n_{jt} \\
& \quad + \eta \left(\ln \left(\frac{\bar{a} - \hat{a}_{bjt}}{\bar{a} - \underline{a}} w_{st+1} + \frac{\hat{a}_{bjt} - \underline{a}}{\bar{a} - \underline{a}} w_{ut+1} \right) \right. \\
& \quad \left. + \phi \ln \left(\frac{\bar{a} - \hat{a}_{gjt}}{\bar{a} - \underline{a}} w_{st+1} + \frac{\hat{a}_{gjt} - \underline{a}}{\bar{a} - \underline{a}} w_{ut+1} \right) \right).
\end{aligned} \tag{A.6}$$

Taking the first order condition of the above objective function with respect to n_{jt} leads to:

$$\frac{- \left(\tau w_{jt}^f + 0.5 \left(\frac{2\bar{a} - \hat{a}_{bjt} - \hat{a}_{gjt}}{\bar{a} - \underline{a}} \right) v_{jt} \right)}{\left(w_{jt}^m + w_{jt}^f - n_{jt} \left(\tau w_{jt}^f + 0.5 \left(\frac{2\bar{a} - \hat{a}_{bjt} - \hat{a}_{gjt}}{\bar{a} - \underline{a}} \right) v_{jt} \right) \right)} + \frac{\gamma}{n_{jt}} = 0. \tag{A.7}$$

Taking the first order condition of the above objective function with respect to \hat{a}_{bjt} leads to:

$$\frac{\frac{0.5n_{jt}v_{jt}}{\bar{a} - \underline{a}}}{\left(w_{jt}^m + w_{jt}^f - n_{jt} \left(\tau w_{jt}^f + 0.5 \left(\frac{2\bar{a} - \hat{a}_{bjt} - \hat{a}_{gjt}}{\bar{a} - \underline{a}} \right) v_{jt} \right) \right)} + \frac{\eta \frac{-w_{st+1} + w_{ut+1}}{\bar{a} - \underline{a}}}{\left(\frac{\bar{a} - \hat{a}_{bjt}}{\bar{a} - \underline{a}} w_{st+1} + \frac{\hat{a}_{bjt} - \underline{a}}{\bar{a} - \underline{a}} w_{ut+1} \right)} = 0. \tag{A.8}$$

Taking the first order condition of the above objective function with respect to \hat{a}_{gjt} leads to:

$$\frac{\frac{0.5n_{jt}v_{jt}}{\bar{a} - \underline{a}}}{\left(w_{jt}^m + w_{jt}^f - n_{jt} \left(\tau w_{jt}^f + 0.5 \left(\frac{2\bar{a} - \hat{a}_{bjt} - \hat{a}_{gjt}}{\bar{a} - \underline{a}} \right) v_{jt} \right) \right)} + \frac{\eta \phi \frac{-w_{st+1} + w_{ut+1}}{\bar{a} - \underline{a}}}{\left(\frac{\bar{a} - \hat{a}_{gjt}}{\bar{a} - \underline{a}} w_{st+1} + \frac{\hat{a}_{gjt} - \underline{a}}{\bar{a} - \underline{a}} w_{ut+1} \right)} = 0. \tag{A.9}$$

We have a system of three equations viz. (A.7), (A.8) and (A.9), in three unknowns $n_{jt}, \hat{a}_{bjt}, \hat{a}_{gjt}$. Solving for the system of equations leads to the following results:

$$n_{jt} = \frac{(w_{jt}^m + w_{jt}^f)(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)}{(1 + \gamma)(\tau w_{jt}^f(w_{st+1} - w_{ut+1}) - v_{jt}w_{ut+1})} \tag{A.10}$$

$$\hat{a}_{bjt} = \frac{v_{jt}(\bar{a}w_{st+1} - \underline{a}w_{ut+1})(\gamma + \eta - \phi\eta) - 2\eta(w_{st+1} - w_{ut+1})(\tau w_{jt}^f(\bar{a} - \underline{a}) + \bar{a}v_{jt})}{v_{jt}(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)} \tag{A.11}$$

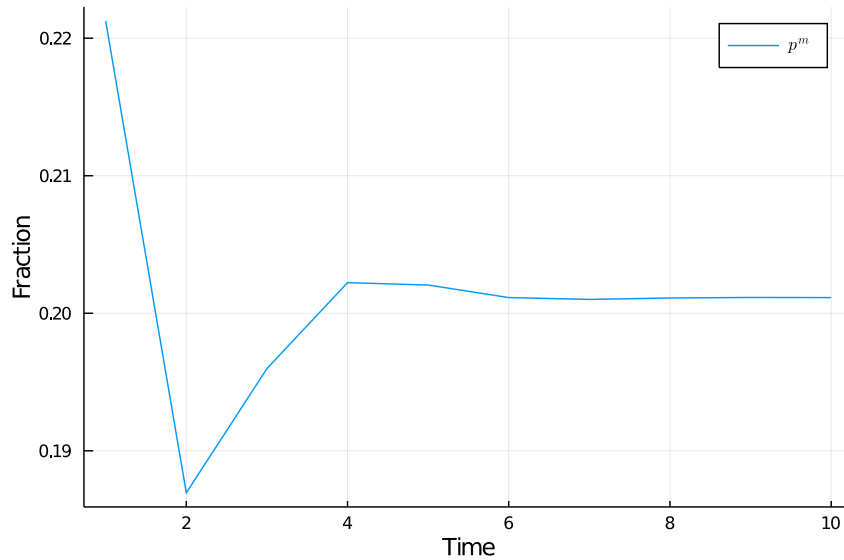
$$\hat{a}_{gjt} = \frac{v_{jt}(\bar{a}w_{st+1} - \underline{a}w_{ut+1})(\gamma - \eta + \phi\eta) - 2\eta\phi(w_{st+1} - w_{ut+1})(\tau w_{jt}^f(\bar{a} - \underline{a}) + \bar{a}v_{jt})}{v_{jt}(w_{st+1} - w_{ut+1})(\gamma - \eta - \phi\eta)}. \quad (\text{A.12})$$

APPENDIX B

DYNAMICS: THE CASE OF DEVIATION OF 10% ABOVE THE STEADY STATE

Dynamics when the economy starts below the steady state with a deviation of 10% for p^m , p^f and β , i.e $p_0^m = 0.2212$, $p_0^f = 0.1586$ and $\beta_0 = 0.2078$ are discussed in this section. Figures B.1, B.2 and B.3 represent time paths for p_t^m , p_t^f and β_t respectively. Similar to -10% deviation, the gap between the deviated value and the steady state value reduces to less than 10% in 3 time periods, which means it takes a period of 75 years for the economy to converge to the steady state when starting with a deviation of 0.02212 percentage point for p^m and 0.01586 percentage point for p^f .

Figure B.1
Time path for p^m : +10% Deviation



The steady-state diagrams for p_t^m , p_t^f and β_t are shown in figures B.4, B.5 and B.6. The economy follows a spiral sink path, as can be seen from the steady-state diagrams.

