A Mechanism for Inducing Cooperation in Non-Cooperative Environments: Theory and Applications.

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February 2000.

Abstract

We construct a market based mechanism that induces players in a non-cooperative game to make the same choices as characterize cooperation. We then argue that this mechanism is applicable to a wide range of economic questions and illustrate this claim using the problems of "The Tragedy of the Commons" and "R&D Spillovers in Duopoly".

JEL: .C72, D62, H40.
Keywords: Cooperation, Externalities, Spillovers, E¢ciency.

For many helpful discussions and comments we wish to thank Emilson Silva, Bill Harbaugh, Richard Cornes, Art Snow, Jurgen Eichberger, and the participants of the Public Economic Theory Conference held at the University of Alabama in May 1998.

1 Introduction.

There are many examples in economics where, in either a Paratian or welfare sense, cooperative behavior by economic agents yields outcomes that are signi…-cantly superior to those achieved under non-cooperation. However, the problem that is frequently encountered is that cooperation cannot be straightforwardly sustained in a non-cooperative environment. When others are behaving cooperatively non-cooperative behavior often yields an individual agent signi…cant gains. Perhaps the most well-known example of this problem is the prisoners' dilemma, although any economic activity that generates external e¤ects, or has some degree of "publicness" attached to it typically su¤ers from similar problems. There are of course many well known methods by which agents may be induced to internalize an externality, or achieve an e¢cient allocation of goods some of which are characterized by a degree of publicness. These solutions include; (i) Pigouvian taxes or subsidies; (ii) quantity rationing; (iii) tradable quotas; (iv) limiting the number of participants in an activity or market; (v) creating appropriate private property rights; and (vi) promoting collusive welfare maximization. Each of these solutions work in the right circumstances[1]. However, each has well-known problems.

The purpose of this paper is to propose a new market based mechanism that induces rational self-interested agents in a non-cooperative environment to make the same choices as characterize the cooperative outcome. The mechanism, once introduced, does not promote overt cooperation, no explicit collusion or joint decision making takes place, rather it provides the appropriate incentives for non-cooperative agents to act exactly as if such activities had occurred. The intuition behind the mechanism we propose is straightforward. Suppose we view an economic activity as a game in which a number of players choose actions which then generate payo¤s according to a given payo¤ matrix. If the mechanism has the characteristic that each player's …nal individual payo¤ is strictly increasing in the sum of the payo¤s from the economic activity, then each has an incentive to choose the action that is jointly maximizing. Thus each behaves as if they are cooperating. To construct a mechanism that can achieve this result is quite simple to do. Suppose we view the economic activity as a two stage game[2]. In the …rst stage the players make choices that generate payo¤s, these payo¤s are then redistributed in the second stage according to some prespeci…ed mechanism. Clearly any mechanism with the property that each player recieves a proportion of the total payo¤ will generate the incentives necessary for total payo¤ maximization in the …rst stage. The purpose of this paper is to propose one speci…c mechanism that has this requisite property. The mechanism we propose is not operated by an outside party determining a redistribution of the payo¤s. Rather, it is a game played by agents who

[1] See for example the discussion in Cornes and Sandler [2].

[2] Guttman and Schnytzer [6] propose that reciprocal externality problems may be solved in a two stage strategic matching game. In the …rst stage the players precommit to matching rates that linearly link their externality causing actions. In the second they choose the actions. Both stages are played non-cooperatively. The resulting allocation is Pareto e¢cient.

are trying to 'claim' as much of the payo¤s as possible. We argue that non-cooperative agents may be given the incentives to make the cooperative choices by the creation of a new type of property right, termed a "pooling property right". The key characteristic of a pooling property right is that it may be used to claim a share of any payo¤ in a game. This might work as follows. Consider a two stage game in which two players each hold pooling property rights. In the …rst stage the players play a classic prisoners' dilemma. In the second stage the players play a non-cooperative game in which they allocate their pooling property rights to the payo¤s generated in stage 1. Each payo¤ is then redistributed between the players in the same proportions as the share of the property right they have allocated to that payo¤. We are able to show that the Nash equilibrium in this second stage of the game has the property of proportionality. It follows that in a subgame perfect equilibrium the players have an incentive to maximize total payo¤s in the …rst stage. The outcome described as cooperative in a one-shot prisoners dilemma game becomes a non-cooperative equilibrium in this two stage game.

We concentrate on pooling property rights as the mechanism for achieving the cooperative outcome both because of their theoretical appeal, and because they may have a wide range of policy implications. Theoretically any problem of externalities may be thought of as arising because there is no market for the externality. Interestingly in our analysis the second stage of the game operates by introducing an extra market, however this is not directly a market for the externality. Further, we are able to show that even with a small number of players our model works as if there is a market for the externality which is perfectly arbitraged. This is the key theoretical contribution of our paper. It is well known that if the players of a game recieve …xed shares of the total payo¤ then cooperation and thus e¢ciency will result, our paper provides a decentralized (market) mechanism for achieving this. From a policy perspective our results may be applicable to real world externality problems. Suppose that a pair of …rms that imposed external costs or bene…ts on each other swapped standard equity for pooling equity, shares that represent claims on either (but not both) …rms. This, which amounts to little more than a relabeling exercise, would generate the same setup as in our theoretical model. Notice that this mechainsm may be introduced without the need to create new property right or to agree on any type of sharing scheme, thus it may avoid many of the pitfalls of alternative solutions. We view this as the major advantage of our mechanism over simpler ones such as those that involve specifying for each player a …xed share of the …rst stage payo¤s.

The rest of our paper is organized as follows. In section 2 we present a general model of an N player two stage non-cooperative game and demonstrate formally the results discussed above. In section 3 we apply our mechanism to two well known externality problems, the tragedy of the commons, and R&D spillovers in duopoly. In each case we demonstrate how our mechanism induces cooperation. We adopt these examples for speci…c reasons. Both are well known, well understood, and have clear real world implications. However they provide us with vehicles to demonstrate di¤erent aspects of the mechamism. In the commons problem there is only one source of distortion, the negative cost externalities each firm bestows on the others. For this problem the application of our mechanism produces an efficient solution. In the R&D spillovers problem there are distortions due both to the spillovers themselves and the market power of the duopoly firms. This is a second best world in which the application of our mechanism cannot yield an efficient solution. However, we show how it can be used to promote maximal R&D or maximal output dependent on when the pooling property rights are used as claims on profits. This example also allows us to explore the implications of pooling property rights that are permanently flexible, i.e. can be reallocated each time the game is played, or are temporarily flexible i.e. must remain as allocated after the first time they are used. Finally in section 4 we supply a conclusion.

2 The Model.

In this section we develop a formal model of our cooperation inducing mechanism. This consists of a two-stage game in which each of the two stages consists of a (non-cooperative) game in strategic form.

Stage 1: The players play a non-cooperative game $\Gamma = \langle M; (A_k)_{k \in M}; (p_k)_{k \in M} \rangle$ with player set $M = \{1, 2, ..., m\}$ in which each player $k \in M$ has an action set A_k and a positive payoff function p_k. If the players choose an action tuple $a = (a_k)_{k \in M} \in (A_k)_{k \in M}$ in this game, then every player $k \in M$ generates a payoff $p_k(a) > 0$.

Stage 2: There is a player set of $N = \{1, 2, ..., n\}$, where $n \geq m$ and $n \geq 2$. We assume each player $i \in N$ holds positive property rights (shares) S^i; which may be used as claims on the m positive payoffs $p_1(a), p_2(a), ..., p_m(a)$ that result from the first stage of the game. Denote the shares contributed by player i to payoff $p_k(a)$ by s^i_k. A strategy for player i in the second stage is a set of contributions $\left(s^i_k\right)_{k=1}^m$ such that $s^i_k \geq 0$ for each k and $\sum_{k=1}^m s^i_k = S^i$. If the players all determine their contributions, then for each payoff $p_k(a)$ player i gets the share $\frac{s^i_k}{\sum_{j \in N} s^j_k}$ of $p_k(a)$ if $\sum_{j \in N} s^j_k > 0$, i.e. if a positive amount of property right is contributed to $p_k(a)$, and if $\sum_{j \in N} s^j_k = 0$ then payoff $p_k(a)$ is divided among the players in some predetermined way (we will show that in equilibrium there will never be a payoff to which no property rights are contributed, no matter what the division). At the end of the game, every player $i \in N$ has a payoff of $\sum_{k=1}^m \frac{s^i_k}{\sum_{j \in N} s^j_k} p_k(a)$.