Figure B.2
Time path for p^f : +10% Deviation

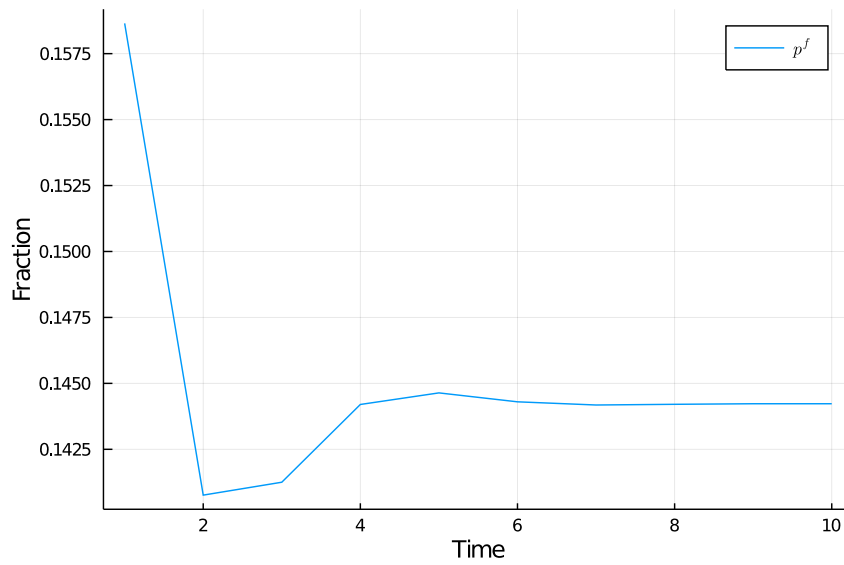


Figure B.3
Time path for β : +10% Deviation

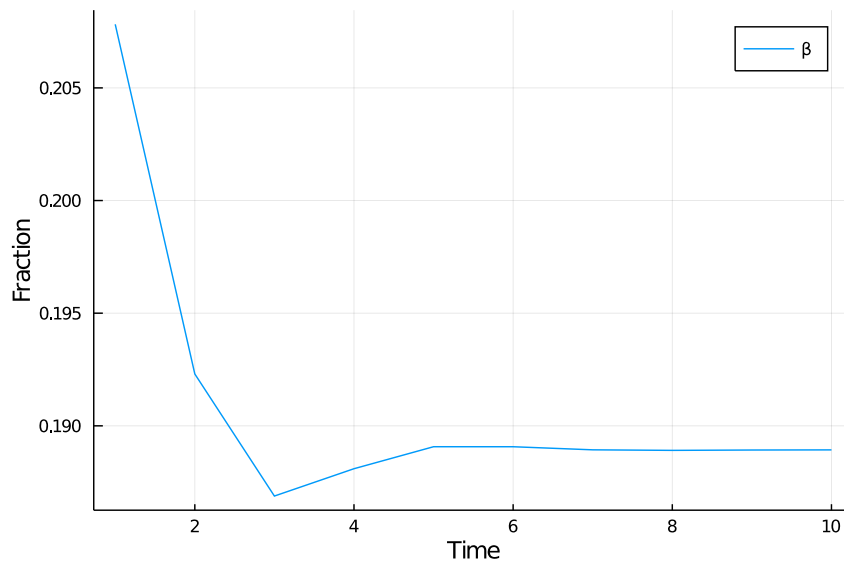


Figure B.4
Steady-state Diagram for p^m : +10% Deviation

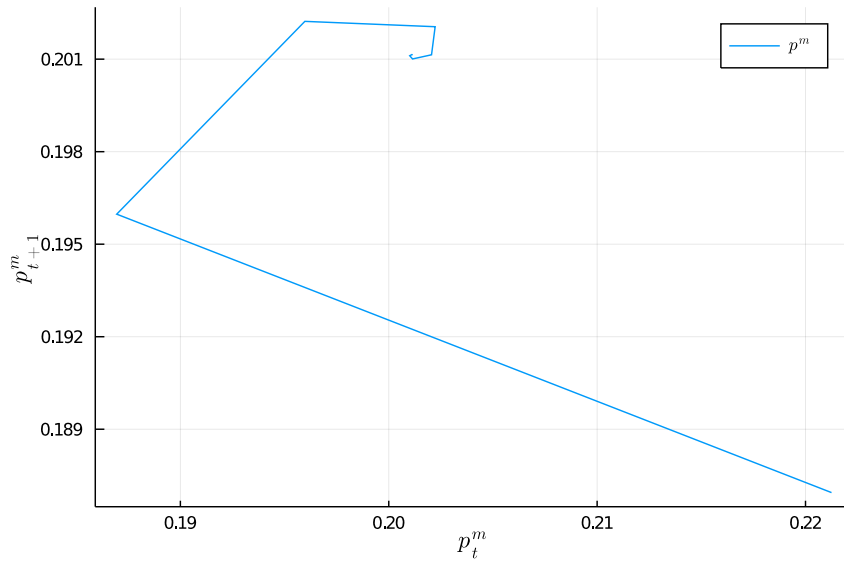


Figure B.5
Steady-state Diagram for p^f : +10% Deviation

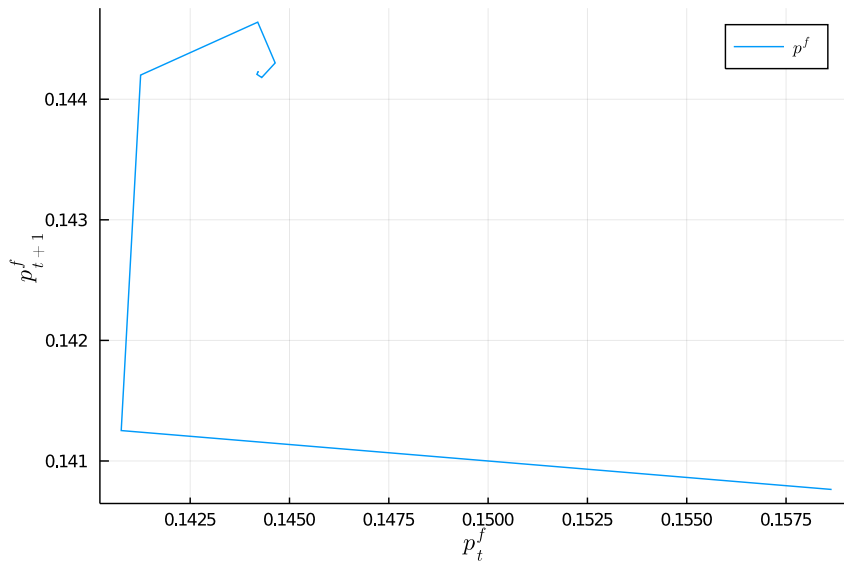
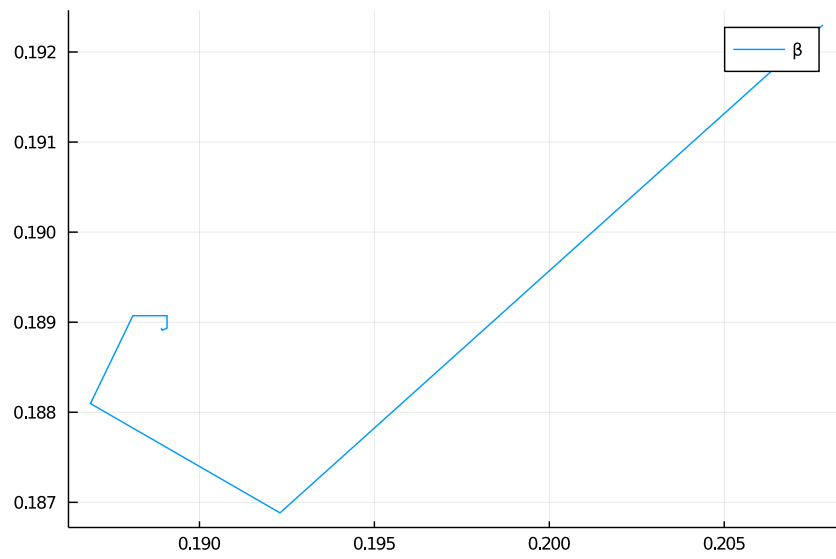


Figure B.6
Steady-state Diagram for β : +10% Deviation



APPENDIX C

COMPARATIVE DYNAMICS: INCREASE IN MARITAL SORTING

The time paths and steady-state diagrams when α changes from 0.3234 to 0.2 are presented from figures C.1 to C.6.