Notice that $M \subseteq N$; all players who play the first stage also play the second, but some players play only the second stage. Those that play both stages may be thought of as both owners and producers, those that play only the second stage are pure owners. [3] Having defined the game, we may now proceed to solve it for its subgame perfect Nash equilibria[4]. Consequently we apply backward induction and analyze first the second stage of the game, and then use the equilibria from the second-stage (sub)games to solve for optimal behavior in the first stage of the game.

2.1 The Second Stage.

In this subsection, we solve for the equilibria of the second stage of the game. We show that for every outcome of the first stage of the game, the second-stage game has a unique Nash equilibrium.

Suppose that in the first stage the action tuple a was played. Then, there would be the payoffs $p_1(a), p_2(a), \ldots, p_m(a)$ to which property rights can be applied. To simplify notation and to try and avoid confusion, we will denote $p_k(a)$ by P_k for every $k \in M$. Lower indices correspond to the different payoffs of the first stage that can be (partially) claimed and upper indices correspond to players in the second stage. We will see later that in a Nash equilibrium there will be no payoff P_k such that $\sum_{i\in N} s_k^i = 0$. Anticipating this, we will ignore the possibility that $\sum_{i\in N} s_k^i = 0$ for some k in our formulation of the maximization problem that each player faces.

A Nash equilibrium is obtained if all players i simultaneously solve the following maximization problem:

$$\text{Maximize} \quad \sum_{k=1}^{m} \frac{s_k^i}{\sum_{j\in N} s_k^j} P_k$$

$$\text{s.t.} \quad \sum_{k=1}^{m} s_k^i = S^i$$

$$\text{and } s_k^i \geq 0 \text{ for each } k$$

If a strategy $\left(s_k^i\right)_{k=1}^m$ solves the maximization problem of player $i \in N$, then there exists a multiplier λ^i such that for every $k = 1, 2, \ldots, m$

$$\frac{\sum_{j\in N} s_k^j - s_k^i}{\left(\sum_{j\in N} s_k^j\right)^2} P_k = \frac{\sum_{j\in N\setminus\{i\}} s_k^j}{\left(\sum_{j\in N} s_k^j\right)^2} P_k \leq \lambda^i, \text{ with equality if } s_k^i > 0 \qquad (1)$$

[3] We assume all players that play the first stage also play the second. This seems to accord well with the idea of managers representing the interests of their shareholders. In a companion paper Ellis and Van den Nouweland [5] we allow some players to play only the first stage as agents of those that play the second. The agents exclusively care about their own "effort". We find that the optimal agency contracts combined with the cooperation inducing mechanism induce an efficient allocation.

[4] Actually, as will become clear in the analysis of the first stage of the game, we consider a subset of the set of subgame perfect equilibria.

We shall proceed in two stages, first we shall demonstrate what a Nash equilibrium looks like, provided one exists, then we shall demonstrate that there exists a unique Nash equilibrium.

Theorem 1. If $\left(\left(s_k^i\right)_{k=1}^m\right)_{i\in N}$ is a Nash equilibrium. Then it has the following properties.

1. Each player makes a positive claim on each payoff, i.e. $s_k^i > 0$ for every $i \in N$ and $k = 1, 2, ..., m$.
2. Each player divides their property rights between the payoffs such that their share in each payoff is in the same proportions as their share of total property rights, i.e. $\frac{s_k^i}{\sum_{j\in N} s_k^j} = \frac{S^i}{\sum_{j\in N} S^j}$ for every $i \in N$ and $k = 1, 2, ..., m$.
3. Each player divides their property rights between the payoffs such that the proportion of their shares allocated to each payoff is the same as the proportion of that payoff to total payoffs, i.e. $\frac{s_k^i}{S^i} = \frac{P_k}{\sum_{l=1}^m P_l}$ for every $i \in N$ and $k = 1, 2, ..., m$.

The proof of this and all subsequent theorems, propositions and lemmas may be found in the appendix.

Theorem 1 may be most easily understood by examining the players incentives to allocate their property rights across the payoffs. Consider first the third part of the theorem and notice that this may be rewritten

$$\frac{\sum_{l=1}^m P_l}{S^i} = \frac{P_k}{s_k^i}.$$

Cross multiplying this expression, summing over the $i \in N$ and then cross multiplying again we obtain

$$\frac{\sum_{l=1}^m P_l}{\sum_{i\in N} S^i} = \frac{P_k}{\sum_{i\in N} s_k^i} \tag{2}$$

Expression (2) tells us that in the second stage the payoff per unit of property right is equalized across all first stage payoffs, and is essentially an arbitrage condition. Given the number of property rights allocated to each payoff no player may reallocate their shares and raise their total payoff. This has the further implication that more shares are placed on the larger payoffs, and in strict proportion to their size.

Next consider part 2, this tells us that each player holds the same percentage of the shares allocated to each payoff. This is an optimality condition that tells us that the reallocation of a property right by a player between first stage payoffs cannot raise their total payoff, this recognizes the effect of the reallocation both on the numerators and denominators of the relevant terms $\frac{s_k^i}{\sum_{j\in N} s_k^j}$. This is the proportionality property that we have already claimed will induce cooperative behavior.

Now we know what a Nash equilibrium looks like if one exists. It remains to be shown that there exists a unique Nash equilibrium.

Theorem 2. Let $\left(\left(s_k^i\right)_{k=1}^m\right)_{i \in N}$ be the set of strategies defined by

$$s_k^i = S^i \left(\frac{P_k}{\sum_{l=1}^m P_l} \right) \tag{3}$$

for every $i \in N$ and $k = 1, 2, \ldots, m$. This set of strategies is the unique Nash equilibrium of the second stage of the game. Moreover, for every player $i \in N$ his payoff according to the Nash equilibrium is

$$\frac{S^i}{\sum_{j \in N} S^j} \left(\sum_{k=1}^m P_k \right):$$

The unique Nash equilibrium to the second stage of the game is characterized by each player receiving a share of the sum of the payoffs from the first stage. Further, this share is equal to the ratio of each players' property rights to total property rights.

2.2 The First Stage.

We now know that for every action tuple a played in the first stage the subgame played in the second stage has a unique Nash equilibrium. We next analyze the first stage and solve for the subgame perfect equilibria of the two stage game. The payoffs that were exogenous in the second stage, are determined in the first stage. As we have seen in our analysis of the second stage, in equilibrium these payoffs will be re-distributed among the players in proportion to their share of total property rights. Hence, after the second stage, player $i \in N$ will end up with a payoff of

$$\frac{S^i}{\sum_{j \in N} S^j} \left(\sum_{k \in M} p_k(a) \right).$$

Since every player in the first stage is also a player in the second, the expression above gives us the payoff that every player in the first stage expects to get at the end of the second stage. Hence, the players in the first stage are playing a weighted potential game (cf. Monderer and Shapley [7]). Each gets a share of the total payoff obtained in the first stage where their shares are determined by their property rights, and are independent of the (relative) payoffs in the first stage. The incentive therefore is for each to maximize total payoffs rather than their own payoff in the first stage. Now, there may be Nash equilibria that do not result in the maximal total payo¤ obtainable (due to the fact that it might take more than one player deviating to get to an action tuple with a higher total payo¤), but it seems reasonable to restrict attention to the Nash equilibria that do result in the maximal total payo¤ possible. In the terminology of Monderer and Shapley (op. cit.): we consider Nash equilibria that are in the argmax set of the weighted potential. As Monderer and Shapley show, every action tuple that maximizes total payo¤s is a Nash equilibrium of the weighted potential game and restricting the set of Nash equilibria to those maximizing total payo¤s brings about a sensible re…nement of Nash equilibrium.[5]

2.3 Equilibria of the Two-Stage Game.

Combining the results we obtained so far, we conclude that the most interesting subgame perfect equilibria of the two-stage game are those in which the players in the …rst stage choose an action tuple that maximizes the total payo¤s from the game played in the …rst stage and in which the players in the second stage then play the strategies as described in theorem 2. Hence, in such a subgame perfect equilibrium, the total payo¤s from the …rst-stage game will be maximized, and in the second stage each player will receive a share of this amount as determined by his proportion of the property rights. Using total payo¤ maximization as our de…nition of cooperation, we have thus described a mechanism that, once introduced, induces the cooperative outcome in a non-cooperative environment without any communication, explicit collusion, agreement on or imposition of payo¤ shares, or joint decision making. This mechanism is fully decentralized, at each stage the players simply choose their own best replies, cooperation is not imposed, but rather arises as a consequence of the private incentives generated.[6]

While we believe this mechanism is in itself theoretically interesting, we also believe that it can be practically applied to a range of real world problems in which there is some kind of external e¤ect or spillover that requires internalization. We explore this issue in the next section by applying our theory to two well-known economic models.

3 Applications.

Below we illustrate the usefulness of our mechanism by analyzing it's application to Cornes, Mason and Sandler's [1] in‡uential analysis of the "Common Pool Resource Problem" and d'Aspremont and Jacquemin's [3] seminal model of R&D spillovers in duopoly. In the …rst example we demonstrate how in a

[5] Monderer and Shapley (op. cit.) point out that the argmax set of a weighted potential does not depend on the particular weighted potential chosen to represent the game. This shows that the argmax set of a weighted potential constitutes a well-de…ned unambiguous Nash equilibrium re…nement.

[6] In a related work Roemer [8] examines how the levels of provision of a public bad are related to how egalitarian is public share ownership. Our work di¤ers from this line of inquiry in that our "solution" to externality problems has the Coasian property that the outcome is independent of the initial distribution of property rights.

problem where there is only one distortion preventing the achievement of an e¢cient allocation, the application of our mechanism can achieve a …rst best equilibrium. In the second example we show how in problem with multiple distortions, and where there are di¤erent potential objectives, the timing of the application of the cooperation inducing mechanism may be exploited to achieve di¤erent cooperative outcomes. In each of these examples cooperative outcomes may be achieved by a simple modi…cation of the property rights system. If we assume that the players in these games are shareholder owned …rms, then these problems may be transformed into ones where our mechanism operates by the simple expedient of an equity swap. Holders of standard equity are o¤ered the opportunity to swap their existing holdings dollar-for-dollar for "pooling equity" which has the characteristic that it may be presented to any …rm for a share in pro…ts. Notice that no equity holder has an incentive to unilaterally refuse this swap as it allows them to exactly mimic their previous wealth holding position, or, if they desire, costlessly switch their assets to another …rm. Notice also that the simplicity of how the mechanism might be introduced is one of its primary attractions, property rights are rede…ned not redistributed, and no shares need be agreed upon. In this sense its introduction poses no distributional issues.

3.1 Cornes, Mason, and Sandler's Model of The Commons.

Cornes, Mason and Sandler's (op. cit.) model of the "Tragedy of the Commons" provides a tractable transparent exposition of the problem of the over exploitation of a common pool resource. In their analysis there are two sources of distortions, the externality associated with the commons problem, and the distortion associated with an imperfectly competitive output market. They demonstrate that these two distortions can be o¤setting, such that if the "correct" number of …rms extract from the resource then the e¢cient rate of extraction may be achieved[7]. We wish to focus on our mechanism rather than market structure as a potential solution to the commons problem and so follow Weitzman [9] in assuming that the output market is competitive. With this modi…cation we are able to illustrate how our cooperation inducing mechanism can induce an e¢cient rate of extraction from the resource. We shall follow Cornes, Mason and Sandler's example and discuss the problem of the exploitation of a common access …shery, applications to similar problems such as extraction from an oil pool should be fairly obvious to the reader.

3.1.1 The Model of a Common Access Fishery.

The industry consists of $k = 1; :::; m$ …shing …rms who's objective is to maximize the pro…t received by their h_k shareholders. The …rms are assumed to sell their output on a perfectly competitive market at the price P. The total catch, or

[7] It is not exactly clear how the correct number of …rms is achieved or how this number is varied over time to achieve the e¢cient time path for extraction.

output of the commons, is denoted C and is determined by the total size of the fleet, R, according to the production technology

$$C = F(R)$$

where $F(R)$ is assumed to be strictly increasing and strictly concave while the input R is assumed to be essential, i.e. $F(0) = 0$. Further the total catch is bounded above by the fish population. These assumptions ensure that $F(R)/R > F'(R)$ and $\lim_{R \to \infty} F(R)/R = 0$.

We examine the symmetric or "pure" commons case in which the fish population is distributed evenly across the commons, so that the catch per vessel is equal. It then follows that each firm's catch can be represented by

$$c_k = \left(\frac{r_k}{r_k + \tilde{R}_k} \right) F(r_k + \tilde{R}_k)$$

where r_k is the number of vessels of any given firm, k, and $\tilde{R}_k = R - r_k$ is the size of the rest of the fleet. Under the assumption of non-cooperative Nash behavior each firm chooses its fleet size to maximize profit per equity taking $\tilde{R}_k$ as given, that is

$$\underset{r_k}{Max} \frac{\pi_k}{h_k} = \left(\frac{PF(r_k + \tilde{R}_k)}{r_k + \tilde{R}_k} - w \right) \frac{r_k}{h_k}$$

where w is the rental rate per vessel. With free access entry drives profit to zero

$$\frac{PF(R)}{R} - w = 0$$

Denote the solution to this problem R^f.

3.1.2 Socially Efficient Fishing.

Given that the firms' output sells on a competitive market at a given price P, and if we assume social welfare to be given by the sum of consumer and producer surplus then the efficient level of fishing simply involves the maximization of industry profit, or

$$\underset{R}{Max} W = PF(R) - wR$$

with the first order condition

$$PF'(R) - w = 0$$

Denote the solution to this problem R^W: This immediately reveals the classic commons problem.

Proposition 1. With free access the commons is overexploited $R^f > R^W$:

3.1.3 Introduction of the Cooperation Inducing Mechanism.

Suppose now that instead of equity representing a fixed claim on a particular firm it may instead be used as a claim on any firm in the industry. Adopting the same notation for this pooling equity as before we write s_k as the claims made on the profits of firm k; and $S = \sum_k s_k$ as total equity claims. The optimization of an individual firm k now involves

$$\operatorname*{Max}_{r_k} \frac{\pi_k}{s_k} = \left(\frac{PF(r_k + \bar{R}_k)}{R} - w \right) \frac{r_k}{s_k}.$$

Proposition 2. The introduction of pooling equities induce a socially efficient level of fishing.

In this model the only deviation from the efficient rate of extraction from the resource arises as a consequence of the crowding externality that fishing vessels impose on each other[8]. Introducing the pooling equities causes profits per share to be equalized across firms. When choosing the number of vessels in its fleet each firm knows that the negative external effects it imposes on other firms will cause a decline in their total profits and thus lead to a redistribution of equity across firms. The increase in equity claims on an expanding firm's profits reduce profit per share and thus cause it to fully internalize the external effects it has on the rest of the industry[9].

While this example demonstrates the potential of our cooperation inducing mechanism to achieve an efficient allocation for a set of well-known problems, there are other circumstances in which there are different dimensions in which cooperation may occur, and where cooperation and efficiency are not immediately synonymous. To explore some of these issues, and to demonstrate the importance of timing of the application of the cooperation inducing mechanism we next examine applications to d'Aspremont and Jacquemin's Model of R&D Spillovers in Duopoly.

3.2 D'Aspremont and Jacquemin's Model of R&D Spillovers in Duopoly.

In their paper D'Aspremont and Jacquemin (hereafter D&J) analyze the behavior of a pair of duopolists that engage in R&D expenditures prior to production. These R&D expenditures reduce the duopolists own cost and also spill over to reduce their rival's costs. For various combinations of cooperation and competition in the two stages of the game D&J obtain a ranking of R&D expenditures

[8] It can be shown (see Ellis [4]) that in a dynamic model our mechanism induces the internalization of both the dynamic and static externalities associated with the commons problem.