Figure C.1
Time path for p^m : Increase in Marital Sorting

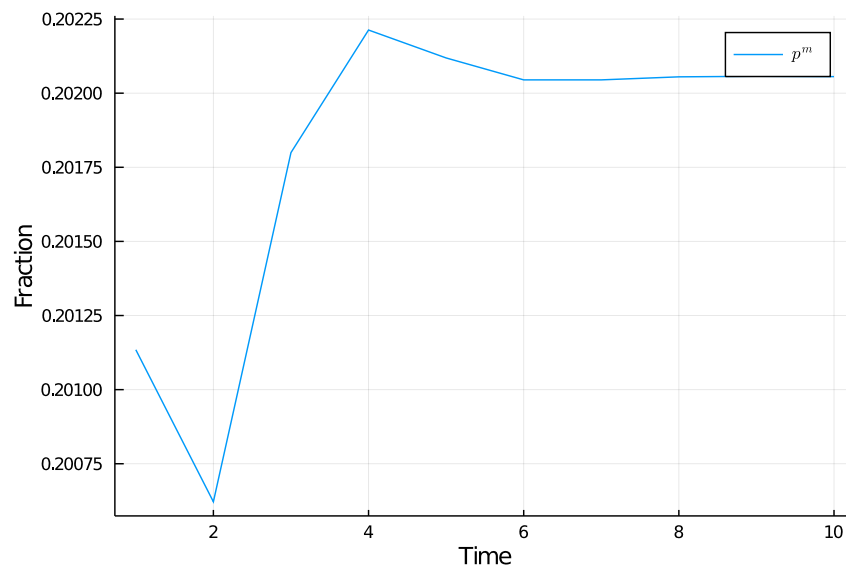


Figure C.2
Time path for p^f : Increase in Marital Sorting

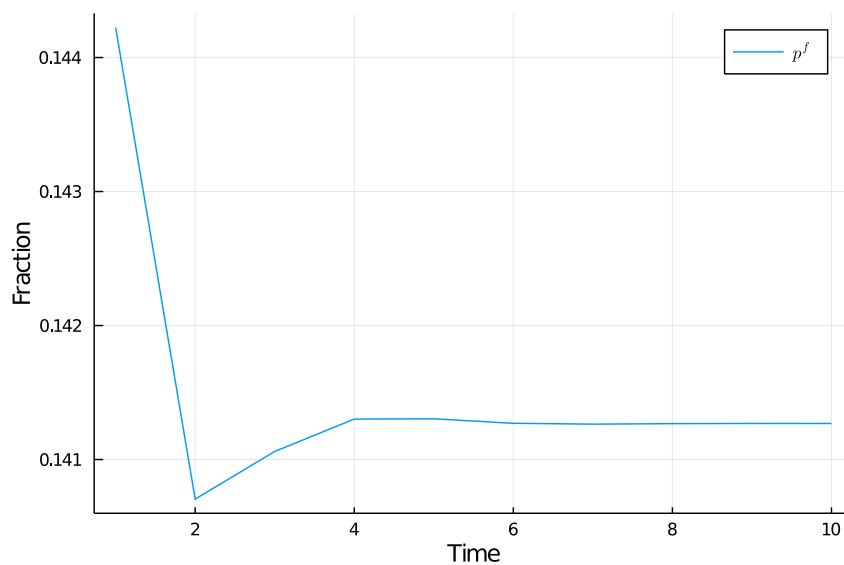


Figure C.3
Time path for β : Increase in Marital Sorting

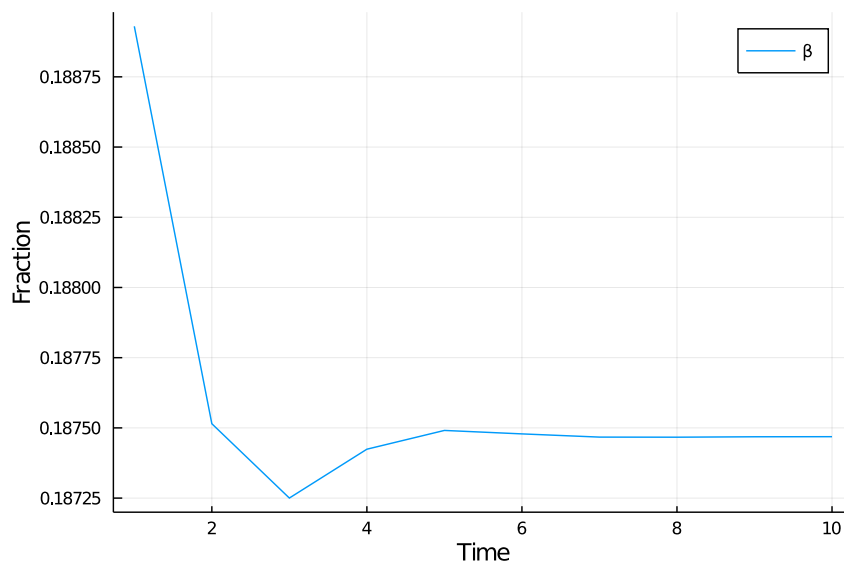


Figure C.4
Steady-state Diagram for p^m : Increase in Marital Sorting

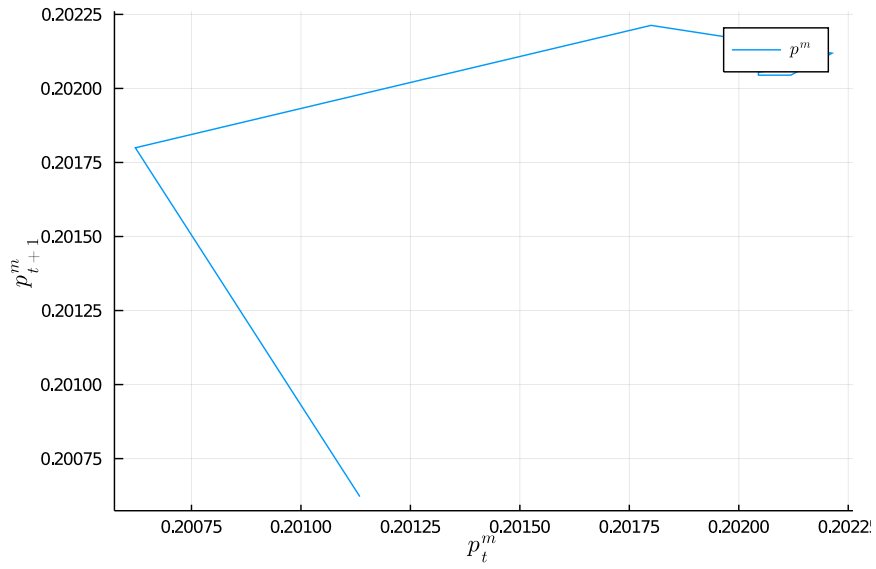


Figure C.5
Steady-state Diagram for p^f : Increase in Marital Sorting

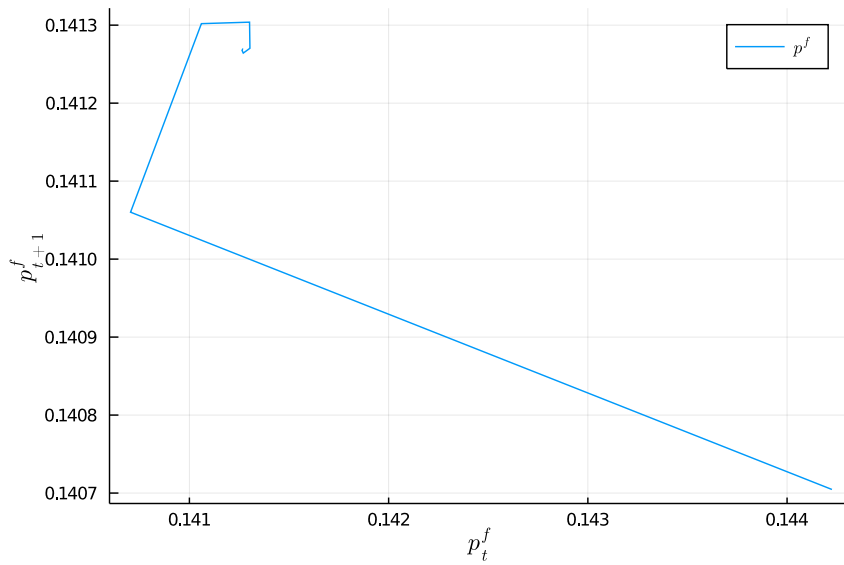
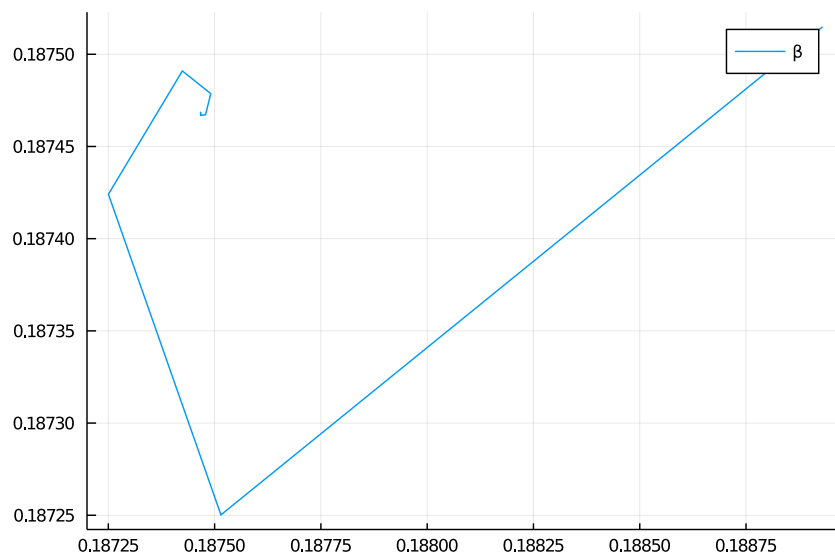


Figure C.6
Steady-state Diagram for β : Increase in Marital Sorting



APPENDIX D

COMPARATIVE DYNAMICS: INCREASE IN RETURNS TO EDUCATION

Figures D.1 to D.6 show results from comparative dynamic analysis when μ increases from 0.5357 to 0.6.

Figure D.1
Time path for p^m : Increase in Returns to Education

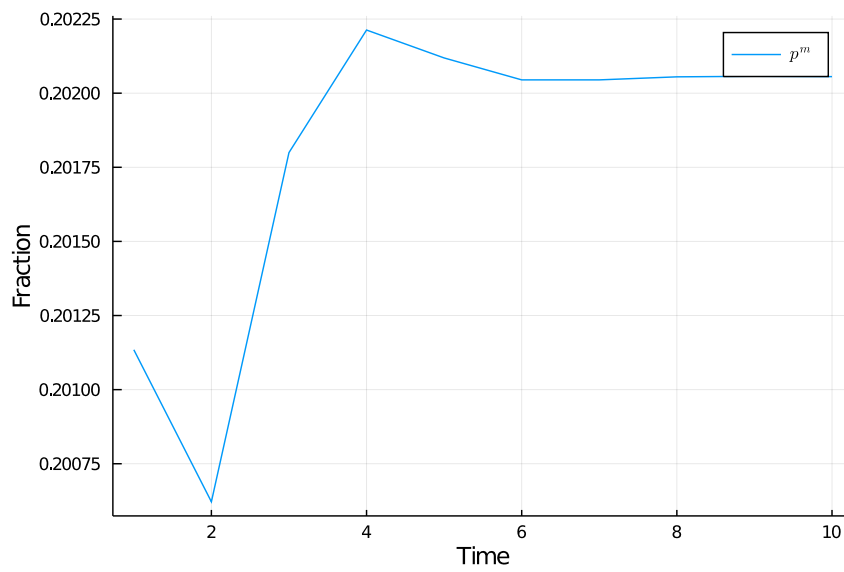


Figure D.2
Time path for p^f : Increase in Returns to Education

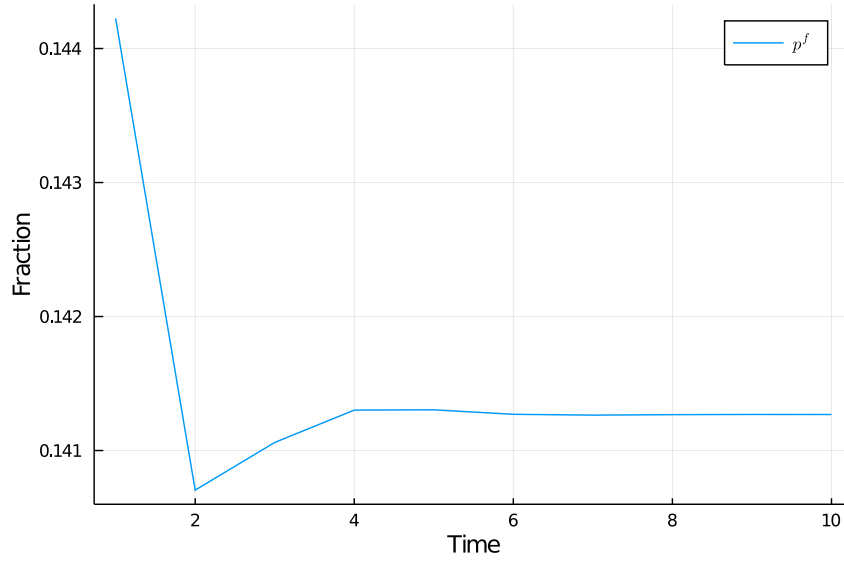


Figure D.3
Time path for β : Increase in Returns to Education

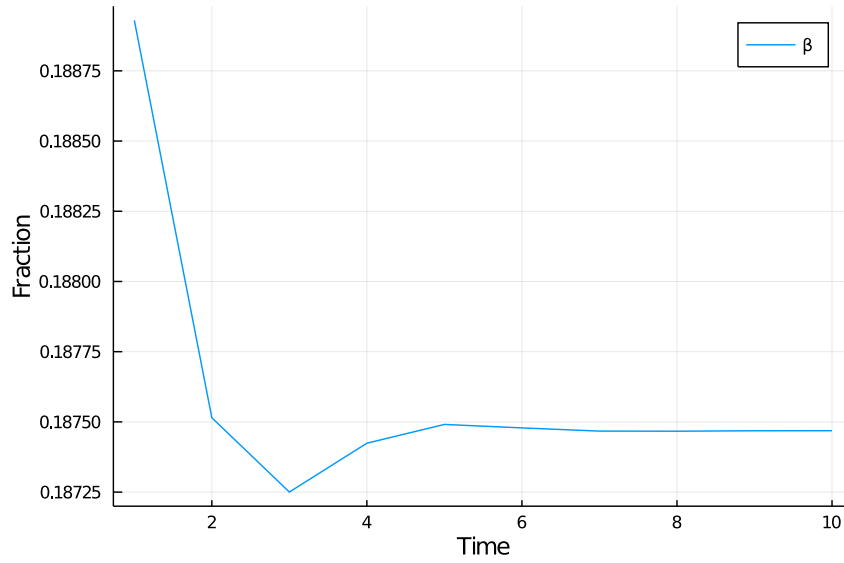


Figure D.4
Steady-state Diagram for p^m : Increase in Returns to Education

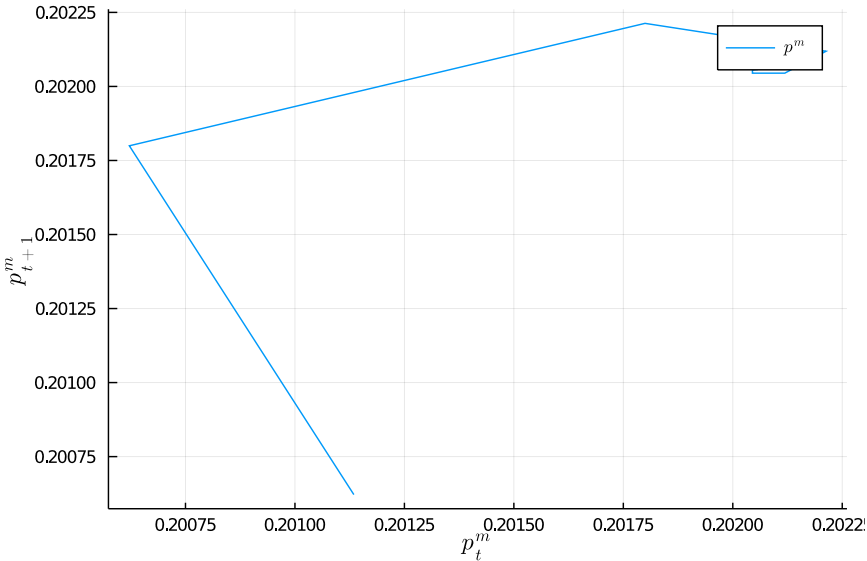


Figure D.5
Steady-state Diagram for p^f : Increase in Returns to Education

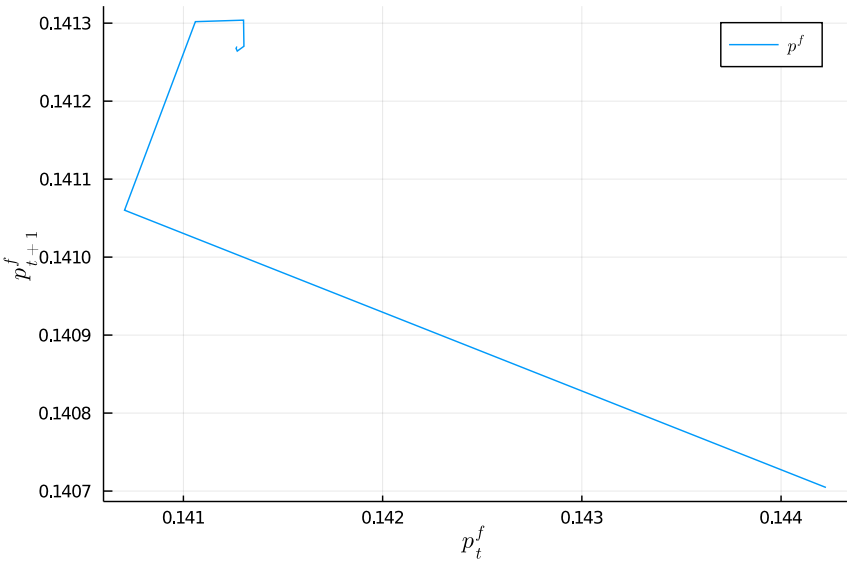
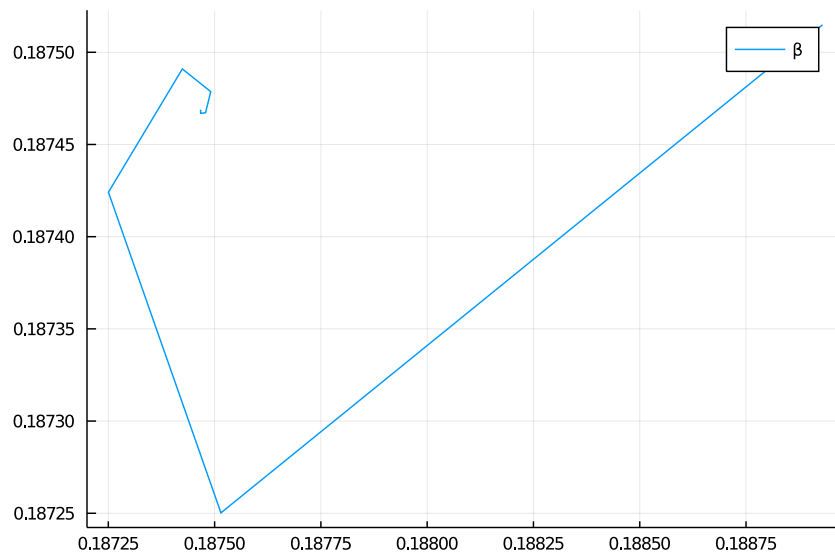


Figure D.6
Steady-state Diagram for β : Increase in Returns to Education



APPENDIX E

COMPARATIVE DYNAMICS: INCREASE IN SKILLED-LABOR PRODUCTIVITY

Comparative dynamics are presented in figures E.1 to E.6 when skilled-labor productivity increases by 10%.