[9] Our mechanism causes firms to behave as if they have merged. However, this solution may be superior to a merger. If individual firms productions technologies were concave then a merger may well increase marginal and average costs. ie. if $f(x_0) = f(x_1) + f(x_1) = A$, then $f'(x_0) < f'(x_1)$.

and output levels relative to those at the social welfare optimum. Specifically they examine; (i) Non-cooperative Nash behavior in both stages of the game; (ii) Cooperation in the R&D stage combined with Nash behavior in the production stage; (iii) Cooperation in both the R&D and production stages. Where, in their terminology, non-cooperation is characterized by individual profit maximizing behavior, and cooperation by joint profit maximizing behavior. D&J show that, provided that spillovers are sufficiently large, out of the three cases considered, R&D expenditure is highest in the fully cooperative scenario, while production is highest when there is cooperation in the R&D stage but competition in the production stage[10].

Our purpose here is to demonstrate what our cooperation inducing mechanism can bring to analyses such as D&J's. In their model cooperation in the different stages of the game occurs by assumption. We first show that neither cooperation in the R&D stage nor in both stages of their game can be supported as a subgame perfect equilibrium. We then introduce our mechanism and show how it can be used to implement the cooperative outcomes in either the R&D stage or both stages of the game dependent on when the mechanism is applied. The mechanism can be used to implement as a non-cooperative equilibrium both the maximal R&D or output results as in D&J, but further can be used to implement a second best welfare optimum.

3.2.1 The Duopoly Model.

We first outline the D&J model which consists of a pair of duopolists indexed $k = 1, 2$; who face the inverse demand function

$$D^{-1} = a - bQ \quad a, b > 0$$

where $Q = \sum_k q_k$ is the sum of the two firms' outputs. Each firm's costs consist of two components (i) R&D costs incurred in the first stage, and (ii) production costs incurred in the second. Summing over the two stages a firm's total costs are given by the function

$$C_k(q_k, x_k, x_j) = [A - x_k - \beta x_j]q_k + \frac{\gamma}{2}x_k^2 \qquad j \neq k;\ k, j = 1, 2.$$

where q_k is its production level, x_k its expenditure on R&D and x_j is the R&D expenditure undertaken by its rival. Following D&J we assume $0 < A < a; 0 < \beta < 1; x_k + \beta x_j \leq A; Q \leq \frac{a}{b}; 0 < b.$

The profit of firm k may now be written

$$\pi_k = [a - bQ]q_k - [A - x_k - \beta x_j]q_k - \frac{\gamma}{2}x_k^2.$$

We assume that each firm is owned by h_k shareholders and that the objective of the firm's managers is to maximize dividend payments

$$\max_{x_k, q_k} \frac{\pi_k}{h_k} = \frac{1}{h_k}\left[(a - bQ)q_k - (A - x_k - \beta x_j)q_k - \frac{\gamma}{2}x_k^2\right].$$

[10] A sufficient condition for their main conclusions being that at least 50% of one firm's expenditure spills over reducing the production costs of its rival.

With fixed equity holdings this is identical to maximizing each firm's individual profits.

Social welfare can be characterized as the sum of consumer plus producer surplus, or the area under the demand curve less firms total costs. Integrating under the demand curve and subtracting the firms' costs provides

$$W = aQ - \frac{b}{2}Q^2 - (A - x_1 - \beta x_2)q_1 - (A - x_2 - \beta x_1)q_2 - \frac{\gamma}{2}x_1^2 - \frac{\gamma}{2}x_2^2.$$

Notice that this also includes the costs of R&D incurred prior to production.

In symmetric equilibria, $(x_1 = x_2 = x; q_1 = q_2 = q)$, which following D&J we focus on here, the social welfare function reduces to

$$W = 2aq - 2bq^2 - 2[A - (1+\beta)x]q - \gamma x^2. \qquad (4)$$

3.2.2 Solutions to the Model.

D&J derive three symmetric solutions to this model involving (i) Non-Cooperative behavior in both the R&D and production stages of the game, (ii) Cooperative behavior in the R&D stage with non-cooperative behavior in the production stage, (iii) Cooperative behavior in both stages. The solutions obtained are as follows[11].

(i) Non-Cooperative behavior in both stages of the game.

$$x_k^* = \frac{(a - A)(2 - \beta)}{4.5b\gamma - (2 - \beta)(1 + \beta)} \qquad k = 1, 2 \qquad (5)$$

$$q_k^* = \frac{(a - A)}{3b}\left[\frac{4.5b\gamma}{4.5b\gamma - (2 - \beta)(1 + \beta)}\right] \qquad k = 1, 2 \qquad (6)$$

(ii) Cooperative behavior in the R&D stage with non-cooperative behavior in the production stage.

$$\hat{x}_k = \frac{(a - A)(1 + \beta)}{4.5b\gamma - (1 + \beta)^2} \qquad k = 1, 2 \qquad (7)$$

$$\hat{q}_k = \frac{(a - A)}{3b}\left[\frac{4.5b\gamma}{4.5b\gamma - (1 + \beta)^2}\right] \qquad k = 1, 2 \qquad (8)$$

(iii) Cooperative behavior in both stages.

$$\tilde{x}_k = \frac{(a - A)(1 + \beta)}{4b\gamma - (1 + \beta)^2} \qquad k = 1, 2 \qquad (9)$$

[11] To obtain these solutions D&J first solve the second stage for the levels of output given R&D expenditure. Then the first stage is solved for the subgame perfect equilibria to determine the level of R&D. Non-cooperation at a stage then involves individual profit maximization, cooperation involves joint profit maximization.

$$q_k = \frac{(a - A)}{4b}\left[\frac{4b\gamma}{4b\gamma - (1+\beta)^2}\right] \qquad k = 1; 2 \qquad (10)$$

Immediately we have

Proposition 3. Cooperative behavior in either R&D or output cannot be supported as a subgame perfect equilibrium in the D&J model.

While this proposition is trivial it remains important. It tells us that even if some degree of cooperation is socially desirable it cannot be achieved given the incentives faced by the firms in the game as currently constructed.

3.2.3 Introduction of the Cooperation Inducing Mechanism.

We now introduce our cooperation inducing mechanism. Our method will be to utilize the fact that our cooperation inducing mechanism can be rewritten as an arbitrage condition which in turn may be shown to induce joint profit maximizing behavior. To facilitate this assume now that the property rights in the two firms are now "Pooling Equities" with the characteristic that they may be used as claims on either of the firms' profits. Let S be the total stock of pooling equities, and s_k be the total equity assigned to claims on the profit of firm k. Rewriting (2) in terms of the notation of this section provides

$$\frac{\pi_1 + \pi_2}{s_1 + s_2} = \frac{\pi_1}{s_1} = \frac{\pi_2}{S - s_1}$$

which is a simple arbitrage condition that we may exploit to demonstrate how our mechanism induces the cooperative allocations in D&J's model.

The optimization problem of firm k is now

$$\max_{x_k; q_k} \frac{\pi_k}{s_k} = \frac{1}{s_k}\left[(a - bQ)\,q_k - (A - x_k - \beta x_j)\,q_k - \frac{\gamma}{2}x_k^2\right]$$

We may now show that our mechanism induces cooperation.

Proposition 4. When the cooperation inducing mechanism is applied the two duopolists make the same choices non-cooperatively as those that characterize joint profit maximization.

The intuition should now be transparent. Our mechanism induces the arbitrage of profits, the payment to each equity is equalized across firms. The firms managers are aware of this, and know that it implies that the profit per share that they pay can only be increased by actions that raise joint profit[12]. They do not explicitly cooperate, but rather are provided by the cooperation inducing mechanism with individual incentives that cause them to selfishly make the cooperative choices.

To induce the outcomes examined by D&J now requires only that the timing of the application of the mechanism be specified. We have the following

[12] We assume that there is no conflict of interest between managers and shareholders, such that the mangement of a firm always acts as a perfect agent of the shareholders

Proposition 5. If the pooling equity is presented to the firms at the end of the production stage of the game, then this induces the firms to cooperate in both the R&D and production stages of the game. This yields the maximal equilibrium level of R&D.

Proposition 6. If the pooling equity is presented to the firms at the end of the R&D stage, then this induces the firms to cooperate in the R&D stage. However, non-cooperation will still characterize the production stage. This yields the maximal equilibrium level of output.

These propositions, 5 and 6, indicate that either expected or actual profits may be arbitraged dependent on when, relative to the production stage, the pooling equities may be used as claims on the firms.

3.2.4 The Desirability of Cooperation.

In the preceding section we demonstrated how the cooperation inducing mechanism implies a profit arbitrage condition that generates incentives for joint profit maximization. Cooperation may be induced either in the R&D stage or in both the R&D and production stages of the game. Here we investigate the welfare properties of the different equilibria. Substituting the solutions (5)-(10) into the social welfare function (4) allows us to obtain the following expressions for social welfare[13]. We shall describe the equilibrium that generates the highest social welfare as the second best welfare optimum. The first best welfare optimum would be the outcome chosen by a social planner choosing R&D expenditures and output so as to maximize social welfare (4).

(i) Non-Cooperative behavior in both stages of the game.

$$W^* = \frac{\gamma(a-A)^2\left[9\beta^2\gamma - b(\beta-2)^2\right]}{b\left[2+\beta-\beta^2-4.5\beta\gamma\right]^2} \tag{11}$$

(ii) Cooperative behavior in the R&D stage with non-cooperative behavior in the production stage.

$$\widetilde{W} = \frac{\gamma(a-A)^2\left[9\beta^2\gamma - b(1+\beta)^2\right]}{b\left[1+\beta^2+\beta(2-4.5\gamma)\right]^2} \tag{12}$$

(iii) Cooperative behavior in both stages.

$$\widehat{W} = \frac{\gamma(a-A)^2\left[6\beta^2\gamma - b(1+\beta)^2\right]}{b\left[1+\beta^2+\beta(2-4\gamma)\right]^2} \tag{13}$$

[13] The following expressions are close approximations generated using Mathematica. The programs are available from the authors on request.

It is not possible to make simple algebraic statements about which of these outcomes is socially superior[14]. Hence we revert to numerical methods. For different values of the parameters β, γ and b figures 1 and 2 describe a ranking of the various cooperative and non-cooperative outcomes according to the social welfare function (4)[15].

[Figures 1, 2 and the legend about here.]

Inspection of the diagrams reveals that when the inverse demand curve is steeply sloped $10 \geq b \geq 3$; and if spillovers are large $\beta > 0.5$ then the second best welfare outcome arises under Nash behavior in both stages of the game. Whereas if the inverse demand curve is relatively flat $1.0 \geq b \geq 0.3$ and if spillovers are large $\beta > 0.5$ then the second best welfare outcome arises under cooperative behavior in the R&D stage with Nash behavior in the production stage. Also if the inverse demand function is relatively steep $10 \geq b \geq 0.9$, and spillovers are small $0.5 \geq \beta \geq 0.2$ then the second best welfare outcome arrises under cooperative behavior in the R&D stage with Nash behavior in the production stage. The second best welfare optimum involves cooperation in both stages only for a small subset of the parameter space, where $\gamma = 2$ and the inverse demand function very flat $0.2 \geq b \geq 0.1$[16].

4 Conclusion.

In this paper we have proposed a novel mechanism for inducing agents playing a non-cooperative game to choose the cooperative outcome. Our mechanism adds a second non-cooperative stage to the game. In the unique Nash equilibrium of this second stage, the payoffs generated in the first stage are reallocated between the players according to the allocation of shares. We show that this effectively converts the first stage into a weighted potential game (cf. Monderer and Shapley (op.cit.)), the players of which have incentives to maximize the total payoff. If we follow Monderer and Shapley (op. cit.) further, and restrict attention to those Nash equilibria that lie in the argmax set of the weighted potential then the mechanism implements the cooperative outcome[17].

This mechanism has applications to a wide range of economic problems, as any situation in which external effects or spillovers are present may be viewed

[14] In the parameter space the boundaries between the regions in which the different regimes are socially superior are higher order polynomials (6th order in β!).

[15] The remaining parameters of the model were set at a=2 and A=1, as an inspection of (11)-(13) immediately reveals deviations from these values just serve to rescale all the results . The calculations were performed using Mathematica. The programs used and raw numbers are available from the authors on request.

[16] In this case the demand function is very steep and thus the deadweight loss triangles associated with monopoly are small.

[17] The reader can probably think of several stories that might justify this refinement. However, these typically involve specifying the beliefs of the players prior to play. This issue is not our focus in this paper.

as one where cooperative behavior can potentially produce welfare improvements. We believe the mechanism is both of theoretical interest and raises some interesting possibilities for policy.

Theoretically, as the arbitrage conditions (2) indicate, it is as if a new market has been created. Establishing property rights and a competitive market on which an external e¤ect may be traded is a well known solution to an externality problem. What is perhaps of interest is that the "market" in the current paper is not directly for the external e¤ect but for the returns generated by the activity that produces it. In at least some examples the arbitrage of pro…ts perfectly substitutes for a competitive market in the externality.

An alternative way to view our theoretical contribution is that it decentralizes a payo¤ sharing scheme. In the subgame perfect equilibrium each player recieves a proportion of the total payo¤. It is as if …xed proportionate payo¤ shares have somehow been agreed in advance of play. However, such prior agreements are unecessary precisely because proportionate payo¤s are a characteristic of the non-cooperative equilibrium.

From a policy perspective our proposed mechanism has several advantages over alternative solutions to externality problems. Once implemented it requires no regulatory body to oversee it, there are no information requirements such as those needed to implement tax or quota based solutions, and no new property rights are established so there are no equity issues such as those that arise when, for example, pollution permits are introduced. Further our solution requires no monitoring. Despite these advantages there are of course some caveats and issues that require further study. The value of our mechanism depends on the possibility of its practical implementation. To introduce the second stage of the game requires that it is possible to introduce the pooling property rights. In the examples we have discussed this is achieved via the swap of standard equity shares for pooling equity shares. This form of ownership structure seems the best suited to our mechanism[18]. The introduction of pooling property rights clearly needs further study and may represent a key role for public policy[19]. In situations like those described in our common pool resource example there may be issues similar to those encountered with the stability of cartels. One individual …rm may have an incentive to refuse to accept the equity (if this is legal) and thus free ride on the cooperative behavior of the others. It may be necessary for the government to mandate the initial acceptance of the pooling property right[20]. This is an issue that we hope to return to in the future.

In other applications, such as our R&D spillovers example, cooperation may bene…t the participants, in this case the duopolists, but harm a third party, here

[18] All claims are thus priced in dollars which satis…es the requirement that payo¤s be transferable. This does not seem to us to be unduly restrictive for many of the applications we might consider.

[19] It might however be noted that the swap of pooling equity for standard equity should raise stock prices. If, as is common, managerial compensation is linked to stock prices then this would provide an incentive for the scheme's adoption.

[20] However, the continued acceptance of the pooling equility seems to us to be no greater a problem than those involved with making a …rm honor its standard contractual obligations to its shareholders.

the consumers. Obviously this is not a direct caveat to our mechanism, but a familiar type of warning that if there are potentially multiple distortions in an economy then limited cooperation may be worse in a welfare sense than none. This is a standard problem frequently encountered in a second best world where if two adjustments are required to move the economy to the Pareto frontier, one of the two may actually move the economy away from it. However, from a policy perspective this is still a very interesting issue. As the R&D example illustrates the timing of when the pooling property right may be used as claims can make a di¤erence to the incentives they induce. Furthermore, we might suspect that whether the pooling property rights are multiple or single use (and thereafter commitment to a particular payo¤) will also be of signi…cance. This suggests the idea of pooling property rights might further be re…ned to induce cooperation when desirable and then allow for a return to competition.

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A Appendix.

Proof of Theorem 1. We prove our first theorem via a sequence of three parts that correspond to the parts of the theorem.

Let $\left(\left(s_k^i\right)_{k=1}^m\right)_{i\in N}$ be a Nash equilibrium, then the following hold.

Part 1. Each player makes a positive contribution to each payoff, i.e. $s_k^i > 0$ for every $i \in N$ and $k = 1, 2, \ldots, m$.

Proof. If there is only one payoff ($m = 1$), then the unique Nash equilibrium is for every player to contribute his entire endowment of property rights to this payoff and the lemma is true. Hereafter, assume that $m \geq 2$.

First we prove that for any payoff there is at least one player who contributes a positive amount to that payoff, and that this is true for each payoff.