Figure E.1
Time path for p^m : Increase in Skilled-Labor Productivity

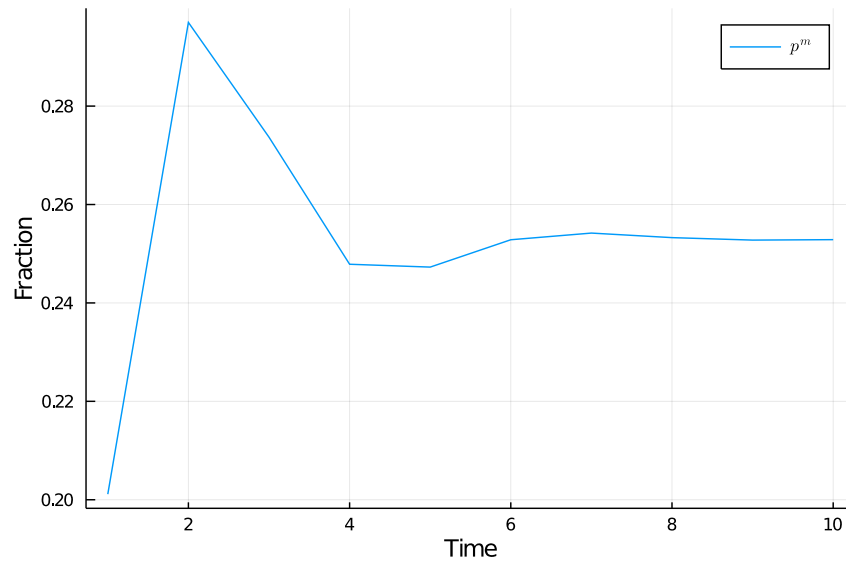


Figure E.2
Time path for p^f : Increase in Skilled-Labor Productivity

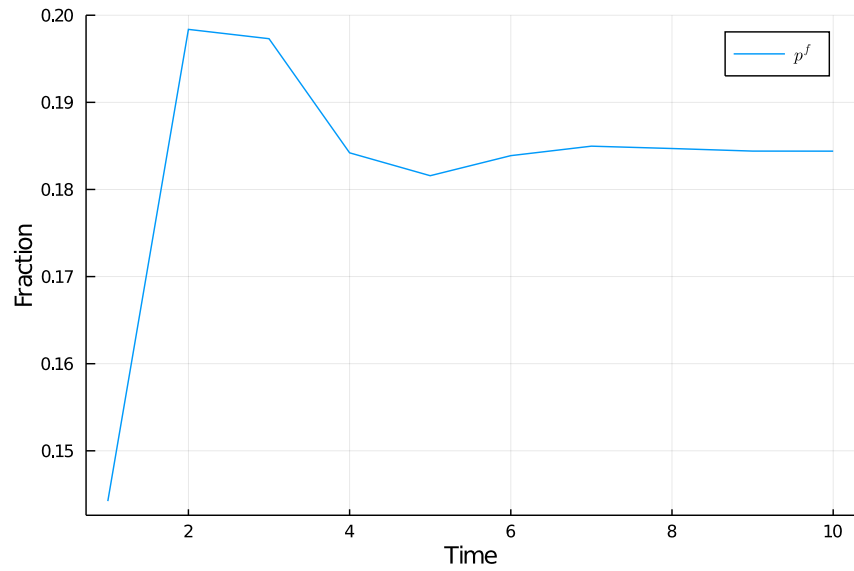


Figure E.3
Time path for β : Increase in Skilled-Labor Productivity

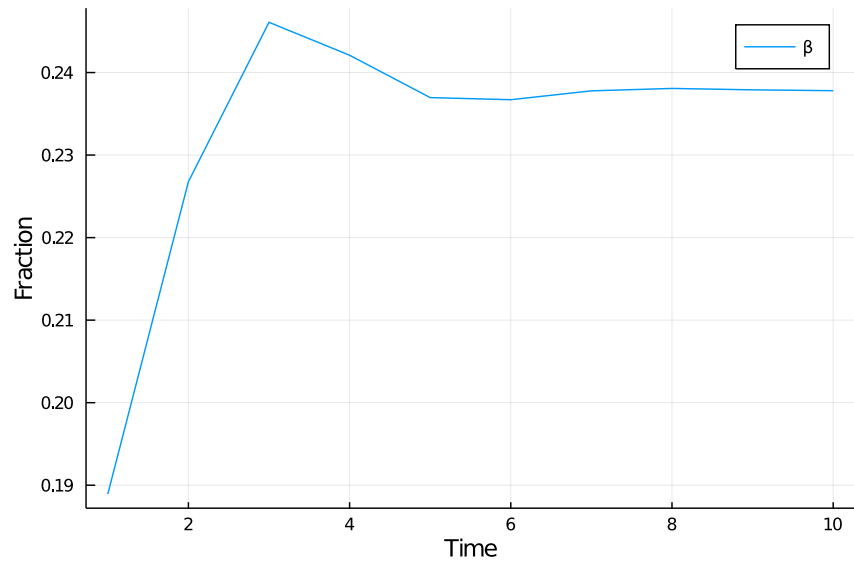


Figure E.4
Steady-state Diagram for p^m : Increase in Skilled-Labor Productivity

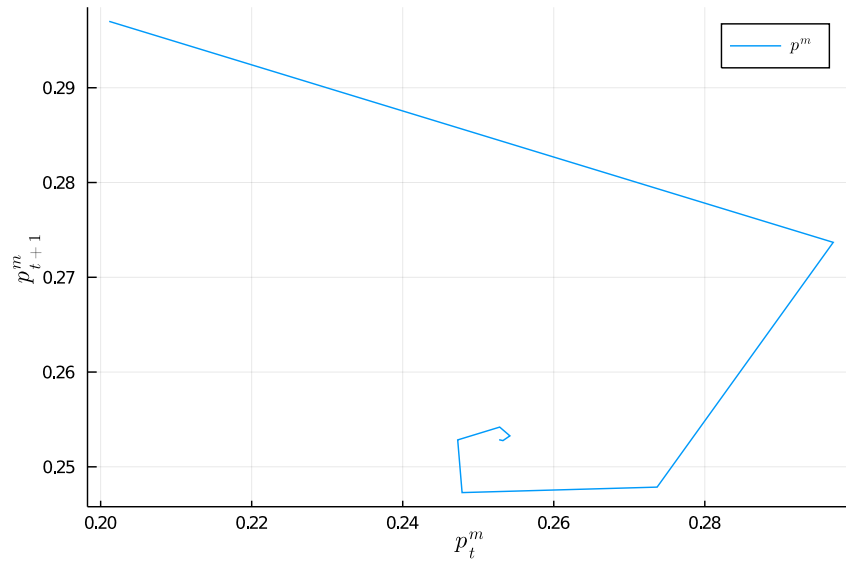