Suppose $k \in \{1, 2, \ldots, m\}$ is such that $s_k^i = 0$ for every $i \in N$. Then payoff P_k is divided among the players in some predetermined way. Since there are at least 2 players ($n \geq 2$), there is at least one player $i \in N$ who gets a part $P_k^i < P_k$ from this payoff. Take such a player i. If player i would make an arbitrarily small positive contribution ε^i to payoff P_k, then he would get the entire payoff and increase his payoff by $P_k - P_k^i$. The contribution ε^i would have to be taken away from some other payoff. Since $\sum_{l=1}^m s_l^i = S^i > 0$, there exists an $l \in \{1, 2, \ldots, m\}$, $l \neq k$, such that $s_l^i > 0$. Take such an l. Player i gets a proportion $\frac{s_l^i}{\sum_{j\in N} s_l^j} > 0$ of payoff P_l. Notice that the proportion that i gets from payoff P_l is a continuous function of i's contribution to this payoff as long as his contribution is positive. Hence, player i can reduce his contribution to payoff P_l by an amount ε^i in such a way that he loses less than $P_k - P_k^i$ from payoff P_l, i.e. $\frac{s_l^i}{\sum_{j\in N} s_l^j} P_l - \frac{s_l^i - \varepsilon^i}{\sum_{j\in N\setminus\{i\}} s_l^j + (s_l^i - \varepsilon^i)} P_l < P_k - P_k^i$. So, if player i reduces his contribution to payoff P_l by such an amount ε^i and increases his contribution to payoff P_k from 0 to ε^i, then he increases his total payoff. Hence, the initial contributions did not form a Nash equilibrium.

Next we show that for every payoff at least two players contribute a positive property rights to that payoff.

Suppose $k \in \{1, 2, \ldots, m\}$ and $i \in N$ are such that $s_k^i > 0$ and $s_k^j = 0$ for every $j \in N\setminus\{i\}$. Then player i gets the entire payoff P_k, because he is the only player contributing to this payoff. However, if he reduces his contribution to payoff P_k by an amount $\varepsilon^i < s_k^i$, then he would still get the entire payoff P_k. Then he can increase his contribution s_l^i to some other payoff P_l by the amount ε^i. If he chooses a payoff P_l for which it holds that there is some other player $j \in N\setminus\{i\}$ such that $s_l^j > 0$ (note that such a payoff exists), then increasing s_l^i will increase the proportion that i gets of payoff P_l (note that the share that i gets from payoff P_l is a continuously increasing function of i's contribution to this payoff). Hence, the initial contributions of the players did not form a Nash equilibrium.

We now are ready to prove that $s_k^i > 0$ for every $i \in N$ and $k = 1, 2, \ldots, m$.

Note that for each payoff at least two players make a positive contribution and that this is true for every payoff, this implies that $\sum_{j \in N \setminus \{i\}} s_k^j > 0$ for every $k = 1, 2, \ldots, m$ and $i \in N$. Suppose that there exists a payoff to which not every player contributes a positive amount. Let $k \in \{1, 2, \ldots, m\}$ and $i \in N$ such that $s_k^i = 0$ and let $l \in \{1, 2, \ldots, m\}$, $l \neq k$, such that $s_l^i > 0$. Define $S \subset N$ by $S = \{h \in N \mid s_k^h > 0\}$. Note that $i \notin S$. Then we find using condition (1) that

$$\frac{\sum_{j \in N \setminus \{i\}} s_k^j}{\left(\sum_{j \in N} s_k^j\right)^2} P_k \leq \frac{\sum_{j \in N \setminus \{i\}} s_l^j}{\left(\sum_{j \in N} s_l^j\right)^2} P_l$$

and

$$\frac{\sum_{j \in N \setminus \{h\}} s_k^j}{\left(\sum_{j \in N} s_k^j\right)^2} P_k \geq \frac{\sum_{j \in N \setminus \{h\}} s_l^j}{\left(\sum_{j \in N} s_l^j\right)^2} P_l \text{ for every } h \in S.$$

From this we derive that

$$\frac{\sum_{j \in N \setminus \{i\}} s_k^j}{\sum_{j \in N \setminus \{i\}} s_l^j} \leq \frac{\left(\sum_{j \in N} s_k^j\right)^2}{\left(\sum_{j \in N} s_l^j\right)^2} \frac{P_l}{P_k}$$

and

$$\frac{\sum_{j \in N \setminus \{h\}} s_k^j}{\sum_{j \in N \setminus \{h\}} s_l^j} \geq \frac{\left(\sum_{j \in N} s_k^j\right)^2}{\left(\sum_{j \in N} s_l^j\right)^2} \frac{P_l}{P_k} \text{ for every } h \in S.$$

Notice that the right-hand sides of the last two inequalities are identical. Hence, we find that

$$\frac{\sum_{j \in N \setminus \{i\}} s_k^j}{\sum_{j \in N \setminus \{i\}} s_l^j} \leq \frac{\sum_{j \in N \setminus \{h\}} s_k^j}{\sum_{j \in N \setminus \{h\}} s_l^j} \text{ for every } h \in S,$$

which can be re-written as

$$\frac{\sum_{j \in N \setminus \{h\}} s_l^j}{\sum_{j \in N \setminus \{i\}} s_l^j} \leq \frac{\sum_{j \in N \setminus \{h\}} s_k^j}{\sum_{j \in N \setminus \{i\}} s_k^j} \text{ for every } h \in S.$$

We use this result to obtain[21]

$$(|S|-1)+\frac{\sum_{j\in N\setminus S}s_l^j}{\sum_{j\in N\setminus\{i\}}s_l^j}<\frac{(|S|-1)\sum_{j\in N\setminus\{i\}}s_l^j+\sum_{j\in N\setminus S}s_l^j+(|S|-1)s_l^i}{\sum_{j\in N\setminus\{i\}}s_l^j}$$
$$=\frac{\sum_{h\in S}\sum_{j\in N\setminus\{h\}}s_l^j}{\sum_{j\in N\setminus\{i\}}s_l^j}=\sum_{h\in S}\left(\frac{\sum_{j\in N\setminus\{h\}}s_l^j}{\sum_{j\in N\setminus\{i\}}s_l^j}\right)$$
$$\leq\sum_{h\in S}\left(\frac{\sum_{j\in N\setminus\{h\}}s_k^j}{\sum_{j\in N\setminus\{i\}}s_k^j}\right)=\frac{\sum_{h\in S}\sum_{j\in N\setminus\{h\}}s_k^j}{\sum_{j\in N\setminus\{i\}}s_k^j}$$
$$=\frac{(|S|-1)\sum_{j\in N}s_k^j+\sum_{j\in N\setminus S}s_k^j}{\sum_{j\in N}s_k^j}=(|S|-1).$$

This shows that

$$\frac{\sum_{j\in N\setminus S}s_l^j}{\sum_{j\in N\setminus\{i\}}s_l^j}<0.$$

However, we know $\sum_{j\in N\setminus\{i\}}s_l^j>0$, $i\in N\setminus S$, and $s_l^i>0$. Hence, we have a contradiction and conclude that every player contributes a positive amount to each payoff. This proves part 1. ■

Part 2. Each player divides their property rights between the payoffs such that their share in each payoff is in the same proportion as their share of total property rights, i.e. $\frac{s_k^i}{\sum_{j\in N}s_k^j}=\frac{S^i}{\sum_{j\in N}S^j}$ for every $i\in N$ and $k=1,2,\ldots,m$:

Proof. From part 1 it follows that $\sum_{j\in N\setminus\{i\}}s_k^j>0$ for every $k=1,2,\ldots,m$ and $i\in N$ and, consequently, that condition (1) implies that $0<\frac{\sum_{j\in N\setminus\{i\}}s_k^j}{(\sum_{j\in N}s_k^j)^2}P_k=\lambda^i$ for every $i\in N$. Dividing condition (1) corresponding to player i and payoff P_k by that corresponding to player h and payoff P_k gives

$$\frac{\left(\frac{\sum_{j\in N\setminus\{i\}}s_k^j}{(\sum_{j\in N}s_k^j)^2}\right)}{\left(\frac{\sum_{j\in N\setminus\{h\}}s_k^j}{(\sum_{j\in N}s_k^j)^2}\right)}\frac{P_k}{P_k}=\frac{\sum_{j\in N\setminus\{i\}}s_k^j}{\sum_{j\in N\setminus\{h\}}s_k^j}=\frac{\lambda^i}{\lambda^h}.$$

From this we derive

$$\sum_{j\in N\setminus\{i\}}S^j=\sum_{j\in N\setminus\{i\}}\sum_{k=1}^{m}s_k^j=\sum_{k=1}^{m}\sum_{j\in N\setminus\{i\}}s_k^j$$
$$=\sum_{k=1}^{m}\left[\frac{\lambda^i}{\lambda^h}\sum_{j\in N\setminus\{h\}}s_k^j\right]=\frac{\lambda^i}{\lambda^h}\sum_{j\in N\setminus\{h\}}\sum_{k=1}^{m}s_k^j=\frac{\lambda^i}{\lambda^h}\sum_{j\in N\setminus\{h\}}S^j$$

[21] We remind the reader that $i\notin S$, $s_k^i=0$, $s_l^i>0$, and $s_k^j=0$ for each $j\in N\setminus S$.

and then

$$\frac{\sum_{j\in N\setminus\{i\}} s_k^j}{\sum_{j\in N\setminus\{i\}} S^j} = \frac{\frac{\lambda^i}{\lambda^h}\sum_{j\in N\setminus\{h\}} s_k^j}{\frac{\lambda^i}{\lambda^h}\sum_{j\in N\setminus\{h\}} S^j} = \frac{\sum_{j\in N\setminus\{h\}} s_k^j}{\sum_{j\in N\setminus\{h\}} S^j}.$$

It follows that for each $k = 1, 2, \ldots, m$ we can define a constant C_k such that

$$\frac{\sum_{j\in N\setminus\{i\}} s_k^j}{\sum_{j\in N\setminus\{i\}} S^j} = C_k \text{ for all } i \in N.$$

Now, for each $i \in N$,

$$(n-1)s_k^i = \sum_{j\in N\setminus\{i\}}\left(\sum_{h\in N\setminus\{j\}} s_k^h\right) - (n-2)\left(\sum_{h\in N\setminus\{i\}} s_k^h\right)$$

$$= \sum_{j\in N\setminus\{i\}}\left(C_k\sum_{h\in N\setminus\{j\}} S^h\right) - (n-2)\left(C_k\sum_{h\in N\setminus\{i\}} S^h\right) = (n-1)C_k S^i.$$

Hence,

$$\frac{s_k^i}{S^i} = C_k \text{ for each } i \in N$$

and

$$\frac{s_k^i}{s_k^j} = \frac{C_k S^i}{C_k S^j} = \frac{S^i}{S^j} \text{ for all pairs } i, j \in N.$$

Then, for each $i \in N$ and $k = 1, 2, \ldots, m$,

$$\frac{s_k^i}{\sum_{j\in N} s_k^j} = \frac{\left(\frac{s_k^i}{s_k^i}\right)}{\sum_{j\in N}\left(\frac{s_k^j}{s_k^i}\right)} = \frac{\left(\frac{S^i}{S^i}\right)}{\sum_{j\in N}\frac{S^j}{S^i}} = \frac{S^i}{\sum_{j\in N} S^j}.$$

This proves part 2. ∎

Part 3. Each player divides their property rights between the payoffs in the same proportions as each has to total payoffs, i.e. $\frac{s_k^i}{S^i} = \frac{P_k}{\sum_{l=1}^m P_l}$ for every $i \in N$ and $k = 1, 2, \ldots, m$.

Proof. From the proof of part 2 we know that for every $k = 1, 2, \ldots, m$ there exists a C_k such that $\frac{s_k^i}{S^i} = C_k$ for each $i \in N$. Let $i \in N$. The Nash-equilibrium strategy $\left(s_k^i\right)_{k=1}^m$ satisfies condition (1) and, by part 1 we know that $s_k^i > 0$ for each $k = 1, 2, \ldots, m$. This implies that

$$\frac{\sum_{j\in N\setminus\{i\}} s_k^j}{\left(\sum_{j\in N} s_k^j\right)^2} P_k = \frac{\sum_{j\in N\setminus\{i\}} s_l^j}{\left(\sum_{j\in N} s_l^j\right)^2} P_l$$

for each $k, l = 1, 2, \ldots, m$. We use this to find

$$\frac{\sum_{j\in N\setminus\{i\}} S^j}{\left(\sum_{j\in N} S^j\right)^2}\frac{P_k}{C_k} = \frac{\sum_{j\in N\setminus\{i\}} C_k S^j}{\left(\sum_{j\in N} C_k S^j\right)^2} P_k = \frac{\sum_{j\in N\setminus\{i\}} s_k^j}{\left(\sum_{j\in N} s_k^j\right)^2} P_k$$

$$= \frac{\sum_{j\in N\setminus\{i\}} s_l^j}{\left(\sum_{j\in N} s_l^j\right)^2} P_l = \frac{\sum_{j\in N\setminus\{i\}} C_l S^j}{\left(\sum_{j\in N} C_l S^j\right)^2} P_l = \frac{\sum_{j\in N\setminus\{i\}} S^j}{\left(\sum_{j\in N} S^j\right)^2}\frac{P_l}{C_l}$$

and, consequently, $\frac{P_k}{C_k} = \frac{P_l}{C_l}$ for all $k, l \in \{1, 2, \ldots, m\}$. Hence, $C_k = \frac{P_k C_1}{P_1}$ for all $k = 1, 2, \ldots, m$ Now we derive

$$1 = \frac{\sum_{l=1}^m s_l^i}{S^i} = \sum_{l=1}^m \frac{s_l^i}{S^i} = \sum_{l=1}^m C_l = \sum_{l=1}^m \frac{P_l C_1}{P_1} = \frac{C_1}{P_1}\sum_{l=1}^m P_l$$

and

$$C_1 = \frac{P_1}{\sum_{l=1}^m P_l}.$$

Hence, C_k is uniquely determined for each $k \in \{1, 2, \ldots, m\}$ by

$$C_k = \frac{P_k C_1}{P_1} = \frac{P_k}{\sum_{l=1}^m P_l}.$$

This yields

$$\frac{s_k^i}{S^i} = C_k = \frac{P_k}{\sum_{l=1}^m P_l},$$

which concludes the proof ■

Theorem 2. Let $\left((s_k^i)_{k=1}^m\right)_{i\in N}$ be the set of strategies defined by

$$s_k^i = S^i\left(\frac{P_k}{\sum_{l=1}^m P_l}\right)$$

for every $i \in N$ and $k = 1, 2, \ldots, m$. This set of strategies is the unique Nash equilibrium of the second stage of the game. Moreover, for every player $i \in N$ his payoff according to the Nash equilibrium is

$$\frac{S^i}{\sum_{j\in N} S^j}\left(\sum_{k=1}^m P_k\right).$$

Proof. Let $\left((s_k^i)_{k=1}^m\right)_{i\in N}$ be the set of strategies defined by

$$s_k^i = S^i\left(\frac{P_k}{\sum_{l=1}^m P_l}\right)$$

for every $i \in N$ and $k = 1, 2, \ldots, m$. This set of strategies is the unique Nash equilibrium of the second stage of the game. Moreover, for every player $i \in N$ his payoff according to the Nash equilibrium is

$$\frac{S^i}{\sum_{j\in N} S^j}\left(\sum_{k=1}^{m} P_k\right).$$

From part 3 of the proof of theorem 1 we derive that if the strategies defined in (3) form a Nash equilibrium, then this is the unique Nash equilibrium. To prove that the strategies defined in (3) form a Nash equilibrium, let $i \in N$. We will prove that $\left(s_k^i\right)_{k=1}^m$ maximizes player i's payoff given the strategies $\left(\left(s_k^j\right)_{k=1}^m\right)_{j\in N\setminus\{i\}}$ of the other players. First we prove that the strategy $\left(s_k^i\right)_{k=1}^m$ satisfies condition (1). It is immediately clear that $s_k^i = S^i\left(\frac{P_k}{\sum_{l=1}^m P_l}\right) > 0$ for every $k = 1, 2, \ldots, m$. Therefore, it is sufficient to prove that

$$\frac{\sum_{j\in N\setminus\{i\}} s_k^j}{\left(\sum_{j\in N} s_k^j\right)^2} P_k = \frac{\sum_{j\in N\setminus\{i\}} s_l^j}{\left(\sum_{j\in N} s_l^j\right)^2} P_l \text{ for all } k, l \in \{1, 2, \ldots, m\}.$$