Figure E.5
Steady-state Diagram for p^f : Increase in Skilled-Labor Productivity

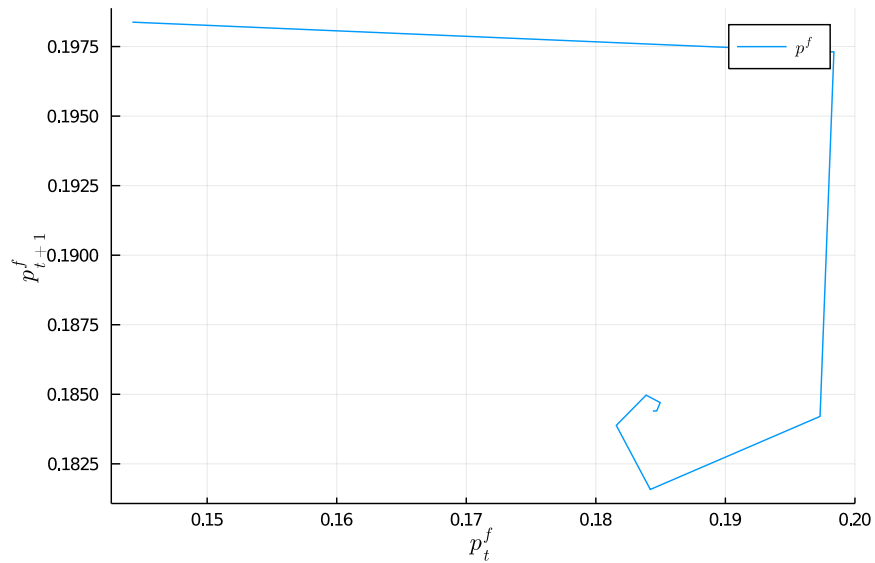
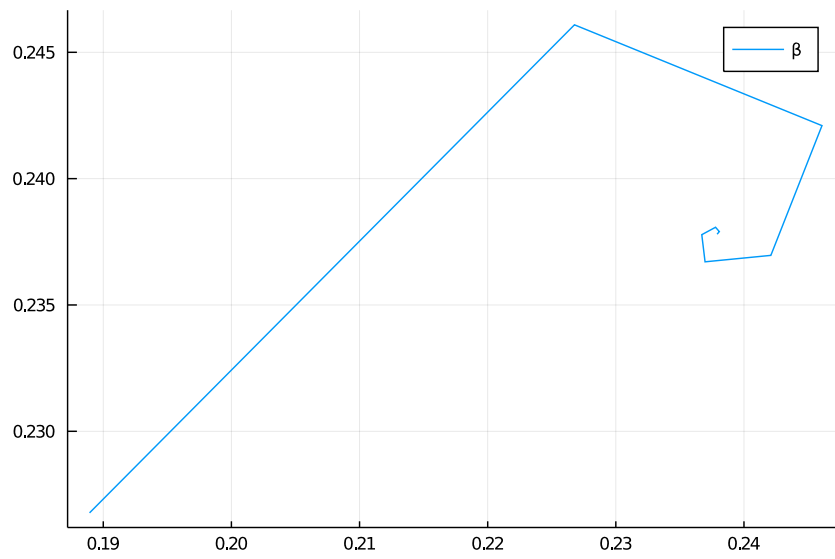


Figure E.6
Steady-state Diagram for β : Increase in Skilled-Labor Productivity



APPENDIX F

COMPARATIVE DYNAMICS: INCREASE IN UNSKILLED-LABOR PRODUCTIVITY

Time paths and steady-state diagrams for the case of increase in unskilled-labor productivity by 10% are shown in figures F.1 to F.6.

Figure F.1
Time path for p^m : Increase in Unskilled-Labor Productivity

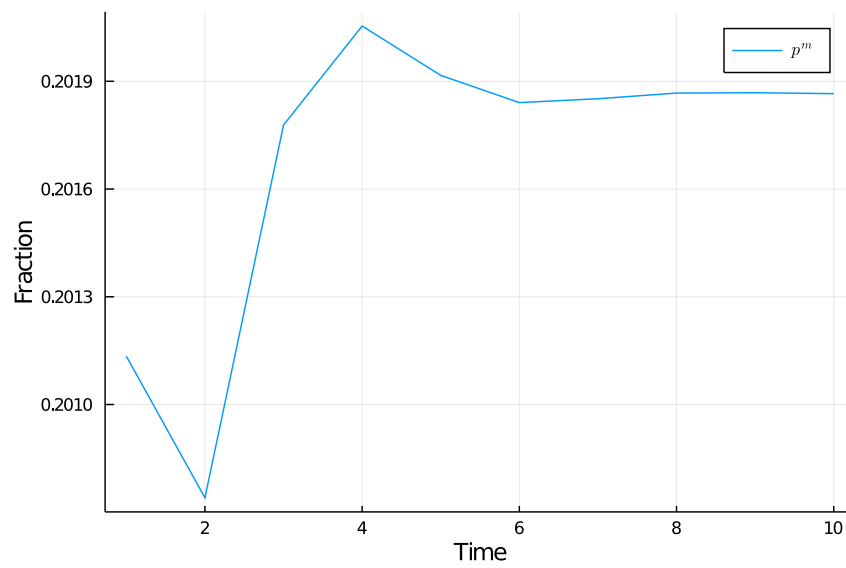


Figure F.2
Time path for p^f : Increase in Unskilled-Labor Productivity

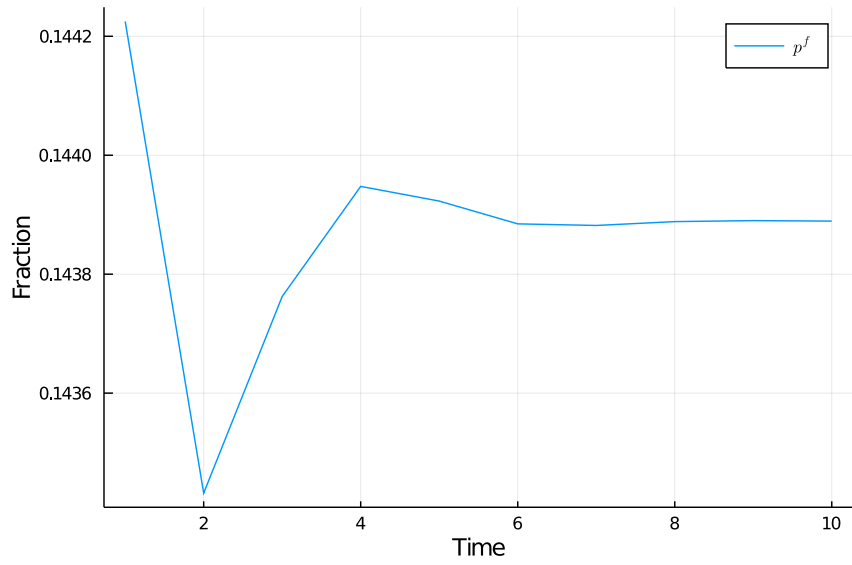


Figure F.3
Time path for β : Increase in Unskilled-Labor Productivity

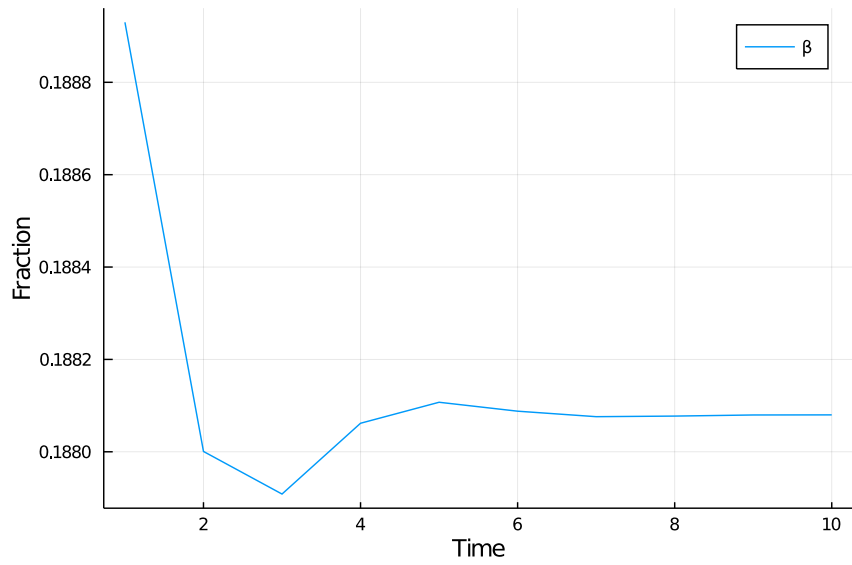


Figure F.4
Steady-state Diagram for p^m : Increase in Unskilled-Labor Productivity

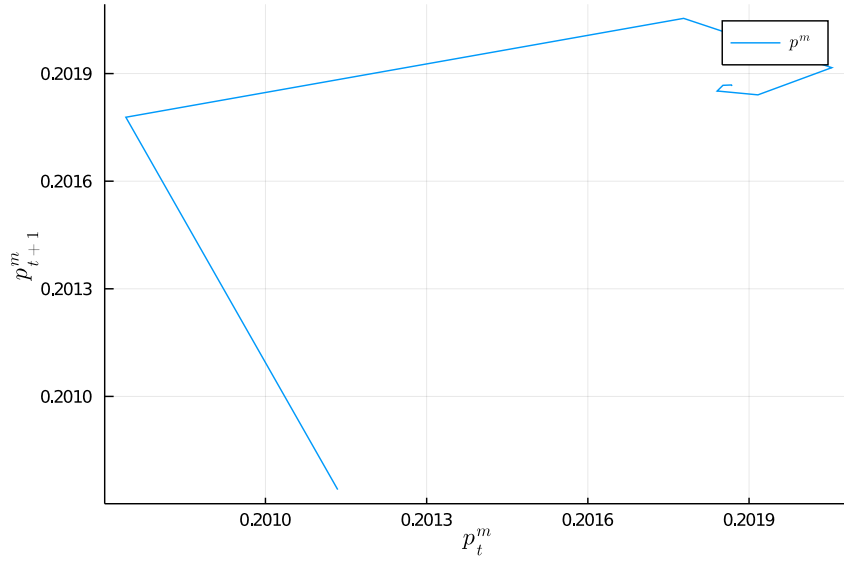


Figure F.5
Steady-state Diagram for p^f : Increase in Unskilled-Labor Productivity

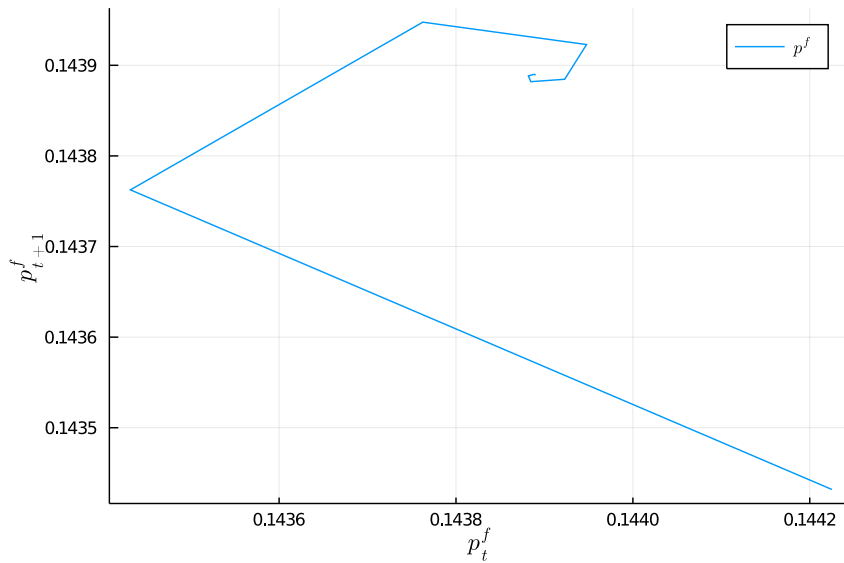
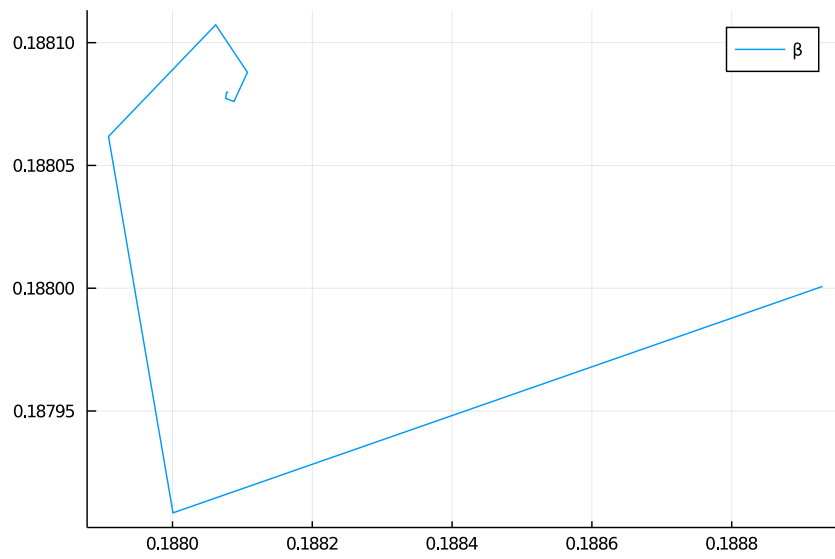


Figure F.6
Steady-state Diagram for β : Increase in Unskilled-Labor Productivity



APPENDIX G

COMPARATIVE DYNAMICS: DECREASE IN THE COST OF EDUCATION

As the cost of education for low-skilled household decreases from 0.1252 to 0.11894 (a 5% decrease), the comparative dynamics resulting from change in the steady state are given in figures G.1 to G.6.