So let $k, l \in \{1, 2, \ldots, m\}$. Then

$$\begin{aligned}
\frac{\sum_{j\in N\setminus\{i\}} s_k^j}{\left(\sum_{j\in N} s_k^j\right)^2} P_k &= \frac{\sum_{j\in N\setminus\{i\}} S^j\left(\frac{P_k}{\sum_{t=1}^m P_t}\right)}{\left(\sum_{j\in N} S^j\left(\frac{P_k}{\sum_{t=1}^m P_t}\right)\right)^2} P_k \\
= \frac{\sum_{j\in N\setminus\{i\}} S^j}{\left(\sum_{j\in N} S^j\right)^2\left(\frac{P_k}{\sum_{t=1}^m P_t}\right)} P_k &= \frac{\sum_{j\in N\setminus\{i\}} S^j}{\left(\sum_{j\in N} S^j\right)^2\left(\frac{P_l}{\sum_{t=1}^m P_t}\right)} P_l \\
= \frac{\sum_{j\in N\setminus\{i\}} S^j\left(\frac{P_l}{\sum_{t=1}^m P_t}\right)}{\left(\sum_{j\in N} S^j\left(\frac{P_l}{\sum_{t=1}^m P_t}\right)\right)^2} P_l &= \frac{\sum_{j\in N\setminus\{i\}} s_l^j}{\left(\sum_{j\in N} s_l^j\right)^2} P_l.
\end{aligned}$$

To simplify notation we define i's objective function as f, where we collapse the contributions made by the other players to a payoff P_k to $s_k^{-i} = \sum_{j\in N\setminus\{i\}} s_k^j$: for all of player i's strategies $(\sigma_k^i)_{k=1}^m$ it holds that

$$f((\sigma_k^i)_{k=1}^m) := \sum_{k=1}^{m} \frac{\sigma_k^i}{s_k^{-i} + \sigma_k^i} P_k.$$

Notice that it follows from definition 3 that $s_k^{-i} > 0$ for every $k = 1, 2, \ldots, m$, so that the objective function f is well-defined and continuous in all $(\sigma_k^i)_{k=1}^m$, even the strategies with some σ_k^i equal to 0. From the fact that $(s_k^i)_{k=1}^m$ satisfies condition (1), we know that $(s_k^i)_{k=1}^m$ is either a local maximum or a local minimum location of f. If we prove that the function f is strictly concave, then it follows that $(s^i_k)_{k=1}^m$ is a unique global maximum location. To show that f is strictly concave, we take two different strategies $(\beta^i_k)_{k=1}^m$ and $(\gamma^i_k)_{k=1}^m$ of player i and an $\alpha \in (0;1)$ and show that $f(\alpha(\beta^i_k)_{k=1}^m + (1-\alpha)(\gamma^i_k)_{k=1}^m) > \alpha f((\beta^i_k)_{k=1}^m) + (1-\alpha) f((\gamma^i_k)_{k=1}^m)$. To prove this, it is sufficient to prove that $\frac{\beta^i_k}{s_k^{-i}+\beta^i_k} P_k$ is a strictly concave function of β^i_k for every $k \in \{1;2;\dots;m\}$. This is easily seen by taking the second derivative of this function with respect to β^i_k, which is clearly negative. This proves that the strategies defined in (3) form the unique Nash equilibrium. ■

Proof. To prove the second part of the theorem, let $i \in N$. Player i's payoff according to the Nash equilibrium is

$$\sum_{k=1}^{m} \frac{s^i_k}{\sum_{j\in N} s^j_k} P_k = \sum_{k=1}^{m} \frac{S^i \left(\frac{P_k}{\sum_{l=1}^m P_l}\right)}{\sum_{j\in N} S^j \left(\frac{P_k}{\sum_{l=1}^m P_l}\right)} P_k = \frac{S^i}{\sum_{j\in N} S^j} \left(\sum_{k=1}^{m} P_k\right).$$

■

Proof of Proposition 1. From the first order conditions $\frac{F(R^f)}{R^f} = \frac{w}{P} = F'(R^W)$: Our earlier assumptions ensure that for any given R, $F(R)/R > F'(R)$, so as $F''(R) < 0$ it follows that $\frac{F(R^f)}{R^f} = F'(R^W)$ implies $R^f > R^W$. ■

Proof of Proposition 2. We know from theorem 2 (and using the same notation as in the text) that

$$s_k = \frac{\pi_k S}{\sum_{\ell=1} \pi_\ell}$$

so the individual firm's optimization problem may be rewritten

$$\underset{r_k}{\mathrm{Max}} \frac{\pi_k}{s_k} = \frac{\pi_k}{\left(\frac{\pi_k S}{\sum_{\ell=1} \pi_\ell}\right)} = \frac{\sum_{\ell=1} \pi_\ell}{S}$$

$$= \frac{1}{S} \sum_{\ell=1} \left(\frac{P F\left(\sum_{j=1} r_j\right)}{\sum_{j=1} r_j} - w \right) r_\ell :$$

The first order condition to this optimization problem is now

$$\frac{\partial\left(\frac{\pi_k}{s_k}\right)}{\partial r_k} = \frac{1}{S}\left\{\frac{PF\left(\sum_{j=1} r_j\right)}{\sum_{j=1} r_j} - w + \sum_{\ell=1}\left[\frac{PF'\left(\sum_{j=1} r_j\right)\left(\sum_{j=1} r_j\right) - PF\left(\sum_{j=1} r_j\right)}{\left(\sum_{j=1} r_j\right)^2}\right] r_\ell\right\} = 0$$

$$= \frac{1}{S}\left[\frac{PF(R)}{R} - w + PF'(R) - \frac{PF(R)}{R}\right] = 0.$$

This reduces to

$$PF'(R) - w = 0$$

which is precisely the first order condition for a social optimum. ■

Proof of Proposition 3. Follows immediately from noting that the solutions $\{\hat{x}_k; \hat{q}_k\}$ and $\{\tilde{x}_k; \tilde{q}_k\}$ do not correspond to the Nash solution $\{x_k^*; q_i^*\}$. ■

Proof of Proposition 4. Note first that as shown above the cooperation inducing mechanism implies the profit arbitrage condition $\frac{\pi_1}{s_1} = \frac{\pi_2}{S - s_1}$. This condition may be rewritten as $s_1 = \frac{\pi_1}{\pi_1 + \pi_2} S$. Now this implies $\frac{\pi_1}{s_1} = \frac{\pi_1}{\left(\frac{\pi_1}{\pi_1+\pi_2}\right) S} = \frac{\pi_1 + \pi_2}{S}$. Since S is a constant this implies maximizing $\frac{\pi_1}{s_1}$ yields the same outcome as maximizing $\pi_1 + \pi_2$ (an identical argument holds for $\frac{\pi_2}{s_2}$). ■

Proof of Proposition 5. The proof follows immediately from noting that if profits are arbitraged after the production stage then the firms maximize $\underset{x_k;q_k}{\mathrm{Max}} \frac{\pi_k}{s_k} = \underset{x_k;q_k}{\mathrm{Max}} \frac{\pi_1+\pi_2}{S}$ hence both the levels of x_k and q_k chosen are equal to their cooperative levels. That this involves the maximal level of R&D was demonstrated by D&J. ■

Proof of Proposition 6. Follows immediately from noting that in the second stage the equity has been committed to a specific firm so the firms objective is to $\underset{q_k}{\mathrm{Max}} \frac{\pi_k}{s_k}$; and they choose non-cooperative production levels. However, in the first stage the equity has not yet been committed so the arbitrage condition implies that both firms' objectives are $\underset{x_k}{\mathrm{Max}} \frac{\pi_k}{s_k} = \underset{x_k}{\mathrm{Max}} \frac{\pi_1+\pi_2}{S}$, thus R&D expenditures are chosen cooperatively. ■