Figure G.1
Time path for p^m : Decrease in the Cost of Education

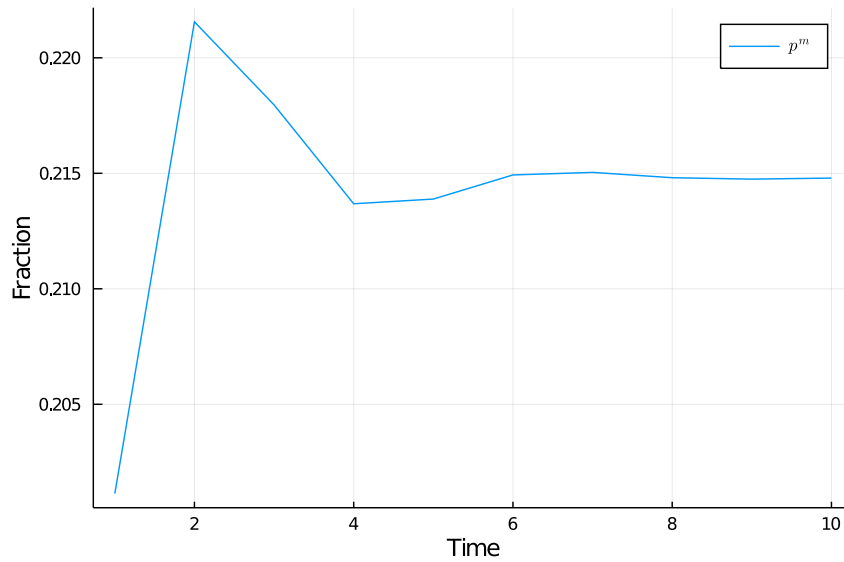


Figure G.2
Time path for p^f : Decrease in the Cost of Education

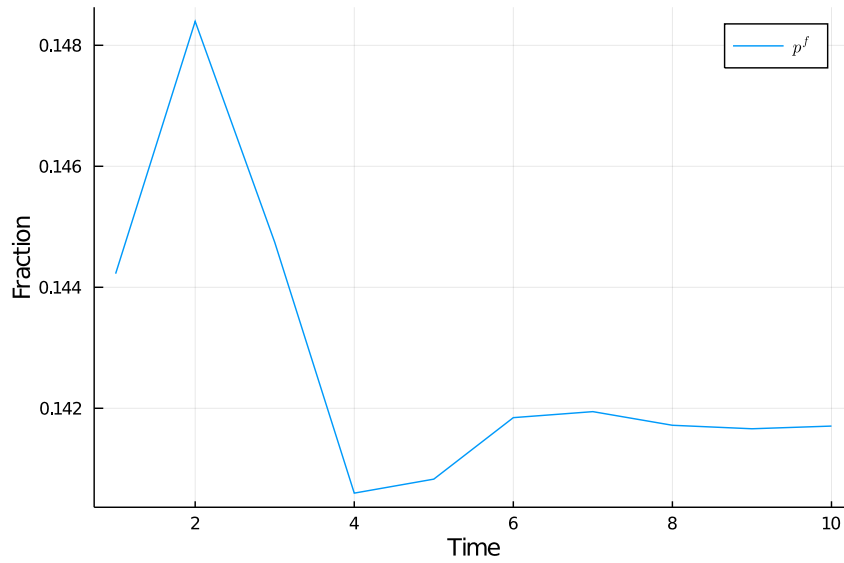


Figure G.3
Time path for β : Decrease in the Cost of Education

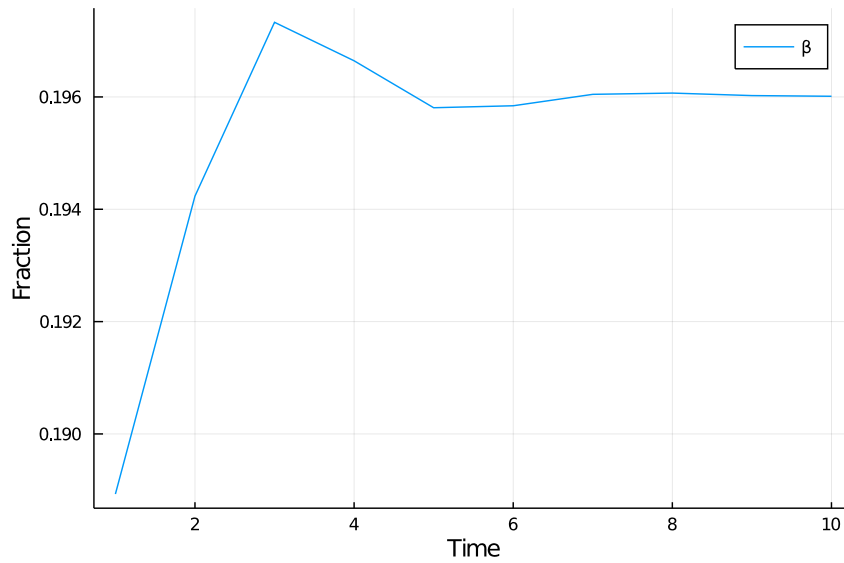


Figure G.4
Steady-state Diagram for p^m : Decrease in the Cost of Education

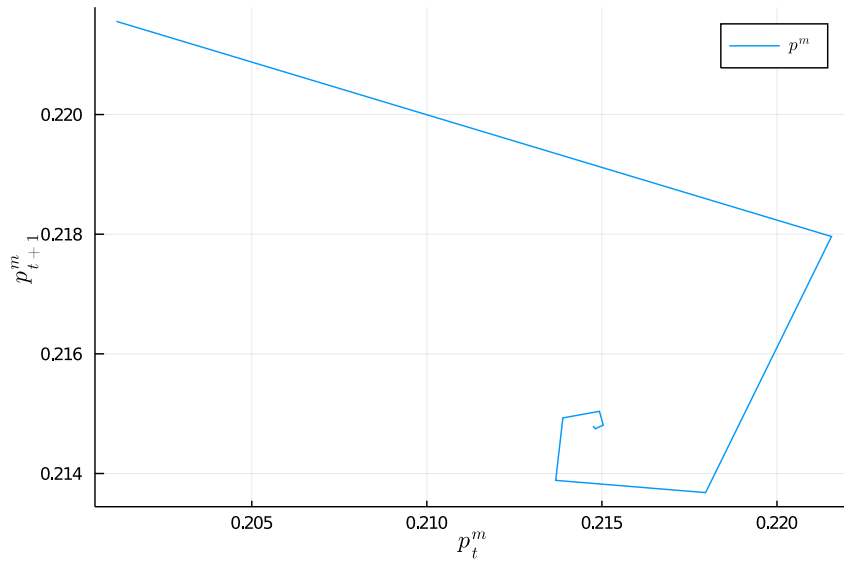


Figure G.5
Steady-state Diagram for p^f : Decrease in the Cost of Education

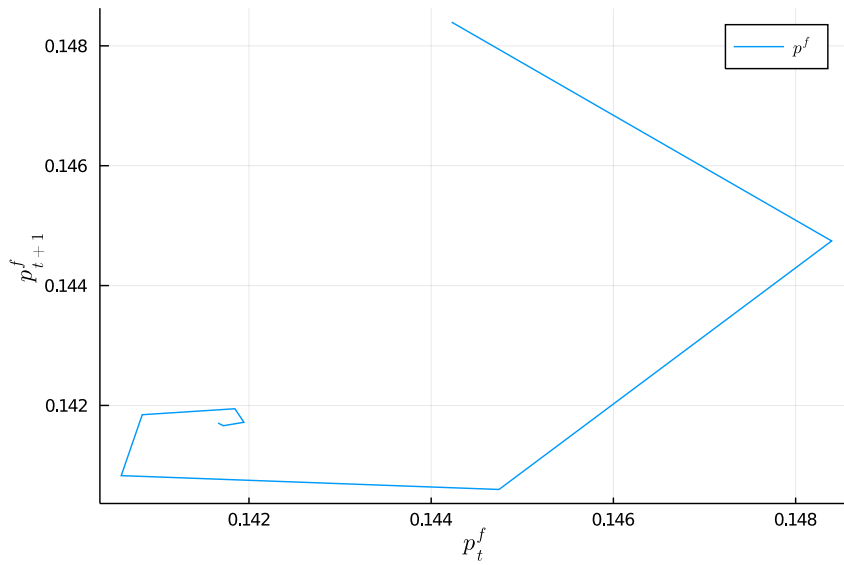
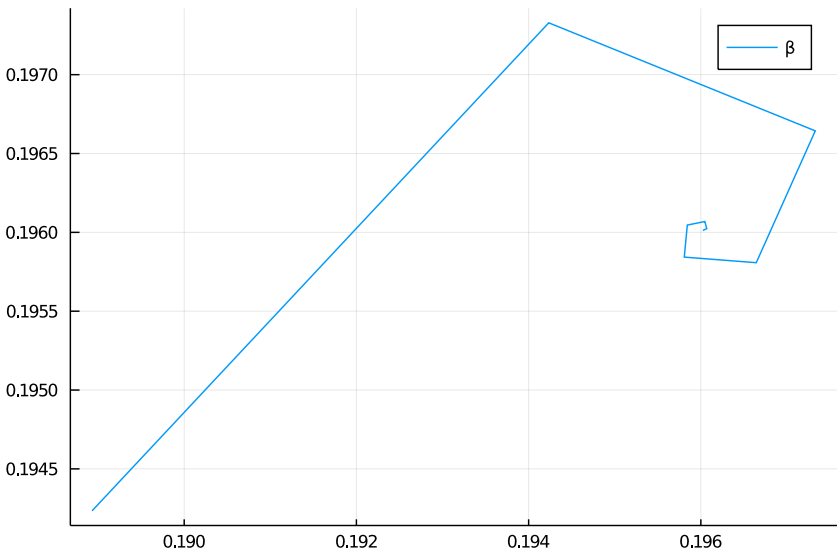


Figure G.6
Steady-state Diagram for β : Decrease in the Cost of Education



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